$\mathbb{R}^3 \quad \beta = \left\{ \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}3\\1\\0\end{bmatrix} \right\}$ $\rightarrow \beta_1, \beta_2 \;\&\; \beta_3$

$\text{rref}\begin{pmatrix}1 & 1 & 3\\ 1 & 0 & 1\\ 1 & 2 & 0\end{pmatrix} = I$, therefore β is linearly independent set.

$$\text{rref}\begin{pmatrix}1 & 1 & 3 & x\\ 1 & 0 & 1 & y\\ 1 & 2 & 0 & z\end{pmatrix} = I \left|\begin{matrix} -\frac{2}{5}x + \frac{6}{5}y + \frac{z}{5} \\ \frac{1}{5}x - \frac{3}{5}y + \frac{2}{5}z \\ \frac{2\cdot x - y - z}{5} \end{matrix}\right.$$

β spans $\mathbb{R}^3 \therefore \rightarrow \mathbb{R}^3 = \text{span}(\beta)$

Linear independency means there is only 1 combination for which you can express a vector in $\mathbb{R}^3$

$$\boxed{\underset{\beta_1}{4\begin{vmatrix}1\\1\\1\end{vmatrix}} \; \underset{\beta_2}{-1\begin{vmatrix}1\\0\\2\end{vmatrix}} \quad \underset{\beta_3}{5\begin{vmatrix}3\\1\\0\end{vmatrix}} = \begin{vmatrix}18\\9\\2\end{vmatrix} = \vec{v}}$$

Identical statements

$$\boxed{\left|\vec{v}\right|_\beta = \begin{vmatrix}4\\-1\\5\end{vmatrix}}$$

$$\left|\vec{v}\right|_S = \begin{vmatrix}18\\9\\2\end{vmatrix}, \quad S = \left\{ \begin{vmatrix}1\\0\\0\end{vmatrix}, \begin{vmatrix}0\\1\\0\end{vmatrix}, \begin{vmatrix}0\\0\\1\end{vmatrix} \right\}$$

$M_{2\times 2}(\mathbb{R})$ $\quad S = \left\{ \begin{vmatrix} 1&0\\0&0 \end{vmatrix}, \begin{vmatrix} 0&1\\0&0 \end{vmatrix}, \begin{vmatrix} 0&0\\1&0 \end{vmatrix}, \begin{vmatrix} 0&0\\0&1 \end{vmatrix} \right\}$

$$\vec{w} = \begin{vmatrix} 3&4\\5&-1 \end{vmatrix} = 3\cdot\begin{vmatrix} 1&0\\0&0 \end{vmatrix} + 4\begin{vmatrix} 0&1\\0&0 \end{vmatrix} + 5\begin{vmatrix} 0&0\\1&0 \end{vmatrix} + -1\begin{vmatrix} 0&0\\0&1 \end{vmatrix}$$

$$|\vec{w}|_S = \begin{vmatrix} 3\\4\\5\\-1 \end{vmatrix}$$

$P_2(\mathbb{R})$ $\quad S = \{t^2, t, 1\}$

$\vec{v} = 3t^2 + 5t - 4$ $\quad |\vec{v}|_S = \begin{vmatrix} 3\\5\\-4 \end{vmatrix}$ $\Big\}$ The same

Ex: $\beta = \{t^2+t,\ t^2-1,\ t^2+t+1\}$ basis of $P_2(\mathbb{R})$

$S = \{t^2, t, 1\}$

$$[\beta_1]_S = \begin{vmatrix} 1\\1\\0 \end{vmatrix} \quad |\vec{\beta}_2|_S = \begin{vmatrix} 1\\0\\-1 \end{vmatrix} \quad |\vec{\beta}_3|_S = \begin{bmatrix} 1\\1\\1 \end{bmatrix}$$

$\vec{w} = 7t^2 + 9t + 8$ How would we write this w/ resp. to standard basis? $\longrightarrow$ $\begin{vmatrix} 7\\9\\8 \end{vmatrix} = |\vec{w}|_S$

How about basis β?

$\text{rref}\left(\beta_1 \,|\, \beta_2 \,|\, \beta_3 \,\middle|\, \begin{matrix} 7\\9\\8 \end{matrix} \right)$ ~~to~~ solve $\quad |\vec{w}|_\beta = \begin{vmatrix} 3\\-2\\6 \end{vmatrix}$

The rest of the lecture (so far) is covering examples

Properties of linear

① $\varphi(\vec{v} + \vec{w}) = \varphi(\vec{v}) + \varphi(\vec{w})$

$$\left|\vec{v}\right|_{\beta} = {}_{\beta}C_{\alpha} = \left|\vec{v}\right|_{\alpha}$$

// stopped @ 1:08