Math 461 Lecture 9 9/24

Homework 2 due at start of wednesday's class

Office Hours today 2:30-3:30 LGRT 1235H

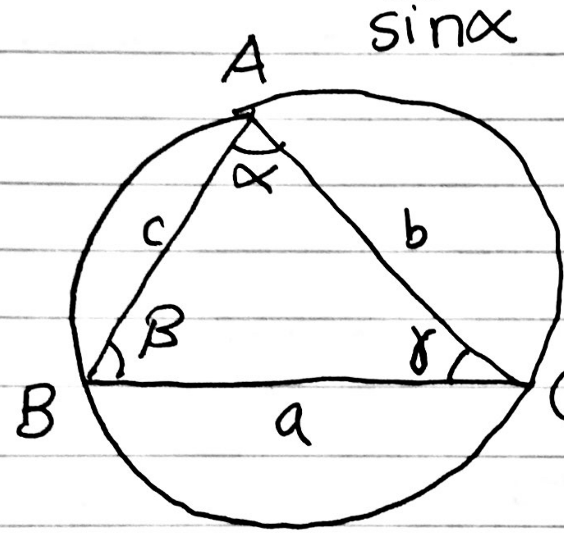
and tomorrow 4:00-5:00

For homework problems there can be more than one way to prove

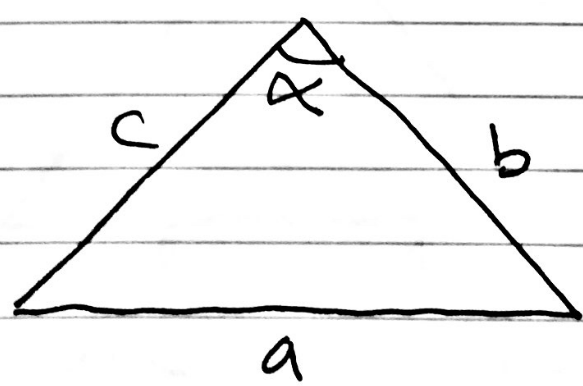
please write complete proofs like in Math 300

Last time:

sine rule $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$



cosine rule $a^2 = b^2 + c^2 - 2bc\cos\alpha$



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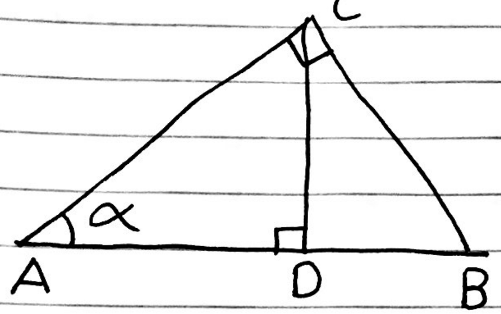
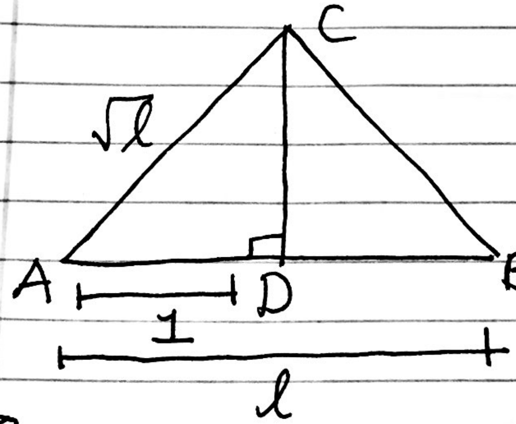
Construction of square roots: Correction!



$\triangle ABC \sim \triangle ACD$

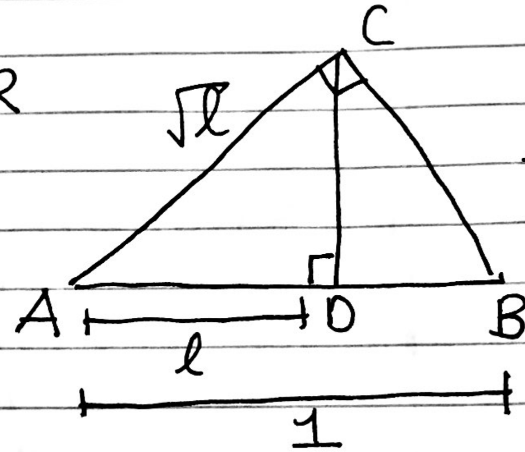
$$\frac{|AC|}{|AB|} = \frac{|AD|}{|AC|} = \cos\alpha$$

$\rightsquigarrow$ $|AC|^2 = |AB| \cdot |AD|$



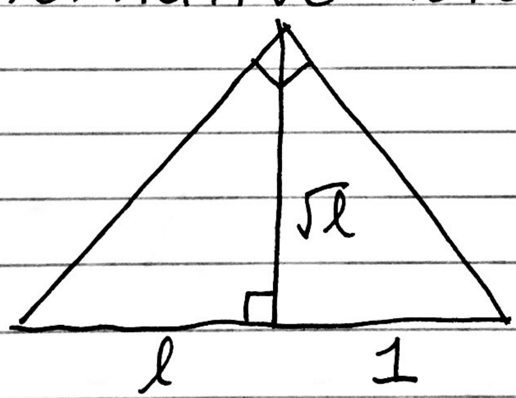
$l > 1$

OR



$l < 1$

Today:
construction of regular pentagons
center of mass
Alternative notation in textbook



Remark: constructible lengths
we're shown we can perform
operations $+, -, \times, \div, \sqrt{\ }$
geometrically using ruler and compass
so any length determined by 1 by
these operations can be constructed

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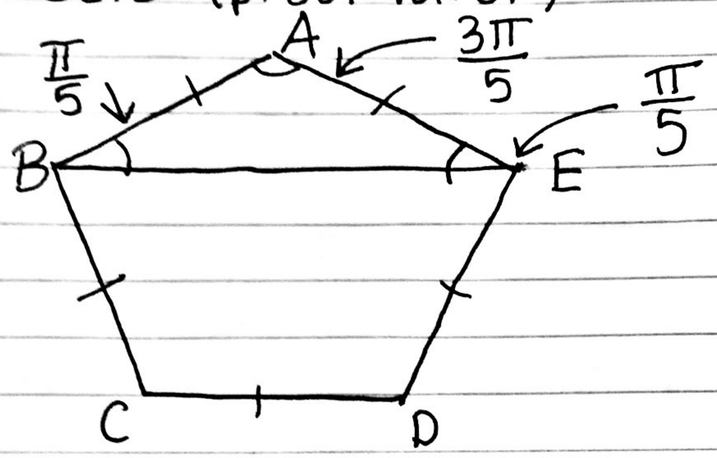
e.g. $\sqrt{4+\sqrt{13-\sqrt{7}}}$

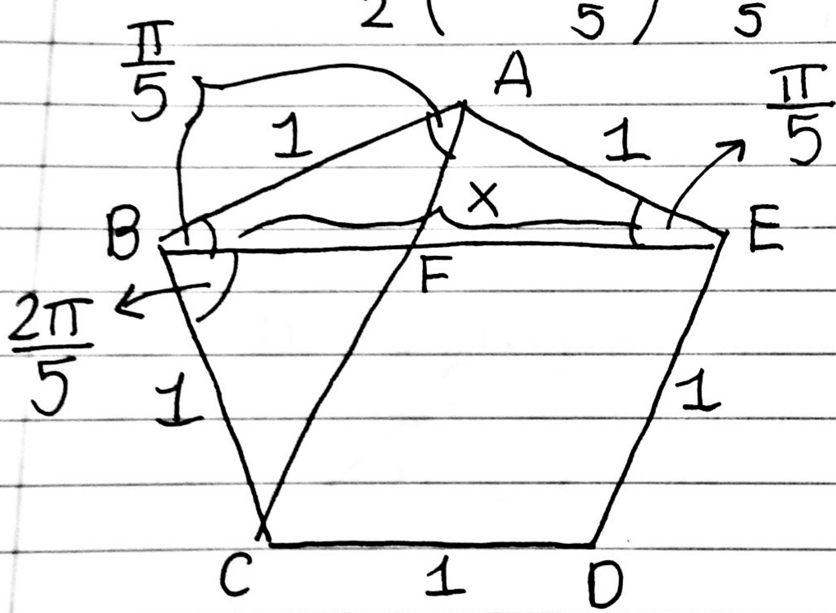
in fact these are all the constructible numbers (proof later)



sum of angles of pentagon = 3π

so each angle of regular pentagon = $\frac{3\pi}{5}$ $\triangle ABE$ is isosceles, angle sum of $\triangle ABE$ is π so

$\angle ABE = \frac{1}{2}\left(\pi - \frac{3\pi}{5}\right) = \frac{\pi}{5}$



picture blown up →

$\triangle BCA \cong \triangle BAE$ $\triangle BAF \sim \triangle BEA$

try to compute the length of x of the diagonals of the pentagon

$\frac{|BE|}{|AB|} = \frac{|AB|}{|BF|} \rightsquigarrow |BE| \cdot |BF| = |AB|^2$

$x \cdot |BF| = 1$

$\triangle BEA \sim \triangle BAF$

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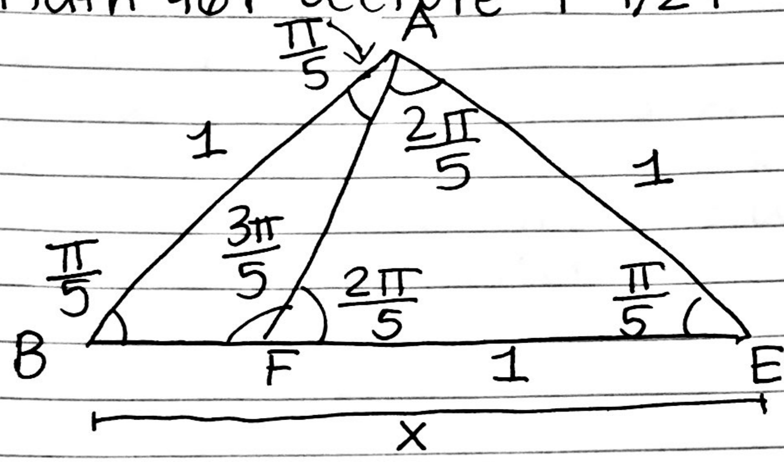


$\triangle EAF$ is isosceles

$\angle EAF = \angle EFA = \frac{2\pi}{5} \Rightarrow |FE| = |AE| = 1$

$x \cdot (|BE| - |FE|) = 1$ $\quad x(x-1) = 1$

$x^2 - x = 1$

$x^2 - x - 1 = 0$

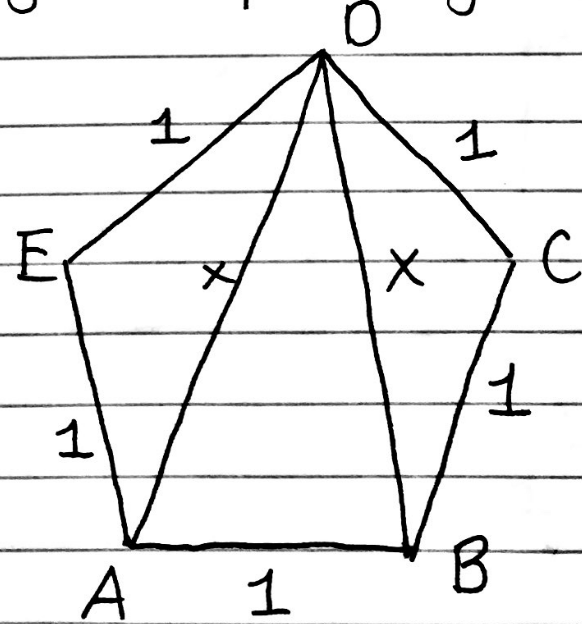
$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

$x > 0 \Rightarrow x = \frac{1+\sqrt{5}}{2}$ "golden ratio"

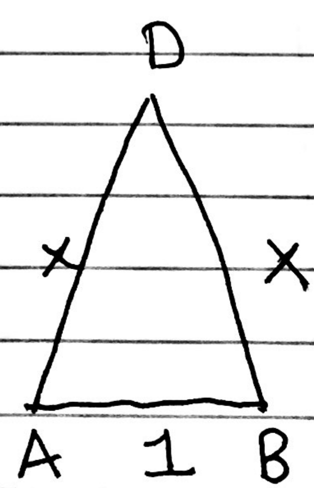
Note: x is constructible

(obtained from 1 by $+, -, \times, \div, \sqrt{\ }$)

given a construction of x, construct regular pentagon



directions →



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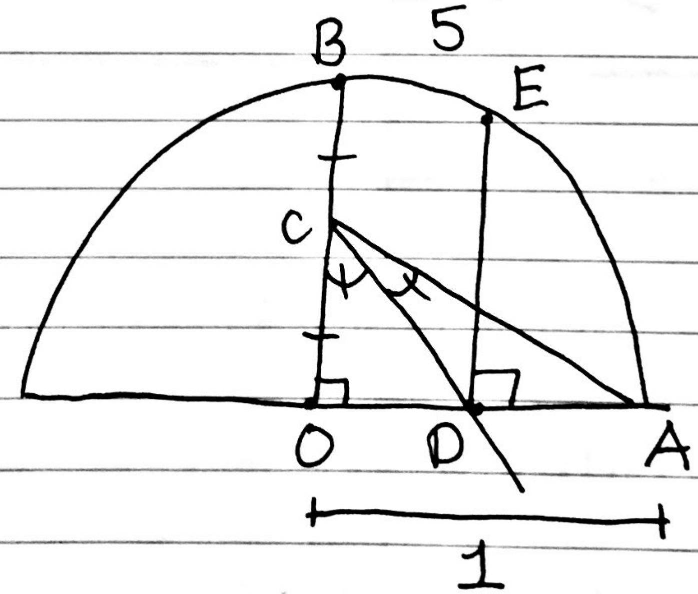
given line segment AB (length 1)
draw circles centers A and B with radius x, intersect at point D
draw circles centers B and D radius 1 intersect at point C
Similarly, circles center A and D with radius 1 intersect at point E

Alternative construction from first principles (quicker)
we'll construct a regular pentagon such that the radius of the circumscribed circle is equal to 1

1. circle center O, radius $|OA| = 1$
2. draw perpendicular to the line OA at point O
3. bisect line segment OB
4. bisect angle $\angle OCA$, bisector intersects OA at D
5. draw perpendicular to OA at D, intersecting circle at E

Claim: $\angle AOE = \frac{2\pi}{5}$



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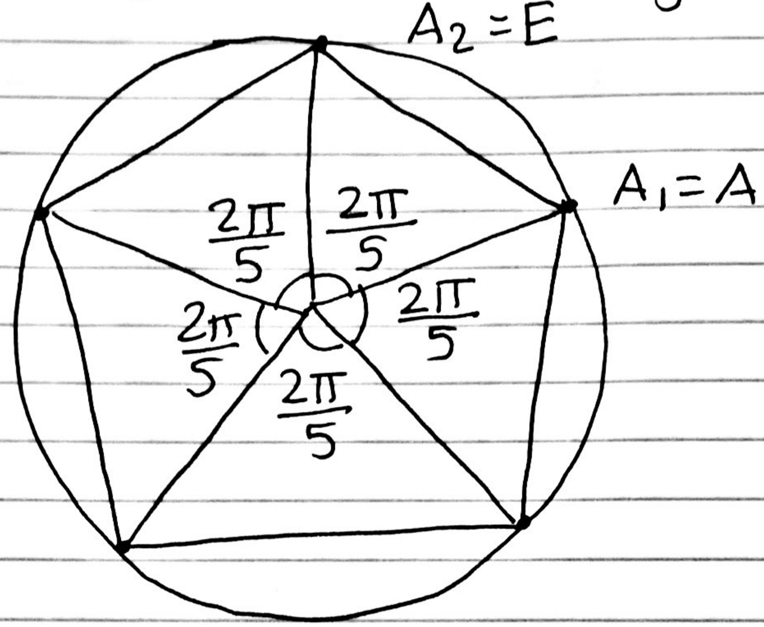
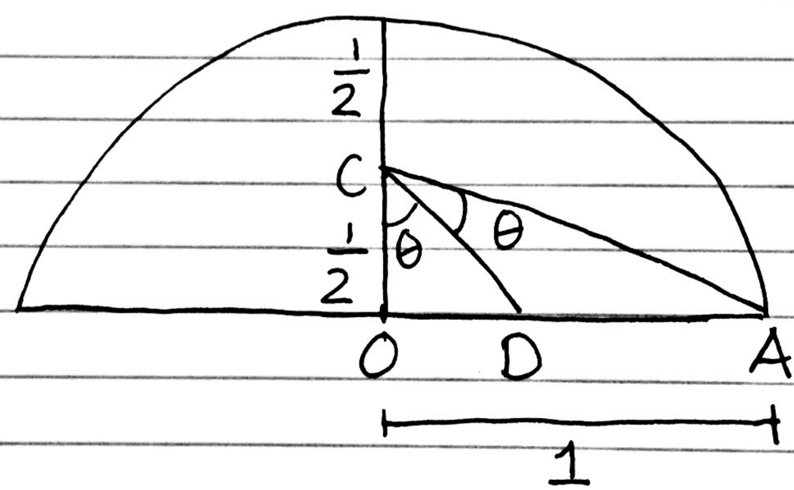
assuming the claim, can construct point $A_1 = A$, $A_2 = E$, A_3, A_4, A_5 on the circle such that $|A_1A_2| = |A_2A_3| = |A_3A_4| = |A_4A_5|$

then $A_1A_2A_3A_4A_5$ is a regular pentagon





Claim: $|OD| = \cos\left(\frac{2\pi}{5}\right)$

know $\frac{|OD|}{1/2} = \frac{\text{opp}}{\text{adj}} = \tan\theta$ for $\triangle COD$

and $\tan 2\theta = \frac{|OA|}{|OC|} = \frac{1}{1/2} = 2$

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$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\dfrac{\sin 2\theta}{\cos 2\theta} = \dfrac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$t = \tan\theta$

$2 = \dfrac{2t}{1-t^2}$ $\quad 2-2t^2 = 2t$

$2t^2 + 2t - 2 = 0$ $\quad t = \dfrac{-1 \pm \sqrt{5}}{2}$ $\quad t > 0$

$t^2 + t - 1 = 0$

$|OD| = \dfrac{1}{2}\tan\theta = \dfrac{\sqrt{5}-1}{4}$