

Math 461

9/4/19

OH: Mon & Tues 4-5pm L6RT 1235H

First hw due 9/18/19

{get compass}

## 1. Euclidean geometry / plane geometry

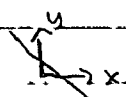


\* Axiomatic approach - Euclid 300BC

(Theorems and proofs starting from axioms)

\* Rules and compass constructions

## 2. Coordinate geometry



$$y = mx + c / Ax + By = C$$

\* Descartes (~1500, France)



$$(x-a)^2 + (y-b)^2 = r^2$$

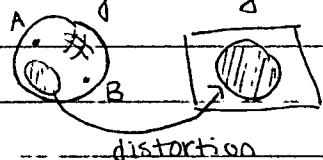
\* Symmetries (Rigid motions) E. Klein (~1800)

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

preserves distances  $d(p, q)$  = distance from  $p$  to  $q$

$$d(T(p), T(q)) = d(p, q) \text{ for all } p, q \in \mathbb{R}^2$$

## 3. Spherical geometry



Shortest path from A to B?

## 4. Hyperbolic geometry (~1800)

\* Parallel axiom (Euclidean geometry)  $\rightarrow$  doesn't work in hyperbolic geometry

P M



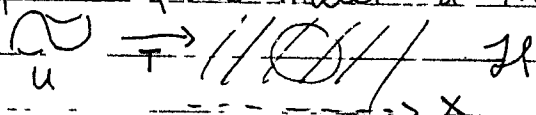
Given line  $L$  and point  $P$  not in  $L$ , there's a unique line  $M$  through  $P$  parallel to  $L$  (meaning  $L \cap M = \emptyset$ )

$\hookrightarrow$  don't intersect

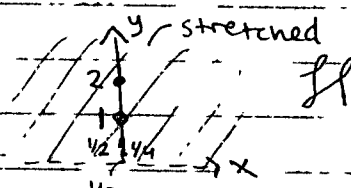
\* infinite surface 1. looks the same at every point and in every direction

$\hookrightarrow$  hyperbolic plane

\* Upper half plane model: a "map" of the hyperbolic plane



1. doesn't preserve distances
2. does preserve angles



distance infinite to x-axis  
 $\ln(b/a)$

Euclid's Elements - 13 volumes ~300 BC

Plane geometry, Number theory, 3-D geometry (e.g. Platonic solids)

\* Model of deductive reasoning

i.e. start w/

• definitions

• assumptions (axioms)

and deduce other properties by logical deduction

5 regular polyhedrons



cube



tetrahedron

icosahedron (20 triangular faces)



dodecahedron (12 pentagonal faces)



octahedron (8 triangular faces)

Why do we study Euclidean geometry?

1. It's useful! x2 & fun
2. It's an excellent foundation for other types of geometry (3-D, spherical, hyperbolic)
3. Practice logical reasoning / proofs
4. To help (some of you) prepare to teach geometry in high school

Ruler & Compass constructions

Q: What geometric figures can be constructed by ruler & compass?

Euclid's construction axioms:

It is possible to

1. Draw a straight line between any 2 points (and extend it indefinitely)
2. Draw a circle with center a point & given radius

Notes: Ruler can't be used for measuring ("straight-edge")

Compass can be used to copy a length from one location to another