NBER WORKING PAPER SERIES

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Working Paper 32436
http://www.nber.org/papers/w32436

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 2024

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NBER Working Paper No. 32436
May 2024
JEL No. E44,F31,G15

ABSTRACT

Financial markets play two roles with implications for the exchange rate: they accommodate risk sharing and act as a source of shocks. In prevailing theories, these roles are seen as mutually exclusive and individually face challenges in explaining exchange rate dynamics. However, we demonstrate that this is not necessarily the case. We develop an analytical framework that characterizes the link between exchange rates and finance across all conceivable market structures. Our findings indicate that full market segmentation is not necessary for financial shocks to explain exchange rates. Moreover, financial markets can accommodate a significant extent of international risk sharing without leading to the classic exchange rate puzzles. We identify plausible market structures where both roles coexist, addressing challenges faced when examined separately.

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Introduction

A wide body of evidence indicates that exchange rate movements have minimal or no correlation with macroeconomic aggregates (Meese and Rogoff, 1983; Obstfeld and Rogoff, 2001). This finding has spurred researchers to seek the sources of these fluctuations elsewhere, shifting attention to financial markets. While this approach has yielded important insights, it also presents its own set of challenges. In this paper, we conduct a general analysis of how the financial sector of an equilibrium model interacts with the exchange rate. This perspective clarifies the root cause of the main challenges to existing theories and allows us to identify frameworks that overcome these challenges.

We focus on the duality between two roles of financial markets in the determination of the exchange rate:

– Financial markets are where sharing of macroeconomic risks across countries takes place. Investors use financial claims to line up their marginal rates of substitution, and this determines the exchange rate that smoothes macro shocks (e.g., Backus and Smith, 1993, Cole and Obstfeld, 1991).

– Financial markets are also a source of shocks to the exchange rate. The exchange rate responds to shifts in demand and supply in the currency market, whether they are of financial or macroeconomic origin (e.g., Gabaix and Maggiori, 2015, Itskhoki and Mukhin, 2021, Jiang, Krishnamurthy, and Lustig, 2021).

The literature usually adopts market structures — that is, a combination of assumptions about what assets are traded and who trades them — in which only one of these roles is emphasized. Each of these roles runs into significant challenges. On the one hand, models of risk sharing typically assume integrated markets, complete markets, or both.[1] Such market structures lead to a tight connection of the exchange rate with the macroeconomy, at odds with the classic evidence of the disconnect.[2] On the other hand, models of financial shocks typically focus on limited and segmented market structures. These assumptions are at odds

[1] Markets are complete when every state of the world can be traded or spanned; markets are integrated when every agent can trade the available assets with everyone else.

[2] Overcoming this difficulty often leads to strong implications for difficult-to-measure aspects of macroeconomic dynamics, such as long-run risks or rare disasters.

with the existence of widely accessible local financial markets and of many global multi-market intermediaries.

Given the existing literature, one is led to believe that the dual roles of financial markets are mutually exclusive. Moreover, the two challenges seem inherent to each role of the financial market. Our main contribution is to show that, while there is a tension between the two roles, both conjectures are incorrect. We highlight plausible market structures in which risk sharing and shocks affecting financial markets jointly determine the exchange rate, overcoming both challenges. In particular, extreme market segmentation is not a prerequisite for shocks in the financial sector to have a significant impact on the exchange rate. Further, risk sharing can be substantial without resulting in puzzles implied by the macro disconnect.

We develop joint restrictions between macroeconomic conditions in two countries, asset returns and the exchange rate to reach these conclusions. Importantly, for any market structure, these restrictions fully characterize the equilibrium implications of risk sharing for the exchange rate. As is standard in models of risk sharing, the notion of macroeconomic conditions relevant for financial markets is the intertemporal marginal rate of substitution (IMRS) of each country's representative household.[3] These IMRSs connect local economies to international financial markets through Euler equations. Changes in market structure alter the set of Euler equations that hold, and hence vary equilibrium restrictions on the exchange rate.

For example, in the familiar case of complete and integrated markets, risk sharing between local households completely pins down the exchange rate. The log home exchange rate depreciation Δs must equal the difference between their log nominal IMRSs m and m^*, reflecting the change in the marginal utility of local currency (see, e.g., Backus, Foresi, and Telmer, 2001):

$$\Delta s_{t+1} = m^*_{t+1} - m_{t+1}. \tag{1}$$

This relation illustrates the tension between the two roles of financial markets.

[3] We use IMRSs to consider the implications of our analysis in the context of equilibrium models, including those with heterogenous agents. Our theoretical results do not require structural IMRSs and equally apply to any pair of local stochastic discount factors (SDFs).

First, this expression highlights why complete and integrated markets often struggle with the macro disconnect: many models relate the discount factors m and m^*, and hence the exchange rate, to macroeconomic aggregates. This results in a mismatch between several moments of the model-based and empirical exchange rates — the volatility (Brandt, Cochrane, and Santa-Clara, 2006), cyclicality (Backus and Smith, 1993), and risk premium (Fama, 1984) puzzles, as jointly analyzed in Lustig and Verdelhan (2019). Second, the relation in equation (1) implies that risk sharing leaves no room for the role of financial markets as a source of shocks: even if financial frictions are modeled, they have no impact on the depreciation rate beyond what can be learned from households' marginal utility.

Our framework generalizes this baseline case to all possible market structures including deviations from market completeness, market integration, or both simultaneously. We assume that households in each country trade a potentially distinct set of assets in their local currency. That is, Euler equations hold with respect to each country's IMRS for these assets.[4] We summarize the remainder of what happens in international financial markets by the assumption of no arbitrage for assets traded in these markets. This is equivalent to assuming the existence of an international stochastic discount factor (SDF) which may or may not coincide with one of the two local IMRSs. In the latter case, it could be the discount factor of a global intermediary.

In this setting, we characterize all restrictions imposed on the exchange rate coming from the households' discount factors. These restrictions are determined by the market structure and the statistical properties of asset returns, which we show are jointly summarized by the notion of globally-traded risks. Globally-traded risks are shocks that both home and foreign households can trade in their respective currency.[5] Using this notion, the risk-sharing restrictions take the form of two simple relations, which generalize equation (1). First, *innovations* (denoted by $\sim$) to the depreciation rate coincide with innovations in the

[4]In circumstances when Euler equations do not hold with equality for some assets (e.g., in the presence of constraints or convenience yields), standard risk-sharing conditions do not constrain the relation of these asset returns with the exchange rate.

[5]For example, a productivity shock (or a financial shock) is globally traded if households can trade the same asset, or different local portfolios, spanning the shock. Therefore, altering either the assets that can be traded (i.e., the market structure) or the correlation between asset returns changes whether the productivity shock is globally traded.

relative discount factors across countries when projected on globally-traded risks $\boldsymbol{\epsilon}^g$:

$$\text{proj}(\,\widetilde{\Delta s}_{t+1}\,|\boldsymbol{\epsilon}^g_{t+1}) = \text{proj}(\,\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}\,|\boldsymbol{\epsilon}^g_{t+1}). \tag{2}$$

Second, the *expected* depreciation rate is similar to the one under complete markets when asset returns span the exchange rate; otherwise, it is unconstrained by local discount factors.

These two results exhaust all possible restrictions imposed by local SDFs on the exchange rate, i.e. they are necessary and sufficient for precluding international arbitrage opportunities. In general, these constraints do not pin down the exchange rate completely. This leaves space for the second role of financial markets — as a source of shocks — to determine the remainder of the exchange rate.

We use these results to address the conjectures about the interaction between the two roles of finance and the challenges associated with them. We fix households' IMRSs m and m^*, and study how variation in market structure affects the exchange rate.[6] Guided by equation (2), we classify market structures according to the size of the globally-traded component of the exchange rate $g_{t+1} \equiv \text{proj}(\,\widetilde{\Delta s}_{t+1}\,|\boldsymbol{\epsilon}^g_{t+1})$.

First, we show that the volatility and cyclicality puzzles are present for all market structures with a large globally-traded component. That is, if applying equation (1) to our chosen m and m^* implies counterfactual volatility and cyclicality of the exchange rate, applying equation (2) under a market structure with large $var(g_{t+1})$ leads to the same puzzles. This situation arises when markets are integrated or approximately complete. Our framework clarifies the relevant notion of approximately complete: when both households can trade assets that nearly span their IMRSs. The converse of this result is that avoiding the puzzles requires a market structure where markets are sufficiently incomplete and not integrated (but instead intermediated or segmented).

Second, we show that the ability of segmented-market models of financial propagation of shocks to rationalize exchange rate dynamics comes from their lack of globally-traded shocks, $var(g_{t+1}) = 0$. Without a globally-traded component, risk sharing does not constrain the ex-

[6] Hence, this exercise is complementary to the large literature which fixes financial markets to be complete and integrated, and alters IMRSs by varying assumptions about preferences (CRRA, habits, or recursive utility) and aggregate dynamics (random walk, long-run risks, or disasters).

change rate. Usually, these models assume extreme segmentation: households can only trade their respective risk-free bonds, and intermediaries engage in the carry trade only. We show that the lack of globally-traded shocks, and hence the flexibility of intermediated models, does not hinge on such a strong form of segmentation. Intermediaries can be sophisticated and trade an arbitrarily large set of assets. Each household can trade many local assets of their own country only, as long as the risks in the two countries are only weakly related.

Third, we show that there exist appealing market structures in which risk sharing and propagation of shocks coexist. This occurs in intermediated markets when the local assets of the two countries have common shocks. When $var(g_{t+1})$ is positive but not very large, risk sharing plays a substantial role in determining the exchange rate, but there is also enough room for the financial propagation of shocks in the currency markets to avoid the puzzles.

In such market structures, the tightness of the constraints on the exchange rate depends on the empirical properties of returns. We show how to quantify these properties and illustrate this approach for a situation in which households trade local stocks and sovereign bonds of various maturities. We find that these asset returns do not span the exchange rate fully, explaining at most half of its variation. Furthermore, common shocks across countries explain no more than 25% of the variation in exchange rates. These results suggest that this market structure leaves room for risk sharing without introducing exchange rate puzzles.

Beyond addressing these three insights on the roles of finance, our analysis provides the full menu of constraints on the exchange rate across market structures. Our findings highlight that many issues with existing models of the exchange rate come from taking stark stylized views of market structure. Departing from these limiting cases and incorporating more realistic features of how financial markets are organized is a promising avenue for understanding the exchange rate.

Contribution to the literature While the literature has explored a number of market structures, general results have been elusive as each case seemingly requires a separate analysis. We are able to make progress by focusing on restrictions on the behavior of the exchange rate, as opposed to fully solving the equilibrium. As such, we follow the tradition of Hansen and Jagannathan (1991). We similarly apply these restrictions to simple moments of exchange rates, such as their volatility and weak relation to business cycles. The key difference is that the international setting leads to a preponderant role of the financial market structure in determining these constraints.

Much of the literature explores one of the two polar cases in terms of market structure. In particular, a large body of work maintains the complete and integrated markets assumption of Backus, Foresi, and Telmer (2001) and varies assumptions about preferences and aggregate dynamics to obtain a realistic exchange rate. Some prominent examples of this line of work include Verdelhan (2010, habits), Colacito and Croce (2011, long-run risk), and Farhi and Gabaix (2016, disasters), among many others.[7] At the other end of the spectrum, Jeanne and Rose (2002), Gabaix and Maggiori (2015), Itskhoki and Mukhin (2021, 2022, 2023), and Kekre and Lenel (2024) take a strongly segmented view of markets with trade in short-term bonds only. Gourinchas, Ray, and Vayanos (2022) and Greenwood, Hanson, Stein, and Sunderam (2022) maintain segmentation but add the term structure. We map out the entire space between these approaches. We find that some interim market structures retain the desirable properties of these polar cases without facing their main drawbacks.

A number of papers consider the exchange rate implications of specific market structures in this interim space. Alvarez, Atkeson, and Kehoe (2002, 2007, 2009), Kocherlakota and Pistaferri (2007, 2008), Zhang (2021) and Marin and Singh (2023) emphasize heterogeneity across households in access to financial markets. Corsetti, Dedola, and Leduc (2008), Engel and Matsumoto (2009), Benigno and Thoenissen (2008) and Lewis and Liu (2022) focus on incomplete markets. Jiang, Krishnamurthy, and Lustig (2021, 2023a), Jiang, Krishnamurthy, Lustig, and Sun (2022), and Kekre and Lenel (2023) study the implications of convenience yield of safe assets. These papers derive results by fully specifying the economic environment. In a similar spirit to our approach, Lustig and Verdelhan (2019), Jiang, Krishnamurthy, and Lustig (2023b), and Orłowski, Tahbaz-Salehi, Trojani, and Vedolin (2023) derive results robust to the details of the macroeconomic environment for the case of partial integration. Our analysis provides the common thread of how financial markets constrain the exchange rate across all situations of incompleteness, partial integration, and intermediation.

[7] Hassan, Mertens, and Wang (2024) identify a tension between calibrated single-country economies and the exchange rate risk premium under complete markets.

Our interest in intermediate market structures is motivated by evidence of weak connection of both asset prices and macroeconomic quantities across countries. Bansal (1997) and Backus, Foresi, and Telmer (2001) find a weak relation between the relative behavior of the yield curve across countries and the exchange rate. Chernov and Creal (2023) highlight that this evidence can be consistent with the absence of arbitrage opportunities.[8] Hau and Rey (2004, 2006) find a stronger, yet incomplete, connection of the exchange rate with cross-country equity returns. Our analysis shows that these facts *per se* do not constitute a test of market structure. However, they are informative about the restrictions that are imposed on the exchange rate within specific market structures.

On the real side, Backus, Kehoe, and Kydland (1992), Backus and Smith (1993) and Kollmann (1995) highlight the low correlation of consumption across countries that cannot be explained by variation in the real exchange rate in standard complete market models. Furthermore, the literature documents a pervasive home bias in portfolios (see e.g. French and Poterba, 1991; Lewis, 1999), providing further evidence suggestive of imperfect risk sharing. Heathcote and Perri (2014) provide an overview of the literature on the efficiency of international risk sharing.

Lastly, we connect to the large literature exploring the sources of shocks to exchange rates. For example, Engel and West (2005) and Chahrour, Cormun, De Leo, Guerrón-Quintana, and Valchev (2023) emphasize news shocks about future macro fundamentals, Gourinchas and Rey (2007) and Pavlova and Rigobon (2007) focus on trade shocks and imbalances, Chen and Rogoff (2003) and Ayres, Hevia, and Nicolini (2020) study commodity shocks, Stavrakeva and Tang (2019), Eichenbaum, Johannsen, and Rebelo (2021) and Fukui, Nakamura, and Steinsson (2023) emphasize monetary shocks and regimes, Adrian, Etula, and Shin (2010) and Lilley, Maggiori, Neiman, and Schreger (2022) focus on financial shocks, and Itskhoki and Mukhin (2024) provide a review. We demonstrate that to understand how a specific source of shocks affect the exchange rate, it is crucial to know whether financial markets allow this shock to be traded globally.

[8] Maurer and Tran (2021) and Sandulescu, Trojani, and Vedolin (2021) consider the problem of recovering SDFs compatible with asset returns expressed in two different currencies.

1 Framework

The aim of our analysis is to characterize the joint restrictions imposed by financial risk sharing on the behavior of three endogenous objects: local SDFs, asset returns, and the exchange rate. We start from local financial markets: the returns of assets available in each country are priced by a local SDF. For the results in this section, we do not need to take a stand on the mechanism by which local SDFs emerge, but for ease of exposition we assume they arise from IMRSs of representative households. Next, trading in international markets leads to constraints on the exchange rate.[9] We characterize these constraints across all market structures, that is, assumptions about which assets are traded and who trades them.

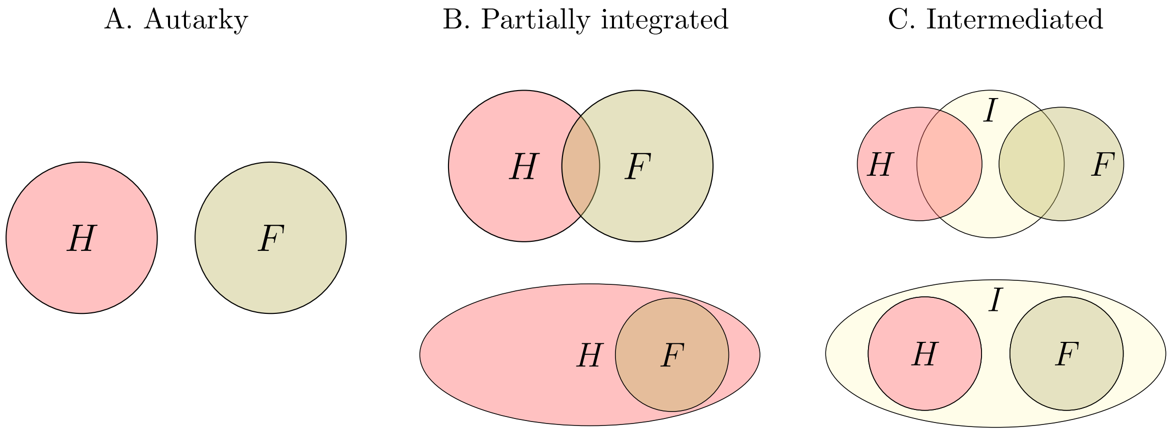
Formally, our exploration of financial markets revolves around two sets of equilibrium restrictions. First, Euler equations must hold with respect to local SDFs for each asset households invest in, in their respective currencies. These conditions are the point of contact of local economies with financial markets. Second, we assume that there are no arbitrage opportunities in international markets, after converting asset returns to the same currency. This section introduces our representation of alternative market structures, the corresponding equilibrium conditions, and then defines the concept of globally-traded shocks that play a central role in our analysis of the exchange rate.

1.1 Market structure

We consider settings with two representative households — home and foreign — each trading a set of assets, H and F, respectively. These sets can contain subsets of local assets and foreign assets converted to local currency. Figure 1 demonstrates some examples. For instance, in autarky H contains domestic stocks and bonds, while F contains the foreign ones, and there is no intermediary that can trade across these sets of assets. Markets are fully integrated when H and F contain the same set of assets, while in partially integrated

[9] Naturally, these are equilibrium links rather than a causal chain. A researcher can start from their favorite model of local asset pricing and characterize what risk sharing implies for the exchange rate's behavior across market structures; we follow this path in Section 3. Alternatively, one can start from data on the exchange rate and returns and consider implications for SDFs; see Section 4.

Figure 1: Examples of Market Structures



The figure illustrates different market structures. H and F are the sets of assets invested in by the home and foreign household. Panel A corresponds to financial autarky. Panel B corresponds to partial integration, symmetric or asymmetric. Panel C corresponds to an intermediated market, with an intermediary I trading some or all assets.

markets H and F only have a subset of assets in common.[10] Markets are complete when H and F contain (or span) the full set of Arrow-Debreu securities.

Further, we consider a set I of assets traded in international markets. Assets can be included in this set for two reasons. First, it could be that home and foreign households trade some assets in common, as in partially or fully integrated cases. Then, either the home or foreign household can be considered as an international intermediary, with $I = H$ or $I = F$, respectively. Second, it could be that financial intermediaries (one or many) trade across borders, even if households do not, as in the examples in panel C of Figure 1. In this case, I contains the assets from H and F that intermediaries can trade.

Our main result is that, in this large family of market structures, restrictions on the exchange rate coming from risk sharing between households are detemined by the properties of returns in $H \cap I$ expressed in domestic currency and returns in $F \cap I$ expressed in foreign currency. To continue our examples, if markets are partially integrated and $I = H$, then $H \cap I = H$ are the assets traded by the domestic household and $F \cap I = F \cap H$ are the assets traded by both households. In intermediated markets, $H \cap I$ is the set of assets traded both by the domestic household and the intermediaries; ditto for $F \cap I$.

[10] An example of a fully integrated market structure is when H and F both contain a domestic sovereign bond and a foreign equity index. As another example, adding a foreign sovereign bond to F, but not to H makes the market structure partially integrated. We discuss more examples in Appendix B.2.

The base assets in the set $H \cap I$ have log returns $\boldsymbol{r}_{t+1} = (r_{1,t+1}, \ldots, r_{N,t+1})$ in home currency. We assume that this collection includes a risk-free asset with return r_{ft} known at time t. We consider all feasible portfolios that can be constructed from these assets. The corresponding set of log returns is:

$$\boldsymbol{r}_{p,t+1} = \left\{ r_{p,t+1} = \log\left(\boldsymbol{w}_t' \exp(\boldsymbol{r}_{t+1})\right) \,\middle|\, \exists \boldsymbol{w}_t \in \mathbb{R}^N : \boldsymbol{w}_t'\boldsymbol{\iota} = 1 \right\}.$$

Furthermore, for our analysis in the main text, we assume that asset returns are log-normal, that is $\boldsymbol{r}_{t+1}$ is multivariate normal with mean $\boldsymbol{\mu}_t$ and variance-covariance matrix $\boldsymbol{\Sigma}_t$. Similarly, the returns of base assets in $F \cap I$ are $\boldsymbol{r}^*_{t+1}$ in foreign currency, log-normal of size N^*, and contain a foreign-currency risk-free rate r^*_{ft}.[11] The corresponding set of portfolio returns is $\boldsymbol{r}^*_{p,t+1}$. Throughout the paper, we use the Campbell and Viceira (2002) approximation for log portfolio excess returns with the relevant derivations described in Appendix A. In Appendix D, we show how our results generalize to an environment with an arbitrary distribution of returns without portfolio approximation.

1.2 Pricing Assumptions

We introduce two sets of assumptions, which enable us to characterize how risk sharing between households constrains the behavior of the exchange rate.

Local Euler equations We specify valuation mechanisms for each of the households with SDFs m at home and m^* abroad. These SDFs value assets as follows.

Assumption 1. *The domestic (log) stochastic discount factor m_{t+1} prices all assets in H in domestic currency. In particular, it satisfies the Euler equation:*

$$\forall r_{t+1} \in \boldsymbol{r}_{p,t+1} : \quad E_t \exp(m_{t+1} + r_{t+1}) = 1. \tag{3}$$

Similarly, the foreign log SDF m^*_{t+1} prices all assets in F in foreign currency, and

$$\forall r^*_{t+1} \in \boldsymbol{r}^*_{p,t+1} : \quad E_t \exp(m^*_{t+1} + r^*_{t+1}) = 1. \tag{4}$$

[11] Our analysis generalizes to an environment without risk-free assets. Specifically, one can prove versions of Proposition 1 and 2 that do not rely on the presence of risk-free assets; see Appendix C.

These Euler equations imply that the expectation of an excess return is related to its covariance with the respective SDF. In particular, under the log-normal assumption, equations (3) and (4) become:

$$\forall r_{t+1} \in \boldsymbol{r}_{p,t+1}: \qquad E_t\, r_{t+1} - r_{ft} + \frac{1}{2} var_t(r_{t+1}) = -cov_t(m_{t+1}, r_{t+1}), \tag{5}$$

$$\forall r^*_{t+1} \in \boldsymbol{r}^*_{p,t+1}: \qquad E_t\, r^*_{t+1} - r^*_{ft} + \frac{1}{2} var_t(r^*_{t+1}) = -cov_t(m^*_{t+1}, r^*_{t+1}). \tag{6}$$

Recall that $\boldsymbol{r}_{p,t+1}$ and $\boldsymbol{r}^*_{p,t+1}$ are the sets of feasible portfolio returns constructed from assets in $H \cap I$ and $F \cap I$, respectively, which is all we need for the derivation of our formal results.

The Euler equations (3) and (4), or respectively their log-normal versions (5) and (6), act as the point of contact of financial markets with the respective local economies. These conditions hold irrespective of the remainder of the economic environment and connect local asset returns with local SDFs. Our results apply to any admissible pair of home and foreign SDFs that are consistent with equilibrium and observed asset returns. In this paper, we are interested in situations when these SDFs are equal to IMRSs of representative households and, thus, reflect local aggregate macroeconomic conditions.[12] Appendix G discusses how our results apply to SDFs constructed using asset returns only without connection to the broader economic environment.

Assumption 1 clarifies what it takes for an asset to be included in H or F. Households have to freely trade this asset so that the corresponding Euler equation holds. Equilibrium in the financial market may involve borrowing or short-sale constraints, infrequent portfolio adjustment, or convenience yield on certain assets.[13] In all these cases, the Euler equation does not always hold with equality, i.e. it features a wedge. Therefore, such assets are not included in the sets H and F (for a given time period t). This exclusion does not mean that these assets are not part of the equilibrium, nor that they are irrelevant for the exchange rate. It simply implies that standard risk-sharing conditions do not apply to such assets.

[12] For example, with CRRA utility, a representative household's IMRS is $m_{t+1} = -\rho - \gamma\Delta c_{t+1} - \pi_{t+1}$ where ρ is the preference discount rate, γ is the coefficient of risk aversion, c_t is log aggregate domestic consumption, and π_t is CPI inflation.

[13] Some constraints are not readily observable in the data. For instance, even though currencies may seem easily tradable, households seldom engage with them in practice. This observation suggests the existence of underlying frictions in such investments, perhaps stemming from a lack of sophistication or a home currency bias (see French and Poterba, 1991, and the literature that follows).

Actually, the associated wedges in Euler equations can be an important source of exchange rate shocks.

International arbitrage So far, none of our analysis involved the exchange rate as we described the constraints imposed on equilibrium by local asset pricing. In order to characterize the interaction between the exchange rate and local finance, one has to take a stand on how international financial markets operate. To isolate the risk-sharing role of these markets, we make a minimal assumption about them: we assume that there are no arbitrage opportunities for assets in I.

Formally, the set of international returns in I combines the domestic and foreign subsets of returns converted to the domestic currency. Our conclusions are unchanged if we focus on international arbitrage in foreign currency. Following our notation, international portfolios are generated by the base assets $\boldsymbol{r}^I_{t+1} = (\boldsymbol{r}_{t+1}, \boldsymbol{r}^*_{t+1} + \Delta s_{t+1})$, where Δs_{t+1} is the log home currency depreciation rate. We denote by $\boldsymbol{r}^I_{p,t+1}$ the set of international portfolios generated by these base assets.

Assumption 2. *There are no arbitrage opportunities in the set of international returns $\boldsymbol{r}^I_{p,t+1}$. In the log-normal setting, this assumption is equivalent to:*

$$\forall r_{p,t+1} \in \boldsymbol{r}^I_{p,t+1}: \quad var_t(r_{p,t+1}) = 0 \quad \Rightarrow \quad E_t\, r_{p,t+1} = r_{ft}. \tag{7}$$

In words, any portfolio that has no risk must earn the risk-free rate of return.[14] This condition is equivalent to the existence of an international SDF m^I. For example, this SDF could be the discount factor of one of the households (in markets with partial integration) or of an international arbitrageur (in intermediated markets). However, unlike for households, we do not assume any knowledge of this SDF beyond its existence.

Assumption 2 reflects our focus on characterizing the set of cross-equation restrictions imposed by no arbitrage on $(m_{t+1}, m^*_{t+1}, \Delta s_{t+1})$ and asset returns $(\boldsymbol{r}_{t+1}, \boldsymbol{r}^*_{t+1})$ without taking a stand on a specific m^I that supports no arbitrage. Importantly, this assumption does not imply that the characteristics of international arbitrageurs do not matter for the exchange

[14] In a log-normal setting, condition (7) is equivalent to the absence of arbitrage opportunities. In more general settings, it is a necessary condition for no arbitrage (Campbell, 2017, p.92).

rate. Just like wedges in the household Euler equations, shifts in m^I can act as a source of exchange rate shocks.

From the perspective of models with fully integrated markets, the introduction of m^I may appear redundant, as either the home or the foreign households can act as international intermediaries. In such market structures, local Euler equations and a simple currency conversion of asset returns fully characterize risk sharing. This insight does not extend to the much larger class of market structures that we consider: our formalism unifies predictions for models of partial integration and intermediation.

Coming from theories of intermediation, it may also be tempting to make away with local households altogether. This corresponds to replacing both households in Assumption 1 by the arbitrageur from Assumption 2 who prices every asset. Such an approach does not introduce informative risk-sharing restrictions as it effectively considers the same investor twice. Mechanically, the conversion of an intermediary's SDF from domestic to foreign currency is $m^{I*} = m^I + \Delta s$, irrespective of market structure — an accounting relation, not an equilibrium one (see Appendix G for further discussion).

Taking stock, the sets H, F and I together with Assumptions 1 and 2 constitute our representation of risk sharing. This representation does not rule out any equilibrium models. Instead, any model can be mapped into this structure by appropriately identifying the asset sets that respect the properties postulated in the two assumptions. This includes theories with segmented markets or Euler equation wedges. In such models, when households can freely trade only local risk-free bonds, we typically have $H \cap F \cap I = \emptyset$. Then, risk-sharing forces may impose no constraints on the exchange rate, in which case it is entirely determined by other equilibrium conditions. Of course, we are mainly interested in circumstances when risk sharing imposes some constraints on the equilibrium exchange rate behavior.

1.3 Globally-traded, locally-traded and unspanned shocks

For our analysis of the exchange rate below, we introduce the concept of globally-traded shocks. We use tilde to denote the innovation (or shock) to any variable x, that is, $\widetilde{x}_{t+1} \equiv x_{t+1} - E_t x_{t+1}$ and hence $var_t(\widetilde{x}_{t+1}) = var_t(x_{t+1})$.

Definition 1. *The set of globally-traded shocks is*

$$\boldsymbol{\epsilon}^g_{t+1} = \left\{ \epsilon^g_{t+1} \,\middle|\, \exists \boldsymbol{\lambda} \in \mathbb{R}^N,\, \boldsymbol{\lambda}^* \in \mathbb{R}^{N^*} :\; \epsilon^g_{t+1} = \boldsymbol{\lambda}' \widetilde{\boldsymbol{r}}_{t+1} = \boldsymbol{\lambda}^{*\prime} \widetilde{\boldsymbol{r}}^*_{t+1} \right\}. \tag{8}$$

Globally-traded shocks can be traded by local investors in their local currency in both countries. Not only must such shocks affect returns in the two countries, but investors must also have access to a trading strategy in each country that isolates the shock from other sources of risk. Appendix B shows how to construct a basis of $\boldsymbol{\epsilon}^g_{t+1}$ using the covariance matrix of $\boldsymbol{r}_{t+1}$ and $\boldsymbol{r}^*_{t+1}$.

Globally-traded shocks can arise for two reasons. First, they can emerge from common underlying economic shocks (e.g., productivity) that determine returns as long as such shocks can be replicated by local-currency portfolio returns in the two countries. Second, globally-traded shocks can emerge without common fundamental shocks as a result of asset trading across countries — either directly by households or via an intermediary. Note that an asset traded by both households (e.g., an individual stock) does not immediately constitute a globally-traded risk due to currency conversion. However, a commonly traded *excess* return does result in a globally-traded risk as the exposure to currency risk is eliminated in the construction of the excess return. We provide examples of globally-traded shocks in Appendix B.2.

Locally-traded shocks $\boldsymbol{\epsilon}_{t+1}$ and $\boldsymbol{\epsilon}^*_{t+1}$ are the residuals of return innovations $\widetilde{\boldsymbol{r}}_{t+1}$ and $\widetilde{\boldsymbol{r}}^*_{t+1}$, respectively, after controlling for globally-traded shocks $\boldsymbol{\epsilon}^g_{t+1}$.[15] Intuitively, $\boldsymbol{\epsilon}^g_{t+1}$ corresponds to the intersection of the spaces of returns $\tilde{\boldsymbol{r}}_{p,t+1} \cap \tilde{\boldsymbol{r}}^*_{p,t+1}$, while $(\boldsymbol{\epsilon}^g_{t+1}, \boldsymbol{\epsilon}_{t+1}, \boldsymbol{\epsilon}^*_{t+1})$ is the union of these spaces $\tilde{\boldsymbol{r}}_{p,t+1} \cup \tilde{\boldsymbol{r}}^*_{p,t+1}$. Finally, we refer to any other sources of variation orthogonal to asset returns $(\widetilde{\boldsymbol{r}}_{t+1}, \widetilde{\boldsymbol{r}}^*_{t+1})$ — or equivalently, orthogonal to locally-traded and globally-traded shocks $(\boldsymbol{\epsilon}^g_{t+1}, \boldsymbol{\epsilon}_{t+1}, \boldsymbol{\epsilon}^*_{t+1})$ — as unspanned shocks. Unspanned shocks are not traded in either home or foreign financial market.

The classification into globally-traded, locally-traded and unspanned shocks characterizes how risks can be shared in a given financial market structure, as opposed to identifying the origin of these shocks. The latter would require a fully specified structural model. Each group

[15] Similarly to Definition 1 for globally-traded shocks, locally-traded shocks are formally defined by $\boldsymbol{\epsilon}_{t+1} = \{\epsilon_{t+1} | \exists \boldsymbol{\lambda}_\ell \in \mathbb{R}^N : \epsilon_{t+1} = \boldsymbol{\lambda}_\ell' \tilde{\boldsymbol{r}}_{t+1} \perp \boldsymbol{\epsilon}^g_{t+1}\}$.

of shocks can, in general, contain aggregate or idiosyncratic shocks, as well as macroeconomic or financial shocks.

Exchange rate decomposition We decompose the exchange rate depreciation Δs_{t+1} into four components. First, we partition the depreciation rate into its expectation $E_t\Delta s_{t+1}$ and the depreciation shock $\widetilde{\Delta s}_{t+1} = \Delta s_{t+1} - E_t\Delta s_{t+1}$. Next, we use the taxonomy of shocks introduced above, and decompose the depreciation shock into a globally-traded, locally-traded and unspanned components:

$$\widetilde{\Delta s}_{t+1} = g_{t+1} + \ell_{t+1} + u_{t+1}. \tag{9}$$

We denote with $g_{t+1} \in \boldsymbol{\epsilon}^g_{t+1}$ the component of $\widetilde{\Delta s}_{t+1}$ that is spanned by globally-traded risks. Correspondingly, the local component ℓ_{t+1} is spanned by locally-traded risks, i.e. it is a linear combination of $\boldsymbol{\epsilon}_{t+1}$ and $\boldsymbol{\epsilon}^*_{t+1}$ and orthogonal to $\boldsymbol{\epsilon}^g_{t+1}$. Finally, u_{t+1} is the component of the depreciation shock unspanned by asset returns, i.e. orthogonal to $(\boldsymbol{\epsilon}^g_{t+1}, \boldsymbol{\epsilon}_{t+1}, \boldsymbol{\epsilon}^*_{t+1})$.

The decomposition (9) is unique and specific to a given market structure, and it plays a central role in our characterization of restrictions on the behavior of the exchange rate. In particular, it applies across broad classes of market structure as follows.

Lemma 1. (a) *In integrated markets, the exchange rate is spanned by asset returns, $u_{t+1} = 0$.* (b) *In fully integrated markets, the exchange rate and all asset returns are globally-traded risks, $\widetilde{\Delta s}_{t+1} = g_{t+1}$ and $\widetilde{\boldsymbol{r}}_{p,t+1} = \widetilde{\boldsymbol{r}}^*_{p,t+1} = \boldsymbol{\epsilon}^g_{t+1}$.* (c) *Intermediated markets can support any decomposition of the exchange rate risk into g_{t+1}, ℓ_{t+1} and u_{t+1} components.*

To understand this lemma, note that the spanned and unspanned components of the depreciation rate in decomposition (9) can be constructed from asset returns:

$$\widetilde{\Delta s}_{t+1} = \widetilde{r}_{p,t+1} - \widetilde{r}^*_{p,t+1} + u_{t+1}, \tag{10}$$

where $r_{p,t+1} \in \boldsymbol{r}_{p,t+1}$ and $r^*_{p,t+1} \in \boldsymbol{r}^*_{p,t+1}$ are the returns on the pair of portfolios that maximize the R^2 for explaining the exchange rate. In (partially) integrated markets, there exists at least one asset traded in both countries, and hence $r_{i,t+1} = r^*_{i,t+1} + \Delta s_{t+1}$ holds for this asset by simple conversion of currency units. Using this asset in equation (10) in home and foreign currency as $\widetilde{r}_{p,t+1}$ and $\widetilde{r}^*_{p,t+1}$ respectively, we see that $u_{t+1} = 0$, point (a). When markets are fully integrated, all assets are traded across countries and the restriction becomes stronger: innovations to all asset returns and the exchange rate are globally-traded shocks, point (b).[16] In contrast, intermediated markets generally have no such restrictions and the unspanned component u_{t+1} can play an important role in the exchange rate shock, point (c).

2 The general risk-sharing view of exchange rates

In this section, we characterize the restrictions on the behavior of the exchange rate imposed by the absence of international arbitrage and given the properties of returns on traded assets, $\boldsymbol{r}$ and $\boldsymbol{r}^*$, and local SDFs m and m^* that price them. We show that Assumptions 1 and 2 impose two sets of necessary restrictions on the depreciation rate: one on the depreciation shocks $\widetilde{\Delta s}_{t+1}$, and another on the expected depreciation $E_t \Delta s_{t+1}$. In Appendix C, we further show that these restrictions are sufficient as well, that is they characterize all constraints imposed by the risk-sharing role of financial markets on the behavior of the exchange rate.

In a complete market setting, these two sets of restrictions lead to the well-known asset market view of exchange rates. Our analysis spells out the implications of these restrictions in a much larger set of market structures. All the proofs are in Appendix C. Appendix D derives exact non-linear versions of the results which do not require any distributional assumptions and hence include the case of disasters.

2.1 Exchange rate shocks

It is natural to think that thanks to risk-sharing, relative marginal utilities of the two households in their respective currencies $m^* - m$ line up with the depreciation rate Δs, as is the case under complete markets. We now show how this logic is altered in a general market structure, and only applies along the dimensions of risk that both households trade.

[16] This result relies on the fact that both home- and foreign-currency risk-free assets are available to all investors, which is sufficient to make the exchange rate risk a globally-traded shock by means of a simple carry trade strategy. Without risk-free assets, the exchange rate risk is not necessarily globally traded even in fully integrated markets, whereas all excess returns are still globally traded.

Proposition 1. *Under Assumptions 1 and 2, the globally-traded component of the depreciation shock, g_{t+1} in decomposition (9), must coincide with the component of the SDF differential that is spanned by the globally-traded shocks:*

$$\text{proj}(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1} | \boldsymbol{\epsilon}^g_{t+1}) = \text{proj}(\widetilde{\Delta s}_{t+1} | \boldsymbol{\epsilon}^g_{t+1}) = g_{t+1}. \tag{11}$$

Said differently, start from the pair of local SDFs and regress them on all globally-traded shocks. The predicted value of this regression must equal the globally-traded component of the exchange rate, g_{t+1}:

$$\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1} = g_{t+1} + v_{t+1} \qquad \text{with} \;\; v_{t+1} \perp \boldsymbol{\epsilon}^g_{t+1}. \tag{12}$$

Because globally-traded shocks are constructed from asset returns alone, this means that Proposition 1 allows to determine a component of the exchange rate by no arbitrage without any knowledge of its statistical properties. In other words, it is sufficient to know local finance summarized by (m_{t+1}, m^*_{t+1}) and $(\boldsymbol{r}_{t+1}, \boldsymbol{r}^*_{t+1})$ to construct g_{t+1}.

In fully integrated markets, $\widetilde{\Delta s}_{t+1} = g_{t+1}$ by Lemma 1, and hence the entire exchange rate shock can be constructed from local finance state-by-state. This is a powerful result that generalizes the asset market view (AMV) of the exchange rate beyond the case of complete markets (Backus, Foresi, and Telmer, 2001; Brandt, Cochrane, and Santa-Clara, 2006). When the exchange rate risk is a globally-traded shock, Proposition 1 generalizes the familiar complete market relationship $\Delta s_{t+1} = m^*_{t+1} - m_{t+1}$ to:

$$\widetilde{\Delta s}_{t+1} = \text{proj}(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1} | \boldsymbol{\epsilon}^g_{t+1}), \tag{13}$$

where the right-hand side is still fully revealed in the local financial market.[17]

However, what is missing from Proposition 1 is just as important as what is there. Risk sharing and no-arbitrage do not impose any restrictions on the local component of the depre-

[17] In complete markets, the pair of Arrow-Debreu state prices in respective local currencies pins down the value of the exchange rate depreciation in that state by no-arbitrage: any deviation from this value would compel an investor to buy the state where it is cheaper and sell where it is more expensive, after conversion into the same currency. Proposition 1 extends this logic to circumstances with more sparse asset spaces, replacing the concept of state prices with a more general concept of globally-traded shocks ϵ^g. The case of an Arrow-Debreu security corresponds to ϵ^g that is an indicator random variable for a given state of the world.

ciation rate ℓ_{t+1} or its unspanned component u_{t+1}. In partially integrated or intermediated markets, these two components may dominate the dynamics of the exchange rate over and above the shared component g_{t+1}. For example, many popular segmented market models correspond to the case where $g_{t+1} = \ell_{t+1} = 0$ and $\widetilde{\Delta s}_{t+1} = u_{t+1}$, with the equilibrium exchange rate behavior unconstrained by risk-sharing forces, and also consistent with the general prediction of Proposition 1. In Section 3, we discuss the implications of this proposition in the interim cases where neither $g_{t+1} = 0$ nor $\widetilde{\Delta s}_{t+1} = g_{t+1}$.

How does the absence of arbitrage lead to this result? In fully integrated markets, local and foreign investors must agree on the price of all payoffs after conversion to a common currency, resulting in $cov_t(m^*_{t+1} - m_{t+1} - \Delta s_{t+1}, r_{t+1}) = 0$ for every $r_{t+1} \in \boldsymbol{r}_{p,t+1}$, and indeed $\boldsymbol{\epsilon}^g_{t+1} = \widetilde{\boldsymbol{r}}_{p,t+1}$ in this case by Lemma 1. Proposition 1 is a generalization of this result that holds across the full range of market structures. To preclude arbitrage opportunities, there must be an agreement between home and foreign investors' pricing of risks that they both trade in their respective currencies, which constrains the behavior of the depreciation shock.

Without a change of currency, the argument is standard: an intermediary can buy the globally-traded risk ϵ^g_{t+1} in the home market (valued by m_{t+1}) and sell it in the foreign market (valued by m^*_{t+1}), hence no-arbitrage requires that $cov_t(m^*_{t+1} - m_{t+1}, \epsilon^g_{t+1}) = 0$. This logic extends to the case with currency conversion, and no arbitrage requires the so-called quanto adjustment $cov_t(\Delta s_{t+1}, \epsilon^g_{t+1})$ to expected returns. This implies that the comovement of the depreciation rate with globally-traded shocks must be the same as that of the relative SDFs, $cov_t(m^*_{t+1} - m_{t+1}, \epsilon^g_{t+1}) = cov_t(\Delta s_{t+1}, \epsilon^g_{t+1})$. In other words, if the projection g of Δs onto $\boldsymbol{\epsilon}^g$ is different from that of $m^* - m$, there exists an arbitrage strategy for the international intermediary.[18]

Conversely, for shocks that are not traded by both investors in their respective currencies, it is impossible to construct candidate arbitrage portfolios that relate the conditional properties of the exchange rate shock to those of the local SDFs (see Appendix C.2). By

[18] To prove Proposition 1, in Appendix C we use a zero-cost differential carry trade which is long one unit of a home risk, and short one unit of a foreign risk, with both legs of the trade financed at the respective local risk-free rates. Unlike the conventional carry trade, differential carry eliminates the direct exposure to the exchange rate risk. As a result, this trade acts as the arbitrage strategy for globally-traded risks that pins down the projection of the exchange rate on ϵ^g_{t+1}.

consequences, risk sharing and no-arbitrage do *not* constrain the components ℓ and u of the exchange rate that are orthogonal to $\boldsymbol{\epsilon}^g$.

2.2 Expected depreciation rate

We turn to restrictions on the behavior of the expected depreciation rate $E_t \Delta s_{t+1}$. Start from the projection of the exchange rate on asset returns, represented by the two portfolios $r_{p,t+1} \in \boldsymbol{r}_{p,t+1}$ and $r^*_{p,t+1} \in \boldsymbol{r}^*_{p,t+1}$ in equation (10). We define δ_t as the difference between the two portfolios' expected returns given by (5) and (6):

$$\delta_t \equiv E_t\, r_{p,t+1} - E_t\, r^*_{p,t+1} = \Big[r_{ft} - cov_t(m_{t+1}, r_{p,t+1}) - \frac{1}{2} var_t(r_{p,t+1})\Big] \\ - \Big[r^*_{ft} - cov_t(m^*_{t+1}, r^*_{p,t+1}) - \frac{1}{2} var_t(r^*_{p,t+1})\Big]. \tag{14}$$

The following proposition relates the behavior of the expected depreciation rate to spanning of the exchange rate and this quantity δ_t, which only depends on local finance — local asset returns and SDFs.

Proposition 2. *The expected depreciation rate is pinned down by no-arbitrage iff the exchange rate is spanned by asset returns, that is when $u_{t+1} = 0$. In this case:*

$$E_t \Delta s_{t+1} = \delta_t = \underbrace{r_{ft} - r^*_{ft}}_{UIP} - \underbrace{cov_t(m_{t+1}, \Delta s_{t+1})}_{\text{exchange rate risk premium}} - \underbrace{\frac{1}{2} var_t(\Delta s_{t+1})}_{convexity} + \theta_t, \tag{15}$$

where $\theta_t \equiv cov_t(m^*_{t+1} - m_{t+1} - \Delta s_{t+1}, r^*_{p,t+1})$, which becomes $\theta_t = 0$ when the exchange rate is spanned by globally-traded shocks and $\widetilde{\Delta s}_{t+1} = g_{t+1}$. Otherwise, if $u_{t+1} \neq 0$, no-arbitrage does not constrain the value of $E_t \Delta s_{t+1}$.

The central implication of Proposition 2 is that it delineates two cases depending on the relation of the exchange rate with asset returns: either local market pricing determines expected depreciation exactly, or it says nothing about it. The expected depreciation rate is closely related to the exchange rate risk premium. Exposure to this risk can be obtained by engaging in the carry trade using the spanning portfolios. This risk premium is pinned down by pricing in local financial markets if and only if the international arbitrageur can use locally traded assets to sell off this risk. Conversely, the absence of arbitrage has no bearing on this quantity if the exchange rate is not spanned by asset returns, that is when $u_{t+1} \neq 0$.

Spanned exchange rate When the exchange rate is spanned by local asset returns, there exists a unique value of the expected depreciation $E_t \Delta s_{t+1}$ given by (15) which is consistent with no international arbitrage. The international arbitrageur uses the two local markets to price the exchange rate risk. Hence, the two local SDFs play a role in the expected depreciation rate. This insight explains the presence of the novel adjustment term θ_t in equation (15) relative to the standard complete market formula with $\theta_t = 0$. It also leads to a symmetric expression to equation (15) which emphasizes the foreign SDF m^*_{t+1}:

$$\delta_t = r_{ft} - r^*_{ft} - cov_t(m^*_{t+1}, \Delta s_{t+1}) + \frac{1}{2} var_t(\Delta s_{t+1}) + \theta^*_t, \tag{16}$$

where $\theta^*_t = cov_t(m^*_{t+1} - m_{t+1} - \Delta s_{t+1}, r_{p,t+1})$.

It is only when the local investors are able to replicate the exchange rate on their own that their individual Euler equations, and hence individual SDFs, are enough to obtain the expected depreciation. If the home (foreign) investor can trade the entire spanning portfolio, then $\theta_t = 0$ ($\theta^*_t = 0$), and the standard complete market formula for the home (foreign) investor applies. For example, this situation occurs in settings in which the home (foreign) investor acts as an international arbitrageur or has access to a currency carry trade.[19] Both home and foreign investors price the exchange rate risk, and hence $\theta_t = \theta^*_t = 0$, when they are both able to trade it — that is, when the exchange rate is a globally-traded risk, $\widetilde{\Delta s}_{t+1} = g_{t+1}$.

Unspanned exchange rate In intermediated markets, the exchange rate shock may not be spanned by traded assets (recall Lemma 1). In this case, the expectation depreciation can deviate from δ_t in (14) by an arbitrary wedge ψ_t, that is:

$$E_t \Delta s_{t+1} = \delta_t + \psi_t. \tag{17}$$

[19]Note that by Proposition 1, $\theta_t = 0$ when $r^*_{p,t+1}$ is a globally-traded risk, that is $\widetilde{r}^*_{p,t+1} \in \boldsymbol{\epsilon}^g_{t+1} = \widetilde{\boldsymbol{r}}_{t+1} \cap \widetilde{\boldsymbol{r}}^*_{t+1}$. This means that the home household can trade both $r_{p,t+1}$ and $r^*_{p,t+1}$ risks which span the exchange rate shock, giving the household access to an effective currency carry trade.

This flexibility might lead to implausibly large trading profits for the international investor. One can be more informative about these deviations ψ_t by imposing a condition that is stronger than the absence of arbitrage in Assumption 2.

Assumption 3 (No quasi-arbitrage)**.** *There is an upper bound B on Sharpe ratios in international markets:*

$$\forall r^I_{p,t+1} \in \boldsymbol{r}^I_{p,t+1}: \quad \left| E_t\, r^I_{p,t+1} + \frac{1}{2} var_t(r^I_{p,t+1}) - r_{ft} \right| \leq B\sqrt{var_t(r^I_{p,t+1})}. \tag{18}$$

This assumption restricts the Sharpe ratio of trades in international markets. Such bounds have a long tradition in finance, going back to Ross (1976), Cochrane and Saa-Requejo (2000), and Kozak, Nagel, and Santosh (2020). Intuitively, it can be motivated by the view that if trades that are too profitable emerged in equilibrium, new financial institutions would step in to take advantage of them. Under this view, we obtain the following condition.

Proposition 3. *Under Assumption 3, the wedge ψ_t in the expected depreciation rate in (17) must satisfy:*

$$\left| \psi_t + \frac{1}{2} var_t(u_{t+1}) \right| \leq B\sqrt{var_t(u_{t+1})} \equiv B\sqrt{(1-R^2) var_t(\Delta s_{t+1})}, \tag{19}$$

where R^2 is the R-squared in the regression of Δs_{t+1} on $\boldsymbol{r}_{t+1}$ and $\boldsymbol{r}^*_{t+1}$.

This result generalizes Proposition 2 by limiting the range of possible expected depreciations in the case of an unspanned exchange rate. It indicates that the deviation ψ_t from the risk premium in the spanned case in (15) is bounded by the volatility of the unspanned shock u_{t+1} in the depreciation rate decomposition (9). In turn, u_{t+1} may also be shaped by the properties of ψ_t as the result of an equilibrium fixed point (see Itskhoki and Mukhin, 2021, as well as our discussion below).

2.3 Relationship between the results

When globally-traded risks are the result of common trading of assets in $H \cap F$, e.g. in (partially) integrated markets, Proposition 2 obtains as a standard asset pricing result from local Euler equations in Assumption 1. Furthermore, in this case, Proposition 1 is a direct consequence of Proposition 2. Indeed, any two globally-traded assets $i, j \in H \cap F$ provide two alternative ways to span the exchange rate in Proposition 2, and we can write condition (15) in two ways with respective θ_{it} and θ_{jt}. Hence, it must be that $\theta_{it} - \theta_{jt} = 0$, which corresponds to the projection requirement in Proposition 1 for the globally-traded excess return of asset i over j.[20]

When globally-traded risks arise from spanning of the same shocks by distinct assets in H and F, e.g. in intermediated markets, Propositions 1 and 2 are not directly linked. Proposition 2 remains a standard asset pricing result that links the expected return of a trade to its risk measured by the covariance of its return with SDF, provided the caveat that both home and foreign SDFs are generally required for no-arbitrage pricing of the exchange rate return. Proposition 1 is less conventional, as it characterizes the exchange rate shock rather than its expected return. Furthermore, Proposition 1 applies even when there is no spanning of the exchange rate in Proposition 2 and thus equation (15) does not hold. In other words, the finance exchange rate disconnect (in the sense that $u_{t+1} \neq 0$) does not imply absence of international risk sharing ($g_{t+1} = 0$), as we study next.

3 The dual role of financial markets

The general results of the previous section inform our understanding of how finance interacts with the exchange rate beyond the standard market structures studied in the literature. In this section, we apply these results to address three questions suggested by the implications of these standard market structures. First, does risk sharing necessarily lead to the currency puzzles? Second, is extreme segmentation necessary for financial markets to be a source of

[20] By definition of θ_t in (15), $\theta_{it} - \theta_{jt} = cov_t(m^*_{t+1} - m_{t+1} - \Delta s_{t+1}, r_{i,t+1} - r_{j,t+1}) = 0$ where $r_{i,t+1} - r_{j,t+1} = r^*_{i,t+1} - r^*_{j,t+1}$ are the return differentials (excess returns) in home and foreign currency, respectively, forming a globally-traded risk. Note that this condition corresponds to the condition in Lustig and Verdelhan (2019) when i and j are the two risk-free assets. In this case, $r_{i,t+1} - r_{j,t+1} = \Delta s_{t+1}$, and hence $var_t(\Delta s_{t+1}) = cov_t(m^*_{t+1} - m_{t+1}, \Delta s_{t+1})$.

exchange rate volatility? Third, are these two roles of financial markets mutually exclusive? We answer all three questions by the negative. Furthermore, we characterize which features of the market structure offer solutions to these challenges.

Propositions 1 and 2 emphasize the role of spanning of the exchange rate by asset returns and the prominence of the globally-traded shocks captured by g_{t+1} in this decomposition. We show that the size of globally-traded shocks $var_t(g_{t+1})$ is a sufficient statistic pinning down the properties of a given market structure. Changing the market structure changes $var_t(g_{t+1})$ and the model's implication for the exchange rate for a given structural pair of SDFs. All models that share the same $var_t(g_{t+1})$ have similar predictions — even when the market structures may seem very different, as we see below.

We focus our analysis on specific properties of the exchange rate — namely, its volatility, cyclicality and the currency risk premium — which have proved puzzling for traditional models in macro-finance. In doing so, we take a complementary approach to a broad literature which has made progress by altering preferences or aggregate dynamics but maintained the assumptions that markets are complete and integrated. In contrast, we hold the model of household IMRSs and the aggregate data that inform their properties fixed, hence taking m and m^* as given. We use our general theoretical results and vary the market structure — namely, what assets are traded and who can trade them — to study its implications for the currency puzzles.

3.1 The currency puzzles

The asset market view (AMV) of the exchange rate under complete and integrated markets results in:

$$\Delta s_{t+1} = m^*_{t+1} - m_{t+1}, \tag{20}$$

which characterizes the entire exchange rate depreciation — both its expectation $E_t \Delta s_{t+1}$ and shocks $\widetilde{\Delta s}_{t+1}$ — from the local SDFs. When the local SDFs are given by the conventional IMRSs of representative households disciplined with macro-data on aggregate consumption growth and inflation, the AMV results in three seminal finance puzzles about the behavior of the exchange rate.

First, consider the variance of the exchange rate depreciation rate implied by (20):

$$
\begin{aligned}
var_t(\Delta s_{t+1}) &= var_t(m^*_{t+1} - m_{t+1}) \\
&= var_t(m^*_{t+1}) + var_t(m_{t+1}) - 2cov_t(m_{t+1}, m^*_{t+1}).
\end{aligned} \tag{21}
$$

Brandt, Cochrane, and Santa-Clara (2006) argue that this equation leads to the *volatility puzzle*, with the exchange rate being not volatile enough. Typically observed Sharpe ratios on domestic assets imply highly volatile IMRSs, much more so than exchange rate depreciation. The mild correlation of macroeconomic quantities across countries suggests that the IMRSs are not correlated enough for the last term of equation (21) to offset this high variance and obtain realistic exchange rate risk.[21]

Second, equation (20) also implies:

$$
var_t(\Delta s_{t+1}) = cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}), \tag{22}
$$

and $corr_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}) = 1$. Changes in exchange rates must be perfectly correlated with changes in relative marginal utilities of the domestic and foreign households, that is, the home currency depreciates in relatively good times for home investors. As pointed out by Backus and Smith (1993), this implication is counterfactual for various measures of good times, leading to the *cyclicality puzzle*.

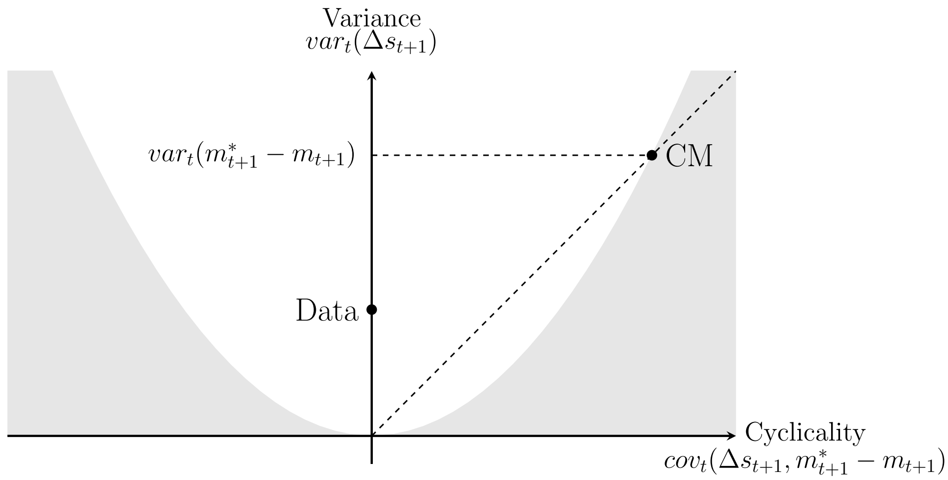
Finally, the expected depreciation rate is:

$$
E_t \Delta s_{t+1} = r_{ft} - r^*_{ft} \; - \frac{1}{2} var_t(\Delta s_{t+1}) - cov_t(m_{t+1}, \Delta s_{t+1}) \tag{23}
$$

The last term, a premium for currency risk, generates deviations from uncovered interest parity (UIP), a well-documented empirical feature. However, standard international models struggle with generating the empirically observed magnitude and dynamics of currency risk premium, resulting in the *risk premium puzzle* (see Engel, 2014, for a review).

[21] A typical annual standard deviation of the exchange rate is 0.1, while a typical annual Sharpe ratio is of the order of 0.5, which is a lower bound on the standard deviation of SDFs because of the Hansen and Jagannathan (1991) bound. Then, according to (21), the correlation between m and m^* must be at least $1 - \frac{1}{2}\frac{0.1^2}{0.5^2} = 0.98$, which is much in excess of any empirical measures of cross-country comovement.

Figure 2: Volatility, cyclicality, and currency puzzles



The grey area represents the infeasible combinations of volatility and cyclicality of depreciation rates due to the Cauchy-Schwarz inequality. The point labeled as CM illustrates the implications of the complete market setting for the properties of the depreciation rate. The point labeled Data is a stylized representation of the exchange rate puzzles summarized in (24).

We assume that the household IMRSs that define m and m^* are such that the exchange rate's cyclicality, volatility, and risk premium are counterfactual under complete and integrated markets, resulting in the currency puzzles. That is, we assume that for given SDFs m and m^* and data on the exchange rate, we have:

$$0 \approx cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}) \ll var_t(\Delta s_{t+1}) \ll var_t(m^*_{t+1} - m_{t+1}) \tag{24}$$

and a currency risk premium $r_{ft} - r^*_{ft} - E_t \Delta s_{t+1}$ that considerably exceeds $cov_t(m_{t+1}, \Delta s_{t+1})$ in absolute value — all conflicting with the AMV in (20).

In what follows, we ask how market structures beyond complete and integrated markets constrain the properties of the exchange rate. Propositions 1 and 2 lead us to consider volatility and cyclicality separately from the risk premium.

We use Figure 2 to visualize the exchange rate cyclicality $cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1})$ and volatility $var_t(\Delta s_{t+1})$, where we hold the statistical properties of m and m^*—and hence $var_t(m^*_{t+1} - m_{t+1})$—as given. The point labeled 'CM' shows the prediction of the AMV under complete and integrated markets summarized by (22), hence it lies on the 45-degree line at a point with $var_t(\Delta s_{t+1}) = var_t(m^*_{t+1} - m_{t+1})$. The point labeled 'Data' is a stylized representation of the empirical properties of the exchange rate summarized by (24). The distance between CM and Data represents the first two currency puzzles — volatility on the y-axis and cyclicality on the x-axis.

Finally, the gray area in Figure 2 indicates all combinations of exchange rate volatility and cyclicality that are infeasible due to the mechanical Cauchy-Schwarz inequality:

$$cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}) \leq \sqrt{var_t(\Delta s_{t+1}) \cdot var_t(m^*_{t+1} - m_{t+1})}. \tag{25}$$

Correspondingly, the white cone reflects all mathematically feasible combinations.

3.2 The volatility-cyclicality tradeoff

Proposition 1 provides a general characterization of all restrictions imposed by international risk sharing on the exchange rate shock $\widetilde{\Delta s}_{t+1}$ that, in particular, determine its volatility and cyclicality properties. It has the following implications:[22]

Proposition 4. *The volatility and cyclicality of the exchange rate must satisfy*

$$\overbrace{var_t(\Delta s_{t+1})}^{volatility} \geq var_t(g_{t+1}) + \frac{\left(\overbrace{cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1})}^{cyclicality} - var_t(g_{t+1})\right)^2}{var_t(m^*_{t+1} - m_{t+1}) - var_t(g_{t+1})} \tag{26}$$

when $var_t(g_{t+1}) < var_t(m^*_{t+1} - m_{t+1})$*, and*

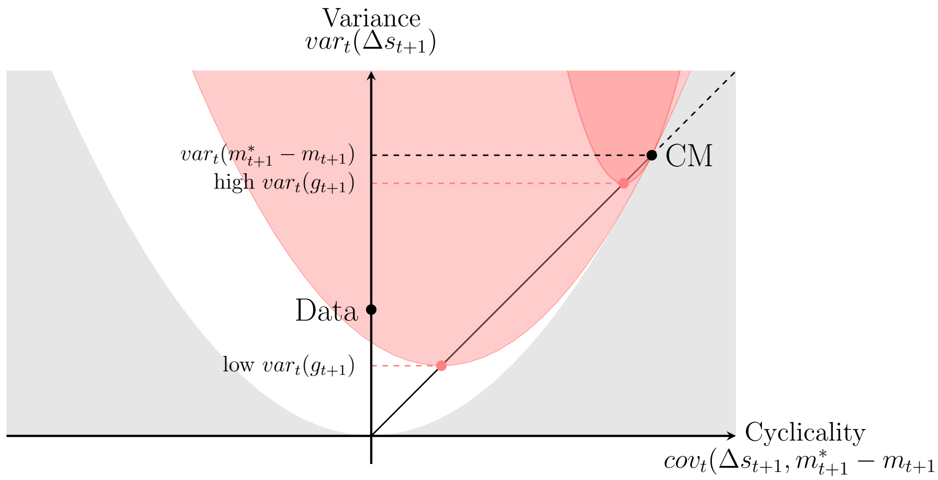
$$var_t(\Delta s_{t+1}) \geq var_t(g_{t+1}) = cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}). \tag{27}$$

when $var_t(g_{t+1}) = var_t(m^*_{t+1} - m_{t+1})$*.*

We highlight three properties that follow from Proposition 4. First, for a given value of $var_t(g_{t+1})$, the proposition introduces a tradeoff between volatility and cyclicality. The red cones in Figure 3 illustrate this tradeoff for values of $var_t(g_{t+1})$ away from the boundaries as described by condition (26): $var_t(g_{t+1})$ is larger for the upper dark cone than for the lower

[22]Proposition 4 is a consequence of (11) and the Cauchy-Schwartz inequality applied to the non-global components of the exchange rate and SDF differential, $\Delta s_{t+1} - g_{t+1}$ and $m^*_{t+1} - m_{t+1} - g_{t+1}$, respectively. The formal proof is in Appendix C.3.

Figure 3: The volatility-cyclicality tradeoff



The figure illustrates the trade-off between volatility and cyclicality of the exchange rate as characterized by (26) in Proposition 4. The dark red cone corresponds to a high value of $var_t(g_{t+1})$, while the light red cone corresponds to a low value. See notes to Figure 2.

light one. The vertex (trough) of the cone corresponds to the minimum level of exchange rate volatility $var_t(\Delta s_{t+1}) = var_t(g_{t+1})$. At this point, volatility coincides with cyclicality, $cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}) = var_t(g_{t+1})$. Thus, the vertex is on the 45-degree line segment between the origin and CM. Reducing cyclicality away from this value comes at the cost of increasing volatility.

Second, when the globally-traded component is nil, $var_t(g_{t+1}) = 0$, financial risk sharing imposes no additional economic constraints on the exchange rate moments. Specifically, condition (26) in this case recovers the mechanical Cauchy-Schwartz inequality (25) which excludes the gray area in Figures 2 and 3.

Third, the CM point is feasible for any value of $var_t(g_{t+1})$, and hence under any market structure. Furthermore, the range of possible exchange rate moments increases from the measure zero CM point to the full white area permitted by the Cauchy-Schwartz inequality as $var_t(g_{t+1})$ declines from its maximal value equal to $var_t(m^*_{t+1} - m_{t+1})$ to its minimal value of 0.[23] In this sense, reducing the span of globally-traded shocks $var_t(g_{t+1})$ does not rule possibilities out, but instead allows possibilities in — more outcomes in terms of

[23]Note that by Proposition 1, and in particular by its corollary in (12), $var_t(g_{t+1})$ cannot exceed $var_t(m^*_{t+1} - m_{t+1})$, and by construction it cannot exceed $var_t(\Delta s_{t+1})$, hence $var_t(g_{t+1}) \leq \min\{var_t(\Delta s_{t+1}), var_t(m^*_{t+1} - m_{t+1})\}$.

second moments of the exchange rate can be consistent with equilibrium risk sharing and no-arbitrage. Therefore, it is easier to match the data with less dense globally-traded shocks.

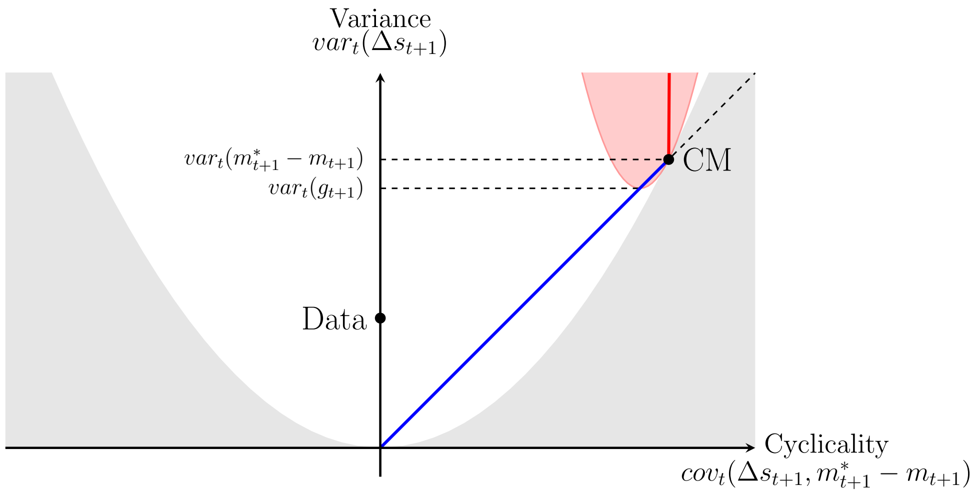
Proposition 4 and this discussion make clear that the range of possible exchange rate moments is controlled by the volatility of the globally-traded component of the exchange rate $var_t(g_{t+1})$. In turn, this value is determined by the structure of financial markets. That is, the variance of the globally-traded component serves as a sufficient statistic for how the market structure disciplines the volatility and cyclicality of the exchange rate. We explore the interplay between market structures and the globally-traded component next.

3.3 When does risk sharing lead to the currency puzzles?

Proposition 4 suggests that the currency puzzles are not unique to complete and integrated markets, but emerge by continuity in all models with a dominant globally-traded component g_{t+1}. Specifically, the puzzles arise when $var_t(g_{t+1})$ is large, imposing tight lower bounds on both volatility and cyclicality of the exchange rate. As the upper bound for $var_t(g_{t+1})$ is given by $\min\{var_t(\Delta s_{t+1}), var_t(m^*_{t+1} - m_{t+1})\}$, we start by exploring two types of market structures which yield the corresponding limiting cases. In these cases the volatility-cyclicality tradeoff takes a different shape than the cones of Figure 3. We then discuss the generic case with a large $var_t(g_{t+1})$.

Spanned marginal utilities We consider first the case when the relative IMRS is spanned by globally-traded shocks, $var_t(g_{t+1}) = var_t(m^*_{t+1} - m_{t+1})$. This situation is close to market completeness in the sense that households in each country are able to trade shocks to their marginal utility. This case frequently arises in models with a small number of common macro risks that are traded in both countries. International real business cycle (IRBC) models would fall into this category. One could also consider various settings popular in finance, such as habits, long-run risk, or rare disasters as long as they feature a small set of shocks that are traded in both countries. However, this situation does not require integrated markets. For example, it can arise when households cannot trade with each other, $H \cap F = \emptyset$, and the markets are intermediated, as long as households in each country have access to a set of assets that is sufficiently rich to span the risks that affect both of their marginal utilities.

Figure 4: The cyclicality-volatility tradeoff with a large globally-traded component



The red ray depicts all possible volatility-cyclicality combinations when globally-traded shocks span household marginal utility (IMRS) and $var_t(g_{t+1}) = var_t(m^*_{t+1} - m_{t+1})$, as in equation (28). The blue 45° line segment corresponds to the case when the exchange rate is spanned by globally-traded shocks, as in equation (29). The red cone corresponds to condition (26) for a high value of $var_t(g_{t+1})$, but away from the two limiting cases. See notes to Figures 2 and 3.

By Proposition 4, condition (27) applies in this case and yields

$$var_t(\Delta s_{t+1}) \geq var_t(m^*_{t+1} - m_{t+1}) = cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}). \tag{28}$$

The volatility of the relative IMRS puts a lower bound on the volatility and pins down the cyclicality of the exchange rate. Hence, this setting deepens the volatility puzzle and leaves the cyclicality puzzle unchanged relative to complete markets. In Figure 4, the vertical red ray emanating upwards from the CM point represents the volatility-cyclicality tradeoff in this situation.

A testable implication of the spanned IMRS assumption is that a regression of the exchange rate depreciation Δs_{t+1} on the relative IMRS $m^*_{t+1} - m_{t+1}$ yields a coefficient of 1, while the reverse regression yields a coefficient (weakly) less than 1.

Spanned exchange rate The other limiting case arises when the exchange rate risk is globally-traded, $var_t(g_{t+1}) = var_t(\Delta s_{t+1})$. This immediately applies when markets are integrated but not necessarily complete, and full market integration is not necessary. For example, it is sufficient for households in both countries to have access to the two risk-free bonds as in Lustig and Verdelhan (2019). This case can also arise under intermediated markets when the depreciation shock is spanned by globally-traded risks.

The implication of condition (26) in Proposition 4 in this case is:

$$cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}) = var_t(\Delta s_{t+1}) = var_t(g_{t+1}). \tag{29}$$

The constraint on the volatility of the exchange rate is weaker, because g can be less volatile than the relative IMRS $m^* - m$. However, just like in the complete and integrated markets case, there is a cyclicality puzzle. This constraint on the cyclicality and volatility of the depreciation rate corresponds to the blue 45-degree line segment between the origin and the complete markets point in Figure 4. Each point on this segment corresponds to the vertex of a cone for a given value of $var_t(g_{t+1})$.[24]

A testable implication of the spanned exchange rate assumption is that a regression of the relative IMRS $m^*_{t+1} - m_{t+1}$ on the exchange rate depreciation Δs_{t+1} yields a coefficient of 1, while the reverse regression yields a coefficient (weakly) less than 1. This implication is symmetric to the one in the spanned IMRS case.

Large globally-traded component The two cases above indicate that the exchange rate continues to be tightly constrained by the properties of the household IMRSs as long as there is only a single departure from either market completeness or market integration. When we modify who can trade assets by allowing for imperfect market integration or intermediation, as long as the available set of assets is rich, we end up in the spanned IMRS scenario. When we limit which assets can be traded while still ensuring market integration for these assets, we find ourselves in the spanned exchange rate scenario. Both situations yield tight constraints on the possible properties of the exchange rate, captured by the respective line segments in Figure 4 and away from the Data point.

Therefore, relaxing the constraints on the behavior of the exchange rate requires departing from both market completeness and market integration at once. This is necessary, but not sufficient. Even intermediated incomplete markets can feature exchange rate puzzles when

[24] In this case, constraints imposed by other traded assets create a lower bound on $var_t(g_{t+1})$. This restricts the set of feasible exchange rate moments to the sub-segment of the blue line above this value.

the set of traded assets in each county is sufficiently broad to get close to span the relative IMRS $m^* - m$. This situation corresponds to a large value of $var_t(g_{t+1})/var_t(m^*_{t+1} - m_{t+1})$, albeit less than 1. This is the dark red cone in Figure 4 that is tight around the red vertical ray. In other words, the constraints on the properties of the exchange rate are continuous in the value of $var_t(g_{t+1})$, and are only sufficiently relaxed when $var_t(g_{t+1})$ is small, as we study next.

3.4 Market structures without currency puzzles

The complementary implication of Proposition 4 is that market structures with a small globally-traded component can accommodate various exchange rate dynamics without encountering currency puzzles. Specifically, when $var_t(g_{t+1}) = 0$, all combinations of volatility and cyclicality are feasible. This situation arises in popular intermediation-based models of exchange rate which assume extreme segmentation and no globally-traded risks. We demonstrate that this can be significantly relaxed: intermediated market structures can allow meaningful risk sharing while $var_t(g_{t+1})$ remains low enough to accommodate the empirical properties of the exchange rate.

No globally-traded risks In many intermediation-based models of exchange rate, households in each country have access to the local risk-free asset only. Intermediaries trade both of these assets and bear the currency risk. Because neither H nor F contain risky assets, there are no globally-traded risks, and hence $g_{t+1} = 0$. All combinations of cyclicality and volatility in the white cone in Figure 2 are compatible with risk-sharing conditions.

One can then ask whether an equilibrium model can generate these combinations of volatility and cyclicality, in particular the Data point. Appendix E answers positively: there exist foundations rationalizing any point in the cone as the equilibrium of an intermediation-based model with exchange rate shocks arising in the currency market as a result of shifts in currency demand or intermediation capacity.

More formally, start from an arbitrary point in the cone defining the volatility and comovement of the exchange rate with a given relative SDF $m^* - m$. Select a series of shocks u_{t+1} that exhibit this volatility and cyclicality. We then construct a series of primitive sources of fluctuation in the currency market — e.g., exogenous liquidity demand for currency or intermediary risk-bearing capacity — such that the equilibrium exchange rate features these conjectured innovations, $\widetilde{\Delta s}_{t+1} = u_{t+1}$. We rely on the fact that the intermediary clears the currency market and hence prices the exchange rate risk. As a result, shocks affecting the demand and supply curves in the currency market pass through to the equilibrium exchange rate. In the absence of vehicles for household risk sharing, m and m^* do not impose any additional constraints.

The extreme form of market segmentation assumed in such models might not be appealing. In practice, households trade more than one asset and intermediaries participate in more than one market. We ask whether one can consider additional markets in the intermediated setup without losing the empirical flexibility afforded by extreme segmentation — that is, keeping $var_t(g_{t+1}) = 0$.

Two such dimensions are particularly helpful in moving towards a more realistic market structure. First, because the trading opportunities of intermediaries do not affect the set of the globally-traded shocks, these intermediaries can be as sophisticated and active in as many markets as one wants. Formally, one can enrich arbitrarily set I and price all assets in this set with m^I without affecting the risk-sharing restrictions on the exchange rate that are shaped by the sets $H \cap I$ and $F \cap I$.

Second, households can trade rich sets of their respective local assets as long as their returns are not related enough to create globally-traded shocks. Formally, the sets $H \cap I$ and $F \cap I$ may include a rich set of local assets such as equities and bonds provided that the set of globally-traded risks $\boldsymbol{\epsilon}^g = \widetilde{\boldsymbol{r}}_p \cap \widetilde{\boldsymbol{r}}_p^*$ remains empty. In this case, local financial markets still impose no risk-sharing restrictions on the exchange rate, affording the same full flexibility to the intermediation model of the currency market.

Small globally-traded component When the set of local assets is sufficiently rich, there may be commonly-traded shocks across countries. Then, the globally-traded component becomes significant, $var_t(g_{t+1}) > 0$; risk-sharing imposes constraints on the exchange rate. This does not necessarily introduce the puzzles. If $var_t(g_{t+1})$ is low enough, as for example in the light red cone in Figure 3, the Data point is still feasible.

The upper bound on the volatility of the globally-traded component such that the corresponding red cone contains the Data point is (see Appendix C.3):

$$\frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})} \leq \frac{var_t(m^*_{t+1} - m_{t+1})/var_t(\Delta s_{t+1})}{1 + var_t(m^*_{t+1} - m_{t+1})/var_t(\Delta s_{t+1})}. \tag{30}$$

This relation highlights an important nuance in how such models can address the puzzles. International risk sharing must be sufficiently weak in the sense that $var_t(g_{t+1}) \ll var_t(m^*_{t+1} - m_{t+1})$, but it can still contribute substantially to exchange rate variation, $var_t(g_{t+1})/var_t(\Delta s_{t+1}) \gg 0$. To see this, notice that under our maintained assumption of a volatility puzzle in complete markets, that is $var_t(\Delta s_{t+1}) \ll var_t(m^*_{t+1} - m_{t+1})$, the right-hand-side of (30) converges to 1.[25]

Just like in the case of $var_t(g_{t+1}) = 0$, any combination of volatility and cyclicality in the relevant feasibility cone — now any red cone in Figure 3 — can be reached in equilibrium (see Appendix E). In this case, the two roles of financial markets coexist. Because $(\boldsymbol{r}, \boldsymbol{r}^*)$ is richer, risk sharing now determines a non-trivial globally-traded component of the exchange rate g_{t+1}, which corresponds to the vertex of the red cone. One can then apply the same reasoning as in the unconstrained case to the variance of $\Delta s_{t+1} - g_{t+1}$ and its covariance with $m^*_{t+1} - m_{t+1} - g_{t+1}$ to find shocks in the currency market that reach any point in the red cone. These shocks add to the volatility of the exchange rate above the base level $var(g_{t+1})$. If the additional source of variation in the exchange rate is designed to comove with the relative IMRS, cyclicality of the exchange rate is also altered, allowing to move towards the Data point. The lower is $var_t(g_{t+1})/var_t(\Delta s_{t+1})$, the lower is the required comovement.[26]

Market structures with intermediation and many locally traded assets may be particularly appealing. They help overcome the limitations that arise in models that consider each role of the financial market in isolation. Specifically, they allow to avoid both the over-reliance of exchange rate dynamics on macroeconomic factors via international risk sharing and the excessive segmentation in market participation for households and intermediaries.

[25] Using the same values as in footnote 21 and assuming conservatively a high correlation of the two IMRSs of 0.9, we obtain an upper bound for $var_t(g_{t+1})/var_t(\Delta s_{t+1}) \leq 0.83$.

[26] Note that the comovement between $\Delta s_{t+1} - g_{t+1}$ and $m^*_{t+1} - m_{t+1} - g_{t+1}$ may be due to either the ℓ_{t+1} or u_{t+1} component of the exchange rate.

3.5 The currency risk premium

We turn to the properties of the currency risk premium, guided by Proposition 2. This proposition indicates a sharp delineation between market structures regarding the currency risk premium, and hence the constraints on the expected depreciation $E_t \Delta s_{t+1}$. On the one hand, if the exchange rate is spanned by asset returns, $u_{t+1} = 0$, expected depreciation is given by equation (15). Because this expression is close to equation (23), the currency risk premium puzzle arises in such market structures as well. On the other hand, if the exchange rate is not spanned by asset returns, expected depreciation can deviate arbitrarily from this tight risk-sharing relation.

When does spanning occur in the market structures discussed above? In integrated markets, as in the blue line in Figure 4, the exchange rate is a globally-traded risk. Because globally-traded shocks are constructed from asset returns, the exchange rate is fully spanned. Therefore, these structures, which already wrestle with the volatility and cyclicality puzzles also face the risk premium puzzle.

In market structures with less risk sharing, like those of Sections 3.4, the exchange rate may or may not be spanned. For example, lack of spanning occurs naturally if local asset returns, such as those of stocks and bonds, are not affected by currency-market shocks that transmit to the exchange rate. Interestingly, the equilibrium model in Appendix E, which is able to reach any point in a given red cone in Figure 3, generically features unspanned exchange rates. Therefore, Proposition 2 implies an unconstrained risk premium for this class of models with $E_t \Delta s_{t+1}$ determined by forces other than international risk sharing.

If we further impose an upper bound on Sharpe ratios, Proposition 3 applies, and risk premium variations cannot be too large relative to exchange rate innovations, constraining the possible range of values of $E_t \Delta s_{t+1}$. In equilibrium models, expected depreciation and exchange rate shocks are additionally constrained by transversality-type conditions. For example, in some models, the exchange rate is stationary and innovations in the exchange rate $\widetilde{\Delta s}_{t+1}$ must be offset by long-term changes in future expected depreciations $(E_{t+1} - E_t)[\Delta s_{t+j+1}]$.[27] The models of Appendix E have an equilibrium reaching any

[27] When the exchange rate is not long-run mean reverting, fundamental shocks in goods and financial

point in the red cone that also satisfy Proposition 3. Still, Proposition 3 restricts some properties of the exchange rate: it requires that the equilibrium features sufficiently small but persistent innovations to the currency risk premium in order to sustain the unspanned exchange rate shocks.

4 Empirical Analysis

We now turn to the data on asset returns and exchange rates to evaluate the implied restrictions on equilibrium SDFs. Our theoretical results show that restrictions between the exchange rate and SDFs depend on the assumed market structure, and that this question cannot be answered with asset return data alone. In particular, given the data on returns, one needs to take a stand on who can trade what assets, that is the sets H, F, and I.[28]

For some market structures, return data is not necessary to characterize the restrictions. For example, when markets are complete, risk sharing implies that the relative household IMRS must equal the observed exchange rate depreciation. Or, in the other extreme of segmented markets, when households only trade their own risk-free asset, there are no restrictions on shocks to household IMRSs from observed asset returns and the exchange rate.

Therefore, for the purposes of this section, we focus on a market structure where data on asset returns and exchange rates plays a meaningful role in characterizing the implied risk-sharing constraints on the local SDFs. Specifically, we consider the case of intermediated markets in which households of each country trade a broad collection of local bonds and equities, as in Section 3.4. In this market structure, spanning of the exchange rate by asset returns and globally-traded risks cannot be taken as given and needs to be evaluated empirically by combining the data on asset returns with the assumptions about H and F.

We find that, through the lens of this market structure, exchange rates appear to have a large component u_{t+1} unspanned by asset returns. Then, we provide methods to characterize

markets can cause shifts in the long-run exchange rate expectations, $\bar{s}_t^{\infty} \equiv \lim_{j\to\infty} E_t s_{t+j}$, resulting in $\Delta\bar{s}_{t+1}^{\infty} \neq 0$. Then the general intertemporal restrictions on the exchange rate shocks and expected depreciations can be written as $(\Delta s_{t+1} - E_t\Delta s_{t+1}) + (E_{t+1} - E_t)\left[\sum_{j=1}^{\infty} \Delta s_{t+j+1}\right] = \Delta\bar{s}_{t+1}^{\infty}$, where $\bar{s}_t^{\infty}$ is determined in the full equilibrium of the model.

[28] Appendix G also shows that one has to take a stand on market structure for an economic interpretation of the SDFs recovered from return data.

globally-traded shocks. We find that these shocks explain a modest share of exchange rate fluctuations. Taken together, these empirical results suggest that, under this assumed market structure, international risk-sharing restrictions are relatively weak and therefore a wide range of household IMRSs is compatible with empirical exchange rate dynamics.

4.1 Data

We consider countries corresponding to G10 currencies between 2/1988 and 12/2022. We consider Germany as the representative country for the euro. Prior to the introduction of the euro, we use the Deutsche mark and splice these series together beginning in 1999. Our analysis focuses on the monthly frequency. We obtain exchange rates from WM/Reuters. Government bond yields are from each country's central bank website. Monthly bond returns are computed from bond yields using a second-order Taylor approximation. We obtain equity indices from Morgan Stanley Capital International (MSCI). For each country, 10 different industry indices and 3 different style equity indices (Large + Mid Cap, Value, Growth) are sourced. Risk-free rates are approximated by dividing the 1-year yield by 12.

4.2 Is the exchange rate spanned?

Motivated by Proposition 2, we ask whether the depreciation rate is spanned by combination of domestic and foreign asset returns. We estimate regressions that implement the construction of the maximum spanning portfolio in (10) as follows:

$$\Delta s_{t+1} = \alpha + \beta' \boldsymbol{r}_{t+1} + \beta^{*\prime} \boldsymbol{r}^*_{t+1} + u_{t+1}, \tag{31}$$

where $\boldsymbol{r}_{t+1}$ and $\boldsymbol{r}^*_{t+1}$ represent asset returns available to home and foreign investors in their respective currencies.[29] Specifically in the case of the assumed market structure, U.S. equities and bond returns are expressed in U.S. dollars, while foreign equities and bond returns are expressed in the currency of that country. The residual u_{t+1} is a direct estimate of the unspanned component of the depreciation rate in equation (9).

[29] We implement these regressions unconditionally for the whole sample at monthly frequency. Alternatively, spanning could be evaluated in sub-samples or at longer horizons.

Table 1: Spanning of depreciation rates by asset returns: R^2

Dependent Variable	AU	CA	DE	JP	NO	NZ	SE	CH	UK
Bonds									
10Y	0.3	0.3	7.5	5.4	4.7	1.1	4.8	4.0	0.9
All Maturities	7.2	7.9	15.7	10.2	13.7	5.7	14.0	11.5	13.7
Stocks									
Mkt	21.7	26.6	7.0	4.4	11.2	16.6	16.2	12.3	12.7
Mkt + Value/Growth	21.6	28.0	6.8	5.1	12.5	17.2	15.9	12.7	13.7
Mkt + Value/Growth + Ind.	35.1	41.6	18.6	22.8	29.4	24.5	24.0	19.6	26.9
Bond + Equity	36.7	45.1	26.8	29.1	36.6	28.0	30.6	25.3	33.8
N	419	395	419	419	406	419	414	419	419

The table reports the adjusted R^2 of a regression of the depreciation rate on various subsets of asset returns, as in equation (31). Domestic asset returns are in domestic currency; foreign asset returns are in foreign currency. Each column is a different country's currency relative to the U.S. dollar. The first row uses only 10-year bonds, while the second entertains maturities between 2 and 10 years, obtained from various central banks. The next three row consider various stock portfolios: the market (a combination of large and mid-cap stocks), plus value and growth portfolios, plus 10 industry portfolios (all from MSCI). The final row considers all assets simultaneously.

Table 1 reports the adjusted R^2 from these regressions for individual countries vis-à-vis the United States. Each row reflects a particular combination of assets used in the regression: we consider bonds and equities separately and in combination. Exact spanning corresponds to an R^2 of 1, and Proposition 3 highlights that this R^2 is an appropriate measure of economic distance to the case of complete spanning.[30]

Our key finding is that these major asset classes do not span the exchange rate. When looking at all assets together, the R^2s range from 25% for Switzerland to 45% for Canada (in each case vis-à-vis the U.S.). Consistent with the evidence in Chernov and Creal (2023), bond returns explain only a modest amount of variation in exchange rates: between 0.2% and 7% for the 10-year bond alone, and between 7% and 14% for the combination of bonds at all maturities. Most of the explanatory power comes from equities. While the market alone gets to a substantial amount of variation, the addition of industry returns is particularly informative.

[30] Campbell, Serfaty-De Medeiros, and Viceira (2010) focus on currency hedging of equity and bond portfolios, so they essentially implement reverse regressions with a focus on the sign and significance of the associated betas. The documented insignificant betas for bond portfolios are suggestive of low R^2.

The economic magnitude of the unspanned component $var(u_{t+1})$ is substantial. According to Proposition 3, even the largest R^2 we measure implies a bound for the expected depreciation that is only $\sqrt{1-0.45} \approx 0.74$ of the bound with $R^2 = 0$, not much tighter. We conclude that, under the intermediated market structure we consider, the expected depreciation rate is weakly constrained by the relative local SDFs m and m^*. Conversely, the observed path of $E_t \Delta s_{t+1}$ and the currency risk premium can be consistent with a wide range of conventional household IMRSs.

4.3 Identifying globally-traded shocks

We quantify the importance of globally-traded shocks $\boldsymbol{\epsilon}^g_{t+1}$, which play the key role in Proposition 1. We follow an undirected approach and use canonical correlation analysis (CCA) to identify these shocks from the asset return data. In Appendix F, we also consider a directed approach starting from candidates for globally-traded shocks proposed in the literature, such as global macro and financial variables. The results in that setting are qualitatively similar.

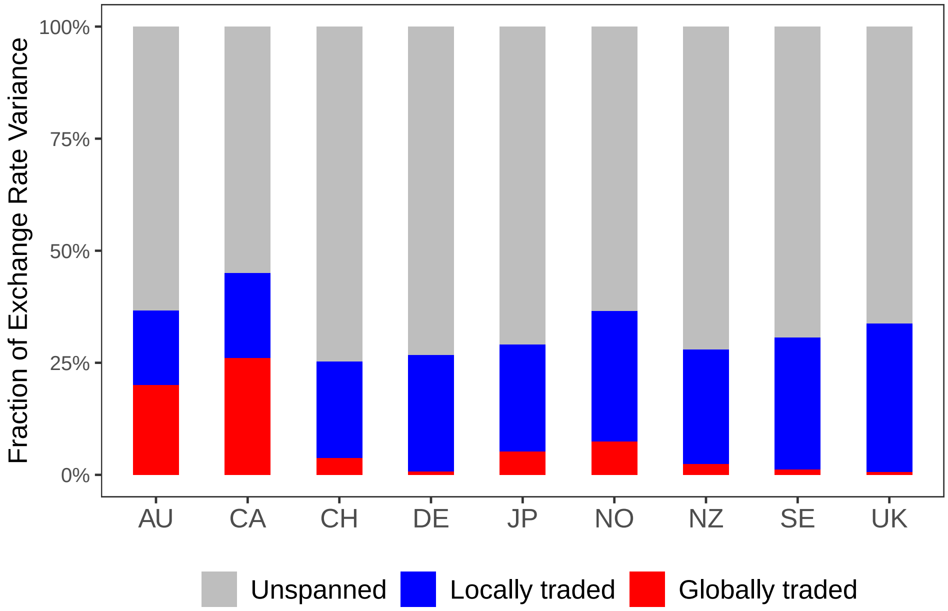
According to Definition 1, globally-traded shocks are innovations to portfolios of asset returns consisting of $\boldsymbol{r}_{t+1}$ and $\boldsymbol{r}^*_{t+1}$, respectively, with perfect correlation. The CCA procedure constructs US and foreign portfolios with the highest correlation possible in sample. Conditional on finding this pair, the procedure then looks for the next maximally correlated pair of portfolios that are orthogonal to their first pair. And so on. See Appendix B for details.

The values of the largest correlations range from 64% for New Zealand to 90% for Canada. The detailed results are reported in Appendix Table A1. We are generous with interpreting the evidence, and assume that portfolios with a correlation over 60% are sufficiently close to each other to constitute a measure of a globally-traded shock.

We ask how much variation in the depreciation rate is explained by globally-traded shocks. Denote the matrix of foreign portfolio weights by $\boldsymbol{w}^*$ so that $\boldsymbol{w}^{*\prime}\boldsymbol{r}^*_{t+1}$ is our basis of globally-traded risks $\boldsymbol{\epsilon}^g_{t+1}$. We implement regressions of the form:

$$\Delta s_{t+1} = \alpha + \beta^{g\prime}(\boldsymbol{w}^{*\prime}\boldsymbol{r}^*_{t+1}) + \varepsilon_{t+1}. \tag{32}$$

Figure 5: Decomposition of exchange rate innovations



The figure reports the fraction of variance in exchange rates explained by globally-traded and locally-traded shocks, and shocks that are not spanned by asset returns, under the assumption of an intermediated market structure. Each bar is a different country's currency relative to the U.S. dollar; globally-traded shocks are measured using CCA for stock and sovereign bond returns.

The R^2 of such a regression is the fraction of variance in exchange rate explained by the globally-traded component, $var(g_{t+1})/var(\Delta s_{t+1})$. The regression residual is a direct estimate of the contribution of locally-traded and unspanned shocks to the depreciation rate, $\varepsilon_{t+1} = \ell_{t+1} + u_{t+1}$.

Combining with the results of regression (31), we can decompose variation in the depreciation rate into the contribution of globally-traded, locally-traded, and unspanned shocks. Specifically, we have $var(\beta^{g\prime}(\boldsymbol{w}^{*\prime}\boldsymbol{r}^*_{t+1}))$ for globally-traded shocks, and $var(\varepsilon_{t+1}) - var(u_{t+1})$ for locally-traded shocks, where u_{t+1} was obtained at the previous step from regression (31). Figure 5 reports these quantities as fraction of the variation in depreciation rate; the contributions mechanically add up to 1.

For all currencies, at least half of the variation in exchange rates is unspanned by asset returns. Globally-traded risks contribute up to 25% to variation in the depreciation rates (e.g., Australia and Canada), and frequently much less. These estimates should be seen as an upper bound on the role of globally-traded risks; remember that we include any pair of portfolios with correlation above 60%, far from the strict Definition 1.

In light of Proposition 4, the relatively modest role of globally-traded shocks $var(g_{t+1})$ in this intermediated market structure implies weak restrictions between exchange rate risks and IMRSs. In particular, models of this kind are capable of resolving the cyclicality and volatility puzzles, as illustrated in Figure 3 with the lower red cone.

5 Conclusion

In this paper, we propose a general framework for understanding how financial markets determine the behavior of exchange rates. Our theory accommodates many settings: complete or incomplete markets, arbitrary forms of market integration, or situations in which international financial trade happens through intermediaries. We characterize all restrictions on the behavior of exchange rates due to the absence of international arbitrage. These restrictions can be summarized by two conditions that share the simplicity of the complete market result while having richer implications.

We use these results to study many different market structures, which leads to new insights on the interaction of financial markets and the exchange rate. First, we show that the puzzles arising in settings with complete and integrated markets are still present when markets are either incomplete or imperfectly integrated. Second, we demonstrate that financial markets can be a source of shocks to the exchange rate without the extreme market segmentation featured in standard models emphasizing this mechanism. Finally, we show that the two roles of financial markets, facilitating risk sharing and transmitting shocks, are not mutually exclusive and both can play an important role in shaping exchange rate dynamics and avoid the currency puzzles. These results highlight that departing from the polar cases of market structure and incorporating realistic features of how financial markets are organized is a promising avenue for understanding the exchange rate.

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Online Appendix

A Portfolio algebra

A.1 Portfolio approximation

To maintain tractability, we follow Campbell and Viceira (2002) and approximate the log portfolio excess returns relative to a risk-free rate r_{ft}:

$$\begin{aligned} r_{p,t+1} - r_{ft} &= \log\left(\boldsymbol{w}_t' \exp(\boldsymbol{r}_{t+1} - r_{ft})\right) \\ &\approx \boldsymbol{w}_t'(\boldsymbol{r}_{t+1} - r_{ft}) + \frac{1}{2}\boldsymbol{w}_t' \operatorname{diag}(\boldsymbol{\Sigma}_t) - \frac{1}{2}\boldsymbol{w}_t'\boldsymbol{\Sigma}_t\boldsymbol{w}_t, \end{aligned} \tag{A1}$$

where $\boldsymbol{\Sigma}_t$ is the $N \times N$ variance-covariance matrix of log returns and $\boldsymbol{w}_t$ is the vector of portfolio weights such that $\boldsymbol{w}_t'\boldsymbol{\iota} = \sum_{i=1}^N w_{it} = 1$. Note that by convention we use the vector notation where, for example, $\boldsymbol{w}_t' \exp(\boldsymbol{r}_{t+1} - r_{ft}) = \sum_{i=1}^N w_{it} R_{i,t+1}/R_{ft}$ and $R_{i,t+1}/R_{ft} = \exp(r_{i,t+1} - r_{ft})$.

The approximation in (A1) allows us to represent portfolios returns as linear combination of log returns. Importantly, it is stable by recombination, leading to the same result when applied in two steps or all at once for a portfolio of portfolios. The approximation becomes exact as time becomes continuous and the underlying data-generating process for returns converges to a purely diffusive stochastic process.

A.2 Two international portfolios

Two international portfolios are useful for the derivation of our main results.

Carry trade. One zero-cost portfolio, often referred to as carry, entails taking long and short positions in related assets:

$$R_{\text{carry},t+1} = R_{t+1} - R^*_{t+1} \cdot S_{t+1}/S_t, \tag{A2}$$

where R_{t+1} and R^*_{t+1} denote asset returns in levels and S_t denotes the level of the exchange rate.

Traditionally, traded assets are taken to be domestic and foreign risk-free (one-period) bonds. But carry does not have to be limited to that. For instance, Lustig, Stathopolous, and Verdelhan (2019) consider long-term bonds. More generally, one could use any pair of assets, e.g. risky assets that are close to each other with $corr_t(r_{t+1}, r^*_{t+1}) \approx 1$. The key characteristic of the carry trade is that it exposes the arbitrageur to currency risk.

Lemma 2. *The conversion from foreign to home returns in the carry portfolio introduces exposure to currency risk,* $\widetilde{r}_{\text{carry},t+1} = \widetilde{r}_{t+1} - \widetilde{r}^*_{t+1} - \widetilde{\Delta s}_{t+1}$.

Proof. To apply the log approximation in equation (A1), we convert the zero-cost portfolio (A2) to a funded portfolio by adding a unit position in the risk-free asset:

$$R_{p,t+1} \equiv R_{\text{carry},t+1} + R_{f,t} = R_{t+1} - R^*_{t+1} \cdot S_{t+1}/S_t + R_{f,t}.$$

The portfolio $R_{p,t+1}$ corresponds to the weights $w_1 = 1$ in the domestic risky asset R_{t+1}, $w_2 = -1$ in the foreign risky asset converted to local currency, $R^*_{t+1} \cdot S_{t+1}/S_t$, and $w_3 = 1$ in the domestic risk-free asset with $\boldsymbol{w}_t = (w_1, w_2, w_3)'$. These weights lead to an expression for the log gross return relative to the risk-free rate $R_{p,t+1}/R_{f,t}$:

$$\begin{aligned} r_{\text{carry},t+1} &\equiv r_{p,t+1} - r_{ft} \\ &= r_{t+1} - r^*_{t+1} - \Delta s_{t+1} + cov_t(r_{t+1} - r^*_{t+1} - \Delta s_{t+1}, r^*_{t+1} + \Delta s_{t+1}). \end{aligned} \tag{A3}$$

This confirms the claim in Lemma 2 about the carry return innovation $\widetilde{r}_{\text{carry},t+1}$ as the covariance term is part of the expected return at time t. ■

Thus, we conclude that the carry portfolio return is exposed to the exchange rate risk. Note that in the special case of carry based on risk-free assets, $r_{\text{carry},t+1} = r_{ft} - r^*_{ft} - \Delta s_{t+1} - var_t(\Delta s_{t+1})$, and thus $\widetilde{r}_{\text{carry},t+1} = -\widetilde{\Delta s}_{t+1}$, that is the carry risk equals the negative of the exchange rate risk. This property holds for any carry with risky assets such that $\widetilde{r}_{t+1} = \widetilde{r}^*_{t+1}$.

Differential carry. The fact that carry is exposed to currency risk prompts us to consider another zero-cost portfolio, labeled as differential carry, which is long one unit of the domestic asset, and short one unit of the foreign asset, financed at the respective risk-free rates:

$$R_{\text{diff},t+1} = (R_{t+1} - R_{ft}) - (R^*_{t+1} - R^*_{ft}) \cdot S_{t+1}/S_t. \tag{A4}$$

Intuitively, this portfolio does not introduce additional currency exposure because, in contrast to carry, only the foreign excess return is converted to the home currency. We demonstrate this formally in the following lemma.

Lemma 3. *The conversion from foreign- to home-currency returns in the differential carry does not introduce additional exposure to currency risk,* $\widetilde{r}_{\text{diff},t+1} = \widetilde{r}_{t+1} - \widetilde{r}^*_{t+1}$.

Proof. To apply the log approximation in equation (A1), we convert the zero-cost portfolio (A4) to a funded portfolio by adding a unit position in the risk-free asset:

$$R_{p,t+1} \equiv R_{\text{diff},t+1} + R_{f,t} = R_{t+1} - (R^*_{t+1} - R^*_{ft}) \cdot S_{t+1}/S_t.$$

The portfolio $R_{p,t+1}$ corresponds to the weights $w_1 = 1$ in the domestic risky asset R_{t+1}, $w_2 = -1$ in the foreign risky asset converted to local currency, $R^*_{t+1} \cdot S_{t+1}/S_t$, and $w_3 = 1$ in the foreign risk-free asset converted to local currency, $R^*_{ft} \cdot S_{t+1}/S_t$, with $\boldsymbol{w}_t = (w_1, w_2, w_3)'$. These weights lead to an expression for the relative log return:

$$\begin{aligned} r_{\text{diff},t+1} &\equiv r_{p,t+1} - r_{ft} \\ &= (r_{t+1} - r_{ft}) - (r^*_{t+1} - r^*_{ft}) + cov_t(r^*_{t+1}, r_{t+1} - r^*_{t+1} - \Delta s_{t+1}). \end{aligned} \tag{A5}$$

Thus, only the covariance of the foreign return with the exchange rate has a material impact on portfolio performance, not the shocks to the exchange rate. ■

The disappearance of exchange rate risk for the differential carry return is in part due to our portfolio approximation. In Appendix Section H, we confirm that this approximation is very tight empirically. We compare the excess returns on various stock portfolios and sovereign bonds in their origin currency, $R^*_{t+1} - R^*_{ft}$, and after conversion to home currency (USD), $(R^*_{t+1} - R^*_{ft})S_{t+1}/S_t$. The correlation between the two monthly series is always around 99.9%. Also, see Daniel, Hodrick, and Lu (2017, Online Appendix C) and Chernov, Dahlquist, and Lochstoer (2023, Internet Appendix II). Appendix D derives exact versions of our results which do not rely on the portfolio return approximation in (A1).

B Globally-traded shocks

We first provide a technical construction of globally-traded shocks. To see specific examples of globally-traded shocks in various market structure, skip to Appendix B.2.

B.1 Identification and construction

We show how to identify a basis for the set of globally-traded shocks $\boldsymbol{\epsilon}^g_{t+1}$ from the base returns $\boldsymbol{r}_{t+1}$ and $\boldsymbol{r}^*_{t+1}$. We drop time indices and tildes for parsimony.

First, recall what canonical correlation analysis does.

Definition 2. *Canonical correlation analysis identifies pairs $(\boldsymbol{\lambda}_i, \boldsymbol{\lambda}^*_i)$ for $i = 1, .., K$ for some K such that:*

1. $\forall i \quad var(\boldsymbol{\lambda}'_i \boldsymbol{r}) \neq 0$;
2. $\forall i \quad \boldsymbol{\lambda}'_i \boldsymbol{r} = \boldsymbol{\lambda}^{*\prime}_i \boldsymbol{r}^*$;
3. $\forall i \neq j \quad \boldsymbol{\lambda}'_i \boldsymbol{r} \perp \boldsymbol{\lambda}'_j \boldsymbol{r}$;
4. $\forall r \in span(\boldsymbol{r}), \forall r^* \in span(\boldsymbol{r}^*)$: *if* $\forall i \ r \perp \boldsymbol{\lambda}'_i \boldsymbol{r}$ *and* $r^* \perp \boldsymbol{\lambda}^{*\prime}_i \boldsymbol{r}^*$, *then* $r \neq r^*$.

Condition 1 says that each canonical component is non-degenerate, that is, has non-zero variance. Condition 2 says that each component can be expressed using only local returns and only foreign returns. Condition 3 indicates that the various components must be orthogonal to each other. Condition 4 indicates that the analysis exhausts all possible components: one cannot find pairs of portfolios satisfying condition 2 in the space orthogonal to the canonical components.

We show that this procedure identifies a basis of $\boldsymbol{\epsilon}^g$.

Lemma 4. *The collection $(\boldsymbol{\lambda}'_1 \boldsymbol{r}, .., \boldsymbol{\lambda}'_K \boldsymbol{r})$ identified by canonical correlation analysis is a basis of $\boldsymbol{\epsilon}^g$.*

Proof. By Definition 1 and by point 2 of Definition 2, all $\boldsymbol{\lambda}'_i \boldsymbol{r}$ are in $\boldsymbol{\epsilon}^g$. Thus, $span(\boldsymbol{\lambda}'_1 \boldsymbol{r}, .., \boldsymbol{\lambda}'_K \boldsymbol{r}) \subset \boldsymbol{\epsilon}^g$.

Let us show the other direction. Assume that $\exists r \in \boldsymbol{\epsilon}^g$ such that $r \notin span(\boldsymbol{\lambda}'_1 \boldsymbol{r}, .., \boldsymbol{\lambda}'_K \boldsymbol{r})$. We can orthogonalize r to all the $\boldsymbol{\lambda}'_i \boldsymbol{r}$ and obtain $\hat{r}$. Because $\hat{r}$ is a linear combination of r and $\boldsymbol{\lambda}'_i \boldsymbol{r}$ which are all in $\boldsymbol{\epsilon}^g$, it is also in $\boldsymbol{\epsilon}^g$, and therefore in $span(\boldsymbol{r})$ and $span(\boldsymbol{r}^*)$. By substituting $\hat{r}$ for both r and r^* in point 4 of Definition 2, we immediately obtain a contradiction. Therefore, $span(\boldsymbol{\lambda}'_1 \boldsymbol{r}, .., \boldsymbol{\lambda}'_K \boldsymbol{r}) \supset \boldsymbol{\epsilon}^g$, and the two sets are equal. By point 3 of Definition 2, $dim(span(\lambda'_1 r, \ldots \lambda'_K r)) = K$, so $(\lambda'_1 r, \ldots \lambda'_K r)$ is indeed a basis of $\boldsymbol{\epsilon}^g$. ■

Furthermore, we relate the dimension of $\boldsymbol{\epsilon}^g$ to the rank of covariance matrices of $\boldsymbol{r}$, $\boldsymbol{r}^*$, and the two combined.

Lemma 5. *The dimension of $\boldsymbol{\epsilon}^g$ is:*

$$dim(\boldsymbol{\epsilon}^g) = rank(var(\boldsymbol{r})) + rank(var(\boldsymbol{r}^*)) - rank(var(\boldsymbol{r}, \boldsymbol{r}^*)).$$

Proof. Observe that, by construction,

$$dim(span(\boldsymbol{r}, \boldsymbol{r}^*)) = \underbrace{dim(span(\boldsymbol{\epsilon}^g)) + dim(span(\boldsymbol{\epsilon}))}_{=dim(span(\boldsymbol{r}))} + \underbrace{dim(span(\boldsymbol{\epsilon}^*))}_{=dim(span(\boldsymbol{r}^*)) - dim(\boldsymbol{\epsilon}^g)}.$$

Therefore,

$$dim(\boldsymbol{\epsilon}^g) = dim(span(\boldsymbol{r})) + dim(span(\boldsymbol{r}^*)) - dim(span(\boldsymbol{r}, \boldsymbol{r}^*)),$$

which yields the result. ■

B.2 Examples of globally-traded, locally-traded and unspanned risks

B.2.1 Alternative market structures with bonds and equities

Consider a world with four assets:

1. a home risk-free bond with return r_{ft} in home currency;
2. a foreign risk-free bond with return r^*_{ft} in foreign currency;
3. a home equity index with return $r_{H,t+1}$ in home currency;
4. a foreign equity index with return $r^*_{F,t+1}$ in home currency,

and a nominal exchange rate depreciation rate Δs_{t+1}.

We consider a variety of international market structures with various subsets of these assets, assumingthat an international intermediary can trade all of them so that:

$$\boldsymbol{r}^I_{t+1} = (r_{ft}, r^*_{ft} + \Delta s_{t+1}, r_{H,t+1}, r^*_{F,t+1} + \Delta s_{t+1}). \tag{A6}$$

Case 1 Consider the non-integrated market where H contains the home bond and home equity index, while F contains the foreign bond and foreign equity index. In this case the sets of return in $H \cap I$ and $F \cap I$ in local currency are respectively given by:

$$\boldsymbol{r}_{t+1} = (r_{ft}, r_{H,t+1}) \qquad \text{and} \qquad \boldsymbol{r}^*_{t+1} = (r^*_{ft}, r^*_{F,t+1}).$$

By Definition 1, assuming the two equity indexes do not have perfectly correlated returns in their respective currencies, $|corr_t(r_{H,t+1}, r^*_{F,t+1})| < 1$, the set of globally-traded shocks in this case is empty, $\boldsymbol{\epsilon}^g_{t+1} = \emptyset$. Otherwise, there exists $\lambda \neq 0$ such that $\widetilde{r}_{H,t+1} = \lambda \widetilde{r}^*_{F,t+1}$, and thus $\boldsymbol{\epsilon}^g_{t+1} = \widetilde{r}_{H,t+1} = \lambda \widetilde{r}^*_{F,t+1}$. This may be the case when both $r_{H,t+1}$ and $r^*_{F,t+1}$ are driven by the same fundamental shock (e.g., relative productivity). Note importantly that these are returns expressed in different currencies. If $\widetilde{\Delta s}_{t+1} = \delta r_{H,t+1}$, then the exchange rate is spanned by the globally-traded shock, $\widetilde{\Delta s}_{t+1} \in \boldsymbol{\epsilon}^g_{t+1}$; otherwise, it is not.[31]

Case 2 Now allow both households to trade both risk-free bonds so that:

$$\boldsymbol{r}_{t+1} = (r_{ft}, r^*_{ft} + \Delta s_{t+1}, r_{H,t+1}) \qquad \text{and} \qquad \boldsymbol{r}^*_{t+1} = (r^*_{ft}, r_{ft} - \Delta s_{t+1}, r^*_{F,t+1}).$$

In this case, independently of the statistical properties of the equity returns $(r_{H,t+1}, r^*_{F,t+1})$, the exchange rate is a globally-traded risk, $\widetilde{\Delta s}_{t+1} \in \boldsymbol{\epsilon}^g_{t+1}$. Therefore, $\widetilde{\Delta s}_{t+1} = g_{t+1}$ and $u_{t+1} = \ell_{t+1} = 0$ according to decomposition (9). This is because both households can trade the exchange rate risk.

[31] If $\nexists \lambda : \widetilde{r}_{H,t+1} \neq \lambda \widetilde{r}^*_{F,t+1}$ and $\widetilde{\Delta s}_{t+1}$ is spanned by $(\widetilde{r}_{H,t+1}, \widetilde{r}^*_{F,t+1})$, then $\widetilde{\Delta s}_{t+1} = \ell_{t+1}$ is a locally-traded shock.

If, additionally, the exchange rate is spanned by $(r_{H,t+1}, r^*_{F,t+1})$, then $\boldsymbol{\epsilon}^g_{t+1} = (\widetilde{\Delta s}_{t+1}, \widetilde{r}_{H,t+1}, \widetilde{r}^*_{F,t+1})$ irrespective of the correlation between $r_{H,t+1}$ and $r^*_{F,t+1}$. This is because using $r^*_{ft} + \Delta s_{t+1}$ and $r_{H,t+1}$, the home households can construct a portfolio that spans $r^*_{F,t+1}$, making it a globally-traded risk; and symmetrically for $r_{H,t+1}$. This situation arises naturally when $r_{H,t+1}$ reflects home productivity, $r^*_{F,t+1}$ reflects foreign productivity, and Δs_{t+1} in proportional to relative productivity. However, the presence of additional shocks may disrupt such spanning.

Case 3 Consider now that both households can trade every asset such that

$$\boldsymbol{r}_{t+1} = (r_{ft}, r^*_{ft} + \Delta s_{t+1}, r_{H,t+1}, r^*_{F,t+1} + \Delta s_{t+1}), \tag{A7}$$
$$\boldsymbol{r}^*_{t+1} = (r^*_{ft}, r_{ft} - \Delta s_{t+1}, r^*_{F,t+1}, r_{H,t+1} - \Delta s_{t+1}). \tag{A8}$$

This is the case of a fully integrated market. According to Definition 1, such a case always features a full set of globally-traded shocks $\boldsymbol{\epsilon}^g_{t+1} = (\widetilde{\Delta s}_{t+1}, \widetilde{r}_{H,t+1}, \widetilde{r}^*_{F,t+1})$ irrespective of statistical properties of the returns. This is because excess returns for every asset are a globally traded. Therefore, fully integrated markets imply $\widetilde{\Delta s}_{t+1} = g_{t+1}$ and $u_{t+1} = \ell_{t+1} = 0$ according to decomposition (9).

Case 4 Consider now three asymmetric partially integrated scenarios. In all of these cases, the foreign household only trades the foreign bond and the foreign equity index, $\boldsymbol{r}^*_{t+1} = (r^*_{ft}, r^*_{F,t+1})$.

(i) The home household trades both bonds and the home equity, $\boldsymbol{r}_{t+1} = (r_{ft}, r^*_{ft} + \Delta s_{t+1}, r_{H,t+1})$. In this case, assuming $r^*_{F,t+1}$ is not spanned by $(\Delta s_{t+1}, r_{H,t+1})$, there are no globally-traded shocks, $\boldsymbol{\epsilon}^g_{t+1} = \emptyset$. However, the exchange rate is spanned by local returns, as the home household has an access to a carry trade with risk-free bonds, and thus $\widetilde{\Delta s}_{t+1} = \ell_{t+1}$ and $u_{t+1} = g_{t+1} = 0$ according to decomposition (9).

(ii) The home household trades the home bond and both equity indexes, $\boldsymbol{r}_{t+1} = (r_{ft}, r_{H,t+1}, r^*_{F,t+1} + \Delta s_{t+1})$. Similarly, assuming $r^*_{F,t+1}$ is not spanned by $(\Delta s_{t+1}, r_{H,t+1})$, there are no globally-traded shocks, $\boldsymbol{\epsilon}^g_{t+1} = \emptyset$. This case is more interesting, however, because the same risky asset (the foreign equity index) is traded by both households. Nonetheless, there are still no globally-traded risks as $r^*_{F,t+1}$ is not traded in local currency, but instead is converted to home currency using the exchange rate, which itself is not a globally-traded risk. Furthermore, the exchange rate is not spanned in this case even for the home household as they do not have access to a carry trade with risk-free bonds. However, it is spanned by the joint set of asset returns $\boldsymbol{r}^I_{t+1} = (\boldsymbol{r}_{t+1}, \boldsymbol{r}^*_{t+1})$, and therefore it is a locally-traded risk, $\widetilde{\Delta s}_{t+1} = \ell_{t+1}$ and $g_{t+1} = u_{t+1} = 0$ according to decomposition (9).

(iii) The home households can trade all asset as in (A7). Assuming $|corr_t(r_{H,t+1}, r^*_{F,t+1})| < 1$, the set of globally-traded shocks in this case is $\boldsymbol{\epsilon}^g_{t+1} = \widetilde{r}_{F,t+1}$. The home households can trade the exchange rate risk using the carry trade as in (i), making it a locally-traded risk ($\widetilde{\Delta s}_{t+1} = g_{t+1} + \ell_{t+1}$ and $u_{t+1} = 0$), and hence they can also trade the $r^*_{F,t+1}$ risk in home currency by combining the foreign equity and the carry trade.

Note that any partial integration makes the exchange rate risk locally traded, but generally not globally-traded without both risk-free bonds being globally-traded, as we illustrate further in the next case.

Case 5 Finally, we consider the case of a symmetrically partially integrated markets where each household can hold both equities, but not the bond of the other country:

$$\boldsymbol{r}_{t+1} = (r_{ft}, r_{H,t+1}, r^*_{F,t+1} + \Delta s_{t+1}) \qquad \text{and} \qquad \boldsymbol{r}^*_{t+1} = (r^*_{ft}, r^*_{F,t+1}, r_{H,t+1} - \Delta s_{t+1}).$$

Irrespective, of the statistical properties of the risky returns, $\widetilde{r}_{H,t+1} - \widetilde{r}^*_{F,t+1} - \Delta s_{t+1} \in \boldsymbol{\epsilon}^g_{t+1}$. Furthermore, if $(r_{H,t+1}, r^*_{F,t+1}, \Delta s_{t+1})$ has statistically a full rank (i.e., there is no linear dependence between these random variables), then $\boldsymbol{\epsilon}^g_{t+1} = (\widetilde{r}_{H,t+1} - \widetilde{r}_{F,t+1} - \Delta s^*_{t+1})$, and the exchange rate is not spanned by globally-traded shocks. However, it is locally-traded due to partial integration with risky assets. That is: $\widetilde{\Delta s}_{t+1} = g_{t+1} + \ell_{t+1}$ and $u_{t+1} = 0$ according to decomposition (9). This is because each household can construct a carry trade using the two risky assets, and the return on this carry trade is the globally-traded shock (see Appendix A). No other random variable is spanned by both $\boldsymbol{r}_{t+1}$ and $\boldsymbol{r}^*_{t+1}$. By induction, the presence of n risky assets traded in common by home and foreign households will introduce $n-1$ globally-traded risks (excess returns), but in general do not make the exchange rate a globally-traded risk. This contrasts with the cases 2 and 3, where the exchange rate risk was immediately globally traded due to a carry trade strategy with two risk-free bonds.

B.2.2 Partial integration: a commonly traded asset

Consider an asset i with return $R_{i,t+1}$ in home currency and corresponding return $R^*_{i,t+1} = R_{i,t+1}\frac{S_t}{S_{t+1}}$ after conversion to foreign currency. The corresponding log returns are $r_{i,t+1} = \log R_{i,t+1}$ and $r^*_{i,t+1} = r_{i,t+1} - \Delta s_{t+1}$. Therefore, when $R_{i,t+1}$ is available to the home household and $R^*_{i,t+1}$ is available to the foreign household (that is, $i \in H \cap F$), then a simple one-asset pair of portfolios $r_{p,t+1} = r_{i,t+1}$ and $r^*_{p,t+1} = r^*_{i,t+1}$ spans the exchange: $\Delta s_{t+1} = r_{p,t+1} - r^*_{p,t+1}$ and, therefore, $u_{t+1} = 0$ in equation (10).[32]

If asset i is the only asset traded in common (that is, $\{i\} = H \cap F$), then there is no globally-traded risk (assuming $\widetilde{\Delta s}_{t+1} \neq 0$), that is $\boldsymbol{\epsilon}^g_{t+1} = \emptyset$ and $g_{t+1} = 0$ in equation (9). Indeed, in this case, the only risk that can be spanned in H is $\widetilde{r}_{i,t+1}$, and the only risk that can be spanned in F is $\widetilde{r}_{i,t+1} - \widetilde{\Delta s}_{t+1} \neq \widetilde{r}_{i,t+1}$.

Traded excess return Consider now a traded excess return $R_{i,t+1} - R_{j,t+1}$ on a zero-cost portfolio, with the foreign currency excess return given by $R^*_{i,t+1} - R^*_{j,t+1} = (R_{i,t+1} - R_{j,t+1})S_t/S_{t+1}$.[33] Following the same steps as in the proof of Lemma 3 in Appendix A.2, one can show that the risk of the log excess return is given by $\widetilde{r}_{i,t+1} - \widetilde{r}_{j,t+1}$ in the home currency and by $\widetilde{r}^*_{i,t+1} - \widetilde{r}^*_{j,t+1} = (\widetilde{r}_{i,t+1} - \widetilde{\Delta s}_{t+1}) - (\widetilde{r}_{j,t+1} - \widetilde{\Delta s}_{t+1}) = \widetilde{r}_{i,t+1} - \widetilde{r}_{j,t+1}$ in the foreign currency. Therefore, unlike a traded asset, a traded excess return is a globally-traded risk.

[32] Note that it is of no significance whether asset i is the same stock traded by both households or there are two assets with identical returns (in a common currency) traded separately in the home and foreign markets (e.g., as might be the case with ADRs or stocks like Royal Dutch Shell), as long as there is an intermediary with access to both assets in the latter case.

[33] One example of such return can be a commodity forward. In general, the return is defined as $R_{i,t+1} = (P_{i,t+1} + D_{i,t+1})/P_{it}$, where P is the price of the asset and D is the dividend. Then the pay-out on a commodity forward is given by the following excess return: $R_{i,t+1} - R_{j,t+1} = P_{i,t+1} - P_{j,t+1}$, where $P_{it} = P_{jt} = 1$, $D_{i,t+1} = D_{j,t+1} = 0$, and $P_{i,t+1}$ is the realized spot commodity price next period and $P_{j,t+1} = F_{it}$ is the forward commodity price.

One way to construct an excess return for a traded asset i is to subtract a risk-free rate, $R_{j,t+1} = R_{ft}$. However, to obtain a globally-traded risk, the risk-free rate should be in the same currency in both markets, so that $R^*_{j,t+1} = R_{ft}S_t/S_{t+1}$. The same works with a risk-free rate in foreign currency, in which case $R_{j,t+1} = R^*_{ft}S_{t+1}/S_t$ and $R^*_{j,t+1} = R^*_{ft}$. In the former case, the globally-traded risk is $\widetilde{r}_{i,t+1}$, and in the latter it is $\widetilde{r}_{i,t+1} - \widetilde{\Delta s}_{t+1}$. Note that this requires that a risk-free bond (either in one or the other currency) is also traded by both households, in addition to risky asset i.

B.2.3 Globally-traded risks vs. common risks

It may be intuitively appealing to think about sources of common variation in domestic and foreign assets as globally-traded shocks. There is a critical difference between such intuition and the formal definition of globally-traded shocks, which requires replication of the exposure to such shock solely using assets of either country.

As an example, consider economies with N risky assets each, with all of these assets having exposure to a shock ϵ_{t+1}: $\widetilde{r}_{i,t+1} = \alpha_i\epsilon_{t+1} + \beta_i\epsilon_{i,t+1}$, and $\widetilde{r}^*_{i,t+1} = \alpha^*_i\epsilon_{t+1}$, and ϵ_{t+1} and all the $\epsilon_{i,t+1}$ are orthogonal to each other. If $\beta_i = 0$ for at least one domestic asset i, then ϵ_{t+1} is a globally-traded shock. If none of the β_i are equal to zero, then ϵ_{t+1} is not a globally-traded shock because it cannot be isolated from $\widetilde{\boldsymbol{r}}_{t+1}$. It is only when $N \to \infty$ than one can construct a portfolio of $\widetilde{r}_{i,t+1}$ to isolate ϵ_{t+1} via diversification.

C Derivation of the main results

C.1 Proof of propositions in Section 2

Proof of Proposition 1 Consider one of the globally-traded shocks, ϵ^g_{t+1}. By definition 1, there exist two portfolios $r_{p,t+1} \in \boldsymbol{r}_{p,t+1}$ and $r^*_{p,t+1} \in \boldsymbol{r}^*_{p,t+1}$ such that $\epsilon^g_{t+1} = \widetilde{r}_{p,t+1} = \widetilde{r}^*_{p,t+1}$.

The differential carry portfolio of Lemma 3 is in $\boldsymbol{r}^I_{p,t+1}$. In this case, the portfolio has no risk because $\widetilde{r}_{p,t+1} = \widetilde{r}^*_{p,t+1}$. The shocks to foreign and domestic return perfectly offset each other. By assumption 2, the portfolio must have expected returns equal to the risk-free rate. That is:

$$0 = E_t[r_{p,t+1} - r_{ft}] - E_t[r^*_{p,t+1} - r^*_{ft}] - cov_t(r^*_{p,t+1}, \Delta s_{t+1}) + cov_t(r^*_{p,t+1}, r_{p,t+1} - r^*_{p,t+1}).$$

The last term is equal to 0 because $r_{p,t+1} - r^*_{p,t+1}$ has no risk. We can replace the first two terms by covariances with the SDFs using the domestic and foreign Euler equations (5) and (6),

$$\begin{aligned} 0 = &-cov_t(m_{t+1}, r_{p,t+1}) - \frac{1}{2}var_t(r_{p,t+1}) + cov_t(m^*_{t+1}, r^*_{p,t+1}) + \frac{1}{2}var_t(r^*_{p,t+1}) \\ &- cov_t(r^*_{p,t+1}, \Delta s_{t+1}). \end{aligned}$$

Remembering that both portfolio shocks are equal to ϵ^g_{t+1}, this expression simplifies to:

$$cov_t(m^*_{t+1} - m_{t+1} - \Delta s_{t+1}, \epsilon^g_{t+1}) = 0.$$

This equation is equivalent to

$$cov(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1} - \widetilde{\Delta s}_{t+1}, \epsilon^g_{t+1}) = 0,$$

which implies equation (11). Furthermore, under log-normality, this condition is equivalent to the equality of respective conditional expectation, $E\big(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}|\epsilon^g_{t+1}\big) = E\big(\widetilde{\Delta s}_{t+1}|\epsilon^g_{t+1}\big)$.

Because this result holds for any globally-traded shock, it must also hold in terms of multivariate projections on all globally-traded shocks $\boldsymbol{\epsilon}^g_{t+1}$. ■

Proposition 1 without risk-free assets The differential carry portfolio from Lemma 3 that we use in the proof of Proposition 1 relies on the availability of both risk-free rates in the intermediaries set of returns r^I_{t+1}. Proposition 1 generalizes to environments without risk-free assets. A globally-traded risk requires the existence of a jointly spanned excess return in $H \cap I$ and $F \cap I$ in respective local currencies. In our baseline setting, we obtained excess returns by subtracting the respective local risk-free rates from a given spanned return, forming a leg of the differential carry portfolio. More generally, we need to focus on excess returns of zero-cost portfolios which we denote with $r_{z,t+1}$ and $r^*_{z,t+1}$, respectively. By analogy with the definition of $\boldsymbol{r}_{p,t+1}$, we have:

$$\boldsymbol{r}_{z,t+1} = \big\{ r_{z,t+1} = \log\big(\boldsymbol{w}'_t \exp(\boldsymbol{r}_{t+1})\big) \,\big|\, \exists \boldsymbol{w}_t \in \mathbb{R}^N : \boldsymbol{w}'_t \boldsymbol{\iota} = 0 \big\}. \tag{A9}$$

Then a globally-traded shock is defined as $\boldsymbol{\epsilon}^g_{t+1} \equiv \widetilde{\boldsymbol{r}}_{z,t+1} \cap \widetilde{\boldsymbol{r}}^*_{z,t+1}$, where formally $\widetilde{\boldsymbol{r}}_{z,t+1}$ is the set of all spanned risks of zero-cost portfolios $\widetilde{r}_{z,t+1}$. Proposition 1 then holds under this generalized definition of globally-traded shocks $\boldsymbol{\epsilon}^g_{t+1}$. Note that in the presence of risk-free assets, this definition coincides with Definition 1. In the absence of risk-free assets, we need to find a pair of assets in each set $H \cap I$ and $F \cap I$ that have a perfectly correlated excess return in respective local currencies (see examples in Appendix B.2).

Proof of Proposition 2 Consider the carry portfolio of Lemma 2 constructed with a pair of portfolios $r_{p,t+1} \in \boldsymbol{r}_{p,t+1}$ and $r^*_{p,t+1} \in \boldsymbol{r}^*_{p,t+1}$ which span the exchange rate (equation (10)). In this case, the portfolio has no risk because $\widetilde{r}_{p,t+1} - \widetilde{r}^*_{p,t+1} = \widetilde{\Delta s}_{t+1}$. The shocks to foreign and domestic return perfectly offset exchange rate risk. By assumption 2, the portfolio must have expected returns equal to the risk-free rate. This corresponds to

$$0 = E_t[r_{p,t+1} - r^*_{p,t+1} - \Delta s_{t+1}] + cov_t(r_{p,t+1} - r^*_{p,t+1} - \Delta s_{t+1}, r^*_{p,t+1} + \Delta s_{t+1}).$$

The covariance term is equal to 0, because $r_{p,t+1} = r^*_{p,t+1} - \Delta s_{t+1}$ has no risk. We can replace expected returns using the domestic and foreign Euler equations (5) and (6):

$$\begin{aligned} E_t \Delta s_{t+1} = r_{ft} - cov_t(m_{t+1}, r_{p,t+1}) - \frac{1}{2} var_t(r_{p,t+1}) \\ - r^*_{ft} + cov_t(m^*_{t+1}, r^*_{p,t+1}) + \frac{1}{2} var_t(r^*_{p,t+1}) = \delta_t. \end{aligned}$$

We replace $\widetilde{r}_{p,t+1} = \widetilde{r}^*_{p,t+1} + \widetilde{\Delta s}_{t+1}$:

$$\begin{aligned} E_t \Delta s_{t+1} &= r_{ft} - r^*_{ft} - cov_t(m_{t+1}, \Delta s_{t+1}) + cov_t(m^*_{t+1} - m_{t+1}, r^*_{p,t+1}) \\ &\quad + \frac{1}{2} var_t(r^*_{p,t+1}) - \frac{1}{2} var_t(\Delta s_{t+1}) - \frac{1}{2} var_t(r^*_{p,t+1}) - cov_t(\Delta s_{t+1}, r^*_{p,t+1}) \\ &= r_{ft} - r^*_{ft} - cov_t(m_{t+1}, \Delta s_{t+1}) - \frac{1}{2} var_t(\Delta s_{t+1}) \\ &\quad + cov_t(m^*_{t+1} - m_{t+1} - \Delta s_{t+1}, r^*_{p,t+1}). \end{aligned}$$

This proves part b) of Proposition 2. If markets are fully integrated, all asset returns are globally-traded shocks, and proposition 1 implies that the last term in the equation above is equal to 0, part a) of the proposition. If the exchange rate is not spanned by asset returns, it is impossible to construct a trade with expected returns involving the expected depreciation rate that is risk-free. Therefore, no-arbitrage imposes no restriction on the expected depreciation rate. ■

Proposition 2 without risk-free assets Note that no step in the proof requires the existence of risk-free assets, as the carry trade in Lemma 2 builds on a pair of arbitrary portfolios $r_{p,t+1} \in \boldsymbol{r}_{p,t+1}$ and $r^*_{p,t+1} \in \boldsymbol{r}^*_{p,t+1}$ that spans the exchange rate risk. The rates r_{ft} and r^*_{ft} in the definition of δ_t in (10) (and in asset pricing equations (5) and (6)) can in general be replaced by shadow risk-free rates defined as $r_{ft} \equiv -E_t m_{t+1} - \frac{1}{2} var_t(m_{t+1})$ and similarly for r^*_{ft}, even when risk-free assets are not available.

One situation must be handled separately: when the replicating portfolio features $\widetilde{r}_{p,t+1} = \widetilde{\Delta s}_{t+1}$ and hence $\widetilde{r}^*_{p,t+1} = 0$ when $r^*_{p,t+1} = r^*_{ft}$ is unavailable (or, symmetrically, without r_{ft}). Note, however, that $r_{p,t+1}$ with $\widetilde{r}_{p,t+1} = \widetilde{\Delta s}_{t+1}$ is equivalent to a foreign-currency risk-free asset traded by the domestic household, and therefore we can use this asset to define a shadow foreign risk-free rate: $r_{p,t+1} = r^*_{ft} + \Delta s_{t+1}$, where r^*_{ft} can be backed out using the home household's asset pricing condition (5):

$$r^*_{ft} + E_t \Delta s_{t+1} \equiv E_t\, r_{p,t+1} = r_{ft} - \frac{1}{2} var_t(\Delta s_{t+1}) - cov_t(m_{t+1}, \Delta s_{t+1}),$$

which coincides with the prediction of Proposition 2. Note that, again, in the absence of risk-free assets, r_{ft} and r^*_{ft} are shadow rates. The contrast with the general case is that in this case both of them are defined by the properties of domestic SDF m_{t+1}.

Proof of Proposition 3 Recall our decomposition of the depreciation rate into spanned and unspanned components, $\Delta s_{t+1} = E_t \Delta s_{t+1} + g_{t+1} + \ell_{t+1} + u_{t+1}$. Because $g_{t+1} + \ell_{t+1}$ is spanned by asset returns, there exists $r_{p,t+1} \in \boldsymbol{r}_{p,t+1}$ and $r^*_{p,t+1} \in \boldsymbol{r}^*_{p,t+1}$ such that $\widetilde{r}_{p,t+1} - \widetilde{r}^*_{p,t+1} = g_{t+1} + \ell_{t+1}$. Using Lemma 2, we see that the risk of this portfolio is equal to $var_t(u_{t+1})$. We apply Assumption 3 to relate this risk to the expected return of the carry trade.

$$\begin{aligned}&\left| E_t[r_{p,t+1} - r^*_{p,t+1} - \Delta s_{t+1}] + cov_t(r_{p,t+1} - r^*_{p,t+1} - \Delta s_{t+1}, r^*_{p,t+1} + \Delta s_{t+1}) + \frac{1}{2} var_t(u_{t+1}) \right| \\ &\quad \le B\sqrt{var_t(u_{t+1})}.\end{aligned}$$

Examining the terms in the left-hand-side, we have:

$$\begin{aligned} E_t[r_{p,t+1} - r^*_{p,t+1} - \Delta s_{t+1}] &= \delta_t - E_t[\Delta s_{t+1}] = -\psi_t, \\ cov_t(r_{p,t+1} - r^*_{p,t+1} - \Delta s_{t+1}, r^*_{p,t+1} + \Delta s_{t+1}) &= cov(-u_{t+1}, \widetilde{r}_{p,t+1} + u_{t+1}) \\ &= -var(u_{t+1}), \end{aligned}$$

where the last equality uses the fact that $u_{t+1} \perp (\boldsymbol{\epsilon}^g_{t+1}, \boldsymbol{\epsilon}_{t+1}, \boldsymbol{\epsilon}^*_{t+1}) \ni \widetilde{r}_{p,t+1}$. Plugging these two results on the left-hand side of the inequality above, we obtain:

$$\left| \psi_t + \frac{1}{2} var_t(u_{t+1}) \right| \le B\sqrt{var_t(u_{t+1})}.$$

Finally, by construction, the pair of portfolios $(r_{p,t+1}, r^*_{p,t+1})$, maximizes $R^2 = 1 - \frac{var_t(\Delta s_{t+1} - r_{p,t+1} + r^*_{p,t+1})}{var_t(\Delta s_{t+1})} = 1 - \frac{var_t(u_{t+1})}{var_t(\Delta s_{t+1})}$, and hence $var_t(u_{t+1}) = (1-R^2)var_t(\Delta s_{t+1})$. ■

C.2 Propositions 1 and 2 are sufficient for no-arbitrage

We show that the results of Propositions 1 and 2 are not only necessary for the absence of international arbitrage — Assumption 2 — but also sufficient. Specifically we show the following.

Proposition 5. *If:*

1. *Assumption 1 holds,*
2. $E\left(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1} | \epsilon^g_{t+1}\right) = E\left(\widetilde{\Delta s}_{t+1} | \epsilon^g_{t+1}\right)$,
3. *(a) either* $\exists r^s_{p,t+1} \in \boldsymbol{r}_{p,t+1}, r^{s*}_{p,t+1} \in \boldsymbol{r}^*_{p,t+1}$ *such that* $\widetilde{\Delta s}_{t+1} = \widetilde{r}^s_{p,t+1} - \widetilde{r}^{s*}_{p,t+1}$ *and*

$$E_t \Delta s_{t+1} = r_{ft} - r^*_{ft} - cov_t(m^*_{t+1}, \Delta s_{t+1}) + \frac{1}{2} var_t(\Delta s_{t+1}) + cov_t\left(m^*_{t+1} - m_{t+1} - \Delta s_{t+1}, r_{p,t+1}\right),$$

   (b) or $\forall r^s_{p,t+1} \in \boldsymbol{r}_{p,t+1}, r^{s*}_{p,t+1} \in \boldsymbol{r}^*_{p,t+1}, \widetilde{\Delta s}_{t+1} \neq \widetilde{r}^s_{p,t+1} - \widetilde{r}^{s*}_{p,t+1}$,

then there are no arbitrage opportunities in international markets, that is Assumption 2 holds.

Proof: We proceed by contradiction. Assume that there exists an international arbitrage:

$$\exists r^I_{p,t+1} \in \boldsymbol{r}^I_{p,t+1}: \; var_t(r^I_{p,t+1}) = 0 \;\text{ and }\; E_t r^I_{p,t+1} \neq r_{ft},$$

and denote $\boldsymbol{w}$ and $\boldsymbol{w}^*$ the set of weights of such a portfolio on $\boldsymbol{r}_{t+1}$ and $\boldsymbol{r}^*_{t+1} + \Delta s_{t+1}$. Remember that $\boldsymbol{\iota}'_N \boldsymbol{w} + \boldsymbol{\iota}'_{N^*} \boldsymbol{w}^* = 1$. We consider the cases of 3a and 3b in turn.

Assume condition 3a holds. As a preliminary, note that this condition is equivalent to saying that a carry portfolio constructed with $r^s_{p,t+1}$ and $r^{s*}_{p,t+1}$ has no risk and no average excess return. Consider the following portfolio: long $\boldsymbol{w}'\boldsymbol{r}_{t+1}$, long $(\boldsymbol{\iota}'_{N^*}\boldsymbol{w}^*)\, r^s_{p,t+1}$, long $\boldsymbol{w}^{*\prime}\left(\boldsymbol{r}^*_{t+1} + \Delta s_{t+1}\right)$, short $(\boldsymbol{\iota}'_{N^*}\boldsymbol{w}^*)\,(r^{s*}_{p,t+1} + \Delta s_{t+1})$. Because we have added and subtracted the same total weights, the new weights still add up to 1, so this is still a portfolio. Because this portfolio combines two risk-free portfolios — our assumed arbitrage and the risk-free carry trade — its expected return is the sum of the two expected returns, that is $E_t\, r^I_{p,t+1}$. The total weight on foreign returns in the portfolio are $\boldsymbol{\iota}'_{N^*}\boldsymbol{w}^* - \boldsymbol{\iota}'_{N^*}\boldsymbol{w}^* = 0$. Therefore, this trade is a differential carry portfolio (defined in Appendix A.2). Because it has no risk, its home and foreign legs offset each other. They form a globally-traded shock. Applying condition 1 in the proposition and Lemma 3 leads immediately to the result that the portfolio return must equal the risk-free rate. This contradicts the assumption that $E_t\, r^I_{p,t+1} \neq r_{ft}$.

Now assume that condition 3b holds. If $\boldsymbol{\iota}'_{N^*}\boldsymbol{w}^* \neq 0$, then the arbitrage portfolio has a non-zero loading on Δs_{t+1} in addition to the home and foreign returns. Because the portofolio is riskless this implies that we can find a pair of home and foreign returns that spans the depreciation rate, a contradiction of condition 3b. If $\boldsymbol{\iota}'_{N^*}\boldsymbol{w}^* = 0$, then the two legs of the portfolio in their home currency perfeclty offset each other. Their innovations constitute a globally-traded shock and applying condition 1 in the proposition jointly with Lemma 3 implies that the arbitrage portfolio has 0 expected return, a contradiction as well. ■

C.3 Proofs of the results of Section 3

Proof of Proposition 4 Apply the Cauchy-Schwarz inequalty to $\widetilde{\Delta s}_{t+1} - g_{t+1}$ and $(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}) - g_{t+1}$:

$$\begin{aligned} &cov_t(\widetilde{\Delta s}_{t+1} - g_{t+1}, (\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}) - g_{t+1})^2 \\ &\qquad \leq var_t(\widetilde{\Delta s}_{t+1} - g_{t+1}) var_t((\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}) - g_{t+1}). \end{aligned} \tag{A10}$$

Proposition 1 implies that both $\widetilde{\Delta s}_{t+1} - g_{t+1}$ and $(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}) - g_{t+1}$ are orthogonal to g_{t+1}. Therefore:

$$\begin{aligned} var_t(\widetilde{\Delta s}_{t+1} - g_{t+1}) &= var_t(\Delta s_{t+1}) - var_t(g_{t+1}) \\ var_t((\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}) - g_{t+1}) &= var_t(m^*_{t+1} - m_{t+1}) - var_t(g_{t+1}) \\ cov_t(\widetilde{\Delta s}_{t+1} - g_{t+1}, (\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}) - g_{t+1}) &= cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}) - var_t(g_{t+1}) \end{aligned}$$

When $var_t((\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}) - g_{t+1}) > 0$, plugging in and rearranging the terms in equation (A10) gives equation (26). When $var_t((\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}) - g_{t+1}) = 0$, we get $var_t(g_{t+1}) = cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1})$, with the first term being no greater than $var_t(\Delta s_{t+1})$ by definition of g_{t+1}, yielding (27). ■

Maximum $var_t(g_{t+1})$ without puzzles We ask what is the largest value of $var_t(g_{t+1})$ so that the Data point falls within the red cone in Figure 3. This is the value such that the frontier of the cone reaches exactly that point. The parabola is defined by taking condition (26) with equality. The Data point is characterized by the empirical value of $var_t(\Delta s_{t+1})$ and cyclicality 0. Plugging in, this corresponds to solving:

$$var_t(\Delta s_{t+1}) = var_t(g_{t+1}) + \frac{var_t(g_{t+1})^2}{var_t(m^*_{t+1} - m_{t+1}) - var_t(g_{t+1})} \tag{A11}$$

Dividing by $var_t(\Delta s_{t+1})$ gives:

$$\begin{aligned} 1 = \frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})} + \left(\frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})}\right)^2 \Big/ \left(\frac{var_t(m^*_{t+1} - m_{t+1})}{var_t(\Delta s_{t+1})} - \frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})}\right) \\ \frac{1 - \frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})}}{\left(\frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})}\right)^2} = 1\Big/\left(\frac{var_t(m^*_{t+1} - m_{t+1})}{var_t(\Delta s_{t+1})} - \frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})}\right) \\ \frac{\left(\frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})}\right)^2}{1 - \frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})}} = \frac{var_t(m^*_{t+1} - m_{t+1})}{var_t(\Delta s_{t+1})} - \frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})} \\ \frac{\frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})}}{1 - \frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})}} = \frac{var_t(m^*_{t+1} - m_{t+1})}{var_t(\Delta s_{t+1})}. \end{aligned}$$

Because $x/(1-x) = a \Leftrightarrow x = a/(1+a)$, this gives:

$$\frac{var_t(g_{t+1})}{var_t(\Delta s_{t+1})} = \frac{\frac{var_t(m^*_{t+1} - m_{t+1})}{var_t(\Delta s_{t+1})}}{1 + \frac{var_t(m^*_{t+1} - m_{t+1})}{var_t(\Delta s_{t+1})}}. \tag{A12}$$

Notice that the deeper the volatility puzzle in complete markets, that is, the larger the value of $var_t(m^*_{t+1} - m_{t+1})/var_t(\Delta s_{t+1})$ is, the larger the possible contribution of globally-traded risks to the exchange rate $var_t(g_{t+1})/var_t(\Delta s_{t+1})$.

D Exact non-linear version of the propositions

Our proofs rely on returns being log-normal and a log-linearization of portfolio returns as described in Appendix A.1. In this section we address the question of how the propositions change without distributional assumption and approximation.

D.1 A version of Proposition 1

Consider two portfolios, domestic with returns $R_{p,t+1}$ and foreign with returns $R^*_{p,t+1}$ such that their innovations coincide with one of the globally-traded shocks, that is, they can be represented as $R_{p,t+1} = \alpha_t + R^*_{p,t+1}$. The local Euler equations imply:

$$\begin{aligned} E_t(M_{t+1}R_{p,t+1}) &= 1, \\ E_t(M^*_{t+1}R^*_{p,t+1}) &= 1. \end{aligned}$$

The local Euler equations can be re-written as

$$E_t(R_{p,t+1}) = R_{ft} - cov_t\left(\frac{M_{t+1}}{E_t(M_{t+1})}, R_{p,t+1}\right) \tag{A13}$$

$$E_t(R^*_{p,t+1}) = R^*_{ft} - cov_t\left(\frac{M^*_{t+1}}{E_t(M^*_{t+1})}, R^*_{p,t+1}\right). \tag{A14}$$

Now consider an intermediary whose SDF expressed in the units of domestic currency, M^I_{t+1}, satisfies the following Euler equations:

$$E_t(M^I_{t+1}R_{ft}) = 1, \tag{A15}$$

$$E_t(M^I_{t+1}R^*_{ft}S_{t+1}/S_t) = 1, \tag{A16}$$

$$E_t(M^I_{t+1}(R_{p,t+1} - R_{ft})) = 0, \tag{A17}$$

$$E_t(M^I_{t+1}(R^*_{p,t+1} - R^*_{ft})S_{t+1}/S_t) = 0. \tag{A18}$$

The intermediary trades the zero-cost differential carry portfolio:

$$\begin{aligned} 0 &= E_t\left(M^I_{t+1}\left[(R_{p,t+1} - R_{ft}) - (R^*_{p,t+1} - R^*_{ft}) \cdot S_{t+1}/S_t\right]\right) \\ &= E_t\left(M^I_{t+1}\left[(R_{p,t+1} - R_{ft}) - (R_{p,t+1} - \alpha_t - R^*_{ft}) \cdot S_{t+1}/S_t\right]\right) \\ &= E_t\left(M^I_{t+1}\left[(R_{p,t+1} - R_{ft})(1 - S_{t+1}/S_t) + (\alpha_t + R^*_{ft} - R_{ft}) \cdot S_{t+1}/S_t\right]\right). \end{aligned}$$

Replace the risk-free rates by the expressions from the local Euler equations (A13) and (A14), divide the equation by $E_t(M^I_{t+1})$, and define

$$cov^I_t\left(\frac{S_{t+1}}{S_t}, R^*_{p,t+1}\right) \equiv E_t\left(\frac{M^I_{t+1}}{E_t(M^I_{t+1})}(R^*_{p,t+1} - R^*_{ft})\frac{S_{t+1}}{S_t}\right) - \underbrace{E_t\left(\frac{M^I_{t+1}}{E_t(M^I_{t+1})}(R^*_{p,t+1} - R^*_{ft})\right)}_{0} \cdot E_t\left(\frac{M^I_{t+1}}{E_t(M^I_{t+1})}\frac{S_{t+1}}{S_t}\right),$$

$$E^I_t\left(\frac{S_{t+1}}{S_t}\right) \equiv E_t\left(\frac{M^I_{t+1}}{E_t(M^I_{t+1})}\frac{S_{t+1}}{S_t}\right) = \frac{R_{ft}}{R^*_{ft}}.$$

Then

$$0 = -cov^I_t\left(\frac{S_{t+1}/S_t}{E^I_t(S_{t+1}/S_t)}, R^*_{p,t+1}\right) + cov_t\left(\frac{M^*_{t+1}}{E_t(M^*_{t+1})} - \frac{M_{t+1}}{E_t(M_{t+1})}, R^*_{p,t+1}\right).$$

(We replace $R_{p,t+1}$ with $R^*_{p,t+1}$ in the cov^I_t term because of our assumption about $R_{p,t+1}$ and $R^*_{p,t+1}$.)

This expression implies

$$cov_t\left(\frac{M^*_{t+1}}{E_t(M^*_{t+1})} - \frac{M_{t+1}}{E_t(M_{t+1})} - \frac{S_{t+1}/S_t}{E_t(S_{t+1}/S_t)}, R^*_{p,t+1}\right) = \underbrace{cov^I_t\left(\frac{S_{t+1}/S_t}{E^I_t(S_{t+1}/S_t)}, R^*_{p,t+1}\right) - cov_t\left(\frac{S_{t+1}/S_t}{E_t(S_{t+1}/S_t)}, R^*_{p,t+1}\right)}_{W} \tag{A19}$$

As we noted in section A.1, the log approximation that we use in Proposition 1 becomes exact if time is continuous and the data-generating process converges to a pure diffusion. Under such scenario, the covariance in the equation above is observable, and, thus, has the same value with and without risk adjustment (via M^I_{t+1}). As a result, $W = 0$. Also, each Arrow-Debreu claim makes the corresponding state globally-traded. For such a globally-traded risk $W = 0$.

Further, the projection result depends on the knowledge of intermediary's SDF, M^I_{t+1} via the term with cov^I_t. The log approximation relies only on the existence of such SDF, due to Assumption 2, and allows us to be agnostic about its actual values.

The first term in the second line is equal to $R^*_{ft} \cdot QRP_t$, where QRP_t is the quanto-implied risk premium of Kremens and Martin (2019). Its role in our paper is different from that of these authors. They use it to approximate the currency risk premium assigned by the intermediary, $R^*_{ft}E_t(S_{t+1}/S_t) - R_{ft} = -R_{ft}cov_t(M^I_{t+1}, R^*_{ft} \cdot S_{t+1}/S_t)$. Here it measures the gap in projections of the relative discount factor and the depreciation rate on globally-traded risks.

D.2 A version of Proposition 2

Consider two portfolios, domestic with returns $R_{p,t+1}$ and foreign with returns $R^*_{p,t+1}$ such that their innovations span the exchange rate, that is, they can be represented as $R_{p,t+1} = \alpha_t + R^*_{p,t+1}S_{t+1}/S_t$. The local Euler equations (A13) and (A14) hold for these portfolios. Also, we consider a (domestically funded) intermediary whose SDF, M^I_{t+1}, satisfies the following Euler equations:

$$
\begin{aligned}
E_t(M^I_{t+1}R_{p,t+1}) &= 1,\\
E_t(M^I_{t+1}R^*_{p,t+1}S_{t+1}/S_t) &= 1.
\end{aligned}
$$

First, we show that $\alpha_t = 0$. The intermediary can form a zero-cost carry portfolio:

$$
0 = E_t\left(M^I_{t+1}\left[R_{p,t+1} - R^*_{p,t+1}S_{t+1}/S_t\right]\right) = \alpha_t E_t\left(M^I_{t+1}\right).
$$

Therefore, the expected return of the carry portfolio is equal to zero:

$$
\begin{aligned}
0 &= E_t\left(R_{p,t+1} - R^*_{p,t+1}S_{t+1}/S_t\right)\\
&= R_{ft} - cov_t\left(\frac{M_{t+1}}{E_t(M_{t+1})}, R_{p,t+1}\right)\\
&- E_t(R^*_{p,t+1})E_t(S_{t+1}/S_t) - cov_t(R^*_{p,t+1}, S_{t+1}/S_t)\\
&= R_{ft} - cov_t\left(\frac{M_{t+1}}{E_t(M_{t+1})}, R_{p,t+1}\right) - cov_t(R^*_{p,t+1}, S_{t+1}/S_t)\\
&- \left[R^*_{ft} - cov_t\left(\frac{M^*_{t+1}}{E_t(M^*_{t+1})}, R^*_{p,t+1}\right)\right]E_t(S_{t+1}/S_t),
\end{aligned}
$$

where we substituted the local Euler equations (A13) and (A14) in lines 2 and 5, respectively. This equation implies the currency risk premium:

$$
\begin{aligned}
R^*_{ft}E_t\left(\frac{S_{t+1}}{S_t}\right) - R_{ft} &= -cov_t\left(\frac{M_{t+1}}{E_t(M_{t+1})}, R^*_{p,t+1}\frac{S_{t+1}}{S_t}\right)\\
&+ cov_t\left(\frac{M^*_{t+1}}{E_t(M^*_{t+1})}, R^*_{p,t+1}\right)E_t\left(\frac{S_{t+1}}{S_t}\right) - cov_t\left(R^*_{p,t+1}, \frac{S_{t+1}}{S_t}\right)\\
&= -R^*_{ft}cov_t\left(\frac{M_{t+1}}{E_t(M_{t+1})}, \frac{S_{t+1}}{S_t}\right) - cov_t\left(\frac{M_{t+1}}{E_t(M_{t+1})}, [R^*_{p,t+1} - R^*_{ft}]\frac{S_{t+1}}{S_t}\right)\\
&+ cov_t\left(\frac{M^*_{t+1}}{E_t(M^*_{t+1})}E_t\left(\frac{S_{t+1}}{S_t}\right) - \frac{S_{t+1}}{S_t}, R^*_{p,t+1}\right)\\
&= \underbrace{-R_{ft}cov_t\left(M_{t+1}, R^*_{ft}\frac{S_{t+1}}{S_t}\right)}_{\text{complete markets}}\\
&+ \underbrace{cov_t\left(\frac{M^*_{t+1}}{E_t(M^*_{t+1})} - \frac{M_{t+1}}{E_t(M_{t+1})} - \frac{S_{t+1}/S_t}{E_t(S_{t+1}/S_t)}, R^*_{p,t+1}\right)E_t\left(\frac{S_{t+1}}{S_t}\right)}_{A}\\
&- \underbrace{cov_t\left(\frac{M_{t+1}}{E_t(M_{t+1})}, [R^*_{p,t+1} - R^*_{ft}]\left[\frac{S_{t+1}}{S_t} - E_t\left(\frac{S_{t+1}}{S_t}\right)\right]\right)}_{B},
\end{aligned}
$$

where in the first line we take advantage of spanning and replace $R_{p,t+1}$ with $R^*_{p,t+1}S_{t+1}/S_t$; the third line is obtained from the first by adding and subtracting the leading term in line 3; the fourth line is obtained by combining the two terms in the second line; the 6th and 7th lines are obtained by adding and subtracting $cov_t(M_{t+1}/E_t(M_{t+1}), R^*_{p,t+1})$.

The term B in the seventh line is the domestic household's risk premium for quanto exposure and disappears in the log-normal approximation. Also, $B = 0$ if $R^*_{p,t+1}$ happens to be R^*_{ft}, that is, domestic household can trade foreign risk-free bond. The term A in the sixth line is equal to zero in this case as well.

Next, if financial markets are integrated then the innovation to $R^*_{p,t+1}$ is a globally-traded shock. Then, equation (A19) from the non-linear version of Proposition 1 implies that

$$A = W \cdot E_t(S_{t+1}/S_t).$$

As is the case for Proposition 1, the log approximation treats this term as close to zero.

It might appear that the departure from log-normality in the case of integrated markets leads to two extra terms, A and B. In fact, when markets are integrated $A - B$ can be simplified to a term with a single source of departures from zero. Indeed, we obtain

$$\begin{aligned}
\frac{A-B}{E_t(S_{t+1}/S_t)} &= cov_t^I\left(\frac{S_{t+1}/S_t}{E_t^I(S_{t+1}/S_t)}, R^*_{p,t+1}\right) - cov_t\left(\frac{S_{t+1}/S_t}{E_t(S_{t+1}/S_t)}, R^*_{p,t+1}\right) \\
&\quad - cov_t\left(\frac{M_{t+1}}{E_t(M_{t+1})} - 1, [R^*_{p,t+1} - R^*_{ft}]\left[\frac{S_{t+1}/S_t}{E_t(S_{t+1}/S_t)} - 1\right]\right) \\
&= E_t\left(\frac{M^I_{t+1}}{E_t(M^I_{t+1})}[R^*_{p,t+1} - R^*_{ft}]\left[\frac{S_{t+1}/S_t}{E_t(S_{t+1}/S_t)} - 1\right]\right) \\
&\quad - E_t\left(\frac{M_{t+1}}{E_t(M_{t+1})}[R^*_{p,t+1} - R^*_{ft}]\left[\frac{S_{t+1}/S_t}{E_t(S_{t+1}/S_t)} - 1\right]\right).
\end{aligned}$$

Thus, $A - B$ is close to zero when the intermediary prices the globally-traded (quanto) risk the same way as the domestic household.

If there is no spanning, $R_{p,t+1} \neq \alpha_t + R^*_{p,t+1}S_{t+1}/S_t$, then it is impossible to find a risk-free strategy and derive restrictions on the currency risk premium.

E An Example of a Financial Sector

In this section, we study exchange rate dynamics within a complete specification of the financial sector. The goal of this analysis is two-fold. First, it illustrates the second role of financial markets as a conduit of shocks to the exchange rate. Second, we show that this second role complements the risk-sharing role: there exists a foundation for what happens in international financial markets that can justify any exchange rate process satisfying Propositions 1 and 2.

E.1 Setting

We maintain Assumption 1 about local financial markets. That is, the two fixed sets of returns $\boldsymbol{r}_{t+1}$ and $\boldsymbol{r}^*_{t+1}$ are priced by the household discount factors m_{t+1} and m^*_{t+1}. Then, we posit the remainder of international markets in such a way that Assumption 2 is satisifed.

First, we assume that there are noise traders with exogenous demand, D_t^{noise}, for the carry trade. That is, each period, they go long D_t^{noise} of the home currency in the foreign risk-free asset and go short the same amount in the domestic risk-free asset.

Second, we assume that, there are overlaping generations of intermediaries in the market. Each period, an intermediary enters the market with wealth $W_{0,t}$ and lives until the next period. The intermediary maximizes their utility of next period wealth,

$$E_t\left(\frac{1}{1-\gamma_t}W_{1,t+1}^{1-\gamma_t}\right).$$

The coefficient of relative risk aversion γ_t can be viewed as a stand-in for various frictions limiting the risk-bearing capacity of the intermediary (see Haddad and Muir, 2021 for a discussion of this interpretation). The intermediary has access to all assets in I, and takes these assets' returns as given in their optimization problem. We denote by D_t^I the optimal position of the intermediary in the carry trade.

The presence of these intermediaries guarantees that Assumption 2 is satisfied: if there were an arbitrage opportunity, the intermediary would choose an infinitely large position in the corresponding portfolio, which would be incompatible with it being an equilibrium.

Third, we include a market-clearing condition for the carry trade. Positions of noise traders and intermediaries must offset the imbalance of positions between households, that is, the net foreign assets NFA_t:

$$D_t^{\text{noise}} + D_t^I + NFA_t = 0. \tag{A20}$$

Taking the household SDFs as given is equivalent to assuming that the assets are provided perfectly elastically to the intermediary. This explains why we do not need market-clearing conditions for assets other than the carry trade nor specification of the demand from noise traders for these assets.

All these relations hold period by period and characterize changes in exchange rate Δs_{t+1}. We need an additional restriction at infinite horizon to close the model and determine the asymptotic level of the exchange rate, and hence its current level s_t. We focus on a generic form of such a restriction originating from the combination of the budget constraint and the transversality condition:

$$\Delta s_{t+1} - E_t\Delta s_{t+1} + (E_{t+1}-E_t)\left[\sum_{j=1}^{\infty}\beta^j\Delta s_{t+j+1}\right] = N_{x,t+1}, \tag{A21}$$

where $\beta < 1$ is a linearization constant and $N_{x,t+1} \equiv (E_{t+1}-E_t)\left[\sum_{j=0}^{\infty}\beta^j\Delta x_{t+j+1}\right]$ is a present value of innovations to Δx_t, a change in some real quantity x_t (for example, relative TFP across countries). By definition, $N_{x,t+1}$ is unpredictable. Appendix E.3 derives the restriction (A21) from the budget constraint.

An alternative approach to the asymptotic behavior of the exchange rate which some researchers have used is to impose stationarity of the exchange rate. This case is encompassed in equation (A21) by assuming $N_{x,t+1} = 0$ for all t and taking the limiting case of $\beta \to 1$, which implies

$$0 = (E_{t+1}-E_t)\left[\sum_{j=0}^{\infty}\Delta s_{t+j+1}\right] = (E_{t+1}-E_t)\left[\lim_{j\to\infty}s_{t+j} - s_t\right],$$

that is $\lim_{j\to\infty}E_t s_{t+j} = \bar{s}$ for all t, with $\bar{s}$ a constant.

E.2 Which exchange rate processes can be rationalized by financial shocks?

Because this model respects Assumptions 1 and 2, any equilibrium exchange rate process must satisfy Propositions 1 and 2. We now show the converse: there exist values for the inputs of the model — noise trader demand, intermediary risk aversion — that justify any exchange rate process satisfying the two propositions.

Proposition 6. *For any sequence of shocks $\{\zeta_t\}$ such that:*

1. *$proj(\zeta_{t+1}|\epsilon^g_{t+1}) = proj\left(m^*_{t+1} - m_{t+1}|\epsilon^g_{t+1}\right)$, and*
2. *ζ_{t+1} is not fully spanned by asset returns,*

there exist processes for noise trader demand $\{D^{noise}_t\}$ and intermediary risk aversion $\{\gamma_t\}$ such that an equilibrium exchange rate shock satisfies:

$$\widetilde{\Delta s}_{t+1} = \zeta_{t+1} \qquad \forall t. \tag{A22}$$

We prove this result in three steps. First, because of condition 2 in Proposition 6, Proposition 2 does not impose any restrictions on the expected depreciation $E_t\Delta s_{t+1}$ at any t. Yet, it has to satisfy the infinite-horizon restriction (A21). Thus, we derive a process for $\eta_t \equiv E_t\Delta s_{t+1}$ which satisfies this restriction at infinity in order to complete the candidate exchange process from equation (A22).[34] Second, we derive the intermediary demand D^I_t as a function of the characteristics of the exchange rate process. Finally, we clear the market.

Candidate depreciation rate satisfying equation (A21). We guess that exchange rate dynamics have the following structure:

$$\Delta s_{t+1} = \eta_t + \zeta_{t+1}, \tag{A23}$$

$$\eta_{t+1} = \rho\eta_t + \theta(\zeta_{t+1} - N_{x,t+1}), \tag{A24}$$

where $0 \leq \rho < 1$ and θ are two fixed parameters to be chosen.

Iterating forward the auto-regressive dynamics of η_t gives the innovation

$$(E_{t+1} - E_t)[\eta_{t+j}] = \rho^{j-1}\theta[\zeta_{t+1} - N_{x,t+1}].$$

We can then compute the present value of these innovations:

$$\begin{aligned}(E_{t+1} - E_t)\left[\sum_{j=1}^{\infty}\beta^j\eta_{t+j}\right] &= \sum_{j=1}^{\infty}\beta^j\rho^{j-1}\theta[\zeta_{t+1} - N_{x,t+1}] \\ &= \beta\theta\frac{1}{1-\beta\rho}[\zeta_{t+1} - N_{x,t+1}].\end{aligned}$$

[34] Note that given the definition of δ_t in equation (14), we can recover the wedge in equation (17) as $\psi_t = \eta_t - \delta_t$.

The restriction at infinite horizon of equation (A21) is satisfied if:

$$\frac{\beta\theta}{1-\beta\rho} = -1 \qquad \Leftrightarrow \qquad \theta = -\frac{1-\beta\rho}{\beta},$$

which pins down θ as a function of ρ.

The parameter ρ is free, and controls the persistence of expected depreciation shocks. As β and ρ approach 1, this exchange rate process converges to a random walk, and therefore it becomes difficult to distinguish it from a random walk in finite samples (Itskhoki and Mukhin, 2021). Choosing a value of ρ close to 1 will therefore generate empirically realistic dynamics.[35]

Intermediary demand. We derive the optimal portfolio choice of the intermediary. We apply the Campbell and Viceira (2002) approximation, summarized in Appendix A.1, for this portfolio problem. The approximation reduces to the intermediary optimization problem:

$$\max_{r_{p,t+1} \in \boldsymbol{r}^I_{p,t+1}} E_t(r_{p,t+1}) + \frac{1}{2}(1-\gamma_t) var_t(r_{p,t+1}) \tag{A25}$$

We represent returns $\boldsymbol{r}^I_{p,t+1}$ by the risk-free asset and a basis $\boldsymbol{r}^b_{t+1}$ with mean $E_t(\boldsymbol{r}^b_{t+1})$, and covariance matrix $\Sigma_{b,t}$. Then the optimal vector of portfolio weights on the risky assets $\boldsymbol{w}_t$ is:

$$\boldsymbol{w}_t = \frac{1}{\gamma_t}\Sigma^{-1}_{b,t}\left(E_t(\boldsymbol{r}^b_{t+1}) - r_{f,t+1}\boldsymbol{\iota} + \text{diag}(\Sigma_{b,t})/2\right) \tag{A26}$$

We construct a basis of all asset returns and the carry trade. First, we can represent both the domestic and foreign asset spaces using assets that load on the locally-traded and globally-traded shocks. For the home country, returns on such assets are:

$$\begin{cases} r_{f,t} + \boldsymbol{\epsilon}^g_{t+1} - cov_t(m_{t+1}, \boldsymbol{\epsilon}^g_{t+1}) - var_t(\boldsymbol{\epsilon}^g_{t+1})/2, \\ r_{f,t} + \boldsymbol{\epsilon}_{t+1} - cov_t(m_{t+1}, \boldsymbol{\epsilon}_{t+1}) - var_t(\boldsymbol{\epsilon}_{t+1})/2, \end{cases} \tag{A27}$$

using the log-normal Euler equation (5). For the foreign country, we similarly have:

$$\begin{cases} r^*_{f,t} + \boldsymbol{\epsilon}^g_{t+1} - cov_t(m^*_{t+1}, \boldsymbol{\epsilon}^g_{t+1}) - var_t(\boldsymbol{\epsilon}^g_{t+1})/2, \\ r^*_{f,t} + \boldsymbol{\epsilon}^*_{t+1} - cov_t(m^*_{t+1}, \boldsymbol{\epsilon}^*_{t+1}) - var_t(\boldsymbol{\epsilon}^*_{t+1})/2, \end{cases} \tag{A28}$$

using the Euler equation (6). We convert the corresponding excess returns to the home currency by using the differential carry portfolio of Lemma 3 (with the home risk-free rate constituting the domestic leg of the trade). This gives the home currency returns:

$$\begin{cases} r_{f,t} + \boldsymbol{\epsilon}^g_{t+1} - cov_t(m^*_{t+1} - \Delta s_{t+1}, \boldsymbol{\epsilon}^g_{t+1}) - var_t(\boldsymbol{\epsilon}^g_{t+1})/2 \\ r_{f,t} + \boldsymbol{\epsilon}^*_{t+1} - cov_t(m^*_{t+1} - \Delta s_{t+1}, \boldsymbol{\epsilon}^*_{t+1}) - var_t(\boldsymbol{\epsilon}^*_{t+1})/2. \end{cases} \tag{A29}$$

[35] Furthermore, in this limit, $var(\eta_{t+1})/var(\zeta_{t+1}) \to 0$ so Proposition 3 is generally satisfied as well for large values of ρ.

We combine these returns with the home assets and the carry portfolio based on risk free assets to obtain all international asset returns. This gives the basis:

$$\begin{cases} r_{f,t} + \boldsymbol{\epsilon}^g_{t+1} - cov_t(m_{t+1}, \boldsymbol{\epsilon}^g_{t+1}) - var_t(\boldsymbol{\epsilon}^g_{t+1})/2 \\ r_{f,t} + \boldsymbol{\epsilon}_{t+1} - cov_t(m_{t+1}, \boldsymbol{\epsilon}_{t+1}) - var_t(\boldsymbol{\epsilon}_{t+1})/2 \\ r_{f,t} + \boldsymbol{\epsilon}^g_{t+1} - cov_t(m^*_{t+1} - \Delta s_{t+1}, \boldsymbol{\epsilon}^g_{t+1}) - var_t(\boldsymbol{\epsilon}^g_{t+1})/2 \\ r_{f,t} + \boldsymbol{\epsilon}^*_{t+1} - cov_t(m^*_{t+1} - \Delta s_{t+1}, \boldsymbol{\epsilon}^*_{t+1}) - var_t(\boldsymbol{\epsilon}^*_{t+1})/2 \\ r_{f,t} + (r_{f,t} - r^*_{f,t}) - var_t(\Delta s_{t+1}) - \Delta s_{t+1}. \end{cases} \tag{A30}$$

Two sets of assets are exposed to the globally-traded shocks ϵ^g_{t+1}: the first and third line in (A30). The first assumption of Proposition 6 ensures that they are exactly equivalent: $cov_t(m_{t+1}, \boldsymbol{\epsilon}^g_{t+1}) = cov_t(m^*_{t+1} - \Delta s_{t+1}, \boldsymbol{\epsilon}^g_{t+1})$. Because they are redundant, we can eliminate one of the two. The second assumption of Proposition 6 ensures that the carry trade is not spanned by all other asset returns.

Applying the optimal portfolio formula of equation (A26), we obtain the portfolio weight on the carry trade:

$$w_{\text{carry},t} = \frac{UIP_t - \left(\boldsymbol{\beta}^{g\prime}_t cov_t(m_{t+1}, \boldsymbol{\epsilon}^g_{t+1}) + \boldsymbol{\beta}'_t cov_t(m_{t+1}, \boldsymbol{\epsilon}_{t+1}) + \boldsymbol{\beta}^{*\prime}_t cov_t(m^*_{t+1} - \zeta_{t+1}, \boldsymbol{\epsilon}^*_{t+1})\right)}{\gamma_t var_t(u_{t+1})}, \tag{A31}$$

where $UIP_t \equiv (r_{f,t} - r^*_{f,t}) - \frac{1}{2} var_t(\Delta s_{t+1}) - E_t \Delta s_{t+1} = (r_{f,t} - r^*_{f,t}) - \frac{1}{2} var_t(\Delta \zeta_{t+1}) - \eta_t$ (with η_t defined in equation (A24)) and the coefficients $\boldsymbol{\beta}^g$, $\boldsymbol{\beta}$, and $\boldsymbol{\beta}^*$ as well as the shock u_{t+1} come from regressing the exchange rate on globally-traded and locally-traded shocks,

$$\zeta_{t+1} = \boldsymbol{\beta}^{g\prime} \boldsymbol{\epsilon}^g_{t+1} + \boldsymbol{\beta}' \boldsymbol{\epsilon}_{t+1} + \boldsymbol{\beta}^{*\prime} \boldsymbol{\epsilon}^*_{t+1} + u_{t+1}. \tag{A32}$$

Intuitively, the numerator and denominator of equation (A31) correspond to the expected return and variance of a portfolio with a unit exposure to the carry trade hedged as much as possible using the other traded assets. Notice also that except γ_t, all the terms in equation (A31) can be constructed from the properties of m, m^*, asset returns (through $\boldsymbol{\epsilon}^g$, $\boldsymbol{\epsilon}$, and $\boldsymbol{\epsilon}^*$), and ζ_{t+1}.

The intermediary's demand is $D^I_t = W_{0,t} w_{\text{carry},t}$.

Market clearing. To have an equilibrium, all that is required is to satisfy the market-clearing condition (A20) each period. This corresponds to:

$$D^{\text{noise}}_t = -NFA_t \tag{A33}$$

$$- W_{0,t} \frac{UIP_t - \left(\boldsymbol{\beta}^{g\prime}_t cov_t(m_{t+1}, \boldsymbol{\epsilon}^g_{t+1}) + \boldsymbol{\beta}'_t cov_t(m_{t+1}, \boldsymbol{\epsilon}_{t+1}) + \boldsymbol{\beta}^{*\prime}_t cov_t(m^*_{t+1} - \zeta_{t+1}, \boldsymbol{\epsilon}^*_{t+1})\right)}{\gamma_t var_t(u_{t+1})}. \tag{A34}$$

All the terms of the right-hand-side only depend on the properties of ζ_{t+1} and primitives of the model excluding D^{noise}_t. This concludes the proof: if we assume that D^{noise}_t is equal to this right-hand-side expression, our conjectured exchange rate is an equilibrium.

E.3 Deriving the transversality condition

We show how to derive the condition of equation (A21). The transversality condition together with the sequence budget constraint within a country often gives the following relation:

$$b_t + \sum_{j=0}^{\infty} \beta^j s_{t+j} - \sum_{j=0}^{\infty} \beta^j x_{t+1} = 0 \quad \forall t, \tag{A35}$$

where b_t is net foreign assets. Taking a first difference, this leads to:

$$\Delta b_t + \sum_{j=0}^{\infty} \beta^j \Delta s_{t+j} - \sum_{j=0}^{\infty} \beta^j \Delta x_{t+1} = 0. \tag{A36}$$

We can take the conditional expectation of this expression from the point of view of date t and from the point of view of date $t+1$, respectively:

$$\Delta b_t + \Delta s_t - \Delta x_t + \beta E_t(\Delta s_{t+1} - \Delta x_{t+1}) + \sum_{j=2}^{\infty} \beta^j E_t(\Delta s_{t+j} - \Delta x_{t+j}) = 0 \tag{A37}$$

$$\Delta b_t + \Delta s_t - \Delta x_t + \beta \Delta s_{t+1} - \Delta x_{t+1} + \sum_{j=2}^{\infty} \beta^j E_{t+1}(\Delta s_{t+j} - \Delta x_{t+j}) = 0 \tag{A38}$$

Subtracting the second relation from the first one gives:

$$\beta \left[\Delta s_{t+1} - \Delta x_{t+1} - E_t(\Delta s_{t+1} - \Delta x_{t+1})\right] + (E_{t+1} - E_t)\left[\sum_{j=2}^{\infty} \beta^j \Delta s_{t+j} - \Delta x_{t+j}\right] \tag{A39}$$

Rearranging this expression leads to

$$(\Delta s_{t+1} - E_t \Delta s_{t+1}) + (E_{t+1} - E_t)\left[\sum_{j=1}^{\infty} \beta^j \Delta s_{t+j+1}\right] = (E_{t+1} - E_t)\left[\sum_{j=0}^{\infty} \beta^j \Delta x_{t+j+1}\right], \tag{A40}$$

the condition of equation (A21).

F Empirical analysis: additional results

CCA analysis for the undirected approach Table A1 reports the results. Each column represents a foreign country. For a given country, each row reports the canonical correlation between the assets of that country and the US assets, reported in order of importance, starting from the largest.

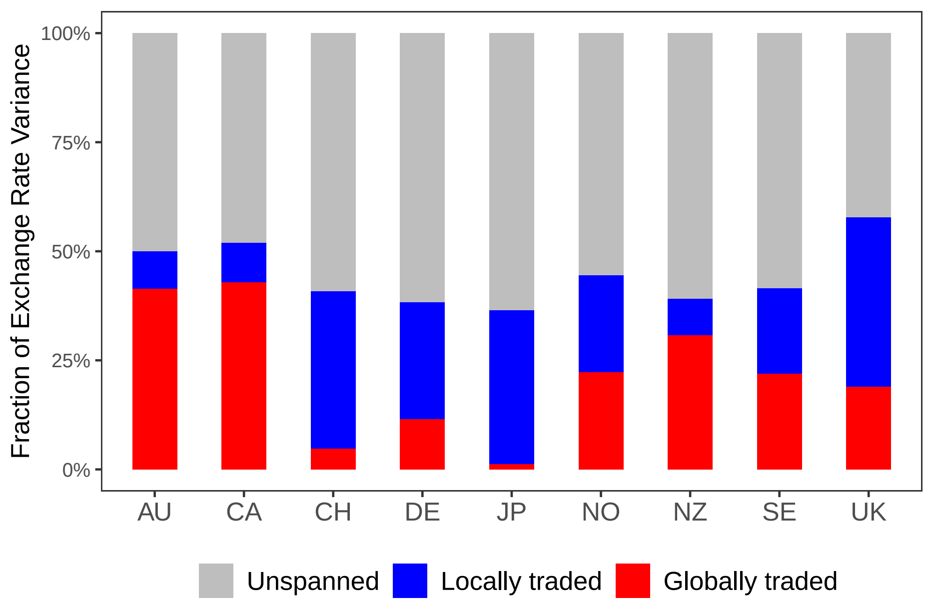
The values of the largest correlations range from 64% for New Zealand to 90% for Canada. In some cases lower ranked correlations are similar to the largest one, like for Canada or the UK. In other cases, the magnitude of correlation drops off quickly, e.g., for New Zealand or Norway. Strictly speaking, the evidence suggests that there are no globally-traded shocks amongst the assets that we consider.

Table A1: Maximally correlated shocks across asset markets

	AU	CA	DE	JP	NO	NZ	SE	CH	UK
Rank 1	75.27	89.82	83.07	75.01	79.47	64.31	78.33	82.95	85.87
Rank 2	65.00	85.06	74.17	64.43	63.49	53.95	65.72	62.62	78.70
Rank 3	61.16	83.44	66.70	58.71	57.14	41.73	59.57	60.41	73.55
Rank 4	57.04	78.79	64.90	51.31	45.86	35.98	55.55	56.12	68.02
Rank 5	51.01	76.82	52.80	46.81	41.74	31.44	49.63	52.32	65.85
Rank 6	41.67	70.79	44.19	46.62	33.59	25.33	38.94	46.83	62.21
Rank 7	34.19	62.84	42.30	41.94	26.88	22.99	38.20	41.16	55.83
Rank 8	31.57	56.20	36.66	39.57	25.80	14.58	33.82	35.18	51.39
N	419	395	419	419	406	419	414	419	419

The table reports the correlation in % between the maximally correlated portfolios of asset returns between the U.S. and each country. The successive pairs of portfolio are orthogonal to each other, and obtained by canonical correlation analysis. Domestic asset returns are in domestic currency; foreign asset returns are in foreign currency. Each column is for a different country's assets relative to the U.S. assets. The assets include government bonds of maturities between 2 and 10 years (obtained from various central banks) and various stock portfolios: the market (a combination of large and mid-cap stocks), value and growth portfolios, and 10 industry portfolios (from MSCI).

Figure A1: Decomposition of exchange rate innovations: directed approach



The figure reports the fraction of variance in exchange rates explained by globally-traded shocks, locally-traded shocks, and shocks that are not spanned by asset returns, under the assumption of an intermediated market structure described in Section 4. Each bar is a different country's currency relative to the U.S. dollar. Globally-traded shocks are measured using exposures to changes in VIX, GFC, and EBP.

Directed approach Instead of being agnostic about the nature of globally-traded shocks we rely on macroeconomic research and assume that they are known. Specifically, we take VIX, GFC (Miranda-Agrippino and Rey, 2020), and EBP (Gilchrist and Zakrajsek, 2012) as such shocks. This approach requires a strong assumption that portfolios of traded assets in each economy can span these shocks.

For each country, we regress its depreciation rate vs USD on these measures of globally-traded shocks. The R^2 from such a regression produce the fraction of the exchange rate variation due to globally-traded shocks. Next, we implement the regression in Equation (31) where the set of returns is complemented by the three global shocks to obtain the unspanned component. Naturally, it is going to be smaller than that in the previous section. The knowledge of the variation due to global and unspanned shocks delivers the variation due to locally-traded shocks.

Figure A1 reports the resulting decomposition of the variation in the exchange rate into the three types of shocks. The directed approach delivers somewhat larger contribution of global shocks, but qualitatively the conclusions are unchanged. The unspanned shocks represent the largest share of shocks. Contribution of the global shocks is the largest for Australia and Canada, which approach 50%.

G What do return data say about the exchange rate?

Our propositions have implications for various exercises using return data and the assumption of no-arbitrage only. We first show how to use our formal framework in this context, then discuss these implications and their relation to the literature.

An alternative interpretation of the framework Instead of representing the assets that investors in each country can access like in Section 1.1, the sets H and F can simply represent different subsets of return data expressed in different currencies. In this interpretation H is a set of assets for which we have data on returns $\boldsymbol{r}_{t+1}$ in the home currency, F is a set of assets for which we have data on returns $\boldsymbol{r}^*_{t+1}$ in the foreign currency. If we assume that neither of these sets of returns feature arbitrage opportunities, one can construct minimum (log) variance SDFs by combining asset returns in the spirit of Hansen and Jagannathan (1991): $m_{t+1} = \lambda' \boldsymbol{r}_{t+1}$ and $m^*_{t+1} = \lambda^{*\prime} \boldsymbol{r}^*_{t+1}$. This setting is equivalent to Assumption 1.

Furthermore, one might want to assume that there are no arbitrage opportunities when a combination of these assets is traded together. We call I this joint set of assets, so that this lack of arbitrage across datasets coincide with Assumption 2. For example, if one assumes that there are no arbitrage opportunities between the two sets of returns, this corresponds to $I = H \cup F$.

In this interpretation of the framework, Propositions 1 and 2 convey all restrictions between the return data, the corresponding minimum variance SDFs, and the exchange rate. Some combinations of assumptions about which data are observed coincide with assumptions about the market structures we study in the paper, and hence we can use the implications of our propositions for these structure. Still, note that this equivalence is only mathematical and the interpretation is different: here we are simply isolating different datasets as opposed to making assumptions about what different groups of investors have access to.

Recovering the exchange rate using local currency returns only. A first classic question is whether the exchange rate can be recovered from return data on local assets only.

For example, one might want to estimate SDFs for the dollar yield curve and the pound yield curve, respectively, and use their ratio to recover the exchange rate. This exercise, followed by Bansal (1997) and Backus, Foresi, and Telmer (2001) should work if markets are complete and local returns in each country span each state of the world. However, their estimates are at odds with the empirical behavior of the exchange rate. Chernov and Creal (2023) propose a model of SDFs including shocks that are not spanned by the yield curves that can price both yield curves and be consistent with the exchange rate.

We consider the general version of this exercise. H and F each contain distinct local assets in their local currency and by m and m^* are the minimum variance SDFs constructed from each set of asset returns. The assumption that there are no international arbitrage opportunities between all these assets corresponds to $I = H \cup F$.

Mathematically, this situation coincides exactly with the intermediated models we consider in Section 4. In this case, globally-traded shocks can be constructed using CCA on the local asset returns $\boldsymbol{r}_{t+1}$ and $\boldsymbol{r}^*_{t+1}$. Proposition 1 immediately says that only the global component of the exchange rate g_{t+1} is pinned down. If the local asset returns do not have common shocks, nothing can be said about exchange rate movements. With globally-traded shocks, the projection of $m^*_{t+1} - m_{t+1}$ on these shocks reveals the projection of Δs_{t+1} on these shocks. Furthermore, the local component ℓ_{t+1} and u_{t+1} can be arbitrary, so the exchange rate can have any amount of excess volatility above this projection-based component g_{t+1}.

Chernov and Creal (2023) find that at most 10% of exchange rate variation is explained by common shocks. Our empirical results suggest that, even after adding stocks to sovereign bonds, the variance of the globally-traded component g_{t+1} is small relative to the variance of the depreciation rate. This implies that a modest component of the exchange rate can be recovered by observing local returns and using the assumption of no-arbitrage alone.

This conclusion does not rule out that the depreciation rate might exhibit substantial correlation with specific assets, as long as it is through locally-traded shocks. Finally, to the extent that the exchange rate features an unspanned component — like we find empirically for stocks and bonds — Proposition 2 indicates that no-arbitrage does not pin down expected depreciation.

Finally, this conclusion also does not rule out that markets might be complete and integrated. Throughout this exercise we maintain the assumption that there exists an SDF pricing all assets (Assumption 2), which might coincide with the IMRSs of both home and foreign households.

Constructing pairs of SDFs satisfying the AMV. Alternatively, one might want to find SDFs that price the same assets in each currency so that $m^*_{t+1} - m_{t+1} = \Delta s_{t+1}$.[36] The minimum variance log SDFs solve this question, and our framework demonstrates why.

In this exercise, we have $H = F = I$, and $\boldsymbol{r}_{t+1} = \boldsymbol{r}^*_{t+1} + \Delta s_{t+1}$. We denote by m_{t+1} the minimum variance SDF pricing $\boldsymbol{r}_{t+1}$, and symmetrically m^*_{t+1} prices $\boldsymbol{r}^*_{t+1}$. Mathematically, this case coincides with a situation of fully integrated markets. Therefore, by Lemma 1, $\text{proj}(\widetilde{\Delta s}_{t+1} | \boldsymbol{\epsilon}^g_{t+1}) = \widetilde{\Delta s}_{t+1}$. Furthermore, because all asset returns are globally-traded shocks and hence the minimum variance SDFs are spanned by asset returns by construction, we also have $\text{proj}(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1} | \boldsymbol{\epsilon}^g_{t+1}) = \widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}$. So, Proposition 1 implies that shocks to the relative minimum variance SDF coincide with

[36] Constructing the exchange rate using observation of the same asset returns in their original currency and in another currency is in general trivial. Naturally, if one has "labels" on the return data, the depreciation rate is simply the ratio of a return in its own and in foreign currency. Even without labels, one can generically recover a unique depreciation rate so that the ratio of returns in home and foreign currency are equal across asset pairs.

shocks to the depreciation rate. Similarly, Proposition 2 implies that the means are equalized as well. Therefore, $\Delta s_{t+1} = m^*_{t+1} - m_{t+1}$.

Interestingly, Sandulescu, Trojani, and Vedolin (2021) show that, away from the log-normal case, this recovery result generalizes by focusing on minimum entropy SDFs: in a log-normal setting, mimimizing the entropy of the SDF is equivalent to minimize the variance of the log SDFs.[37]

Of course, this is not the only way to construct pairs of SDFs pricing the assets in two currencies. Under the assumption of no arbitrage, for any SDF m_{t+1} that prices $\boldsymbol{r}_{t+1}$, the SDF $m_{t+1} + \Delta s_{t+1}$ prices $\boldsymbol{r}^*_{t+1} = \boldsymbol{r}_{t+1} - \Delta s_{t+1}$.

This observation highlights that these recovery exercises do not necessarily lead to economically meaningful SDFs. For example, the world might be well-described by an intermediated market structure where a global intermediary trades all asset but households face arbitrary types of frictions. Then, all that this exercise is doing is recovering the projection of m^I_{t+1} and $m^{I*}_{t+1} = m^I_{t+1} + \Delta s_{t+1}$ on asset returns as opposed to the IMRSs of local investors. This observation echoes the conclusion of our main theoretical analysis: data on returns and the depreciation rate alone are in general not enough to identify the financial market structure.

H Evaluating the portfolio approximation

We report the correlation (in %) between the excess return on various stock portfolios —Table A2— and bonds of different maturities —Table A4— in their origin currency and converted to U.S. dollars. Tables A3 and A5 start from the U.S. version of these portfolios and converts them to foreign currency. These correlations are pervasively extremely high, almost all over 99.9%.

[37] Sandulescu, Trojani, and Vedolin (2021) also study cases with asymmetric data observations across countries, but with both risk-free assets observed in each country. Mathematically, this corresponds to the case of partial integration in which the exchange rate is spanned by globally-traded shocks.

Table A2: Correlation between excess returns converted in different currencies: foreign stocks

	AU	CA	DE	JP	NO	NZ	SE	CH	UK
Market	99.88	99.91	99.93	99.96	99.88	99.89	99.91	99.94	99.94
Value	99.92	99.94	99.93	99.96	99.89	99.85	99.92	99.93	99.94
Growth	99.82	99.88	99.93	99.96	99.9	99.93	99.92	99.95	99.94
Oil, Gas, Coal	99.89	99.93	NA	99.96	99.92	99.92	99.93	NA	99.96
Basic Material	99.84	99.94	99.94	99.95	99.88	99.91	99.91	99.96	99.91
Consumer Discretionary	99.91	99.95	99.93	99.96	99.92	99.94	99.94	99.93	99.96
Consumer Products, Services	99.88	99.96	99.97	99.95	NA	NA	99.94	99.93	99.98
Industrials	99.90	99.91	99.94	99.95	99.89	99.92	99.92	99.94	99.94
Health Care	99.91	99.97	99.96	99.96	NA	99.91	99.93	99.96	99.97
Financials	99.92	99.95	99.94	99.96	99.89	99.93	99.91	99.93	99.92
TeleCom	99.92	99.95	99.96	99.96	99.92	99.84	99.93	99.94	99.96
Technology	99.91	99.88	99.96	99.96	99.86	NA	99.94	99.95	99.95
Utilities	99.93	99.91	99.94	99.97	NA	99.93	NA	99.95	99.97

The table reports the correlation (in %) between the excess return on various stock indices expressed in their home currency and converted to U.S. dollar. The portfolios include the market (a combination of large and mid-cap stocks), value and growth portfolios, and 10 industry portfolios, all from MSCI. Each column corresponds to a different country.

Table A3: Correlation between excess returns converted in different currencies: U.S. stocks

	AU	CA	DE	JP	NO	NZ	SE	CH	UK
US Market	99.88	99.94	99.95	99.96	99.87	99.90	99.92	99.94	99.94
US Value	99.90	99.95	99.96	99.96	99.87	99.91	99.92	99.95	99.95
US Growth	99.87	99.93	99.94	99.96	99.88	99.90	99.92	99.94	99.94
US Oil, Gas, Coal	99.90	99.96	99.97	99.98	99.92	99.92	99.94	99.96	99.96
US Basic Material	99.81	99.90	99.92	99.95	99.85	99.88	99.90	99.93	99.93
US Consumer Discretionary	99.91	99.95	99.95	99.96	99.9	99.91	99.92	99.95	99.95
US Consumer Products, Services	99.93	99.97	99.97	99.97	99.92	99.93	99.94	99.96	99.96
US Industrials	99.86	99.93	99.94	99.96	99.84	99.90	99.90	99.94	99.94
US Health Care	99.90	99.96	99.95	99.96	99.88	99.93	99.93	99.95	99.96
US Financials	99.91	99.95	99.95	99.94	99.87	99.93	99.91	99.92	99.94
US TeleCom	99.87	99.93	99.95	99.95	99.9	99.91	99.93	99.96	99.95
US Technology	99.88	99.93	99.94	99.96	99.89	99.91	99.92	99.94	99.94
US Utilities	99.84	99.92	99.94	99.96	99.85	99.88	99.91	99.96	99.94

The table reports the correlation (in %) between the excess return on various stock indices expressed in the U.S. dollars and converted to foreign currency. The portfolios include the market (a combination of large and mid-cap stocks), value and growth portfolios, and 10 industry portfolios, all from MSCI. Each column corresponds to a different country.

Table A4: Correlation between excess returns converted in different currencies: foreign bonds

	AU	CA	DE	JP	NO	NZ	SE	CH	UK
2Y Bond	99.86	99.97	99.92	99.97	NA	99.85	99.91	99.91	99.95
3Y Bond	99.86	99.97	99.92	99.97	99.91	NA	NA	99.93	99.96
4Y Bond	NA	99.97	99.93	99.97	NA	NA	NA	99.94	99.96
5Y Bond	99.87	99.97	99.93	99.97	99.91	99.85	99.91	99.93	99.96
6Y Bond	NA	99.96	99.93	99.97	NA	NA	NA	99.92	99.96
7Y Bond	NA	99.96	99.93	99.96	NA	NA	99.91	99.91	99.96
8Y Bond	NA	99.96	99.92	99.96	NA	NA	NA	99.90	99.96
9Y Bond	NA	99.96	99.92	99.96	NA	NA	NA	99.89	99.96
10Y Bond	99.87	99.96	99.93	99.96	99.91	99.88	99.91	99.88	99.96

The table reports the correlation (in %) between the excess return on government bonds of different maturity expressed in their home currency and converted to U.S. dollars. Bond returns are constructed from yields obtained from each country's central bank. Each column corresponds to a different country.

Table A5: Correlation between excess returns converted in different currencies: U.S. bonds

	AU	CA	DE	JP	NO	NZ	SE	CH	UK
US 2Y Bond	99.9	99.95	99.95	99.97	99.91	99.93	99.95	99.93	99.96
US 3Y Bond	99.91	99.96	99.95	99.97	99.92	99.93	99.95	99.92	99.96
US 4Y Bond	99.92	99.96	99.94	99.96	99.92	99.94	99.95	99.91	99.96
US 5Y Bond	99.91	99.97	99.93	99.96	99.91	99.94	99.95	99.89	99.95
US 6Y Bond	99.91	99.97	99.93	99.96	99.89	99.94	99.94	99.88	99.95
US 7Y Bond	99.9	99.96	99.92	99.96	99.88	99.94	99.94	99.86	99.95
US 8Y Bond	99.89	99.96	99.91	99.96	99.86	99.93	99.93	99.85	99.95
US 9Y Bond	99.88	99.96	99.9	99.96	99.85	99.93	99.93	99.84	99.95
US 10Y Bond	99.88	99.96	99.9	99.96	99.84	99.93	99.92	99.83	99.94

The table reports the correlation (in %) between the excess return on U.S. government bonds of different maturity expressed in U.S.. dollars and converted to foreign currency. Bond returns are constructed from yields obtained from the Federal Reserve. Each column corresponds to a different country.