Calculations

1) What is chance that neither settle the dispute if they both try independently

for manager 'X' :-

$P(X \text{ doesn't settle}) = P(\bar{A}) = \frac{8}{14} = \frac{4}{7}$

$P(X \text{ settles}) = P(A) = \frac{6}{14} = \frac{3}{7}$

for manager 'Y' :-

$P(Y \text{ doesn't settle}) = P(\bar{B}) = \frac{16}{30} = \frac{8}{15}$

$P(Y \text{ settles}) = P(B) = \frac{14}{30} = \frac{7}{15}$

$\therefore$ Prob. the neither settles $= P(\bar{A} \cap \bar{B})$

$= \frac{4}{7} \times \frac{8}{15} = \frac{32}{105}$

2) Prob that the dispute is settled

$P(\text{dispute is settled}) = 1 - P(\text{neither settles}) = 1 - \frac{32}{105}$

$= \frac{73}{105}$

PRACTICAL - 1

Aim: To obtain the probability when two variables are independent.

Experiment: We are analyzing a wage dispute scenario where manager 'X' has odds of 8:6 against setting the dispute and 'Y' has odds of 14:16 in favour of setting the dispute.

Theory: If the odds against the event A are m:n, then:

$P(A) = n/(m+n)$

$P(\bar{A}) = m/(m+n)$

If odds are in favour of an event A, then

$P(A) = m/(m+n)$

$P(\bar{A}) = n/(m+n)$

For independent events A & B, the Prob. is

$P(A \cap B) = P(A) \cdot P(B)$

Result:

1) The Prob. that neither manager settles the dispute if they both try independently is $\frac{32}{105}$ or $\simeq 30.5\%$.

2) The Prob. that the dispute will be settled is $\frac{73}{105}$ or $\simeq 69.5\%$.

Calculations

For person X :-

$P(\text{X speaks truth}) = P(A) = 3/5$

$P(\text{X doesn't speak truth}) = P(\bar{A}) = \frac{2}{5}$

For Y :-

$P(\text{Y speaks truth}) = P(B) = \frac{5}{8}$

$P(\text{Y doesn't speak truth}) = P(\bar{B}) = \frac{3}{8}$

Contradiction occurs when :-

X speaks truth & Y doesn't $= P(A \cap \bar{B})$

or $= P(A) \cdot P(\bar{B}) = \frac{3}{5} \times \frac{3}{8} = \frac{9}{40}$

X doesn't speak truth but Y does $= P(\bar{A} \cap B) = P(\bar{A}) \cdot P(B)$

$= \frac{2}{5} \times \frac{5}{8} = \frac{10}{40}$

Total $= \frac{9}{40} + \frac{10}{40} = \frac{19}{40}$

PRACTICAL - 2

Aim : To determine the percentage of cases where two people are likely to contradict each other on an identical point.

Experiment : Person 'X' has odds of 3:2 that they speak the truth, and 'Y' has odds of 5:3 that they speak the truth.

Theory : For independent events, we can calculate the prob. of contradiction by finding cases when one person speaks the truth while the other doesn't.

Result : The percentage of cases where two people are likely to contradict each other on an identical point is $\frac{19}{40} = 0.475 = 47.5\%$

Calculation

Let A denote the event of drawing 4 gold coin in 1st case & B denote the events of drawing 4 silver coins in 2nd draw

∴ Need to find $P(A \cap B)$

(i) Draws with replacement - if coin drawn in 1st draw are replaced back in the bag before the 2nd draw these events A & B are independent i.e.

$P(A \cap B) = P(A) \cdot P(B)$ — ①

1st draw - 4 coins are drawn out of 18

no. of ways $= {}^{18}C_4$

$$P(A) = \frac{{}^{10}C_4}{{}^{18}C_4}$$

2nd draw - similarly, when 4 silver are drawn they should be from 8 coins means

$$P(B) = \frac{{}^{8}C_4}{{}^{18}C_4}$$

$$\text{Total Prob.} = P(A) \cdot P(B) = \frac{{}^{10}C_4}{{}^{18}C_4} \times \frac{{}^{8}C_4}{{}^{18}C_4} = \frac{49}{31212}$$

$$= 0.00157$$

PRACTICAL - 3

Aim: To obtain conditional Probability

Experiment: A bag contains 10 gold & 8 silver coins 2 successive drawings of 4 coins are made such that

i) coins are replaced before the second trial

ii) Coins are not replaced before second trial

find the prob. that the 1st drawing will be given 4 gold & second 4 silver coins.

Theory: Conditional probability is prob of an event occuring given that another event has already occured. It is denoted by:-

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where, $P(A|B)$ is prob of event A occuring given that event B has already occured.

$P(A \cap B)$ is the prob. of both events A & B occuring

$P(B)$ is the prob. of event B occuring assuming $P(B) > 0$

- When A & B are independent:

if the occurence of one doesn't affect the prob. of the other. then

$$P(A \cap B) = P(A) \cdot P(B)$$

this means

$P(A|B) = P(A)$, $P(B|A) = P(B)$

(ii) Draws without replacement

1st draw. – same as before

$$P(A) = \frac{{}^{10}C_4}{{}^{18}C_4} = \frac{210}{3060}$$

2nd draw – now compute the prob. that, for remaining 14 coins:-

$$P(B) = \frac{{}^{8}C_4 \cdot {}^{6}C_0}{{}^{14}C_4} = \frac{70}{1001}$$

Total prob = $P(A) \cdot P(B) = \frac{210}{3060} \times \frac{70}{1001} = \frac{49}{10302} \frac{245}{51051}$

$= 0.00479$

Result: Prob. of the first drawing will be given 4 gold & 2nd 4 silver coins in case:-

(1) Coins are replaced before 2nd trial = 0.00157 or 0.157 %

(2) Coins are not replaced before 2nd trial = 0.00479 or 0.479 %

Calculations

Let A :-

A = introduction of Co-ed

E_1 = Mr. Chatterjee selected

E_2 = Mr. Iyengar selected

E_3 = Dr. Singh selected

thus:-

$P(A|E_1) = 0.3$, $P(A|E_2) = 0.5$, $P(A|E_3) = 0.8$

(i) P(A: introduction of Coed)

$= P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)$

$= \frac{4}{9} \times 0.3 + \frac{2}{9} \times 0.5 + \frac{3}{9} \times 0.8 = \frac{4.6}{9} = 51.1\%$

(ii) P(if there is Coed the Dr. Singh is Principle)

$= P(E_3|A) = \frac{P(E_3) \cdot P(A|E_3)}{P(A)}$

$= \frac{\frac{3}{9} \times 0.8}{\frac{4.6}{9}} = \frac{12}{23} = 0.5217$

$= 52.17\%$

PRACTICAL - 4

Aim: To obtain the conditional Probability using Baye's theorem

Experiment: In 2027, there will be 3 candidates for position of principle Mr. Chatterjee, Mr. Iyengar & Dr. Singh whose chances of getting appointment are in ratio 4:2:3 respectively. The Prob. that Mr. Chatterjee if selected would introduce Co-ed is 0.3 & of Mr. Iyengar & Dr. Singh for same are 0.5 & 0.8 respectively.

i) What is probability that there will be co-ed in college

ii) if there is co-ed & what is prob. that Dr. Singh is principle

Theory:- Acc. to Baye's theorem:-

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

where $P(A|B)$ = Prob. of A given B, $P(B|A)$ = likelihood prob. A given B

$P(A)$ = Prob of A, $P(B)$ = Prob of B

Result: (i) The Probability the co-ed would be introduced is 51.1 %

(ii) The Probability for Dr. Singh is principle if there is Co-ed is 52.17 %

Calculation

$E[X] = \sum x \cdot p(x)$

$= 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6}$

$= \frac{21}{6} = \frac{7}{2} = 3.5$

~~Var~~ $E[x^2] = \sum x^2 \cdot p(x)$

$= 1^2\times\frac{1}{6} + 2^2\times\frac{1}{6} + 3^2\times\frac{1}{6} + 4^2\times\frac{1}{6} + 5^2\times\frac{1}{6} + 6^2\times\frac{1}{6}$

$= \frac{91}{6}$

$Var(X) = E[x^2] - [E[X]]^2$

$= \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{91}{6} - \frac{49}{4}$

$= \frac{182 - 147}{12} = \frac{35}{12}$

Aim: To obtain mean and Variance

Experiment: find mean & variance of following uniform distribution obtained by tossing a die

x:	1	2	3	4	5	6
$p(x)$:	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Formula Used: We know that, mean is $E[X]$

$$E[X] = \sum x \cdot p(x)$$

And variance ~~is~~ $= E[x^2] - [E[x]]^2$

Result:

Mean $= \frac{7}{2}$

Variance $= \frac{35}{12}$

Calculation

$E(X) = \sum x \cdot p(x) = np = 4$

$E[X^2] = npq + n^2p^2$

Variance $= npq = \frac{4}{3}$

$4q = \frac{4}{3}$

$q = \frac{1}{3}$ then $p = \frac{2}{3}$

$np = 4$

$n \times \frac{2}{3} = 4$

then $n = \frac{4 \times 3}{2} = 6$

(a) $P(X=2) = {}^6C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4 = 15 \times \frac{4}{9} \times \frac{1}{81} = \frac{60}{729}$

$= \frac{20}{243}$

(b) $P(X>2) = 1 - P(X \le 2) = 1 - [P(0) + P(1) + P(2)]$

$P(0) = {}^6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6 = \frac{1}{729}$

$P(1) = {}^6C_1 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^5 = 6 \times \frac{2}{3} \times \frac{1}{243} = \frac{12}{729}$

$P(2) = \frac{60}{729}$

$P(X>2) = 1 - \left[\frac{1 + 12 + 60}{729}\right] = 1 - \frac{73}{729} = \frac{729 - 73}{729}$

$= \frac{656}{729}$

Aim: To obtain Probability using binomial distribution

Experiment: The mean and variance of binomial distribution are 4 and $4/3$ respectively. Find

(a) Probability of 2 successes

(b) Probability of more than 2 successes

Formula Used: $E(X) = \sum x \cdot p(x) = np$ & $Var = npq$

$P(X=x) = {}^nC_x \, p^x q^{n-x}$

Result: (a) Probability of 2 successes is $\frac{20}{243}$

(b) Probability of more than 2 successes is $\frac{656}{729}$

Calculation

Given $\mu = 4$ & $\sigma^2 = 3$

For $X \sim Bin(n, p)$

$\mu = np$

$\sigma^2 = np(1-p) = npq$

$np = 4$ —(1) & $npq = 3$

using eq (1)

$4q = 3$

$q = \frac{3}{4}$ then $p = \frac{1}{4}$

& $n = 4 \times 4 = 16$

So, $X \sim Bin(\cancel{4}, 16, \frac{1}{4})$

Mode $= \lfloor (n+1)p \rfloor = (16+1)\frac{1}{4} = \frac{17}{4} = \lfloor 4.25 \rfloor = 4$

Aim: To obtain mode of binomial distribution

Experiment: Determine the binomial distribution for which the mean is 4 and variance is 3. Also obtain its mode.

Formula Used: $E(X) = np$ = mean

Variance $= npq$

Mode $= \lfloor (n+1)p \rfloor$

Result: Binomial distribution is

$X \sim Bin(16, \frac{1}{4})$

And, Mode $= 4$

no. of head	f	(n-x/x+1)*p/q	p(x)	E[f]
0	7	7	0.0078125	1
1	6	3	0.0546875	7
2	19	1.666666667	0.1640625	21
3	35	1	0.2734375	35
4	30	0.6	0.2734375	35
5	23	0.333333333	0.1640625	21
6	7	0.142857143	0.0546875	7
7	1		0.0078125	1
	128			128

Mean =	3.3828125

Aim: To fit Binomial distribution

EXPERIMENT: 7 coins are tossed and no. of heads are noted, the experiment is repeated 128 times and following distribution is obtained

No. of head	f
0	7
1	6
2	19
3	35
4	30
5	23
6	7
7	1

fit Binomial dist. assuming Coin is unbiased

Theory - $X \sim B(n, p)$

$P(x) = {}^{n}C_x \, p^x q^{n-x}$; $x = 0, 1, 2 \cdots$ & $q = 1-p$

we know that

$p(x+1) = \frac{n-x}{x+1} \times \frac{p}{q} \times p(x)$ = in excel = $\frac{7-x}{x+1} \times p(x)$

as coin is unbiased so $p = \frac{1}{2}$, $q = \frac{1}{2}$ then $\frac{p}{q} = 1$

$E[f] = N \cdot p(x)$, Mean = SUMPRODUCT(x0:x7, f0:f7)/SUM(f0:f7)

Result:

No. of head	f	E[f]
0	7	1
1	6	7
2	19	21
3	35	35
4	30	35
5	23	21
6	7	7
7	1	1

no. of head	f	(n-x/x+1)*p/q	p(x)	E[f]
0	7	6.546436285	0.0078125	1
1	6	2.805615551	0.051144033	6.546436285
2	19	1.558675306	0.143490496	18.36678344
3	35	0.935205184	0.223655092	28.6278518
4	30	0.56112311	0.209163402	26.7729154
5	23	0.311735061	0.117366418	15.02290156
6	7	0.133600741	0.036587228	4.683165137
7	1		0.004888081	0.62567433
	128			101.645728

Mean =	3.3828125
p =	0.483258929
q=	0.516741071

Aim: To fit Binomial distribution

EXPERIMENT: 7 coins are tossed and no. of heads are noted, the experiment is repeated 128 times and following distribution is obtained

No. of heads	f
0	7
1	6
2	19
3	35
4	30
5	23
6	7
7	1

fit Binomial dist. assuming coin is biased

Theory: $X \sim B(n, p)$

$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}; \quad x = 0, 1, 2 \ldots \ \& \ q = 1-p$$

we know that

$$P(x+1) = \frac{n-x}{x+1} \times \frac{p}{q} \times P(x), \quad E[f] = N \cdot p(x)$$

Mean = SUMPRODUCT (x0 : x7, f0 : f7) / SUM(f0:f7)

Result:

No. of head	f	E[f]
0	7	1
1	6	6.5464
2	19	18.3667
3	35	28.6278
4	30	26.7729
5	23	15.0229
6	7	4.6831
7	1	0.6256

x	f	fx	lambda/(x+1)	p(x)	E [f]
0	56	0	1.972	0.139178	69.58911
1	156	156	0.986	0.274459	137.2297
2	132	264	0.657333333	0.270617	135.3085
3	92	276	0.493	0.177886	88.94279
4	37	148	0.3944	0.087698	43.8488
5	22	110	0.328666667	0.034588	17.29397
6	4	24	0.281714286	0.011368	5.68395
7	0	0	0.2465	0.003202	1.60125
8	1	8	0.219111111	0.000789	0.394708
Total =	500	986			

Mean(lambda) =	1.972

Aim: To fit Poisson distribution

EXPERIMENT: Fit Poisson distribution to following data

x	f
0	56
1	156
2	132
3	92
4	37
5	22
6	4
7	0
8	1

Theory: $X \sim P(\lambda)$

$$P(X) = \frac{e^{-\lambda}\lambda^{x}}{x!} ; x = 0 \ldots$$

$$p(x+1) = \frac{\lambda}{x+1} \cdot p(x), \quad Mean = \frac{\Sigma fx}{\Sigma f}, \quad E[f] = N \cdot P(x)$$

Result:

x	f	P(x)	E(f)
0	56	0.1392	69.58
1	156	0.2744	137.23
2	132	0.2706	135.3
3	92	0.1778	88.94
4	37	0.0876	43.84
5	22	0.3458	17.29
6	4	0.0113	5.68
7	0	0.0032	1.6
8	1	0.0007	0.39

class	f	Lower class (x)	z=(x-mew)/sigma	p(x+1)	p(x)	E[f]
<60	0			0	0.000124642	0.124642398
60-65	3	60	-3.662993572	0.000124642	0.002903486	2.903485574
65-70	21	65	-2.744719927	0.003028128	0.030863405	30.86340501
70-75	150	70	-1.826446281	0.033891533	0.148001977	148.0019767
75-80	335	75	-0.908172635	0.18189351	0.322136142	322.1361418
80-85	326	80	0.01010101	0.504029651	0.319363718	319.3637183
85-90	135	85	0.928374656	0.82339337	0.144207566	144.2075658
90-95	26	90	1.846648301	0.967600936	0.029552244	29.55224361
95-100	4	95	2.764921947	0.997153179	0.002731657	2.73165674
>100		100	3.683195592	0.999884836		
	mean(mew)=	79.945				
	sigma=	5.445				

PRACTICAL -11

Aim: To fit normal distribution

EXPERIMENT: Obtain the equation of normal curve that may be fitted to following data:

Class:	60-65	65-70	70-75	75-80	80-85	85-90	90-95	95-100
Freq:	3	21	150	335	326	135	26	4

Also obtain expected normal frequencies

Theory: · $X \sim Normal(\mu, \sigma)$

$$P(X=x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$z = \frac{(x-\mu)}{\sigma}, \quad p(x+1): = NORMSDIST(z)$$

$$p(x): = p(x+1) - p(x)$$

$$E[f] = N \cdot p(x)$$

Result:

Class	f	Lower class (x)	E(f)
<60	0	$-\infty$	0.1246
60-65	3	60	2.9034
65-70	21	65	30.8634
70-75	150	70	148.0019
75-80	335	75	322.1361
80-85	326	80	319.3637
85-90	135	85	144.2075
90-95	26	90	29.5522
95-100	4	95	2.7316
>100	0	100	

Calculation

Here, the RV X which denotes the no. of demands for a car on any day follows Poisson distribution with mean $\lambda = 1.5$. The proportion of days on which there are x demands for a car is given by:

$$P(X=x) = \frac{e^{-1.5}(1.5)^x}{x!}$$

(i) $$P(X=0) = e^{-1.5} = \left\{1 - 1.5 + \frac{(1.5)^2}{2!} - \frac{(1.5)^3}{3!} \cdots\right\}$$

$$= 0.2231$$

(ii) $$P(X>2) = 1 - P(X \leq 2) = 1 - \{P(X=0,1,2)\}$$

$$= 1 - e^{-1.5}\left\{1 + 1.5 + \frac{(1.5)^2}{2!}\right\} = 1 - 0.2231 \times 3.625$$

$$= 0.19126$$

Aim: To obtain Probability using Poisson distribution

EXPERIMENT: A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which (i) neither car is used (b) the proportion of days on which some demand is refused

Formula Used: $P(X=x) = \frac{e^{-\lambda}(\lambda)^x}{x!}$; $x = 0, 1, 2 \ldots$

Result:

(i) Proportion of days on which neither car is used is

$P(X=0) = 0.2231$

(ii) Proportion of days on which some demand is refused is

$P(X>2) = 0.19126$

Calculation

The avg. no. of typographical errors per page in book is given by $\lambda = 390/520 = 0.75$

$$[P(X=0)]^5 = (e^{-0.75})^5 = e^{-3.75}$$

Aim: To obtain Probability using Poisson distribution

EXPERIMENT: In a book of 520 pages, 390 typo-graphical errors occur. Assuming poisson law for no. of errors per page, find the probability that a random sample of 5 pages will contain no error.

Formula Used: $P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$; $x = 0, 1, 2 \ldots$

Result:

The required probability that a random sample of 5 pages will contain no error is:

$$[P(X=0)]^5 = (e^{-0.75})^5 = e^{-3.75}$$

Calculation

If RV X denote the yield (in kilo) for one-acre plot, given that $X \sim N(\mu, \sigma^2)$, where $\mu = 662$ & $\sigma = 32$

(i) $P(X > 700) = P(Z > 1.19)$, $Z = \frac{700-662}{32} = 1.1875$

$= 0.5 - P(0 \le Z \le 1.19)$

$= 0.5 - 0.3830 = 0.1170$

exp. no. of plot $= 1000 \times 0.117 = 117$

(ii) $P(X < 650) = P(Z < -0.38)$, $Z = \frac{650-662}{32} = -0.375$

$= P(Z > 0.38)$ by symmetry

$= 0.5 - P(0 \le Z \le 0.38) = 0.5 - 0.1480$

$= 0.352$

exp. no. of plot $= 1000 \times 0.352 = 352$

(iii) $P(X > x_1) = \frac{100}{1000} = 0.1$, when $X = x_1$

$Z = \frac{x_1 - \mu}{\sigma} = \frac{x_1 - 662}{32} = z_1$, such that

$P((Z > z_1) = 0.1) = P(0 \le Z \le z_1) = 0.4 = z_1 = 1.28$

$x_1 = 662 + 32z_1 = 662 + 32 \times 1.28$

$= 662 + 40.96 = 702.96 = 703$

100 best plot yield over is 703 ~~.96~~ kilos

Aim: To obtain Probability using normal distribution

EXPERIMENT: The mean yield for one-acre plot is 662 kilos with a S.D. 32 kilos. Assuming normal distribution, how many one-acre plots in a batch of 1000 plots would you expect to have yield (i) over 700 kilos (ii) below 650 kilos (iii) what is lowest yield of best 100 plots?

Theory: $X \sim N(\mu, \sigma^2)$, $\mu = 662$ & $\sigma = 32$

$$P(X > x) = P(Z > \text{[illegible]}), \quad Z = \frac{X - \mu}{\sigma}$$

Result:

(i) exp. no. of plots with yield over 700 kilos is 117

(ii) exp. no. of plots with yield below 650 kilos is 352

(iii) best 100 plots have yield over 703 kilos

Aim: To obtain probability using normal distribution

EXPERIMENT : X is normally distributed and the mean of X is 12 and S.D. is 4. Find following probabilities (i) $X \geq 20$ (ii) $X \leq 20$ (iii) $0 \leq X \leq 12$

Theory : $X \sim N(\mu, \sigma^2)$, $\mu = 12$, $\sigma = 4$

$$Z = \frac{x - \mu}{\sigma}$$

Result : (i) $P(X \geq 20) = 0.0228$

(ii) $P(X \leq 20) = 0.9722$

(iii) $P(0 \leq X \leq 20) = 0.4987$

Calculation

$\mu = 12$, $\sigma = 4$, $X \sim N(12, 16)$

(i) $P(X \geq 20) = P(Z \geq 2)$, $Z = \frac{20-12}{4} = 2$

$= 0.5 - P(0 \leq Z \leq 2) = 0.5 - 0.4772 = 0.0228$

(ii) $P(X \leq 20) = 1 - P(X \geq 20) = 1 - 0.0228 = 0.9722$

(iii) $P(0 \leq X \leq 20) = P(-3 \leq Z \leq 0)$, $Z = \frac{0-12}{4} = -3$

$= P(0 \leq Z \leq 3)$, $Z = \frac{12-12}{4} = 0$

$= 0.4987$