Class 4: Risk and Return II

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Today's Question

You can invest in a fund that's returned 15%/year with 40% volatility,

or one that's returned 10%/year with 15% volatility.

Which is better?

Positioning

Where We've Been	Where We Are	Where We're Going
Measuring returns	Risk-adjusted performance; Sharpe ratios	Diversification and portfolio construction

“No One Was Supposed to Lose This Much Money on Swiss Francs”

Key themes: Tail risk, leverage, and unexpected losses

What happens when the “safe” trade blows up?

Learning Objectives

By the end of today's class, you should be able to:

1. Calculate and interpret the Sharpe ratio
2. Explain why comparing raw returns is misleading
3. Describe how leverage affects risk-adjusted performance

The Two Funds

Fund A: “Aggressive Growth”

- Average return: 15% per year
- Volatility (σ): 40% per year
- *“We swing for the fences!”*

Fund B: “Steady Eddie”

- Average return: 10% per year
- Volatility (σ): 15% per year
- *“Slow and steady wins the race”*

What Makes This Hard?

- Fund A has higher returns... but also higher risk
- Fund B has lower returns... but lower risk

We need a way to compare risk-adjusted returns

- Just looking at average returns is misleading
- Just looking at volatility misses the return picture

The Sharpe Ratio

A single number that captures the risk-return trade-off *in isolation*

A measure of risk-adjusted performance

Formula:

$$\text{Sharpe Ratio} = \frac{E(r) - r_f}{\sigma}$$

- **Numerator:** Excess return above the risk-free rate
- **Denominator:** Risk (volatility)
- **Interpretation:** “How much excess return do I get per unit of risk?”

Calculating Sharpe Ratios for Our Two Funds

Assume risk-free rate = 3%

Fund A (Aggressive Growth):

$$\text{Sharpe}_A = \frac{15\% - 3\%}{40\%} = \frac{12\%}{40\%} = 0.30$$

Fund B (Steady Eddie):

$$\text{Sharpe}_B = \frac{10\% - 3\%}{15\%} = \frac{7\%}{15\%} = 0.47$$

Fund B has a higher Sharpe ratio!

Fund B gives you more return per unit of risk, even though its absolute return is lower.

Why Sharpe Ratio Matters

Fund A: 12% excess return, but you take on 40% volatility to get it

- Sharpe = 0.30 means 0.30% excess return per 1% of volatility

Fund B: 7% excess return, but only 15% volatility

- Sharpe = 0.47 means 0.47% excess return per 1% of volatility

Implication: Fund B is more “efficient” at converting risk into return

If you’re going to take risk, you want to be compensated well for it!

However: this doesn’t mean Fund B is always better. What might it depend on?

But Wait... What About Leverage?

Can I lever up Fund B to adjust its risk to match Fund A?

Leverage and Risk-Adjusted Returns

Setup: You have $100 to invest. Risk-free rate = 3%.

Option 1: Invest $100 in Fund A (no leverage)

- Expected return: 15%
- Volatility: 40%
- Sharpe: 0.30

Option 2: Borrow $167 at 3%, invest $267 in Fund B (2.67x leverage)

- Expected return: $267 \times 10\% - 167 \times 3\% = 26.7\% - 5.0\% = 21.7\%$ on your $100
- Volatility: $2.67 \times 15\% = 40\%$ (same as Fund A!)
- Sharpe: Still 0.47 (leverage doesn't change Sharpe ratio)

The Power of Leverage with Higher Sharpe

Comparison at 40% volatility:

Strategy	Return	Volatility	Sharpe
Fund A (no leverage)	15%	40%	0.30
Fund B (levered 2.67x)	21.7%	40%	0.47

Result: By levering Fund B to match Fund A's risk, you get:

- **6.7% higher return** (21.7% vs 15%)
- **Same risk** (40% volatility)

How Leverage Affects Returns and Risk

General formula for leverage:

If you lever a portfolio by factor L:

$$E(r_{\text{levered}}) = r_f + L \times \big(E(r) - r_f\big)$$

$$\sigma_{\text{levered}} = L \times \sigma$$

$$\text{Sharpe}_{\text{levered}} = \frac{r_f + L\big(E(r) - r_f\big) - r_f}{L \times \sigma} = \frac{E(r) - r_f}{\sigma} = \text{Sharpe}_{\text{unlevered}}$$

Leverage scales both return and risk proportionally, leaving Sharpe ratio unchanged.

Caveats About Leverage

In theory: Leverage is free (borrow at risk-free rate) and Sharpe ratio is all that matters

In practice:

1. You can't actually borrow at the risk-free rate (borrowing costs > r_f)
2. Margin calls and forced liquidations at bad times
3. Leverage amplifies losses as well as gains
4. Many investors face leverage constraints (can't borrow, or shouldn't)

Bottom line: Sharpe ratio is a good way to compare funds, but implementing leverage has real-world frictions that change the investment profile

So far we've focused on volatility (σ) as our measure of risk.

But there are other ways to think about risk:

1. How bad can things get? → **Value at Risk (VaR)**
2. How bad is it when things go really wrong? → **Expected Shortfall**
3. What about tail events that aren't captured by normal distributions?

Downside risk focus

- A measure of loss most frequently associated with extreme negative returns
- VaR is the quantile of a distribution below which lies q% of the possible values
- The 5% VaR tells us how bad returns will be in the worst 5% of times

Example for Fund A (15% return, 40% volatility):

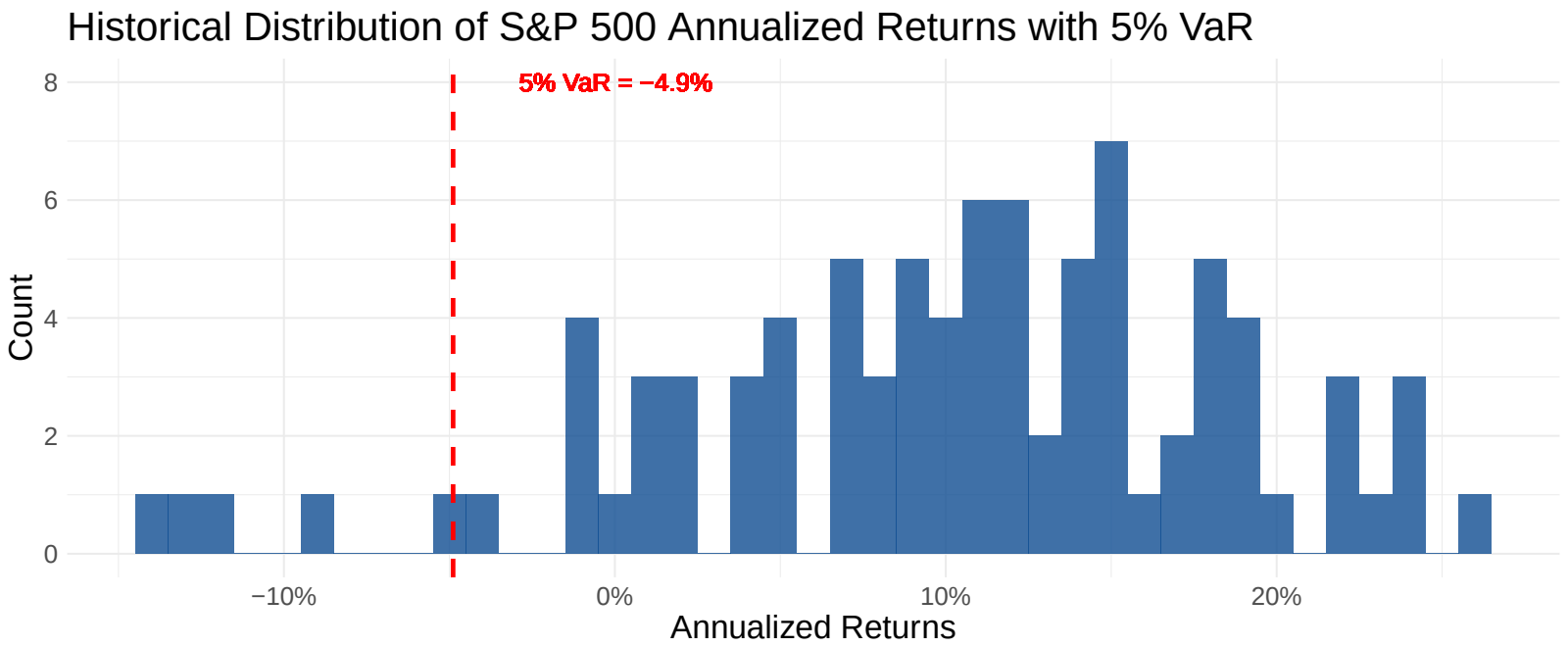
Assuming normal distribution:

$$5\%\ \mathrm{VaR} = E(r) - 1.645 \times \sigma = 15\% - 1.645 \times 40\% = -50.8\%$$

This says: “In the worst 5% of years, you’ll lose at least 50.8%”

VaR: Historical Approach

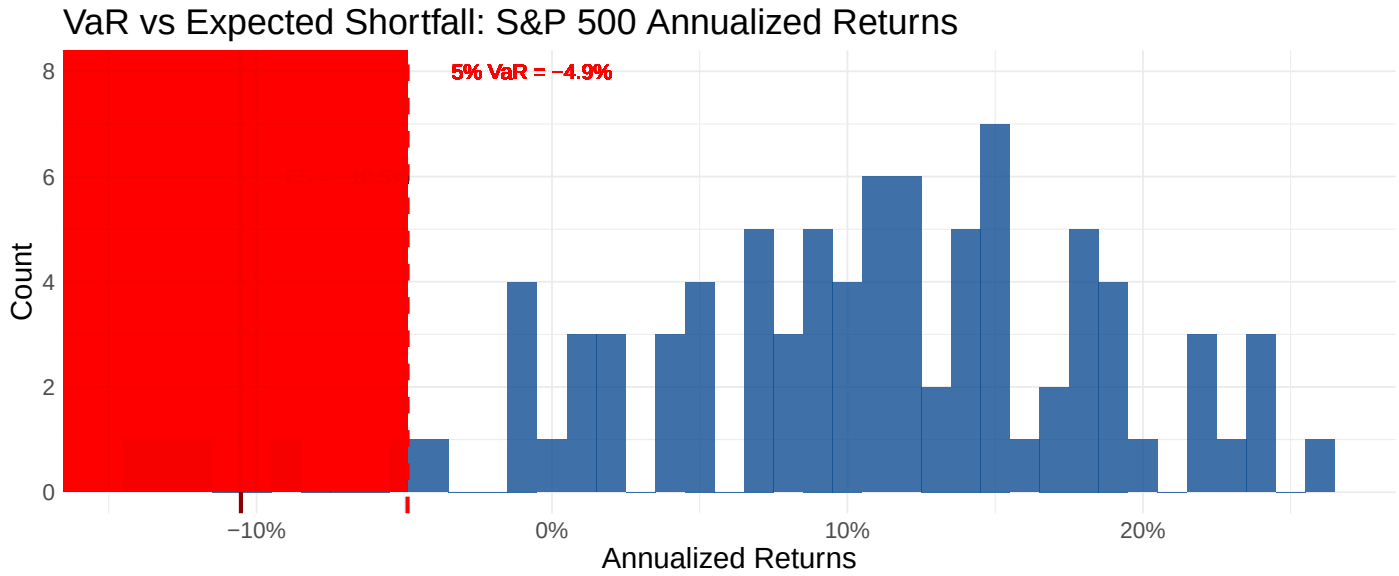
Rather than assume normality, look at actual historical returns:



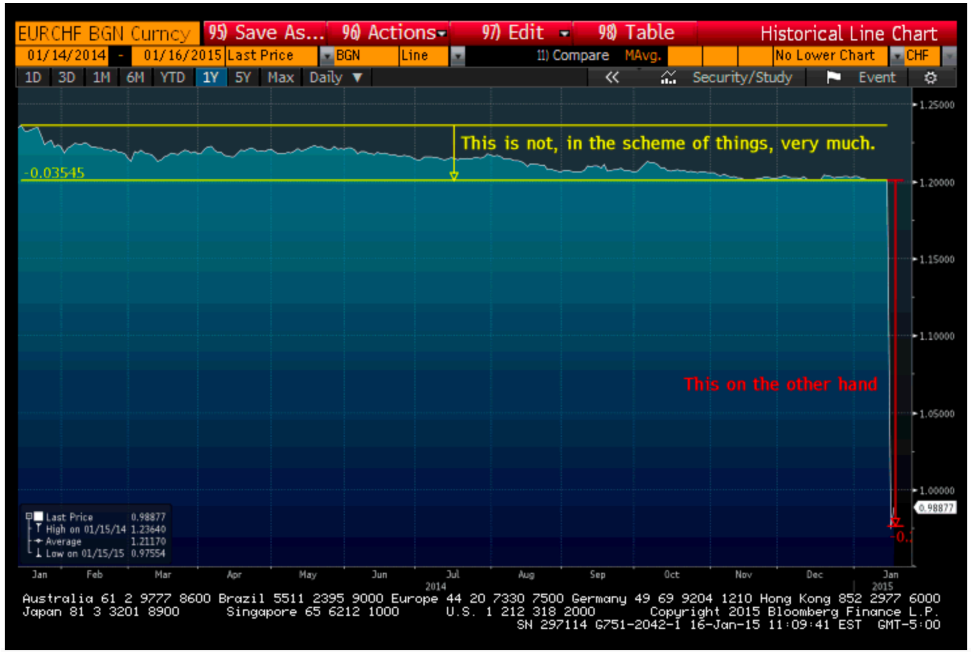
The 5% VaR from historical data may differ from the normal assumption!

Also called Conditional Tail Expectation (CTE)

- More conservative measure of downside risk than VaR
  - VaR takes the highest return from the worst cases
  - ES takes an **average** return of the worst cases



The Swiss Franc Case: When Tail Risk Hits



Goldman Sachs Chief Financial Officer Harvey Schwartz said on this morning's earnings call that this was something like a 20-standard-deviation event, and while the exact number of standard deviations is of course a subjective matter, that's the right ballpark. Over the 12 months ended on Wednesday, the

https://www.bloomberg.com/opinion/articles/2015-01-16/no-one-was-supposed-to-lose-this-much-money-on-swiss-francs 3/7

10/13/22, 10:06 AM No One Was Supposed to Lose This Much Money on Swiss Francs - Bloomberg

annual volatility -- that is, the annualized standard deviation of daily returns -- of the euro/franc relationship was a bit over 1.7 percent; over the last three months of that period the volatility was less than 1 percent. [2] That converts to a *daily* standard deviation of something like 0.1 percent. [3] On Thursday, the euro ended down almost 19 percent, or call it 180 standard deviations, depending on what period you use.

What happens when “impossible” events occur?

Takeaways: Why Tail Risk Matters

The Swiss Franc case shows why VaR and ES matter:

- Normal distributions underestimate tail risk
- “6-sigma events” happen more often than theory predicts
- Leverage + tail risk = disaster

This connects back to our two funds:

Even if Fund A has a lower Sharpe ratio, if it has *much* fatter tails (higher VaR/ES), that’s important information that Sharpe ratio alone doesn’t capture.

Let's look at some data

https://colab.research.google.com/github/paulgp/investment-management-notebooks/blob/main/02_return_calculations.ipynb

https://colab.research.google.com/github/paulgp/investment-management-notebooks/blob/main/04_risk_return.ipynb

Back to Our Opening Question

15%/year with 40% volatility vs 10%/year with 15% volatility.

Which is better?

The Answer: Fund B (Probably)

Reason 1: Higher Sharpe Ratio

- Fund A: Sharpe = 0.30
- Fund B: Sharpe = 0.47
- Fund B is more efficient at converting risk into return

Reason 2: Leverage

- If you want higher returns, lever up Fund B
- Levered 2.67x, Fund B gives 21.7% return with same 40% volatility as Fund A
- You end up with 6.7% more return for the same risk!

The Answer (continued)

Reason 3: Downside Risk

Under symmetric risk:

- Fund A: 5% VaR ≈ −51%
- Fund B: 5% VaR ≈ −15%
- In bad times, Fund A loses much more

So when might you choose Fund A?

The Answer (continued)

When might you choose Fund A?

1. You can't use leverage (regulatory constraints, personal preference)
2. You believe Fund B has higher tail risk than volatility suggests (Swiss Franc case)
3. You have very high risk tolerance and want overall returns
4. *It is a small part of your overall portfolio!*

Comparing raw returns is misleading.

The Answer (continued)

Better to look at risk-adjusted returns (Sharpe ratio) and consider leverage and downside risk as well

Summary: Key Concepts from Today

1. **Sharpe Ratio** = (Return - Risk-free) / Volatility
   - The gold standard for comparing risk-adjusted performance
2. **Leverage** scales both return and risk proportionally
   - Prefer the higher Sharpe ratio fund and adjust risk with leverage
3. **Other risk measures** beyond volatility:
   - VaR: How bad can things get?
   - Expected Shortfall: How bad when things go really wrong?
4. **Tail risk matters**—Swiss Franc case study

Next Class: February 3

- Topics: Building Optimal Portfolios
  - Risk aversion and preferences
  - Capital allocation between risky and risk-free assets
  - Two risky assets and the efficient frontier
- Matt Levine Reading: “SPACs Aren’t Cheaper Than IPOs Yet” / “Should Trump Have Indexed?”