Math 461 Lecture 6 9/17

Homework 1 due wednesday at start of class

Office Hours: today 2:30-3:30
tomorrow 4:00-5:00

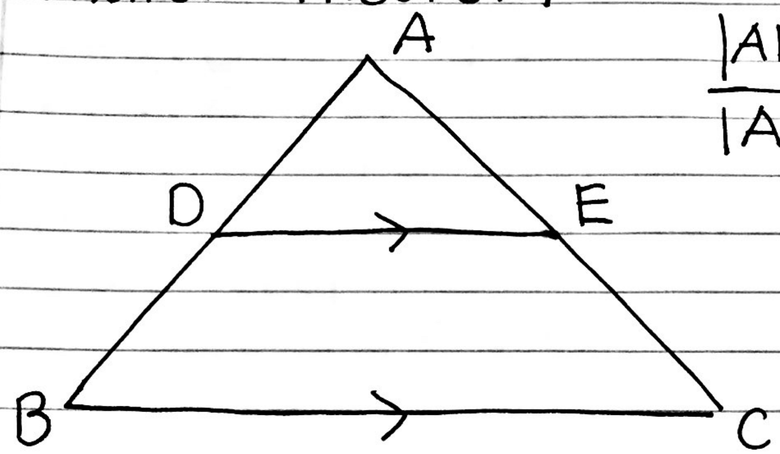
Office: LGRT 1235H

Last time:

area

Pythagoras' Theorem

Thales' Theorem



$$\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|}$$

Today:

finish proof of Thales' Theorem

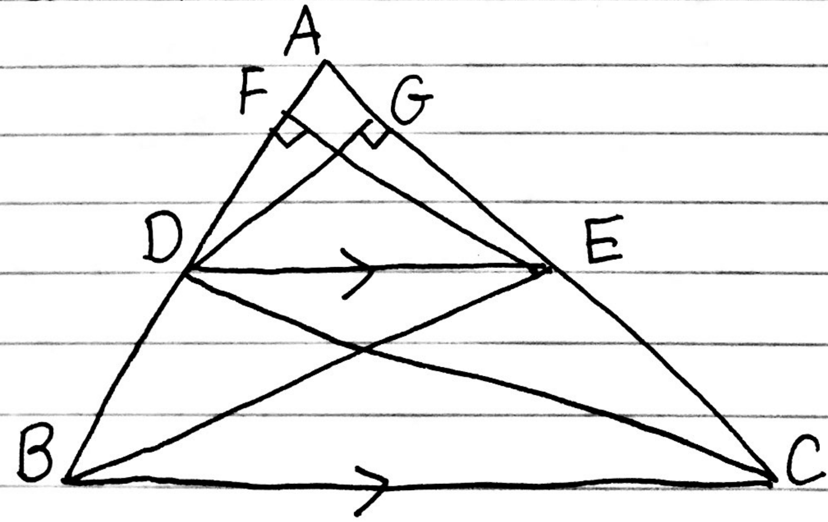
similar triangles

multiplication and division

converse of Thales' Theorem, parallel

Pappus and Desargues Theorems

Last time:



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Area($\triangle DBE$) = Area($\triangle DCE$)

Area($\triangle ABE$) = Area($\triangle ADC$)

Goal: $\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|}$

Area($\triangle ADE$) $= \frac{1}{2}|AD| \cdot |EF|$

Area($\triangle ABE$) $= \frac{1}{2}|AB| \cdot |EF|$

$\div$ two equations $\Rightarrow$

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABE)} = \frac{|AD|}{|AB|}$$

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABE)} = \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ADC)} =$$

$$\frac{\frac{1}{2}|AE| \cdot |DG|}{\frac{1}{2}|AC| \cdot |DG|} = \frac{|AE|}{|AC|}$$

$$\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|} \quad \square$$

Similar Triangles:

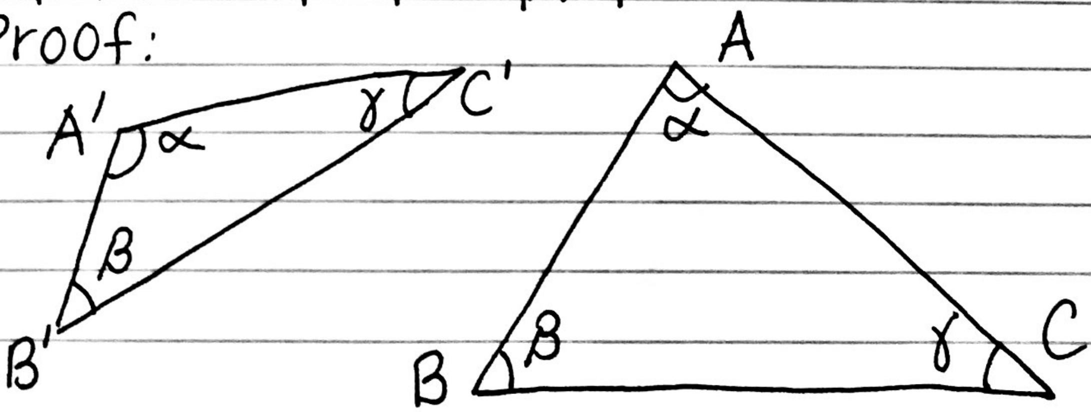
We say two triangles $\triangle ABC$, $\triangle A'B'C'$ are similar if the corresponding angles are equal

Write $\triangle ABC \sim \triangle A'B'C'$

Theorem: If $\triangle ABC \sim \triangle A'B'C'$ then

$$\frac{|A'B'|}{|AB|} = \frac{|B'C'|}{|BC|} = \frac{|A'C'|}{|AC|}$$

Proof:



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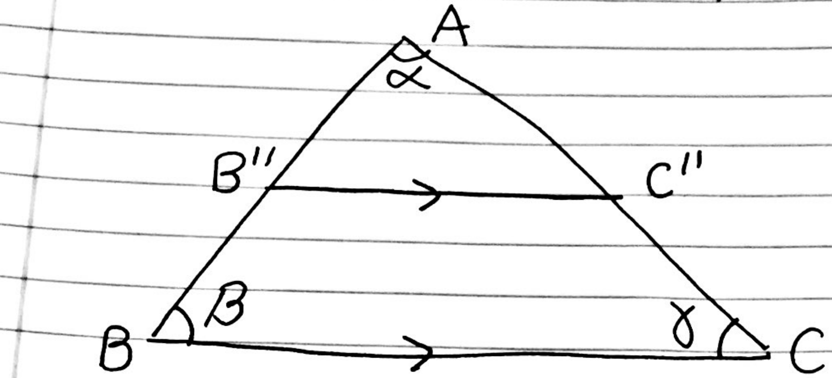


$|AB''| = |A'B'|$

Claim: $\triangle AB''C'' \cong \triangle A'B'C'$

$|AB''| = |A'B'|$

$\angle B'A'C' = \angle B''AC''$

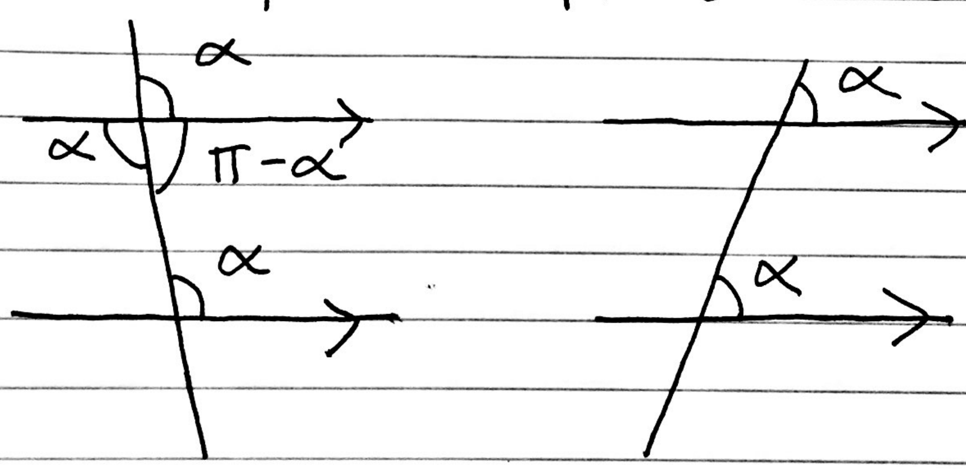
$\angle A'B'C' = \angle AB''C''$

$\triangle AB''C'' \cong \triangle A'B'C'$ by ASA

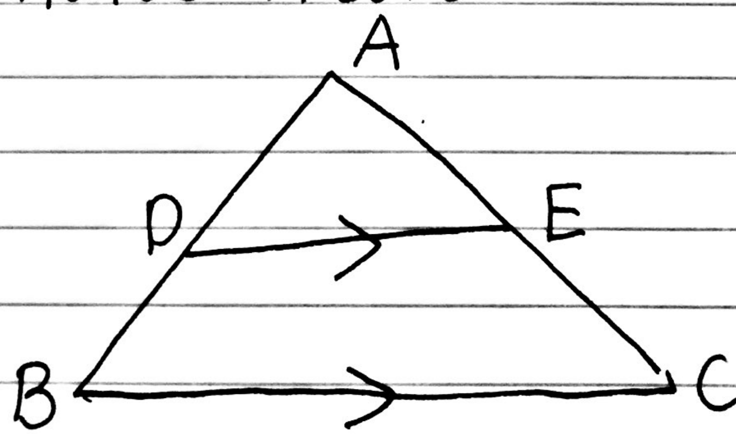
$\frac{|AB''|}{|AB|} = \frac{|AC''|}{|AC|}$ by claim

$\frac{|A'B'|}{|AB|} = \frac{|A'C'|}{|AC|}$ $\square$

other equalities proved in some way



Thales Theorem



$$\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|}$$

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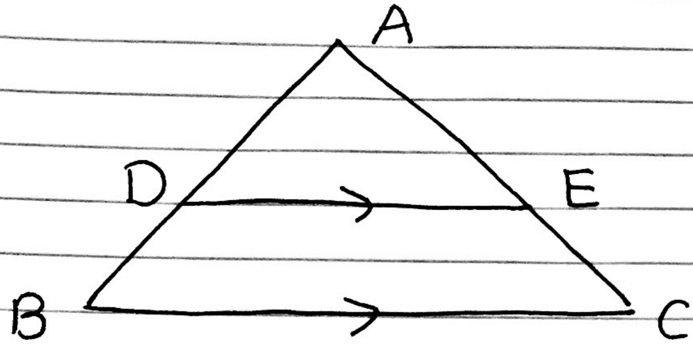
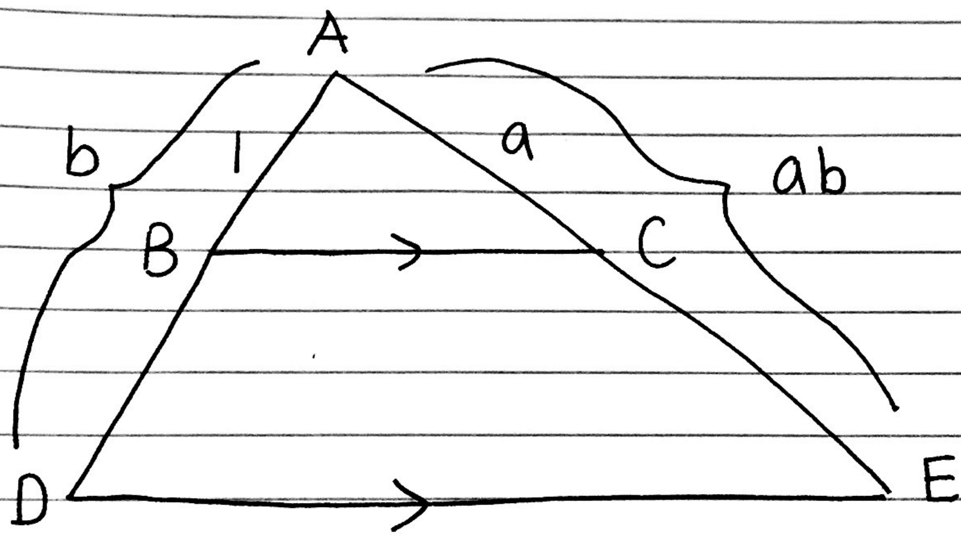
Multiplication and Division:
given lengths $1, a, b$, construct lengths ab, $\frac{a}{b}$

Thales Theorem



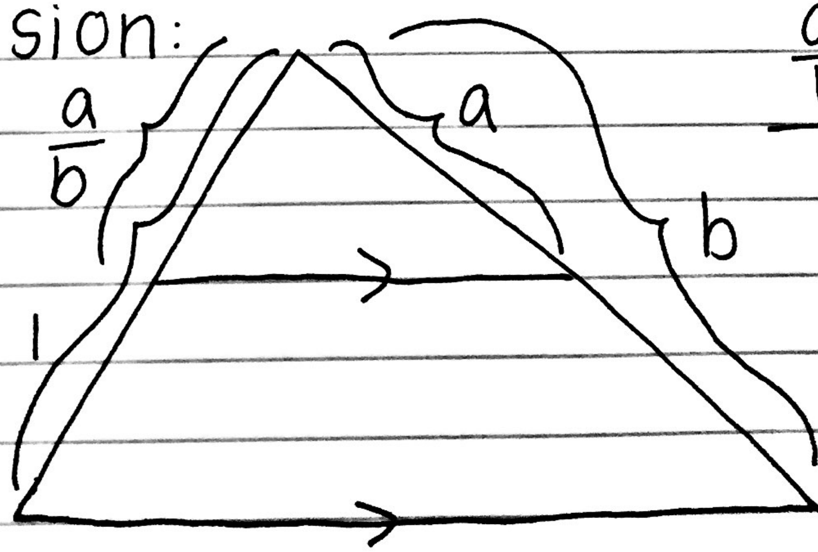
$$\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|}$$



Thales': $\frac{|AB|}{|AD|} = \frac{|AC|}{|AE|}$

$$|AE| = \frac{|AC| \cdot |AD|}{|AB|} = \frac{a \cdot b}{1}$$

Division:



$$\frac{\frac{a}{b}}{1} = \frac{a}{b}$$

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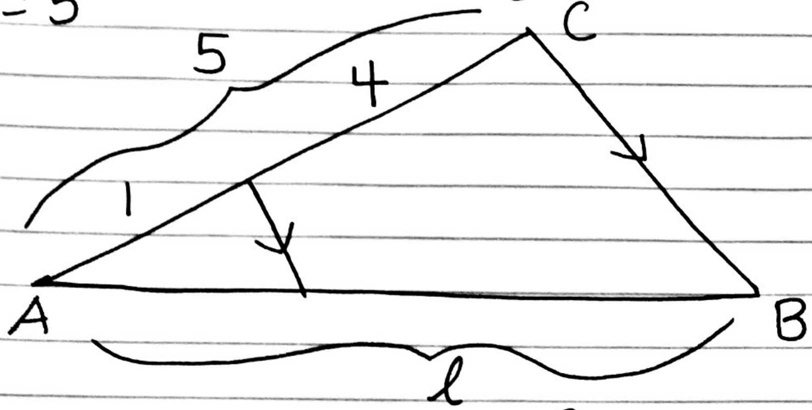
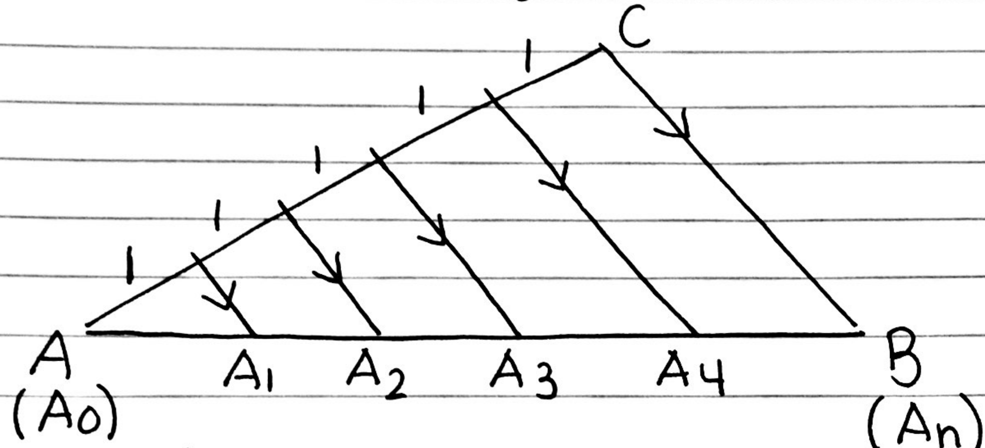
Earlier, bisected a line segment.
Can now divide a line segment into n equal parts (n any positive integer)

$n=5$

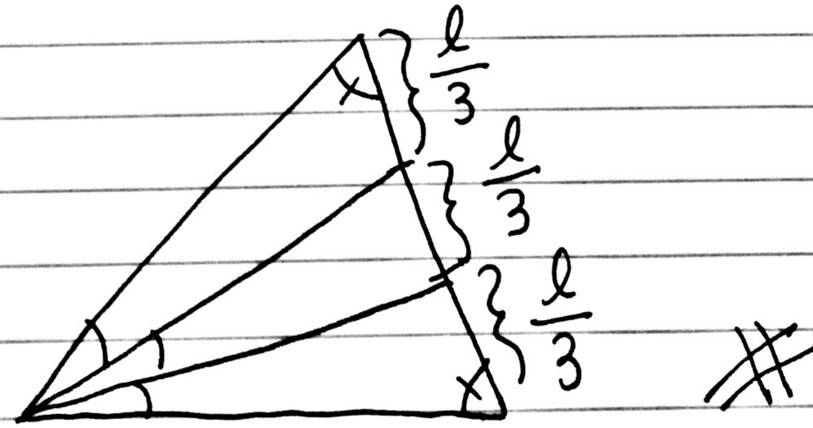




$$\frac{|A_0A_k|}{|A_0A_n|} = \frac{k}{n} \Rightarrow \frac{|A_kA_{k+1}|}{|AB|} = \frac{1}{n} \text{ for each } k$$

On the other hand, it is impossible to divide an angle into n equal parts for some values of n (e.g. $n=3$)

What's wrong with this construction?



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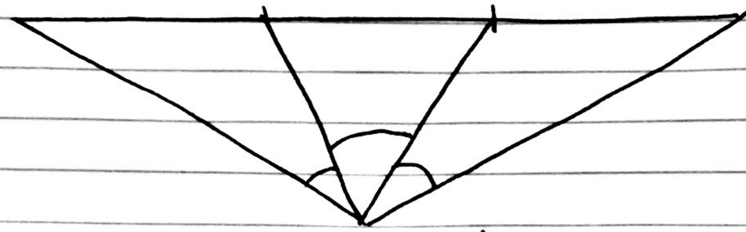
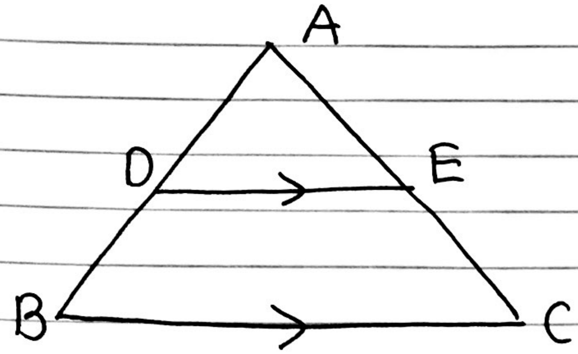
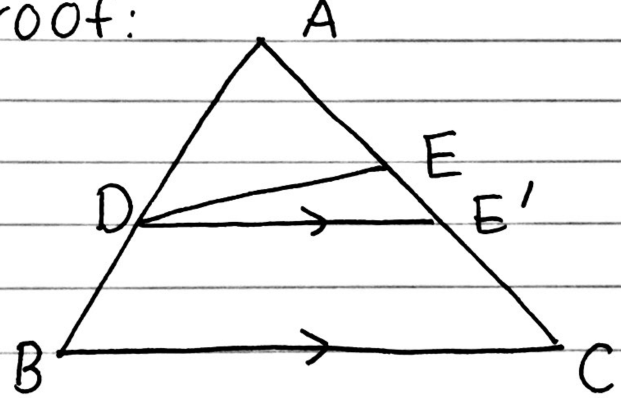


Converse of Thales' Theorem



$$\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|} \Rightarrow DE \text{ and } BC \text{ are parallel}$$

Proof:



Assume $\frac{|AD|}{|AB|} = \frac{|AE|}{|AC|}$, want to show $DE \parallel BC$

Draw line through D parallel to BC

Thales: $\frac{|AD|}{|AB|} = \frac{|AE'|}{|AC|}$

get $|AE| = |AE'| = \frac{|AD||AC|}{|AB|}$

so $E = E'$, $DE = DE' \parallel BC$ $\square$