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Taking a Whipper—The Fall-Factor Concept in Rock Climbing

Dan Curtis



Dan Curtis (curtiswd@cwu.edu; Central Washington University, Ellensburg, WA 98926) earned his Ph.D. from the University of Massachusetts in 1970, and for the next twenty years was on the faculty of Kansas State University. From 1990 to 2000, he was an Associate Technical Fellow at the Mathematics Group at the Boeing Company. Following his retirement from Boeing, he joined the Mathematics Department at Central Washington. His current interests are in applied mathematics, nonlinear dynamics, and probability theory, and also in the use of *Mathematica* in both education and research.

In the world of climbing, “taking a whipper” means taking a long fall, where the climber is whipped around by the rope as it (hopefully) breaks his fall. But experienced climbers are aware that the length of a fall does not tell the complete story with respect to the dangers involved. There are (at least) two aspects to a fall that determine the danger involved. First, the farther the climber falls, the greater the opportunity to hit something hard along the way. Second, there is the impact delivered to the climber by the rope as the fall is arrested. The first aspect is difficult to analyze further, depending as it does on the details of the fall, the particular rock formations involved, and so on. But with respect to the second, some interesting things can be said. And in fact these things have immediate importance for how climbers climb and for how manufacturers test climbing gear.

To focus the discussion, we suppose there is a length L of rope (unstretched) between a climber and his belayer. (For the non-climber, the *belayer* is the person holding the rope to which the climber is attached and whose job it is to stop the rope in the event of a fall.) Then suppose the climber takes a fall, plummeting a distance D_F (free-fall distance) before the rope becomes taut and then falling a further distance after the rope becomes taut and begins to stretch until the fall is finally arrested (that is, the climber’s downward velocity becomes zero). Figure 1 is a schematic of the situation, showing the initial and final positions of the climber, the position of the belayer, and the location of the piece of protection that the climber placed before the fall. Figure 2 shows the various distances involved in the fall. The unstretched length L of the rope is not labeled in the figure, but it should be clear what it is. Let D_T denote the total distance fallen. The force that the rope exerts on the climber varies depending on how much the rope has been stretched beyond its natural length. Suppose the length of the rope at some instant is $l > L$. According to Hooke’s Law, the force F_R exerted on the climber by the rope is proportional to the fractional stretch in the rope. Thus, there is a constant k (whose numerical value depends on the particular rope being used) such that

$$F_R = k \frac{l - L}{L}.$$

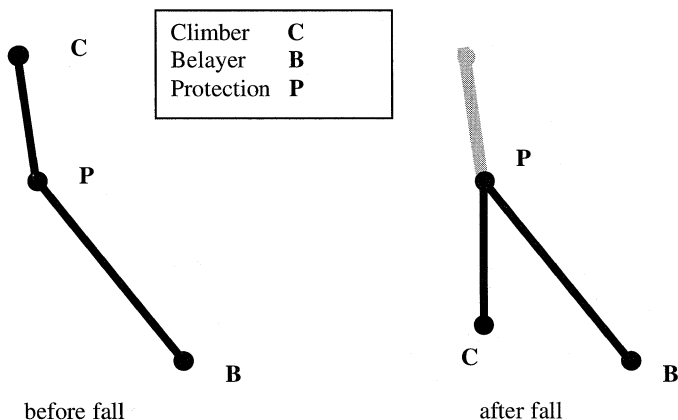


Figure 1.

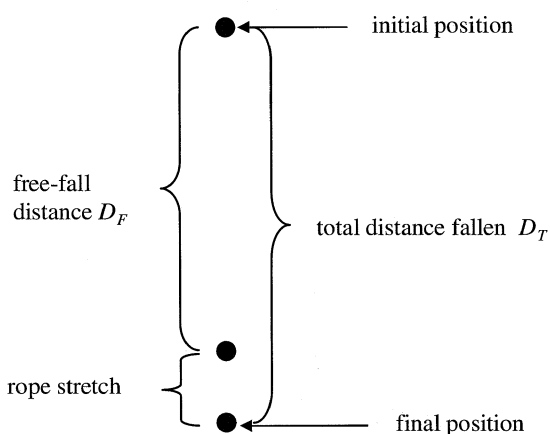


Figure 2.

We set two goals:

- Calculate an expression for the maximum force exerted on the climber by the rope.
- Show how this maximum force depends on the length of the fall D_T and on the (unstretched) length L of rope between the belayer and the climber.

We shall demonstrate an important and unexpected fact, which is part of the folklore of rock climbing:

The maximum force experienced by the climber does not depend on the length of the fall, but rather on the ratio of the distance fallen to the length of rope out; that is, the maximum force depends on the ratio D_T/L .

This ratio is known to climbers as the fall-factor.

In order to solidify the notion, the reader should answer the following questions (for answers, see the Appendix):

1. A climber climbs 25 feet straight up from a belay point and places a piece of protection. He then climbs another 25 feet and falls.

- a. Assuming the protection holds and the belay does not slip, how far does the climber fall? (Ignore the stretch in the rope, for now.)
 - b. What is the fall-factor for this fall?
2. What is the greatest fall factor that could occur in a normal roped climbing situation (again, ignoring the rope stretch)? Describe a situation that would result in such a fall.

The fact that the maximum force experienced by the climber depends only on the fall-factor and not on the actual distance fallen is of great importance in the design and testing of climbing gear. It also has significant implications for how protective anchors should be placed in order to avoid disastrous falls. We discuss these matters briefly below.

Mathematical formulation of the problem

To analyze this problem we set up a coordinate system with the x -axis pointing down. We let the origin be the position of the climber at the moment of the fall (see Figure 3). Then (ignoring air resistance), before the rope becomes taut, the position of the falling climber satisfies the differential equation

$$m \frac{d^2 x}{dt^2} = mg. \quad (1)$$

We are interested in finding the velocity of the falling climber at the instant the rope becomes taut, which is when $x = D_F$. The differential equation (1) can be rewritten using the identity

$$\frac{d^2 x}{dt^2} = v \frac{dv}{dx}$$

(where $v = dx/dt$ is velocity), to read

$$v \frac{dv}{dx} = g.$$

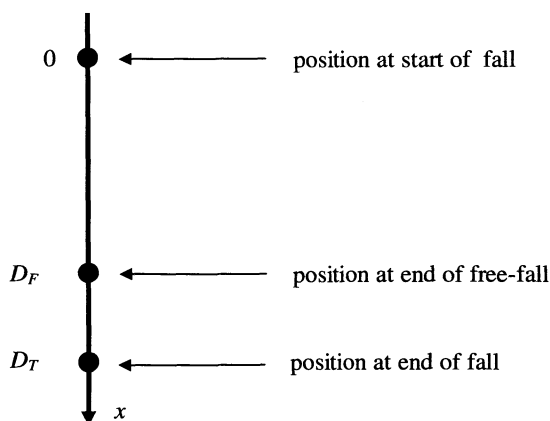


Figure 3.

The general solution is

$$\frac{1}{2}v^2 = gx + C,$$

and since $v = 0$ when $x = 0$, $C = 0$. Therefore, $v^2 = 2gx$ or $v = \sqrt{2gx}$. So the velocity at the instant when $x = D_F$ is $v_F = \sqrt{2gD_F}$.

After the rope becomes taut and begins to stretch, the velocity satisfies the initial-value problem

$$v \frac{dv}{dx} = g - \frac{k}{mL}(x - D_F) \quad \text{with} \quad v(D_F) = \sqrt{2gD_F}.$$

The solution of this is

$$v^2 = 2gx - \frac{k}{mL}(x - D_F)^2. \quad (2)$$

The maximum force felt by the climber will occur when the stretch in the rope is greatest, that is, when $x = D_T$. This will occur at the moment the climber's velocity becomes 0. Thus, we set $v = 0$ and $x = D_T$ in (2) to get

$$0 = 2gD_T - \frac{k}{mL}(D_T - D_F)^2. \quad (3)$$

Then from (3), we see that the maximum force that the rope will exert on the climber is

$$F_{\max} = k \left(\frac{D_T - D_F}{L} \right) = \frac{k}{L} \sqrt{\frac{2mgLD_T}{k}} = \sqrt{\frac{2mgkD_T}{L}},$$

that is,

$$F_{\max} = \sqrt{2mgk \left(\frac{D_T}{L} \right)}. \quad (4)$$

Equation (4) is the desired relationship: we see that F_{\max} depends on the fall distance D_T only through the ratio D_T/L , that is, the maximum force exerted by the rope depends not on the length of the fall, but rather on the fall-factor. We note that, as expected, the maximum force *does* depend on the weight of the climber and on the elasticity of the rope (the constant k). Further discussion of the methods of solution used above can be found in any standard text on elementary differential equations (for example, Boyce and DiPrima [4, pp. 42–59]).

Implications for rock climbers

An immediate implication for rock climbers concerns the situation where a climber is about to climb above a narrow ledge from which he is being belayed. As the climber leaves the ledge, until he has placed a piece of protection, any fall will be a fall-factor 2 fall; that is, the climber will fall (slightly more than) twice the length of the rope that is out, as shown in Figure 4. As soon as a piece of protection is placed, the fall-factor for a subsequent fall will be less than 2, as illustrated in Figure 5. The importance of getting that first piece of protection in is well known to climbers.

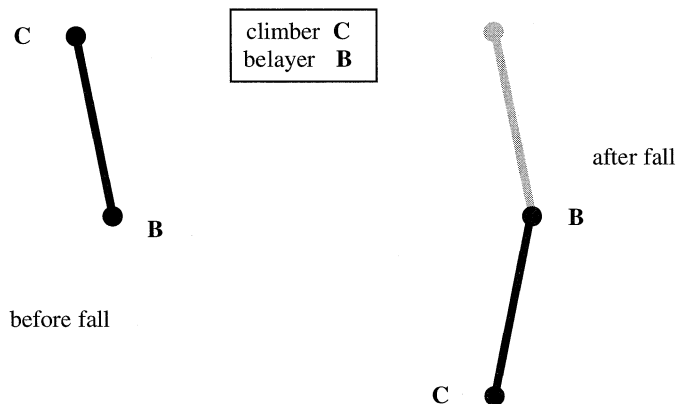


Figure 4. A fall-factor 2 fall

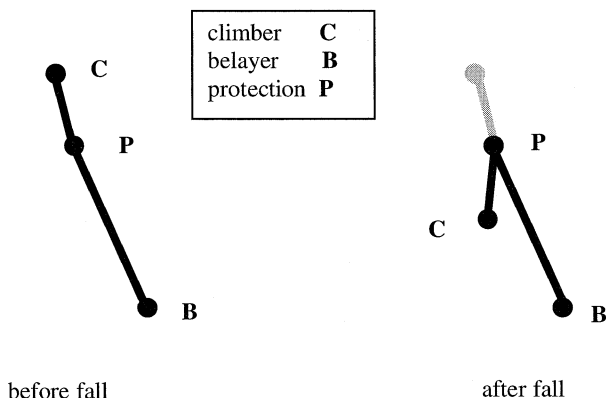


Figure 5. Fall-factor ≈ 0.6

Design and testing of climbing gear

Testing of climbing gear is carried out by the UIAA (Union Internationale des Associations d'Alpinisme). One of the important tests that a climbing rope must pass in order to be UIAA certified is called the Drop Test.

This test goes roughly as follows: A length of rope approximately 2.5 m long is fixed to an anchor at one end. An 80 kg mass (representing a “typical” climber) is attached to the other end of the rope. The mass is then raised vertically 2.5 m above the anchor point and released. The mass then falls slightly more than 5.0 m (slightly more because of rope stretch).

One important requirement is that the maximum force exerted on the mass by the rope in stopping the fall not exceed 12 kiloNewton (kN) [3]. This is because results of tests by the U.S. Air Force have indicated that the human body cannot sustain an impact of greater than 12 kN without serious injury. When climbers read about this test, a common first reaction is “Well, that test only uses a 5-meter fall. What happens if I take a longer fall?” But the point is, the fall used in the test has a fall-factor 2. This is the most severe fall that can occur in normal roped climbing. Since, as was shown above, the maximum force on the mass depends only on the fall-factor and not on the actual distance fallen, if a rope passes the Drop Test, then it will not exert more than

12 kN in stopping any fall, no matter the length, as long as the mass of the climber does not exceed the 80 kg used in the test. (In practice, rope manufacturers build in a considerable safety margin to accommodate heavier climbers.) Therefore, the result we established about the maximum impact force provides justification for the UIAA Drop Test standards. For more information about testing standards, see [2], [3].

Appendix: Answers to questions

1. a. The climber falls 50 ft. Since he is 25 ft above his last piece of protection when the fall occurs, he will end up 25 ft below that protection point and thus will have fallen 50 ft.
- b. The fall-factor is 1: The climber falls 50 ft and there are 50 ft of rope out at the moment of the fall. The ratio is therefore 1.
2. The answer is 2. The longest fall possible with a given length L of rope out is $2L$. This can occur if the climber climbs above the belayer a distance L with no protection placed and then falls to a point a distance L below the belayer.

References

1. UIAA/EN892 Standard for Dynamic Rope, Union Internationale des Associations d'Alpinisme, Monbijoustrasse 61, CH-3007 Bern, Switzerland.
2. <http://home.echoon.net/toadhall/outside/climbing/shock.html>.
3. <http://www.bealropes.com/english.dir/standards.html#testing>.
4. W. Boyce and R. DiPrima, *Elementary Differential Equations*, 8th ed., Wiley, 2004.

Geometry will draw the soul toward truth and create the spirit of philosophy.

—Plato

The great book of nature lies ever open before our eyes and the true philosophy is written in it. But we cannot read it unless we have first learned the language and the characters in which it is written. It is written in mathematical language and the characters are triangles, circles and other geometrical figures.

—Galileo

There are more things in heaven and earth, Horatio, than dreamt of in your philosophy.

—Shakespeare (*Hamlet*)

Now, my suspicion is that the universe is not only queerer than we suppose, but queerer than we can suppose . . . I suspect that there are more things in heaven and earth that are dreamed of in any philosophy.

—J. B. S. Haldane