Math 300.2 Homework 1

Paul Hacking

January 24, 2012

Reading: Gilbert and Vanstone, Chapter 1.

(1) Compute the truth tables for the following statements.

(a) $(\mathrm{NOT}\, P) \Rightarrow Q$.

(b) $(P\, \mathrm{OR}\, Q)\, \mathrm{AND}(\mathrm{NOT}(P\, \mathrm{AND}\, Q))$. [This is sometimes called "exclusive or"].

(c) $(P\, \mathrm{AND}\, Q) \Rightarrow R$.

(d) $(P \Rightarrow Q)\, \mathrm{AND}(Q \Rightarrow R)\, \mathrm{AND}(R \Rightarrow P)$.

(2) In each of the following cases, show that the two statements are equivalent (that is, they are either both true or both false).

(a) $P \Rightarrow Q, \quad \mathrm{NOT}(P\, \mathrm{AND}(\mathrm{NOT}\, Q))$.

(b) $P\, \mathrm{AND}(Q\, \mathrm{OR}\, R), \quad (P\, \mathrm{AND}\, Q)\, \mathrm{OR}(P\, \mathrm{AND}\, R)$.

(3) List the elements of the following sets. (If the set is infinite you can use $\ldots$, but you should list enough elements so that the pattern is clear.)

(a) $\{x \in \mathbb{Z} \mid x^2 + 3 < 11\}$.

(b) $\{x \in \mathbb{N} \mid \sqrt{x} \in \mathbb{N}\}$.

(c) $\{x \in \mathbb{R} \mid x^2 + 1 = 0\}$.

(d) $\{3, 6\} \times \{2, 4, 6, 8\}$.

(4) Let A, B, C be sets. Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ in two ways: first, using a Venn diagram, second, using truth tables. [Hint: For the second approach, express the statement $x \in A \cap (B \cup C)$ as a

compound of the statements $x \in A$, $x \in B$, and $x \in C$, and write out the truth table. Now repeat for the statement $x \in (A \cap B) \cup (A \cap C)$ and compare the truth tables.]

(5) Let A be a subset of a set X. We define the *indicator function* of A to be the function $1_A \colon X \to \{0,1\}$ given by $1_A(x) = 1$ if $x \in A$ and $1_A(x) = 0$ if $x \notin A$.

(a) Draw the graph of the indicator function for $A = [0,1] \subset X = \mathbb{R}$.

(b) Let A and B be two subsets of a set X. Show that

$$1_{A \cap B}(x) = 1_A(x) \cdot 1_B(x)$$

and

$$1_{A \cup B}(x) = 1_A(x) + 1_B(x) - 1_A(x) \cdot 1_B(x)$$

for all $x \in X$. [Hint: There are 4 cases to check given by $x \in A$ or $x \notin A$ and $x \in B$ or $x \notin B$.]