# Math 461 Homework 1

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(1) Given a circle show how to determine the center of the circle using ruler and compass.

(2) Suppose given two triangles $\Delta ABC$ and $\Delta A'B'C'$ such that $\angle CAB = \angle C'A'B'$, $|AB| = |A'B'|$ and $|BC| = |B'C'|$. Does it follow that the triangles are congruent? Give a proof or a counterexample.

[Hint: Given triangle $\Delta ABC$, draw two lines $L$ and $M$ intersecting at a point $A'$ at angle $\alpha = \angle CAB$, mark a point $B'$ on $L$ such that $|A'B'| = c = |AB|$, draw the circle $C$ with center $B'$ and radius $a = |BC|$, and consider the intersection points of the circle $C$ with the line $M$.]

(3) Let $A$,$B$,$C$ be 3 distinct points in the plane.

(a) Show that the perpendicular bisectors of $AB$ and $BC$ intersect if and only if the points $A$,$B$,$C$ do not lie on a line.

(b) Prove that if the points $A$,$B$,$C$ do not lie on a line then there exists a unique circle passing through the points.

(4) Recall that we say a $n$-sided polygon is *regular* if all the sides have equal lengths and all the angles are equal. Given a line segment, describe a ruler and compass construction of a regular $n$-gon with one side the line segment in the cases (a) $n = 4$ and (b) $n = 6$. Prove carefully that your construction is correct in each case.

(5) Let $n \geq 3$ be a positive integer. We say that a $n$-sided polygon $P$ is *convex* if for any two points $A$ and $B$ in $P$ the line segment $AB$ is contained in $P$. (Equivalently, $P$ is convex if all the interior angles of $P$ are less than $\pi$.) Prove by induction that the sum of the interior angles of a convex $n$-sided polygon equals $(n-2)\pi$.

(6) Let $C$ be a circle with center $O$ and $P$ a point on $C$. Let $L$ be a line passing through $P$. We say $L$ is *tangent* to $C$ at $P$ if $L \cap C = \{P\}$. Prove that $L$ is tangent to $C$ if and only if $OP$ is perpendicular to $L$.

[Hint: Suppose $L$ intersects the circle $C$ at another point $Q$. What can you say about the angle $\angle OPQ$?]

(7) (a) Given a triangle $\Delta ABC$, show that if a point $P$ in the triangle lies on the bisector of the angle $\angle BAC$ then the perpendicular distance from $P$ to $AB$ equals the perpendicular distance from $P$ to $AC$.

(b) Show that the three bisectors of the angles of a triangle are concurrent, that is, they all pass through some point $P$.

(c) Show that a triangle has an *inscribed circle*: a circle contained in the triangle which is tangent to each of the sides of the triangle.