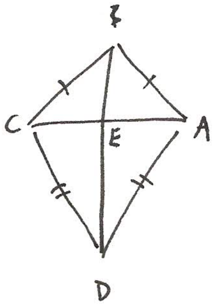
MATH 461 MIDTERM SOLUTIONS.

1. 

$\angle BAC = \angle BCA$ & $\angle DAC = \angle DCA$ (isosceles Δ thm) $\Rightarrow \angle BAD = \angle BCD$

$\triangle BAD \cong \triangle BCD$ (SAS)

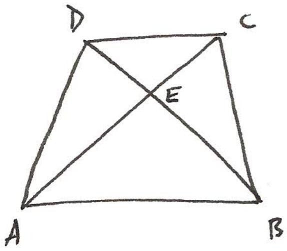
$\Rightarrow \angle ABD = \angle CBD$

$\Rightarrow \triangle ABE \cong \triangle CBE$ (SAS)

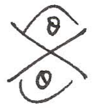
$\Rightarrow |AE| = |CE|$

& $\angle AEB = \angle CEB$

Now $\angle AEB + \angle CEB = \pi \Rightarrow \angle AEB = \pi/2$ □.

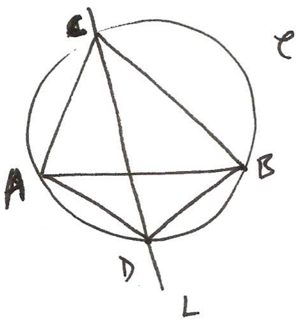
2. 

$$\left.\begin{aligned} \angle ABE &= \angle EDC \\ \angle BAE &= \angle ECD \end{aligned}\right\} \text{(alternate angles)}$$

$\angle AEB = \angle CED$ ()

So $\triangle ABE \sim \triangle CDE$

$$\Rightarrow \frac{|AE|}{|CE|} = \frac{|BE|}{|DE|} \Rightarrow |AE| \cdot |DE| = |BE| \cdot |CE|.$$

3. 

$\angle ACD = \angle DCB$ (L is the bisector of $\angle ACB$)

$\angle ACD = \angle ABD$ (angles subtended by a chord)

$\angle DCB = \angle DAB$ (" " " " ")

$\therefore \angle ABD = \angle DAB$

$\therefore |AD| = |BD|$ (isosceles Δ theorem)

4. a. 1. Draw the perpendicular bisector L of AB

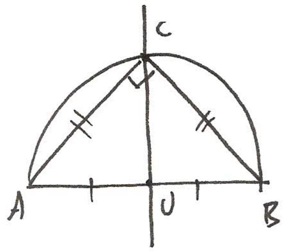
2. Draw the circle $\mathcal{C}$ with center the midpoint O of AB (the intersection of L & AB) and radius $|OA| = |OB|$. Let C be one of the intersection points of $\mathcal{C}$ & L

Then $\angle ACB = \pi/2$ (angle in a semicircle) and $|AC| = |BC|$ (C lies on perpendicular bisector of AB)

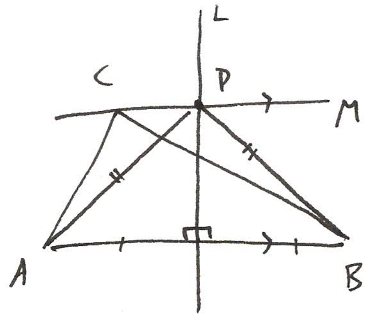
$\Rightarrow$ $\angle ABC = \angle CAB = \frac{1}{2}(\pi - \frac{\pi}{2}) = \frac{\pi}{4}$. $\square$

(isosceles Δ thm)

(angle sum of Δ)



b.



1. Construct the perpendicular bisector L of AB
2. Construct the parallel line M to AB through C.

Let $D = L \cap M$ be the intersection point of L & M.

Then $D \in L \Rightarrow |AD| = |BD|$

$\Rightarrow \angle ABD = \angle BAD$ (isosceles triangle thm.)

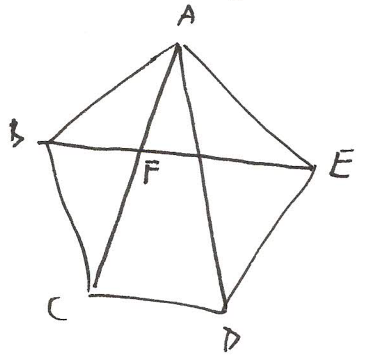
And $\text{Area}(\Delta ABD) = \text{Area}(\Delta ABC)$

because they have the same base $|AB|$ & perpendicular height. $\square$.

5. a. The angle sum of a pentagon equals $(5-2)\cdot\pi = 3\pi$.

So each angle in a regular pentagon equals $3\pi/5$.

b.



ΔABC is isosceles ($|AB| = |BC|$)

so $\angle BAC = \angle BCA = \frac{1}{2}(\pi - \angle ABC) \overset{(a)}{=} \frac{1}{2}(\pi - 3\pi/5) = \pi/5$

isosceles triangle theorem — angle sum of triangle

Similarly, $\angle EAD = \angle EDA = \pi/5$, & $\angle ABE = \angle AEB = \pi/5$

So $\angle CAD = 3\pi/5 - \pi/5 - \pi/5 = \pi/5$

& $\angle ACD = \angle ADC = 3\pi/5 - \pi/5 = 2\pi/5$

Finally, $\angle AEF = \angle AEB = \pi/5$

& $\angle FAE = 3\pi/5 - \pi/5 = 2\pi/5$

so $\angle AFE = \pi - \pi/5 - 2\pi/5 = 2\pi/5$.

angle sum of triangle

Thus $\Delta AFE \sim \Delta CDA$ $\square$.