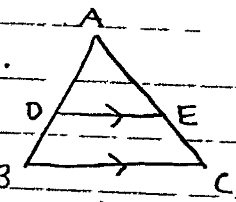
9/23/19

**Announcement:** HW2 due on Wednesday at start of class.
Office hours today & tomorrow 4PM–5PM, LGRT 1235H.

**Last Time:** Thales' theorem.




DE parallel to BC
$\Rightarrow \dfrac{|AD|}{|AB|} = \dfrac{|AE|}{|AC|}$

Similar triangles: We say $\Delta ABC$ & $\Delta A'B'C'$ are similar & write $\Delta ABC \sim \Delta A'B'C'$ if corresponding angles are equal.

**Theorem:** $\Delta ABC \sim \Delta A'B'C' \Rightarrow$ ratios of corresponding sides are equal:
$$\frac{|A'B'|}{|AB|} = \frac{|B'C'|}{|BC|} = \frac{|C'A'|}{|CA|}$$

**Today:**
- Constructible lengths – multiplication & division
- Converse of Thales' thm, Parallel Pappus & Desargues theorems
(Angles in a circle)

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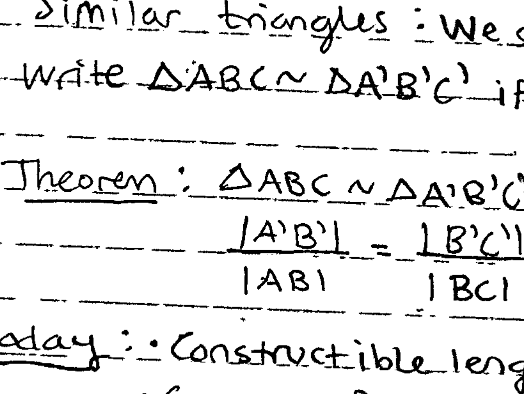
Q: Given line segment of lengths $a$ and $b$, how can we construct line segments of lengths ① $a \cdot b$ and ② $a/b$?
(always have length 1 given)

①

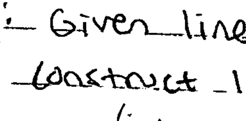



- Mark off length $a$ on a line.
- Draw perp. line through one endpoint of that line segment & make lengths 1 & $b$.
- Draw line $RQ$ & draw line through $S$ parallel to $RQ$, intersecting $AB$ at $T$.

**Claim:** $|QT| = ab$

**Proof** $\dfrac{b}{1} = \dfrac{|QT|}{a}$ by Thales' theorem $\Rightarrow |QT| = ab$. ∎

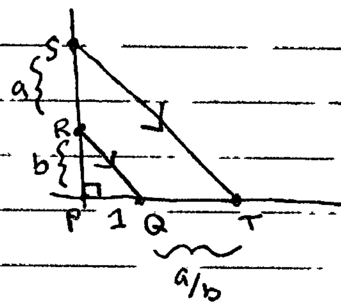
**Aside:**




$\dfrac{|AD|}{|AB|} = \dfrac{|AE|}{|AC|}$ equivalently $\dfrac{|AD|}{|DB|} = \dfrac{|AE|}{|EC|}$

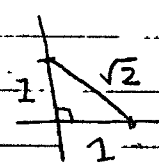
$$\left.\begin{aligned} |AB| &= |AD| + |DB| \\ |AC| &= |AE| + |EC| \end{aligned}\right\} \Rightarrow |AD| \cdot (\cancel{|AE|} + |EC|) = |AE| \cdot (\cancel{|AD|} + |DB|)$$
$$|AD| \cdot |EC| = |AE| \cdot |DB|$$

② Given lengths $a, b$ ( & 1), construct length $a/b$. – Similar to $a \cdot b$.
In particular any fraction (rational number) is constructible.




$$\frac{a}{b} = \frac{x}{1} = x \quad \checkmark$$

Q: Are all constructible lengths rational numbers?



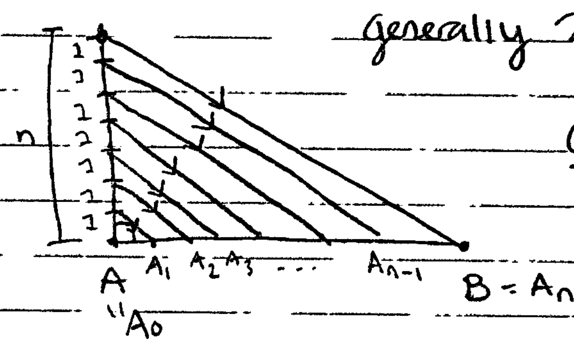

NO. eg. $\sqrt{2}$ is constructible

In fact, we will later show:
The constructible lengths are those real numbers which can be obtained from 1 by $+, -, \times, \div,$ & $\sqrt{\ }$.
(uses coordinates)

EX: $\sqrt{72 + 65\sqrt{29 - \sqrt{5}}}$

Q: Given a line segment $AB$, how can we divide it into $n$ equal parts (where $n$ is a positive integer)?

$n=2$ ✓ (bisect a line segment)
⇝ can divide into $4 = 2^2$ parts or more
generally $2^r$ parts (any power of 2)




Claim: $|A_0A_1| = |A_1A_2| = \dots = |A_{n-1}A_n| = \frac{1}{n}|AB|$

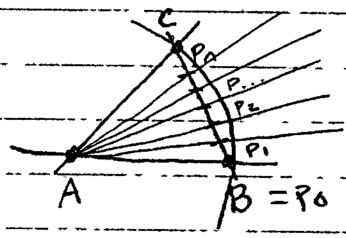
Proof $\dfrac{|AA_1|}{|AB|} = \dfrac{1}{n}$

by Thales' theorem $|AA_1| = \frac{1}{n}|AB|$ ✓

$\dfrac{|AA_i|}{|AB|} = \dfrac{i}{n}$ by Thales' theorem, $|AA_i| = i/n \cdot |AB|$.
↳ for any $i = 1, \dots, n$
Subtracting $|A_{i-1}A_i| = 1/n\,|AB|$. ∎

Alternatively, find $x/n$ by same construction, then use compass to split $x$ in $n$ lengths of $x/n$.

<u>Q</u>: Given an angle, how can we divide it into $n$ equal parts ($n$ a positive integer)?

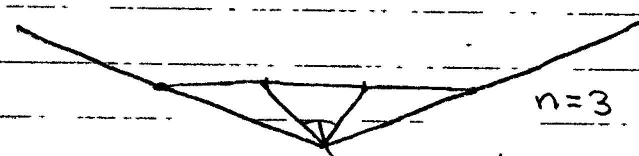



$|AB| = |AC|$

Draw line $BC$, & divide into $n$ equal parts.

<u>Claim</u>: $\angle P_i A P_{i+1} = \frac{1}{n} \cdot \angle BAC$ for $i = 1, \dots, n$

<u>Actually doesn't work</u>:




Actually, it's impossible in general to divide an angle into $n$ equal parts with ruler & compass. ($n=2$ OK, also $n = 2^r$ OK, but $n=3$ fails)

Why? (Galois theory / fields Math 412)

... to be continued.