Math 462: Homework 5

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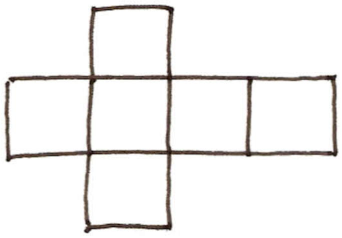
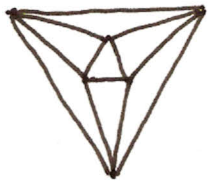
3/21/10

(1) One way to describe a polyhedron is by cutting along some of the edges and folding it flat in the plane. The diagram obtained in this way is called a net. For example here is a net for the cube:



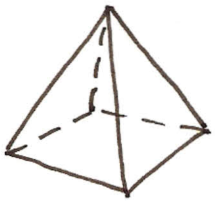
Draw nets for the tetrahedron, octahedron, and dodecahedron.

(2) Another way to describe a polyhedron is as follows: imagine that the faces of the polyhedron are made of glass, look through one of the faces and draw the image you see of the remaining faces of the polyhedron. This image is called a Schlegel diagram. For example here is a Schlegel diagram for the octahedron.



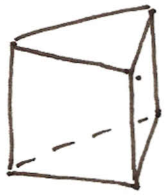
Note that the face you are looking through is not drawn, but the boundary of that face corresponds to the boundary of the diagram. Now draw a Schlegel diagram for the dodecahedron.

pyramid $n=4$



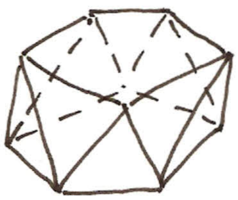
(3) (a) A *pyramid* is a polyhedron obtained from a polygon with some number n of sides by joining every vertex of the polygon to a point lying above the plane of the polygon. The polygon is called the base of the pyramid and the additional vertex is called the apex. For example the Egyptian pyramids have base a square (so $n = 4$) and the tetrahedron is a pyramid with base an equilateral triangle (so $n = 3$). Compute the number of vertices, edges, and faces of a pyramid with base a polygon with n sides.

prism $n=3$



(b) A *prism* is a polyhedron obtained from a polygon (called the base of the prism) as follows: translate the polygon some distance in the direction normal to the plane of the polygon, and join the vertices of the original polygon to the corresponding vertices of the translated polygon. For example, a Toblerone box is a prism with base an equilateral triangle. Compute the number of vertices, edges, and faces of a prism with base a polygon with n sides.

antiprism $n=5$



(c) An *antiprism* is a polyhedron obtained from a regular n-sided polygon (called the base of the antiprism) as follows: translate the polygon some distance in the direction normal to the plane of the polygon, and rotate the polygon through an angle of π/n either clockwise or anticlockwise about its center. (Note that the choice of the direction of rotation does not change the image of the polygon (because rotation by $2\pi/n$ is a symmetry of the polygon), but it does affect the way the vertices are matched up.) Now join the original polygon to the new polygon by connecting each vertex to its image under the two transformations. For example, an octahedron is an antiprism with base an equilateral triangle, and the middle portion of an icosahedron is an antiprism with base a regular pentagon (check for yourself by drawing a picture). Compute the number of vertices, edges, and faces of an antiprism with base a regular polygon with n sides.

(d) Compute the number of vertices, edges, and faces of the regular polyhedrons (the cube, the octahedron, the tetrahedron, the icosahedron, and the dodecahedron).

(e) Do you notice any patterns? Can you explain them?

(4) Recall that if P is a regular polyhedron then the *dual polyhedron* P^* has vertices the centers of the faces of P. Two vertices of P^* are joined by an edge if the corresponding faces of P meet at an edge. Identify the dual of a cube, an octahedron, and a tetrahedron. In each case, find the ratio of the length of the edges of the original polyhedron P to the length of the edges of its dual P^*.

(5) Describe the duals of a pyramid, a prism, and an antiprism (see Q3 above).