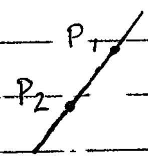
10/9/19

- HW 4 due now
- HW 3 returned
- Midterm 1 next Wednesday 10/16/19 7-9 PM in LGRC A301
- Syllabus: Everything we have covered, up to the end of today's class.
- Lecture notes available at people.math.umass.edu/~hacking/461F19/classlog
- Will post review problems tomorrow
  (No HW due next Wed.)

Last Time
- Coordinates
- Distance formula: $P_1 = (a_1, b_1)$, $P_2 = (a_2, b_2)$
  $\Rightarrow |P_1P_2| = \sqrt{(a_2-a_1)^2 + (b_2-b_1)^2}$
- Eq. of circle: $\mathcal{C}$ center $P = (a,b)$, radius r
  $\Rightarrow \mathcal{C} = \{(x,y) \mid (x-a)^2 + (y-b)^2 = r^2\}$
- Slope of line:



$m = \dfrac{b_2 - b_1}{a_2 - a_1}$

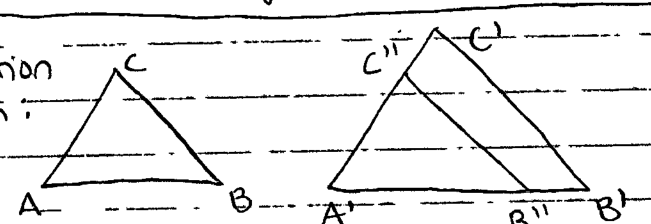
$(\Rightarrow) \checkmark$
$(\Leftarrow)$?

Lemma: two lines are parallel or equal $\Leftrightarrow$ slopes are equal

Thm: $\dfrac{|A'B'|}{|AB|} = \dfrac{|B'C'|}{|BC|} = \dfrac{|C'A'|}{|CA|} \Rightarrow \triangle A'B'C' \sim \triangle ABC$

Today
- Finish proof of Lemma
- Eq. of line
- Perpendicular bisector
- Algebraic interpretation of ruler & compass construction

Continuation of Thm:



$|AB| = |A'B''|$
$|AC| = |A'C''|$

Assume: $\dfrac{|A'B'|}{|AB|} = \dfrac{|B'C'|}{|BC|} = \dfrac{|C'A'|}{|CA|}$

Want to show corresponding angles are equal
(i.e. $\triangle ABC \sim \triangle A'B'C'$)

Showed: $\triangle A'B''C'' \sim \triangle A'B'C'$
(using converse of Thales' thm & corresponding angles).

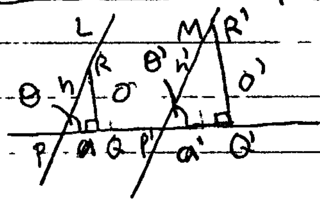
Claim: $\triangle ABC \cong \triangle A'B''C''$ ($\Rightarrow$ ▨)

$\longrightarrow$

Proof. By SSS: $\dfrac{|B''C''|}{|B'C'|} \underbrace{=}_{\text{because } \Delta A'B''C'' \sim \Delta A'B'C'} \dfrac{|A'B''|}{|A'B'|} = \dfrac{|AB|}{|A'B'|} \overset{\text{by assumption}}{=} \dfrac{|BC|}{|B'C'|}$

$\Rightarrow |B''C''| = |BC|$. ▨

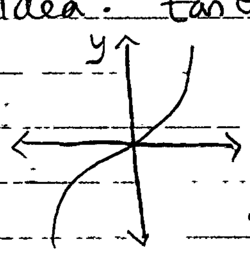
Continuation of proof of lemma ($\Leftarrow$)



Assume $\frac{o}{a} = \frac{o'}{a'}$. Show M & L are parallel or equal.
↳ slopes are equal

Equivalently, show $\theta = \theta'$.

(Idea: $\tan\theta = \frac{o}{a}$. So $\theta = \tan^{-1}\left(\frac{o}{a}\right) = \tan^{-1}\left(\frac{o'}{a'}\right) = \theta'$



$y = \tan\theta$

→ not developed yet

↳ invertible)

$o^2 + a^2 = h^2$, $o'^2 + a'^2 = h'^2$ $\Rightarrow \Delta PQR \sim \Delta P'Q'R'$ by Theorem

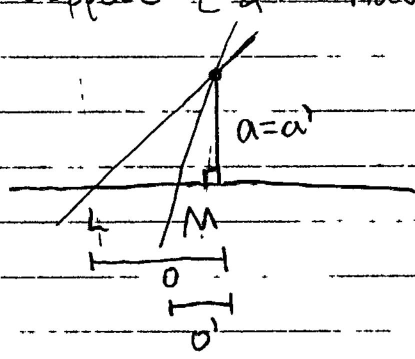
(*) $\frac{o'}{o} = \frac{a'}{a} \overset{?}{=} \frac{h'}{h}$. Say, $\frac{o'}{o} = \frac{a'}{a} = \lambda$.

Then, $h'^2 = (o')^2 + (a')^2 = (\lambda o)^2 + (\lambda a)^2 = \lambda^2(o^2 + a^2) = \lambda^2 h^2$

$h' = \lambda h$, $\frac{h'}{h} = \lambda$. ▨

Alternative proof of lemma ($\Leftarrow$) (Proof by contradiction)

Suppose L & M have same slope but not parallel or equal.



$o \neq o'$ $\Rightarrow$ slopes different. ▨

L line passing through point $P = (a,b)$, with slope m.

$$\Longrightarrow L = \left\{ (x,y) \mid \frac{y-b}{x-a} = m \right\} \cup \{(a,b)\}$$
$$= \{ (x,y) \mid (y-b) = m(x-a) \}$$
$$= \{ (x,y) \mid y = mx + b \} \text{ s.t. } c = b - ma$$

<u>Proof</u> <u>Recall</u>: to show equality $A = B$ of two sets A & B, need to show: $A \subset B$ and $B \subset A$.

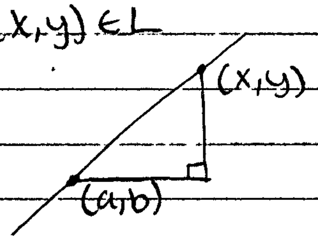
<u>Our case</u>: $L \stackrel{?}{=} \left\{ (x,y) \mid \frac{y-b}{x-a} = m \right\} \cup \{(a,b)\}$

($\subset$) Want to show either $(x,y) = (a,b)$

OR $\frac{y-b}{x-a} = m$

Let $(x,y) \in L$

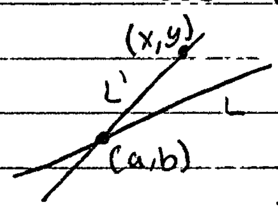
↳ This is the definition of slope of L. ✓



($\supset$) Suppose $(x,y) \in \mathbb{R}^2 \neq (a,b)$

& $\frac{y-b}{x-a} = m$.

Show $(x,y) \in L$



Slope of L is m (our assumption)

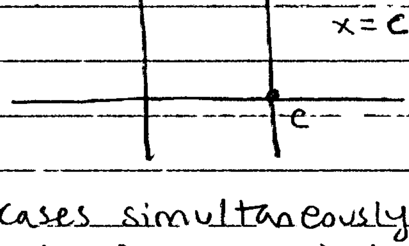
Slope of L' is m (by computation)

So L & L' are either parallel or equal (Lemma before)

intersect at $P = (a,b)$, so equal, in particular $(x,y) \in L$. ∎

<u>Remark</u>: Vertical lines have slope "$m = \infty$"

~~Equation:~~

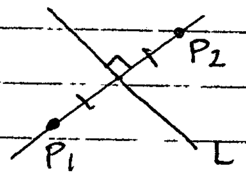


Earlier: $m \neq \infty$; $y = mx + c$

Treat both cases simultaneously: $ax + by = c$ $a,b,c \in \mathbb{R}$ $(a,b) \neq (0,0)$.

$b = 0$: vertical line, $b \neq 0$: slope $m = -a/b$

Q: Given points $P_1 = (a_1, b_1)$, $P_2 = (a_2, b_2)$.
What's the equation of the perpendicular bisector of the line segment P_1P_2?



to be continued...