Wednesday 11/18/15 MATH 611 HW6 Solution Q1b.

1b.

R integral domain.

$0 \neq a \in R$ is <u>irreducible</u> if 1. a is not a unit
& 2. $\nexists$ $a = bc$, b, c not units.

Equivalently 1. $(a) \neq R$
& 2. $\nexists$ b s.t. $(a) \subsetneq (b) \subsetneq R$.

Recall, say an ideal $I \subset R$ of a ring R is <u>Maximal</u> if
$I \neq R$ & $\nexists$ ideal J s.t. $I \subsetneq J \subsetneq R$.

(every ideal is principal)

So, if R is a principal ideal domain, see that an ideal $I = (a)$ is maximal $\Leftrightarrow$ a is irreducible,
assuming $I \neq (0) = \{0\}$.

Finally $\{0\}$ is maximal $\Leftrightarrow$ $R = R/_{\{0\}}$ is a field.

$\therefore$ if R is a PID, not a field, then the maximal ideals are (a) for $a \in R$ irreducible.