# Math 132.5. Series (11.2–11.4)

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## 1 Section 11.2

### 1.1 Definition of series

If $a_1, a_2, a_3, \ldots$ is a sequence of real numbers, the associated *series* is the sum

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$$

which is defined as the limit of the *partial sums* $a_1 + a_2 + \cdots + a_n$ as $n \to \infty$.

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} \sum_{i=1}^{n} a_i = \lim_{n\to\infty} (a_1 + a_2 + \cdots + a_n).$$

If the limit exists (and is finite) we say the series is *convergent*. If the limit does not exist or is infinite, we say the series is *divergent*.

### 1.2 Geometric series

The series

$$\sum_{n=1}^{\infty} ar^{n-1} = \sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots$$

is called a *geometric series* with *initial term* $a$ and *common ratio* $r$. It is convergent if $|r| < 1$ and divergent if $|r| \geq 1$.

If $|r| < 1$ then $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$.

### 1.3 The divergence test

If $\lim_{n\to\infty} a_n \neq 0$ or the limit does not exist then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

## 2 Section 11.3

### 2.1 The integral test

Let $f(x)$ be a positive, decreasing, continuous function which is defined for $x \geq 1$. The improper integral $\int_1^{\infty} f(x)dx$ and the series $\sum_{n=1}^{\infty} f(n)$ are either both convergent or both divergent.

### 2.2 The $p$-series convergence criterion

The *p-series* $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.

## 3 Section 11.4

### 3.1 The comparison test

Let $a_n$ and $b_n$ be two sequences such that $0 \leq a_n \leq b_n$ for $n$ sufficiently large.

(1) If $\sum_{n=1}^{\infty} b_n$ is convergent then $\sum_{n=1}^{\infty} a_n$ is convergent.

(2) If $\sum_{n=1}^{\infty} a_n$ is divergent then $\sum_{n=1}^{\infty} b_n$ is divergent.

### 3.2 The limit comparison test

Let $a_n$ and $b_n$ be two sequences such that $a_n, b_n > 0$ for all $n$ sufficiently large. If the limit $\lim_{n\to\infty} \frac{a_n}{b_n}$ exists and is nonzero and finite then the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are either both convergent or both divergent.