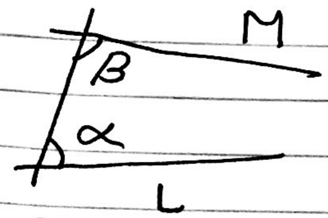
Math 461 Lecture 5 9/14

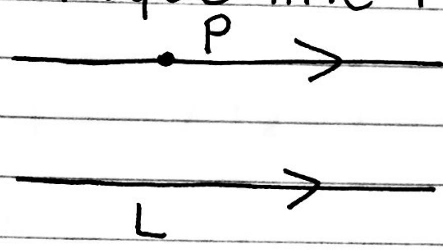
Last time:

1.

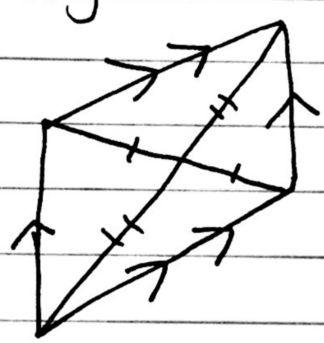


L & M are parallel $\Longleftrightarrow$ $\alpha + \beta = \pi$

2. Playfair's Axiom
given line L, point $P \notin L$, there is a unique line M through P parallel to L



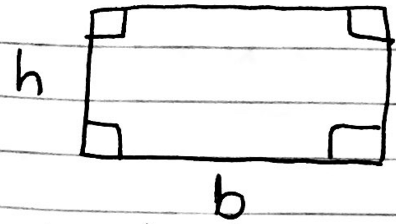
3. angle sum of a triangle $= \pi$
4. parallelograms:
opposite sides have equal lengths
diagonals bisect each other



Today:
area
area of parallelogram and triangle
Pythagoras' Theorem
Thales' Theorem, similar triangles

Area:
we will assume (following Euclid) that one can assign an area $A(P)$ (a positive real number) to any polygon P

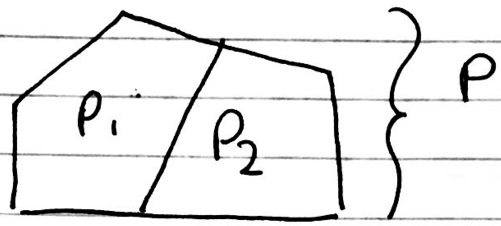
Math 461 Lecture 5 9/14



$A = b \cdot h$

such that

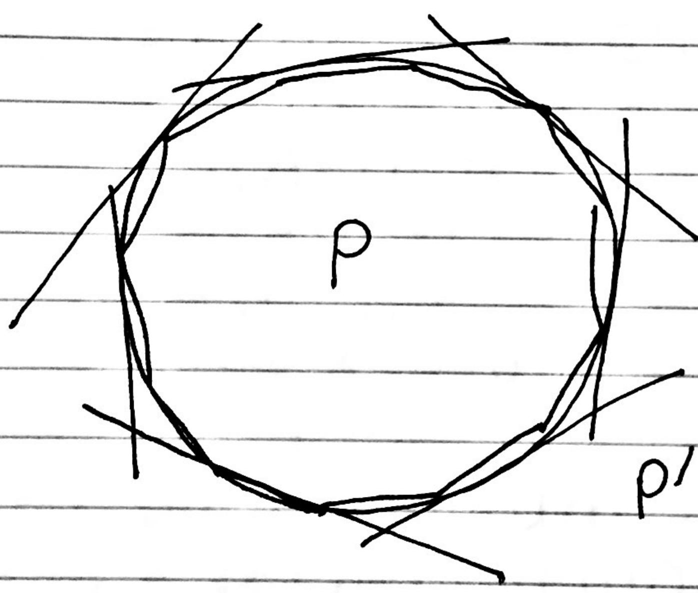
1. $A(\text{rectangle}) = \text{base} \times \text{height}$
2. $A(P) = A(P_1) + A(P_2)$ if



modern approach to area (1600s)

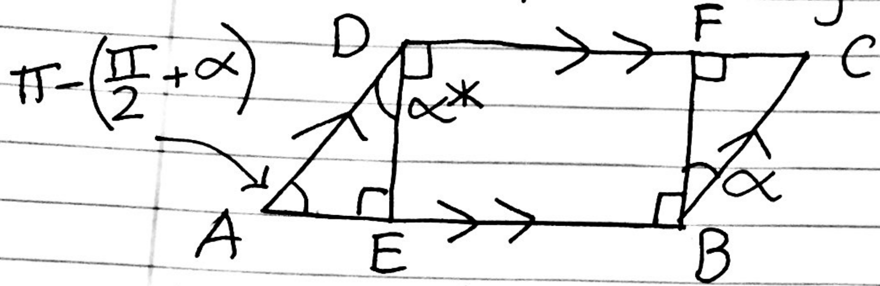
region $R \subset \mathbb{R}^2$ $A(R) = \iint_R dx\,dy$

$A(P) < A(\text{circle}) < A(P')$



Math 461 Lecture 5 9/14

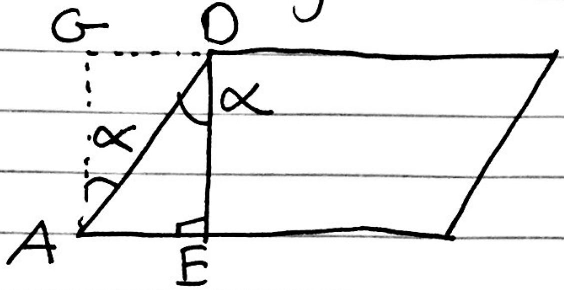
Area of a parallelogram:

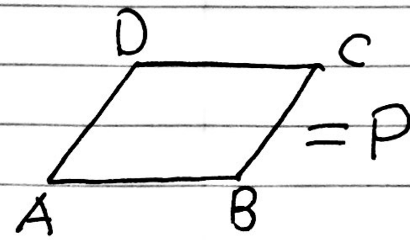


Know $A(BFDE) = |EB| \times |BF|$

*by sum of triangles

$\triangle ADE \cong \triangle CBF$ by ASA



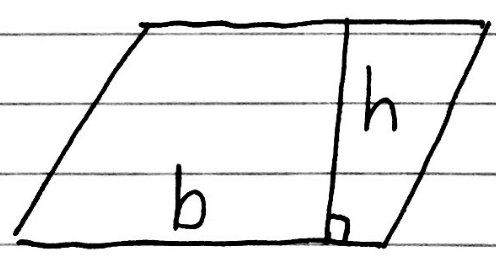


$\triangle BCF \cong \triangle ADG$

$$\begin{aligned}
A(P) &= A(\triangle ADE) + A(EBFD) + A(\triangle BCF) \\
&= A(\triangle ADE) + A(ADG) + A(EBFD) \\
&= A(AEDG) + A(EBFD) \\
&= A(ABFG) \\
&= |AB| \cdot |BF| \\
&= \text{base} \times \text{height}
\end{aligned}$$

Conclusion:

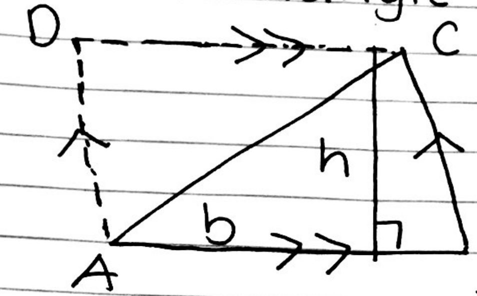
area(parallelogram) = base $\times$ perpendicular height



Remark: also assumed areas of congruent triangles are equal

Math 461 Lecture 5 9/14

Area of a Triangle:

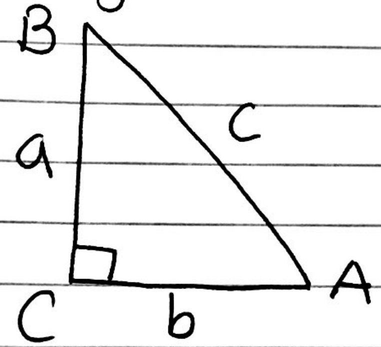


$\triangle ABC \cong \triangle CDA$ proved earlier

$$\text{area}(ABCD) = \text{area}(\triangle ABC) + \text{area}(\triangle CDA)$$
$$= 2\,\text{area}(\triangle ABC)$$
$$\cancel{= \text{base} \times \text{height}}$$

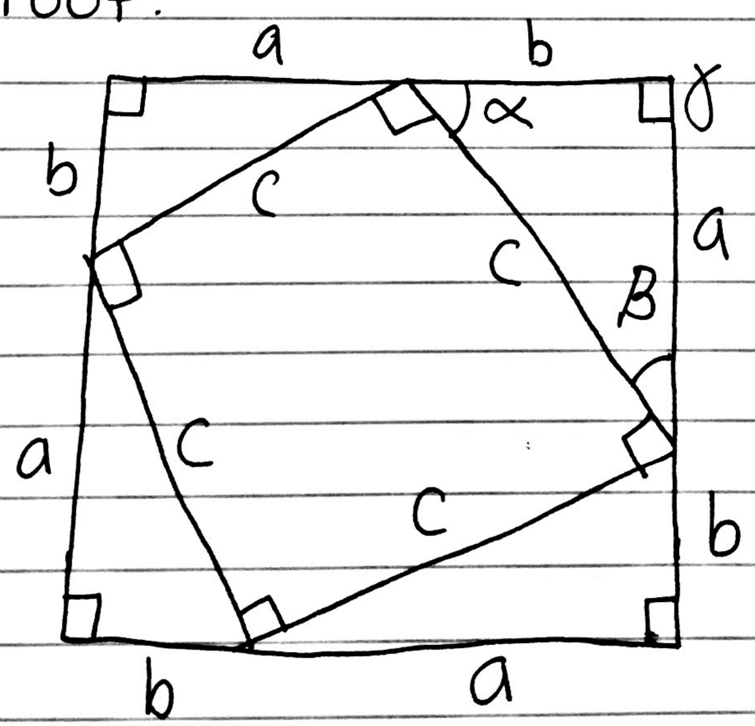
$$\text{base} \times \text{height} = 2\,\text{area}(\triangle ABC)$$
$$\text{area}(\triangle ABC) = \frac{1}{2}(\text{base} \times \text{height})$$

Pythagoras' Theorem:



$$a^2 + b^2 = c^2$$

Proof:

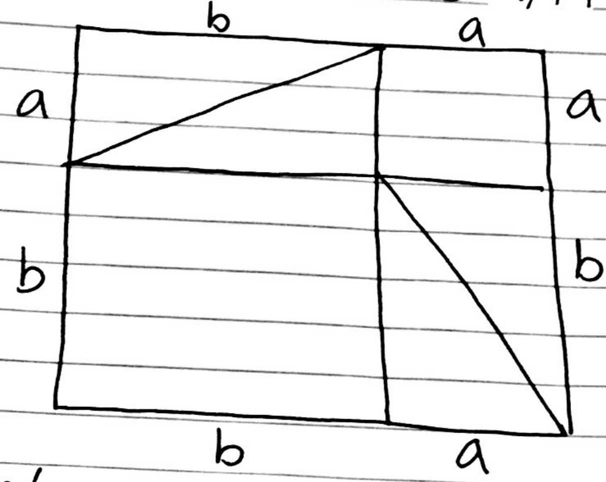


$$\alpha + \beta + \frac{\pi}{2} = \pi$$

angle sum of triangles $\Rightarrow$

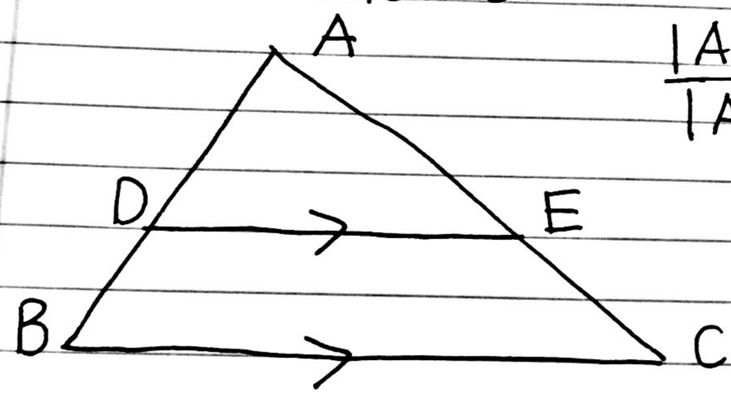
$$\gamma = \pi - (\alpha + \beta) = \frac{\pi}{2}$$

Math 461 Lecture 5 9/14



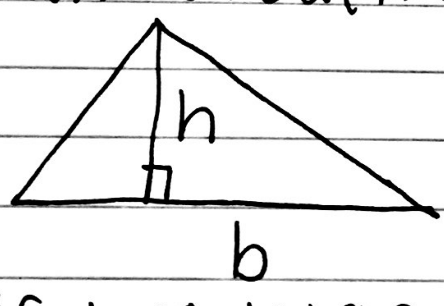
area(square side a+b) =
area(square side c) + 4 area(triangle) =
area(square side a) + area(square side b) +
4 area(triangle) =>
$c^2 = a^2 + b^2$ $\square$

Thales Theorem



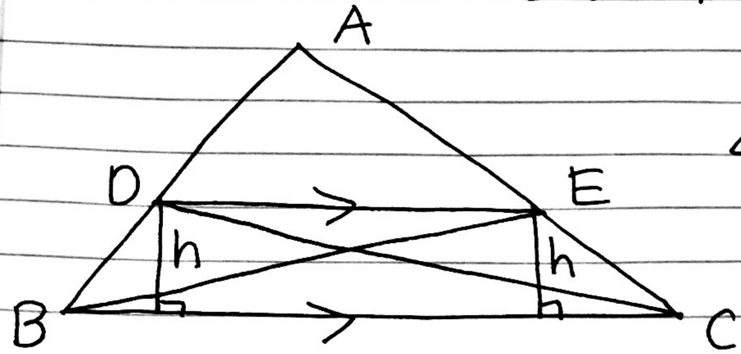
$$\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|}$$

Recall: area(triangle) $= \frac{1}{2} bh$



So if two triangles have same height, areas are in same ratio as their bases

Math 461 Lecture 5 9/14



$\triangle AEB \ncong \triangle ACD$

$\triangle BCD$ & $\triangle BCE$ have same area
same base BC and same height h
$\triangle BED$ & $\triangle CED$ have same area
same base DE and height h

$$\left.\begin{array}{l}\triangle ADE \cup \triangle BED = \triangle ABE \\ \triangle ADE \cup \triangle CED = \triangle ADC\end{array}\right\} \text{have same area}$$