9/18/19

Announcements: HW 1 due now

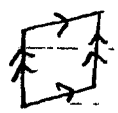
HW 2 available at people.math.umass.edu/~hacking/461F19

Due Wednesday 9/25/19 at start of class

Last Time: • Angle sum of a triangle $= \pi$.

• Parallelograms:

(opposite sides are parallel)

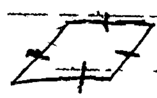


a) opposite sides have equal length

b) diagons bisect each other

• Rhombus

(all side lengths equal)



a) opposite sides are parallel (rhombus $\Rightarrow$ parallelogram)

b) diagonals meet at right angles

Today: • Areas

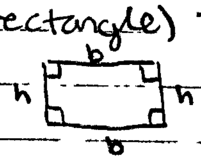
• Pythagoras' theorem

• Similar triangles (Thales' theorem)

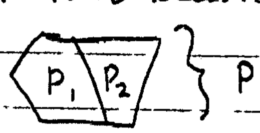
Area

We will assume (following Euclid) that for any polygon P in the plane we can assign an area $A(P)$ (a positive real number) such that:

① $A(\text{rectangle}) = \text{base} \times \text{height}$



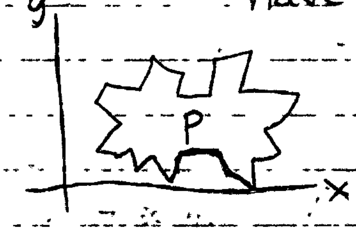
② If P is subdivided into P_1 & P_2



$A(P) = A(P_1) + A(P_2)$

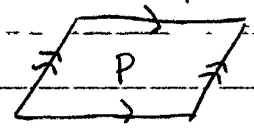
③ If T_1 & T_2 are congruent triangles, then $A(T_1) = A(T_2)$

Remark: Given coordinate geometry & calculus, we have $A(P) = \iint_P 1 \cdot dx\,dy$

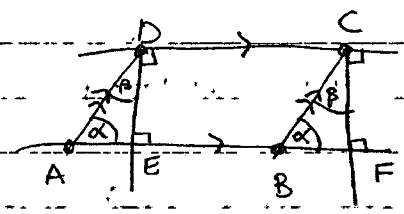


but we will use the axiomatic approach.

1. Area of a parallelogram



$A(P) = ?$

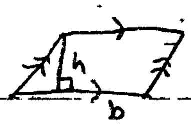


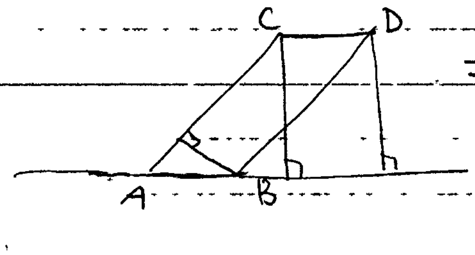
Claim: $\triangle ADE \cong \triangle BCF$

Proof $|AD| = |BC|$ since ABCD is a parallelogram.
$\angle AED = \angle BFC$ by construction
$\angle DAE = \angle CBF$ by (AD & BC are parallel) alternating angles
ASA $\Rightarrow$ AAS because $\beta = \pi - \pi/2 - \alpha$ ✓ ▨

$\text{Area}(EFCD) = |EF| \cdot |FC|$
$\text{Area}(P) = \text{Area}(EFCD) + \text{Area}(\triangle ADE) - \text{Area}(\triangle BCF)$
(precisely in terms of axiom ②)
$\text{Area}(P) + \text{Area}(\triangle BCF)$
$= \text{Area}(\triangle AED) + \text{Area}(EFCD)$
$= \text{Area}(AFCD)$
$\Rightarrow \text{Area}(P) = \text{Area}(EFCD)$ because $\triangle AED \cong \triangle BCF$

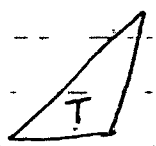
i.e. Theorem Area of parallelogram = base × perpendicular height



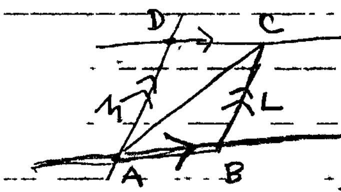


proof $\Rightarrow$ still works

2. Area of a triangle



Area(T) = ?



- Draw line L through C parallel to AB.
- Draw line M through A parallel to BC.
- Let $D = L \cap M$ be the intersection point of L & M

Then ABCD is a parallelogram.

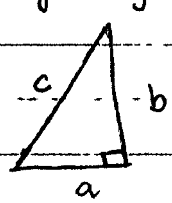
$T = \triangle ABC \cong \triangle CDA$ by SSS

(opposite sides of parallelogram have equal length)

$$\underset{\text{base} \times \text{height}}{\overset{\shortparallel}{\text{Area}(ABCD)}} \overset{②}{=} \text{Area}(\triangle ABC) + \text{Area}(\triangle CDA) \underset{③}{=} 2 \cdot \text{Area}(\triangle ABC)$$

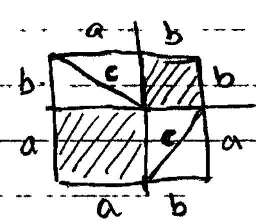
$$\text{Area}(\triangle ABC) = \tfrac{1}{2}\, \text{base} \times \text{height}$$

3. Pythagoras' theorem

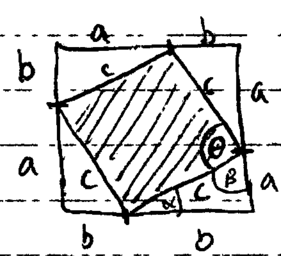
T = 

$a^2 + b^2 = c^2$

Proof



- Square, side length $a+b$
- Divide into 2 squares of side lengths a & b, and 4 triangles, each congruent to T (SAS).
- Area left over $a^2 + b^2$



- Move triangles around
- Area left over c^2

↳ prove it is a square: all triangles are congruent $\Rightarrow$ side lengths of square equal

$\Rightarrow \theta + \alpha + \beta = \pi$, $\alpha + \beta = \pi/2$

$\Rightarrow \theta = \pi/2$ ∎