

Math 461 Homework 2

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- (1) Let $\triangle ABC$ be a triangle such that $\angle ABC = \pi/2$. Let D be the midpoint of AC . Prove that $|BD| = \frac{1}{2}|AC|$.
- (2) Let $\triangle ABC$ be a triangle such that $|AB| = |AC|$. Suppose given points D on AB and E on AC such that $|BC| = |CD| = |DE| = |EA|$. Determine (with proof) $\angle BAC$.
- (3) Let $\triangle ABC$ be a triangle. Prove that if $|AC| > |AB|$ then $\angle ABC > \angle ACB$.
- (4) Let $\triangle ABC$ be a triangle. Let D be the intersection point of the bisector of the angle $\angle BAC$ and the side BC .
 - (a) Prove that $\text{Area}(\triangle ACD)/\text{Area}(\triangle ADB) = |CD|/|DB|$.
 - (b) Prove that $|CD|/|DB| = |AC|/|AB|$.
- (5) Let $ABCD$ be a quadrilateral. Let P, Q, R, S be the midpoints of the sides AB, BC, CD , and DA . Prove that $PQRS$ is a parallelogram.
- (6) Let C be a circle and P a point not lying on C . Let L and L' be two lines passing through P such that L intersects C in two points X and Y and L' intersects C in two points X' and Y' . Prove that $|PX| \cdot |PY| = |PX'| \cdot |PY'|$.
- (7) Suppose the vertices A, B, C, D of a quadrilateral $ABCD$ lie on a circle. Prove that the opposite angles of the quadrilateral sum to π , that is, $\angle ABC + \angle CDA = \angle BCD + \angle DAB = \pi$.

- (8) Let $ABCD$ be a convex quadrilateral such that $\angle ACB$ and $\angle ADB$ are equal. Show that the vertices A, B, C, D of the quadrilateral lie on a circle.
- (9) Given a circle C and a point P outside C give a ruler and compass construction of the two lines through P that are tangent to C .