Math 611 Midterm, Wednesday 10/28/15, 7:00PM–8:30PM.

Instructions: Exam time is 90 mins. There are 6 questions for a total of 50 points. Calculators, notes, and textbook are not allowed. Justify all your answers carefully. If you use a result proved in the textbook or class notes, state the result precisely.

Q1. (6 points) Let G be a group of order 52 and $x \in G$ an element such that the conjugacy class $C(x)$ of x has size $|C(x)| = 4$. What is the order of x?

Q2. (6 points) Let G be a group of order 90. Suppose that there is a nontrivial action of G on a set X of size $|X| = 5$. (Here we say an action of G on X is *nontrivial* if $g \cdot x \neq x$ for some $g \in G$ and $x \in X$.) Prove that G is not a simple group.

Q3. (8 points) Let G be a non-abelian group of order 75. Determine the number of elements of order 3 in G. Justify your answer carefully.

Q4. (14 points) Let G be a non-abelian group of order 44 such that G contains an element of order 4.

(a) (8 points) Describe G in terms of generators and relations.

(b) (4 points) Determine the center $Z(G)$ of G.

(c) (2 points) Identify the quotient $G/Z(G)$ with a standard group.

Q5. (8 points) Let m be a positive integer and let

$$G = \langle a, b \mid a^7 = e,\, b^3 = e,\, bab^{-1} = a^m \rangle$$

be the group generated by a and b subject to the relations $a^7 = e$, $b^3 = e$ and $bab^{-1} = a^m$. Determine the order of G in the following cases.

(a) (4 points) $m = 2$.

(b) (4 points) $m = 3$.

Q6. (8 points) Let p and q be primes such that $q^2 \equiv 1 \bmod p$. Prove that there exists a non-abelian group of order pq^2.