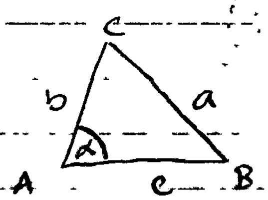
10/7/19

Office hours today and tomorrow 4-5PM LGRT 1235H
HW 4 due Wednesday at start of class.

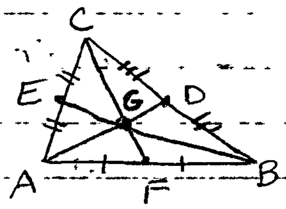
Last Time: • Cosine rule



$a^2 = b^2 + c^2 - 2bc\cos\alpha$

• Center of mass:

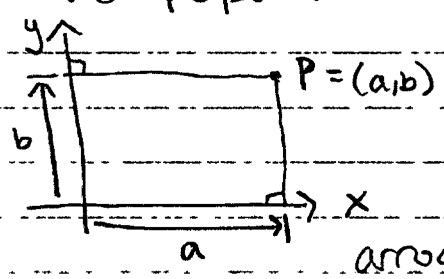
(End of Euclid's Geometry)



$\frac{|AG|}{|GD|} = \frac{|BG|}{|GE|} = \frac{|CG|}{|GF|} = 2$

Today: • Coordinates (~1630)
↳ Fermat & Descartes (Cartesian coordinates)

Introduce x & y axes:
Pick a point, call it the $\boxed{\text{origin}}$.
Draw two perpendicular lines through the origin, x-axis & y-axis as shown.



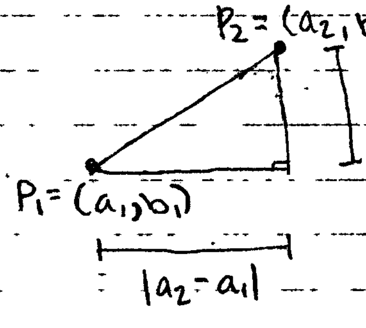
positive or negative
arrow "signed" distance

Remark We make a choice of coordinates.
Often a good choice of coordinates makes a geometric problem much easier to solve.

Geometry <~~~> Algebra

Plane = $\mathbb{R}^2$, given choice of coordinates.

Ex



Length of a line:

$$\boxed{|P_1P_2| = \sqrt{(a_2-a_1)^2 + (b_2-b_1)^2}}$$

$$|P_1P_2|^2 = |a_2 - a_1|^2 + |b_2 - b_1|^2$$

↳ by Pythagoras' Thm

Ex Equation of a circle $\mathcal{C}$, center $P = (a, b)$, radius r:

$$\mathcal{C} = \{ (x,y) \in \mathbb{R}^2 \mid \; ?? \; \}$$

$$|P(x,y)| = r$$

↳ distance from P to (x,y)

$$= \{ (x,y) \in \mathbb{R}^2 \mid \sqrt{(x-a)^2 + (y-b)^2} = r \}$$

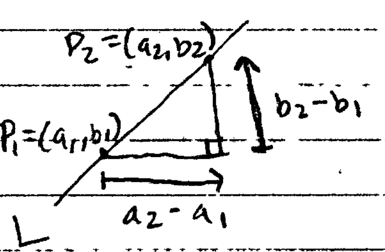
$$= \{ (x,y) \in \mathbb{R}^2 \mid (x-a)^2 + (y-b)^2 = r^2$$

can take square root because radius is always positive $\Rightarrow$ no loss of sign.

Ex Equation of a line:

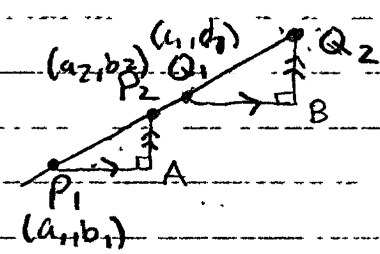
Slope of a line $L = \dfrac{\text{rise}}{\text{run}} = \dfrac{b_2 - b_1}{a_2 - a_1}$

Pick any two points, $P_1 = (a_1, b_1)$ & $P_2 = (a_2, b_2)$ on L



$$L = \{ (x,y) \in \mathbb{R}^2 \mid \; ?? \; \}$$

Q: Is the slope well-defined?



$$\frac{b_2-b_1}{a_2-a_1} \stackrel{?}{=} \frac{d_2-d_1}{c_2-c_1} \quad (*)$$

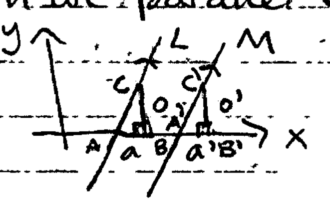
YES b/c $\triangle P_1AP_2 \sim \triangle Q_1BQ_2$

$$\Rightarrow \frac{P_1A}{|Q_1B|} = \frac{|P_2A|}{|Q_2B|} \rightsquigarrow \frac{|P_2A|}{|P_1A|} = \frac{|Q_2B|}{|Q_1B|}$$

which is $(*)$. ▨

Lemma: L & M are parallel $\Longleftrightarrow$ same slope. OR $L=M$

Proof ($\Rightarrow$)



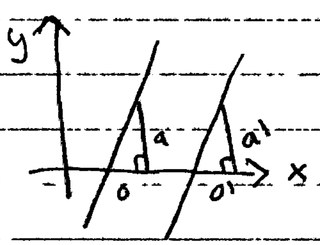
Know L parallel to M,

Want to show slope(L) = slope(M)

$\therefore$ $o/a = o'/a'$ b/c of similar triangles

$\triangle ABC \sim \triangle A'B'C'$

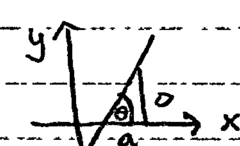
($\Leftarrow$)



We assume $o/a = o'/a'$,

want to show L parallel to M

Remark: Slope $m = \tan\theta$

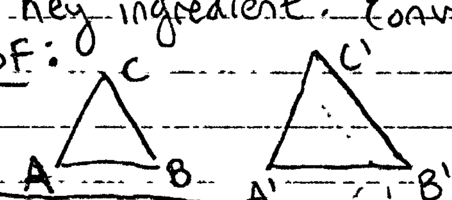


Recall Say $\triangle ABC \sim \triangle A'B'C'$ if corresponding angles are equal. Proved, $\triangle ABC \sim \triangle A'B'C' \Rightarrow \frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$.

The converse ($\Leftarrow$) is also true.

Key ingredient: Converse of Thales' thm

Proof:



Assume $\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}$

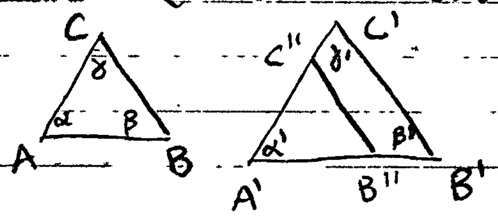
Show $\triangle ABC \sim \triangle A'B'C'$.

(corresponding angles are equal) $\longrightarrow$

Aside: Could use cosine rule and solve for α.

Introduce B'' on $A'B'$ s.t. $|A'B''| = |AB|$

" " C'' on $A'C'$ s.t. $|A'C''| = |AC|$



Converse of Thales' theorem: $\dfrac{|A'B''|}{|A'B'|} = \dfrac{|A'C''|}{|A'C'|}$

$\Rightarrow B''C'' \parallel B'C'$

Now, by corresponding angles, get $\Delta A'B''C'' \sim \Delta A'B'C'$.

Still need to prove: $\Delta ABC \cong \Delta A'B''C''$.

To be continued...