Math 461 Lecture 4 9/12

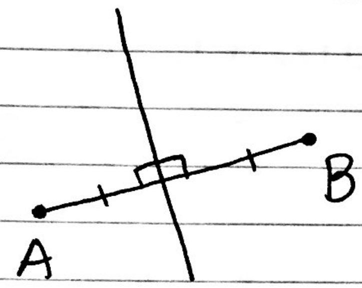
Homework 1 available

people.math.umass.edu/~hacking/461F18

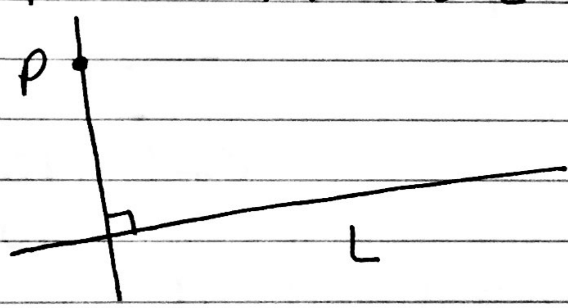
due at start of class next Wednesday 9/19

Last time:

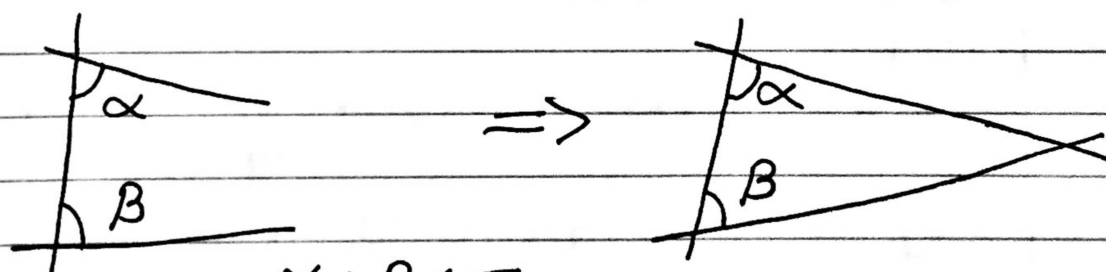
perpendicular bisector



construction: given line L, point P, construct line M through P perpendicular to L



parallel axiom



$\alpha+\beta<\pi$

hyperbolic geometry (brief overview)

$\alpha+\beta+\gamma<\pi$

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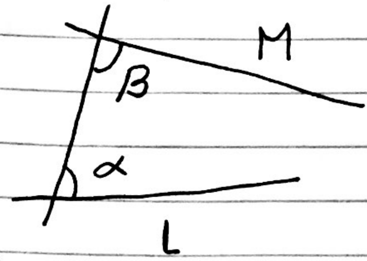
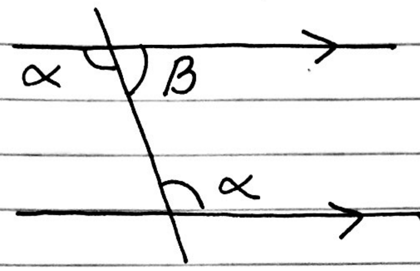
Today:

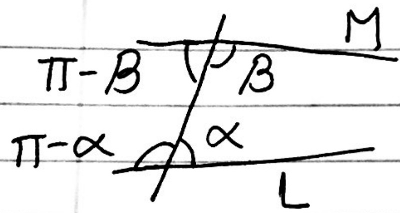
consequences of parallel axiom



L & M parallel $\Leftrightarrow \alpha+\beta=\pi$

"$\Rightarrow$" alternate angles are equal





L & M parallel $\Leftrightarrow \alpha+\beta=\pi$

Proof: "$\Rightarrow$"

axiom: $\alpha+\beta<\pi \Rightarrow L$ & M meet

i.e not parallel

contrapositive: $\alpha+\beta \geq \pi \Leftarrow L$ & M are parallel

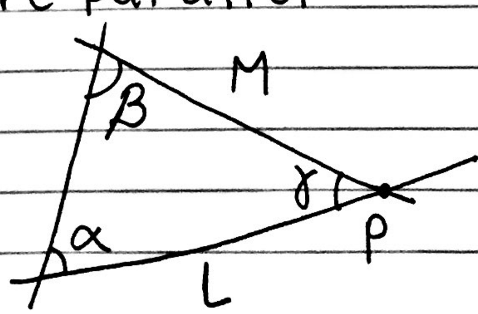
if $\alpha+\beta>\pi$ then $(\pi-\alpha)+(\pi-\beta)<\pi$

so $\alpha+\beta=\pi$

L & M meet on other side ※ □

Proof: "$\Leftarrow$"

assume $\alpha+\beta=\pi$ want to show L & M are parallel



$\alpha+\beta+\gamma=\pi+\gamma>\pi$

attempt at proof by contradiction

haven't proved yet

angle sum of $\Delta=\pi$

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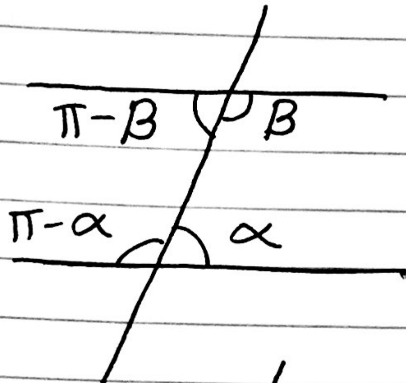
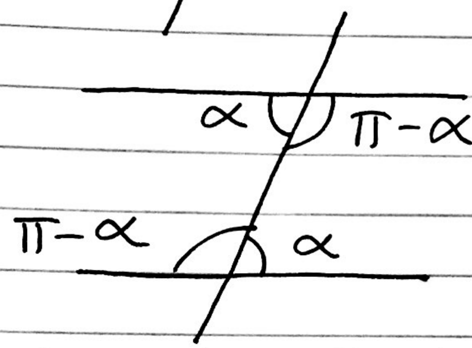
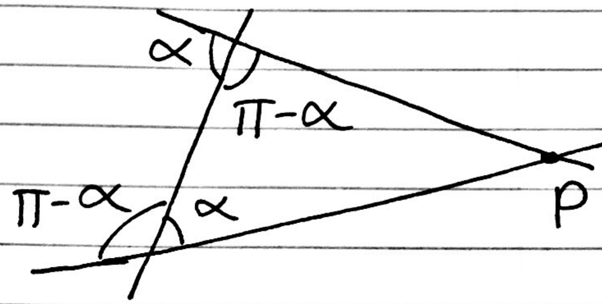
proof by contradiction:
suppose $\alpha+\beta=\pi$ but L & M are not parallel

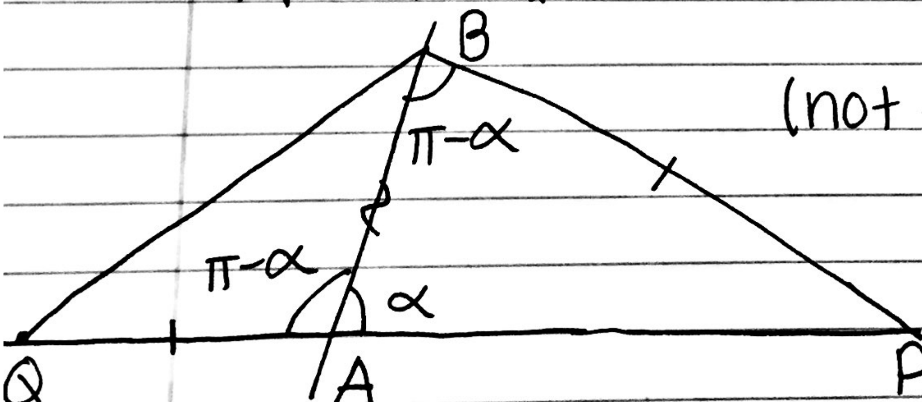


first note $\alpha+\beta=\pi$
$\Longleftrightarrow$
$(\pi-\alpha)+(\pi-\beta)=\pi$





the contradiction is going to be:
L & M intersect on both sides
Euclid assumes: There is a unique line through 2 points
so two lines intersect in at most 1 point ※



(not to scale)

draw point Q on the line L
on opposite side of A to P such that
$|AQ|=|BP|$
$\triangle APB \cong \triangle BQA$ by SAS $\Rightarrow$
$\angle ABQ = \angle BAP = \alpha$

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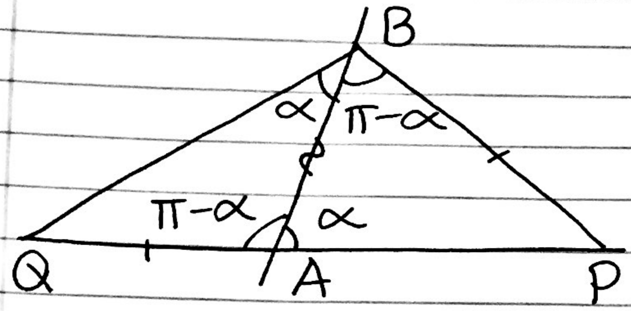
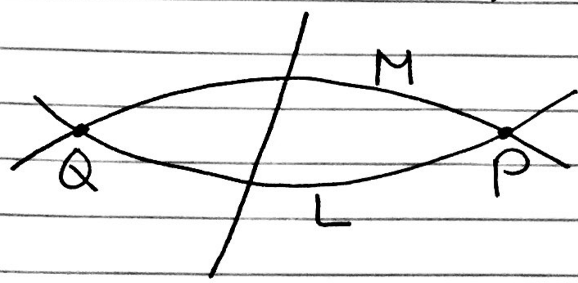
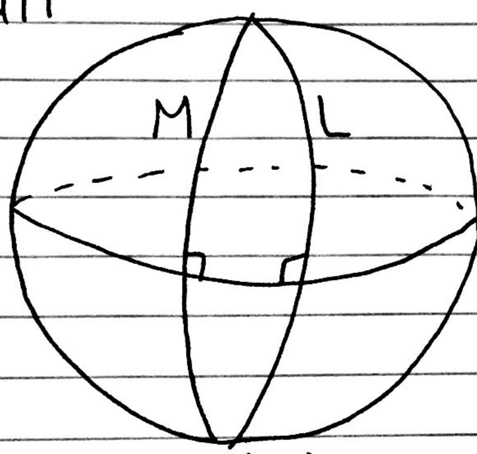


i.e.



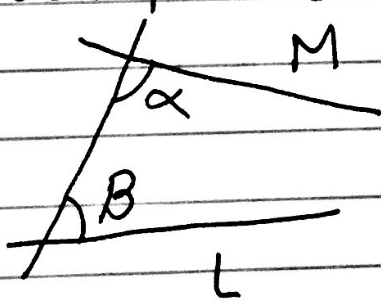
$\#$ $\square$

recall

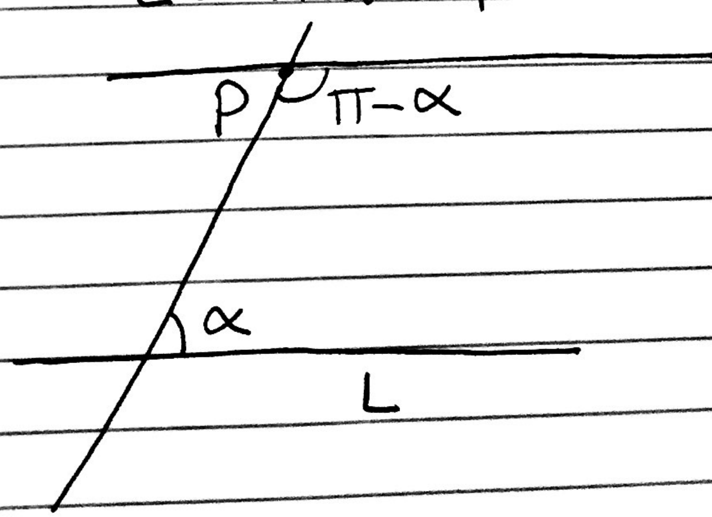


spherical geometry

Playfair's Axiom:
line L, point $P \notin L \Rightarrow$ there is a unique line M through P parallel to L
just proved:



$\alpha + \beta = \pi \iff$ L & M are parallel



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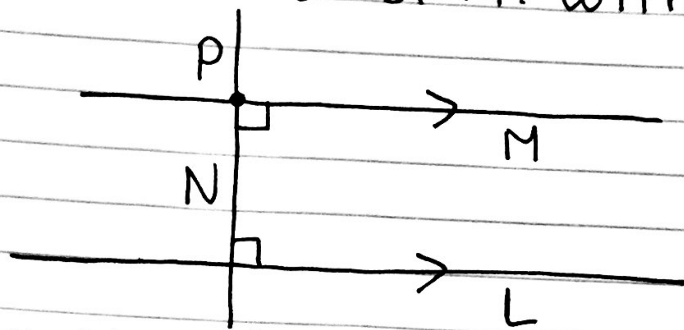
construction of M: with ruler & compass

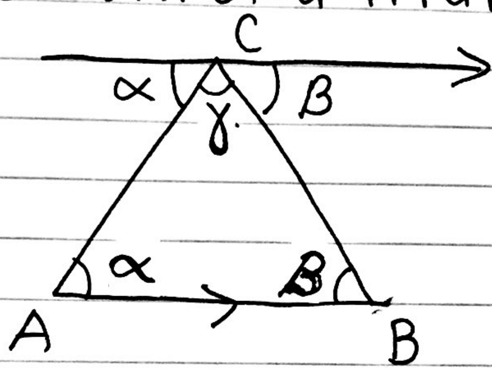


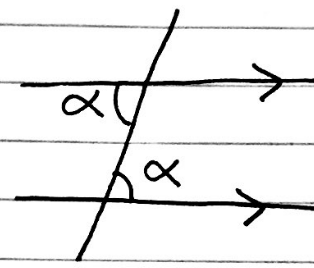
construct perpendicular N to L through P

construct perpendicular M to N through P

then L & M are parallel by (*)

angle sum of a triangle $= \pi$





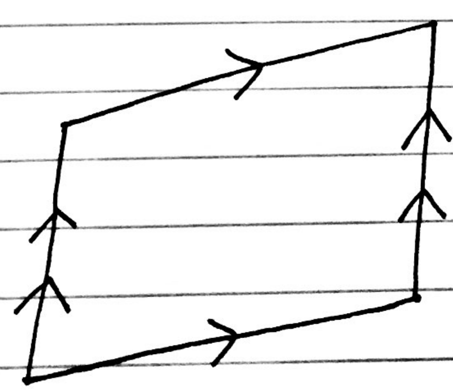
alternate angles

construct parallel to AB through C

use alternate angles two times

$\alpha + \beta + \gamma = \pi$ $\square$

A parallelogram is foursided polygon (quadrilateral) such that the two pairs of opposite sides are parallel



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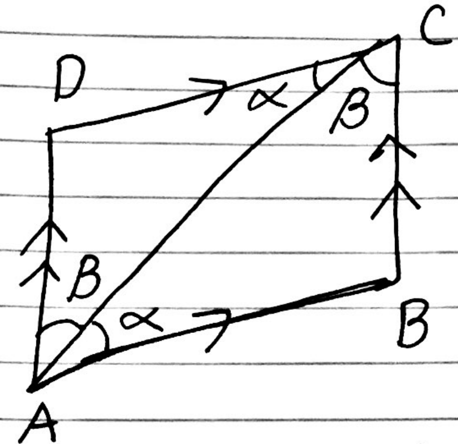
Problems:

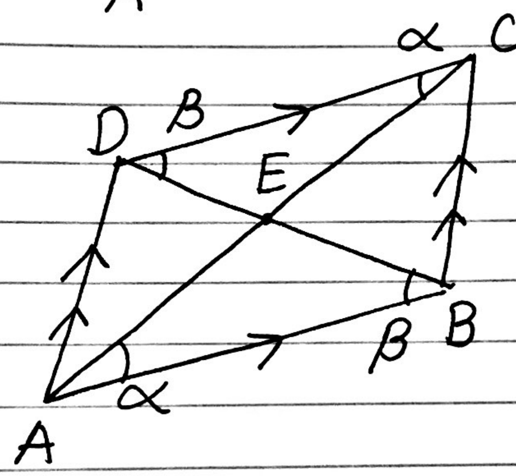
1. opposite sides of a parallelogram have equal lengths
2. diagonals of a parallelogram bisect each other



$\Delta ABC \cong \Delta CDA$ by ASA $\Rightarrow$

$|AB| = |DC|$

$|BC| = |AD|$ $\square$



$|AB| = |DC|$

ASA $\Rightarrow \Delta ABE \cong \Delta CDE$

$\Rightarrow |AE| = |EC|$

$|BE| = |ED|$ $\square$