Math 461 Homework 2

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(1) Let ΔABC be a triangle such that $\angle ABC = \pi/2$. Let D be the midpoint of AC. Prove that $|BD| = \frac{1}{2}|AC|$.

(2) Let ΔABC be a triangle such that $|AB| = |AC|$. Suppose given points D on AB and E on AC such that $|BC| = |CD| = |DE| = |EA|$. Determine (with proof) $\angle BAC$.

(3) Let ΔABC be a triangle. Prove that if $|AC| > |AB|$ then $\angle ABC > \angle ACB$.

(4) Let ΔABC be a triangle. Let D

be the intersection point of the bisector of the angle $\angle BAC$ and the side BC.

(a) Prove that $\text{Area}(\Delta ACD)/\,\text{Area}(\Delta ADB) = |CD|/|DB|$.

(b) Prove that $|CD|/|DB| = |AC|/|AB|$.

(5) Let $ABCD$ be a quadrilateral. Let P, Q, R, S be the midpoints of the sides AB,BC,CD, and DA. Prove that $PQRS$ is a parallelogram.

(6) Let C be a circle and P a point not lying on C. Let L and L' be two lines passing through P such that L intersects C in two points X and Y and L' intersects C in two points X' and Y'. Prove that $|PX| \cdot |PY| = |PX'| \cdot |PY'|$.

(7) Suppose the vertices A, B, C, D of a quadrilateral $ABCD$ lie on a circle. Prove that the opposite angles of the quadrilateral sum to π, that is, $\angle ABC + \angle CDA = \angle BCD + \angle DAB = \pi$.

(8) Let $ABCD$ be a convex quadrilateral such that $\angle ACB$ and $\angle ADB$ are equal. Show that the vertices A, B, C, D of the quadrilateral lie on a circle.

(9) Given a circle C and a point P outside C give a ruler and compass construction of the two lines through P that are tangent to C.