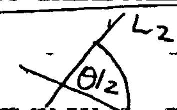
10/28/19

HW 5 due at start of Wednesday's class.

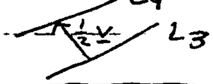
Office hours this week: Tuesday 4-5pm } LGRT 1235H
Tuesday 1-2pm }

Correction

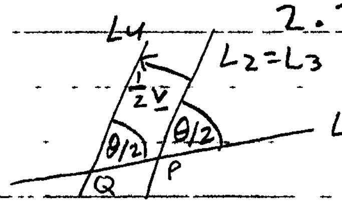


$\mathrm{Rot}(P,\theta) = \mathrm{Refl}_{L_2} \circ \mathrm{Refl}_{L_1}$

$\mathrm{Trans}_{\underline{v}} = \mathrm{Refl}_{L_4} \circ \mathrm{Refl}_{L_3}$



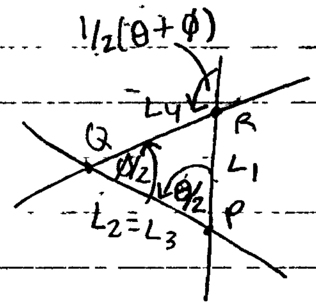
Last Time: • Compositions of Isometries
1. Rotation & translation
2. Two rotations



$\mathrm{Trans}_{\underline{v}} \circ \mathrm{Rot}(P,\theta) = (\mathrm{Refl}_{L_4} \circ \mathrm{Refl}_{L_3}) \circ (\mathrm{Refl}_{L_2} \circ \mathrm{Refl}_{L_1})$

$L_2 = L_3 \rightarrow \; = \mathrm{Refl}_{L_4} \circ \mathrm{Refl}_{L_1}$
$= \mathrm{Rot}(Q,\theta)$

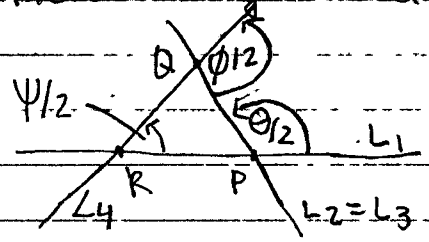
(*)



$\mathrm{Rot}(Q,\phi) \circ \mathrm{Rot}(P,\theta) = \mathrm{Rot}(R, \theta+\phi)$

Today: • GPS Theorem
• Isometries & congruence }
• 3 reflections theorem } did not get to
• Classification of isometries }

Remarks about composite of two rotations.
Picture may look a bit different to (*)



$\mathrm{Rot}(Q,\phi) \circ \mathrm{Rot}(P,\theta) = \mathrm{Rot}(R,\psi)$
$= \mathrm{Rot}(R, \theta+\phi)$

angle sum of triangle

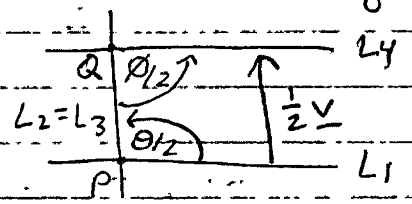
$\psi = ?, \quad \pi - (\pi - \phi/2) - (\pi - \theta/2) = \psi/2$

$\theta/2 + \phi/2 - \pi =$

$\Rightarrow \psi = \theta + \phi - 2\pi$

$= \theta + \phi \mod 2\pi$ (i.e. up to multiples of 2π).

Third possibility:



L_1 & L_4 are parallel

$Rot(Q, \phi) \circ Rot(P, \theta) = Refl_{L_4} \circ Refl_{L_1}$

$= Trans_{\underline{v}}$

What is condition on θ & ϕ here?

$\theta/2 + \phi/2 = \pi$ (parallel axiom)

$\theta + \phi = 2\pi$ $\qquad \phi = 2\pi - \theta$

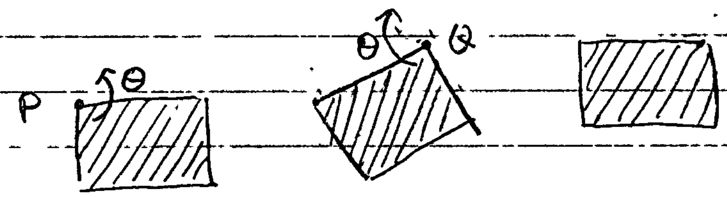
$Rot(P, \theta)$: ccw rotation by θ about P

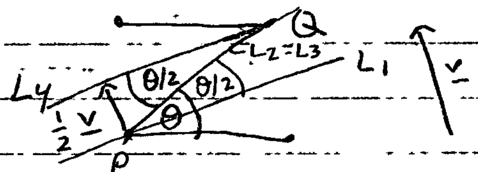
$Rot(Q, 2\pi - \theta)$: ccw rotation by $2\pi - \theta$ about Q

$=$ cw rotation by θ about Q

Application: Moving furniture

"Walk a wardrobe"





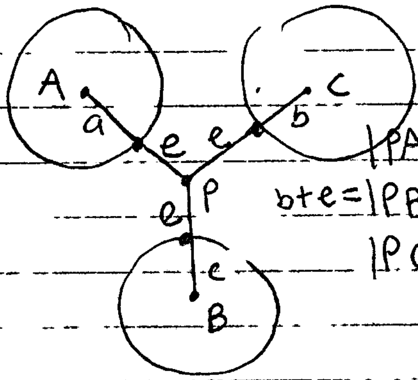
GPS Theorem

A, B, C 3 points in $\mathbb{R}^2$, not collinear (don't lie on a line) &

then a point P in $\mathbb{R}^2$ is uniquely determined by the distances $|PA|, |PB|, |PC|$.

[Actually GPS is even smarter, point P is determined by the differences in $|PA|, |PB|, |PC|$ — will determine the point P (your position) & the correction to your phone's time from this information (A, B, C = satellites).]

Aside:



$|PA| = a+e$

$b+e = |PB|$ known up to a common error e given by

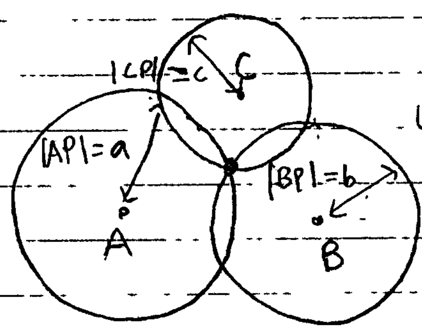
$|PC| = c+e$ difference between your phone's time & true time (distance = speed × time)

Geometrically: Appollonius' Problem

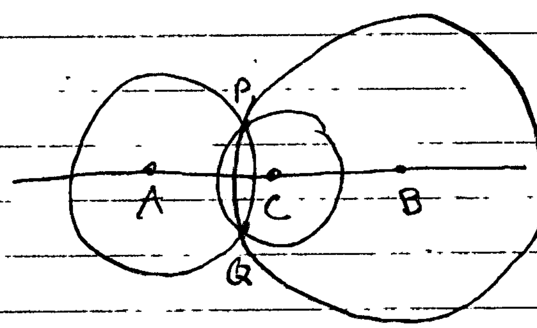
Newton: construction using ruler & compass

Proof of GPS Theorem

We need to show: if P & Q are two points in the plane & $|PA| = |QA|$, $|PB| = |QB|$, $|PC| = |QC|$ then $P = Q$ (assuming A, B, C do not lie on a line).



where all three circles intersect is P?



If all three points are on a line:

P & Q both satisfy conditions, then not uniquely determined $P \neq Q$.

Rephrasing again:

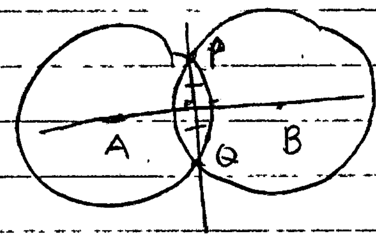
If $P \neq Q$ and $\overbrace{|PA| = |QA|, |PB| = |QB|, |PC| = |QC|}^{X}$

then $\underbrace{A, B, C \text{ must lie on a line}}_{Y}$. $\underbrace{P = Q}_{Z}$

X AND (NOT Y) $\Rightarrow Z$

$\equiv$ (NOT Z) AND $X \Rightarrow Y$

Notice:



$|PA| = |QA|$, $|PB| = |QB|$, $|PC| = |QC| \Rightarrow A, B, C$ lie on the perp. bisector of PQ, in particular they lie on a line. ▨