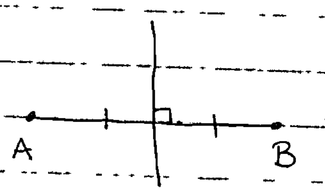
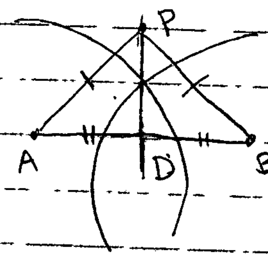
9/11/19

Perpendicular Bisector



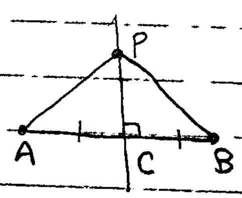
Claim: For any point P in the plane,
$|AP| = |BP| \iff P$ lies on the perpendicular bisector of the line segment AB



Proof: ($\Rightarrow$) Assume $|AP| = |BP|$, required to show: P lies on perp. bisector of AB

Draw PD where D is the midpoint of AB.
$\triangle ADP \cong \triangle BDP$ by SSS
$\Rightarrow \angle ADP = \angle BDP \Rightarrow \angle ADP = \angle BDP = \pi/2$
$\Rightarrow PD$ is perp. bisector of AB i.e. P lies on perp. bisector of AB.

($\Leftarrow$)

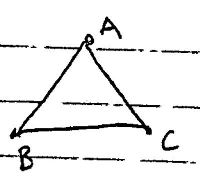


Assume P lies on perp. bisector, required to show: $|AP| = |BP|$

$\triangle ACP \cong \triangle BCP$ by SAS $\Rightarrow |AP| = |BP|$ ∎

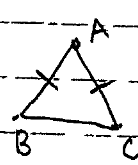
Isosceles triangle theorem

Given $\triangle ABC$
$|AB| = |AC| \iff \angle ABC = \angle ACB$

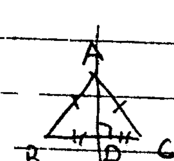


Definition: In this case, we say $\triangle ABC$ is isosceles.

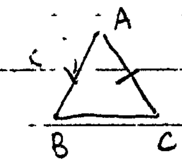
Proof: ($\Rightarrow$)



Required to prove $\angle ABC = \angle ACB$.
By previous claim; $|AB| = |AC|$
$\Rightarrow A$ lies on perp. bisector of BC



$\triangle ABD \cong \triangle ACD \Rightarrow \angle ABD = \angle ACD$.

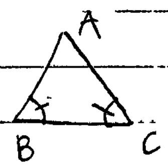


by Pathos (Alternate Proof)

$\triangle ABC \cong \triangle ACB$ by SSS.

$$\left.\begin{array}{l} |AB| = |AC| \\ |BC| = |BC| \\ |AC| = |AB| \end{array}\right\} \Rightarrow \angle ABC = \angle ACB$$

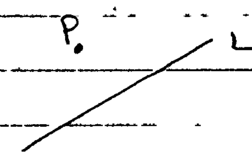
($\Leftarrow$)



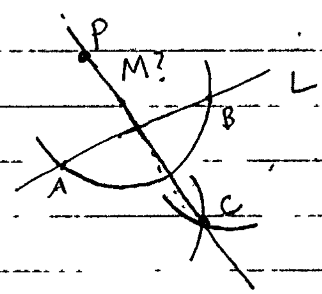
Required to prove $|AB| = |AC|$

$\triangle ABC \cong \triangle ACB$ (two angles and side between them) by ASA

$\Rightarrow |AB| = |AC|$ ∎



Construction: Given point P and a line L, construct a line M through P perp. to L. (using ruler & compass)



1. Draw a circle center P, intersecting L in two points A & B
2. Draw circles center A & B of same radius r intersecting at C on the opposite side of L to P
3. The line M is the line PC

Why does this work?

If you take same radius in steps 1. and 2., see that P and C are the two intersection points of the circles in 2., so 2. and 3. is the usual construction of perp. bisector of AB. So $M \perp L$ ("M is perp. to L")

Proof: $|AP| = |BP|$ & $|AC| = |BC| \Rightarrow$ P & C lie on perp. bisector of AB

$\Rightarrow$ PC is the perp. bisector of AB $\Rightarrow M = PC \perp L = AB$ ∎