PAUL HAZEN
MAT250 LECTURE NOTES: 2021-01-05 #1

12:30PM - 2:00PM Office Hours

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Evie, Hibno & Ryan are classmates

Vector spaces → Lots of shit.

Every Tuesday HW is due

He's Dutch don't call him "Boerkoel", in Dutch it's considered very impolite.

Fields

$\mathbb{Q}$ rational #

$\mathbb{R}$ real #

$\mathbb{N} = \{0, 1, 2, \dots\}$

$\mathbb{Z} = \{\dots, -3, -2, -1, \dots\}$

INCREASING SIZE OF NUMBER SYSTEM

$x + 4 = 7$ → 3

$x + 4 = 2$ → -2

$2x + 7 = 4$

$2x = -3$

$x = -\frac{3}{2}$

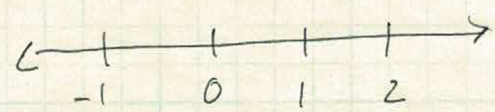
$\mathbb{Q} = \left\{ \frac{a}{b} \;\middle|\; a, b \in \mathbb{Z},\ b \neq 0 \right\}$

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

$\mathbb{R}$



$x^2 = -1$

→ $x = i, -i$

$\boxed{\mathbb{C} \quad i^2 = -1}$

$3x^5 + 7x - 5 = 0$

→ All solutions exist in field of complex numbers

Complex Numbers

$$\mathbb{C} = \{ a + bi \mid a, b \in \mathbb{R} \}$$

$$\boxed{i^2 = -1}$$

Addition: $(3+4i) + (7-5i) = 10 - i$

$-1i$

10

$z = 3 + 4i$

$\to Im(z) = 4$

~~Re(z) = 3~~

Real $(z) = 3$

$$(a + bi) + (A + Bi) = (a + A + (b+B)i$$

Multiplication:

$$(3+4i) \cdot (5-2i)$$

$$= (3+4i) \cdot 5 + (3+4i)(-2i)$$

$$= 15 + 20i + (-6i) - 8 \cdot i^2$$

$$= 15 + 8 + 14i \quad (\text{because } i^2 = -1)$$

$$= 23 + 14i$$

$z = a + bi$ $\quad$ $Re(z) = a$

$Im(z) = b$

$|z| = \sqrt{a^2 + b^2}$ $\quad$ $\bar{z} = a - bi$

complex conjugate of z

$z = 3 + 4i$ $\quad$ $|z| = \sqrt{3^2 + 4^2} = \cancel{2}\ 5$

$\bar{z} = 3 - 4i$

$$z \cdot \bar{z} = (a + bi)(a - bi)$$
$$= a^2 + - abi + abi - b^2 i^2$$
$$= a^2 + b^2$$

$$\boxed{z \cdot \bar{z} = |z|^2}$$

$z \neq 0$

$$\frac{\bar{z}}{|z|^2} = \frac{1}{z}$$

* $(a - bi) - (A + Bi) = (a - A) + (b - B) \cdot i$

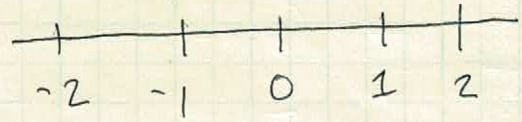
* $\dfrac{3 + 4i}{5 + 12i} = (3 + 4i) \cdot \dfrac{1}{5 + 12i}$

$$\frac{1}{5 + 12i} = \frac{5 - 12i}{13^2}$$
$$= \frac{5}{13^2} - \frac{12}{13^2} i$$
$$= \frac{5}{169} - \frac{12}{165} i$$

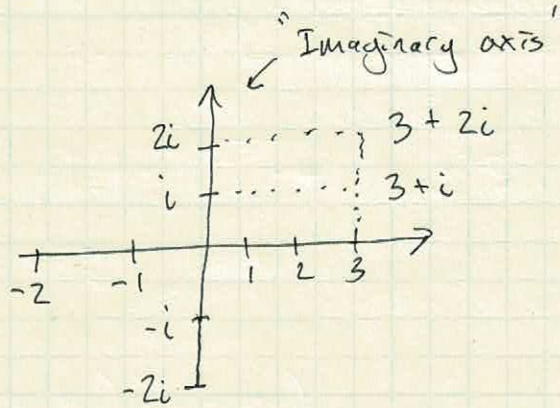
$|5 + 12i| = \sqrt{5^2 + 12^2}$

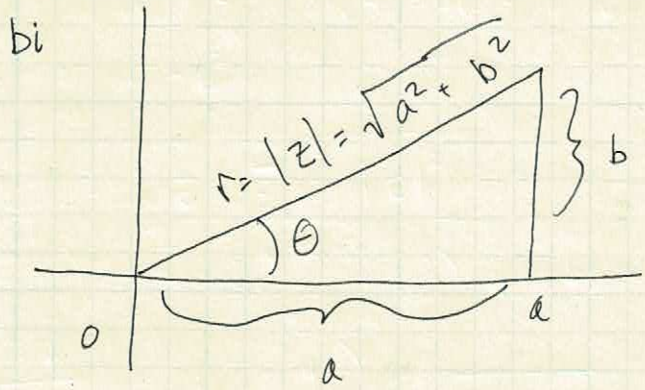
$= \sqrt{169} = 15$

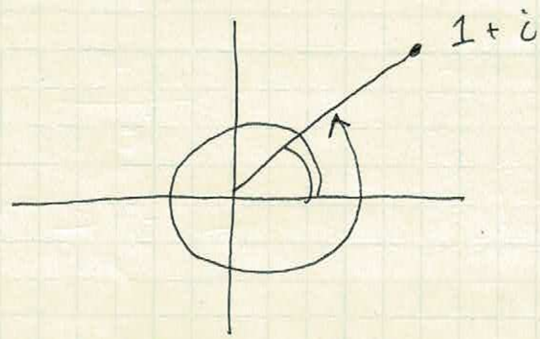
$\mathbb{R}$



$\mathbb{C}$







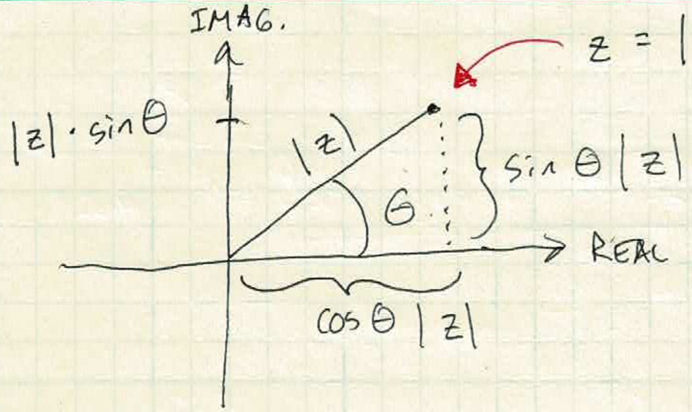
constant

$$\theta = \frac{\pi}{4} + k \cdot 2\pi$$

$\theta = \mathrm{Arg}(z)$

$-\pi < \theta \leq \pi$

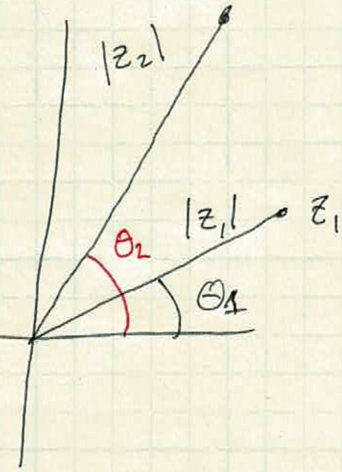
$$z = |z| \cdot \cos\theta + |z| \cdot \sin\theta \cdot i$$



$$z = |z| \cdot (\cos\theta + i \sin\theta)$$

$$z_1 = |z_1| \cdot (\cos\theta_1 + i \sin\theta_1)$$

$$z_2 = |z_2| \cdot (\cos\theta_2 + i \cdot \sin\theta_2)$$

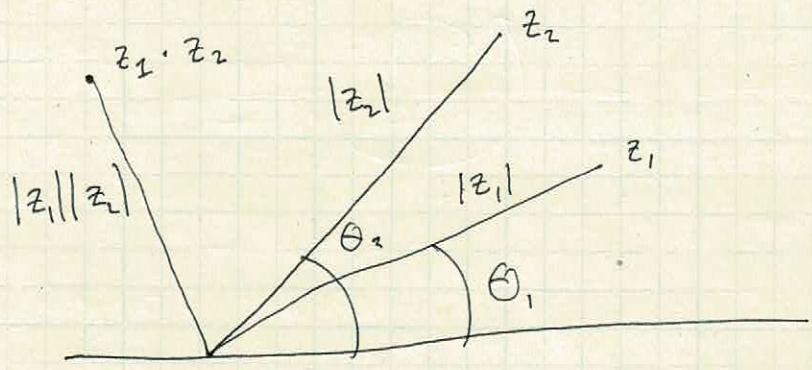


$$z_1 \cdot z_2 = |z_1 \cdot z_2| \cdot (\cos\theta_1 + i \sin\theta_1) \cdot (\cos\theta_2 + i \sin\theta_2)$$

$$= |z_1 \cdot z_2| \cdot \Big((\cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \cdot \sin\theta_2) + (\cos\theta_1 \cdot \sin\theta_2 + \cos\theta_2 + \sin\theta_1) i \Big)$$

Using trig identities:

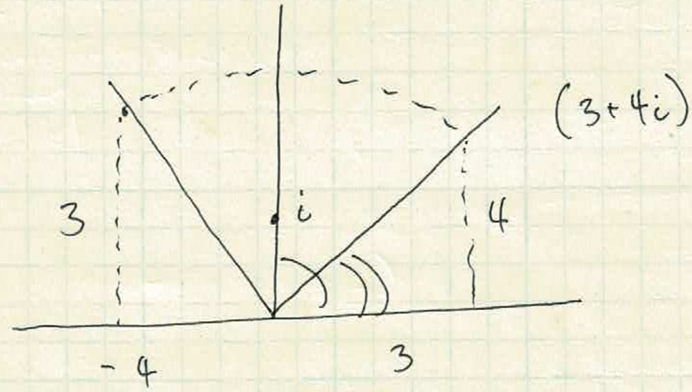
$$= |z_1 \cdot z_2| \cdot \big(\cos(\theta_1 + \theta_2) + \sin(\theta_1 + \theta_2) i \big)$$



$i \cdot (3+4i)$

$\theta_1 = \frac{\pi}{2}$

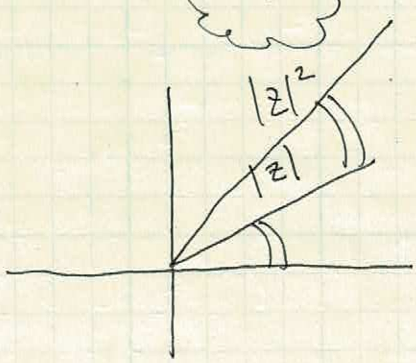
$|i| = 1$



$i \cdot z =$ rotation of z over $90°$

$\sqrt{z} \in \mathbb{R}$

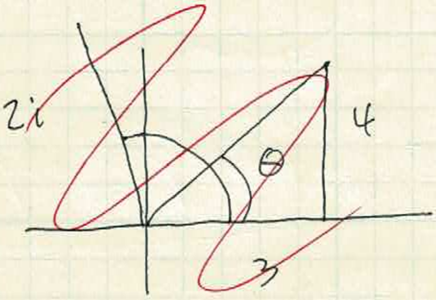
$\sqrt{i}$? $x^2 = i$



$5 + 2i = z$

z^2

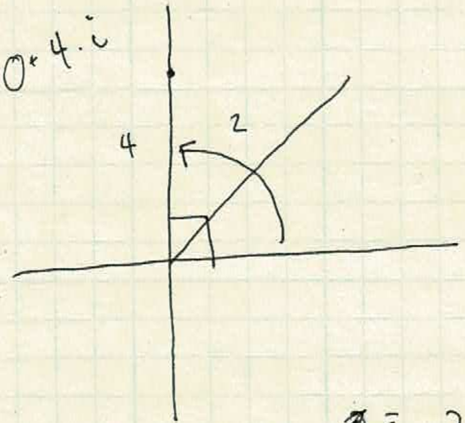
"complex multiplication" → learn more from youtube.
Dr. B is going really fast



$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

→ his numbers are hard to read, this may be wrong

$0 + 4 \cdot i$



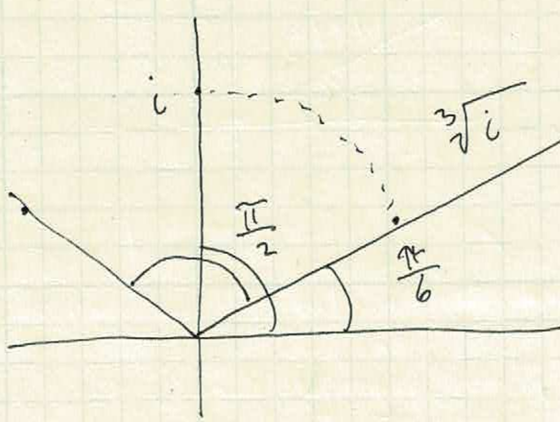
$$\theta = \frac{\pi}{2}$$

$$\frac{\theta}{2} = \frac{\pi}{4}$$

$$\sqrt{4i}$$

$$z = 2 \cdot \left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$$
$$= 2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)$$
$$= \sqrt{2} + \sqrt{2} i$$

$$z = \sqrt{2} + \sqrt{2} i = \sqrt{4i}$$
$$z = -\sqrt{2} - \sqrt{2} i$$



$$\frac{\pi}{2} / 3 = \frac{\pi}{6}$$

$$\frac{\frac{\pi}{2} + 2\pi}{3} = \frac{\pi}{6} + \frac{2\pi}{3}$$

$$\frac{\frac{\pi}{2} + 2 \cdot 2\pi}{3} = \frac{\pi}{6} + \frac{4\pi}{3}$$

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$\mathbb{C}$ = Field of complex #

Finite Fields

$\mathbb{F}_2$ $\mathbb{F}_4$ $\mathbb{F}_7$

$\{0, 1\}$

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

ALL ADDITION/MULTIPLICATION FOR $\mathbb{F}_2$

NOTE THAT 7 = 0

$\mathbb{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

·	0	1	2	3	4	5	6
0	0	6	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

NOTE THAT ALL DIGITS ARE PRESENT ONCE IN EACH ROW/COL

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$\cancel{\mathbb{F}_4 = \{0, 1}, 2,$

$\mathbb{F}_4 = \{0, 1, a, b\}$

All numbers added to themselves are 0

If you add any of 1, a, or b to another, it equals the third

+	0	1	a	b
0	0	1	a	b
1	1	0	b	a
a	a	b	0	1
b	b	a	1	0

·	0	1	a	b
0	0	0	0	0
1	0	1	a	b
a	0	a	b	1
b	0	b	1	a

$a \cdot a \cdot a$

$b \quad a$

$a^2 \cdot b + a \cdot b + a^3 + b + 1$

$= b \cdot b + 1 + 1 + b + 1$

$= a + b + 1$

$= 0$

THIS IS CONFUSING AF

$ab^2 + ab + a =$

$a \cdot a + 1 + a =$

$b + 1 + a = 0$

$\oplus$

$\underline{\underline{0}}$

$$\boxed{\begin{aligned} a + b &= b + a \\ (a+b) + c &= a + (b+c) \\ a + 0 &= a \\ a + (-a) &= 0 \end{aligned}}$$

$\odot$

$\underline{\underline{1}}$

$$\boxed{\begin{aligned} a \cdot b &= b \cdot a \\ (a \cdot b) \cdot c &= a \cdot (b \cdot c) \\ a \cdot 1 &= a \\ a \cdot \frac{1}{a} &= 1 \quad (a \neq 0) \end{aligned}}$$

$a \cdot (b + c) = a \cdot b + a \cdot c$

ALL FIELDS HAVE A SET OF NUMBERS AND THE ABOVE OPERATIONS