DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

MATH 235 **MIDTERM** **Fall 2009**

NAME:____________________________________

Section Number:________________________ Instructor's Name: ________________________

1.	(12)	
2.	(16)	
3.	(16)	
4.	(12)	
5.	(14)	
6.	(14)	
7.	(16)	
Total	(100)	

Instructions:

- **Please use correct notation when writing matrices and vectors**
- **In true/false questions give a proof or counter-example. Examples are not proofs.**
- **You must explain how you arrived at your answers, and show your algebraic calculations.**
- **You must justify your statements. Unsubstantiated answers receive no credit.**

1: Let V, W be a vector spaces and let $F : V \longrightarrow W$ be a linear transformation of vector spaces.
1a: Define ker(F).

1b: Define rank and nullity of F.

2: Let $F : \mathbb{R}^5 \longrightarrow \mathbb{R}^3$ be given by the matrix

$$\begin{pmatrix} 2 & -1 & 1 & 0 & 2 \\ 2 & -1 & -1 & 1 & 0 \\ 6 & -3 & -1 & -2 & 2 \end{pmatrix}.$$

Find a basis of $\ker(F)$ and a basis of $im(F)$. What is the rank of F?

3: Let $A = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\}$ and let $B = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ be two bases of $\mathbb{R}^2$.

3a: For a vector $v \in \mathbb{R}^2$ let v_A denote its expression in terms of A coordinates and let v_B denote its expression in terms of B coordinates. Find a matrix $1_{B \leftarrow A}$ so that

$$1_{B \leftarrow A} v_A = v_B.$$

3b: Find a matrix $1_{A \leftarrow B}$ so that

$$1_{A \leftarrow B} v_B = v_A.$$

3c: Let $M = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ be a matrix representing a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ with respect to the basis B. What is the matrix of this linear transformation with respect to the basis A?

4: True or False. You must explain the reason for your answer.
4a: Let M be a matrix. If the kernel of M is just the zero vector, then the columns of M are linearly independent.

4b: If V is a subspace of $\mathbb{R}^n$ and $u, v, w \in V$, then $2u - 3v + 4w \in V$ as well.

5: Let S be the set of 2×2 matrices A so that

$$A \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Find a basis for S. What is the dimension of S?

6: Let P_2 denote the vector space of all polynomials of degree less than or equal to 2. Find a basis for the image and kernel of the map

$$\begin{aligned} T &: P_2 \longrightarrow \mathbb{R}^2 \\ T &: f \mapsto \begin{pmatrix} f(1) \\ f'(1) \end{pmatrix}. \end{aligned}$$

7: Let $A = \{1, t, t^2\}$ be a basis of P_2, the set of all polynomials of degree less than or equal to 2. Find the matrix of the map

$$\begin{aligned} T &: P_2 \longrightarrow P_2 \\ T &: f \mapsto tf' + 2f'' - f \end{aligned}$$

with respect to the basis A.