Math 461 Lecture 2 9/7

Last time:

Euclidean geometry

ruler and compass constructions

motivating problems

congruence of triangles SAS, ASA, SSS

Today:

constructions:

bisect angles

bisect line segments

construct perpendicular to given line through given point

proofs using congruence

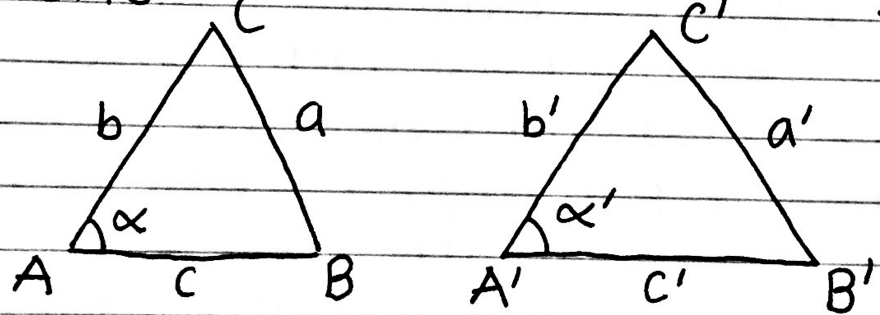
Recap:

we say two triangles ΔABC and $\Delta A'B'C'$ are congruent if the corresponding sides and angles are equal

congruence criteria:

in fact its enough to check SAS or ASA or SSS

SAS: *



$\alpha = \alpha'$, $b = b'$, and $c = c'$

Euclid's proof of SAS:

with notation as in diagram *

want to show $\Delta ABC \cong \Delta A'B'C'$

move ΔABC so $A = A'$ and B lies on line $A'B'$ and C lies on line $A'C'$

using assumption that $\alpha = \alpha'$

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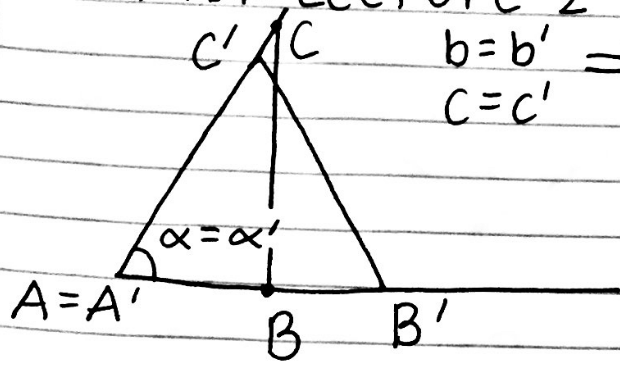


$b = b'$, $c = c'$ $\Rightarrow$ $C = C'$, $B = B'$ $\Rightarrow$ congruent

Problem: Euclid doesn't define what it means to "move" triangle

Modern Approach: either take SAS as an axiom (then SAS $\Rightarrow$ ASA & SSS) or define notion of distance in the plane

$\mathbb{R}^2$ $d((a_1, b_1), (a_2, b_2)) = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

then define isometry of plane to be a function $T: \mathbb{R}^2 \to \mathbb{R}^2$ such that it preserves distances

$\forall p, q \in \mathbb{R}^2$ $d(T(p), T(q)) = d(p, q)$

this is how to "move" the triangle: apply isometry

Remark: in this setup can define congruence as follows:

two triangles are congruent if there exists an isometry T such that

Δ_1 & Δ_2 $T(\Delta_1) = \Delta_2$

Basic constructions:

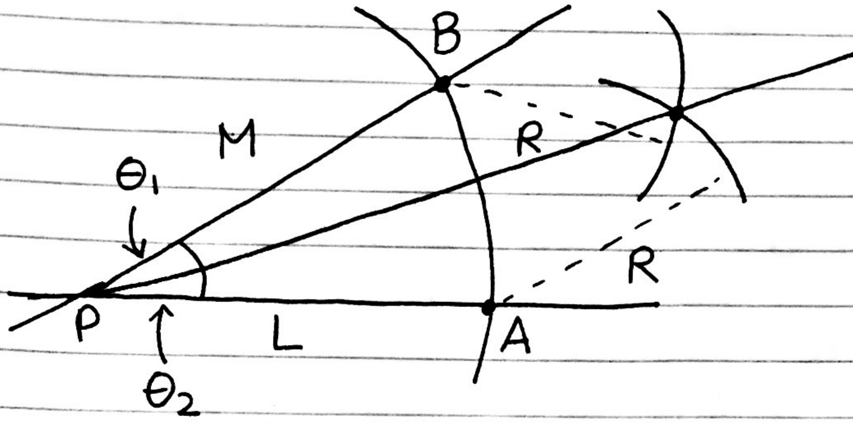
Bisection of an angle:

given an angle construct a line through P dividing the angle into two equal parts

1. draw circle center P, same radius intersects L at A and M at B
2. draw circles centers A and B,

same radius R intersect at C

3. draw line PC



Claim: $\angle APC = \angle BPC$ ie. $\theta_1 = \theta_2$

we claim:

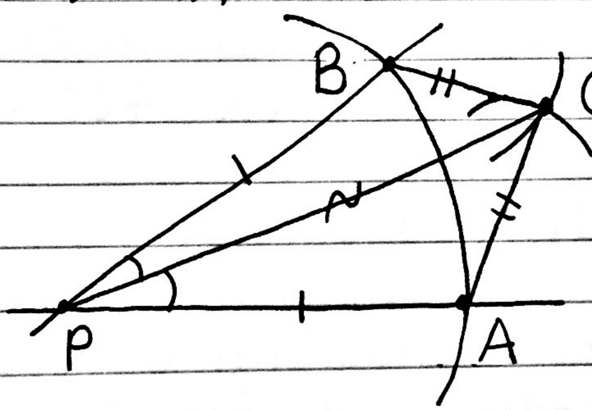
$\triangle APC \cong \triangle BPC$ by SSS

$|AP| = |BP|$ first circle radius

$|CP| = |CP|$

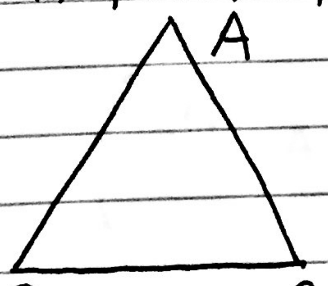
$|AC| = |BC|$ second circle radius

$\Rightarrow \angle APC = \angle BPC$ $\square$ (end of proof)



Theorem: given triangle $\triangle ABC$

if $|AB| = |AC| \Leftrightarrow \angle ABC = \angle ACB$



in this case we say $\triangle ABC$ is an isosceles triangle

first consider "$\Rightarrow$"

assume $|AB| = |AC|$

show $\angle ABC = \angle ACB$

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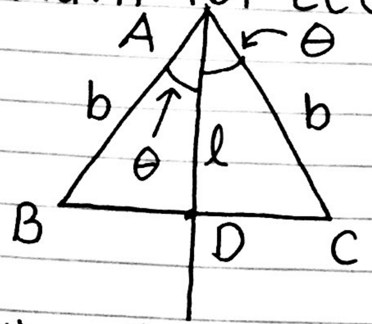
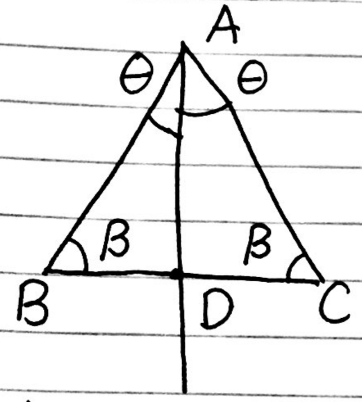


$\triangle BAD \cong \triangle CAD$ by SAS
$\Rightarrow \angle ABD = \angle ACD$ and
$\angle ABC = \angle ACB$

"$\Leftarrow$" assume $\angle ABC = \angle ACB$
want to show $|AB| = |AC|$



angle sum of a triangle
* is π radians $\Rightarrow$
$\angle BDA = \angle CDA$
now $|AD| = |AD| \Rightarrow$
$\triangle ABD \cong \triangle ACD$ by ASA
$\Rightarrow |AB| = |AC|$

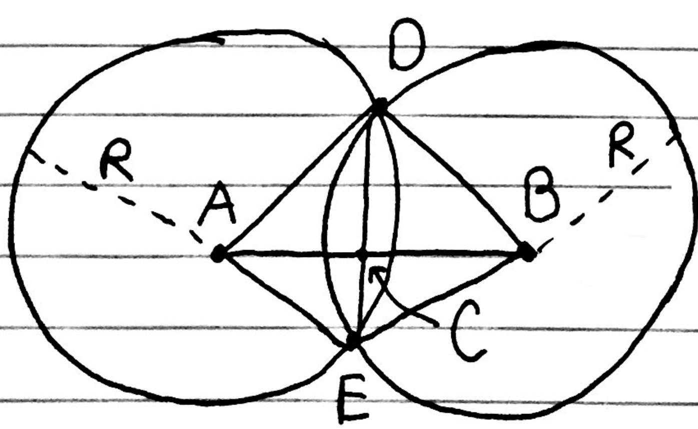
* hasn't been proved yet (will be later)

$\angle ABC = \angle ACB \overset{?}{\Rightarrow} \triangle ABC \cong \triangle ACB$

claim: $\triangle ABC \cong \triangle ACB$

proof: $\angle ABC = \angle ACB$
$|BC| = |CB|$ by ASA
$\angle ACB = \angle ABC \Rightarrow |AB| = |AC|$ $\square$

Next construction:
given a line segment AB, bisect the line segment, ie. construct point C such that $|AC| = |CB|$ on line AB



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Proof that $|AC| = |CB|$

$\angle ADC = \angle BDC$

$|AD| = |BD|$

$|DC| = |DC| \Rightarrow$ by SAS $\triangle ADC \cong \triangle BDC$

$\Rightarrow |AC| = |BC|$