Math 462: Homework 2

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(1) Consider the matrix
$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(a) Check that A is orthogonal.

(b) Determine whether A is a rotation or a reflection / rotary reflection. [Hint: What is the determinant of A?]

(c) Find the eigenvalues and eigenvectors of A (including the complex ones if there are any).

(d) Describe the motion $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ given by $T(\mathbf{x}) = A\mathbf{x}$ geometrically. (If T is a rotation, give the axis and angle of rotation. If T is a reflection in a plane, find the plane.)

(e) Find an orthonormal basis of $\mathbb{R}^3$ such that the matrix B of T with respect to this basis has the form described in the Theorem on p.12 of the textbook, and write down the new matrix B.

(2) Repeat Q1 for the matrix
$$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

(3) The matrix
$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & \sqrt{2} \\ 1 & -1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$
defines a rotation of $\mathbb{R}^3$. Find the axis and angle of rotation (give the angle in radians to 2 decimal places.) [Hint: What are the eigenvectors and eigenvalues of A?]

(4) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a motion of $\mathbb{R}^3$ given by $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$. Describe T geometrically (as a translation, rotation, twist, reflection, glide, or rotary reflection) in the following cases.

(a) A the matrix from Q1 and $\mathbf{b} = (3, 3, -1)^T$.

(b) A the matrix from Q2 and $\mathbf{b} = (0, -3, -3)^T$.

(c) $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.

(5) In this question, we find the group G of all rotational symmetries of the cube. (For concreteness, you might like to use the cube with center the origin and vertices the points $(\pm 1, \pm 1, \pm 1)$.)

(a) Find the order of G. (Hint: Use the orbit-stabilizer theorem.)

(b) Find an element of G of order 4, and an element of order 3.

(c) Describe all elements of G geometrically (give the axis and angle of rotation).

(d) Show that the group G is isomorphic to the symmetric group S_4. (Hint: G permutes the 4 diagonals of the cube joining opposite vertices.)