Math 611 Midterm, Wednesday 10/22/13, 7PM-8:30PM.

Instructions: Exam time is 90 mins. There are 5 questions each worth 10 points for a total of 50 points. Calculators, notes, and textbooks are not allowed. Justify all your answers carefully.

Q1. Let G be a non-abelian group of order 21.

(a) Prove that the center of G is trivial.

(b) Determine the class equation of G.

Q2. Classify all groups G such that $|G| = 44$ and G contains an element of order 4.

Q3. Let $G = \langle x, y \mid x^2, y^2 \rangle$ be the group generated by x and y subject to the relations $x^2 = e$ and $y^2 = e$. Describe an isomorphism θ from G to a semi-direct product of two abelian groups. (Define the semi-direct product and the homomorphism θ precisely, and prove carefully that the homomorphism θ you define is an isomorphism.)

Q4. Describe each of the following quotient rings R/I explicitly. (Establish an isomorphism from R/I to a direct product of standard rings.) Using your description or otherwise, determine whether the ideal $I \subset R$ is prime, maximal, or neither.

(a) $\mathbb{R}[x]/(x^3 - 8)$.

(b) $\mathbb{Z}[\sqrt{-2}]/(1 + 3\sqrt{-2})$.

(c) $\mathbb{C}[x, y]/(x + y^3)$.

Q5. Let $d \in \mathbb{Z}$ and $d \geq 3$. Prove that 2 is irreducible but not prime in the ring $\mathbb{Z}[\sqrt{-d}]$.