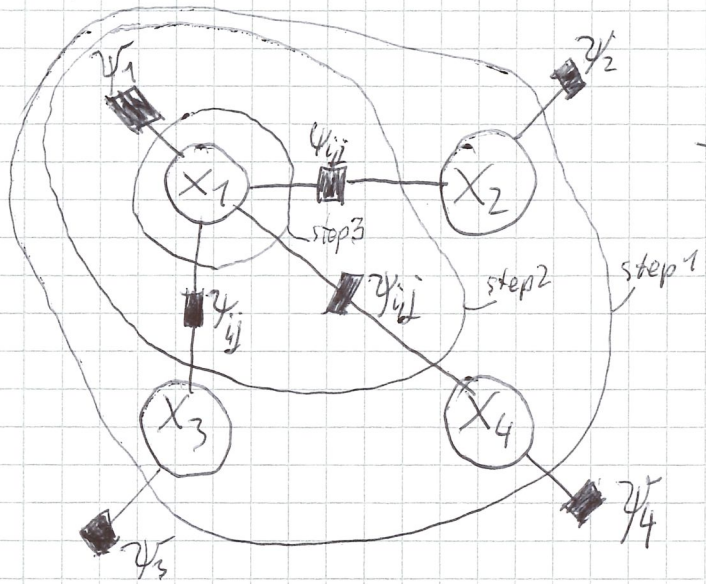
Ex 1:

a)

This is one possibility for the model. The role of x_1, connecting all other nodes could be played by any other variable x_i.



→ x_1 is the root

step1:

$$m_{\psi_2 \to x_2}(x_2) = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$m_{\psi_3 \to x_3}(x_3) = \begin{bmatrix} 1 \\ -1\ 0 \end{bmatrix}$$

$$m_{\psi_4 \to x_4}(x_4) = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

step2:

$$m_{x_2 \to \psi_{ij}}(x_2) = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$m_{x_3 \to \psi_{ij}}(x_3) = \begin{bmatrix} -1 \\ 1\ 0 \end{bmatrix}$$

$$m_{x_4 \to \psi_{ij}}(x_3) = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

step 3:

x_2 thinks its a 1 → so take 2. row of matrix

$$\Rightarrow m_{\psi_{ij} \to x_1}(x_1, x_2 = 1) = \begin{bmatrix} 6 \\ 3 \end{bmatrix} + \begin{bmatrix} -10 \\ 3 \end{bmatrix}$$

x_3 thinks its a 0

$$\Rightarrow m_{\psi_{ij} \to x_1}(x_1, x_3 = 0) = \begin{bmatrix} 1 \\ 1\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ -10 \end{bmatrix}$$

x_4 thinks its a 0

$$\Rightarrow m_{\psi_{ij} \to x_1}(x_1, x_4 = 0) = \begin{bmatrix} 5 \\ 7 \end{bmatrix} + \begin{bmatrix} 3 \\ -10 \end{bmatrix}$$

$$m_{\psi_1 \to x_1}(x_1) = \begin{bmatrix} 11 \\ 3 \end{bmatrix}$$

$$\sum_{m(x_1)} = \begin{bmatrix} -4 \\ 6 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 8 \\ -3 \end{bmatrix} + \begin{bmatrix} 11 \\ 3 \end{bmatrix} = \begin{bmatrix} 19 \\ 6 \end{bmatrix} \Rightarrow x_1 = 1$$

when $x_1 = 1$ take second colum of matrix:

$$\sum_{m(x_2)} = \begin{bmatrix} 6 \\ 3 \end{bmatrix} \begin{bmatrix} -10 \\ 3 \end{bmatrix} = \begin{bmatrix} -4 \\ 6 \end{bmatrix} \Rightarrow x_2 = 0$$

$$\sum_{m(x_3)} = \begin{bmatrix} -1 \\ 1\ 0 \end{bmatrix} + \begin{bmatrix} -10 \\ 3 \end{bmatrix} = \begin{bmatrix} -9 \\ 13 \end{bmatrix} \Rightarrow x_3 = 0$$

$$\sum_{m(x_4)} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} + \begin{bmatrix} -10 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \end{bmatrix} \Rightarrow x_4 = 0$$

$x = [1, 0, 0, 0]$ ⇒ That complies with the program in ex. 1b.