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MAT250 LECTURE NOTES: 2021-01-21 #1

Span $\Rightarrow$ set of all linear combinations

$$\text{Span}\left(\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}\right) = \left\{ \underset{\times 3}{\underset{\uparrow}{\begin{bmatrix}1\\0\\1\end{bmatrix}}}, \underset{\times 4}{\underset{\uparrow}{\begin{bmatrix}2\\1\\0\end{bmatrix}}}, \begin{bmatrix}3\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\-1\\1\end{bmatrix}, \begin{bmatrix}4\\4\\3\end{bmatrix} \cdots \right\}$$

all combinations

inv

$$\underset{\downarrow}{\text{inv}}\; Q \cdot A = \text{rref}(A)$$

$$Q \cdot \underbrace{\begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 3 \\ 0 & 1 & -1 & 0 & 1 & +1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & 2 & 0 & 2 & 1 & 6 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 2 & 2 & 0 & 3 \\ 0 & 1 & -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\text{rref}(A)}$$

ALL DEPENDENCIES ARE MAINTAINED, AND EQUAL BETWEEN A AND rref(A)

A SPAN IS ALSO A VECTOR SPACE

BASIS: $\mathcal{B} = \{\vec{b}_1 \ldots \vec{b}_n\}$

properties of basis:

* They have order
* $\mathcal{B}$ is linearly independent
* $W = \text{span}(\vec{b}_1 \ldots \vec{b}_n)$

"ORDERED LINEARLY INDEPENDENT SPANNING SET"

Let $\beta = \{\vec{b}_1 \cdots \vec{b}_n\}$ be a <u>basis</u> of ω

and $\vec{\omega} \in W$

① $\vec{\omega} = \text{span}(\vec{b}_1 \cdots \vec{b}_n)$

$\Rightarrow \vec{\omega} = \omega_1 \vec{b}_1 + \omega_2 b_2 \cdots \omega_n b_n$

② This is unique, only <u>one</u> way of writing any vector $\vec{\omega}$ ~~aa~~ a linear combination of β vectors

Basis co-ordinates are the coefficience in front of the basis vectors

$$\begin{vmatrix} 4 & 5 \\ 1 & 3 \end{vmatrix} = 4 \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} + 5 \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} + 1 \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} + 3 \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 4 \\ 5 \\ 1 \\ 3 \end{vmatrix} \rightarrow [\vec{\omega}]_\beta = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix}$$

Basis lets us write vectors in unique ways. Ways where every vector in a set can be described by one.

Standard basis is usually easiest form to work with matrices in.

Coordinates of vector → Unique way to describe a vector space

STANDARD BASIS ARE IMPORTANT