9/20/19

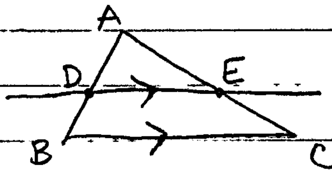
Announcements: - HW 1 solutions available
- people.math.umass.edu/~hacking/461F19
- Lecture note available: Under "Class log" on webpage.

Last Time: - Areas
- Area of parallelogram; Area of triangle
- Pythagoras' thm

Today: - Similar triangles (Thales' theorem)
- Multiplication and division (constructible lengths)
- Converse of Thales' thm. Pappas & Desargues theorems

didn't get to it

Thales' theorem (624-548 BC)



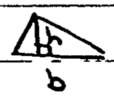
$\underbrace{DE \parallel BC}$ $\Longrightarrow \dfrac{|AD|}{|AB|} = \dfrac{|AE|}{|AC|}$

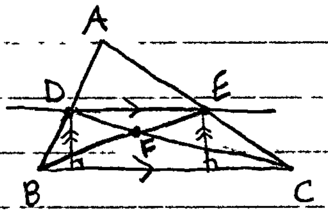
"DE parallel to BC"

$\left(\text{Equivalently, } \dfrac{|AD|}{|DB|} = \dfrac{|AE|}{|EC|} \right)$

Proof We will use:

Area (triangle) = $\frac{1}{2}$(base) × (perpendicular height)





(1) Area($\triangle BEC$) = Area($\triangle BDC$)
because same base and equal heights (parallelogram has equal opposite sides)

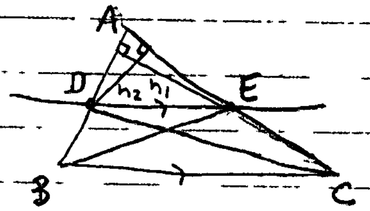
(2) Area($\triangle BDE$) = Area($\triangle CDE$) by same criteria as above.

Because Area($\triangle BEC$) = Area($\triangle BDC$), subtract Area($\triangle BFC$) in order to get Area($\triangle BDF$) = Area($\triangle CEF$) (3).

Now add Area(ADFE) to previous equality and get

Area($\triangle ABE$) = Area($\triangle ADC$) (4)

Using ③ Area(ΔABE) = Area(ΔADC)



i.e. $\boxed{\tfrac{1}{2}|AB|\cdot h_1 = \tfrac{1}{2}|AC|\cdot h_2}$ ④

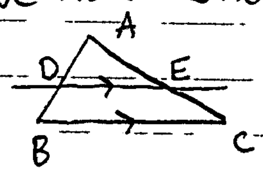
Area(ΔADE) = Area(ΔADE)

$\boxed{\tfrac{1}{2}|AD|\cdot h_1 = \tfrac{1}{2}|AE|\cdot h_2}$ ⑤

⑤ ÷ ④ $\quad \dfrac{|AD|}{|AB|} = \dfrac{|AE|}{|AC|}$. ▨

Notice: We didn't use ② or ④.

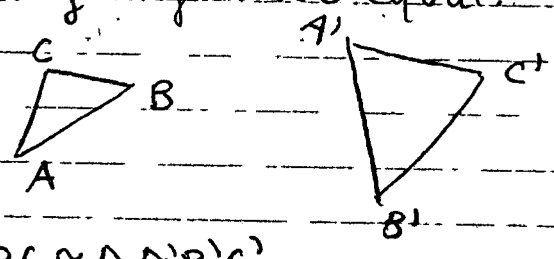
We have shown:



$\Longrightarrow \quad \dfrac{|AD|}{|AB|} = \dfrac{|AE|}{|AC|}$

Similar Triangles

We say triangles ΔABC & $\Delta A'B'C'$ are similar if the corresponding angles are equal.



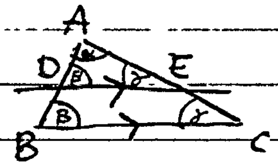
i.e. $\angle CAB = \angle C'A'B'$
$\angle ABC = \angle A'B'C'$
$\angle BCA = \angle B'C'A'$

Notation: $\Delta ABC \sim \Delta A'B'C'$
["tilde", not to be confused with $\Delta ABC \cong \Delta A'B'C'$]

Theorem. If $\Delta ABC \sim \Delta A'B'C'$ are similar,
then $\dfrac{|A'B'|}{|AB|} = \dfrac{|B'C'|}{|BC|} = \dfrac{|C'A'|}{|CA|}$

(corresponding side lengths are in same ratio)

An Aside:

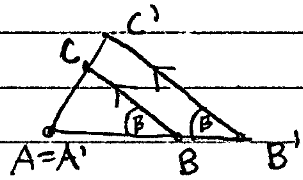


Proof We will use Thales' theorem.

Observe, $\Delta ADE \sim \Delta ABC$ and Thales' theorem says:

$$\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|}$$

(Euclid style) "Move" triangle ΔABC so A coincides with A' and lines AB and $A'B'$ coincide & lines AC and $A'C'$ coincide (using $\angle CAB = \angle C'A'B'$)

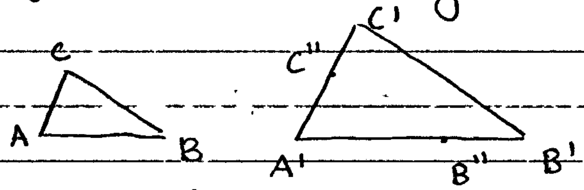


Claim: $\dfrac{|A'B'|}{|AB|} = \dfrac{|A'C'|}{|AC|}$

We know $\angle ABC = \angle A'B'C'$ ($\Delta ABC \sim \Delta A'B'C'$).
So BC & $B'C'$ are parallel (corresponding angles are equal).
Thales' theorem implies $\dfrac{|A'B'|}{|AB|} = \dfrac{|A'C'|}{|AC|}$

Similarly for other inequality (move ΔABC so B coincides w/ B'). ▨

How to avoid "moving"



Construct points C'' on $A'C'$ and B'' on $A'B'$ such that $|A'B''| = |AB|$ and $|A'C''| = |AC|$.

Then $\Delta ABC \cong \Delta A'B''C''$ (by SAS).
Now prove as before, using $\Delta A'B''C''$ & $\Delta A'B'C'$. ▨