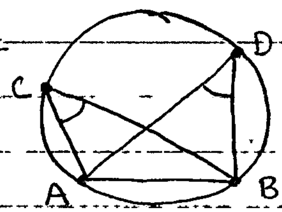
9/27/19

Last Time: • Converse of Thales' theorem
• Impossibility of trisection of angles – sketch of proof

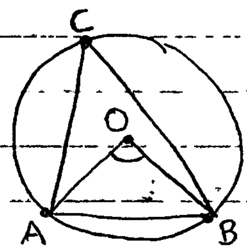
Today: • Angles in a circle
• Construction of square root
(The regular pentagon is constructible) – didn't get to it

Theorem 1



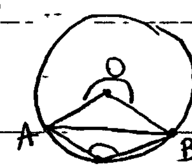
• $\angle ACB = \angle ADB$

Theorem 2



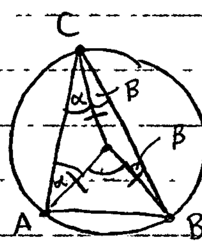
• O = center of circle
• $\angle AOB = 2\angle ACB$

Aside:



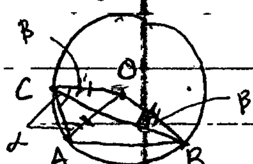
$\angle AOB = 2\angle ACB$

Proof (of Thm 2)



• Draw OC
• Note $|OA| = |OB| = |OC|$ = radius of circle
• $\Delta AOC, \Delta BOC, \Delta AOB$ isosceles
$\angle ACB = \alpha + \beta$, $\angle AOC = \pi - 2\alpha$, $\angle BOC = \pi - 2\beta$
• $\angle AOB = 2\pi - (\pi - 2\alpha) - (\pi - 2\beta)$
$= 2\pi - \pi + 2\alpha - \pi + 2\beta$
$= 2\pi - 2\pi + 2\alpha + 2\beta$
$= 2\alpha + 2\beta = 2(\alpha + \beta)$
• Thus $\angle AOB = 2\angle ACB$

To convince ourselves:

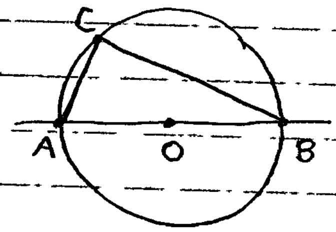


$\angle ACB = \alpha - \beta$
$\angle AOB = (\pi - 2\beta) - (\pi - 2\alpha)$
$= 2\alpha - 2\beta$
$= 2(\alpha - \beta)$

Theorem 2 ⇒ Theorem 1

• O = center of circle, draw AO, BO
• $\angle ACB \underset{(2)}{=} \frac{1}{2}\angle AOB \underset{(2)}{=} \angle ADB$ ▨

Theorem 3

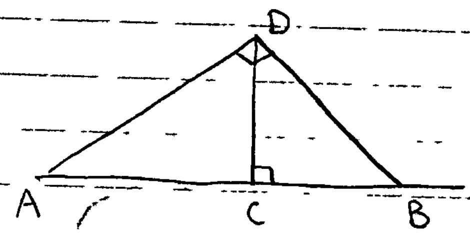


- circle center O
- $\angle ACB = \pi/2$

Proof (Thm 3) $\angle ACB = \frac{1}{2}\angle AOB = \pi/2$. ▨
(by Thm. 2)

Application

Construction of square root : Given length a, construct (w/ ruler + compass) $\sqrt{a}$.



Q: Similar triangles?

$\triangle ADC \sim \triangle ABD$.

by equal corresponding angles

$\angle DAB = \angle DAC$ ✓
$\angle ADB = \angle ACD = \pi/2$
$\angle ABD = \angle ADC$ by angle sum of $\triangle$.

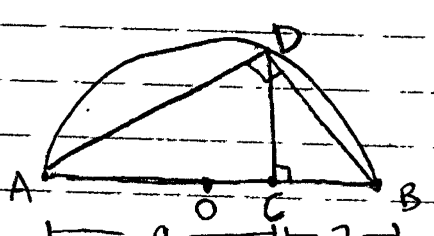
and $\triangle ADC \sim \triangle ABD \sim \triangle DBC$ by same argument.

$\triangle ADC \sim \triangle DBC$

$\Rightarrow \frac{|AC|}{|DC|} = \frac{|DC|}{|BC|} \Rightarrow |AC| \cdot |BC| = |DC|^2$.

in other words $|DC| = \sqrt{|AB||BC|}$

Construction of square root of a

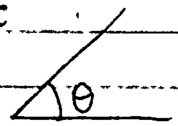


Ruler & compass construction

1. Draw length $a+1$ with $|AC| = a$ and $|CB| = 1$.
2. Bisect AB + let O be the midpoint.
3. Draw circle center O, radius $|OA| = |OB|$.
4. Construct perp. line to AB crossing thru C, intersecting circle at D.
5. Draw $\triangle ADB$. By Thm 3, $\angle ADB = \pi/2$, & $|DC| = \sqrt{a}$. ▨

Recap: We can perform $+, -, \times, \div$ & $\sqrt{\ }$ using ruler & compass
(Later, we'll show this is everything, using coordinates)

(**Aside**:



approximately trisect?

can bisect an angle

$\frac{1}{3} = \frac{1}{4} + \frac{1}{16} + \cdots$ e.g. $\frac{1}{3} \approx \frac{5}{16}$

$= 0.0101\ldots$ (base 2))