

Celula Sumatoare (FAC)

$$\begin{cases} z_i = x_i \oplus y_i \oplus C_i \\ C_{i+1} = x_i y_i + x_i C_i + y_i C_i \end{cases}$$

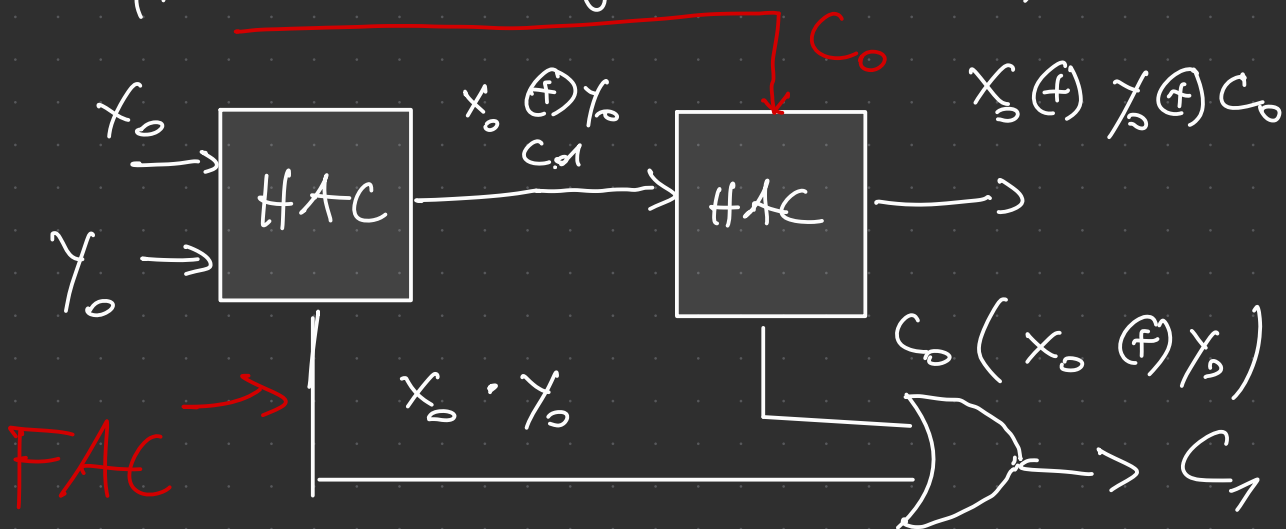
$$a \oplus b = a\bar{b} + \bar{a}b$$

$\oplus \rightarrow$ doar 2 intrări

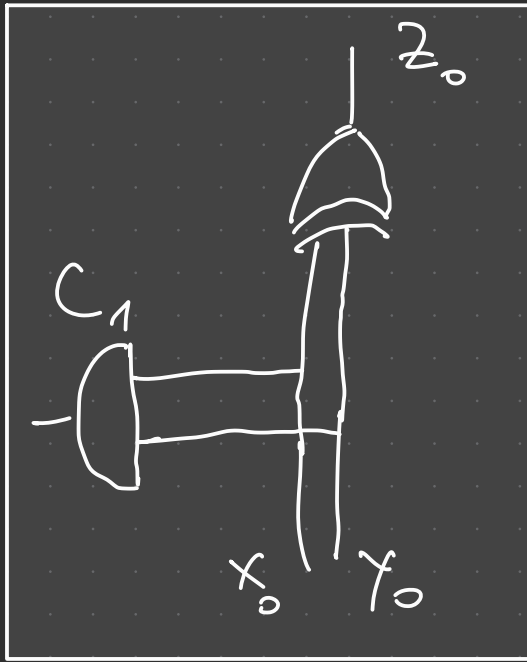
$$C_{in} = C_0 = 0$$

$$\text{FAC: } \begin{cases} z_0 = x_0 \oplus y_0 \oplus 0 = \underline{\underline{x_0 \oplus y_0}} \\ C_1 = x_0 y_0 + x_0 0 + y_0 0 = \underline{\underline{x_0 y_0}} \end{cases}$$

HAC (Half-Adder Cell)



① EXOR ; AND



② NOR

$$z_0 = x_0 \oplus y_0 = x_0 \bar{y}_0 + \bar{x}_0 y_0$$

$$= x_0 (\bar{x}_0 + \bar{y}_0) + (\bar{x}_0 + \bar{y}_0) y_0 =$$

$$= \overline{x_0 + y_0} + \overline{\bar{x}_0 + \bar{y}_0}$$

$$C_1 = \overline{\bar{x}_0 + \bar{y}_0}$$

③ NAND

$$z_0 = x_0 \oplus y_0 = x_0 \bar{y}_0 + \bar{x}_0 y_0$$

$$= x_0 (\bar{x}_0 + \bar{y}_0) + y_0 (\bar{x}_0 + \bar{y}_0)$$

$$= x_0 \overline{x_0 y_0} + y_0 \overline{x_0 y_0} =$$

$$= \overline{x_0 x_0 y_0} + \overline{y_0 x_0 y_0}$$

$$C_1 = \overline{x_0 y_0}$$

CMOS

NAND \rightarrow cea mai eficientă + rapidă
 \downarrow
NOR

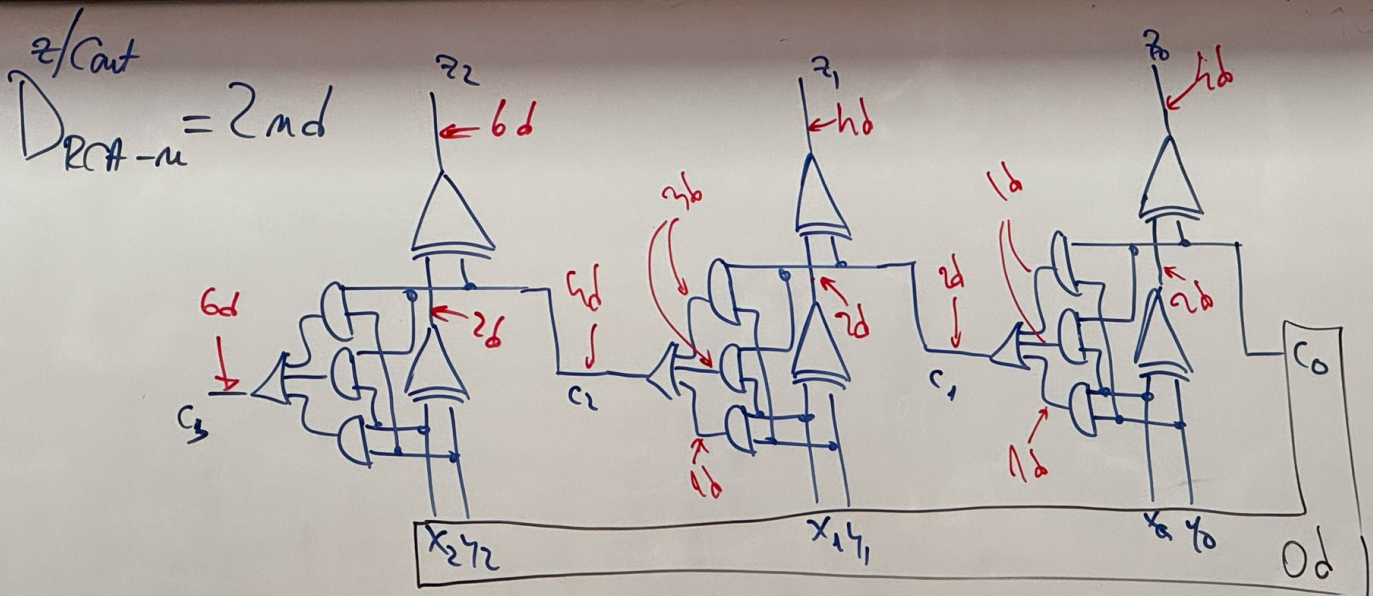
Calea critică

- \rightarrow întârzierea maximă pe o cale
- \rightarrow orice punct primitiv \rightarrow întârziere 1d

AND, NAND
OR, NOR \rightarrow 1d

NOT \rightarrow 0d

EXOR \rightarrow 2d



- carry from MSB
- overflow 2)
- zero
- negative

$$X = x_{n-1} \quad x_{n-2} \quad \dots \quad x_1 \quad x_0$$

$$Y = y_{n-1} \quad y_{n-2} \quad \dots \quad y_1 \quad y_0$$

$$Z = z_{n-1} \quad z_{n-2} \quad \dots \quad z_1 \quad z_0$$

| x_i | y_i | c_i | z_i | 2) (overflow) |
|-------|-------|-------|-------|---------------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 |

$$D = \overline{x_{n-1}} \overline{y_{n-1}} c_{n-1} + x_{n-1} y_{n-1} \overline{c_{n-1}}$$

$$I_1 = (A \oplus B) \cdot C = AC \oplus BC$$

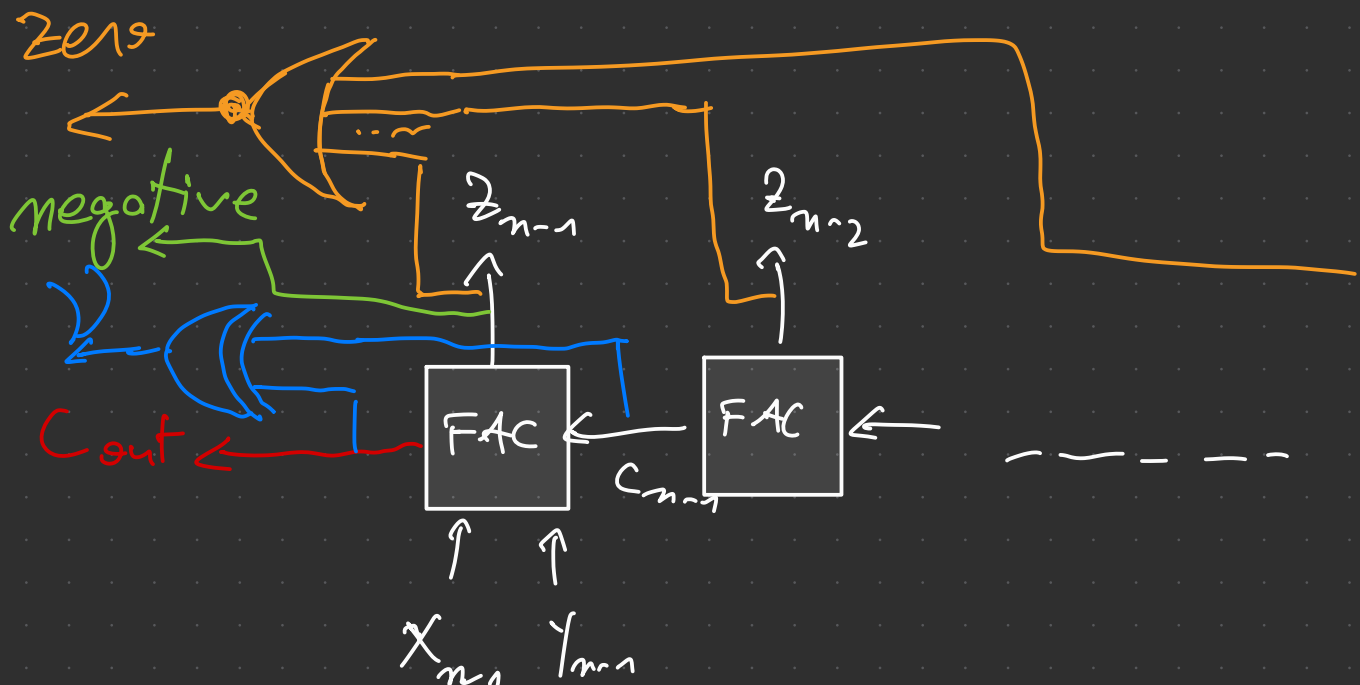
$$I_2 = A \oplus B = (A+B) \oplus AB$$

$$I_2' = A+B = A \oplus B \oplus AB$$

$$X \oplus 1 = \overline{X}$$

$$D = \overline{x_{n-1}} \overline{y_{n-1}} c_{n-1} + x_{n-1} y_{n-1} \overline{c_{n-1}}$$

$$D = C_n \oplus C_{n-1}$$



Adunarea cu o constantă

o se consideră constante impare

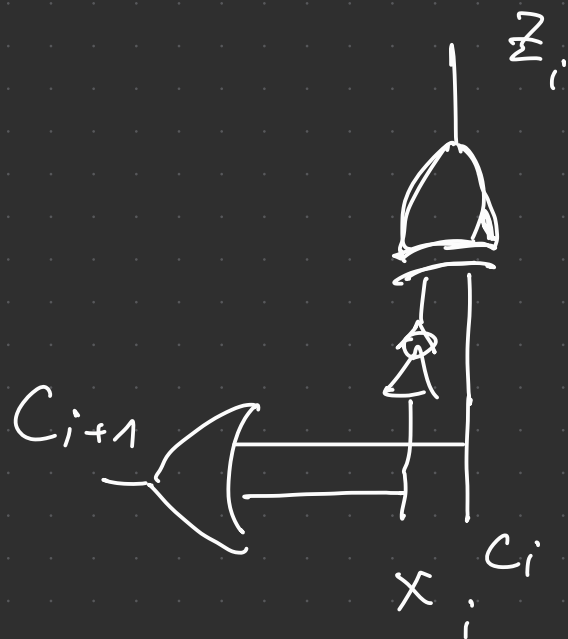
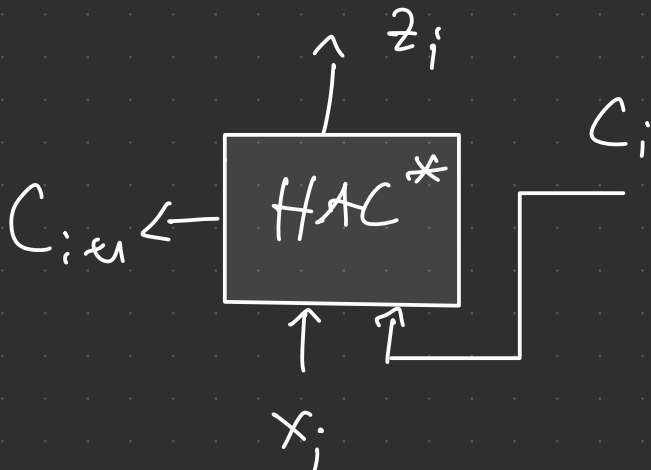
x, y pe n biți

$$z = x + y$$

$y \rightarrow$ constant

$$y_i = 0 \left\{ \begin{array}{l} z_i = x_i \oplus C_i \\ C_{i+1} = x_i \cdot C_i \end{array} \right.$$

HAC



$$k_i = 0 \rightarrow \text{HAC}$$

$$k_i = 1 \rightarrow \text{HAC}^*$$

$$\left\{ \begin{array}{l} z_i = \overline{x_i} \oplus C_i \\ C_{i+1} = x_i + C_i \end{array} \right.$$

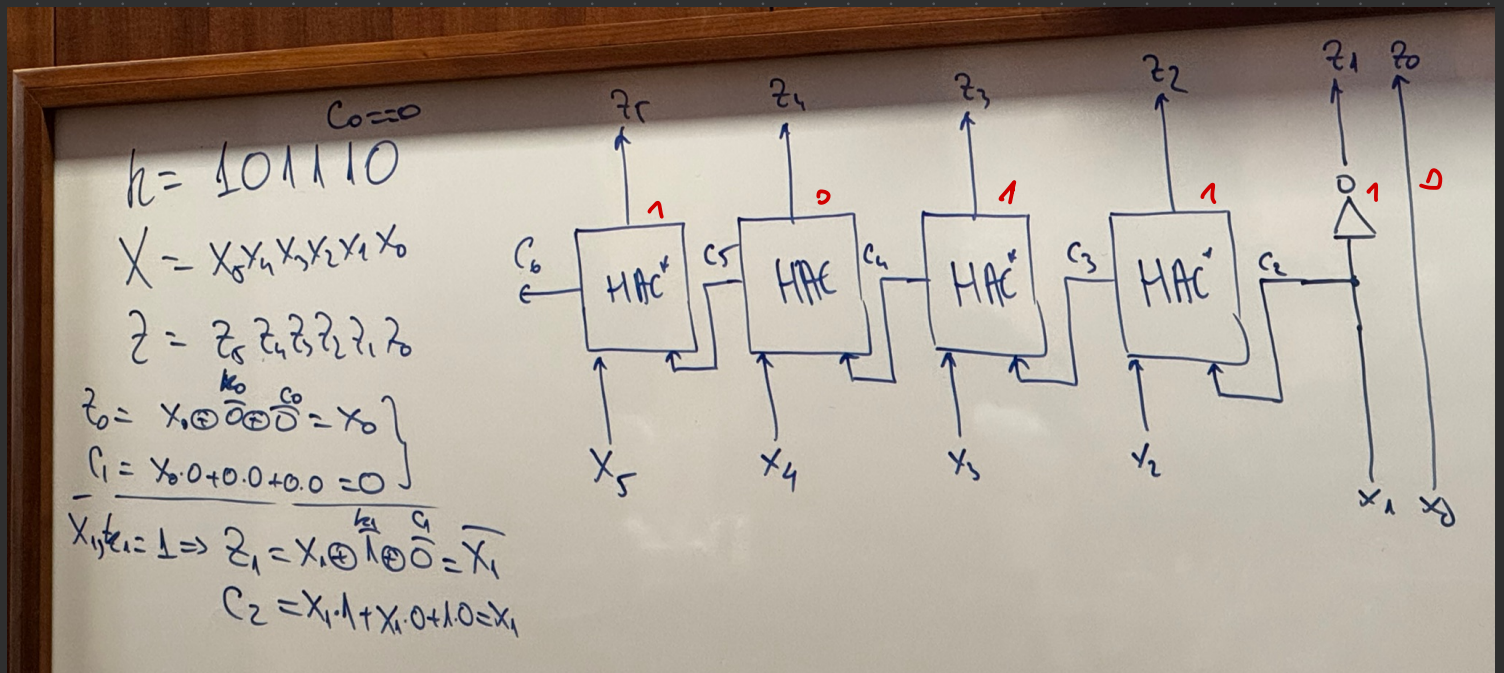
$$k = 101110$$

$$x = x_5 x_4 x_3 x_2 x_1 x_0$$

$$z = z_5 z_4 z_3 z_2 z_1 z_0$$

$$z_0 = x_0 \oplus \frac{k_0}{0} \oplus \frac{c_0}{0} = x_0$$

$$c_1 = x_0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 = 0$$



Adunare BCD

$$x_i + y_i < z_i \quad \text{cifă sumă} \\ c_{i+1} \quad \text{Carry}$$

$$\bullet \quad x_i + y_i < 10 \quad \begin{cases} z_i = x_i + y_i \\ c_{i+1} = 0 \end{cases}$$

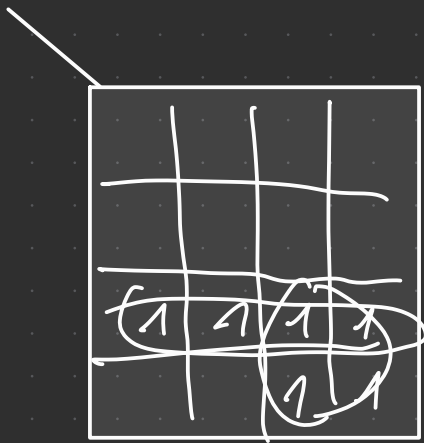
$$\bullet \quad x_i + y_i \geq 10 \quad \begin{cases} z_i = x_i + y_i - 10 \\ c_{i+1} = 1 \end{cases}$$

correction

$$x_i + y_i \geq 10 \quad C^* z_3^* z_2^* z_1^* z_0^* \geq 10$$

$$\begin{cases} 10 \leq \dots < 16 \\ 16 \leq \dots \end{cases} \quad \begin{matrix} (C_1) \text{ OR} \\ (C_2) \end{matrix}$$

$$(C_1) \quad C^* = 0; \quad 10 \leq z_3 z_2 z_1 z_0 < 16$$



$$C^* (z_3^* z_2^* + z_3^* z_1^*)$$

$$(C_2) \quad C^* = 1 \quad x_i + y_i \geq 10$$

$$= C^* + z_3^* \cdot z_2^* + z_3^* z_1^*$$

$$x_i + y_i - 10 \rightarrow \text{cif } 0 \text{ or } 9 \text{ (19-10)}_{10}$$

$$(x_i + y_i - 10) \bmod 2^4 \quad (4 \text{ bits})$$

$$+16-10 = \underline{\underline{+6}}$$

$$\begin{array}{r} 13 - \\ 10 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 1101 - \\ 1010 \\ \hline 0011 \\ \quad \quad \quad \underbrace{\hspace{1cm}}_3 \end{array}$$

$$\begin{array}{r} 13 + \\ 6 \\ \hline 19 \end{array} \quad \begin{array}{l} \% 16 = 3 \\ \underline{\underline{3}} \end{array}$$

$$\begin{array}{r} 1101 \\ 0110 \\ \hline \cancel{X} 0011 \end{array}$$

$$\begin{array}{r} 19 - \\ 10 \\ \hline 9 \end{array}$$

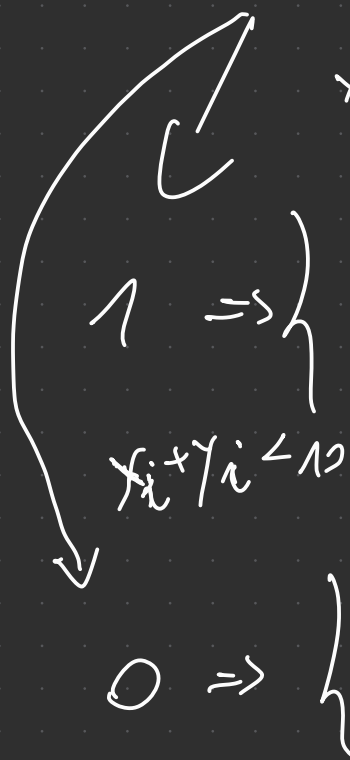
$$\begin{array}{r} 10011 \\ 1010 \\ \hline \cancel{X} 1101 \\ \quad \quad \quad \underbrace{\hspace{1cm}}_9 \end{array}$$

$$\begin{array}{r} 25 \\ \hline 16 = 9 \\ \hline (19+6) \end{array}$$

$$\begin{array}{r} 0011 + \\ 0110 \\ \hline 1001 \end{array}$$

* \rightarrow rez. intermediare al sumei

$$C^* + z_3^* \cdot z_2^* + z_3^* \cdot z_1^*$$



$$x_i + y_i \geq 10$$

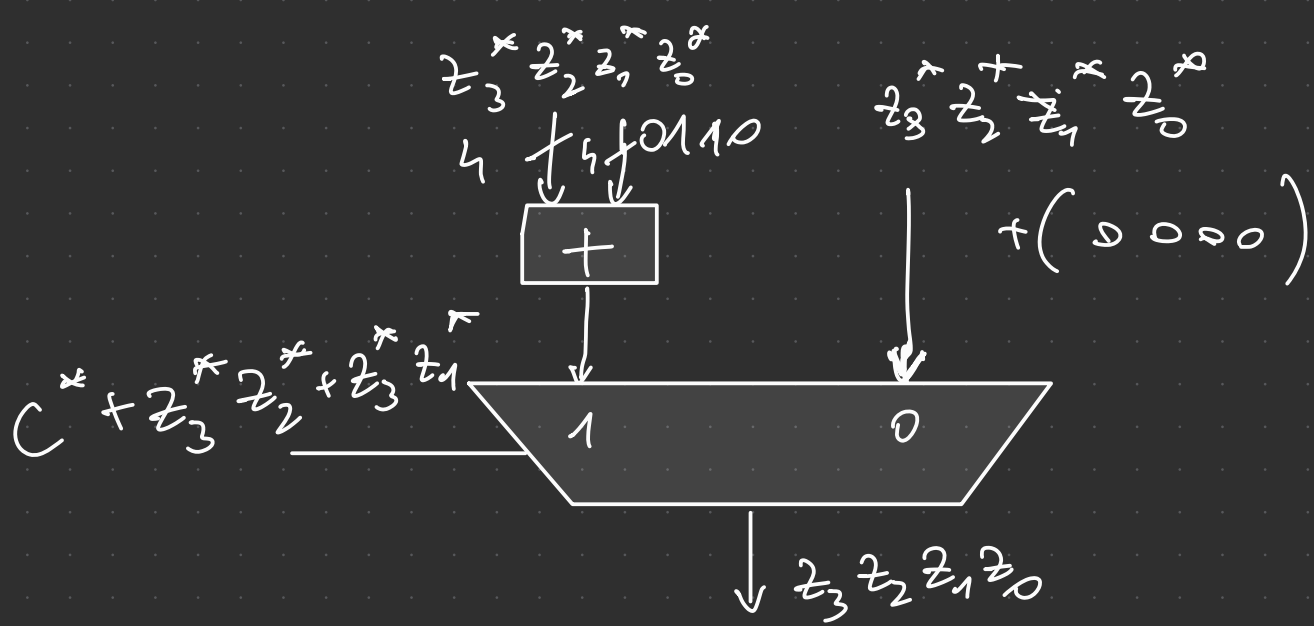
$$z_i = z_3^* z_2^* z_1^* z_0^* +$$

$$0 \quad 1 \quad 1 \quad 0$$

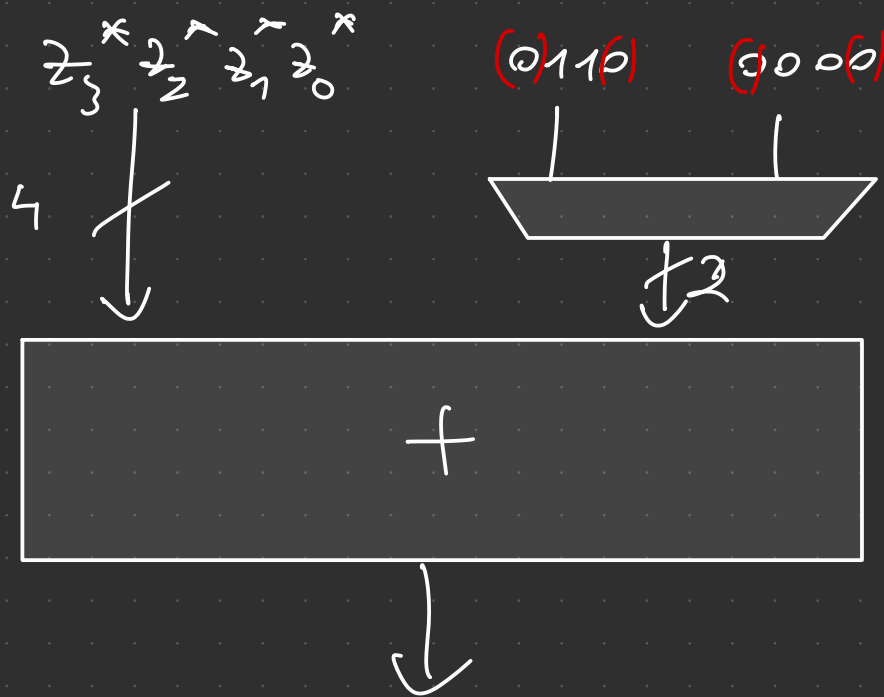
$$C_{i+1} = 1$$

$$z_i = z_3^* z_2^* z_1^* z_0^*$$

$$C_{i+1} = 0$$



→ max pt. suma pe care o vom aduna la z_3^* ---



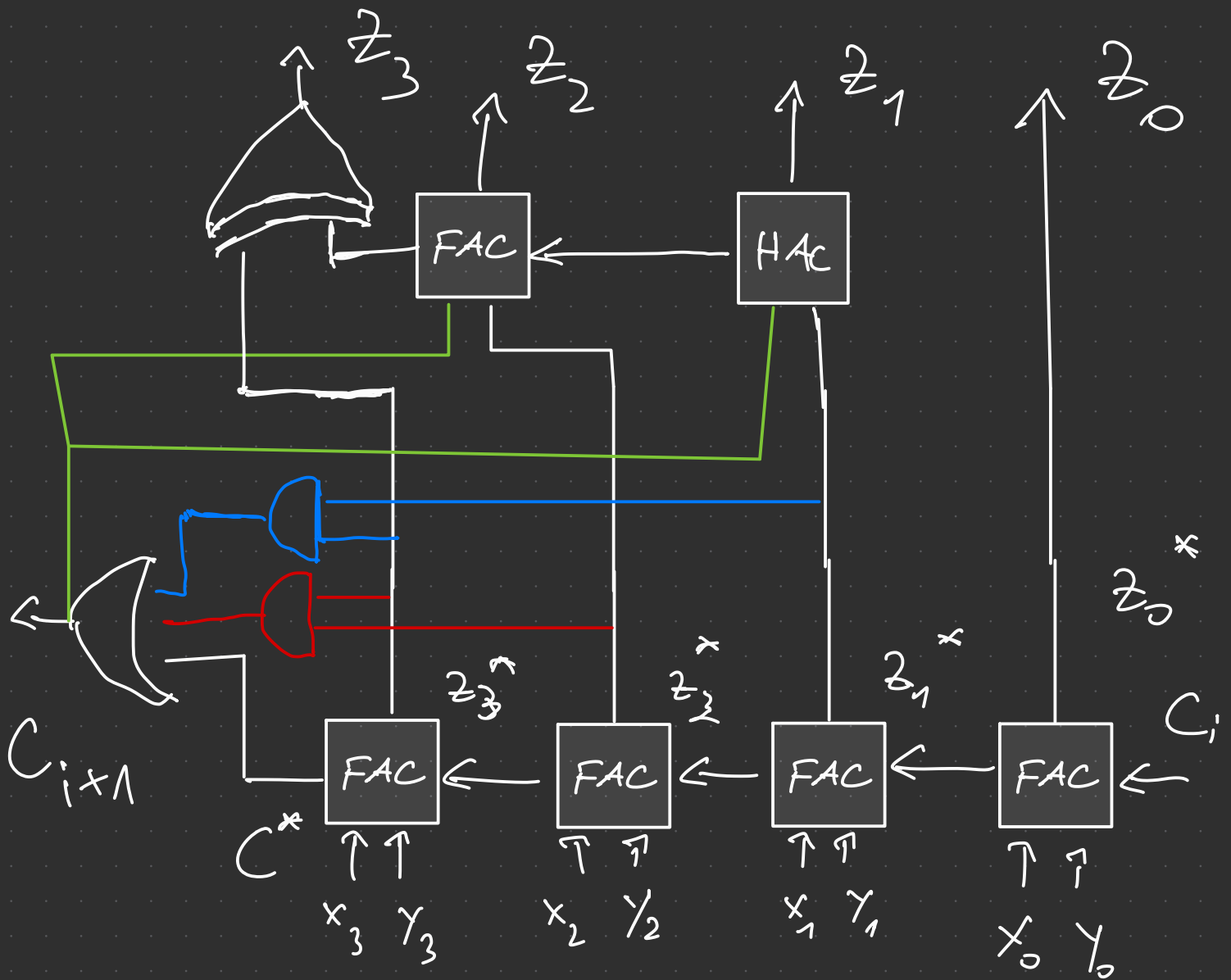
Etajul de corectie la însumarea a două cifre BCD :

$$z_i = z_3^* z_2^* z_1^* z_0^*$$

0 C_{i+1} C_{i+1} 0

$$C_{i+1} = C_i^* + z_3^* z_2^* + z_3^* z_1^*$$

1 DIGIT BCD ADDER



$$\begin{array}{r} 683 + \\ 197 \\ \hline 880 \end{array}$$

$$\begin{array}{r} 0110 \quad +1 \quad 1000 \quad +1 \quad 0011 \quad + \\ 0001 \quad 1001 \quad 0111 \\ \hline 0111 \quad 1000 \quad 1010 \quad +210 \\ 1000 \quad 0110 \quad 0110 \\ \hline \leftarrow 1000 \quad 0000 \\ \leftarrow \quad \leftarrow \quad \leftarrow \\ 8_{BCD} \quad 8_{BCD} \quad 0_{BCD} \end{array}$$

Exces de 3

Z_{E3}

$$X_1 = BCD$$

$$X_{E3} = \text{Exces de 3}$$

$$X_{E3} = X_1 + 3$$

$x_i + y_i \begin{cases} \geq 10 \Rightarrow \begin{cases} z_i = x_i + y_i - 10 \\ C_{i+1} = 1 \end{cases} \\ < 10 \Rightarrow \begin{cases} z_i = x_i + y_i \\ C_{i+1} = 0 \end{cases} \end{cases}$

$$\begin{aligned}
 & x_i + y_i \geq 10 \quad / +6 & x_{i \in E_3} + y_{i \in E_3} & \geq 16 \\
 & x_{i \in E_3} + y_{i \in E_3} = C'' z_3'' z_2'' z_1'' z_0'' \geq 16 = C'' & z_{i \in E_3} & = z_3'' z_2'' z_1'' z_0'' + \\
 & x_i + y_i \geq 10 & & \quad \quad \quad 0 \quad 0 \quad 1 \quad 1 \\
 & \begin{array}{l} C'' \nearrow 1 \\ \searrow 0 \end{array} \Rightarrow \left\{ \begin{array}{l} C_{i+1} = 1 \\ C_{i+1} = 0 \end{array} \right. & z_{i \in E_3} & = z_3'' z_2'' z_1'' z_0'' + \\
 & x_i + y_i \geq 10 & & \quad \quad \quad 1 \quad 1 \quad 0 \quad 1
 \end{aligned}$$