

Math 461 Homework 3

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- (1) Let $\triangle ABC$ be a triangle. Let \mathcal{C} be the unique circle passing through A , B , and C . Let L be the tangent line to the circle at the point A . Prove that the angle between the line L and the line segment AC is equal to the angle $\angle ABC$.
- (2) Let $\triangle ABC$ be a triangle. Let $PQRS$ be a square with vertices P, Q lying on the line BC , vertex R lying on the line segment CA , and vertex S lying on the line segment AB . Given that $|BC| = a$ and the perpendicular height from A to BC equals h , determine the side length of the square $PQRS$.
- (3) Let $ABCD$ be a convex quadrilateral. Let L be the line which bisects the angle $\angle BAD$ and M the line which bisects the angle $\angle BCD$. Suppose that L intersects the line segment BC at a point E , M intersects the line segment AD at a point F , and L is parallel to M . Prove that $\angle ABC = \angle ADC$.
- (4) Let $\triangle ABC$ be a triangle. Let \mathcal{C} be the unique circle passing through A , B , and C . Let L be line through C which bisects the external angle of the triangle $\triangle ABC$ at C . (Here, by the external angle of the triangle $\triangle ABC$ at C , we mean the angle $\angle ACP$ where P is a point on the line BC on the opposite side of C to B .) Let D be the other intersection point of L with the circle \mathcal{C} (besides C). Prove that $|AD| = |BD|$.
- (5) Let ABC be a triangle. Let D be the midpoint of the line segment BC , and let E be a point on the line segment AD such that $|AE|/|ED| = 1/3$. Let L be the line through the point E parallel to the line AC , and let F be the intersection point of L and the line BC . Determine $|BF|/|FC|$.

- (6) Let $ABCDEF$ be a convex hexagon such that AB is parallel to CF , CD is parallel to EB , and EF is parallel to AD . Show that the two triangles $\triangle ACE$ and $\triangle BDF$ have equal area.
- (7) Let $\triangle ABC$ and $\triangle CDE$ be two equilateral triangles with a common vertex C . Determine the angle between the lines AD and BE .
- (8) Let $ABCD$ be a parallelogram of area 1. Let E be a point on the line segment $|BC|$ such that $|BE|/|EC| = 3/2$. Let F be the intersection point of the lines AE and BD . Determine the area of the quadrilateral $CDFE$.