Bargaining on price on behalf of price-insensitive downstream consumers

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Abstract

There are settings in which prices are negotiated by procurement agents and final consumption decisions are made by end users who are indifferent to negotiated prices. For example, a patient seeking medical treatment is indifferent to the treatment's cost, if it is covered by his insurance program. We study bargaining for per-unit prices between suppliers and an intermediary who represents price-insensitive consumers. Under the commonly used simultaneous bargaining framework we show that, if suppliers have sufficiently high bargaining power, the resulting prices will exceed the value of the good (or service) being delivered. This overpricing is solved if simultaneous negotiations are replaced by sequential ones. The theoretical problem with sequential negotiations is that they necessitate treating the suppliers asymmetrically, even if they are symmetric; the empirical problem with sequential negotiations is that the negotiations-order is unobservable. We propose a multi-period model that resolves these issues: overpricing is prevented and all suppliers are treated the same. In this model, the result about sequential negotiations is utilized in order to produce (asymmetric) off-path threats. These threats sustain symmetric on-path play.

Keywords: Bargaining; Nash-in-Nash; Overpricing; Price insensitivity.

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1 Introduction

In most economic settings, agents care about prices and costs. Consumers care about the prices of the goods they buy, firms about their production costs, and so on. However, there are many settings in which linear prices are negotiated by procurement agents and final consumption decisions are made by end users who are indifferent to negotiated prices. For example:

- A fully-insured patient seeking medical treatment that is covered by medical insurance—he ignores the treatment's price when taking his decision since it is paid by the insurer;
- A military commander choosing which (or how much) ammunition to use in battle—he is unlikely to consider its costs.[1]
- The end user of a music subscription service is indifferent to the price the service provider pays for access to any particular music song.

A common feature of these examples is the vertically structured industry in which an intermediary (e.g., an insurance company, Department of Defense, a streaming service) bargains prices with multiple potential suppliers (e.g., hospitals, military contractors), under the assumption that the negotiated prices will have minimal to no effect on the end users' consumption decisions. In turn, final payments are determined by the negotiated price and the quantity consumed.

We study this scenario in the context of several models of negotiations. For concreteness, we consider an insurer who bargains with hospitals for per-treatment prices, and whose goal is to maximize the consumers' expected surplus net of prices paid.

[1] An account (in Hebrew) of ammunition-overuse in the IDF can be found in Shelah (2015), pages 44-45.

This case arises, for example, in the case of an employer providing health insurance to its employees who wants to maximize the benefit provided net of costs.[2] However, our results apply to any environment which is characterized by the aforementioned vertical structure and price-insensitivity.

The first bargaining environment we study follows the literature's dominant approach to multi-lateral negotiations originated in Horn and Wolinsky (1988). In the insurer-and-hospitals terminology, their work is such that the insurer bargains with each hospital in separate, simultaneous negotiations in accordance to the Nash bargaining solution (Nash 1950), taking the prices with all other hospitals as given. Collard-Wexler et al. (2019) call this approach *Nash-in-Nash* (NiN). We will sometimes follow their terminology but also refer to this as the simultaneous bargaining solution to more clearly differentiate it from our alternatives proposed below.

An important distinction between the bargaining environment studied in Collard-Wexler et al. (2019) and our setting is that in their environment bargaining is over a lump sum payment, whereas in our setting bargaining is over linear prices, with quantities determined by downstream consumers. Bargaining over per-unit prices is commonly observed in the real world and has been dominant in applied work. For example, in their analysis of bargaining in the multichannel TV market, Crawford and Yurukoglu (2012) assume distributors and content providers bargain over per-subscriber licensing fees. Similarly, analysis of bargaining between hospitals and insurers assumes bargain over capitation rates rather than lump sum payments (e.g., Gowrisankaran et al. 2015, Ho and Lee 2017). We will refer to our basic model as the

[2]For simplicity, we will assume that the intermediary is a monopolist in the sense that consumers cannot substitute to other intermediaries. This is natural in the case of employer provided insurance or military contracting. In other settings, such as music streaming or the individual health insurance market, intermediaries may compete. Considering competition is a much more complicated problem. In the case of insurance, insurers will compete over both networks and premiums. While the forces underlying Nash Bargaining would be present in this setting, a full analysis of this context would be outside the scope of this paper.

linear NiN model in order to distinguish it from the lump sum approach of Collard-Wexler et al. (2019). As we explain in the following paragraph, the main issue with which we are concerned in this paper can come about only in an environment with linear pricing (as opposed to lump-sum payments).

In the linear NiN model, the insurer bargains with each hospital simultaneously, holding the contracts with all other hospitals fixed. The Nash product is formed by comparing the hospital-network's surplus with and without the bargained-with hospital. To see this more concretely, consider the case of two hospitals, A and B. Consider bargaining with A, given the negotiated price with B. The network's surplus without A depends not only on B's price but also on A's patients that would substitute to B if A left the network, and their value for B. Since patients are price-insensitive, they will go to hospital B even if their value for B is lower than B's price.[3] Thus, in the event that A drops out of the network, these patients may generate negative surplus: the utility they get from B—their second choice hospital—is less than B's price. As a result, A's marginal value per-patient when added to the network (given B's price) is higher than its actual per-patient value, since its addition prevents the aforementioned negative surplus creation. We show that as long as some patients substitute to an in-network hospital when their most preferred hospital is out of the network, prices exceed patient valuations, given that hospitals' bargaining power is sufficiently large. We call this phenomenon *Nash overpricing*. In Theorem 1 we show that if the hospitals' bargaining power is large enough, Nash overpricing will occur for every hospital in the network, which we refer to as *complete Nash overpricing*.

Since overpricing does not pass any reality-check, the meaning of the above result is that, in the present context, assuming simultaneous negotiations is an inappropri-

[3]This substitution is the reason that linear pricing is important, under lump-sum pricing, patient substitution in the event of disagreement with A would not affect the insurer's total payment to hospital B and so negative surplus would not arise.

ate modeling approach. We propose alternative approaches, that fare better.

First, we establish that if negotiations are carried out sequentially rather than simultaneously, overpricing is prevented. With J hospitals ordered in a sequence, given that prices with hospitals $\{1, \cdots, J-1\}$ have been determined, the insurer faces a standard 2-player bargaining problem with hospital J, in which (under mild assumptions) his payoff is positive. In the negotiation with hospital $J-1$ this is taken into account, and so negotiations with hospital $J-1$ are also a standard 2-player bargaining problem in which the insurer's payoff is positive, and so on.

Whereas sequential negotiations prevent overpricing, they present challenges that do not appear under simultaneous negotiations. More specifically, the agents' payoffs may depend on the order of negotiations, hence the order may be of crucial importance. But there seems to be no good reason to prefer one order over another, and, moreover, this is also an issue for applications, because negotiation-orders are unobservable. In our model, the insurer's payoff is independent of the order of negotiations[4] but hospitals care about the order of negotiations, at least under some conditions. In particular, this is the case if the hospitals are symmetric or if there are only two hospitals. The 2-hospital case exemplifies the point easily: here, the first hospital's price is lower than that of the second, because once there is disagreement with the first hospital and it "drops out" of the game, the second hospital becomes a monopolist; by contrast, there is no such effect for the first hospital when the second hospital drops out.

It turns out that, under very general circumstances, the first hospital in the sequence earns a low payoff. Based on this fact, we propose a model in which overpricing is resolved, and hospitals are treated symmetrically. Specifically, we propose

[4]The reason is that the prices that are obtained by maximizing Nash products internalize the effects of earlier negotiations on later ones.

the following multi-period model. Every period the insurer makes simultaneous price offers to all hospitals, to which they simultaneously respond by acceptance or rejection. If all accept, the offered prices are contracted for the period, and the game moves one period ahead. Hospitals discount future payments with a common discount factor, $\delta \in (0,1)$. In equilibrium everybody accepts their offers, so a rejection means a deviation; once there is a rejection by some hospital j^D, the model enters an absorbing phase in which the sequential model is repeated in every period, and the negotiations-order is one in which j^D is placed first. As mentioned above, the price that this first-placed hospital obtains is low, hence it deters the hospitals from rejecting their price offers.

In the multi-period model, we focus on the equilibrium which is best for the insurer. As $(\beta, \delta) \to (1,1)$, the equilibrium price paid to each hospital converges to the above-mentioned "punishment price"—namely, the price that the hospital obtains under sequential negotiations when it is first in the sequence. Interestingly, this price turns out to be the hospital's *standalone value*—what the hospital contributes on average per treated patient, when it is the only hospital in the network. It is smaller than the hospital's value, because when the hospital is the only available option, it also serves patients for whom it is the second-best choice. Therefore, since the price paid to every hospital is below this hospital's value, the insurer's payoff is positive.

The rest of the paper is organized as follows. Section 1.1 reviews the literature. Section 2 lays down the environment, Second 3 considers simultaneous negotiations, Section 4 considers sequential negotiations, Section 5 considers the multi-period model, and Section 6 concludes. The overpricing problem that we identify in the linear NiN model is derived under the assumption that the pool of hospitals is exogenous. In Appendix A we show that being able to exclude some hospital from the pool (*ex ante exclusion*), as well as excluding some hospitals after contracts with them have already been signed (*ex post exclusion*), do not provide a satisfactory solution to overpricing. Thus, the overpricing problem is robust. In Appendix B we discuss the insurer's outside option, Appendix C provides estimating equations for the multi-period model and Appendix D collects proofs.

1.1 Literature

Our paper belongs to a strand of literature that concerns bilateral bargaining in vertically-structured markets.[5] Horn and Wolinsky (1988) and Collard-Wexler et al. (2019) are central references in this regard.

One of our non-trivial findings is that under sequential negotiations, the insurer's payoff is independent of the hospital order. There are bargaining settings in which the order of negotiations matters (Manea 2018, Xiao 2018), and there are settings in which it does not (Marx and Shaffer 2007, Krasteva and Yildirim 2012). A deeper investigation on order-dependence (or independence) in an environment with price-insensitive consumers is beyond the scope of the present paper.

The bargaining externalities in our paper (the price paid to a given hospital influences bargaining with other hospitals) make it relate to the literature on contracting with externalities. However, much of this literature concerns externalities among agents, whereas we focus on the principal who is contracting with them (the insurer).[6]

Finally, though our study is theoretical, our models are inspired by the applied theoretical work from the health economics literature; an important reference in this regard is the handbook chapter by Gaynor and Town (2011). Versions of our linear NiN model are also used in Crawford and Yurukoglu (2012), Gowrisankaran et al.

[5] A more general framework is that of bargaining in networks. E.g., Abreu and Manea (2012), De Fontenay and Gans (2013), Stole and Zwiebel (1996).

[6] See, e.g., Galasso (2008), and Genicot and Ray (2006).

(2015) and Ho and Lee (2017) and recently reviewed by Lee et al (2021).

2 The environment

An insurer bargains with $J \geq 2$ hospitals on behalf of a mass of heterogeneous patients. Under the full network, which comprises all J hospitals, the quantity of patients treated by hospital j is $q_j > 0$. The expected value for a patient who goes to hospital j (given the full network) is $v_j > 0$. More specifically, the mass of patients who go to hospital 1 under the full network is $[0, q_1)$, the mass of those who go to hospital 2 under the full network is $[q_1, q_1 + q_2)$, and so on. The aforementioned expected value v_k is $v_k = \int_{q_{k-1}}^{q_{k-1}+q_k} v_{ki} di$, where v_{ki} is the individual valuation of patient i in the segment $[q_{k-1}, q_{k-1} + q_k)$, where $q_0 = 0$. Regardless of what hospitals are in the network, patients always have the (outside) option of not seeking medical treatment, which is associated with the value zero.

If a hospital, say j, is not part of the network, then it is not available for patients to seek treatment. This event only affects those patients who prefer to be treated at j, who then go to their next preferred hospital. The hospital choice of patients that chose hospital $l \neq j$ when j is in the network does not change. The mass of additional consumers for hospital k when hospital j is dropped from the network and those patients' expected value are denoted by $q_{k,-j}$ and $v_{k,-j}$, respectively.

Hospitals can treat patients with a marginal cost $c_j \geq 0$. Therefore, under the full network hospital j's profit is:

$$\pi_j = (p_j - c_j)q_j.$$

If hospital j is out of the network, it receives zero profit. However, if another hospital, say k, is out of the network, j receives more patients and its profits become:

$$\pi_{j,-k} = (p_j - c_j)(q_j + q_{j,-k}) \ .$$

The hospitals are *symmetric* if $\{q_j, v_j, q_{k,-j}, v_{k,-j}, c_j\}$ are independent of k and j. We make the following assumptions:

- (I) For all j: $\sum_{k \neq j} q_{k,-j} > 0$;
- (II) For all distinct j and k: $q_{j,-k} > 0 \Rightarrow v_j > v_{j,-k}$;
- (III) For all j: $c_j < \min\{v_{j,-k} : q_{j,-k} > 0\}$;
- (IV) For all distinct j and k with $q_{k,-j} > 0$: $v_j - c_j > v_{k,-j} - c_k$.

(I) says that for every hospital j, at least some patients have a second-choice-hospital that they prefer over the outside option. (II) requires that patients whose first choice of a hospital is j, on average value hospital j more than patients for whom j is the second choice. (III) says that the surplus from providing service to patients for whom the service provider is the second choice is positive. This has two important implications. First, it implies—because of (I) and (II)—that $c_j < v_j$, and so the surplus from the full network is positive. Second, it means that negative surplus for the insurer—the thing around which our linear NiN model revolves—is *only* because of overpricing, not because of providing service by inefficient hospitals. Finally, (IV) is a bound that means that the surplus generated by the first-choice hospital is large enough; specifically, it is greater than the surplus that would have been generated had that first choice been removed from the network and then we looked at what surplus the patients who remain in the network generate in any other hospital. This assumption follow from (II) if all hospitals have the same cost. Assuming zero costs, namely $c_1 = \cdots = c_J = 0$, is a convenient normalization. Under zero costs, (III) and (IV) follow from (I) and (II).

We assume the insurer maximizes patient surplus, net of prices paid. Therefore, the insurer's value from the full network, given a price vector $p = (p_1, \ldots, p_J)$, is:

$$F(p) = \sum_{j=1}^{J} (v_j - p_j) q_j. \tag{1}$$

This assumption corresponds to two plausible scenarios. First, the insurer will maximize patient valuation if it is acting as an agent for consumers seeking, as one might suppose of a self-insuring employer offering medical insurance as a benefit to employees. Second, an insurer will do so if it has complete market power over consumers and is able to fully extract the surplus.

Similarly, the insurer's surplus from the network without hospital j, given the remaining hospital prices, is:

$$F_{-j}(p) = \sum_{k \neq j} \left[(v_k - p_k) q_k + (v_{k,-j} - p_k) q_{k,-j} \right]. \tag{2}$$

We study several bargaining protocols between the insurer and the hospitals, to be specified in the next sections; the (asymmetric) Nash bargaining solution is common to all of them, which justifies the following terminology: we say that *Nash overpricing* occurs if there is some hospital j whose price exceeds its value, i.e., $p_j > v_j$. If this is true for every j, then we say that there is *complete Nash overpricing*.

3 Simultaneous negotiations

We start by considering the case where prices between the insurer and each hospital are set following the Nash bargaining solution, holding all other prices fixed. The hospitals' bargaining power parameter is $\beta \in (0,1)$. We refer to this model as the *linear NiN model*, and to its prices as *NiN prices.*

The NiN prices $(p_1^N, \cdots, p_J^N)$ satisfy:

$$p_j^N = \max_{p_j} [F(p_j, p_{-j}^N) - F_{-j}(p_{-j}^N)]^{(1-\beta)} \cdot [q_j(p_j - c_j)]^{\beta}.$$

Maximization of the Nash product gives:

$$p_j^N = \beta[v_j - \frac{\sum_{k \neq j}(v_{k,-j} - p_k^N)q_{k,-j}}{q_j}] + (1-\beta)c_j. \tag{3}$$

A solution to this system of equations is called an *equilibrium.*

Theorem 1. *In the linear NiN model, an equilibrium exists, and it is unique. There exists a* $\bar{\beta} < 1$*, such that if the hospitals' bargaining power parameter satisfies* $\beta \in (\bar{\beta}, 1)$*, then each of the NiN prices exceeds the value of service in the corresponding hospital. That is,*

$$p_j^N > v_j \quad \forall j = 1, \cdots, J.$$

Namely, there is complete Nash overpricing.

Proof. We start by establishing existence and uniqueness. Note that we may assume that prices never exceed some (possibly large) bound M: clearly, no hospital can obtain a price that exceeds its value plus all the "adverse selection prevention" that its addition to the network can bring about. Then the RHS of (3) describes a map from $[0, M]^J$ into itself. Though this is a map of vector-to-vector, it can be viewed as an operator on functions because a vector is a constant function. It is easy to check that this operator—i.e., the RHS of (3)—satisfies Blackwell's sufficient conditions for contraction (monotonicity and discounting). Therefore, (3) has a unique solution; that is, an equilibrium exists, and is unique.

We now turn to complete Nash overpricing. Consider the formulas for NiN prices:

$$p_j^N = \beta v_j + \frac{\beta}{q_j} \sum_{k \neq j} (p_k^N - v_{k,-j}) q_{k,-j} + (1 - \beta) c_j. \tag{4}$$

Let p^0 be the vector of prices that solves the above system (uniquely), when $q_{k,-j} = 0$ for all distinct k and j. That is, $p_k^0 = \beta v_k + (1 - \beta) c_k$. Suppose that β is large enough, so that each p_k^0 is sufficiently close to v_k, so that the following holds: $p_k^0 > v_{k,-j}$ for all $j \neq k$.

Now increase $\{q_{k,-1}\}_{k>1}$ from zero to their true values. These $(J - 1)$ coefficients only appear in the formula for the first price, and because $p_k^0 > v_{k,-1}$ for all $k \neq 1$ this price increases: it changes from p_1^0 to some $\tilde{p}_1^0 > p_1^0$. The change $p_1^0 \mapsto \tilde{p}_1^0$ increases any other price p_k^0, and since prices depend positively on one another, the new price vector that results, call it p^1, satisfies $p_k^1 > p_k^0$ for all k.

Now increase $\{q_{k,-2}\}_{k \neq 2}$ from zero to their true values. By the same logic, the resulting price vector, call it p^2, satisfies $p_k^2 > p_k^1$ for all k. Repeating this process iteratively we end up with the vector of NiN prices, p^N. The analysis above implies that the following holds for all k and $j \neq k$:

$$p_k^N > \cdots > p_k^2 > p_k^1 > p_k^0 > v_{k,-j}.$$

Thus, when $\beta \sim 1$ it follows from (4) that $p_j^N \sim v_j + \frac{1}{q_j} \sum_{k \neq j} (p_k^N - v_{k,-j}) q_{k,-j} > v_j$. $\square$

Empirical Implications Under Nash overpricing, the insurer's payoff is negative, implying that the insurer would have been better off allowing negotiations to break down. As this outcome is unrealistic, using the linear Nash-in-Nash model implies an upper bound on the hospital's bargaining power parameter β to ensure that insurer payoff is non-negative. To see the degree to which the possibility of Nash overpricing limits the range admissible bargaining parameters, we consider a simple example calibrated to resemble the empirical setting of Gowrisankaran et al. (2015, hereafter GNT).

GNT study negotiations a multi-lateral negotiation between a single insurer and 11 hospital systems. While they estimate demand using a conditional multinomial logit demand model with observable consumer heterogeneity and simultaneously estimate bargaining parameters and costs, we assume a simplified structure where hospitals are symmetric, marginal costs are 0 and patients' taste for hospitals follows the logit model.[7] Under these assumptions, each hospital's demand is

$$q_j = M \frac{e^{\psi}}{1 + e^{11\psi}}. \tag{5}$$

where M is market size and ψ represents the baseline utility of a hospital relative to choosing the outside good (selecting a hospital outside the insurance network). As M scales all quantities, we normalize it to 1 without loss of generality. We calibrate ψ so that outside share, $1/(1 + e^{11\psi})$, matches the outside share reported by GNT—1.4 percent of patients choose an out of network hospital. Thus we use $\psi = 1.8567$.

Given, the calibrated demand system, $q_{k,-j}$, v_j, and $v_{k,-j}$ are symmetric and can

[7]That is, patient i's unconditional utility of going to hospital j is $\psi + \varepsilon_{ij}$ and the unconditional utility of forgoing treatment is ε_{i0} where the vector ε_i consists of independent draws from the Type-I extreme value (Gumbel) distribution. Patient i observes ε_i and selects the hospital that maximizes their utility.

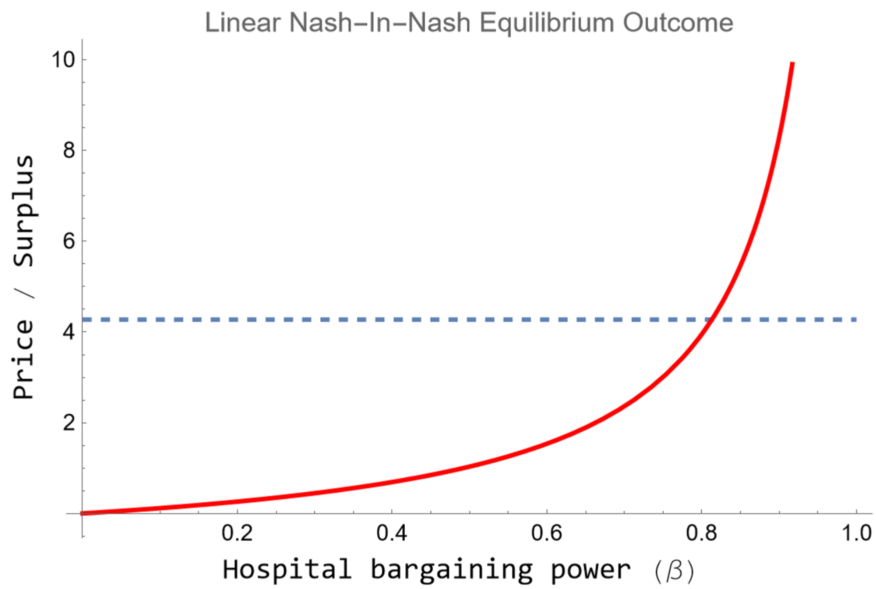

Figure 1: Linear Nash-in-Nash equilibrium outcome as a function of hospital bargaining power, equilibrium price exceeds the total per-consumer surplus generated by the network for $\beta = .82$, implying complete Nash overpricing.

be straightforwardly computed for all j, k.[8] We plug these into (3) and then solve for the linear NiN prices for the full range of $\beta \in [0, 1]$. Figure 1 plots the Nash-in-Nash negotiated price as a function of hospital bargaining power, β.[9] The per-patient surplus provided by the hospital network, which is independent of β, is plotted as the dashed line. Prices become extremely sensitive to the bargaining parameter when the hospital has a higher bargaining weight than the insurer, and for bargaining parameters above .82, complete overpricing occurs.

In this setting, survey results have indicated that hospitals have significant bargaining power over insurers (Devers et al. 2003) and that premiums increase with hospital market concentration (Trish and Herring 2015). In their empirical work,

[8]By the independent of irrelevant alternatives (IIA) property of the logit, $q_{k,-j} = e^{\psi}/(1 + e^{10\psi})$. Further, the logit assumption plus symmetry implies that $v_j = \log(1 + 11e^{\psi})$. and $v_{k,-j} = \log(1 + 10e^{\psi})$.

[9]Imposing symmetry further reduces this system of 11 equations to a single equation. The Mathematica notebook to produce this Figure 1 is available by request.

GNT either fix bargaining power at 0.5 or estimate that insurers have more bargaining power.

Our results suggest that one reason for the finding of relatively high insurer power in GNT is bias due to the effects of Nash over-pricing. To provide some intuition, recall that, as in Figure 1 and Theorem 1, with high enough β, the NiN price will exceed the insurer's valuation, so the NiN bargaining solution should not be accpeted by the insurer. This is because the insurer's disagreement value under linear NiN is calculated holding other prices fixed, but it is unrealistic to assume the insurer will accept this outcome upon disagreement. Suppose instead that some alternative Nash bargaining model was used where the insurer's disagreement value D_{-j} is higher than the one implied by the NiN model, $F_{-j}(p^N_{-j})$, as suggested by renegotiation following disagreement. Following the literature, use the first order condition of the Nash bargaining problem to arrive at the following condition to estimate the bargaining parameter β and hospital marginal costs c,[10]

$$(p_j - c_j) = \frac{\beta}{1-\beta} \frac{F(p) - D_{-j}}{\partial F / \partial p_j}$$

Consider a researcher who observes prices and has estimated demand attempting to learn bargaining parameters and costs. As D_{-j} is unobserved, the researcher employs the mis-specified linear Nash-in-Nash model for estimation by using $F_{-j}(p^N_{-j})$ in place of D_{-j}. Since $D_{-j} > F_{-j}(p^N_{-j})$, the right hand side will be biased upwards. There are two ways to restore the equality by biasing the estimates $(\hat{\beta}, \hat{c}_j)$: either reduce hospital bargaining power from its true value β (decreasing the right hand side), or lower hospital costs and increase markups (increasing the left hand side). We would

[10]This condition is equivalent to Gowrisankaran et al. (2015, equation 16) when we (a) impose linear NiN bargaining so $D_{-j} = F_{-j}(p^N_{-j})$ and (b) allow for price sensitive consumers which we do not consider in this paper. An even more general version that allowing for multi-hospital systems (but still imposing linear NiN) appears as equation 14 in GNT.

expect both biases to be present in an empirical setting.

The overpricing phenomenon comes from the substitution across hospitals in the event of disagreement coupled with the Nash assumption of holding other negotiated prices fixed. It is exacerbated when the utility of consumers that prefer the disagreement hospital is low for their substitute hospital (relative to the disagreement hospital) but still higher than the utility of the outside option. For example, if we reduced the number of hospitals in our example from 11 to 4, the cutoff in hospital bargaining weights for Nash overpricing falls to .76. This is because the gap between the surplus generated by a consumers' first and second option more than doubles, while the share of the outside option only rises to 3.7 percent, so the vast majority of consumers still substitute to another hospital in case of disagreement. This suggests that Nash overpricing may be particularly severe in more concentrated or differentiated markets.

Once demand is estimated, the bargaining cutoff for Nash overpricing can be computed for any cost structure by simply raising bargaining weights until equilibrium prices exceed per-consumer surplus. Doing so would allow empirical researchers to gain an understanding of this implicit restriction on bargain weights imposed by the NiN model. In cases where this restriction on bargain is too severe, we now consider alternative negotiations which relieve the Nash overpricing phenomenon.

4 Sequential negotiations

The difficulty with the linear NiN model lies in the hypothetical "disagreement event" associated with each negotiation. To see this concretely, consider two hospitals, A and B. When A is out of the network (i.e., there is disagreement with A) its entry-contribution exceeds its true value because its absence from the network generates an adverse shift in patients going to B which would be welfare reducing at B's price. Avoiding this outcome depends on constructing a bargaining mechanism where the impact of disagreement in a certain problem on other bargaining problems is taken into account. We now take this approach, in our *sequential Nash model*, which is as follows: the hospitals are ordered in a (commonly known) sequence, and if negotiation breaks down with some hospital j, all subsequent negotiations assume that j is not in the insurer's network. That is, the economic environment is the same as the one considered in the linear NiN model, the only difference is that the Nash products reflect the order of negotiations.

In the case of only two hospitals, it is easy to see that the insurer's surplus is positive under sequential negotiations: Given any outcome with the first-negotiated hospital A, the bargaining problem with B must result in a non-negative addition to the insurer's overall surplus (or else the insurer will not sign a deal with B). Additionally, one can map any possible outcome in the A-negotiations, say o, to the subsequent bargaining problem with B, say $P(o)$. Since the insurer's surplus in $P(o)$ is non-negative (under some mild assumptions) given any possible o, the bargaining problem with A boils down to a standard bargaining problem in which both parties make positive profits.

Generalizing this idea to an arbitrary-length hospital sequence requires taking care of the following subtlety. Suppose that the hospitals are ordered in a sequence, from 1 to J, and consider bargaining with some hospital j. If there is disagreement in these negotiations then j drops out of the game and the insurer continues to bargain with subsequent hospitals. Consider one of these hospitals, say hospital $j + k$. In order to formulate the Nash product for *these* negotiations, it is important to know what happens in case there is disagreement with $j + k$, and, in particular, we need to know what would happen to the patients who had j as their top choice, and would choose

$j+k$ if j is out of the network. We assume that patients leave the insurer if their top two options leave the network, an assumption that automatically resolves the above complication. To use the notation from Section 2's first paragraph: for every patient i there are at most two ks such that $v_{ki} > 0$.

Similarly to the linear NiN model, here, too, an "equilibrium" is a list of prices that maximize the relevant Nash products, the only difference, as mentioned above, is that now these products are given in a particular exogenous order, and once there is disagreement with a certain hospital it leaves the game and not accounted for in the subsequent products.

Theorem 2. *In the sequential Nash model, an equilibrium exists, and it is unique. In equilibrium, the insurer's payoff is positive, the full hospital network is formed , and there is no Nash overpricing.*

Proof. The proof is by induction on the length of the hospital-sequence, $\{1, \cdots, J\}$. For $J = 1$ the theorem's claim is obvious, due to the uniqueness of the Nash bargaining solution's outcome, and the fact that it provides a non-negative payoff for every bargainer. Consider then $J' = J + 1$. Given that there is disagreement with the last hospital in the sequence, $J + 1$, the insurer's payoff, by the combination of the induction's hypothesis and assumption (III), is some $d > 0$.[11] By assumption (IV), striking a deal with hospital J' is worthwhile, and the Nash solution provides the insurer a payoff greater than d is these negotiations.

Therefore, the full network is formed and the insurer's payoff is positive. The equilibrium is unique, because the prices are pinned down uniquely by the sequential maximization of the Nash products. It remains to verify that, in the unique equi-

[11] The assumption that patients leave the insurer if their top two choices are out of network simplifies the calculation of d significantly. This will be evident in Appendix D, which contains such calculations.

librium, there is no overpricing.

Let then J and $J' = J + 1$ be as above. It is clear that there is no overpricing for $J = 1$. Suppose that there is no overpricing for all hospitals up to and including J, but there is overpricing for J'. This, however, contradicts the fact that was established at the last sentence of the proof's first paragraph. $\square$

For any of the $J!$ possible orders, the sequential model's equilibrium is unique. However, these equilibria are not payoff-equivalent. Specifically, while the order of negotiations does not affect the insurer's surplus, it does affect negotiated prices and hospital payoffs, at least in some cases. For example, in the 2-hospital case where A is first, disagreement with A automatically makes B a monopolist.[12] In contrast, disagreement with B cannot have such a favorable effect on A's bargaining position, since it can only happen after the interaction with A has concluded.

Formally, the results about equilibrium-payoffs are as follows.

Proposition 1. *In the equilibrium of the sequential Nash model with J hospitals, the insurer's surplus is independent of the order of negotiations.*

For the result concerning the hospitals' payoffs, we need to assume symmetry. In the result's statement, π^l is the equilibrium-profit of a hospital if it is in the l-th position in the sequence.

Proposition 2. *There exists a $\tilde{\beta} < 1$ such that if $\beta \in (\tilde{\beta}, 1)$ then π^l is strictly increasing in l.*

The proofs of Propositions 1 and 2 involve some tedious algebra, and are therefore relegated to Appendix D.

Recall that the hospitals have names: hospital 1, hospital 2, etc. Suppose that

[12] A detailed account of the 2-hospital case appears in a working paper version of this paper, and is available from the authors upon request.

the negotiation sequence is given by these labels, namely hospital 1 is the first in the sequence, hospital 2 is second, and so on. Then, the price obtained by hospital i is:[13]

$$p_i = \beta \frac{q_i v_i + \beta \sum_{k=i+1}^{J} q_{i,-k} v_{i,-k} - \sum_{j=1}^{i-1} q_{j,-i}(v_{j,-i} - p_j) - (1-\beta) \sum_{j=i+1}^{J} q_{j,-i}(v_{j,-i} - c_j)}{q_i + \beta \sum_{k=i+1}^{J} q_{i,-k}} + \\ + (1-\beta) c_i \quad (6)$$

Setting in the formula $i = 1$ makes the middle term in the numerator disappear.[14] Therefore, when $\beta \to 1$ this price converges to:

$$p_1^{SAV} \equiv \frac{q_1 v_1 + \sum_{k=2}^{J} q_{1,-k} v_{1,-k}}{q_1 + \sum_{k=2}^{J} q_{1,-k}}. \quad (7)$$

This is hospital 1's *standalone value*—what it contributes (per patient) when it is the only hospital in the network. It is easy to see that $p_1^{SAV} < v_1$. Similarly, $p_j^{SAV} < v_j$ for every j.

5 A multi-period model

While the sequential model does not exhibit the Nash overpricing phenomenon, it is not suitable for empirical work because hospital surplus depends on the order of sequential negotiations, which is typically not observed. To address this, we now provide a model where hospitals negotiate simultaneously that alleviates Nash overpricing. We utilize our results from the previous section in the following infinite-horizon model. In each period $t = 1, 2, \cdots$, the insurer makes simultaneous price offers to the hospitals. These are given by a publicly-observed vector $(p_1(t), \cdots, p_J(t))$, where $p_j(t)$

[13]The derivation appears in Appendix D.

[14]This terms refers to i's predecessors, which, for $i = 1$, do not exist.

is the offer made to hospital j. The hospitals react simultaneously by accept/reject responses. If all accept, the prices are implemented and play moves on to the next period. Once there is a rejection by a single hospital, say j^D, the following occurs:

- With every $j \neq j^D$, the price $p_j(t)$ is contracted with for period t, where t is the period being considered (i.e., the one when the deviation occurred).
- The price contracted with j^D is $\tilde{p}_{j^D}$; it is determined by bargaining between this hospital and the insurer, but as we will soon see there is no importance to what exactly this price is, or to the bargaining mechanism that generates it.
- From $t+1$ onwards, prices are set at $(p_1(j^D), \cdots, p_J(j^D))$ in every period, where these are the prices obtained from the sequential negotiations model, where the negotiations order is one of the orders in which j^D is placed first in the sequence. Specifically, the order is selected by a uniform randomization over all orders in which j^D is placed first.

If several hospitals deviate simultaneously, then the deviator to be punished is selected from among these deviators according to the uniform distribution.

We assume that all hospitals share the discount factor $\delta \in (0, 1)$ and we look for a time-independent price vector $(p_1^*, \cdots, p_J^*)$ that will be accepted by all hospitals. Consider hospital 1. Its associated incentive constraint is:

$$(1-\delta)\tilde{p}_{1^D} + \delta p_1 \leq p_1^*, \tag{8}$$

where p_1 is given by (6). The analogous equations hold for all $j = 2, \cdots, J$. We define a solution to this model (an equilibrium) as a vector of prices, $(p_1^*, \cdots, p_J^*)$, that satisfy the J constraints and maximize the insurers' payoff. Clearly, these are the prices under which the J inequalities hold as equalities.[15] As $\delta \to 1$ each p_i^* converges to p_i. By equation (7), as $\beta \to 1$ the price p_i converges to p_i^{SAV}, namely hospital i's standalone value, $\frac{q_i v_i + \sum_{k \neq i} q_{i,-k} v_{i,-k}}{q_i + \beta \sum_{k \neq i} q_{i,-k}}$. Since this standalone value is strictly smaller than v_i, the insurer makes positive profits under the model's solution when $(\beta, \delta) \to (1, 1)$. In particular, there is no over-pricing when $(\beta, \delta) \to (1, 1)$.

In the above analysis, we do not specify an entire game-tree, hence, strictly speaking, our solution is not a (subgame perfect) equilibrium. Instead, it is simply defined to be a vector of prices that satisfies (7). Such prices clearly exist: given whatever exists on the LHS, one can simply pick prices p_i^* that are sufficiently large to satisfy the inequality. On the other hand, there are multiple such prices, so there is no uniqueness here. Nonetheless, this suffices to establish that the infinite horizon model eliminates Nash overpricing.

Implications for Estimation We now explore the possibility of employing the multi-period model in empirical work. We will show that—similar to the linear NiN model—the multi period model can be estimated by minimizing a GMM objective function over costs, bargaining parameters, and δ. Moreover, if we fix $\delta = 0$, this objective function is identical to the GMM objective function used to estimate the linear NiN model.

The linear NiN framework has served as a workhorse model for the empirical investigation of multilateral bargaining, in part due to it's empirical tractability (Crawford and Yurukoglu 2012, Gowrisankaran et al. 2015, Ho and Lee 2017). This tractability stems from the fact that, given observed prices and estimated demand elasticities, it is possible to recover implied costs from the linear NiN model by solving a linear system of equations. This permits straightforward estimation of linear NiN bargaining

[15] If not, then the RHS can be decreased, which increases the insurer's payoff without upsetting any constraint.

parameters and costs via generalized method of moments.

In Appendix C, we propose estimating the multi-period model, which requires a specification of disagreement prices $\tilde{p}_{j^D}$. We assume $\tilde{p}_{j^D}$ is determined by Nash-in-Nash bargaining fixing all other prices at their contracted outcomes $p_j(t)$ for $j \neq j^D$. Under this assumption our multi-period model can also be estimated via GMM. Moreover, the multi-period model represents a statistical generalization of linear NiN with one additional parameter—the discount factor δ. The system of equations that define costs under the multi-period model remains linear (see equation 14 in Appendix C). Thus, there is no increase in computational complexity when estimating our proposed model instead of linear NiN beyond the introduction of an additional parameter δ.

One would like to know what is the relation between the multi-period model and linear NiN. In Remark 1 of Appendix C, we establish that if $\delta = 0$, the moment conditions of our multi-period model are identical to the moment conditions of the linear NiN model. As a result, the linear NiN model can be tested by estimating the more general multi-period model and testing the null hypothesis that $\delta = 0$.

6 Conclusion

We have studied bargaining between an intermediary and suppliers in which the total surplus from a suppliers' network is non-linear in the quantity sold by each supplier. This feature makes sense when the intermediary acts on behalf of price-insensitive users. We showed that under the common NiN approach, suppliers may charge unit prices that surpass the unit value of their service, because of the negative surplus which is created from directing users to second-best choices. If negotiations happen in a sequence, this overpricing problem cannot arise. However, in the sequential model, prices depend on the order of negotiations, which is typically unobserved.

To address this, we have also constructed a multi-period model in which all suppliers are treated identically in terms of their place in the bargaining mechanism (as opposed to one-shot sequential negotiations), and the intermediary makes a positive payoff (as opposed to one-shot simultaneous negotiations). The multi-period model can be estimated via GMM in a manner similar to that of the linear NiN model, albeit with one additional parameter which must be identified via exogeneity restrictions. Using the more complicated multi-period model introduces the flexibility to estimate higher hospital bargain weights while also nesting the linear NiN model. Whether this results in different conclusions than linear NiN is of course an empirical question.

CRediT authorship contribution statement

Guy Arie: Conceptualization, Methodology, Software, Validation, Writing - original draft, Writing - reviewing & editing; **Paul Grieco:** Methodology, Software, Validation, Writing - original draft, Writing - reviewing & editing; **Shiran Rachmilevitch:** Formal Analysis, Methodology, Validation, Writing - original draft, Writing - reviewing & editing.

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7 Appendix A: Exclusion in the linear NiN framework

7.1 Ex ante exclusion

In the *linear NiN model with ex-ante exclusion* a group of J hospitals is selected out of a finite pool of potential hospitals, and only then, in a second stage, the NiN interaction occurs with the selected hospitals. The solution concept for this model is subgame perfect equilibrium: the insurer selects a group of hospitals such that its surplus will be maximized, given the second-stage NiN prices.

Proposition 3. *In the linear NiN model with ex-ante exclusion there exists a $\beta^* < 1$ such that if $\beta \in (\beta^*, 1)$ then in any subgame perfect equilibrium the selected network consists of a single hospital.*

Proof. Let $\mathcal{X}$ denote the hospital pool. By Theorem 1, for each subset of hospitals $X \subset \mathcal{X}$ with at least two hospitals, there is a discount factor $\beta_X < 1$ such that if $\beta > \beta_X$ the insurer makes a negative surplus in the linear NiN model in which the hospital network is X. Since the insurer makes a positive surplus when it Nash bargains with any single hospital, the result follows by taking β^* to be the maximum of the β_X's. $\square$

Proposition 5 implies that this two-stage structure cannot resolve the overpricing problem satisfactorily: within the two-stage framework, the threat of overpricing leads to a monopoly.

7.2 Ex post exclusion

An alternative to ex ante exclusion is *ex-post exclusion linear NiN*, suggested in Crawford and Yurukoglu (2012). Here, the insurer may remove hospitals from the network after the negotiation is complete. Ex-post exclusion guarantees the insurer at least zero surplus—it is also possible to "undo" contracts—and thus avoids complete Nash overpricing. However, *some* Nash overpricing may persist. That is, it is still possible that for a sufficiently large β *some* hospital's price exceeds the value of service. This is illustrated in the following example.

Example: Partial Nash overpricing

Consider two hospitals and two types of patients ab, ba, with the same $u^h = 10$ and $u^l = 5$ as in Example 1. However, in contrast to Example 1, assume a unit measure of patients, with s of the patients type ab and $1 - s$ of the patients of type ba.

The insurer's value from a full network given prices p_A, p_B is:

$$F(p) = \max\{0, 10 - p_A s - p_B(1 - s), 10s + 5(1 - s) - p_A, 10(1 - s) + 5s - p_B\}$$

Without A, the insurer's value given p_B is:

$$F_{-A}(p) = \max\{0, 10(1-s) + 5s - p_B\}$$

The Nash bargaining price responses are:

$$p_A(p_B) = \begin{cases} \beta(p_B + 5) & p_B \leq 10 - 5s \\ \beta\frac{10 - p_B(1-s)}{s} & p_B \geq 10 - 5s \end{cases}; \quad p_B(p_A) = \begin{cases} \beta(p_A + 5) & p_A \leq 5(1+s) \\ \beta\frac{10 - p_A s}{1-s} & p_A \geq 5(1+s) \end{cases}$$

The pair $\hat{p}_A = 10\frac{\beta}{s(1+\beta)}$ and $\hat{p}_B = 10\frac{\beta}{(1-s)(1+\beta)}$ is a solution if β and s are such that $\hat{p}_A > 5(1+s)$ and $\hat{p}_B > 10 - 5s$. Both $\hat{p}_j$ increase with β and are continuous for $\beta \geq 0$. Assume $\beta = 1$. Then for $s \in (0.5 - \hat{\xi}, 0.5 + \hat{\xi})$, with $\hat{\xi} = 1 - \frac{\sqrt{5}}{2}$ we have $\hat{p}_A = \frac{5}{s} > 5(1+s)$ and $\hat{p}_B = \frac{5}{1-s} > 10 - 5s$. Therefore, for any such s which is different from 0.5 one of the prices will exceed 10. By continuity, this is true also for all sufficiently large β's below 1.

8 Appendix B: The outside option

Under complete Nash overpricing, the insurer's objective assumes a negative value. In particular, not signing contracts with some hospitals is not a feasible alternative and the insurer's outside option is not zero, despite the fact that its payoff would have been zero if it did not sign any contract. It should be noted, however, that there is a limit to how low the insurer's payoff can be. We illustrate this for the 2-hospital case, though the idea is more general. Let $\mathcal{P}$ be the set of prices (p_A, p_B) that are consistent with an equilibrium of the model. It is enough to show that there is some bound $\bar{p}$ such that $p_A \leq \bar{p}$ for every $(p_A, p_B) \in \mathcal{P}$ that satisfy $p_B \leq p_A$. Consider then such prices. It holds that $p_A = \beta[v(AB) - v(B)] \leq \beta[\bar{v} + \alpha p_B]$, for some numbers $\bar{v} > 0$ and $\alpha \in (0, 1)$. Therefore, $p_A \leq \beta[\bar{v} + \alpha p_A]$, hence $\bar{p} = \frac{\beta\bar{v}}{1-\alpha\beta}$. In particular, prices are bounded above by $\frac{\beta\bar{v}}{1-\beta}$.

9 Appendix C: Estimating equations for the multi-period model

This appendix shows how the multi-period bargaining model can be estimated jointly with hospital costs using the generalized method of moments.

Estimation of patient hospital demand is independent of the bargaining problem. Therefore, we assume that the demand system is estimated prior to estimating costs and bargaining parameter.[16] This means that we can treat consumer expected val-

[16]For example, demand could be estimated following Capps, Dranove and Satterthwaite (2003). In principle, it would be more efficient to estimate demand and supply jointly in a simultaneous equa-

uations and choices conditional on the hospital network as known. The remaining parameters to estimate are the bargaining parameters (β, δ) and hospital costs. We assume that hospitals face an constant marginal cost per patient which is a function of a set of observable cost shifters, z_j and a hospital specific error term,

$$c_j = \lambda z_j + \omega_j$$

where λ is a vector of cost parameters to estimate.

To estimate (β, δ, λ) must specify the prevailing prices in the event that a hospital does not accept the insurer proposal. Specifically, we assume that the deviating hospital conducts NiN bargaining fixing other hospital prices at their observed (and multi-period equilibrium) outcomes p^*. Under this assumption let $\tilde{p}_{j^D}$ be the negotiated price if the hospital rejects the initial take-it-or-leave. Using equation (3) substituting p^*_{-j} for the other hospitals prices, the disagreement prices can be collected into the vector $\tilde{p}^D$:

$$\tilde{p}^D = \beta\theta^D + \beta\Gamma^D p^* + (1-\beta)c \tag{9}$$

Where, θ^D is a vector and Γ^D is a matrix defined by:

$$\theta^D_j = v_j - \frac{\sum_{\ell \neq j} v_{\ell,-j} q_{\ell,-j}}{q_j} \quad ; \; \Gamma^D_{j,l} = \frac{q_{l,-j}}{q_j} \quad ; \; \Gamma^D_{j,j} = 0$$

We now consider the vector of first sequential prices by hospital, collect these values into the vector p^F. Rewriting the sequential model outcome (5) for $i = 1$ and using matrix notation obtains,

$$p^F = \theta^F(\beta) + (I + \Psi^F(\beta)) \cdot c \tag{10}$$

The matrix $\Psi^F(\beta)$ accounts for the impact of disagreement with hospital j on the cost of treating patients,

$$\Psi^F_{j,j} = \beta, \qquad \Psi^F_{j,k\neq j} = \beta(1-\beta)\frac{q_{k,-j}}{q_j + \beta\sum_{k\neq j} q_{j,-k}}.$$

The vector $\theta^F(\beta)$ is,

$$\theta^F_j(\beta) = \beta\frac{q_j v_j + \sum_{k\neq j}\left[\beta q_{j,-k} v_{j,-k} - (1-\beta) q_{k,-j} v_{k,-j}\right]}{q_j + \beta\sum_{k\neq j} q_{j,-k}}$$

The first term of the numerator represents the hospitals contribution to surplus, the remaining terms are adjustments to hospital j's bargaining position based on cross-hospital substitution.

As discussed in Section 5, prices under the multi-period model are a convex com-

tions framework, which is conceptually straightforward although computationally more demanding.

bination of disagreement prices $\tilde{p}_{j^D}$ and each hospital's "punishment" price from reverting to the sequential model with that hospital negotiating in the first (least-favorable) position p^F weighted by the discount factor to bind the incentive constraint (7). The observed price in the multi-period model is given by

$$p^* = \delta p^F + (1-\delta)\tilde{p}^D$$

Using equations (9) and (10) we have an expression that is linear in prices and costs,

$$p^* = \delta\theta^F(\beta) + \delta(I + \Psi^F(\beta))c + (1-\delta)\beta(\theta^D + \Gamma^D p^*) + (1-\delta)(1-\beta)c \tag{11}$$

To solve for either p^* or c, rewrite as:

$$0 = \Psi(\beta,\delta)c + \theta(\beta,\delta) + \Gamma(\beta,\delta)p^* \tag{12}$$

Here, the terms multiplying the cost vector c and price vector p are aggregated into the matrices Ψ and Γ, and the constant terms are aggregated into the vector θ:

$$\begin{aligned}\Psi(\beta,\delta) &= I(1-\beta(1-\delta)) + \delta\Psi^F(\beta,\delta)\\ \Gamma(\beta,\delta) &= (1-\delta)\beta\Gamma^D - I\\ \theta(\beta,\delta) &= \delta\theta^F(\beta) + (1-\delta)\beta\theta^D\end{aligned} \tag{13}$$

To back out costs from the demand system and bargaining parameters, solve (12) for costs,

$$c(\beta,\delta) = -\Psi(\beta,\delta)^{-1}(\theta(\beta,\delta) + \Gamma(\beta,\delta)p^*). \tag{14}$$

Estimation of the supply side parameters using the multi-period model therefore is similar to the existing method for linear NiN with the additional parameter (δ) and a slightly more complicated non-linear function for costs. Specifically, given a set of bargaining parameters the structural error in costs is,

$$\omega_j(\alpha_0,\alpha_1,\beta,\delta) = c_j(\beta,\delta) - \gamma z_j.$$

To construct the moments, define $h_{j,n}$ as a vector of instruments for hospital j in market n. The data moments are:

$$g_{NJ}(\alpha_0,\alpha_1,\beta,\delta) = \frac{1}{NJ}\sum_{n,j} h_{n,j}\omega_{n,j}(\alpha_0,\alpha_1,\beta,\delta)\ .$$

The GMM estimator is:

$$\underset{\alpha_0,\alpha_1,\beta,\delta}{\operatorname{argmin}}\, g_{NJ}(\alpha_0,\alpha_1,\beta,\delta)' W g_{NJ}(\alpha_0,\alpha_1,\beta,\delta).$$

Where W is a symmetric positive definite weight matrix. We use the standard 2-step

GMM to derive the optimal weight matrix for each dataset.

The observed cost shifters z_j, are available as instruments, but we clearly need two additional instruments to identify the two bargaining parameters. Candidates are most likely to come from the exogenous variation in the demand system, which generates variation in the substitutability of hospitals (e.g., $v_{j,-k}$ and $q_{j,-k}$) that represent the key primitives in the matrices defined in (13). An example of such an instrument could be the distances between hospitals, or the relative weights of different types of observable consumers which vary across markets.

Remark 1. *To conclude this section, note that, the multi-period model specified in* (11) *nests the simultaneous linear NiN model by fixing* $\delta = 0$*, allowing us to test linear NiN against a more general alternative.*

10 Appendix D: Proofs

Lemma 1. *Consider Nash bargaining between the insurer and a hospital, under the following assumptions:*

1. *The insurer's profit without the hospital is* V_0*.*
2. *The hospital's unit cost is* c *and its bargaining power parameter is* β*.*
3. *If the hospital joins the network it serves a population of mass* q*.*
4. *Adding the hospital to the network at unit price* p *increases the insurer's profit by* $K - p \cdot y$*.*

Then the price is:

$$p = \beta \frac{K}{y} + (1 - \beta)c. \tag{15}$$

The lemma's proof boils down to a simple maximization of a Nash product, and is therefore omitted.

In what follows we consider sequential negotiations between the insurer and the hospitals, where the hospitals are order in a particular, commonly known order. Given the order, each hospital has a position in it—the first in line, the second in line, etc. In addition to that, the hospitals have names—hospital 1, hospital 2, etc.—and each name is associated with particular model-parameter-values, such as q_1, q_2, and so on. We use the term **canonical order** to denote the order where the two label-systems coincide; that is, hospital 1 is placed first in the canonical order, hospital 2 is second, and so on.

Lemma 2: Consider sequential negotiations according to the canonical order, $\{1, \cdots, J\}$. Given this order, denote the insurer's surplus after bargaining and signing contracts with hospitals $\{1, \cdots, i\}$ for prices $(p_1, \cdots, p_i)$, and facing "future hospitals" $\{i+1, \cdots, J\}$, by $V(p_1, \cdots, p_i; \{i+1, \cdots, J\})$. This surplus is given by:

$$
\begin{aligned}
&V(p_1, \cdots, p_i; \{i+1, \cdots, J\}) = \\
&\qquad = \sum_{j=1}^{i} [q_j(v_j - p_j) + \beta \sum_{k=i+1}^{J} q_{j,-k}(v_{j,-k} - p_j)] + \\
&\qquad\qquad + (1-\beta) \sum_{j=i+1}^{J} [q_j(v_j - c_j) + \beta \sum_{k=i+1, k\neq j}^{J} q_{j,-k}(v_{j,-k} - c_j)].
\end{aligned}
$$

The price obtained by hospital i in these negotiations is:

$$
\begin{aligned}
p_i = \beta \frac{q_i v_i + \beta \sum_{k=i+1}^{J} q_{i,-k} v_{i,-k} - \sum_{j=1}^{i-1} q_{j,-i}(v_{j,-i} - p_j) - (1-\beta) \sum_{j=i+1}^{J} q_{j,-i}(v_{j,-i} - c_j)}{q_i + \beta \sum_{k=i+1}^{J} q_{i,-k}} + \\
+ (1-\beta) c_i
\end{aligned}
$$

Before we turn to the proof, it is worthwhile to consider the equation for the above-mentioned value function V. The RHS is composed of two terms, one corresponding to the already-contracted-with hospitals and one corresponding to the "future hospitals," and the first term is such that for each element in the summation (each $j = 1, \cdots, i$) we have the "direct value" from the hospital plus β times the value that this hospital generates assuming that all "future hospitals" drop out. It will be useful to bear this meaning in mind later on in the proof.

Proof of Lemma 2: Clearly, $V(p_1, \cdots, p_J; \emptyset) = \sum_{j=1}^{J} q_j(v_j - p_j)$. When negotiating with hospital J, given that contracts with all previous hospitals have been signed, the insurer's outside option is the value $V(p_1, \cdots p_{J-1}; \emptyset) = \sum_{j=1}^{J-1} [q_j(v_j - p_j) + q_{j,-J}(v_{j,-J} - p_j)]$. Thus, the gain from adding J is:

$$
\begin{aligned}
W_J(p_1, \cdots, p_J; \emptyset) \equiv V(p_1, \cdots, p_J; \emptyset) - V(p_1, \cdots, p_{J-1}; \emptyset) = q_J(v_J - p_J) - \sum_{j=1}^{J-1} q_{j,-J}(v_{j,-J} - p_j) = \\
= \underbrace{q_J v_J - \sum_{j=1}^{J-1} q_{j,-J}(v_{j,-J} - p_j)}_{K} - p_J \underbrace{q_J}_{y},
\end{aligned}
$$

where the "K" and "y" are the notations of Lemma 1. Applying this lemma we obtain the price p_J:

$$p_J = \beta \frac{q_J v_J - \sum_{j=1}^{J-1} q_{j,-J}(v_{j,-J} - p_j)}{q_J} + (1-\beta)c_J. \tag{16}$$

Having obtained the price and surplus for the last bargaining problem in the sequence, we turn to the next-to-last bargaining. The value of these negotiations is $V(p_1, \cdots, p_{J-1}; \{J\}) = \sum_{j=1}^{J-1} q_j(v_j - p_j) + q_J(v_J - p_J)$.[17] Note that we slightly abuse notation to have p_J on the RHS is legitimate even though it does not appear as an argument of the V function on the LHS, because of (16)—namely, p_J is pinned down by the previous prices (and the other model parameters). Substituting p_J into the expression gives:

$$V(p_1, \cdots, p_{J-1}; \{J\}) = \sum_{j=1}^{J-1} [q_j(v_j - p_j) + \beta q_{j,-J}(v_{j,-J} - p_j)] + (1-\beta) q_J(v_J - c_J). \tag{17}$$

The outside option in the next-to-last negotiations has the value $V(p_1, \cdots, p_{J-2}; \{J\})$. It follows from (17) that this value is:

$$\begin{aligned} V(p_1, \cdots, p_{J-2}; \{J\}) = \sum_{j=1}^{J-2} [&\underbrace{q_j(v_j - p_j) + q_{j,-(J-1)}(v_{j,-(J-1)} - p_j)}_{L} + \beta q_{j,-J}(v_{j,-J} - p_j)] + \\ &+ (1-\beta) \underbrace{[q_J(v_J - c_J) + q_{J,-(J-1)}(v_{J,-(J-1)} - c_J)]}_{M}. \end{aligned}$$

Here, L is the counterpart of $q_j(v_j - p_j)$ from (17), and M is the counterpart of $q_J(v_J - c_J)$ (expected value minus expected cost at the final hospital).[18]

The gain from the $(J-1)$-th bargaining is $W_{J-1}(p_1, \cdots, p_{J-1}; \{J\}) = V(p_1, \cdots, p_{J-1}; \{J\}) - V(p_1, \cdots, p_{J-2}; \{J\})$, or:

[17] The value when the prices up to and including p_{J-1} have been contracted, and the insurer *expects* p_J to be contracted next is $\sum_{j=1}^{J} q_j(v_j - p_j)$.

[18] This is true because (17) holds also for a sequence of length $J' = J - 1$.

$$W_{J-1}(p_1,\cdots,p_{J-1};\{J\}) =$$

$$= \underbrace{q_{J-1}v_{J-1} + \beta q_{J-1,-J}v_{J-1,-J} - \sum_{j=1}^{J-2} q_{j,-(J-1)}(v_{j,-(J-1)} - p_j) - (1-\beta)q_{J,-(J-1)}(v_{J,-(J-1)} - c_J)}_{K} -$$

$$- p_{J-1}\underbrace{(q_{J-1} + \beta q_{J-1,-J})}_{y}.$$

By Lemma 1,

$$p_{J-1} =$$

$$= \beta\frac{q_{J-1}v_{J-1} + \beta q_{J-1,-J}v_{J-1,-J} - \sum_{j=1}^{J-2} q_{j,-(J-1)}(v_{j,-(J-1)} - p_j) - (1-\beta)q_{J,-(J-1)}(v_{J,-(J-1)} - c_J)}{q_{J-1} + \beta q_{J-1,-J}} +$$

$$+ (1-\beta)c_{J-1}.$$

It follows from equation (17) that:[19]

$$V(p_1,\cdots,p_{J-2};\{J-1,J\}) = \sum_{j=1}^{J-2}[q_j(v_j-p_j)+\beta q_{j,-J}(v_{j,-J}-p_j)]+(1-\beta)q_J(v_J-c_J)+$$

$$+ q_{J-1}v_{J-1} + \beta q_{J-1,-J}v_{J-1,-J} - p_{J-1}(q_{J-1} + \beta q_{J-1,-J}).$$

Combining this with the formula for p_{J-1} gives:

$$V(p_1,\cdots,p_{J-2};\{J-1,J\}) =$$

$$= \sum_{j=1}^{J-2}[q_j(v_j-p_j)+\beta \sum_{k=J-1}^{J} q_{j,-k}(v_{j,-k}-p_j)]+(1-\beta)\sum_{j=J-1}^{J}[q_j(v_j-c_j)+\beta \sum_{k=J-1,k\neq j}^{J} q_{j,-k}(v_{j,-k}-c_j)].$$

Now assume that given the contracted prices $\{p_1,\cdots,p_i\}$, and "future hospitals" $\{i+1,\cdots,J\}$, the insurer's payoff is:

[19] This is simply writing separately the $(J-1)$-th term from the first summation, leaving the first $J-2$ elements in the sum.

$$V(p_1,\cdots,p_i;\{i+1,\cdots,J\}) =$$
$$= \sum_{j=1}^{i}[q_j(v_j-p_j)+\beta\sum_{k=i+1}^{J}q_{j,-k}(v_{j,-k}-p_j)]+(1-\beta)\sum_{j=i+1}^{J}[q_j(v_j-c_j)+\beta\sum_{k=i+1,k\neq j}^{J}q_{j,-k}(v_{j,-k}-c_j)]. \tag{18}$$

As we have shown above, this assumption is indeed correct given a fixed J and $i \in \{J-2, J-1\}$. Basically, the same arguments can be applied given that the hospital sequence is of length $J' = J-1$: the formula still holds with J' replacing J, $i = J'-1$, and also for $i = J'-2$ provided that this is a positive integer. But, one has to be careful in the application and note the role of our assumption that when i drops out, all of its consumers that choose $k \neq i$ as their second choice leave the network if the second choice drops out as well. This is what makes the application work, and hence (18) implies:

$$V(p_1,\cdots,p_{i-1};\{i+1,\cdots,J\}) =$$
$$= \sum_{j=1}^{i-1}[q_j(v_i-p_j)+q_{j,-i}(v_{j,-i}-p_j)+\beta\sum_{k=i+1}^{J}q_{j,-k}(v_{j,-k}-p_j)]+$$
$$+(1-\beta)\sum_{j=i+1}^{J}[q_j(v_j-c_j)+\beta\sum_{k=i+1,k\neq j}^{J}q_{j,-k}(v_{j,-k}-c_j)].$$

Note that, like in the explanation that preceded the proof, each element in the first summation has a direct benefit component and an additional component, when in writing down these components we have invoked the abovementioned assumption regarding what happens when i drops out.

Thus, the gain from bargaining with i is:

$$W_i(p_1,\cdots,p_i;\{i+1,\cdots,J\}) = V(p_1,\cdots,p_i;\{i+1,\cdots,J\}) - V(p_1,\cdots,p_{i-1};\{i+1,\cdots,J\}),$$

or:

$$q_i(v_i-p_i)+\beta\sum_{k=i+1}^{J}q_{i,-k}(v_{i,-k}-p_i)-\sum_{j=1}^{i-1}q_{j,-i}(v_{j,-i}-p_j)-(1-\beta)\sum_{j=i+1}^{J}q_{j,-i}(v_{j,-i}-c_j)=$$

$$=\underbrace{q_iv_i+\beta\sum_{k=i+1}^{J}q_{i,-k}v_{i,-k}-\sum_{j=1}^{i-1}q_{j,-i}(v_{j,-i}-p_j)-(1-\beta)\sum_{j=i+1}^{J}q_{j,-i}(v_{j,-i}-c_j)}_{K}-$$

$$-\,p_i\underbrace{\left(q_i+\beta\sum_{k=i+1}^{J}q_{i,-k}\right)}_{y}.$$

Applying Lemma 1 we obtain:

$$p_i=\beta\frac{q_iv_i+\beta\sum_{k=i+1}^{J}q_{i,-k}v_{i,-k}-\sum_{j=1}^{i-1}q_{j,-i}(v_{j,-i}-p_j)-(1-\beta)\sum_{j=i+1}^{J}q_{j,-i}(v_{j,-i}-c_j)}{q_i+\beta\sum_{k=i+1}^{J}q_{i,-k}}+$$

$$+(1-\beta)c_i \quad (19)$$

With (18) and (19) established, the proof is completed. □

Proof of Proposition 1: It follows from (18) that the insurer's value, before he approaches the first hospital in the canonical order, is:

$$V(\emptyset;\{1,\cdots,J\})=(1-\beta)\sum_{j=1}^{J}[q_j(v_j-c_j)+\beta\sum_{k\neq j}q_{j,-k}(v_{j,-k}-c_j)],$$

and the RHS is independent of the order. □

Proof of Proposition 2: Consider J symmetric hospitals and let $q\equiv q_j$, $\hat{q}\equiv q_{j,-k}$, $v\equiv v_j$ and $\hat{v}\equiv v_{j,-k}$ (recall that symmetry means independence of these quantities of j and k). Let $p_i^*\equiv\lim_{\beta\to1}p_i$, where p_i is given by (19). Setting $\beta=1$ at (19) gives:

$$p_i^*=\underbrace{\frac{qv+\hat{q}\hat{v}(J-i)}{q+\hat{q}(J-i)}}_{A}+\underbrace{\frac{\hat{q}\sum_{j=1}^{i-1}(p_j^*-\hat{v})}{q+\hat{q}(J-i)}}_{B}.$$

Claim 1: A is increasing in i.

Proof of Claim 1: The sign of $\frac{\partial A}{\partial i}$ is the same as the sign of $-\hat{q}\hat{v}[q+\hat{q}(J-i)]+\hat{q}[qv+\hat{q}\hat{v}(J-i)]$, and the latter is positive if and only if $v>\hat{v}$, which is true.

Claim 2: B is increasing in i.

Proof of Claim 2: It is enough to prove that $p_j^* > \hat{v}$. This is true for $j = 1$ in virtue of $v > \hat{v}$, and the fact that it is true for all $j' < j$ implies that it is true also for j. $\square$

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