Math 461 Homework 1

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(1) Given a circle show how to determine the center of the circle using ruler and compass.

(2) Suppose given two triangles ΔABC and $\Delta A'B'C'$ such that $\angle CAB = \angle C'A'B'$, $|AB| = |A'B'|$ and $|BC| = |B'C'|$. Does it follow that the triangles are congruent? Give a proof or a counterexample.

[Hint: Given triangle ΔABC, draw two lines L and M intersecting at a point A' at angle $\alpha = \angle CAB$, mark a point B' on L such that $|A'B'| = c = |AB|$, draw the circle C with center B' and radius $a = |BC|$, and consider the intersection points of the circle C with the line M.]

(3) Let A,B,C be 3 distinct points in the plane.

   (a) Show that the perpendicular bisectors of AB and BC intersect if and only if the points A,B,C do not lie on a line.

   (b) Prove that if the points A,B,C do not lie on a line then there exists a unique circle passing through the points.

(4) Recall that we say a n-sided polygon is *regular* if all the sides have equal lengths and all the angles are equal. Given a line segment, describe a ruler and compass construction of a regular n-gon with one side the line segment in the cases (a) $n = 4$ and (b) $n = 6$. Prove carefully that your construction is correct in each case.

(5) Let $n \geq 3$ be a positive integer. We say that a n-sided polygon P is *convex* if for any two points A and B in P the line segment AB is contained in P. (Equivalently, P is convex if all the interior angles

of P are less than π.) Prove by induction that the sum of the interior angles of a convex n-sided polygon equals $(n-2)\pi$.

(6) Let C be a circle with center O and P a point on C. Let L be a line passing through P. We say L is *tangent* to C at P if $L \cap C = \{P\}$. Prove that L is tangent to C if and only if OP is perpendicular to L.

[Hint: Suppose L intersects the circle C at another point Q. What can you say about the angle $\angle OPQ$?]

(7)(a) Given a triangle ΔABC, show that if a point P in the triangle lies on the bisector of the angle $\angle BAC$ then the perpendicular distance from P to AB equals the perpendicular distance from P to AC.

(b) Show that the three bisectors of the angles of a triangle are concurrent, that is, they all pass through some point P.

(c) Show that a triangle has an *inscribed circle*: a circle contained in the triangle which is tangent to each of the sides of the triangle.