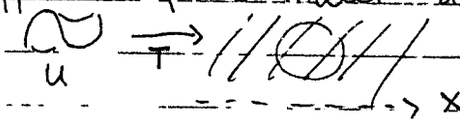
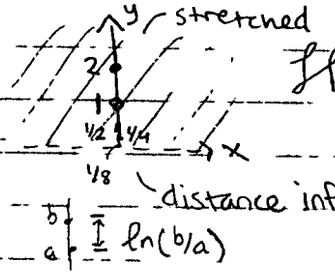
* Upper half plane model: a "map" of the hyperbolic plane



1. doesn't preserve distances
2. does preserve angles



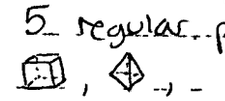
Euclid's Elements = 13 volumes ~300 BC 9/6/19

Plane geometry, Number theory, 3-D geometry (e.g. Platonic solids)

* Model of deductive reasoning
  i.e. start w/
  - definitions
  - assumptions (axioms)

  and deduce other properties by logical deduction

5 regular polyhedrons




octahedron (8 triangular faces)

Why do we study Euclidean geometry?

1. It's useful! x2 $ for
2. It's an excellent foundation for other types of geometry (3-D, Spherical, hyperbolic)
3. Practice logical reasoning / proofs
4. To help (some of you) prepare to teach geometry in high school

Ruler & Compass constructions

Q: What geometric figures can be constructed by ruler & compass?

Euclid's construction axioms:

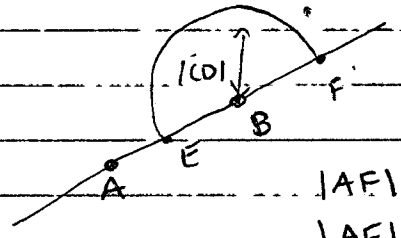
It is possible to

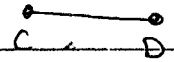
1. Draw a straight line between any 2 points (and extend it indefinitely)
2. Draw a circle with center a point & given radius

Notes: Ruler can't be used for measuring ("straight-edge")

Compass can be used to copy a length from one location to another

Ex 1: Given two line segments AB, CD of lengths $|AB|, |CD|$, construct line segments of lengths $|AB|+|CD|$ and $|AB|-|CD|$ (assuming $|AB| > |CD|$)





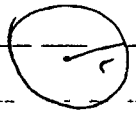
measure w/ compass

$|AF| = |AB| + |CD|$

$|AE| = |AB| - |CD|$

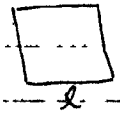
Q1: Suppose we're given a line segment of length 1. What other lengths can be constructed? (use coordinate geometry)

ex: Given a circle, is it possible to construct a square w/ the same area as the circle?



$r=1$

$\text{Area} = \pi r^2 = \pi$



$\text{Area} = \ell^2 = \pi$

$\ell = \sqrt{\pi}$

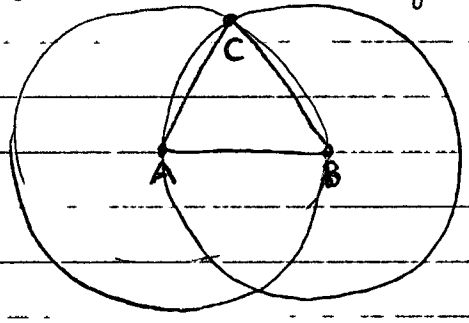
↳ can't construct

$\sqrt{2} = 1.41\ldots$ "irrational" $x^2 - 2 = 0$

$3.1415926\ldots$ "transcendental" $\not\exists$ poly. $p(x)$ w/ integer coeff. w/ $p(\pi) = 0$

Ex 2: Construct an equilateral triangle (a $\triangle ABC$ such that $|AB| = |BC| = |CA|$)

- assume given base AB of triangle



$|AB| = |AC|$ & $|AB| = |BC|$

$\Rightarrow |AB| = |BC| = |AC|$

$\triangle ABC$ is equilateral ▨

Q2: Regular n-sided polygon is a polygon w/ n sides s.t. all sides have same length & all angles are equal ($n \in \mathbb{N}, n \geq 3$). For which n can the regular n-gon be constructed?

↳  or 

$n=3$ ✓	$n=8$ ✓		
$n=4$ ✓	$n=9$ ✗	$n=13$ ✗	$n=17$ ✓
$n=5$ ✓	$n=10$ ✓	$n=14$ ✗	⋮
$n=6$ ✓	$n=11$ ✗	$n=15$ ✓	
$n=7$ ✗	$n=12$ ✓	$n=16$ ✓	Gauss 1796