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Congruence

We say two triangles ΔABC & $\Delta A'B'C'$ are congruent if corresponding sides are equal & corresponding angles are equal.

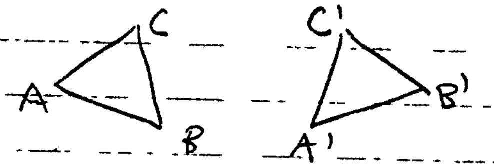
$|AB| = |A'B'|$, $|BC| = |B'C'|$, $|AC| = |A'C'|$

&

$\angle ABC = \angle A'B'C'$, $\angle BCA = \angle B'C'A'$, $\angle CAB = \angle C'A'B'$

Shorthand:

$\Delta ABC \cong \Delta A'B'C'$



Modern Definition:

ΔABC is congruent to $\Delta A'B'C'$ if there is an isometry = rigid motion (transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$ which preserves distances)

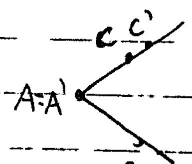
$T: \mathbb{R}^2 \to \mathbb{R}^2$, such that $T(A) = A'$
$T(B) = B'$
$T(C) = C'$

Congruence criteria

- SAS — Euclid gave a "proof" of SAS using implicitly rigid motions
- ASA
- SSS

"Proof" of SAS (Euclid) → doesn't follow from axioms he had written down

- Move ΔABC so point A coincides with point A' & the ray in direction AB coincides w/ the ray A'B', and same for AC and A'C' rays

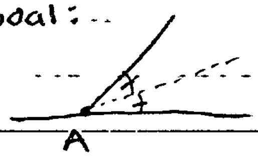


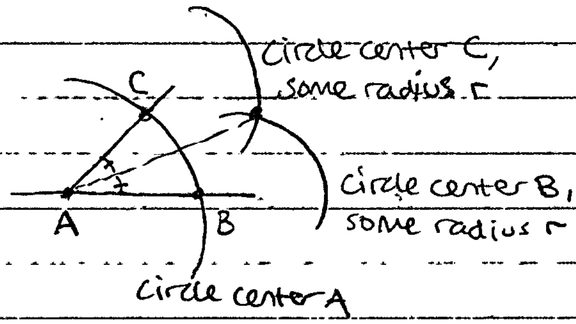
Now $|AB| = |A'B'| \Rightarrow B = B'$
$|AC| = |A'C'| \Rightarrow C = C'$
& $A = A'$ $\Big\} \Rightarrow \Delta ABC \cong \Delta A'B'C'$ ☐

- later David Hilbert (~1900) rewrote Euclid - SAS added as an axiom, (can deduce ASA, SSS.)

Bisect an angle

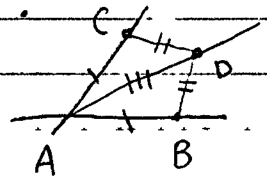
Goal:





Claim: $\angle BAD = \angle CAD$

Proof:



$\triangle ABD \cong \triangle ACD$ by SSS $\Rightarrow \angle CAD = \angle BAD$ ▨

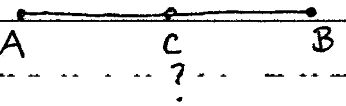
$|AC| = |AB|$

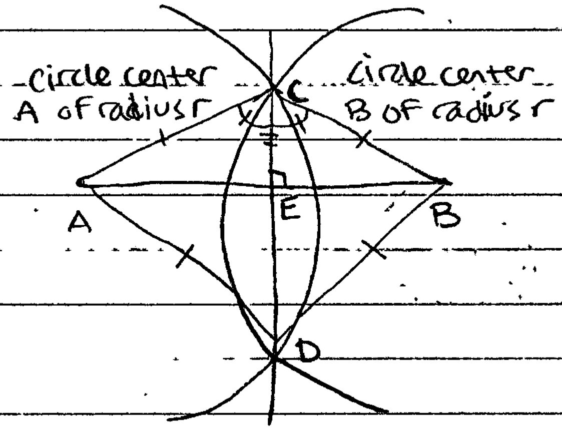
$|AD| = |AD|$

$|CD| = |DB|$

Bisect a line segment

Goal:





$(2r > |AB|)$

Claim: $|AE| = |BE|$

Proof: $\triangle ACD \cong \triangle BCD$ by SSS

$\Rightarrow \angle ACD = \angle BCD$

$\triangle ACE \cong \triangle BCE$ by SAS $\Rightarrow |AE| = |BE|$ ▨

Other properties: $(|CE| = |DE|)$

$\angle AEC = \angle BEC$ $(\triangle AEC = \triangle BEC)$

$\angle AEC + \angle BEC = \pi \Rightarrow \angle AEC = \angle BEC = \pi/2$