Paul's Homework

Paul Gustafson

16/09/2022

Let's look at Binomial data when n is small, and the prior strongly favours rare outcome scenarios

```
set.seed(13)
n <- 10; m <- 10000

### vector of p's from nature's prior
p.tr <- rbeta(m, 1, 14)

### corresponding vector of datapoints
y <- rbinom(m, size=n, prob=p.tr)

### corresponding vector of posterior means
### when investigator's prior matches nature's prior: beta(1,14)
p.hat <- (1+y)/((1+y)+(14+(n-y)))
```

Average squared-error incurred

```
mean((p.hat-p.tr)^2)
```

```
## [1] 0.002262297
```

```
### or for interpretation
sqrt(mean((p.hat-p.tr)^2))
```

```
## [1] 0.04756361
```

Confirm this worsens as we nudge the investigator's prior to the right

```
p.hat.r <- (1.5+y)/((1.5+y)+(13.5+(n-y)))
mean((p.hat.r-p.tr)^2)
```

```
## [1] 0.00270834
```

```
sqrt(mean((p.hat.r-p.tr)^2))
```

```
## [1] 0.05204171
```

And ditto if we nudge it to the left

```
p.hat.l <- (0.5+y)/((0.5+y)+(14.5+(n-y)))
mean((p.hat.l-p.tr)^2)
```

```
## [1] 0.002616254
```

```
sqrt(mean((p.hat.l-p.tr)^2))
```

```
## [1] 0.05114933
```

Due numerical diligence: Are we sure?

```r
tmp <- (p.hat.r - p.tr)^2 - (p.hat - p.tr)^2

c(mean(tmp),sqrt(var(tmp/m)))
```

```
## [1] 4.460427e-04 1.902082e-07
```

```r
tmp <- (p.hat.l - p.tr)^2 - (p.hat - p.tr)^2

c(mean(tmp),sqrt(var(tmp/m)))
```

```
## [1] 3.539573e-04 1.902082e-07
```