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NUCLEAR PASTA

THE SECRET INGREDIENT TO NEUTRON STARS

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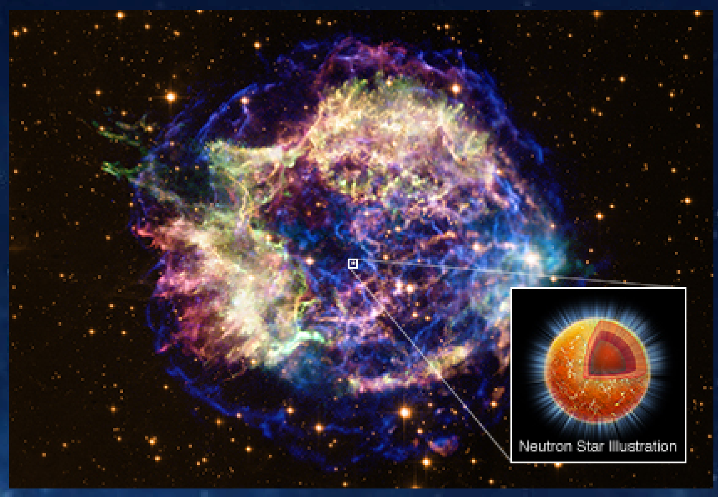
How is Nuclear Pasta formed?

We begin with a star. All stars have a certain life span as they burn the fuel in their core. Towards the end of their lives, they will begin a process called iron fusion. When iron is fused, it absorbs energy. Within hours of this process beginning, the star will die. Since there is no more energy to absorb, the star will begin to collapse. This happens so quickly that the entire star (which was once massive) collapses in 0.25 seconds. The stars are like earth in that they have several layers. The outer layers (crust) cannot collapse as quickly as the core, despite collapsing at supersonic velocities, therefore when the core reaches a critical density, the outer layers collide with the collapsed center and bounce outwards again. This is observed as a large explosion. This explosion after the sudden implosion is called a supernova. [1]



X-ray: NASA/CXC/UNAM/Ioffe/D.Page, P. Shternin et al; Optical: NASA/STScI; Illustration: NASA/CXC/M. Weiss

If the star is a small star (less than 8 solar masses), this is were the process finishes and a white dwarf is formed. If the star is a bigger main-sequence star, (between 8 and 20 solar masses), the pressure of the collapse is so strong that electrons that would otherwise repel each other occupy the same space. This is called surpassing electron degeneracy. During this process, protons are forced to absorb electrons which combine to form neutrons. As the neutrons become more compact and the supernova happens, a neutron star is formed from the degenerate matter left behind. If the star were more than 20 solar masses and the pressure forced the neutrons to combine under pressure, neutron degeneracy pressure would be surpassed and a black hole would form. [2]
The main focus is that of neutron formation when neutron stars are created. The process allows 10^{55} nuclei to combine into one massive nucleus. because of the extreme temperatures of the cores of stars (10^{12} K), the nuclei melt together. [2] Typically, the nucleus of an atom is spherical. But because of the high pressure and other forces coming into play in the formation of a neutron star (electrical forces and surface tension) the nucleus may warp and deform out of its typical spherical shape. If the surface tension of the nuclear matter is larger than any of the electrical repulsion of the protons (i.e. the force of surface tension forces the protons to be pushed together against the electrical forces) the nucleus changes shape. At such high densities and high temperatures, the nuclei form unconventional shapes that resemble various pasta types. Peculiar patterns are formed by the nuclei. The nucleus is in a liquid phase mixture of protons and neutrons and the gaps are bubbles of a gas phase.
Some round shapes are called nuclear gnocchi, the long rods are nuclear spaghetti and the slabs are nuclear lasagna along with each of their counterparts anti gnocchi, anti spaghetti and anti lasagna. Each shape is favourable depending on the energies interacting in the star. Lower densities favour nuclear gnocchi and spaghetti and higher densities favour anti-gnocchi and anti-spaghetti. Whether the nuclear pasta is present or not and which shape it's in depends on complex processes of the formation of nuclear pasta. [4] More of this is described in Nuclear Phases.

Propertites - Density and Shear Strength

If it exists, nuclear pasta would be the strongest material in the universe - roughly 10 billion times stronger than steel [5]. It is formed under incredibly high densities, typically in the range of 10^{14} to 10^{17} g/cm³ - many orders of magnitude denser than any terrestrial matter. While the pasta layer of the neutron star crust may be only 100-250 metres thick, due to its high density it is expected to contain at least half of the total crust mass [6]. This is a result of the immense gravitational pressures of up to 10^{34} Pa in the neutron star crust.

The high density and close packing of atomic nuclei results in strong interactions between them, arising from the conflict of the strong nuclear force holding protons and neutrons together, and the electromagnetic Coulomb force causing like charges to repel each other. The ability of the strong nuclear force to overcome the Coulomb repulsion to some extent and hold the protons and neutrons together to maintain structural integrity gives rise to a very high shear strength - resistance to force acting coplanar with a given cross-section of the material - in nuclear pasta. Magnitude of shear strength varies through descent into the different phases of nuclear pasta, and is predicted to be such that even short-range disorders such as helicoidal defects - filaments connecting the plates that make up the pasta - may support shear stresses between plates [5].

Elastic Properties

Nuclear pasta is tough. It can absorb energy and deform plastically without immediate fracture when neutron stars undergo various dynamic processes, such as pulsar glitches or magnetic field reconfigurations, which subject the inner crust to rapid and intense energy fluctuations. The deformation of atomic nuclei into pasta shapes is necessary in an attempt to balance the competing forces, but once shapes are formed, they maintain their structural integrity over long periods of time and resist permanent structural changes. Pasta can deform on length scales of the order of 10 times its spacing [7].

The ion crust forms a polycrystalline bcc lattice, and has a high breaking strain $\epsilon \approx 0.1$ [8]. The pasta is less well understood. Quantum mechanical simulations suggest that pasta, like the ion crust, does not have a uniform orientation across the star but is composed of many microscopic domains with distinct orientations, so we should consider liquid 'polycrystalline' nuclear pasta with many different domains [5]. The energy of deformation of lasagna phase pasta plates is considered to be
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$$E_d = \frac{B}{2}\left(\frac{\partial u}{\partial z} - \frac{1}{2}(\nabla_\perp u)^2\right)^2 + \frac{K_1}{2}(\nabla^2{}_\perp u)^2.$$

One simulation straining x-y plane 'lasagna' plates apart in the z direction showed that at $\epsilon(z) > 0.05$, plates started to buckle to maintain their initial spacing. Tensile stress along z-direction remains constant from then on, while stress decreases as plates buckle. Deformation continued without breaking all the way to strain $\epsilon(z) = 1$, and no significant shear stresses were observed - unsurprising given the high shear strength discussed [5]. The same study produced large breaking strain for the pasta greater than 0.1, and large shear modulus value of $\mu = 1.1 \times 10$ erg/cm³ [9].

Neutrino Opacity

The presence of nuclear pasta can have implications for the opacity or impenetrability of neutron star matter to neutrinos, and the cooling of neutron stars. The pasta structures can affect the ability of neutrinos to escape from the star by significantly altering their mean free paths. Since neutron star cooling is due mostly to neutrino emission from the core, the interaction between neutrinos and nuclear pasta is relevant in the cooling and evolution of neutron stars. The phases of nuclear pasta structures create low-density regions within their layered shapes, allowing neutrinos, and heat, a way out of the neutron star's core [10]. Furthermore, nuclear pasta can compress, absorbing momentum in a rippling motion. It is thought that neutrino emissions from nuclear pasta are vastly more efficient than neutrino emissions at a neutron star's core, meaning pasta is likely responsible for much of the cooling. This means that neutron stars cool more slowly than previously, and so live longer.

The neutron-neutron static structure factor of nuclear pasta describes coherent neutrino scattering [11]. This is a mathematical description of how the neutrons in the pasta scatters incident radiation, and is the cause of the neutrino opacity for certain wavelengths. At high densities, where very big clusters are expected (spaghetti and lasagna), the wavelength stays relatively constant and maximum opacity is obtained for neutrinos of higher wavelengths, with high energies of roughly $E_\nu \approx 80$ MeV. As density goes down, we move into the gnocchi pasta phase, in which clusters are of finite size - hence the name. In this case, maximum opacity moves to lower energies (so a ranger of longer wavelengths in which the structure is considerably opaque) [12]. This relation - that peak opacity wavelength decreases as density increases and correlation length of the structure is lower as the density increases - is to be expected, as the higher the density, the closer the structures are.

Phases of Nuclear Pasta

In real life we have different type of pasta depending on their shape. We have spaghettis that are thin and long, penne, which are round and have a hole through the middle, and so on. Other forms such as a cross-hatched pattern called nuclear waffles can also be produced. As pointed out by Ravenhall and Hashimoto, the nuclei are stable due to the ongoing competition between nuclear surface energies and coulomb energies.[4] This nuclear pasta is nuclear matter that forms crystal-like structures at low temperatures ($T \lesssim 1$ MeV = 12 K). It also depends on the density that this matter has, it needs to be really low. We can see how this depends on the temperature for a given low density in Figure 1. We see two of the different shapes that are created for this pasta, the lasagna (sheets) and the jungle gym, which is a random really messy distribution of the particles. This represented in the graph are the radial distribution of these nuclear pasta shapes, which can be found by computing the following integral [15],

$$g(r) = \frac{1}{4\pi r^2}\int \frac{\rho_s}{\rho_0}\left(1 - \frac{r|\cos\theta|}{2z_0}\right) r^2 \sin\theta d\theta d\varphi$$

Here ρ_s is the slab density, ρ_0 = N/V is the mean density and z_0 is a constraint. This constraint is made because we consider that ideally the pasta behaves like a lasagna, which can be seen at the lowest temperatures in Figure 1.

The shape that the nuclear matter takes is what gives it its name. However, mathematically it is not correct to say "the ones that are long and thin", and so some specific parameters were created to give it a name. These parameters that measure it are the Minkowski functionals. There are several of them, but in our journey to classify pasta we only need to worry about two, Euler's number χ and the mean breadth B. The mean breadth quantifies how thin or thick the object is; it's similar to measuring how much light gets blocked if we were to shine a light at the object. We consider the mean. From [15] χ on the other hand is defined as:

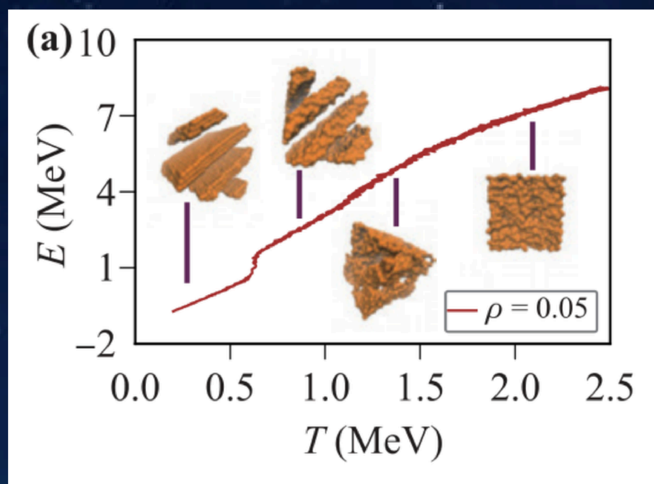

Figure 1: Obtained from [15]

X = isolated regions + cavities - tunnels

We will use this one to see if there are more isolated regions, i.e. solids; or it's just a big mass full of tunnels, or in between. Hence, with this information we can classify them using χ and B in Table 1. This information can also be seen in Figure 2. In Figure 2 we can see that as χ gets negative, we start getting structures with a lot of tunnels, like the jungle gym and the anti-jungle gym. When χ is bigger than 0 we get solid structures, the gnocchi, and finally when χ 0 we get this lasagna structure for really low temperatures, and some anti-spaghetti for higher temperatures. A sketch of the different pastas can be seen in Figure 3.

	$B < 0$	$B \sim 0$	$B > 0$
$\chi > 0$	anti-gnocchi		gnocchi
$\chi \sim 0$	anti-spaghetti	lasagna	spaghetti
$\chi < 0$	anti-jungle gym		jungle gym

Table 1: Obtained from [15].

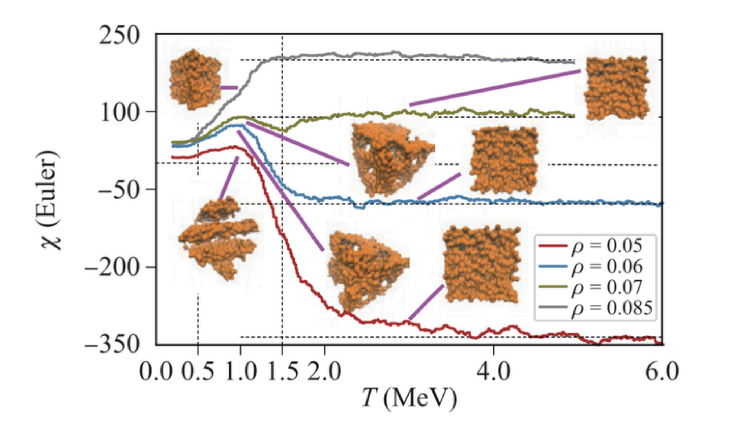

Figure 2: Obtained from [15].

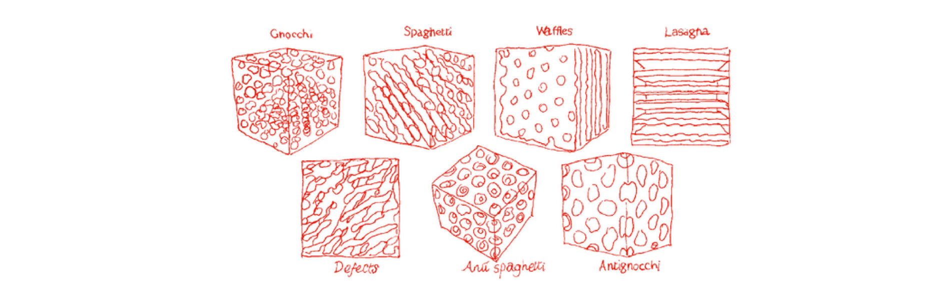

Figure 3: Obtained from https://spark.iop.org/sites/default/files/media/images/147_stories_from_physics_astronomy_and_space-25.png

How do we know it exists?

We know that gravity is trying to collapse the white dwarf (WD) while the degeneracy pressure of electrons acts against gravity. Iron nuclie capture electrons over time via fusion and this eventually causes the WD to collapse. As WDs collapse the density increases to exceeds 1011 g cm−3. The density reaches nuclear density thus the strong force causes the structure to stiffen which stops the collapse. The structure formed is what we call nuclear pasta. It doesn't last long, only lasting for O(10−100) msec until the bouncing of the core melts the pasta. The pasta phases have an influence on the glitch phenomena of pulsars. A pulsar glitch is a sharp decrease in the pulse period due to a sudden spin up of the neutron star's crust [4].

To test for the existence of nuclear pasta researchers, use computer simulations which incorporate the relevant laws of physics such as quantum chromodynamics QCD and general relativity. The simulations tell us about the formation, stability and shapes of the pastas. For the simplest case the density of the pasta is the critical property, so researchers use density functional theory to determine the energy and structure of the pasta. On the right here is the density profile of a nuetron star [7].

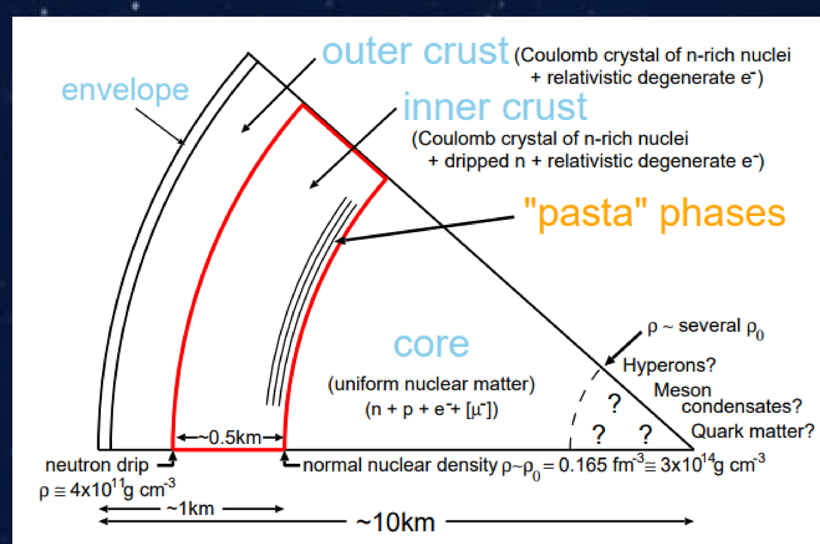

Figure 4: Cross section of Neutron Star [4]

These models help researchers determine the equation of-state EOS of the pasta. The EOS shows the relationship between density, pressure and the energy of the material. In this case we have the Tolman-Oppenheimer-Volkoff equation.

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}\left(1 + \frac{P}{\rho}\right)\left(1 + \frac{4\pi r^3 P}{M(r)}\right)\left(1 - \frac{2GM}{r}\right)^{-1}$$

P is the pressure ρ , ϱ is the density and r is the radial density[13]
Implementing nuclear spaghetti for technological purposes may be a challenge, however, Thor has harnessed this matter to make a cool hammer! Studying nuclear pasta helps us understand the behavior of matter under extreme conditions. We get insight into the forces that determine the behavior of matter at the nuclear scale. We also get insights into the structure of neutron stars [14] which helps explain their features such as mass, radius and the rate of cooling. We measure neutron star merge to detect gravitation waves. Nuclear pasta can help us refine our current models of the universe as the material is an extreme form that matter can exist in. Nuclear pasta may not have practical applications but it is crucial in advancing our understanding of the extreme pressures and temperatures in our universe.

Fun Fact: Thor's hammer is made of nuclear pasta! In norse mythology, Thor's hammer is made from the heart of a dying star and is supposedly the strongest material in the universe. Norse mythology predicted nuclear pasta!

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