Math 461 Lecture 3 9/10

Last time:

congruence criteria (SAS, ASA, SSS)

bisect angle

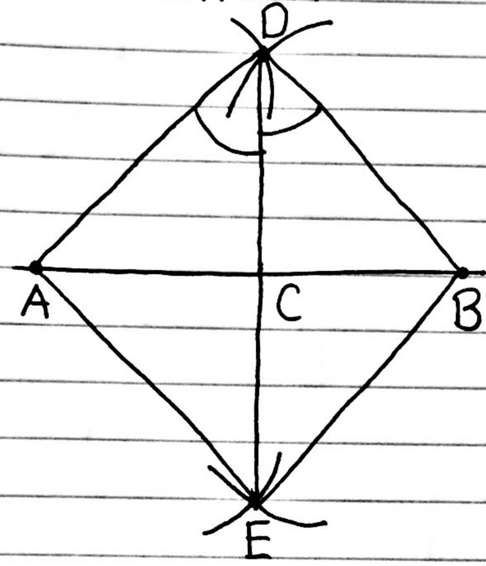
isosceles triangle (2 equal sides $\Leftrightarrow$ 2 equal angles)

bisect line segment

Today:

construct perpendicular to line through given point

parallel axiom



claim: $\angle ADE = \angle BDE$ $\quad *$

by SAS $\Rightarrow \triangle ADC \cong \triangle BDC \Rightarrow |AC| = |BC|$

proof of claim: $\triangle ADE \cong \triangle BDE$ by SSS $\Rightarrow$ $\angle ADE = \angle BDE$ $\square$

remark: notice proof gives

$* \Rightarrow \angle ACD = \angle BCD \quad \angle ACD + \angle BCD = \pi \Rightarrow$ $\angle ACD = \angle BCD = \pi/2$

we call line DE the perpendicular bisector of the line segment AB

claim: given line segment AB, point P

P lies on the perpendicular bisector of AB $\Leftrightarrow |AP| = |BP|$

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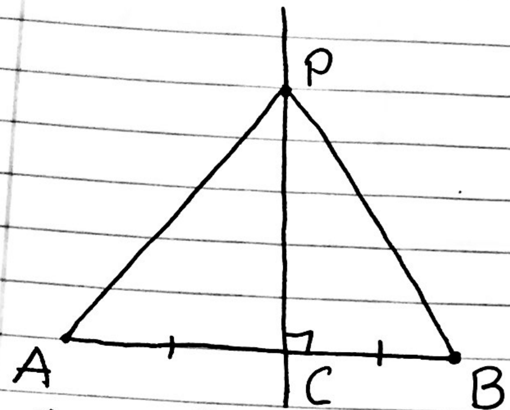
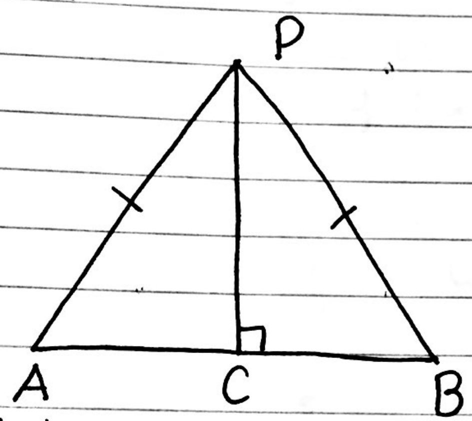
Proof: "$\Rightarrow$"



$\Delta ACP \cong \Delta BCP$ by SAS

$\Rightarrow |AP| = |BP|$ ✓

"$\Leftarrow$"

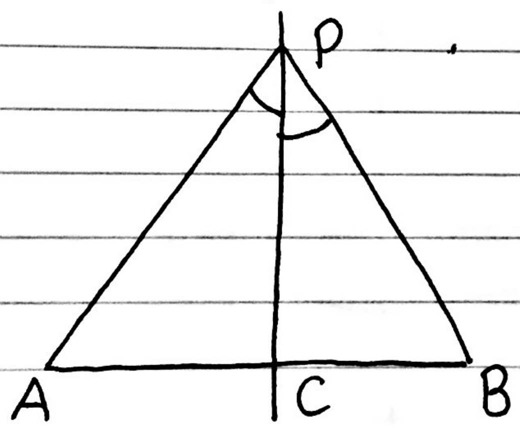


know $|AP| = |BP|$

drop a perpendicular from P to AB

need to show $|AC| = |CB|$

this approach may not work ✗



bisect $\angle APB$

by SAS $\Rightarrow$

$\Delta APC \cong \Delta BPC \Rightarrow$

$|AC| = |BC|$

$\angle ACP + \angle BCP = \pi$

$\angle ACP = \angle BCP \Rightarrow$

$\angle ACP = \pi/2$

so CP is perpendicular bisector of AB (the line through the midpoint of AB, perpendicular to AB)

given a line L and a point P construct a line M through P perpendicular to L

case where P lies on the line L:

1. draw circle center P some radius

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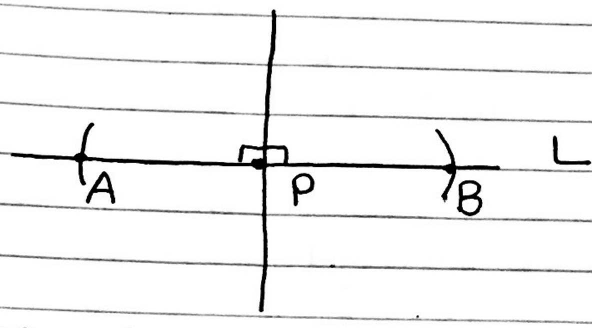
intersects L at A and B

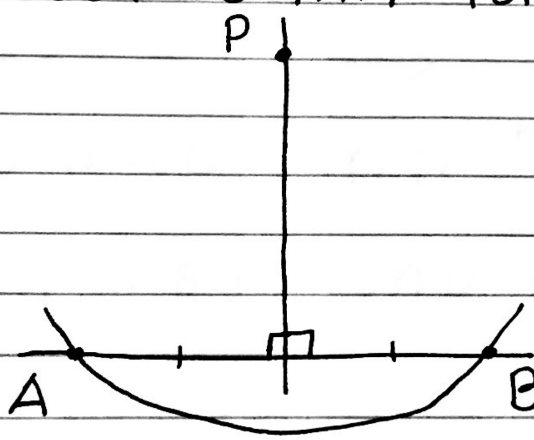
2. draw perpendicular bisector of AB



case where P doesn't lie on the line L:

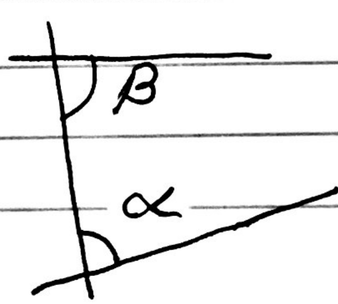
1. draw circle center P some radius sufficiently large intersecting L at A and B
2. draw perpendicular bisector of AB

the perpendicular bisector goes through P because $|AP| = |BP|$ (proved earlier)

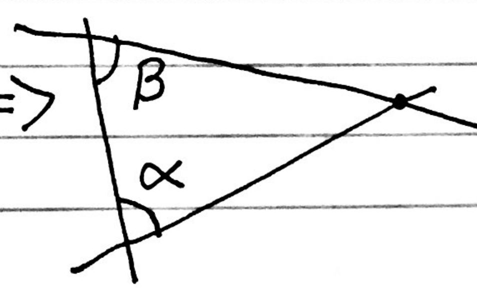


Euclid's Parallel Axiom:

if a line crosses two lines and makes interior angles with sum $< \pi$ on one side, then the two lines meet on that side



$\alpha + \beta < \pi \Rightarrow$



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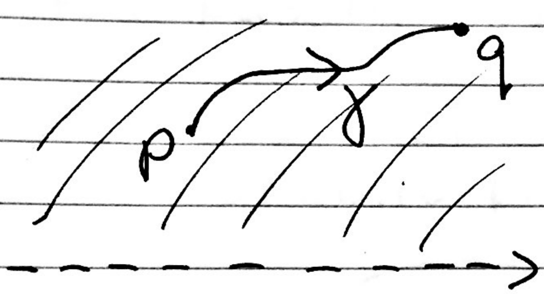
Historically: long discussion whether parallel axiom is necessary

in fact it is: 1810 - 1830
Gauss, Lobachevsky, Bolyai discovered "Hyperbolic Geometry" where all Euclid's axioms except parallel axiom are satisfied

What is hyperbolic geometry?
(more: chapter 8)

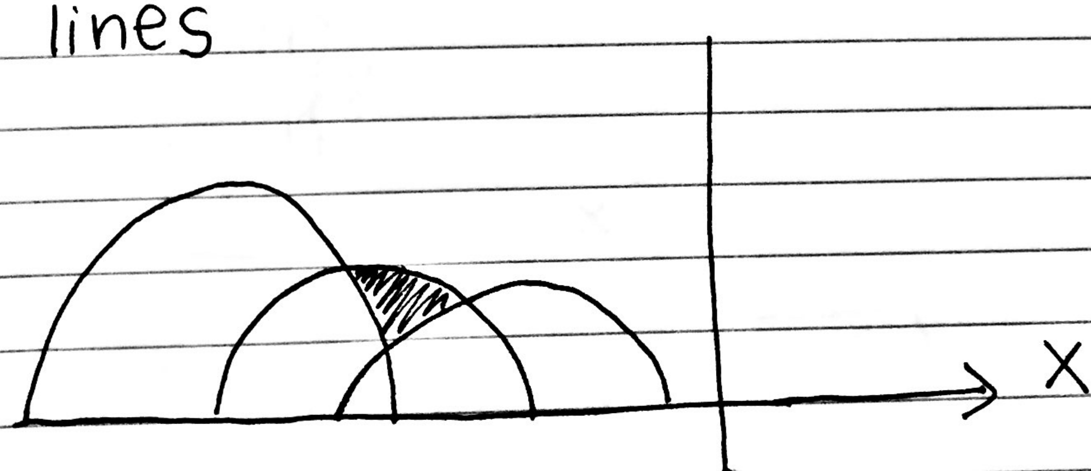
model: ("map" of hyperbolic plane) where distances are distorted

$H = \{(x,y) \mid y > 0\} \subset \mathbb{R}^2$



distance: $d(p,q) = \min_{\gamma} \left(\int_{\gamma} \frac{\sqrt{dx^2 + dy^2}}{y} \right)$

with this notion of distance the shortest paths are given by semicircles with center on x-axis and vertical lines



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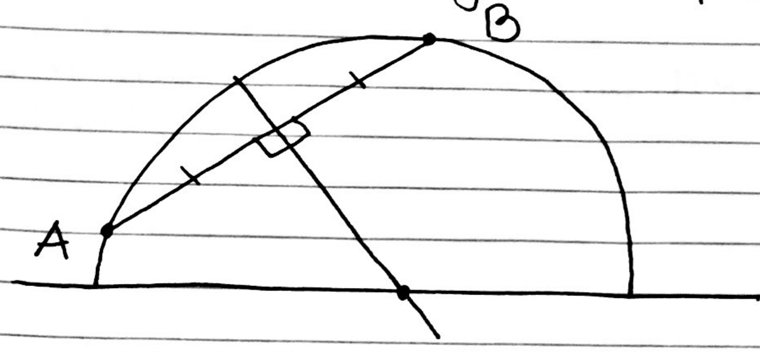
draw "line" through two points

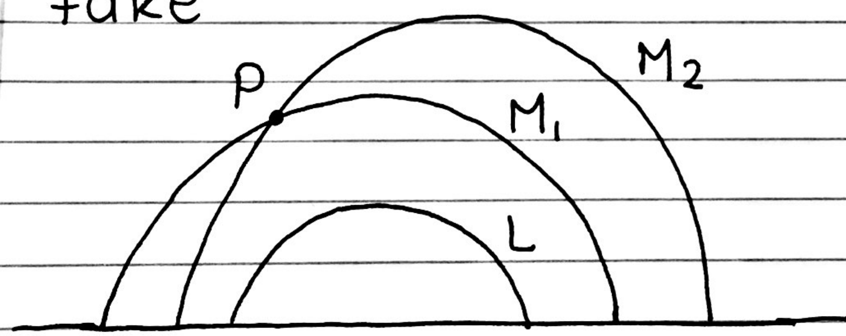


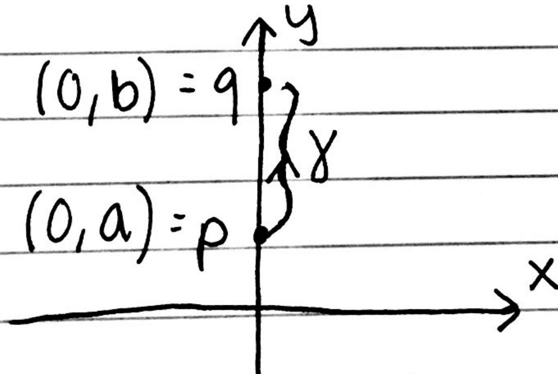
equivalent formulation of parallel axiom (John Playfair 1795)

given a line L and a point P not on L then there is a unique line M parallel to L passing through P

(recall we say two lines are parallel if they do not intersect)

in hyperbolic plane, this is obviously fake





$$d(p,q) = \min\left(\int_\gamma \frac{\sqrt{dx^2+dy^2}}{y}\right) = \int_{pq} \frac{\sqrt{dx^2+dy^2}}{y} =$$

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$$\int_a^b \frac{dy}{y} = [\log]_a^b = \ln\left(\frac{b}{a}\right)$$

each segment has some hyperbolic length