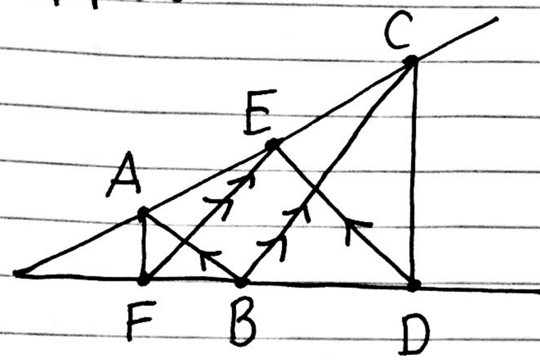
Math 461 Lecture 8 9/21

Solutions to HW 1 available

Last time:

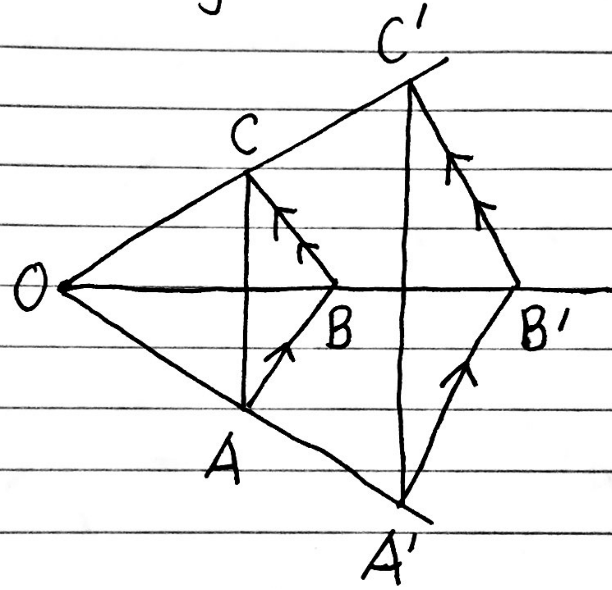
Parallel Pappus and Desargues Thm

Pappus



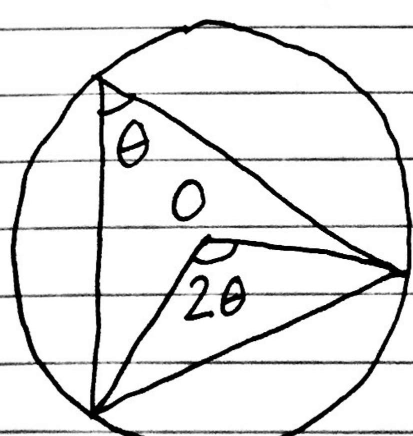
$AB \parallel DE$ & $BC \parallel EF \Rightarrow CD \parallel FA$

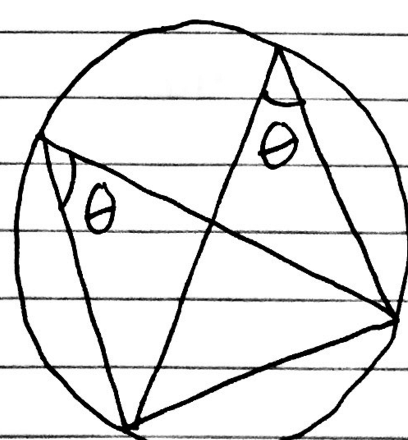
Desargues



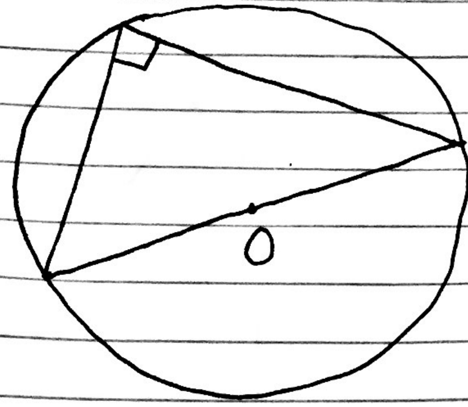
$AB \parallel A'B'$ & $BC \parallel B'C' \Rightarrow AC \parallel A'C'$

Angles in a circle



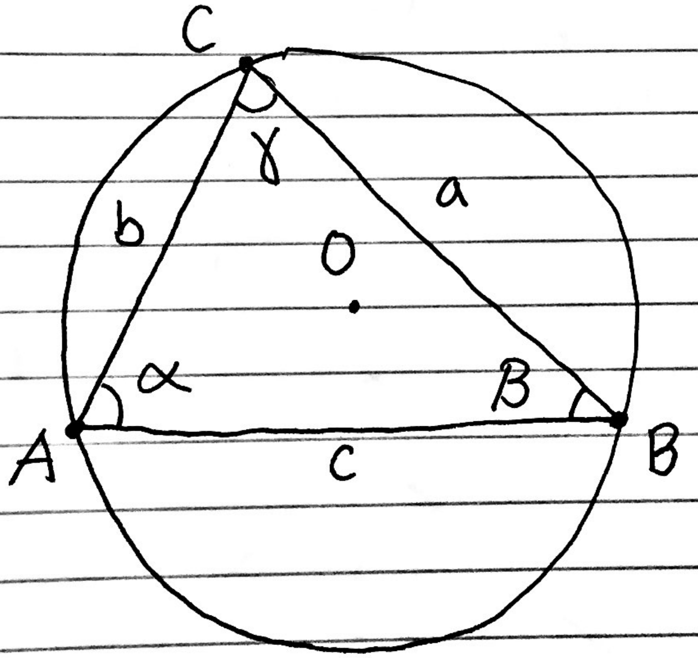


Math 461 Lecture 8 9/21



Today:
sine rule
cosine rule
square roots
regular pentagon

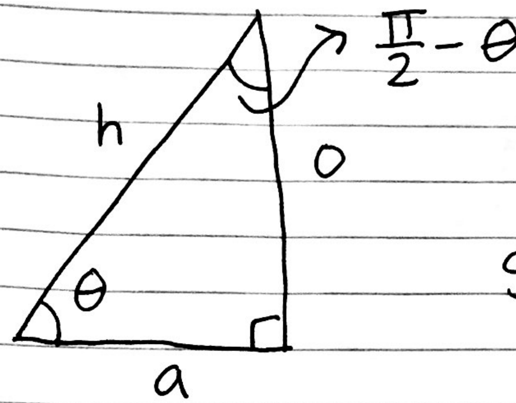
Sine rule



$$\frac{a}{\sin\alpha} = \frac{b}{\sin B} = \frac{c}{\sin\gamma} = 2R$$

where R = radius of circumscribed circle

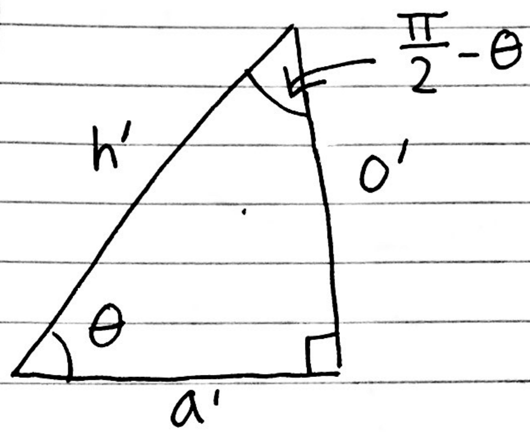
Math 461 Lecture 8 9/21



SOH CAH TOA

$\sin\theta = \frac{o}{h}$ $\cos\theta = \frac{a}{h}$ $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{o}{a}$

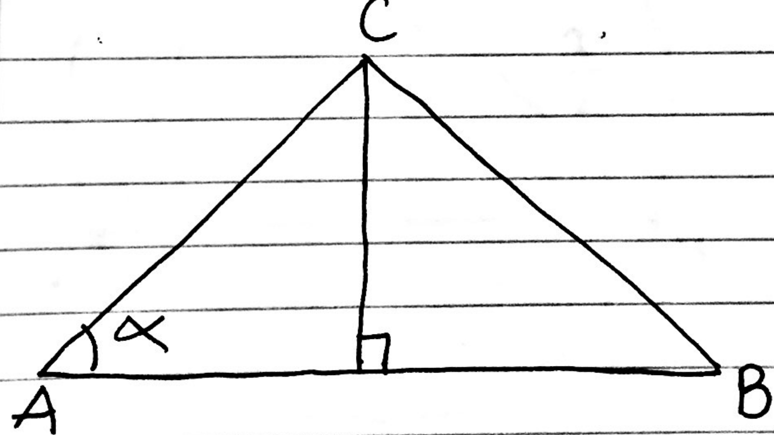
Remark: this is well defined



$\frac{o}{h} = \frac{o'}{h'}$ etc.

these triangles are similar $\Rightarrow$ ratio of sides agree

Proof



$\sin\alpha = \frac{|CD|}{|AC|}$ $\sin\beta = \frac{|CD|}{|BC|}$ $\Rightarrow$

$\sin\alpha |AC| = |CD| = \sin\beta |BC| \Rightarrow$

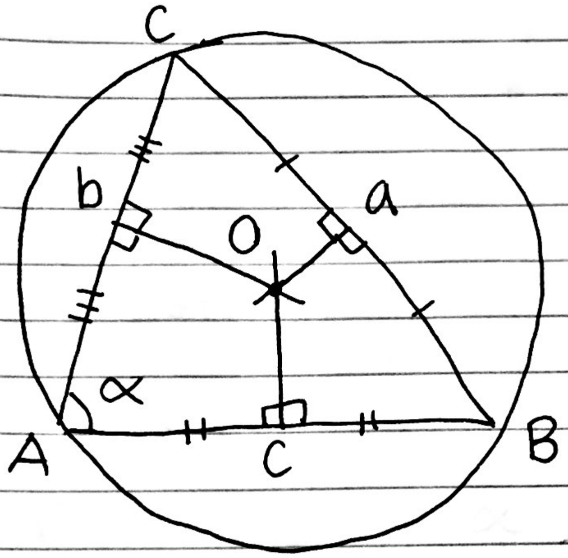
$(\sin\alpha)\, b = (\sin\beta)\, a \Rightarrow \frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

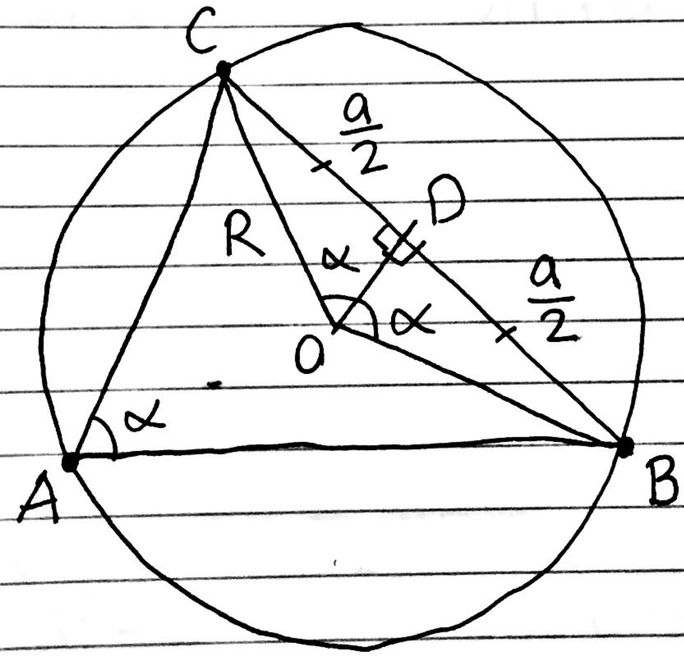
Math 461 Lecture 8 9/21

Similarly, $\dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

so $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

Still need $\dfrac{a}{\sin\alpha} = 2R$





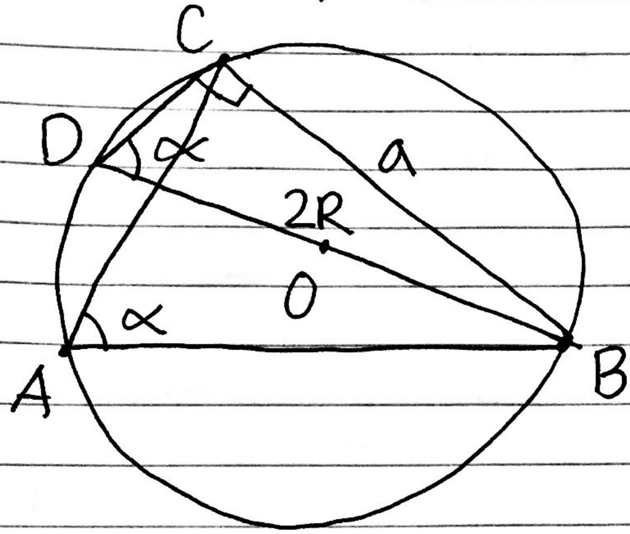
$\Delta ODC \cong \Delta ODB$ by SAS

$|OD| = |OD|$ $|DC| = |DB|$ $\angle ODC = \angle ODB = \frac{\pi}{2}$

$\angle COB = 2\alpha$ angle at center = twice angle at circumference and $\angle COD = \angle BOD \Rightarrow \angle COD = \angle BOD = \alpha$

Math 461 Lecture 8 9/21

$\triangle OCD$ $\quad \sin\alpha = \dfrac{o}{h} = \dfrac{a/2}{R} \qquad 2R = \dfrac{a}{\sin\alpha} \quad \square$



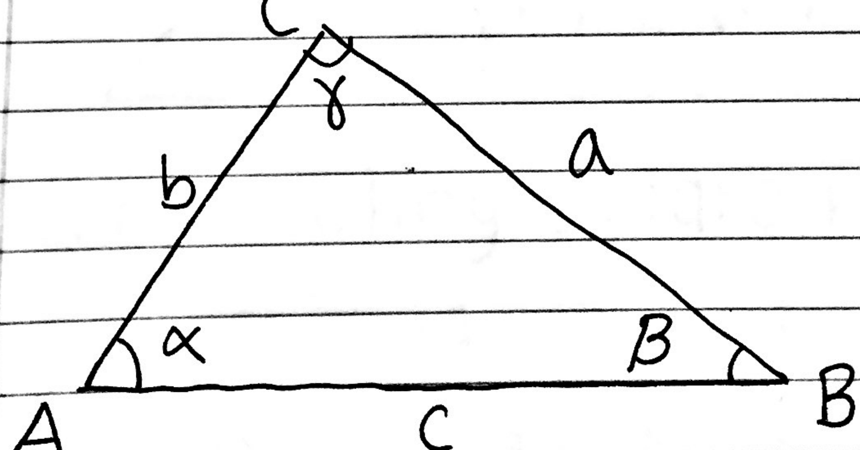
$\triangle BCD$ $\quad \sin\alpha = \dfrac{o}{h} = \dfrac{a}{2R} \qquad \dfrac{a}{\sin\alpha} = 2R$

$\triangle BCD$ $\quad \angle BCD = \dfrac{\pi}{2}$ angle in a semicircle

$\angle COB = \angle CAB = \alpha$

angles at circumferene are equal

cosine rule

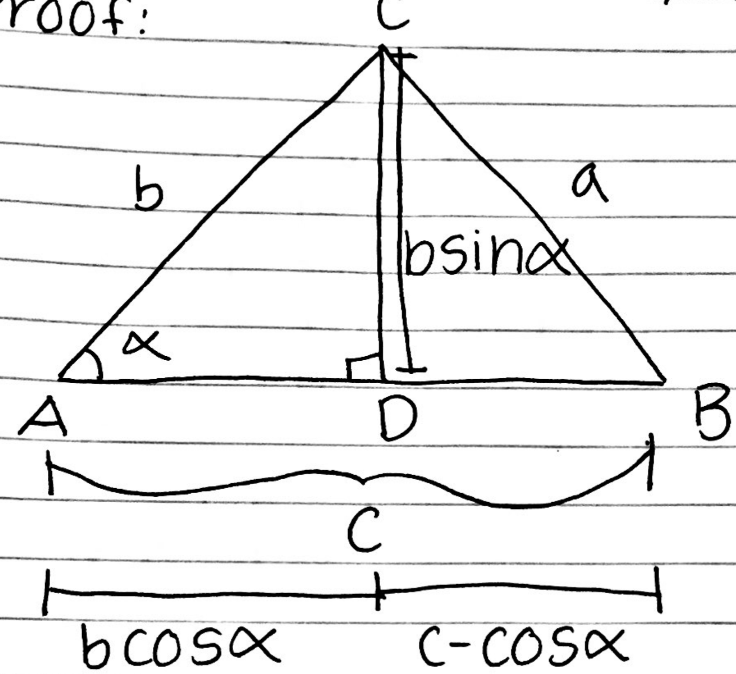


$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$\left(\text{if } \alpha = \frac{\pi}{2},\ a^2 = b^2 + c^2\right)$$

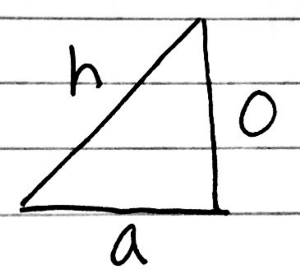
Math 461 Lecture 8 9/21

Proof:



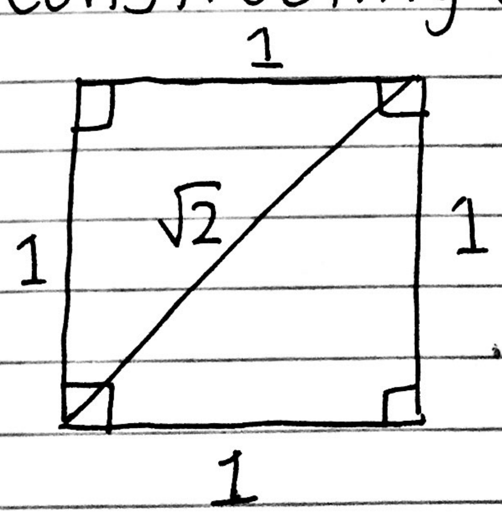
$$a^2 = (c - b\cos\alpha)^2 + (b\sin\alpha)^2$$
$$= c^2 - 2bc\cos\alpha + b^2\cos^2\alpha + b^2\sin^2\alpha$$
$$= b^2(\sin^2\alpha + \cos^2\alpha) + c^2 - 2bc\cos\alpha$$
$$= b^2 + c^2 - 2bc\cos\alpha \quad \square$$

$\sin^2\alpha + \cos^2\alpha = 1$



$$\left(\frac{o}{h}\right)^2 + \left(\frac{a}{h}\right)^2 = \frac{o^2 + a^2}{h^2} = 1$$

Constructing square roots



we know we can construct any rational number
can do $+, -, \times, \div,$
start with length 1
BUT $\sqrt{2} \neq \frac{a}{b}$ for $a, b \in \mathbb{N}$

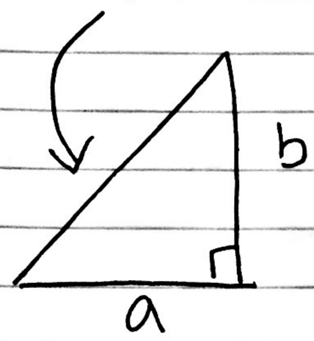
Math 461 Lecture 8 9/21

Is it possible to construct square roots?

That is, given a length ℓ, can I construct $\sqrt{\ell}$?

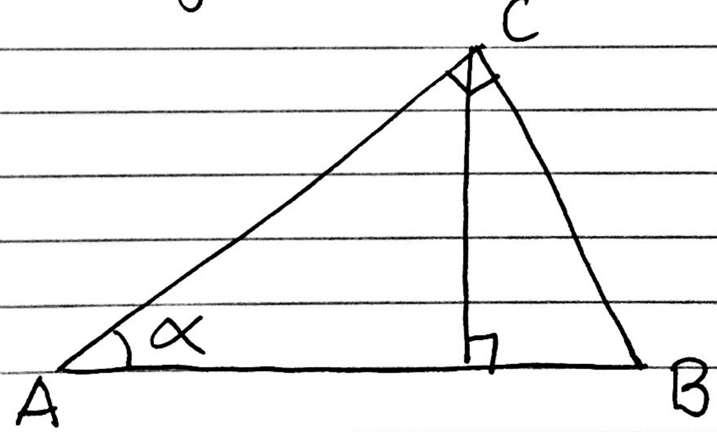
if $\ell = a^2 + b^2$ where a, b are positive integers then

$\sqrt{\ell} = \sqrt{a^2+b^2}$



Remark: a prime p can be written as the sum of two squares $\Leftrightarrow$ $p = 2$ OR $p \equiv 1 \bmod 4$

Fermat probably proved this and a proof using the Gaussian integers $a+bi$ where $a, b \in \mathbb{Z}$



What can we say about the side lengths $|AB|, |BC|, |AC|$ & $|AD|, |DB|$

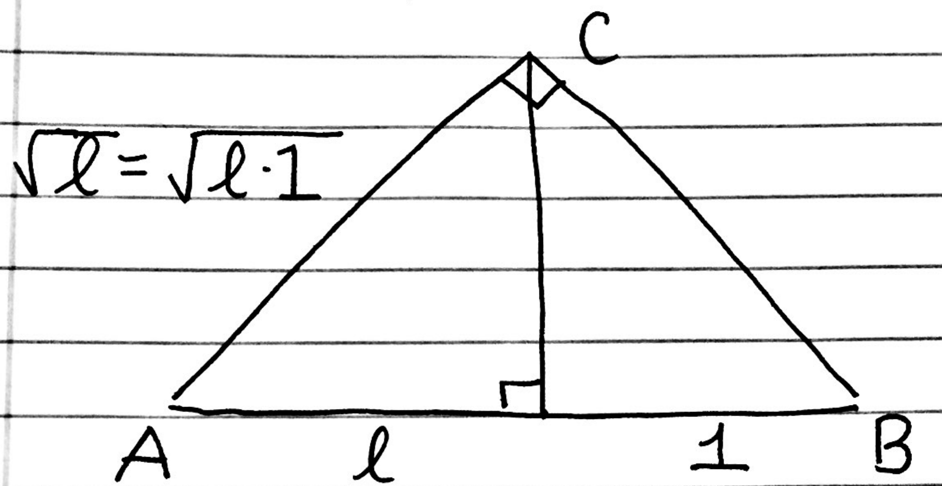
$$\frac{|AC|}{|AB|} = \frac{|AD|}{|AC|} \qquad \triangle ADC \sim \triangle ACB$$

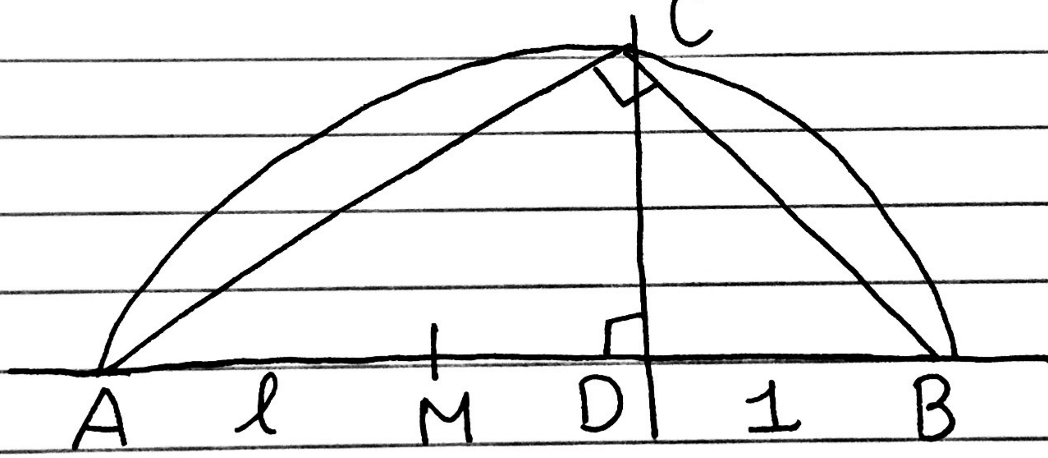
$$|AC|^2 = |AB| \cdot |AD|$$

Math 461 Lecture 8 9/21

given length ℓ, construct $\sqrt{\ell}$

if $\ell = a \cdot b = \ell \cdot 1$





M = midpoint of AB