

KU Leuven Summer School
Segment 5A
Preferential Sampling and Inferring COVID
Lethality

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A second “latent variable model in covid times” tale

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Translator Disclaimer

Bayesian adjustment for preferential testing in estimating infection fatality rates, as motivated by the COVID-19 pandemic

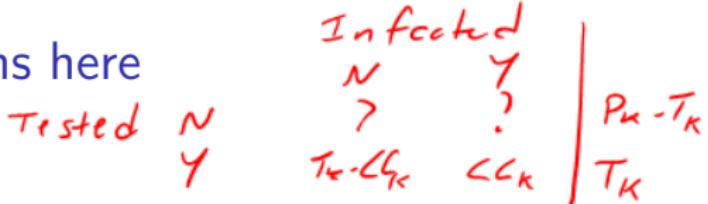
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(see journal website → supplementary material for code/data)

The story begins here



In a given (the k -th) time/place, there are P_k people. Of these:

- ▶ C_k get infected by covid
 - ▶ and of these D_k die from the infection
- ▶ T_k get tested for covid
 - ▶ and of these CC_k test positive

confirmed cases

Observable: (P_k, D_k, T_k, CC_k) . (E.g. ourworldindata.org for country-level)

Latent: C_k .

Can report case fatality rate, $CFR_k = D_k/CC_k$.

Want to report infection fatality rate $IFR_k = D_k/C_k$?

What if testing were done on the basis of random sampling?

E.g., The T_k tested individuals were purely drawn at random from the P_k population members.

Then $\hat{C}_k = (CC_k/T_k)P_k$, hence $\hat{IFR}_k = \frac{D_k/P_k}{(CC_k/T_k)}$.

Helpful to think about this for two reasons:

- ▶ An anchor - can think about how this \hat{IFR}_k breaks as the process of "who gets tested" deviates from random sampling.
- ▶ Then in some instances, testing a random subset of the population did/does happen: "sero-surveys."

Testing of a biased sample

Think of infection rate $IR_k = C_k/P_k$ and infection fatality rate $IFR_k = D_k/C_k$ as parameters.

Generalize from random sampling, $CC_k \sim \text{Bin}(T_k, IR_k)$ to biased sampling:

$$\phi_k = 1$$

$$CC_k \sim \text{Bin}(T_k, 1 - (1 - IR_k)^{\phi_k})$$

where $\phi_k > 1$ describes the bias.

Interpretation? $OR(\text{get tested, are infected}) \approx \phi_k$

public health / disease control perspective
(finite testing capacity) ϕ_k large = success

Take stock

Have parameters (IR_k, IFR_k, ϕ_k)

latent

Convenient to **collapse**: From $C_k \sim \text{Bin}(P_k, IR_k)$ and
 $(D_k|C_k) \sim \text{Bin}(C_k, IFR_k)$ down to:

- ▶ $D_k \sim \text{Bin}(P_k, (IR_k)(IFR_k))$

math / intuition ?

Along with (from prev. slide):

- ▶ $CC_k \sim \text{Bin}(T_k, 1 - (1 - IR_k)^{\phi_k})$

And then recall we are aiming to draw together information from multiple jurisdictions

$$f(IR_{1:K}, IFR_{1:K}, \phi_{1:K} | P_{1:K}, T_{1:K}, CC_{1:K}, D_{1:K}) \propto$$

$$\underbrace{f(IR_{1:K}, IFR_{1:K}, \phi_{1:K})}_{\text{prior}} \times$$

$$\prod_{k=1}^K f(CC_k | T_k, IR_k, \phi_k) f(D_k | P_k, IR_k, IFR_k)$$

*two binomial models
(as per prev. slide)*

Prior distributions (1 of 3): sigh - so much devil in detail

g() a link function e.g. logit

Meta-analysis / random effect idea, e.g.,

$$g(IR_1), \dots, g(IR_K) \sim N(\beta, \sigma^2), (\beta, \sigma^2) \sim \text{prior}$$

$$g(IFR_1), \dots, g(IFR_K) \sim N(\theta, \tau^2), (\theta, \tau^2) \sim \text{prior}$$

- ▶ Science: cross-jurisdiction variation in IFR much less than in IR.
- ▶ So prior that concentrates τ near zero justified
- ▶ In fact, perhaps with addition of covariates, unexplained IFR variation could be *very low*? Getting toward a “biological constant’’?
- ▶ Interpret $g^{-1}(\theta)$ as target parameter, “typical” IFR.

Prior distributions (2 of 3): refining the previous slide a tad

- lockdown timing (z)
- pop. density

$$g(IR_i) \sim N(\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i}, \sigma^2)$$

$$g(IFR_i) \sim N(\theta_0 + \theta_1 z_{1i} + \theta_2 z_{2i}, \tau^2)$$

age hospital capacity

All country-specific covariates centred.

So can interpret $g^{-1}(\theta_0)$ as the typical IFR amongst typical jurisdictions.

↑
target of inference

Prior distributions (3 of 3)

Recall that larger ϕ_k corresponds to more targeted testing (compared to testing a random subset, $\phi_k = 1$).

- ▶ Countries contributing (P_k, CC_k, T_k, D_k):
 - ▶ $\phi_k \sim \text{Unif}(1, 1 + \gamma)$, $\gamma \sim \text{prior}$.
- ▶ Sero-surveys contributing (P_k, CC_k, T_k, D_k):
 - ▶ $\phi_k \equiv 1$.

Trying to keep score - Need JAGS (or comparable) to encode/compute:

$$f(\gamma, \beta, \sigma^2, \theta, \tau^2, \{\phi, IR, IFR\}_{1:K} | \{P, T, CC, D\}_{1:K}) \propto$$

$$f(\gamma)f(\beta)f(\sigma^2)f(\theta)f(\tau^2) \times$$

$$\prod_{k=1}^K f(IR_k | X_k, \beta, \sigma^2) \times$$

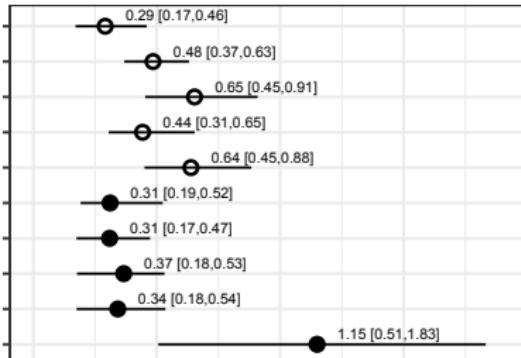
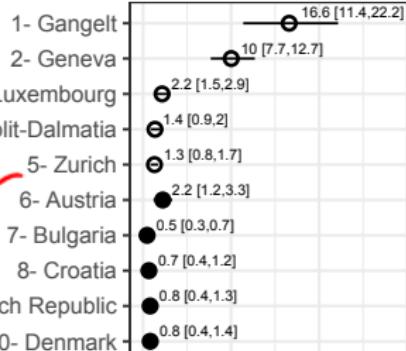
$$\prod_{k=1}^K f(IFR_k | Z_k, \theta, \tau^2) \times$$

$$\prod_{k=1}^K f(\phi_k | \gamma) \times$$

$$\prod_{k=1}^K f(CC_k | IR_k, \phi_k, T_k) f(D_k | IR_k, IFR_k, P_k)$$

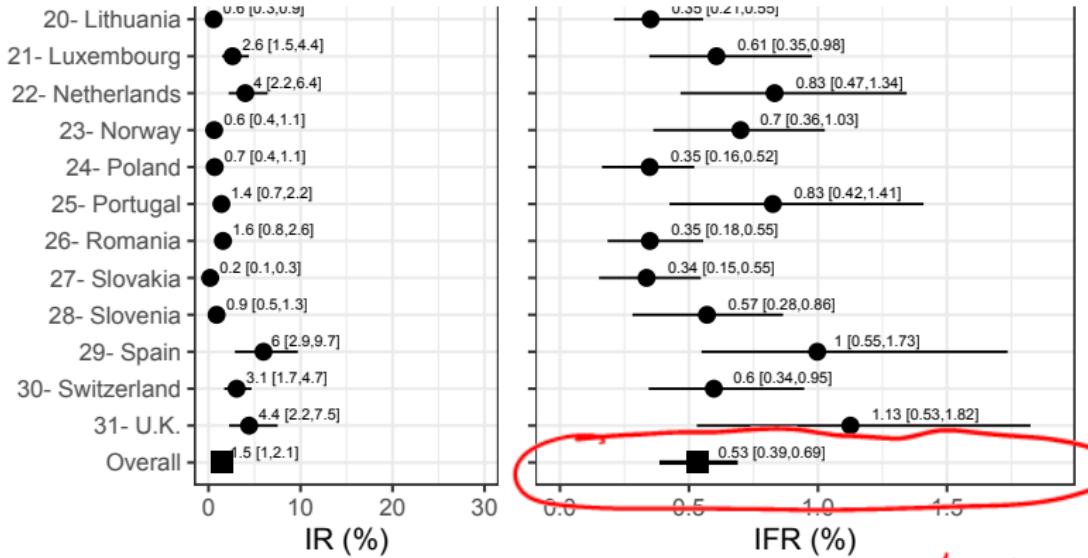
Inference: (head of Fig. 3)

several surveys



• IR_k
• IFR_k
• national
• statistics

Inference: (tail of Fig. 3)



*inference about
 $g^{-1}(\theta_0)$ - typical IFR
for typical jurisdiction*

Second-order issue

We had some high-info datapoints ($\phi_k = 1$ known, sero-surveys), and some lower-info datapoints ($\phi_k \sim$ prior, national statistics), drew them all together in a meta-analysis.

Would it be useless to try to proceed in these sorts of problems with only lower-info data points?

SEGMENT 5B

Appendix: A meta-analysis “life hack”

For the sero-surveys, really helps to organize our thinking that a random sample of size T_k produced CC_k confirmed cases (and $T_k - CC_k$ non-cases).

In actuality, the better surveys did something along the lines of:

after adjusting for one or more potential
biases, a 95% CI for IR_k
was blah

$$(CC_k, T_k) \leftrightarrow \left(\hat{p}_k, \sqrt{\frac{\hat{p}_k(1-\hat{p}_k)}{n_k}} \right)$$

So in fact we take the data to be the effective T_k and CC_k that would produce this 95% credible interval *assuming* random sampling.