Class 3: Risk and Return I

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Your uncle says his stock picks “always go up 20%.”

Why are you skeptical?

Positioning

Where We've Been	Where We Are	Where We're Going
Market structure	Measuring returns and risk	Combining assets into portfolios

"When Everything Is Too Safe, Add Risk"

Key idea: Risk and return are fundamentally linked.

If you want higher returns, you must accept more risk.

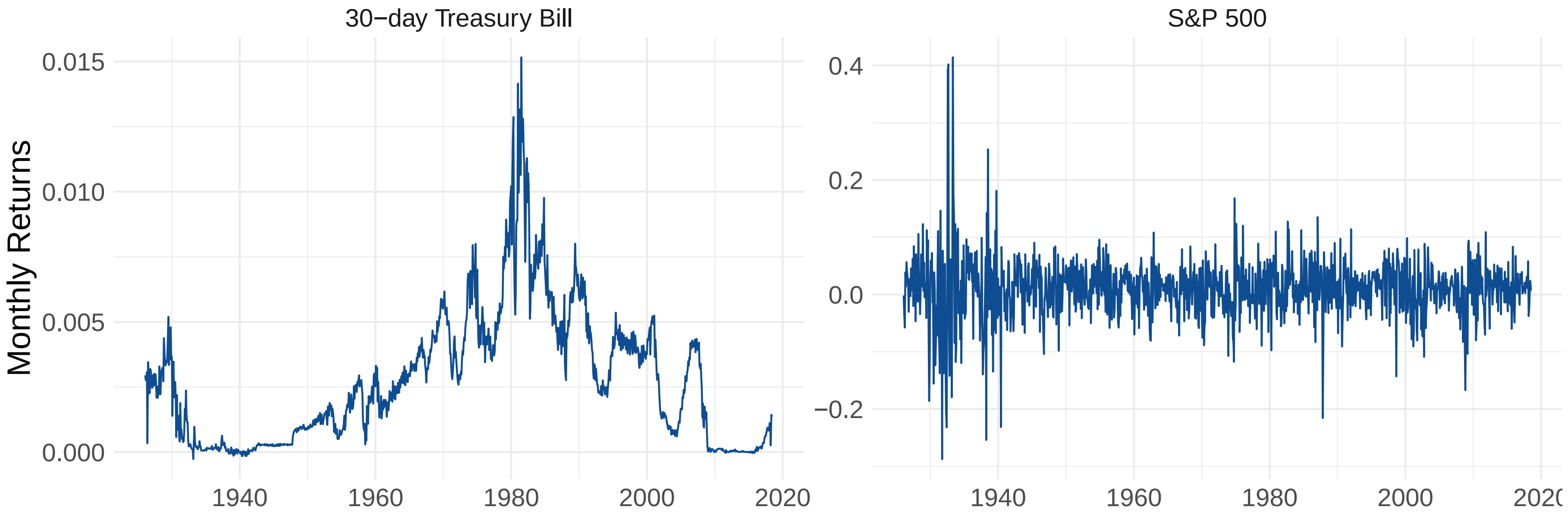
By the end of today's class, you should be able to:

1. Calculate holding period returns, arithmetic vs. geometric means
2. Explain why volatility matters for long-run wealth accumulation
3. Distinguish between ex-ante and ex-post risk measures

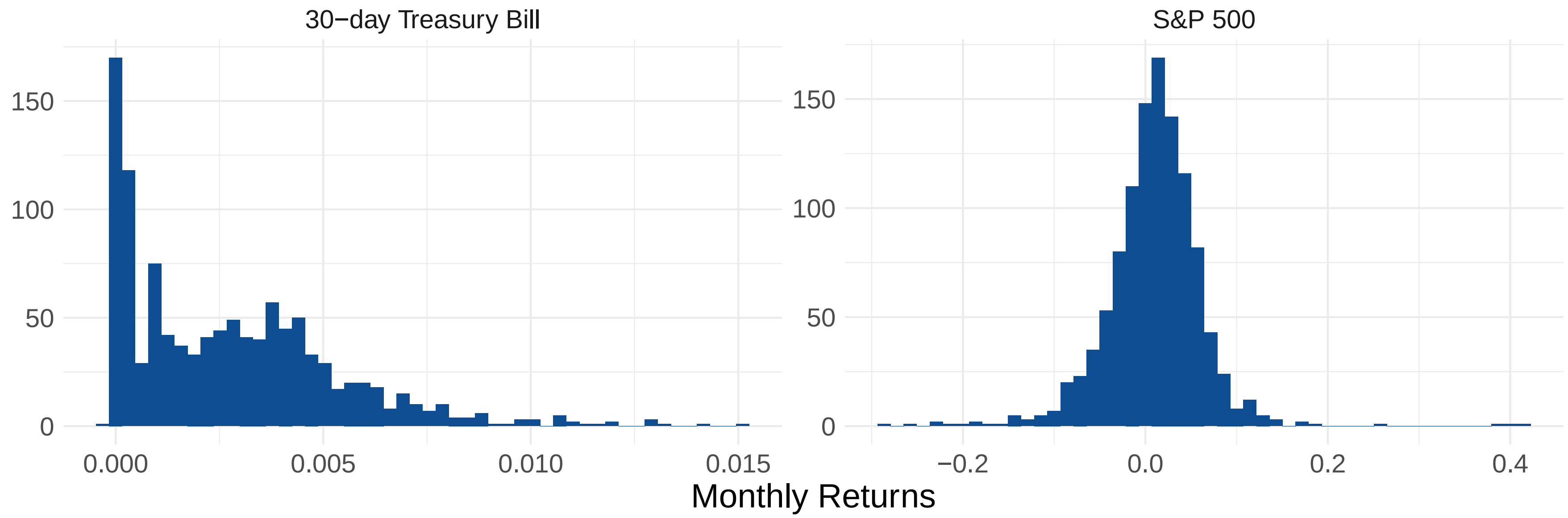
1. **How are you measuring that 20%?**
   - Arithmetic? Geometric? Time-weighted? Dollar-weighted?
2. **How volatile were your returns?**
   - Did you really make 20% *every* year, or is that an average?
3. **Are you cherry-picking your track record?**
   - Survivorship bias? Forgetting the losers?
4. **How does that compare to the market?**
   - Is 20% even realistic?

To answer these questions, we need to understand how to measure and interpret returns.

Measuring Assets' Returns



- Let’s begin by focusing on two major asset classes—stocks and bonds
- How have these assets performed historically?
  - What’s the best way to summarize their risk + return?



- Why does a Treasury bill’s return not fall below zero?
- Is monthly return the best measure? What else could we do?

Return Definitions

- The rate of return is also known as the **“net return”**:

$$r_t = \frac{P_t + D_t}{P_{t-1}} - 1$$

- **Gross returns** (sometimes R_t) is referred to as $1 + r_t$
- The **risk-free rate** will be called r_f
  - How can I get the risk-free rate?
- **Excess returns** above the risk free rate are $r_e = r - r_f$

Return Definitions: Holding Period

A holding-period return of T years is:

$$r_{0,T} = (1 + r_1)(1 + r_2)\cdots(1 + r_T) - 1$$

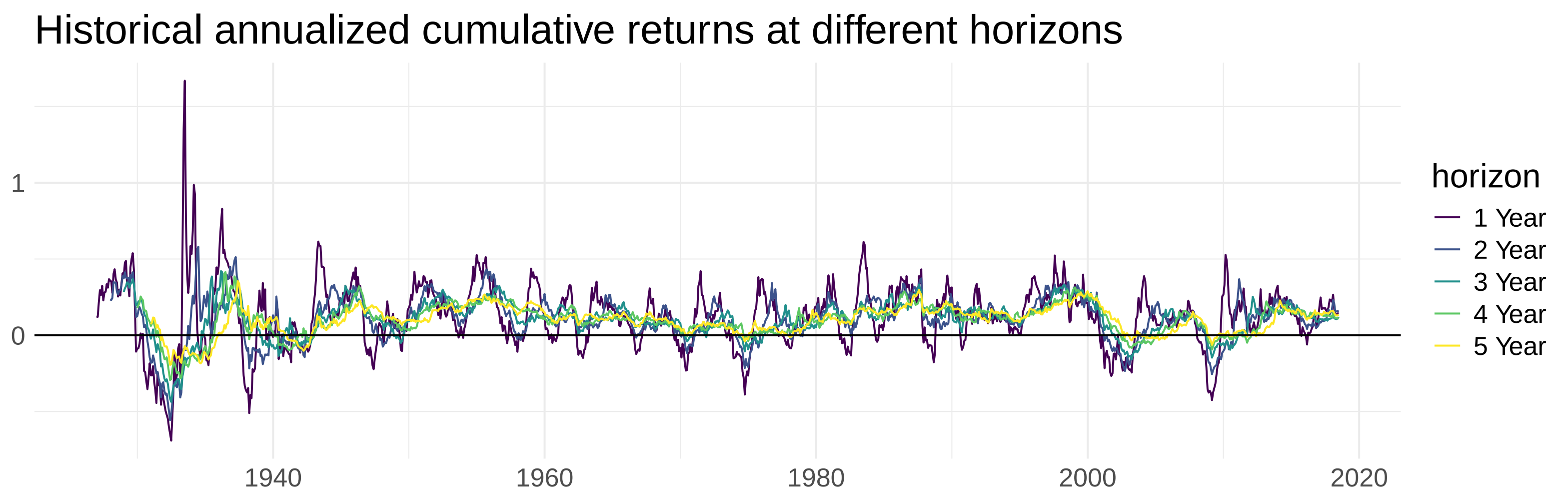
- How you measure the holding period matters a ton!

Forward five−year cumulative returns on the S&P 500

- How do we compare returns across different horizons?
  - Say I want to compare a window of cumulative returns 5 vs 10 years.
- **Annualized returns** on the cumulative return:

$$\tilde{r}_{0,T} = \left(1 + r_{0,T}\right)^{\frac{1}{T_{\text{years}}}} - 1$$

Return Definitions: Annualized Returns



Arithmetic vs. Geometric Average Returns

Arithmetic average returns:

$$\bar{r} = \frac{1}{T}(r_1 + r_2 + \cdots + r_T)$$

Geometric average returns:

$$r_G = [(1 + r_1)(1 + r_2)\cdots(1 + r_T)]^{\frac{1}{T}} - 1 = \left[\frac{P_T}{P_0}\right]^{\frac{1}{T}} - 1$$

Arithmetic average:
Unbiased estimate of 1-period future returns
(expected returns)

Geometric average:
Measure of cumulative past performance
(actual wealth growth)

The “Volatility Drag”

Example: Two investments over 2 years, both starting at \$100

Investment A (Volatile):

- Year 1: +50% $\longrightarrow$ \$150
- Year 2: −30% $\longrightarrow$ \$105
- Arithmetic average: **10%**
- Geometric average: 2.5%
- **Actual wealth: \$105**

Investment B (Stable):

- Year 1: +10% $\longrightarrow$ \$110
- Year 2: +10% $\longrightarrow$ \$121
- Arithmetic average: **10%**
- Geometric average: **10%**
- **Actual wealth: \$121**

Same arithmetic average, but volatile investment ends with \$16 less!

Volatility Drag: The Formula

Key insight: Even with the same arithmetic average return (10%), the volatile investment ends up with less wealth!

$$\text{Geometric Mean} \approx \text{Arithmetic Mean} - \frac{1}{2}\sigma^2$$

- The more volatile your returns, the bigger the drag on long-run wealth
- This is why "20% average returns" with high volatility might not be as good as it sounds

Volatility Drag: The Formula

Back to uncle's claim:

If uncle really averaged 20% but with 40% volatility:

$$\text{Geometric} \approx 20\% - \frac{1}{2}(0.40)^2 = 20\% - 8\% = 12\%$$

His *actual* wealth compound would be closer to 12%, not 20%!

Variance and Standard Deviation

We calculate estimates of **variance** using squared deviations from arithmetic average returns:

$$\sigma^2(r) = \mathrm{VAR}(r) = \frac{1}{T}\Big((r_1 - \overline{r})^2 + (r_2 - \overline{r})^2 + \cdots + (r_T - \overline{r})^2\Big)$$

Standard deviation is the square-root of variance:

$$\sigma(r) = \mathrm{SD}(r) = \sqrt{\frac{1}{T}\Big((r_1 - \overline{r})^2 + (r_2 - \overline{r})^2 + \cdots + (r_T - \overline{r})^2\Big)}$$

Covariance and Correlation

Covariance measures how two returns move together:

$$\sigma_{i,j} = \mathrm{COV}(r_i, r_j) = \frac{1}{T}\left(\left(r_{1,i} - \bar{r}_i\right)\left(r_{1,j} - \bar{r}_j\right) + \cdots + \left(r_{T,i} - \bar{r}_i\right)\left(r_{T,j} - \bar{r}_j\right)\right)$$

Correlations scale the covariance by standard deviations:

$$\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j}$$

Excel provides functions: `AVERAGE`, `GEOMEAN`, `VAR`, `STDEV`, and `COVAR`

A Critical Distinction

Ex-Post (Looking Backward):

- What actually happened
- Historical returns and volatility
- Measured from data
- Example: "The S&P 500 returned 12% last year with 18% volatility"

Ex-Ante (Looking Forward):

- What we expect to happen
- Expected returns and expected volatility
- Forecasted/estimated
- Example: "We expect the S&P 500 to return 8% next year with 20% volatility"

Why This Distinction Matters

When your uncle says "20% returns," which is he talking about?

- Probably ex-post (what happened)
- But you care about ex-ante (what will happen)

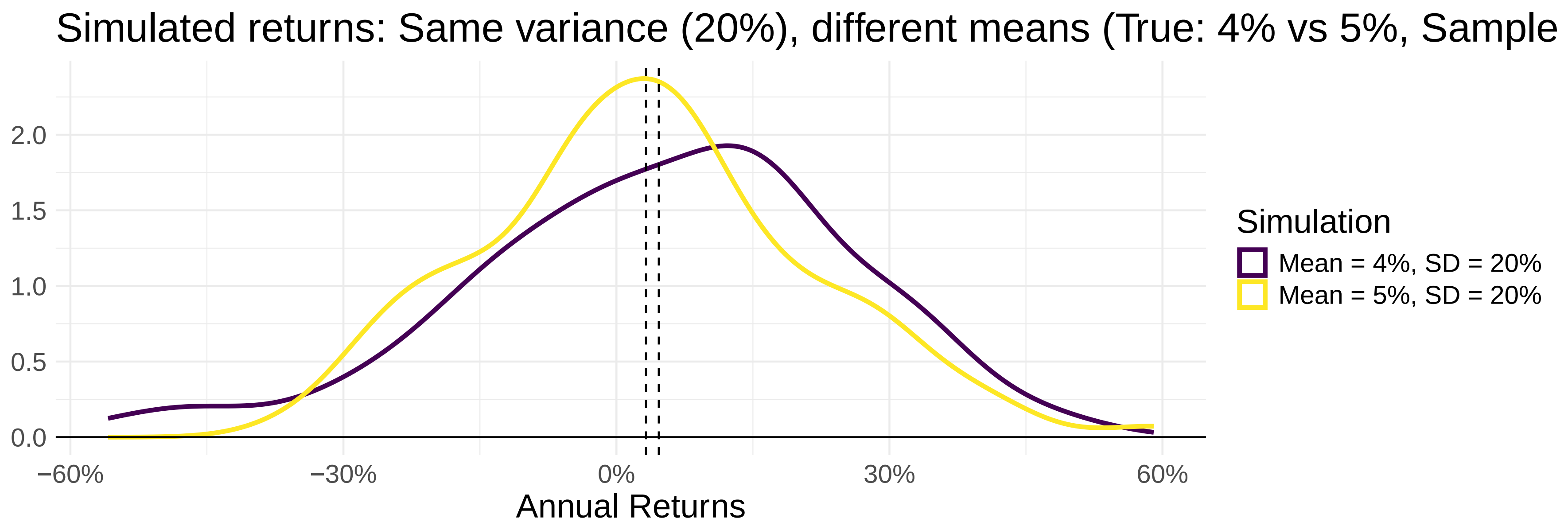
Past performance ≠ Future performance

- Historical returns are noisy estimates of expected returns
- Historical volatility is a better (but still imperfect) estimate of future volatility

Wanted: Information about risk and return

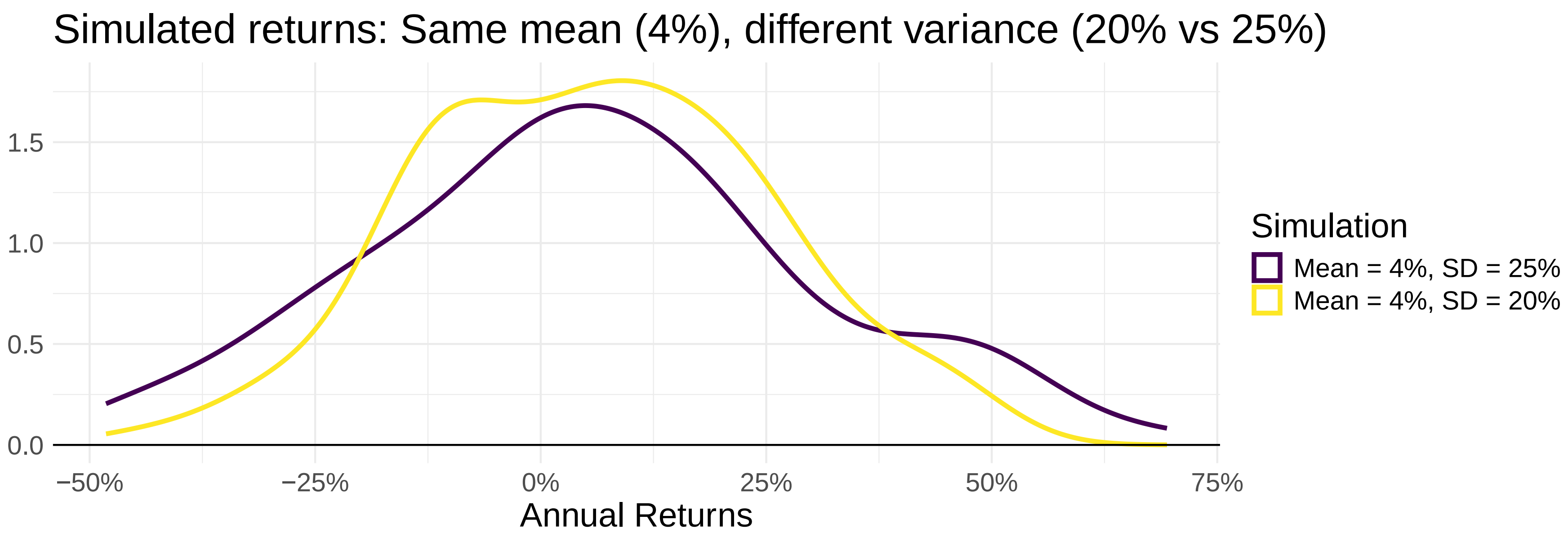
- To make investment decisions, we need to know:
  - ... the expected **future** returns (ex-ante)
  - ... the riskiness of **future** returns (ex-ante)
- We turn to historical return data for these (ex-post)
  - A caveat...
  - The data give a pretty good sense of the risk, but expected returns are hard to measure.
  - Why?

Why Expected Returns Are Hard to Measure



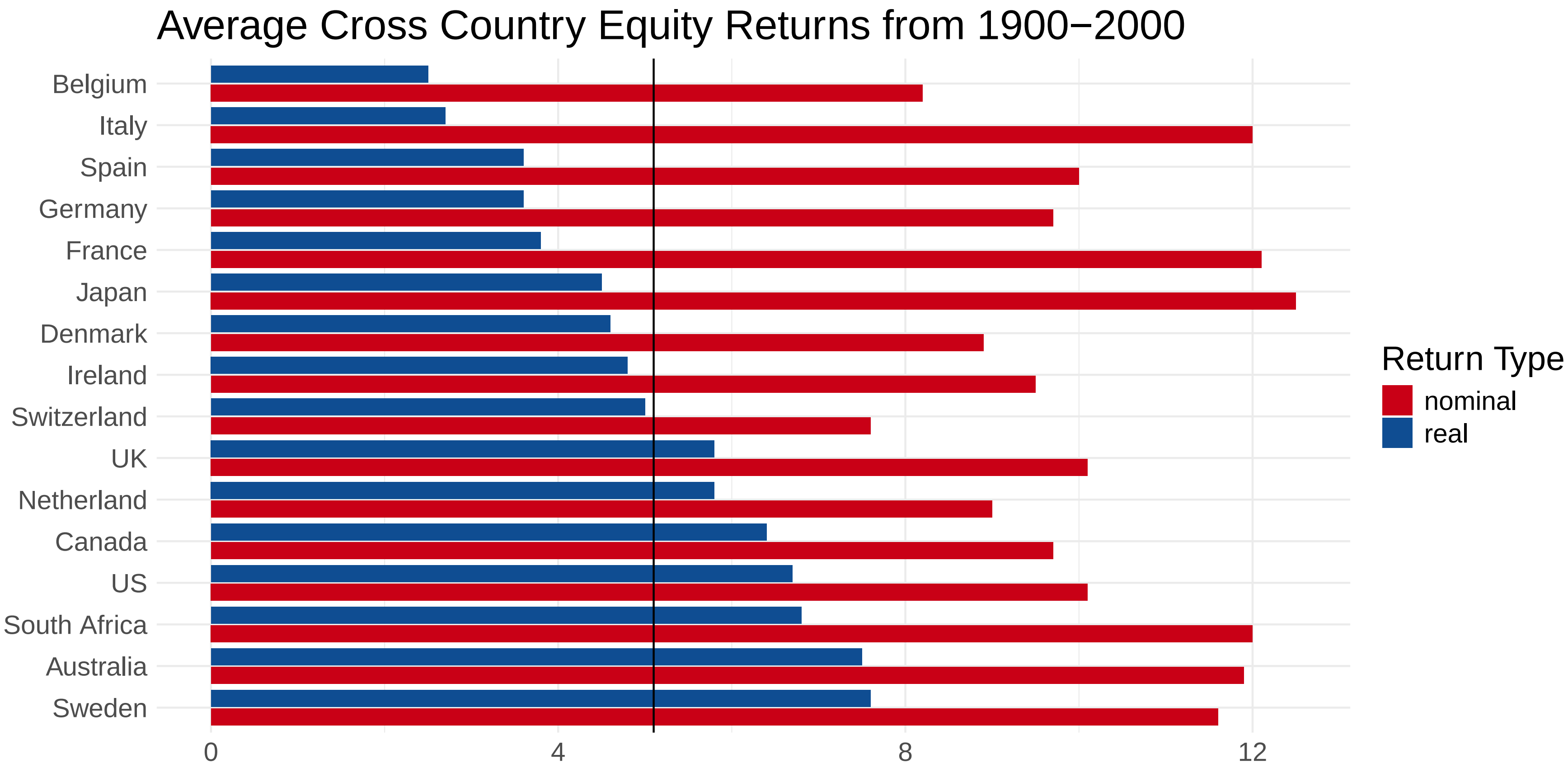
Even with 100 years of data, distinguishing between a 4% vs 5% expected return is nearly impossible with 20% volatility!

Volatility Is Easier to Estimate



Distinguishing between 20% vs 25% volatility is much more feasible.

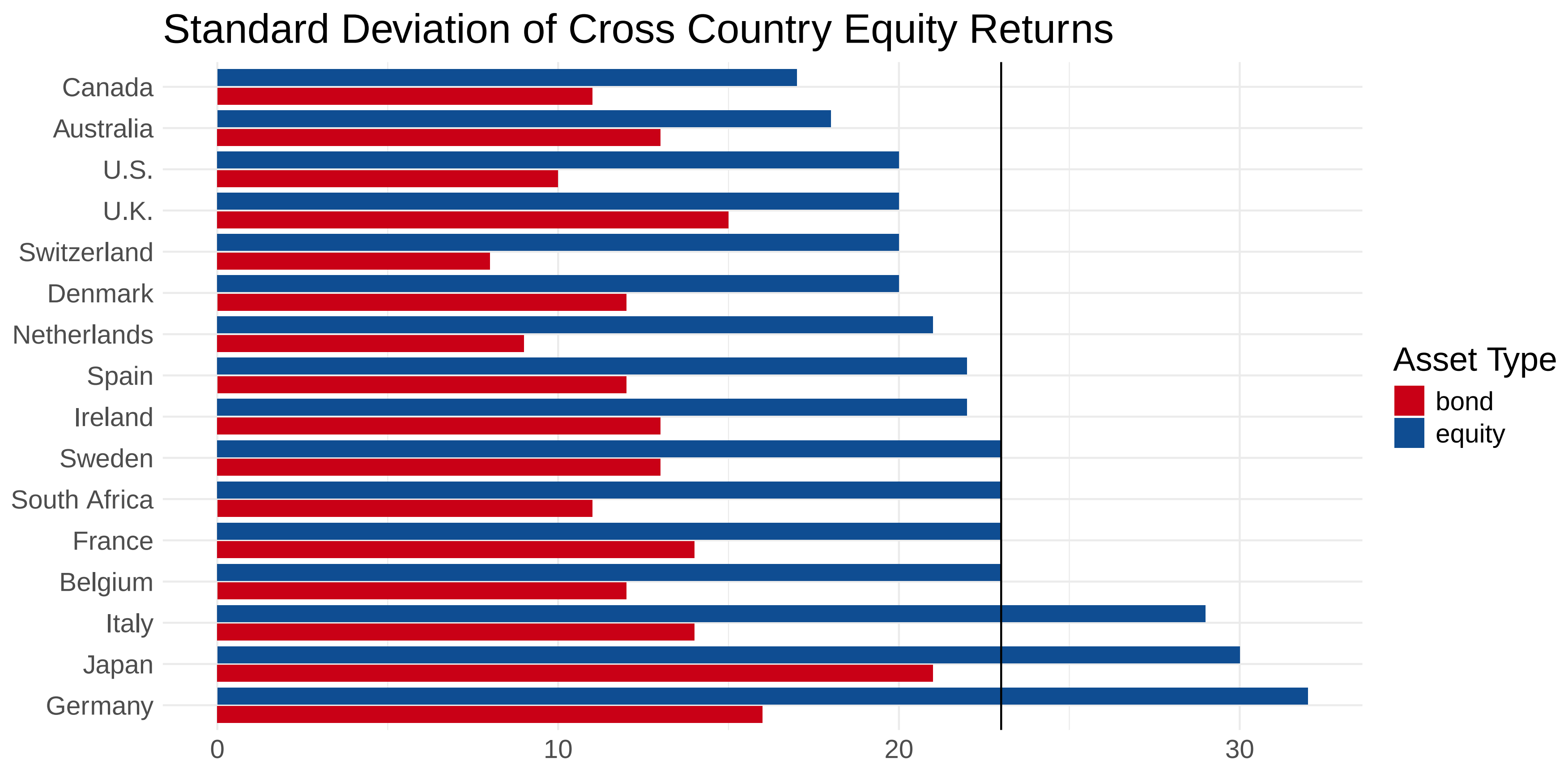
Asset Returns: Historical Record



Asset Returns: Historical Record

- US returns are high, but not an outlier
- Average real return across countries: 5.1%

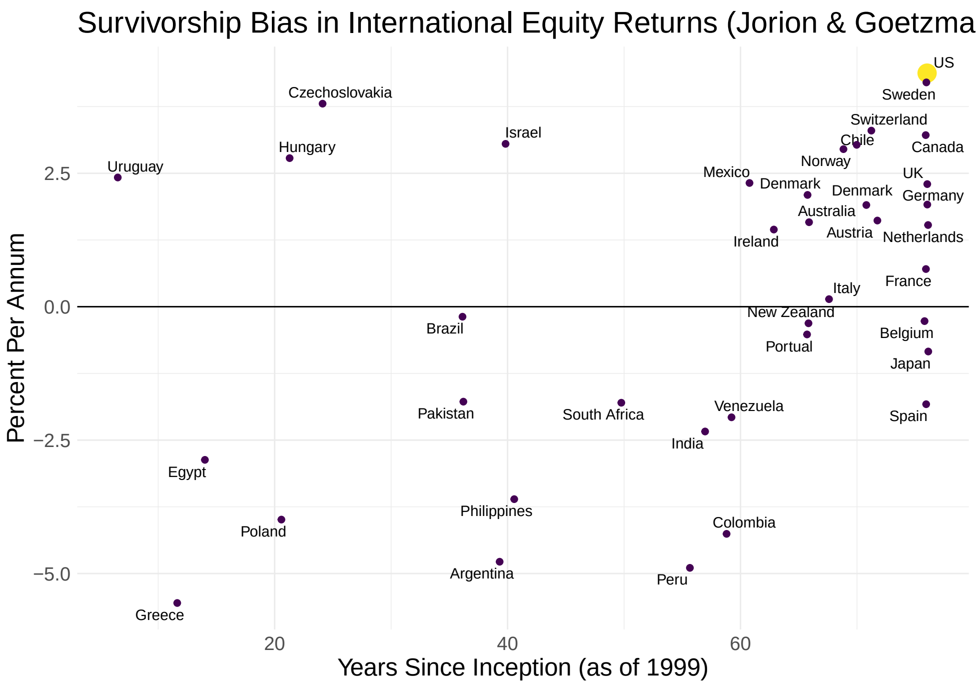
Asset Returns: Volatility



Asset Returns: Volatility

- Average s.d. of returns is 23% (US 20%)
- What's up with Italy, Germany, Japan?

Jorion and Goetzmann (1999)



- Countries with low returns fall out of the analysis
- Median real return of all other countries is 0.75% (compared to 4.3% for US)
- This is **survivorship bias**!

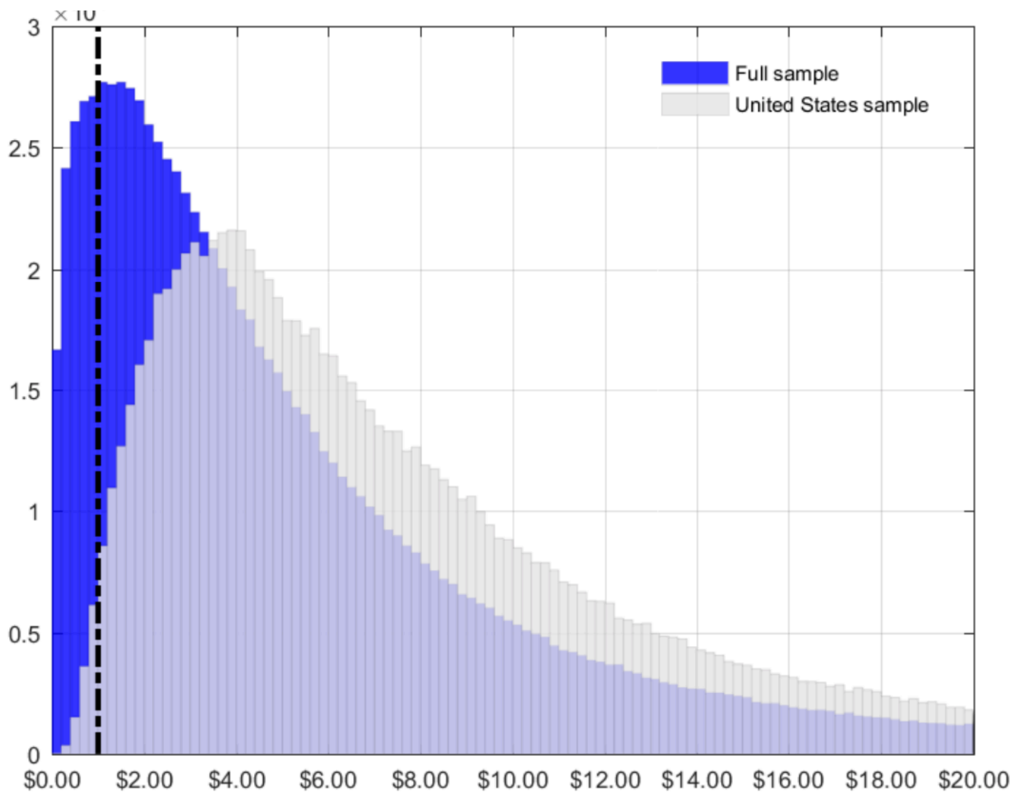


Fig. 1. Cumulative 30-year payoffs. The figure shows histograms of real payoffs across 1,000,000 bootstrap simulations at a return horizon of 30 years. The real payoffs are from the perspective of a domestic investor in a representative country. The underlying sample for the simulated returns is the pooled sample of all developed countries (blue) or the United States sample (gray). The dashed line separates the regions of real

Anarkulova et al. (2020)

- "From 1990 to 2019, a diversified investment in Japanese stocks produced returns of −9% in nominal terms"
- Incorporating the losses from buy-and-hold strategies that were selected away changes the picture dramatically

Back to Our Opening Question

Your uncle says his stock picks "always go up 20%."

Why are you skeptical?

1. How is he measuring 20%?

- Arithmetic vs geometric mean matter
- If volatility is high, geometric (actual wealth) << arithmetic (average)
- Volatility drag: Geometric $\approx$ Arithmetic $- \frac{1}{2}\sigma^2$

2. Is he confusing ex-post with ex-ante?

- Past returns (ex-post) don't guarantee future returns (ex-ante)
- Historical returns are noisy—hard to distinguish skill from luck
- Standard error with 10 years of data is huge!

Why We're Skeptical (continued)

3. Survivorship bias

- Is he only counting the winners and forgetting the losers?
- Jorion & Goetzmann: Countries with bad returns disappear from indices
- Same applies to individual stocks (and uncles!)

4. Is 20% realistic?

- U.S. stocks: 10% nominal, 7% real (historically)
- To consistently beat market by 10%+ would make uncle one of the best investors in history
- More likely: (1) Lucky, (2) Cherry-picking, or (3) Taking massive risk

Ask uncle (fund manager) to show you:

1. Actual portfolio statements (not just memory)
2. Full track record (including losses)
3. Risk metrics (volatility, drawdowns)
4. How it compares to a simple index fund

Bottom line: Extraordinary claims require extraordinary evidence. And understanding how to properly measure returns and risk is essential to evaluating any investment opportunity.

Next Class: January 29

- Topics: Risk and Return (continued)
  - Risk-adjusted performance and Sharpe ratios
  - How leverage affects risk-adjusted performance
  - Value at Risk and Expected Shortfall
- Matt Levine Reading: "No One Was Supposed to Lose This Much Money on Swiss Francs"