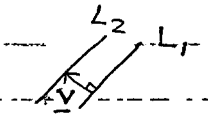
10/25/14

Midterm returned (Average 43/60)
Exam & solutions available (check e-mail/webpage)

$F(\underline{x}) = A\underline{x} + \underline{b}$ ⇝ identity, translation, rotation, reflection, glide reflection

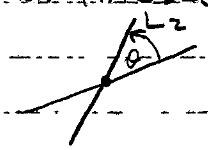
Last Time : Isometries : Algebraic formula ⇝ geometric description
• Compositions : 1. Two reflections

1a. Parallel lines



$Refl_{L_2} \circ Refl_{L_1} = Trans_{2\underline{v}}$

1b. Intersecting lines



$Refl_{L_2} \circ Refl_{L_1} = Rot(P, 2\theta)$

Today : More compositions
• GPS theorem
• Isometries & Congruence } if time
• 3 reflections theorem

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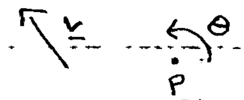
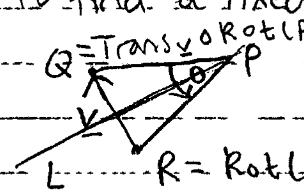
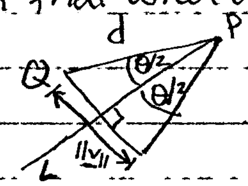
2. Composition of a rotation & a translation.

$Trans_{\underline{v}} \circ Rot(P, \theta) = ?$



Try to find a fixed point : find Q such that when we rotate and translate, end at Q.

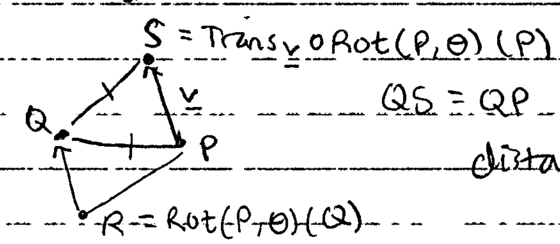




Take line L through P perpendicular to $\underline{v}$.

$\sin \theta/2 = \dfrac{1/2 \|\underline{v}\|}{d}$ ⇝ d ⇝ Q.

Notice that,



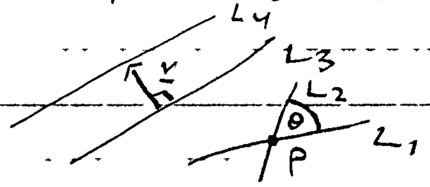
QS = QP because isometries preserve distances.

Summarize: See there's a unique fixed point Q of the composition, so the composition is a rotation about Q (angle ?)

Another approach: We can write both $\text{Trans}_{\underline{v}}$ & $\text{Rot}(P,\theta)$ as a composite of reflections. Can we do this in such a way that the composition can be described geometrically?

$$\text{Trans}_{\underline{v}} \circ \text{Rot}(P,\theta) = (\text{Refl}_{L_4} \circ \text{Refl}_{L_3}) \circ (\text{Refl}_{L_2} \circ \text{Refl}_{L_1})$$

Notice, there are choices for L_1, L_2 & L_3, L_4.

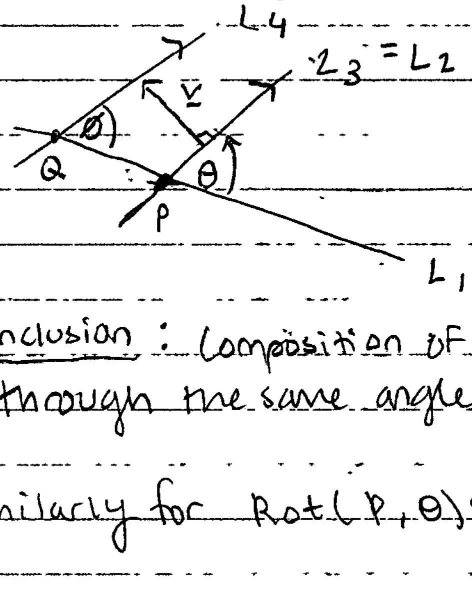


What would be a good choice?

Make $L_2 = L_3$ then, $(\text{Refl}_{L_4} \circ \text{Refl}_{L_3}) \circ (\text{Refl}_{L_2} \circ \text{Refl}_{L_1})$

$\text{Refl}_{L_2} \circ \text{Refl}_{L_3} = \text{identity}$

$$\begin{aligned} &= \text{Refl}_{L_4} \circ (\text{Refl}_{L_3} \circ \text{Refl}_{L_2}) \circ \text{Refl}_{L_1} \\ &= \text{Refl}_{L_4} \circ \text{Refl}_{L_1} \\ &= \text{Rot}(Q, \phi) \end{aligned}$$



Notice, $\phi = \theta$ by "corresponding angles" for parallel lines L_3 & L_4.

Conclusion: Composition of rotation & a translation is a rotation through the same angle about another point (constructed as above).

Similarly for $\text{Rot}(P,\theta) \circ \text{Trans}_{\underline{v}}$ (order switched).

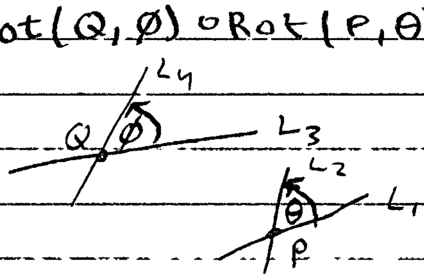
3. Composition of two rotations.

$\mathrm{Rot}(P,\phi)\circ \mathrm{Rot}(P,\theta) = \mathrm{Rot}(P,\phi+\theta)$ ✓.

What about $\mathrm{Rot}(Q,\phi)\circ \mathrm{Rot}(P,\theta)$?

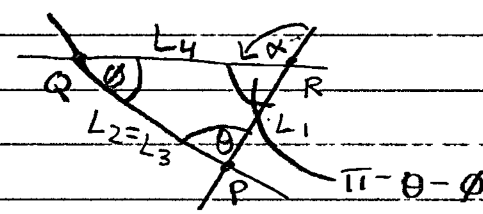
→ rotation about a point R through some angle?

$\mathrm{Rot}(Q,\phi)\circ \mathrm{Rot}(P,\theta) = (\mathrm{Refl}_{L_4}\circ \mathrm{Refl}_{L_3})\circ(\mathrm{Refl}_{L_2}\circ \mathrm{Refl}_{L_1})$



IF $L_2 = L_3$, ↳ $= \mathrm{Refl}_{L_4}\circ \mathrm{Refl}_{L_1}$

rotate ⇝



So... $\alpha = \theta + \phi$