Class 7: The Capital Asset Pricing Model

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Today's Question

Your financial advisor says to “just buy index funds.”

What theory justifies that advice — and what has to be true?

Positioning

Where We've Been	Where We Are	Where We're Going
Optimal portfolios (individual)	Equilibrium: what happens when everyone optimizes	Testing the model

"Buying the Good Stocks Can Be Bad"

Key themes: Why picking "good" companies doesn't guarantee good returns

Learning Objectives

By the end of today's class, you should be able to:

1. Explain why only systematic risk is priced in equilibrium
2. Use the Security Market Line to calculate expected returns
3. State the key assumptions of the CAPM and which seem most problematic

From One Investor to Everyone

Last class, we proved a powerful result: **every investor should hold the same tangency portfolio** (combined with the risk-free asset to taste).

Now run a thought experiment:

1. Imagine everyone in this room solved the Class 6 problem and found the tangency portfolio
2. You all go to your brokerage and place buy orders for the same portfolio weights
3. Some of you are cautious and put 30% in the tangency, 70% in T-bills
4. Some of you are aggressive and borrow to put 150% in the tangency
5. But you all want the **same risky portfolio**

Key Question

If everyone holds the same risky portfolio, and the lending and borrowing in T-bills cancels out in aggregate, what must the tangency portfolio look like?

A Risk You Can Eliminate for Free

Consider two types of risk that Moderna faces:

Risk 1: A recession hits. Consumer spending drops, the stock market falls, and Moderna's stock falls with it. Every stock in your portfolio is affected. You cannot escape this risk no matter how many stocks you hold.

Risk 2: A drug trial fails. Moderna's stock drops, but Pfizer's might rise (less competition). Across 500 stocks, some trials fail and some succeed. This risk washes out through diversification.

Now ask: which risk should investors be **compensated** for bearing?

Risks that can be diversified away are not priced

If a risk can be eliminated for free by diversifying, no rational market will pay you a premium for bearing it.

Only risks that survive diversification — **systematic risks** — earn expected returns above the risk-free rate.

Introduction to the CAPM

- The Capital Asset Pricing Model (CAPM) takes the insights from mean variance analysis and asks “what happens if everyone behaves this way?”
- With a risk-free asset, every investor holds some combination of the risk-free asset and the same tangency portfolio
- CAPM says:

  1. The tangency portfolio has to be the market portfolio
     - We’ll tackle this implication first
  2. Since everyone holds the same tangency portfolio, all assets are judged based on their contribution to the risk and return of the tangency portfolio

The Tangency Portfolio is the Market Portfolio

- Define the *market* portfolio as the **value-weighted portfolio of all assets traded in an economy**
- Assume all assets = 2 stocks and all investors = 2 investors with $50 each

After solving for the MVE/tangency portfolio:

1. both investors hold 20% of their risky portfolio in stock A and 80% in B
2. Lending/borrowing in the risk free asset sums to zero
   - In aggregate, every dollar lent by one investor is borrowed by another

Then, what's the market cap of each stock?

The Tangency Portfolio is the Market Portfolio

Then, what's the market cap of each stock?

- There are 100 dollars in the market
- The market capitalization of A is $0.2 \times \$100 = \$20$ and the market capitalization of B is $0.8 \times \$100 = \$80$
- Then, the market portfolio must be 20% weighted in stock A (\$20/\$100) and 80% (\$80/\$100) weighted in stock B
  - just like the MVE/tangency portfolio

Tangency = Market Portfolio

- We can "free-ride" on the market portfolio, rather than solve for the MVE
  - This logic motivates the growth in indexing over the past 30 years
- This result also allows us to restate the capital allocation line in terms of the market portfolio.

$$E(r_p) = r_f + \sigma_p \frac{E(r_m - r_f)}{\sigma_m}$$

where p denotes any complete portfolio along this line.

- Presented this way, it is commonly called the **Capital Market Line** (CML)

Expected Returns on Individual Assets

- Given that all investors hold the market portfolio, we only care about how asset i contributes to the risk and return of the market portfolio
- The contribution of asset i to the market excess return is its expected excess return, $E(r_i - r_f)$, times its market weight

$$E(r_m - r_f) = \sum_{i=1}^{n} w_i E(r_i - r_f)$$

- E.g. Walmart's contribution to our portfolio's excess return is $w_{\mathrm{WMT}} E(r_{\mathrm{WMT}} - r_f)$

Expected Returns on Individual Assets

- Asset i's contribution to the variance of the market portfolio is proportional to its covariance with the market, $\mathrm{Cov}(r_i, r_m)$

$$
\begin{aligned}
\sigma_m^2 &= \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \ \mathrm{Cov}(r_i, r_j) = \sum_{i=1}^{n} w_i \sum_{j=1}^{n} w_j \ \mathrm{Cov}(r_i, r_j) \\
&= \sum_{i=1}^{n} w_i \sum_{j=1}^{n} \mathrm{Cov}(r_i, w_j r_j) = \sum_{i=1}^{n} w_i \ \mathrm{Cov}\left(r_i, \sum_{j=1}^{n} w_j r_j\right) = \sum_{i=1}^{n} w_i \ \mathrm{Cov}(r_i, r_m)
\end{aligned}
$$

Key step: we can pull the weights w_j inside the covariance because covariance is a linear operator.

- E.g. adding Walmart to our portfolio increases risk by $w_{\mathrm{WMT}} \ \mathrm{Cov}(r_{\mathrm{WMT}}, r_m)$

Expected Returns on Individual Assets

- Next, the CAPM argues that in equilibrium, each security's contribution to return, weighted by its contribution to risk, must be equal
  - Any stock that contributes a higher (lower) return than justified based on its contribution to risk will be bought (sold) until this is no longer the case.
- From prior slide, for assets i and j

$$\frac{E(r_i) - r_f}{\mathrm{Cov}(r_i, r_m)} = \frac{E(r_j) - r_f}{\mathrm{Cov}(r_j, r_m)}$$

- This equality holds for all portfolios, *including the market portfolio*,

$$\frac{E(r_i) - r_f}{\mathrm{Cov}(r_i, r_m)} = \frac{E(r_j) - r_f}{\mathrm{Cov}(r_j, r_m)} = \frac{E(r_m) - r_f}{\mathrm{Cov}(r_m, r_m)} = \frac{E(r_m) - r_f}{\mathrm{Var}(r_m)}$$

- After some manipulation, we have the famous relationship

$$E(r_i) = r_f + \frac{\mathrm{Cov}(r_i, r_m)}{\mathrm{Var}(r_m)}(E(r_m) - r_f) = r_f + \beta_i (E(r_m) - r_f)$$

Interpreting the CAPM

$$E(r_i) = r_f + \frac{\mathrm{Cov}(r_i, r_m)}{\mathrm{Var}(r_m)}\big(E(r_m) - r_f\big) = r_f + \beta_i\big(E(r_m) - r_f\big)$$

- The CAPM makes a statement about how expected returns should vary across assets
- Answers the question "what expected returns assumptions are consistent with the market portfolio?"

Interpreting the CAPM

- Economically, it says investors receive returns proportional to a stock's exposure to broad market risks
  - Investors like stocks that hedge their market exposure and are willing to pay for this feature (or must be paid to forego it)
  - In fact, the only priced measure of risk is beta. Total risk = systematic risk + idiosyncratic risk, but only systematic risk (beta) is priced
  - The premium that investors are paid for accepting beta risk is the market risk premium, $E(r_m - r_f)$
  - Why is beta the only priced risk? Because diversification eliminates everything else...

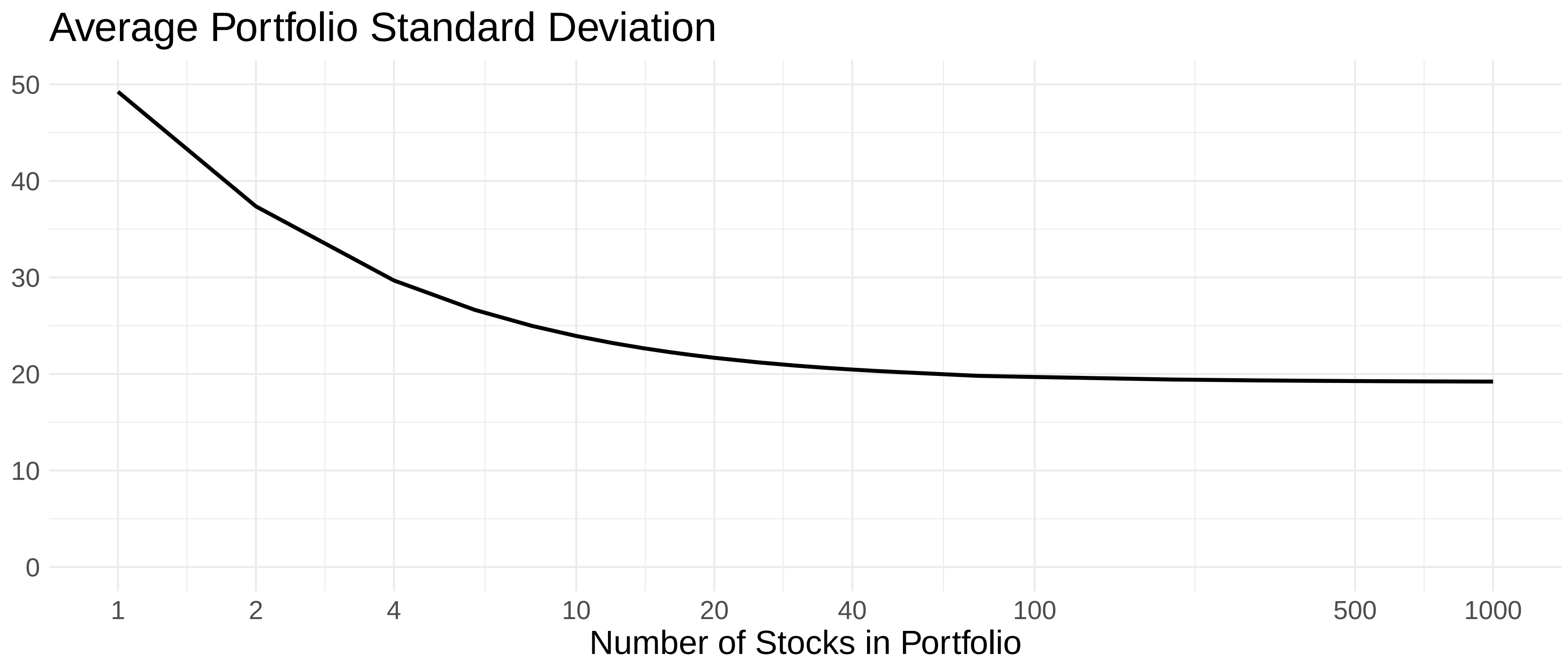
A Note on Diversification

- Suppose all assets have a common variance (σ^2) and equal covariance amongst each other ($\rho\sigma^2$). With weights of $1/n$ in each stock, the portfolio variance is

$$\sigma^2(r_p) = n \times \left(\frac{1}{n^2}\sigma^2\right) + n(n-1) \times \left(\frac{1}{n^2}\rho\sigma^2\right) = \underbrace{\frac{1}{n}\sigma^2}_{\text{idiosyncratic}} + \underbrace{\frac{n-1}{n}\rho\sigma^2}_{\text{systematic}}$$

- As $n \to \infty$: idiosyncratic risk ($1/n \to 0$) vanishes, only covariance (systematic) risk remains
- This result holds much more generally — the equal-weight/equal-variance setup is just for illustration

From Statman (1987)



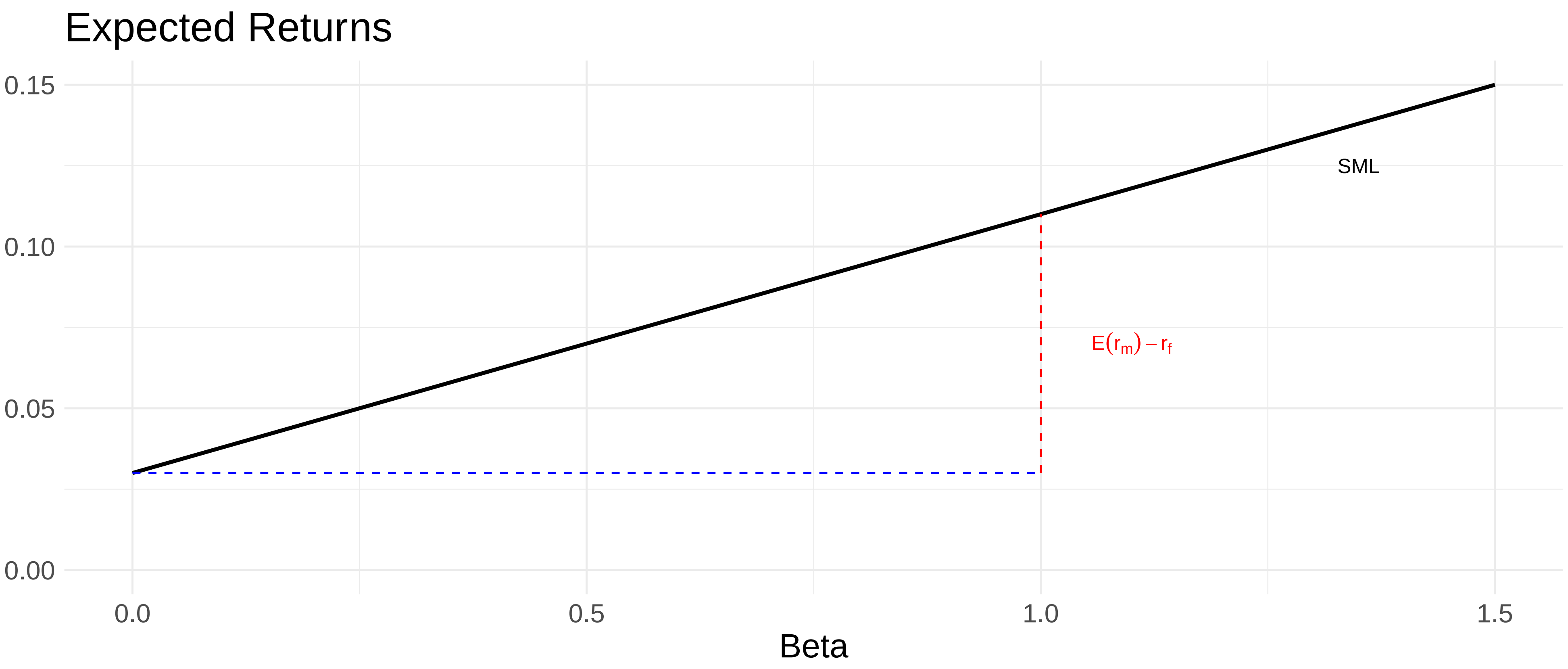
https://colab.research.google.com/github/paulgp/investment-management-notebooks/blob/main/06_diversification_benefits.ipynb

The Security Market Line

$$E(r_i) = r_f + \beta_i E\big(r_m - r_f\big)$$

- The Security Market Line plots the theoretical relationship between firm betas and expected returns
- Note: the CML plots *efficient portfolios* in $E(r)$ vs σ space; the SML plots *all assets* in $E(r)$ vs β space

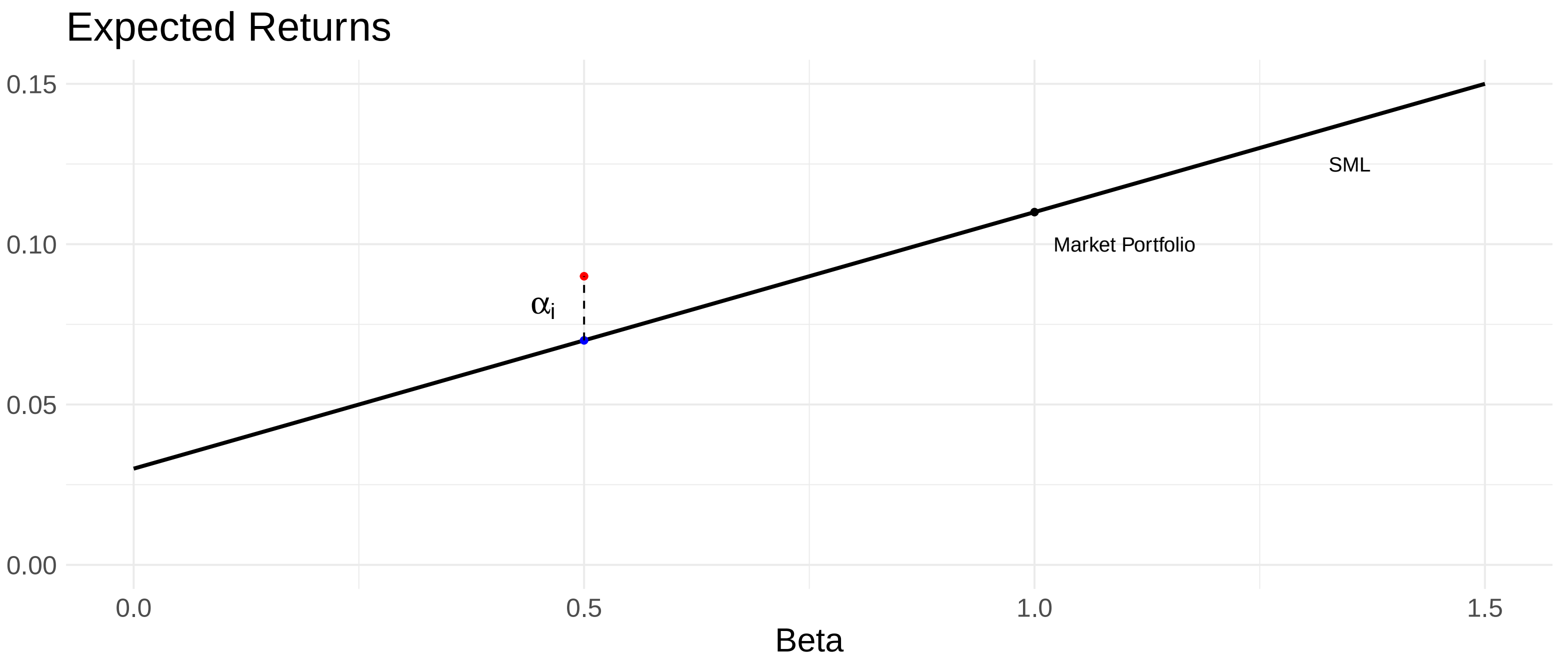
The Security Market Line



The Security Market Line

- A stock's alpha refers to its deviation from the predicted SML or

$$\alpha_i = E(r_i) - r_f - \beta_i E(r_m - r_f)$$



Implementing the CAPM

- Betas can be uncovered by running linear regressions of firm excess returns on market excess returns (LINEST in excel)
- In practice, we prefer market-weighted market indices like the S&P500 to measure r_m
- Monthly returns avoid noise of higher frequency returns, but allow for feasible estimates over relatively short periods
- Because betas may be dynamic, focus on prior 5 years

https://colab.research.google.com/github/paulgp/investment-management-notebooks/blob/main/07_estimating_beta_capm.ipynb

What Do Betas Look Like? Where Do They Come From?

- By definition, the average beta in the economy (weighted by value) is one (why?)
- Industry and capital structure good predictors of equity betas
- Can stocks have negative betas?
  - Put otherwise, would you ever accept a return less than the risk free rate?
- Also, remember, the betas we see are only statistical estimates of the true betas

What's Wrong with the CAPM?

- Accepting the wisdom of the CAPM involves its own trade-offs
- First, requires strong assumptions on distribution of returns, investor utility, information, and beliefs
  - Investors are rational mean-variance optimizers
  - Information is costless and available to all investors
  - There are homogeneous expectations
  - Individual investors are price takers
  - No taxes and transaction costs
- Second, it has faced empirical challenges (we’ll address this later)
- When these assumptions break down, additional risk factors beyond market beta may be priced — which motivates Arbitrage Pricing Theory

Next Class: February 12

Topics: Arbitrage Pricing Theory and Factor Models

- No-arbitrage assumption
- Single and multifactor models
- Fama-French factors

Matt Levine Reading: “The Robots Can Handle the Factors”