

Math 461 Midterm, Wednesday 10/16/19, 7:00PM–9:00PM, LGRC A301.

Instructions:

- Exam time is 2 hours.
- There are 6 questions for a total of 60 points.
- You are allowed one sheet of notes (letter size, both sides).
- Calculators, phones, other electronic devices, additional notes, and textbooks are *not* allowed.
- Justify all your answers carefully. Complete proofs are expected as in MATH 300. Results proved in class, the textbook, or the homework may be stated without proof.

Q1 (10 points). Let $ABCD$ be a convex quadrilateral with vertices labelled in counterclockwise order. Suppose that $|AB| = |BC|$ and $|CD| = |DA|$. Let E be the intersection point of the diagonals AC and BD . Prove that $|AE| = |EC|$ and $\angle AEB = \pi/2$. (That is, the diagonal BD bisects the diagonal AC and the diagonals meet at right angles.)

Q2 (10 points). Let $ABCD$ be a quadrilateral with vertices labelled in counterclockwise order such that AB is parallel to DC . Let E be the intersection point of the diagonals AC and BD . Prove that $|AE| \cdot |DE| = |BE| \cdot |CE|$.

Q3 (10 points). Let $\triangle ABC$ be a triangle and \mathcal{C} the unique circle passing through A , B , and C . Let L be the bisector of the angle $\angle ACB$ and D the other intersection point of L and \mathcal{C} (besides C). Prove that $|AD| = |BD|$.

Q4 (10 points). Suppose given a line segment AB . Describe a ruler and compass construction of a triangle $\triangle ABC$ with vertices A , B and a third point C such that $\angle ABC = \pi/4$, $\angle BCA = \pi/2$, and $\angle CAB = \pi/4$. Prove that your construction is correct.

[You may use ruler and compass constructions from class or the textbook as components of your constructions.]

Q5 (10 points). Suppose given a triangle $\triangle ABC$. Describe a ruler and compass construction of a triangle $\triangle ABD$ such that $\text{Area}(\triangle ABD) = \text{Area}(\triangle ABC)$ and $\angle ABD = \angle BAD$. Prove that your construction is correct.

[You may use ruler and compass constructions from class or the textbook as components of your constructions.]

Q6 (10 points). Let $ABCDE$ be a regular pentagon with vertices labelled in counterclockwise order.

- (a) (3 points). What are the interior angles of the pentagon $ABCDE$?
- (b) (7 points). Let F be the intersection point of AC and BE . Prove that $\triangle ACD$ is similar to $\triangle EAF$.