Math 461 Lecture 12 10/1

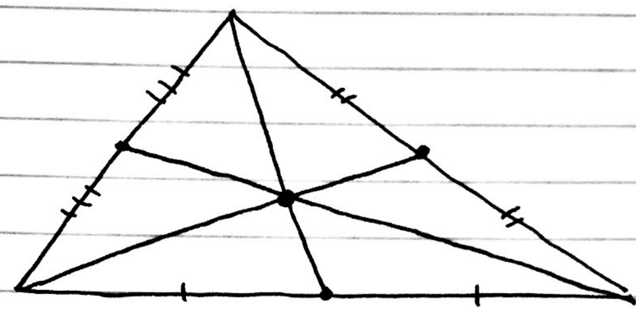
Homework 3 due Wednesday
at start of class

Office Hours: today 2:30 - 3:30
tomorrow 4:00 - 5:00

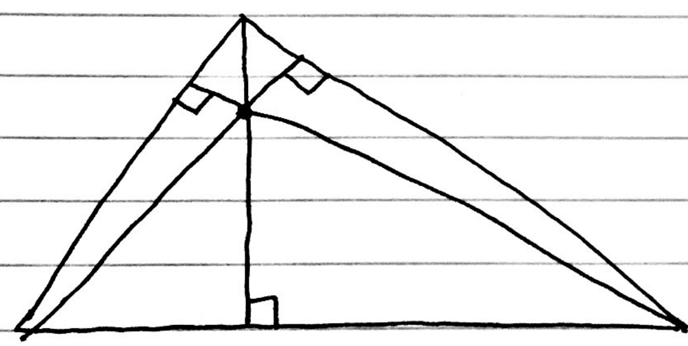
Office: LGRT 1235H

Last time:

Center of Mass:



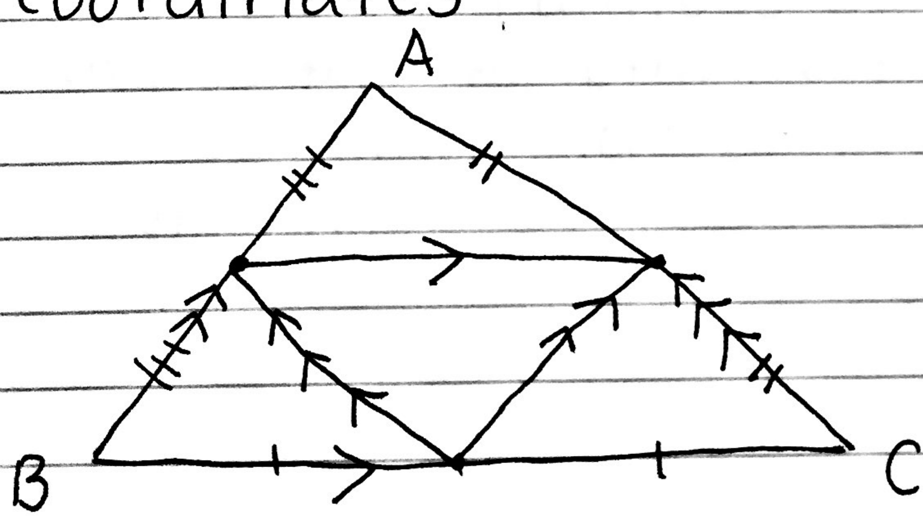
Orthocenter:



Today:

Finish orthocenter

coordinates



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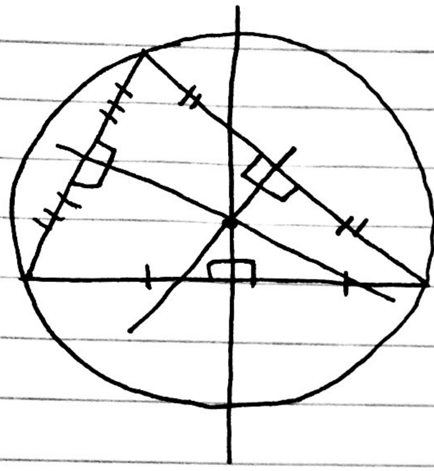
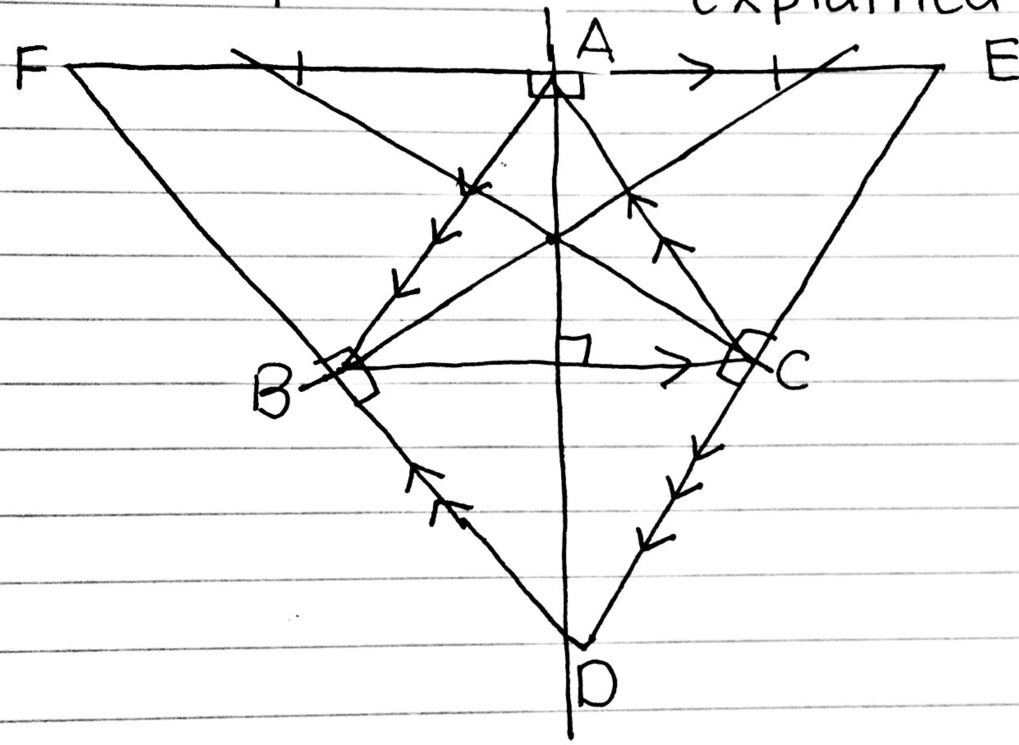


perpendicular bisectors of sides of a triangle are concurrent
common point is center of circumscribed circle
explained in HW 1




Note: $|CD| = |CE|$ (c is the midpoint of DE)
BDCA & BCEA are parallelograms $\Rightarrow$
$|DC| = |BA| = |CE|$ ✓
Now perpendicular bisector of EF coincides with altitude of ABC through A
same is true for other altitude

of $\Delta DEF$

now perpendicular bisectors are concurrent $\Rightarrow$ altitudes are concurrent $\square$

of $\Delta ABC$

coordinates:

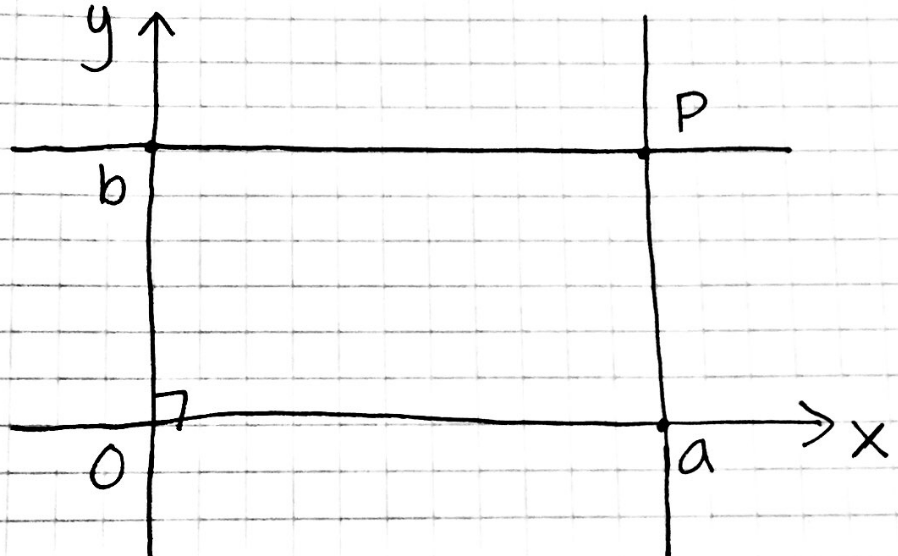
~1630 Fermat, Descartes (cartesian)

In the Euclidean Plane:

Introduce two perpendicular lines meeting at a point O

lines: x & y axes  O: origin

choose a direction for each axis, so that angle from x-axis to y-axis is $\frac{\pi}{2}$ radians counter clockwise

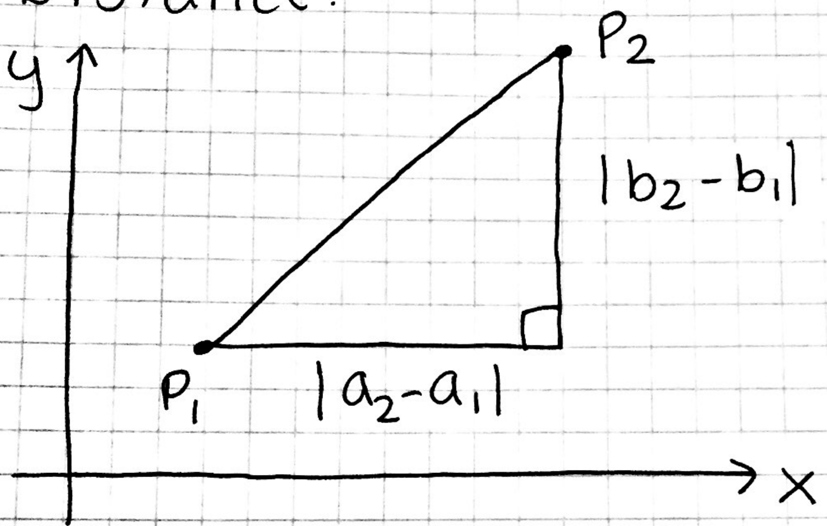



Now can set up coordinates:

Given point P in the plane, draw lines through P parallel to the axes, intersecting the x-axis at signed distance a from O and intersecting the y-axis at signed distance b from O

This gives an identification of the plane with $\mathbb{R}^2 = \{(a,b) \mid a,b \in \mathbb{R}\}$
why is this useful?
solve geometric problems using algebra (and calculus etc.)

Distance:




$P_1 = (a_1, b_1) \quad P_2 = (a_2, b_2)$

$|P_1P_2| = \sqrt{(a_2-a_1)^2 + (b_2-b_1)^2}$

$P.T. = |P_1P_2| = \sqrt{|a_2-a_1|^2 + |b_2-b_1|^2}$

Equation of a circle:

Circle $C$ center $P = (a,b)$, radius $r$

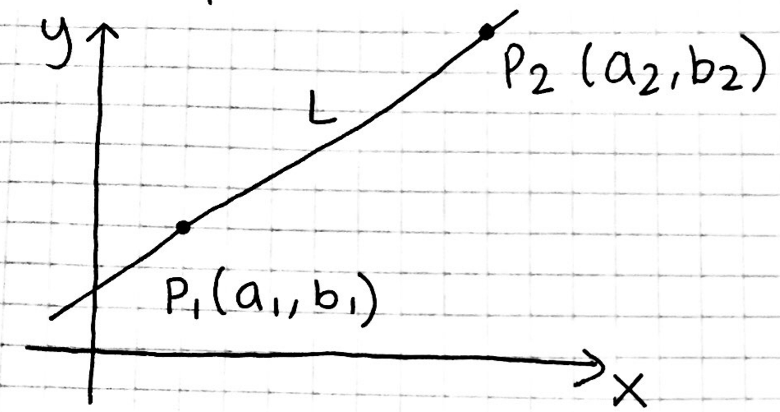
Equation of $C$: $(x-a)^2 + (y-b)^2 = r^2$

$C = \{Q \mid |PQ| = r\}$ definition of a circle

$= \{(x,y) \mid \sqrt{(x-a)^2 + (y-b)^2} = r\}$

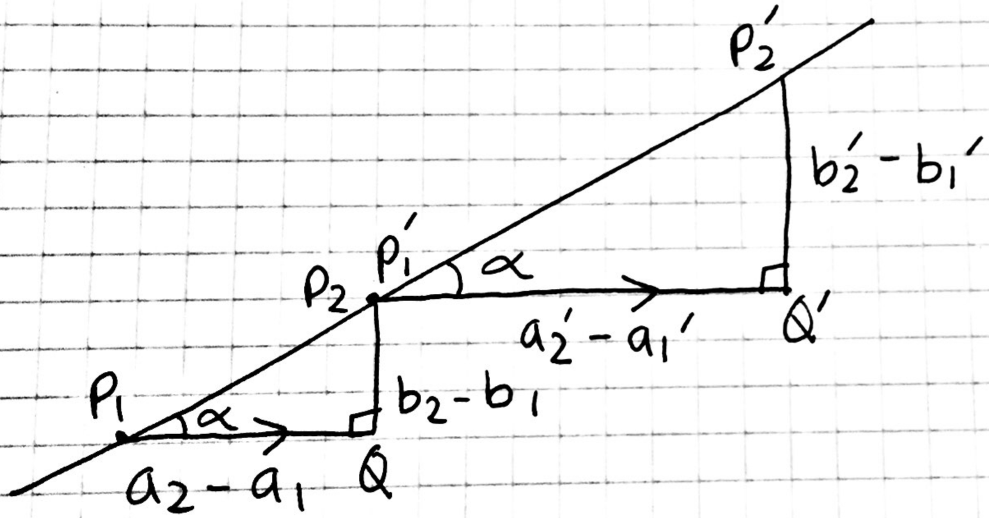
$= \{(x,y) \mid (x-a)^2 + (y-b)^2 = r^2\}$

Ex. Equation of a line.




A line has a slope (with respect to the choice of coordinates) given by $m = \frac{b_2 - b_1}{a_2 - a_1}$

Question: why is this well defined, i.e. independent of choice of points $P_1$ & $P_2$ on line L




slope $m = \frac{|P_2 Q|}{|P_1 Q|} = \frac{|P_2' Q'|}{|P_1' Q'|}$ because $\Delta P_1 P_2 Q \sim \Delta P_1' P_2' Q'$

Equation of a line through point $P = (a, b)$ with slope m

Question: when are two lines parallel?
(Recall say two lines are parallel if they don't intersect)

Answer: when they have the same slope (including the case $m = \infty$)

Proof: want to prove $L_1$ & $L_2$ are parallel $\iff$ slopes are equal

$L_1$ $a_1x + b_1y = c_1$ ①

$L_2$ $a_2x + b_2y = c_2$ ②

Claim becomes: Equations ① & ② have no common solutions $\iff$ $(a_1, b_1)$ & $(a_2, b_2)$ are proportional ⊛

i.e. $(a_1, b_1) = \lambda(a_2, b_2)$, some $\lambda \neq 0 \in \mathbb{R}$

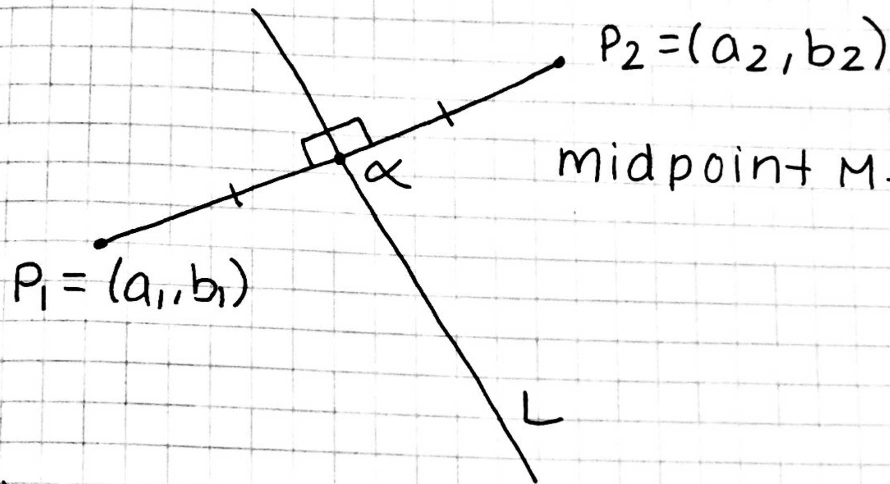
Linear Algebra (Math 235)

⊛ $\iff$ $\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ is not invertible

$\iff$ The equations don't have a solution for $c_1, c_2$ in general

In fact, will have solution only when $L_1 = L_2$ (equations ① & ② are multiples of each other)

Ex. Equation of perpendicular bisector?




midpoint $M = \frac{1}{2}(a_1 + a_2, b_1 + b_2)$

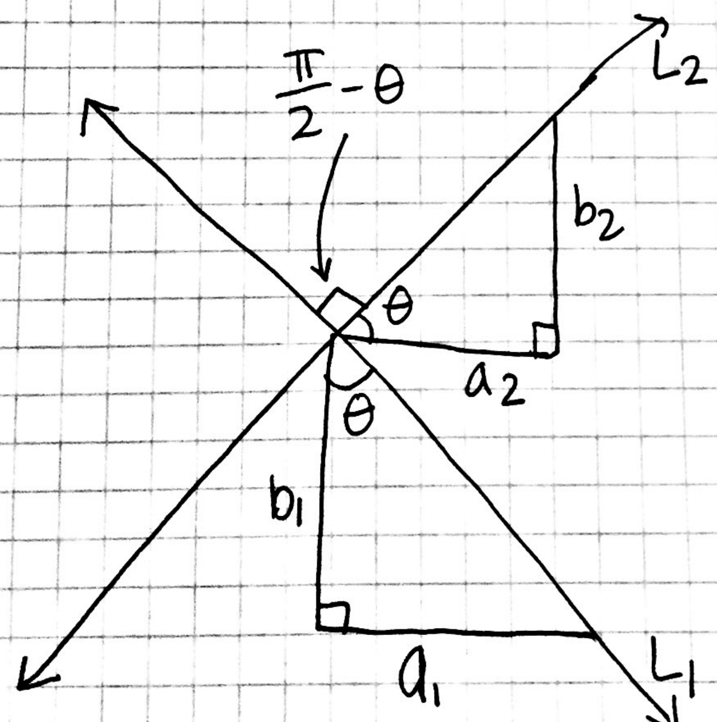
Useful facts:

$L_1$ & $L_2$ perpendicular <=> slopes $m_1, m_2$ satisfy $m_1 m_2 = -1$

$m_1 = -\frac{b_1}{a_1}$ $\quad m_2 = \frac{b_2}{a_2}$

If $L_1$ & $L_2$ are perpendicular by similar triangles $\frac{b_2}{a_2} = \frac{a_1}{b_1}$, $b_1 b_2 = a_1 a_2$

$m_1 m_2 = \frac{-b_1 b_2}{a_1 a_2} = -1$ ✓




this gives => direction of proof