Math 461 Midterm, Wednesday 10/24/18, 7:00PM-9:00PM.

Instructions:

- Exam time is 2 hours.
- There are 6 questions for a total of 100 points.
- You are allowed two sheets of notes (letter size, both sides).
- Calculators, phones, other electronic devices, additional notes, and textbooks are *not* allowed.
- Justify all your answers carefully.

Q1 (10 points). Let $\mathcal{C}$ be a circle and P be a point outside $\mathcal{C}$. Let L and M be the two tangent lines to the circle $\mathcal{C}$ that pass through the point P. Let A be the intersection point of L and $\mathcal{C}$ and let B be the intersection point of M and $\mathcal{C}$. Prove that $|PA| = |PB|$.

Q2 (10 points). Let ΔABC be a triangle such that $\angle ACB = \pi/2$. Let L be the line through the point C perpendicular to the the line AB. Let D be the intersection point of L and the line AB. Prove that $|AB| \cdot |BD| = |BC|^2$.

Q3 (20 points). Describe a ruler and compass construction in each of the following cases.

(a) (10 points) Suppose given a line segment AB. Construct a triangle ΔABC

with vertices A, B and a third point C such that $\angle ABC = \pi/3$, $\angle BAC = \pi/6$, and $\angle ACB = \pi/2$.

(b) (10 points) Suppose given a triangle ΔABC. Construct a triangle ΔABD such that $\angle ADB = \angle ACB$ and $\angle ABD = \angle BAD$.

[You may use ruler and compass constructions from class or the textbook as components of your constructions.]

Q4 (20 points).

(a) (10 points) Let A, B, C be three points and let L be the bisector of the angle $\angle BAC$ (that is, the line through A that divides the angle $\angle BAC$ into two equal parts). Let P be a point on L. Let M be the line through P

perpendicular to the line AB, and let Q be the intersection point of M with the line AB. Similarly, let N be the line through P perpendicular to the line AC, and let R be the intersection point of N with the line AC. Prove that $|PQ| = |PR|$.

(b) (5 points) Let $ABCD$ be a convex quadrilateral and suppose that the bisectors of the angles $\angle ABC$, $\angle BCD$, $\angle CDA$, and $\angle DAB$ of the quadrilateral all meet at a point P. Using part (a) or otherwise, prove that there is a circle $\mathcal{C}$ such that the sides AB, BC, CD, and DA of the quadrilateral are all tangent to $\mathcal{C}$.

[Recall that a polygon is *convex* if all the interior angles are less than

π.]

(c) (5 points) Do the angle bisectors of a convex quadrilateral always meet at a point? Give a proof or a counterexample.

Q5 (25 points). Give a precise geometric description of each of the following isometries $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ as a translation, rotation, reflection, or glide reflection. (For a translation, give the translation vector. For a rotation, give the center, angle, and sense (counterclockwise or clockwise) of rotation . For a reflection, give the line of reflection. For a glide reflection, give the line of reflection and the translation vector.)

(a) (10 points) $T(x, y) = (-y+3, x-3)$.

(b) (5 points) $T(x,y) = \frac{1}{5}(-3x - 4y + 4, -4x + 3y + 2)$.

(c) (10 points) $T(x,y) = (y+5, x+1)$.

Q6 (15 points). Give a precise geometric description of each of the following compositions of isometries as a translation, rotation, reflection, or glide reflection.

(a) (5 points) Reflection in the line L_1 with equation $y = 3$ followed by reflection in the line L_2 with equation $y = x + 1$.

(b) (10 points) Rotation about the point $(1,4)$ through angle π counterclockwise followed by rotation about the point $(3,4)$ through angle $\pi/2$ counterclockwise.