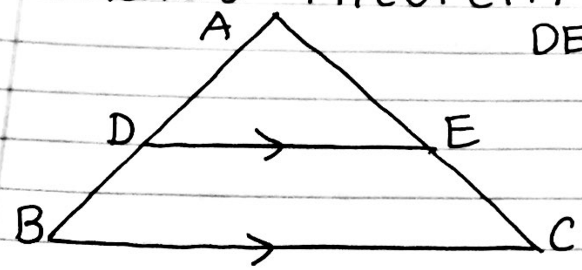
Math 461 Lecture 7 9/19

Homework 2 available and due next Wednesday 9/26 at start of class

Last time:

Thales' Theorem and its converse



$DE \parallel BC \iff \frac{|AD|}{|AB|} = \frac{|AE|}{|AC|}$

Similar Triangles

$\triangle ABC$ is similar to $\triangle A'B'C'$ if the corresponding angles are equal

$\triangle ABC \sim \triangle A'B'C' \Rightarrow \frac{|A'B'|}{|AB|} = \left|\frac{B'C'}{BC}\right| = \frac{|A'C'|}{|AC|}$

Geometric multiplication and division, subdivision into n equal parts however trisection of angles is impossible with ruler and compass

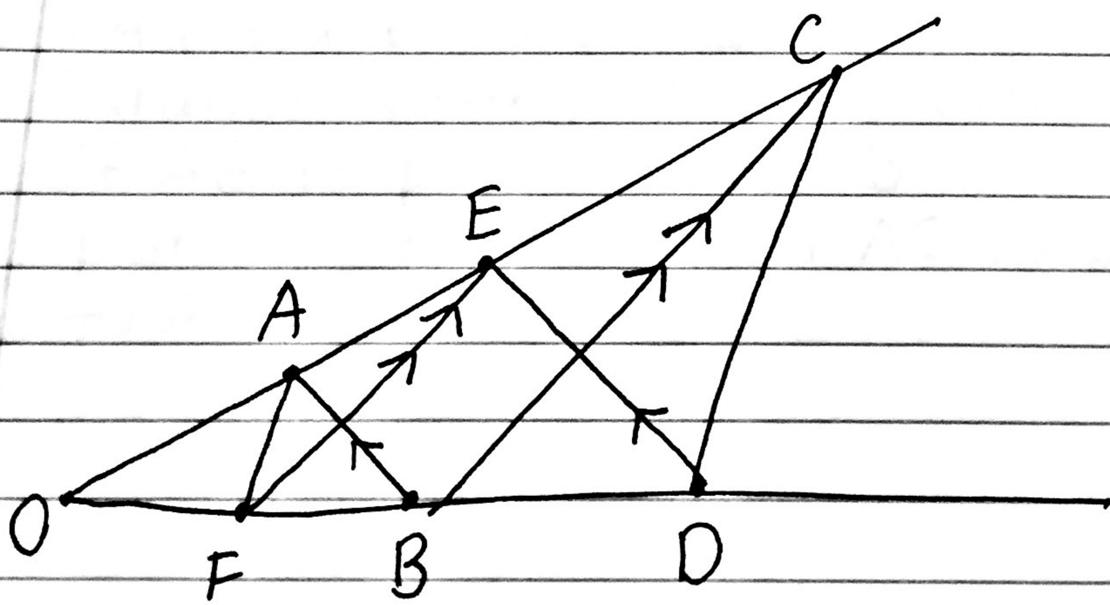
Today:

Parallel Pappas and Desargues Theorems

Angles in a circle

Sine rule and cosine rule

Parallel Pappas Theorem



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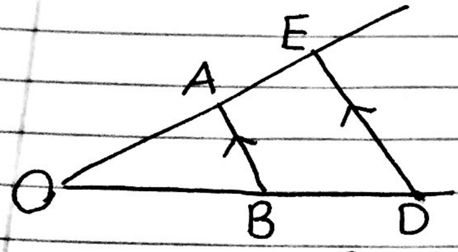
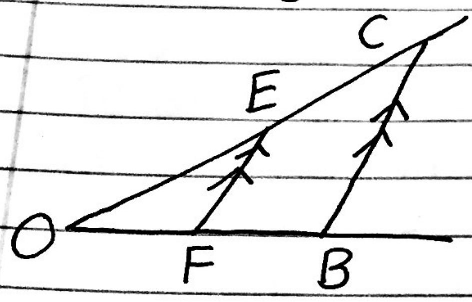
Theorem:

If AB is parallel to DE and BC is parallel to EF then CD is parallel to FA

Proof of Parallel Pappus Theorem

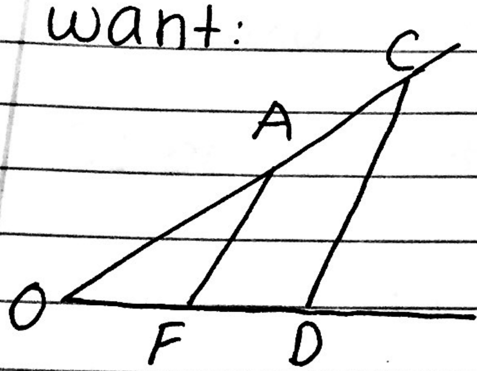


$$\frac{|OB|}{|OD|} = \frac{|OA|}{|OE|}$$



$$\frac{|OF|}{|OB|} = \frac{|OE|}{|OC|}$$

want:



CD parallel to FA

multiply $\Rightarrow \dfrac{|OF|}{|OD|} = \dfrac{|OA|}{|OC|} \Rightarrow$

by converse Thales' Theorem

FA parallel to CD □

Parallel Desargues Theorem

If AB is parallel to A′B′ and BC is parallel to B′C′ then AC is parallel to A′C′

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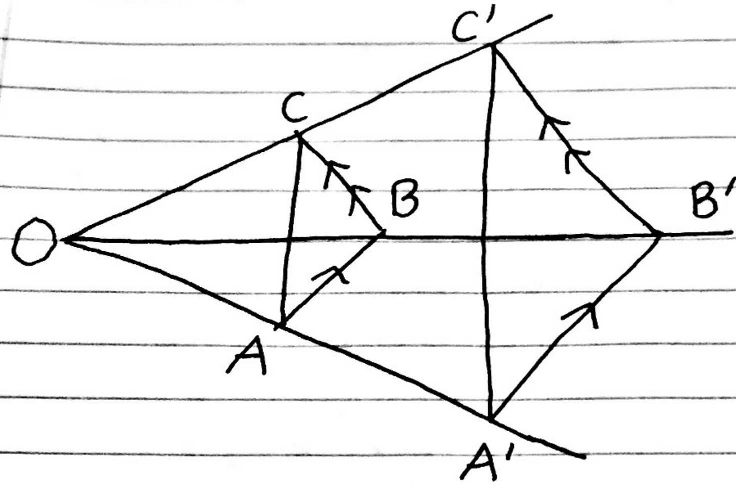

Proof: Use Thales' Theorem
To show AC is parallel to A'C' by converse Thales' Theorem, need to show $\frac{|OA|}{|OA'|} = \frac{|OC|}{|OC'|}$

AB parallel to A'B' $\Rightarrow \frac{|OA|}{|OA'|} = \frac{|OB|}{|OB'|}$

BC parallel to B'C' $\Rightarrow \frac{|OB|}{|OB'|} = \frac{|OC|}{|OC'|}$

combining, $\frac{|OA|}{|OA'|} = \frac{|OB|}{|OB'|} = \frac{|OC|}{|OC'|}$ $\square$

General Pappus Theorem
The intersection points
$AB \cap DE$, $BC \cap EF$, and $CD \cap FA$ are collinear (lie on a line)

Pappus Theorem

picture →

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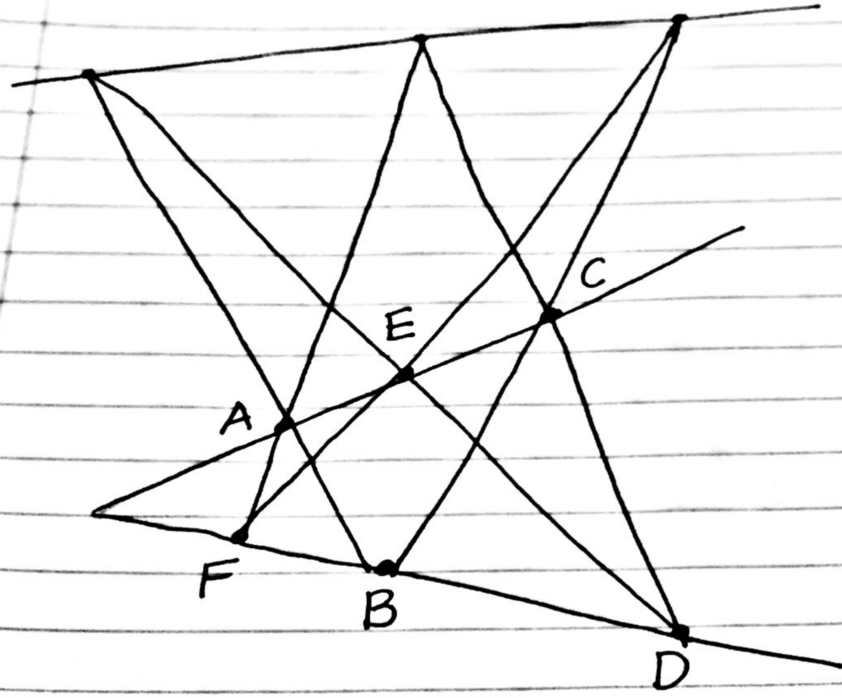
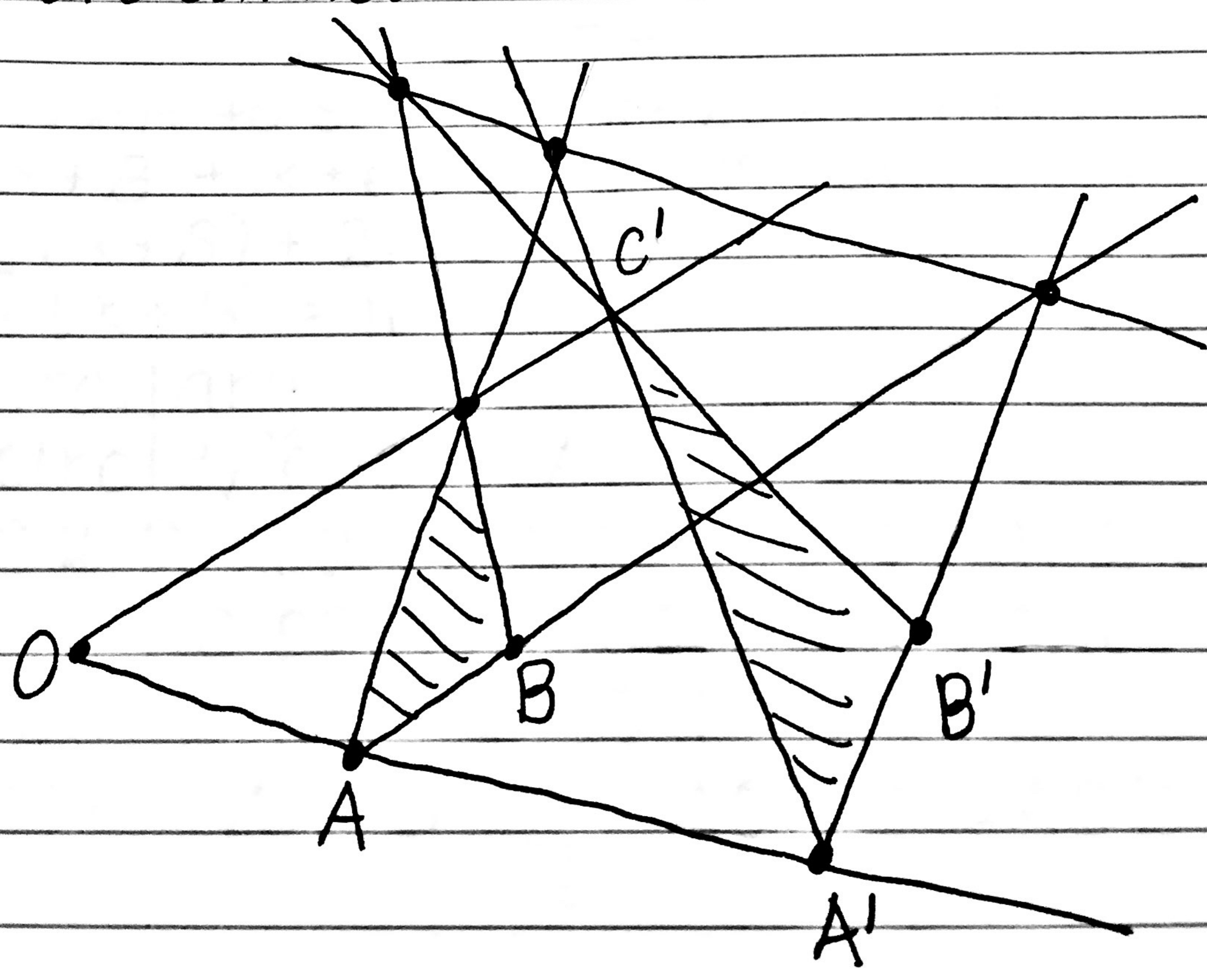


General Desargues Theorem

$AB \cap A'B'$, $BC \cap B'C'$, and $CA \cap C'A'$ are collinear



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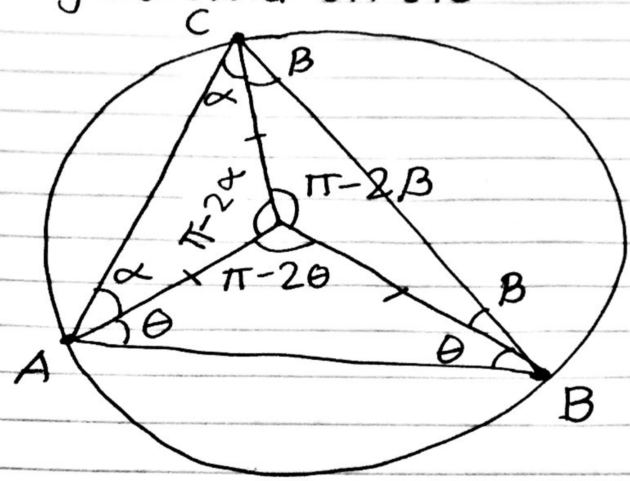
Angles in a circle



O is the center of the circle
Theorem: $\angle AOB = 2\angle ACB$
Proof: $\angle AOB = \pi - 2\theta$
$\angle AOC = \pi - 2\alpha$
$\angle BOC = \pi - 2\beta$
want to show: $\pi - 2\theta = 2(\alpha+\beta)$
$\alpha+\beta+\alpha+\theta+\beta+\theta = \pi$ (angle sum of $\triangle ABC$)
$2(\alpha+\beta)+2\theta = \pi$
$2(\alpha+\beta) = \pi - 2\theta$ $\square$

Corollary:
circle, chord AB
C & D are two points on the circle, on the same side of the chord
then $\angle ACB = \angle ADB$

Proof: $\angle ACB = \frac{1}{2}\angle AOB = \angle ADB$ $\square$

picture →

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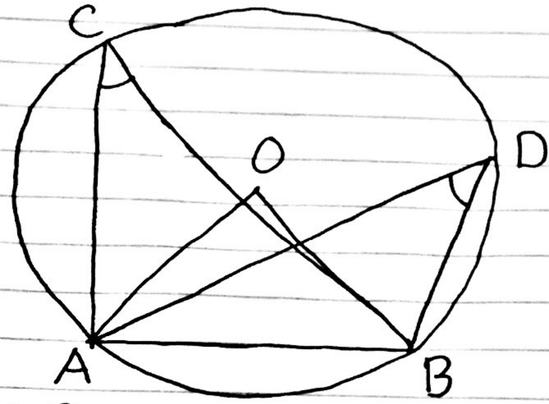
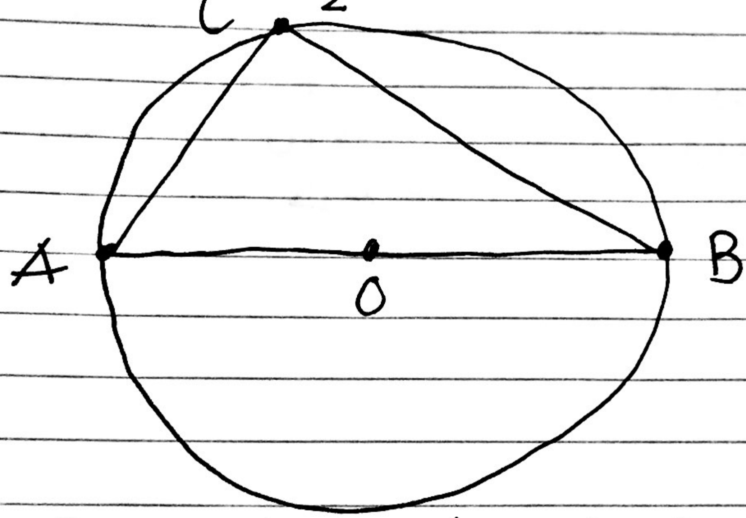


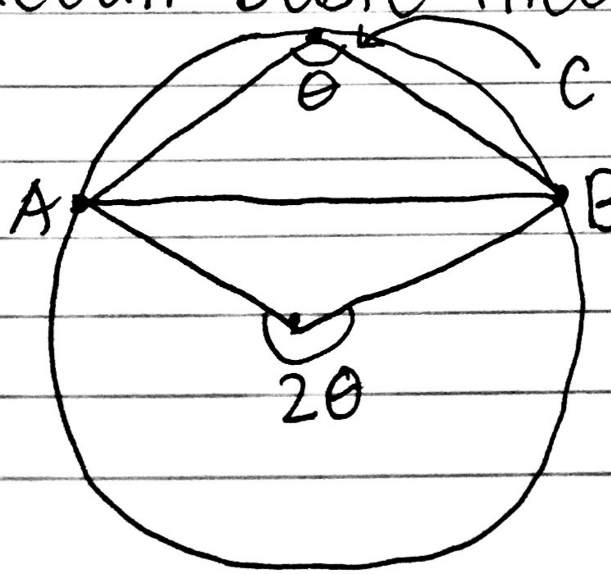
Corollary 2:
If AB is a diameter of a circle, and C is a point on the circle, then $\angle ACB = \frac{\pi}{2}$



Proof: $\angle ACB = \frac{1}{2} \angle AOB$ by Theorem

$= \frac{1}{2}\pi$ $\square$

Recall basic theorem



angle at circumference $= \frac{1}{2}$ (angle at center)

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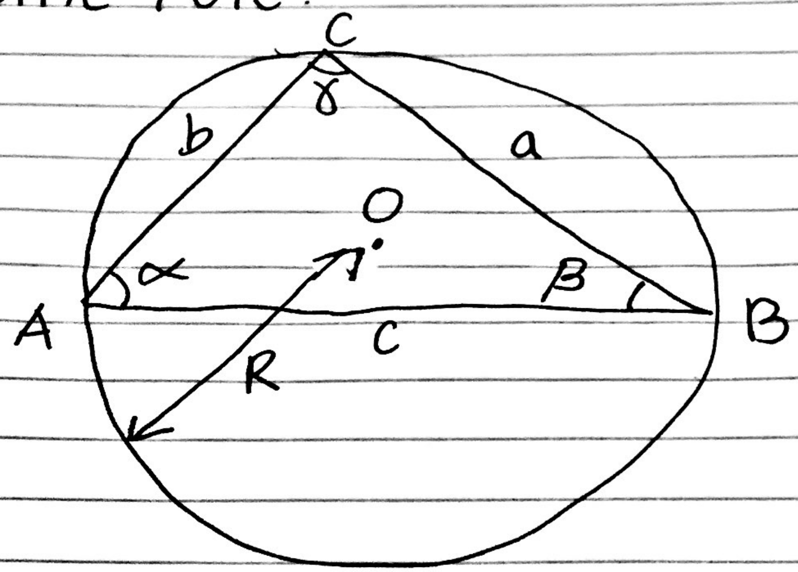
Sine rule:



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$$

where R is the radius of the circle passing through A, B, and C

the circumcircle or circumscribed circle of ΔABC