

CLASS 9: EVIDENCE FROM CAPM AND APT I

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Your portfolio manager says “high-beta stocks are the best way to earn higher returns.”

Is she right?

Where We've Been	Where We Are	Where We're Going
APT: multiple factors beyond the market	Testing CAPM predictions against real data	Anomalies and the search for new factors

How well does the CAPM hold up to the data?

We'll examine:

- 1. Time-series tests** — Does alpha equal zero?
- 2. Cross-sectional tests** — Does the security market line fit?
- 3. The Fama-Macbeth procedure** — How to test when beta is noisy

Key question: If CAPM fails, what does that teach us about which risks are priced?

To test the validity of a model, we need to test its predictions.

CAPM predicts:

1. A linear relationship between expected excess returns and beta
2. No other variable has marginal explanatory power / alpha is zero
3. The risk premium for beta is positive and equal to the market risk premium

These predictions can be tested in various ways.

Jensen's alpha: From the CAPM regression,

$$r_{i,t} - r_f = \alpha_i + \beta_i(r_{m,t} - r_f) + \varepsilon_{i,t}$$

Prediction: α_i should be jointly zero, i.e., $\alpha_1 = 0, \alpha_2 = 0, \dots$

Historical tests have not typically been favorable towards the CAPM:

- Reject that alphas are jointly zero ($p=0.02$) (Campbell, Lo, MacKinlay 1997)

Can show joint tests of alpha are equivalent to:

1. Find the Sharpe ratio of the market portfolio and compare it to the portfolio with the highest realized Sharpe ratio over a given period
2. Time-series tests provide statistical comparison of the predicted MVE (the market) and the actual MVE portfolio

Said otherwise, non-zero alphas suggest the market portfolio lags the realized maximum Sharpe ratio portfolios by more than the CAPM would suggest.

Generally, it is useful to think about CAPM tests/factor model tests as figuring out if the market/factor portfolios are “**efficient**”.

Big question: which stocks should we overweight in the market portfolio to make it more efficient?

Cross-sectional tests have been somewhat more favorable.

Rather than regressing returns on returns, we now regress **returns on betas**.

- Hope to find that betas and returns line up as predicted by security market line

Fama-Macbeth (1973) provides the standard framework for these tests.

Two-step procedure:

Step 1: Run time-series regressions to estimate beta for all stocks

$$r_{i,t} - r_f = \alpha_i + \beta_i(r_{m,t} - r_f) + \varepsilon_{i,t}$$

Step 2: Run cross-section regression of average excess returns $\overline{r_i - r_f}$ on estimated betas

$$\overline{r_i - r_f} = \lambda_0 + \lambda_1 \hat{\beta}_i + u_i$$

Prediction: $\lambda_0 = 0$ and $\lambda_1 = E(r_m - r_f)$

However, note that **beta estimates are noisy.**

- Regressing any variable on a noisy proxy will flatten the slope coefficient

Why?

- Imagine noise is added to x so that you observe $x + e$
- True OLS coefficient is $\lambda_1 = \frac{\text{Cov}(x, y)}{\text{Var}(x)}$
- Estimated OLS coefficient is:

$$\lambda_1 = \frac{\text{Cov}(x + \varepsilon, y)}{\text{Var}(x + \varepsilon)} = \frac{\text{Cov}(x, y)}{\text{Var}(x) + \text{Var}(\varepsilon)}$$

So, we expect any regression estimate of the security market line (SML) to be **too flat.**

In response, we **form portfolios** of stocks and hope that idiosyncratic noise in beta estimates disappears.

- Why might this help?

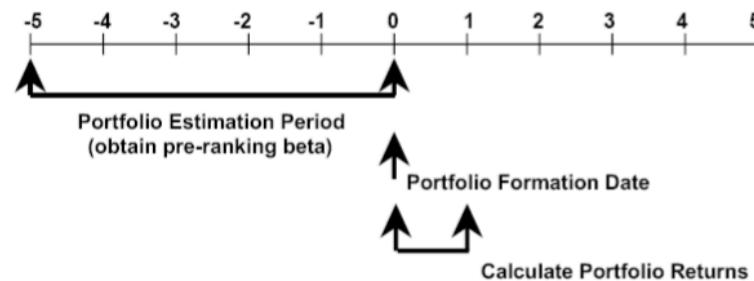
Portfolios are formed based on firm betas.

- Why not random assignment?

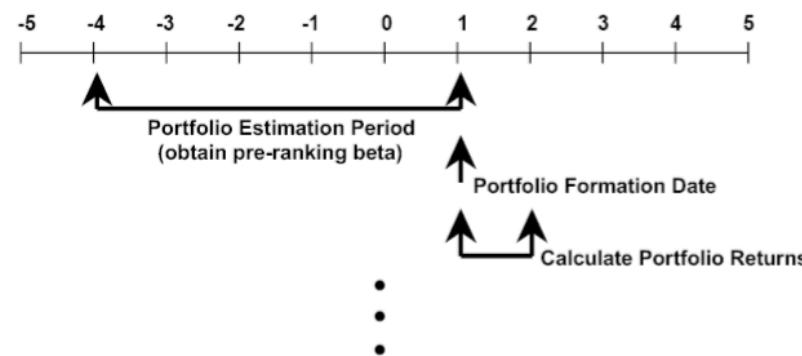
Actual procedure:

1. Each year, calculate betas for all firms (past five years data – 60 months)
2. Form 10 decile portfolios based on estimated betas
3. Calculate realized portfolio returns and betas for 10 portfolios

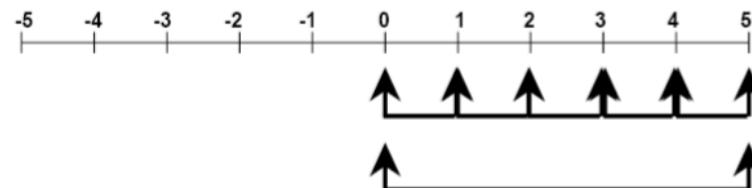
Beta Estimation Process



Second Year:

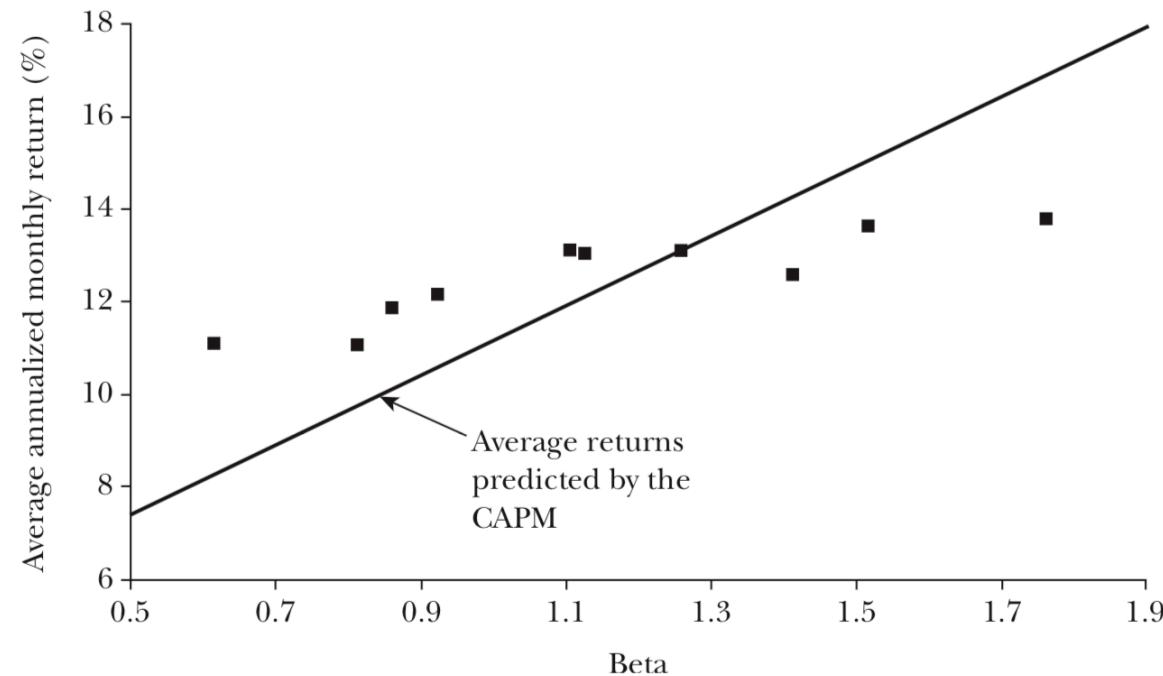


Combine Sets of Returns:



Positive relation between beta and portfolio returns, but **fitted line too flat**.

Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003



The empirical security market line is **flatter** than predicted by CAPM:

- High-beta stocks earn **less** than CAPM predicts
- Low-beta stocks earn **more** than CAPM predicts

Possible explanations:

1. **Measurement error** — Even portfolios don't fully eliminate noise in beta
2. **Market portfolio proxy** — We use S&P 500, not the true market portfolio
3. **Other risk factors** — Beta alone doesn't capture all systematic risk (leads to APT)
4. **Behavioral frictions** — Leverage constraints, attention, limits to arbitrage

This motivates the search for additional factors beyond the market.

Time-series tests:

- Reject joint hypothesis that all alphas equal zero
- Suggests market portfolio is not mean-variance efficient

Cross-sectional tests:

- Positive relation between beta and returns (good news)
- But relationship is too flat (puzzling)

Fama-Macbeth procedure:

- Standard way to handle measurement error in betas
- Form portfolios to reduce noise
- Still find deviations from CAPM predictions

Next class: We'll explore what **other** factors might help explain returns — size effects, value effects, and more anomalies.

Topics: Evidence from CAPM and APT II

- Other CAPM predictions
- The search for anomalies
- Size and book-to-market effects