Math 461 Lecture 1 9/5
Professor: Paul Hacking
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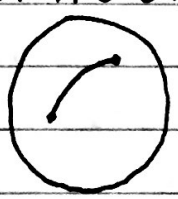
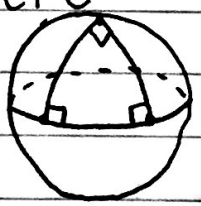
Office Hours: Mon + Tues 4-5 pm
Office: LGRT 1235H
Textbook: J. Stillwell, The four pillars of Geometry (can download PDF for free from UMass Libraries)
Homework: due weekly at start of Wednesday's class (HW1 due 9/19) (drop lowest grade)
Exams: Midterm Wed 10/24 7-9 pm
Final Fri 12/14 8-10 am LGRT 141
Grading: Midterm 30%
Final 35%
Homework and Quizzes 35%
Overview of Course:
1. Euclidean geometry
ruler and compass constructions
axiomatic approach
2. coordinates and transformations
3. projective geometry ex. shadow of a kite discovered by artists
4. spherical geometry
finding the shortest path between two points on a sphere





spherical triangle sum of angles is more than 180°

strait edge and compass:
ruler is used to make a straight line not for measurement therefore its refered to as a straight edge

Math 461 Lecture 1 9/5

compass is used to draw a circle

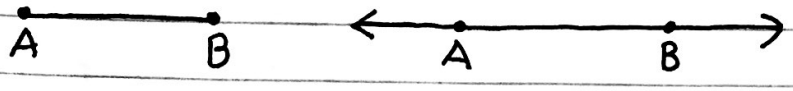
question: what geometric figures can be constructed using ruler and compass?

Euclid ~ 300 BC

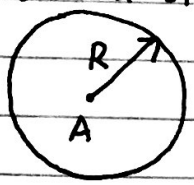
wrote the "Elements (of geometry)" that is 13 volumes

Euclid's construction axioms

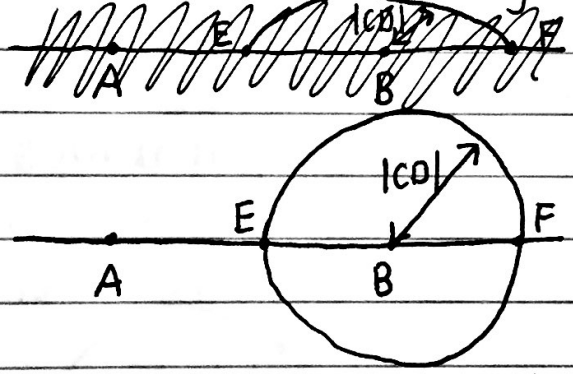
1. draw a straight line segment between two points and extend it indefinitely



2. draw a circle with given center and radius



ex. if given line segment AB and CD of lengths $|AB|$ and $|CD|$ then we can construct line segments with lengths $|AB| + |CD|$ and $|AB| - |CD|$ assuming $|AB| > |CD|$

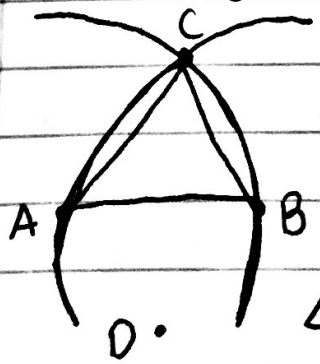


$|AE| = |AB| - |CD|$

$|AF| = |AB| + |CD|$

ex. given line segment AB, construct an equilateral triangle with base AB

↑ sides have equal lengths



1. draw circle center A, radius AB
2. draw circle center B, radius AB intersect at points C and D
3. draw line segments AC, BC

$\triangle ABC$ equilateral $|BC| = |AB| = |AC|$

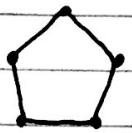
Math 461 Lecture 1 9/5

motivating problems

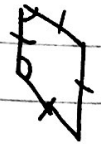
1. suppose we are given a line segment of length 1, what other lengths can be constructed?
2. for which $n \geq 3$ can we construct a regular polygon with n sides

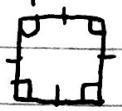
regular: all side lengths are equal and all angles are equal

polygon $n=5$ (pentagon)



rhombus. regular $n=4$ (square)

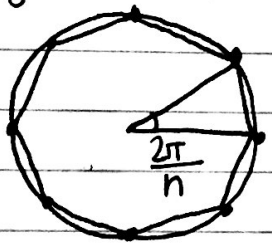




Euclid $n = 3, 4, 5, 6$ $15 = 3 \cdot 5$

(and multiples of there by power of 2)

given regular n-gon and get regular 2n-gon by intersecting angles



regular n-gon

Gauss 1796 (19 years old)

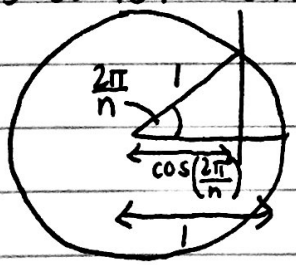
complete solution

ex. $n=17$ is possible

depends on Gauss theory (1811-1832)

regular n-gon is constructible $\Leftrightarrow$ $\cos\left(\frac{2\pi}{n}\right)$ is constructible

$\uparrow$ iff



congruence of triangles:

we say two triangles ΔABC and $\Delta A'B'C'$ are congruent if the corresponding sides have equal lengths and the corresponding angles are equal

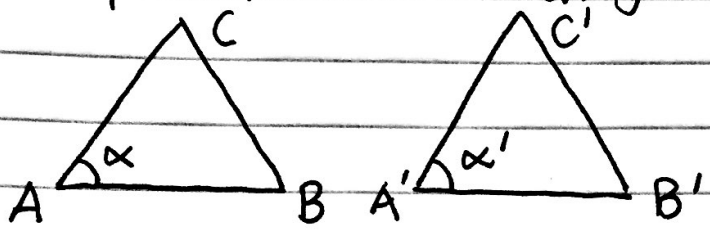
Math 461 Lecture 1 9/5

$|AB| = |A'B'|$, $|BC| = |B'C'|$, $|AC| = |A'C'|$ and $\angle ABC = \angle A'B'C'$, $\angle BCA = \angle B'C'A'$, $\angle CAB = \angle C'A'B'$ have congruence criteria:

SAS, ASA, SSS

ex. SAS (side-angle-side)

if two sides and the included angle are equal, then triangles are congruent



$|AB| = |A'B'|$, $|AC| = |A'C'|$, $\alpha = \alpha' \Rightarrow$

$\triangle ABC \cong \triangle A'B'C'$

↖ is congruent to