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Homework 2 solutions available

Last time:

Gauss' Theorem

Regular n-gon is constructible <=>

$n = 2^k p_1 p_2 \cdots p_r$ $k \geq 0, p_1, \ldots, p_r$ distinct,

Fermat primes (primes of form $2^{(2^m)}+1$)

Remarks:

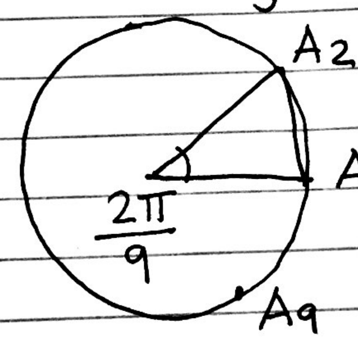
1. Equivalently, $\phi(n) = 2^\ell$ for some ℓ where ϕ is Euler's function

$\phi(n) = |\{a \in \mathbb{N}) \mid 1 \leq a \leq n, \gcd(a,n) = 1\}|$

2. modern proof using Galois Theory (MATH 412)

3. Theorem => some angles can't be trisected by ruler and compass

e.g. 9-gon is not constructible => $\frac{2\pi}{9}$ is not constructible => $\frac{2\pi}{3}$ can't be trisected



Today:

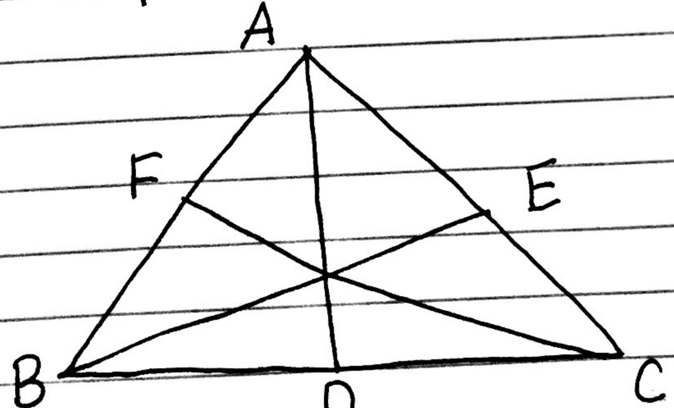
center of mass

orthocenter

coordinates

Center of Mass:

Given triangle ΔABC, let D,E,F be the midpoints of BC, CA, AB



Claim:

The lines AD, BE, CF (the medians of ΔABC) are concurrent (meet at 1 point)

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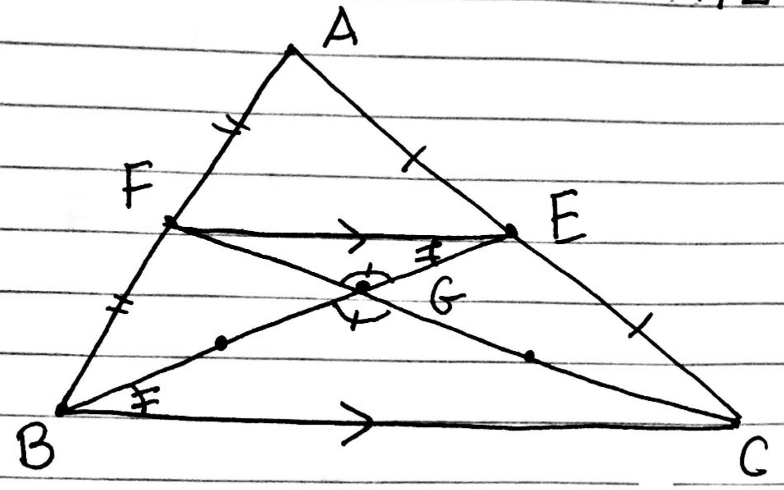


What are the ratios $\frac{|BG|}{|GE|}$, $\frac{|CG|}{|GF|}$?

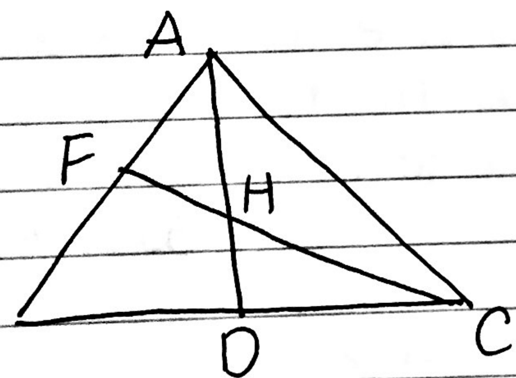
$\frac{|AF|}{|FB|} = \frac{|AE|}{|EC|} \Rightarrow EF \parallel BC$ by the converse Thales Theorem

$\Rightarrow \triangle BCG \sim \triangle EFG$

so $\frac{|BG|}{|GE|} = \frac{|CG|}{|GF|} = \frac{|BC|}{|FE|} = \frac{|AB|}{|AF|} = 2$

$\triangle AFE \sim \triangle ABC$

Now deduce the claim (that all 3 medians meet at the point G)



by same argument

$\frac{|CH|}{|HF|} = \frac{2}{1} = 2$

so $H = G$ $\square$

G is the center of mass of the triangle if we make the triangle from a sheet of uniform density and thickness (the point at which the triangle will balance on a pin)

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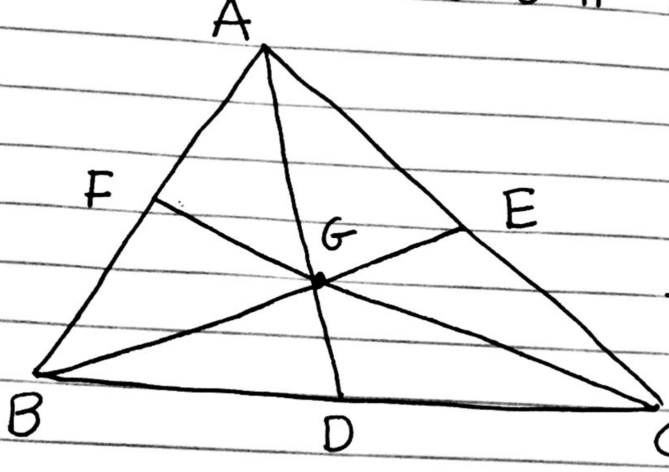
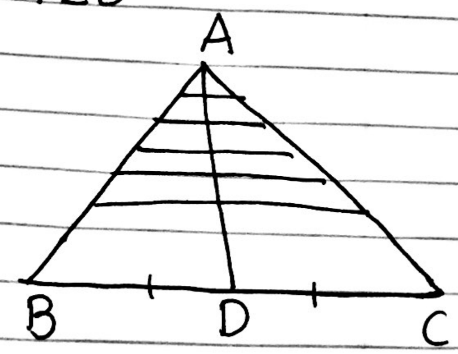
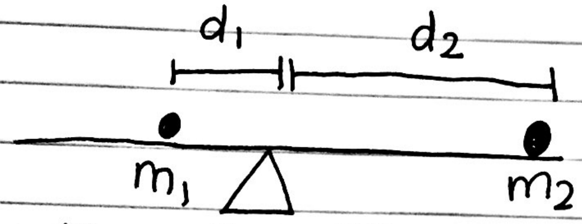






$m_1 d_1 = m_2 d_2$

Calc II: $R \subset \mathbb{R}^2$

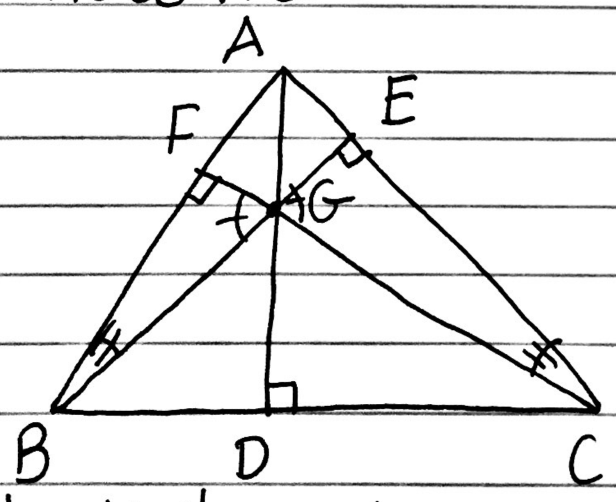
center of mass = $(\bar{x}, \bar{y})$ where

$\bar{x} = \int_R x\,dx\,dy$, $\bar{y} = \int_R y\,dx\,dy$

weighted by the distances from the axes

(exception is a washer where you can't balance it on a pin because the center of mass is in space)

Orthocenter:



Claim: the perpendicular heights (altitudes) of $\triangle ABC$ are concurrent

$\angle FGB = \angle EGC$

$\angle FBG = \angle ECG$

$\triangle BGF \sim \triangle CGE$

$\frac{|BG|}{|CG|} = \frac{|GF|}{|GE|} = \frac{|BF|}{|CE|}$ might work but not sure

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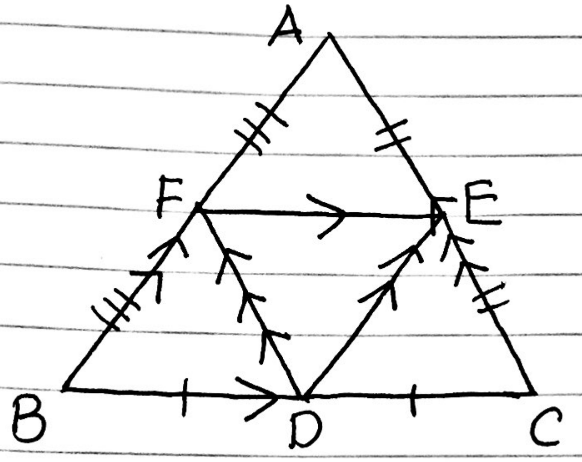
Easier problem:



$\triangle ABC$

D, E, F midpoints of BC, CA, AB

$\triangle AFE \sim \triangle ABC$
$\triangle ECD \sim \triangle ACB$

$\Rightarrow \triangle AFE \sim \triangle EDC$

and in fact

$\triangle AFE \cong \triangle EDC$

because $|AE| = |EC|$

$\triangle AFE \cong \triangle EDC \cong \triangle FBD \cong DEF$

Alternative proof:

Converse Thales' Theorem

$FE \parallel BC$, $ED \parallel AB$, $DF \parallel AC$

Fact: parallelograms have opposite sides of equal lengths

$|BD| = |FE|$, $|CE| = |DF|$, $|AF| = |DE|$ $\Rightarrow$

$\triangle AFE \cong \triangle EDC \cong \triangle FBD \cong \triangle DEF$ by SSS

Recall: (maybe Homework 1)

Perpendicular bisectors of the sides of a triangle are concurrent