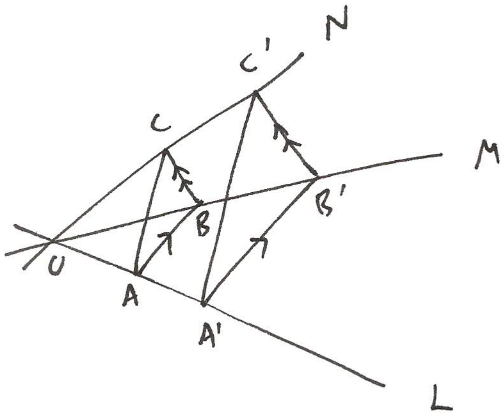
Wednesday 10/2/19.

MATH 461 HW3 SOLUTIONS.

1.



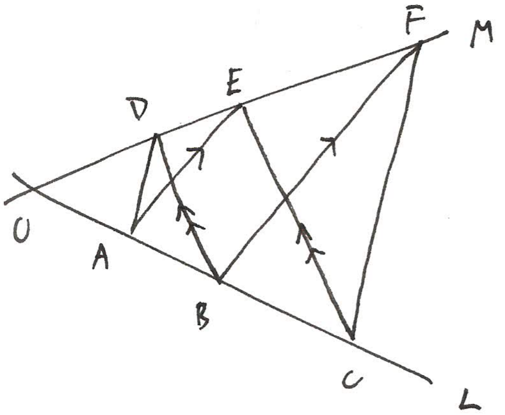
AB parallel to A'B' $\Rightarrow$ $\frac{|OA|}{|OA'|} = \frac{|OB|}{|OB'|}$ (Thales' thm)

BC parallel to B'C' $\Rightarrow$ $\frac{|OB|}{|OB'|} = \frac{|OC|}{|OC'|}$ (" ")

Combining, $\frac{|OA|}{|OA'|} = \frac{|OC|}{|OC'|}$.

So AC is parallel to A'C' by the converse of Thales' thm. □

2.



AE parallel to BF $\Rightarrow$ $\frac{|OA|}{|OB|} = \frac{|OE|}{|OF|}$ (Thales' thm)

BD parallel to CE $\Rightarrow$ $\frac{|OB|}{|OC|} = \frac{|OD|}{|OE|}$ (" ")

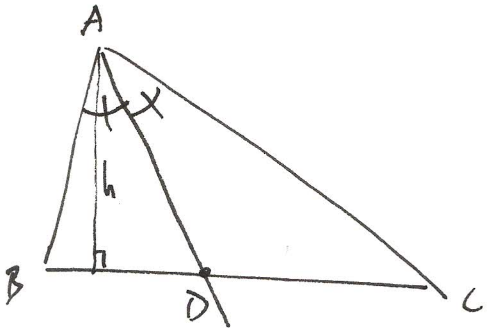
Multiplying,

$$\frac{|OA|}{\cancel{|OB|}} \cdot \frac{\cancel{|OB|}}{|OC|} = \frac{\cancel{|OE|}}{|OF|} \cdot \frac{|OD|}{\cancel{|OE|}}$$

So $\frac{|OA|}{|OC|} = \frac{|OD|}{|OF|}$.

So AD is parallel to CF by the converse of Thales' thm. □.

3a.



Recall : Area (triangle) = $\frac{1}{2}$ base × height

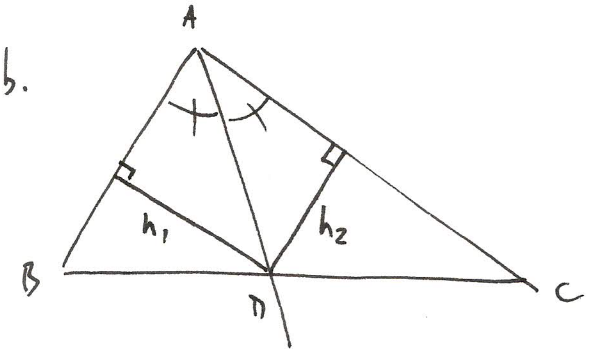
$\text{Area}(\triangle ACD) = \frac{1}{2}|CD| \cdot h$

$\text{Area}(\triangle ABD) = \frac{1}{2}|BD| \cdot h$

where h is the perpendicular distance from A to the line BC.

Dividing, $\dfrac{\text{Area}(\triangle ACD)}{\text{Area}(\triangle ABD)} = \dfrac{|CD|}{|BD|}.$

b.



$\text{Area}(\triangle ACD) = \frac{1}{2}|AC| \cdot h_2$ where h_2 is the perpendicular distance from D to AC

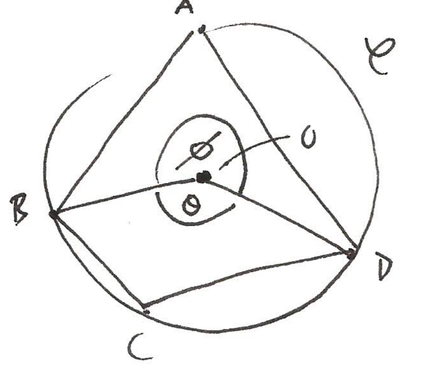
$\text{Area}(\triangle ABD) = \frac{1}{2}|AB| \cdot h_1$ " h_1 " " " " " D to AB.

By HW2Q3a, $h_1 = h_2$ (because D lies on the bisector of angle $\angle BAC$)

Dividing $\dfrac{\text{Area}(\triangle ACD)}{\text{Area}(\triangle ABD)} = \dfrac{|AC|}{|AB|}.$

Combining with (a), $\dfrac{|CD|}{|BD|} = \dfrac{|AC|}{|AB|}$. $\square$.

4.



$2\angle BAD = \phi$

$2\angle BCD = \theta$

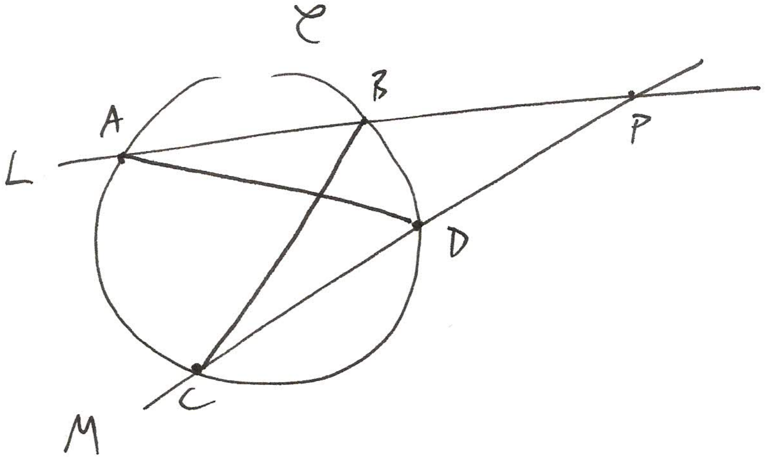
(angle at center = twice angle at circumference)

So $2\angle BAD + 2\angle BCD = \phi + \theta = 2\pi$.

$\div 2$ $\quad \angle BAD + \angle BCD = \pi$. □.

O denotes the center of the circle $\mathcal{C}$.

5.



$\angle BAD = \angle BCD$ (angles subtended by same chord BD at the circumference are equal)

~~$\angle ABC = \angle ADC$ (.. .. .. AC .. ..)~~

$\Delta PAD \sim \Delta PCB$:

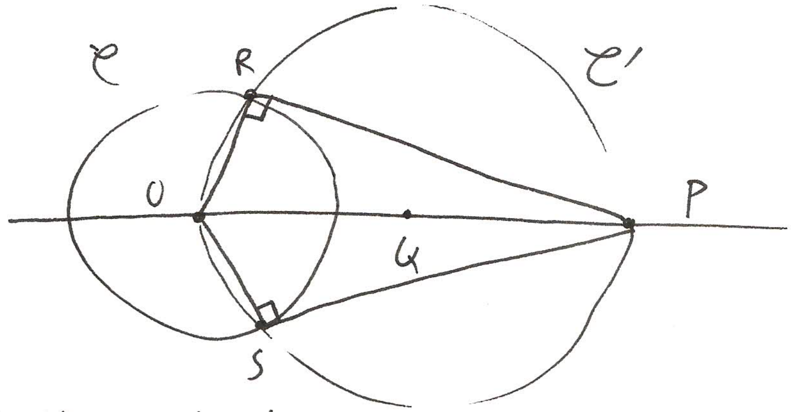
$\angle PAD = \angle PCB$ (see above)

$\angle APD = \angle CPB$ (same angle)

$\therefore \angle ADP = \angle CBP$ by angle sum of $\Delta = \pi$.

So $\dfrac{|PA|}{|PC|} = \dfrac{|PD|}{|PB|}$, or $|PA| \cdot |PB| = |PC| \cdot |PD|$ □.

6.



1. Draw the line OP.
2. Bisect the line segment OP; let Q be its midpoint.
3. Draw the circle $\mathcal{C}'$ with center Q and radius OQ;

   let R & S be the intersection points of this circle with $\mathcal{C}$.
4. Draw the lines PR & PS.

<u>Claim</u>: These lines are tangent to $\mathcal{C}$.

<u>Proof</u>: $\angle ORP = \angle OSP = \pi/2$ (angle in a semicircle $= \pi/2$ applied to circle $\mathcal{C}'$)

So PR is perpendicular to the radius OR of $\mathcal{C}$.

So PR is tangent to $\mathcal{C}$ at R by HW1Q5.

Similarly PS is tangent to $\mathcal{C}$ at S. $\square$.