Math 300.2 Homework 9

Paul Hacking

November 27, 2017

Reading: Sundstrom, Sections 7.1, 7.2, 7.3, and 7.4.

Justify your answers carefully.

(1) Let S be a set and R a relation on S. What does it mean to say that R is an equivalence relation? In each of the following cases, determine whether R is an equivalence relation.

(a) $S = \mathbb{R}$, $aRb \iff a \leqslant b$.

(b) $S = \mathbb{R}$, $aRb \iff b = a + \pi n$ for some $n \in \mathbb{Z}$.

(c) $S = \mathbb{N}$, $aRb \iff a \mid b$.

(d) $S = \mathbb{Z}$, $aRb \iff 7 \mid a - b$.

(e) $S = \mathbb{R}$, $aRb \iff |a - b| \leqslant 2$.

(f) $S = \mathbb{R}^2$, $(a_1, a_2)R(b_1, b_2) \iff \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2} < 5$.

(g) $S = \mathbb{Q}$, $aRb \iff b = 3^n a$ for some $n \in \mathbb{Z}$.

(h) $S = \mathbb{N}$, $aRb \iff ab = n^2$ for some $n \in \mathbb{N}$.

(i) $S = \mathbb{Z}^2$, $(a_1, a_2)R(b_1, b_2) \iff a_1 b_2 = a_2 b_1$.

(2) Let $S = \{1, 2, 3, 4, 5, 6\}$ and let R be the relation on S defined by the following table. (In row a and column b of the table we write Y if aRb and N otherwise.) Determine whether R is an equivalence relation and if so list the equivalence classes.

R	1	2	3	4	5	6
1	Y	N	N	N	Y	N
2	N	Y	Y	N	N	Y
3	N	Y	Y	N	N	Y
4	N	N	N	Y	N	N
5	Y	N	N	N	Y	N
6	N	Y	Y	N	N	Y

(3) Let $S = \mathbb{R}^2$ and let R be the relation on S defined by

$$(x_1, y_1)R(x_2, y_2) \iff x_1^2 + y_1^2 = x_2^2 + y_2^2.$$

(a) Show that R is an equivalence relation.

(b) Draw a picture showing the equivalence classes of R in $\mathbb{R}^2$.

(4) Let $S = \mathbb{R}^2 \backslash \{(0, 0)\}$ and R be the relation on S defined by

$$(x_1, y_1)R(x_2, y_2) \iff (x_2, y_2) = \lambda(x_1, y_1) \text{ for some positive real number } \lambda.$$

(a) Show that R is an equivalence relation.

(b) Draw a picture showing the equivalence classes of R in the plane $\mathbb{R}^2$.

(c) Let $C = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\} \subset \mathbb{R}^2$ be the circle with center the origin and radius 1. Let f be the function

$$f : C \to S/R$$

from the circle C to the set S/R of equivalence classes of R given by $f(x, y) = [(x, y)]$ (that is, $f(x, y)$ is the equivalence class of (x, y)). Show that f is a bijection.