Math 462: Homework 1

Paul Hacking

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(1) Suppose a, b, c are positive real numbers satisfying the inequalities

$$a < b + c, \quad b < c + a, \quad c < a + b.$$

Give a geometric construction using a ruler and compass for a triangle with side lengths a, b, c. What happens if one of the inequalities becomes an equality, say $a = b + c$?

(2) Can you find 4 points A, B, C, D in $\mathbb{R}^2$ such that the distance between A and D equals 2, but the distance between every other pair of points equals 1? Explain. Can you find 4 such points on a sphere? [You will need to choose the radius carefully. Here is a hint: An explorer leaves base camp, walks 2 miles south, sees a bear, runs 2 miles east, then 2 miles north, and arrives back at base. What colour was the bear?]

(3) Find the eigenvectors of the matrix $A = \begin{pmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{pmatrix}$. Describe the transformation $T \colon \mathbb{R}^2 \to \mathbb{R}^2$, $T(\mathbf{x}) = A\mathbf{x}$ geometrically.

(4) Describe the following motions of $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ geometrically (as a translation, rotation, reflection, or glide reflection).

(a) $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x+2 \\ y+1 \end{pmatrix}$.

(b) $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y+3 \\ -x+1 \end{pmatrix}$.

(c) $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$, where A is as in Q3 and $\mathbf{b} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$.

(d) $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$, where $A = \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

(5) (a) Two lines l and m in $\mathbb{R}^2$ intersect at a point P and meet at an angle θ, $0 < \theta \leq \pi/2$. Describe geometrically the composition of the reflections in l and m. [Hint: You can argue geometrically or work in coordinates — choosing coordinates carefully makes it easier.]

(b) (Harder). Suppose $\theta = \pi/m$ for some integer $m \geq 2$. What is the group G generated by reflections in l and m? Describe a fundamental domain for the action of G on $\mathbb{R}^2$, that is, a region $R \subset \mathbb{R}^2$ such that each orbit of G contains exactly one point of R.