Round to Zero

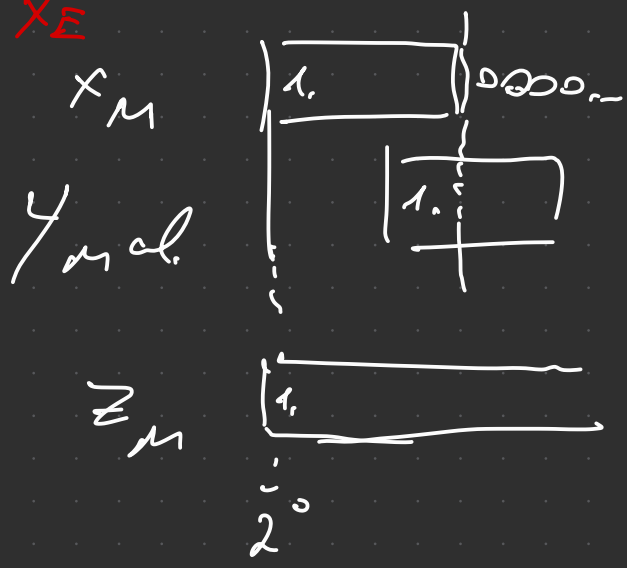
Significand x_M | 1. |

! m. fără semn

y_M aliniat

$$x+y = \left[x_M + y_M \cdot 2^{-(y_E - x_E)} \right] \cdot 2^{x_E}, \text{ if } x_E \geq y_E$$

$z_M \cdot 2^{x_E}$



Ex. $x = +3.625$

$x = 0011.101_{SM}$

$x^* = 0011 = 3$ (trunc)

$x = -5.625$

$x = 1101.101_{SM}$

$x^* = 1101_{SM} = -5$ (trunc)

$|x^*| \leq |x|$

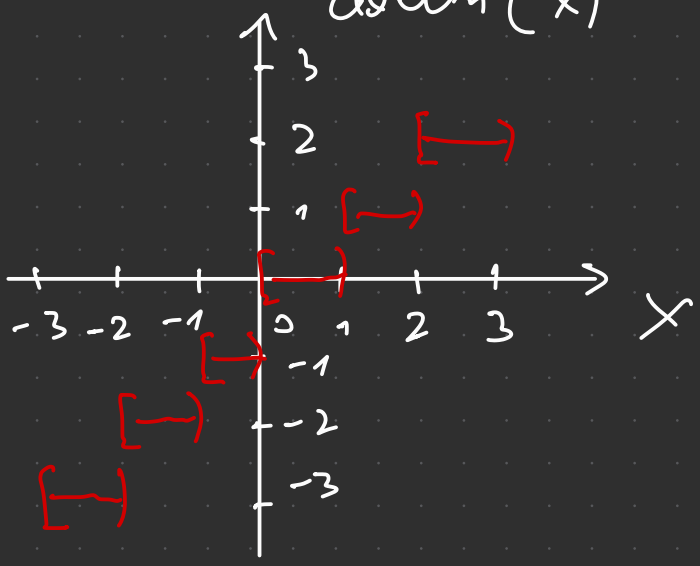
$3.625 \rightarrow 3$

$-5.625 \rightarrow -5$

(B) $x^* \leq x$ floor

$x \leq x^*$ ceil

odsum(x)



$-1.625 \rightarrow -2$

$1.625 \rightarrow 1$

C in C_2 !!!

$x = 3.625 = 0011.101_{C_2}$

$x^* = 3 = 0011_{C_2}$

$x = 5.625 = 0101.101_{C_2}$

$x = -5.625 = 1010.011_{C_2}$

$x^* = 1010_{C_2} \rightarrow x^* = -6$

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in SM if $x \leq 0$ x^* { $x_{n-1} x_{n-2} \cdots x_0$ if $0\, x_{-1} x_{-2} \cdots x_{-m} = 0$

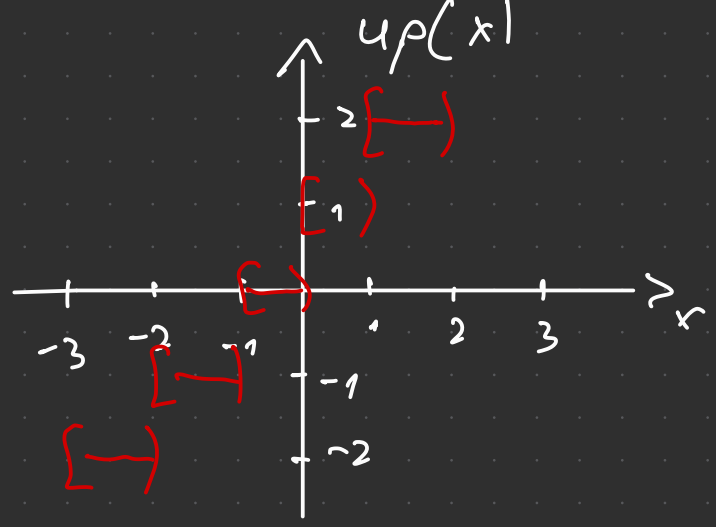
$x_{n-1} x_{n-2} \cdots x_0 . x_{-1} x_{-2} \cdots x_{-m} = 1$

if $x \geq 0$: $x_{n-1} x_{n-2} \cdots x_1 x_0$

(c) $x^* \geq x$

$+3.625 \to 4$

$-5.625 \to -5$



$$x^* \begin{cases} \text{if } x \leq 0 \\ \text{if } x > 0 \end{cases}$$

$$x^* = x_{n-1}\, x_{n-2} \cdots x_1\, x_0$$

$$\begin{cases} x^* = x_{n-1}\, x_{n-2} \cdots x_1\, x_0 \text{ if} \\ \quad \cdot x_{-1}\, x_{-2} \cdots = 0 \\ x^* = x_{n-1}\, x_{n-2} \cdots x_1\, x_0 + 1 \text{ if} \\ \quad \cdot x_{-1}\, x_{-2} \cdots \neq 0 \end{cases}$$

(d) rtn

round to nearest

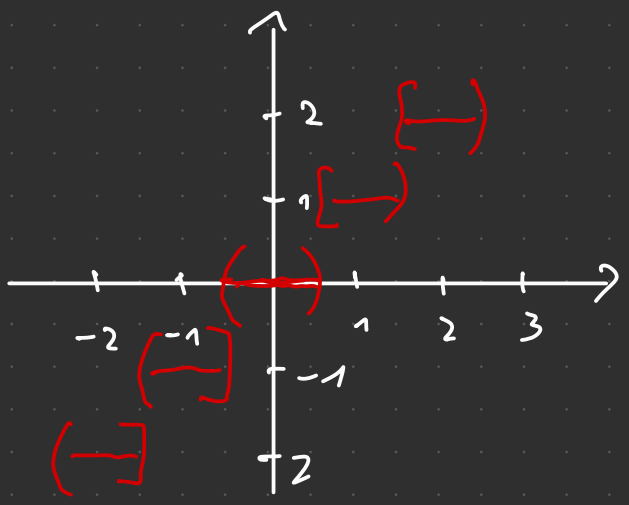
$+2.2 \to 2$

$+2.99 \to 3$

$+2.5 \to 3$

$+4.5 \to 5$

$$x^* = \begin{cases} x_{m-1}\, x_{m-2} \cdots x_0 \,, & \text{if } \cdot x_{-1} x_{-2} \cdots x_{-m} < \frac{1}{2} \\ x_{m-1} \cdots x_0 \,+1, & \text{if } \cdot x_{-1} x_{-2} \cdots \geq \frac{1}{2} \end{cases}$$



$$x = x_{m-1} \cdots x_1 x_0 \bullet x_{-1} x_{-2}$$

2^{-1} 2^{-2}

$\bullet x_{-1} x_{-2}$ $\qquad x^*$ $\qquad \varepsilon = x^* - x$

$\cdot 0_{(10)}$	0	0	$x_{m-1} \cdots x_1 x_0$,	0
$\cdot 25_{(10)}$	0	1	$x_{m-1} \cdots x_0 \bullet$	$-\frac{1}{4}$
$\cdot 5_{(10)}$	1	0	$x_{m-1} \cdots x_0 + 1$	$+\frac{1}{2}$
$\cdot 75_{(10)}$	1	1	$x_{m-1} \cdots x_0 + 1$	$+\frac{1}{4}$

$$\varepsilon_{medio} = \frac{0 + \frac{1}{4} + \frac{1}{2} + \frac{1}{4}}{4} = \frac{1}{8}$$

x_{-1}	x_{-2}	x_{-3}	ε
0	0	0	0
0	0	1	$-\frac{1}{8}$
0	1	0	$-\frac{1}{4}$
0	1	1	$-\frac{3}{8}$
1	0	0	$+\frac{1}{2}$
1	0	1	$+\frac{3}{8}$
1	1	0	$+\frac{1}{4}$
1	1	1	$+\frac{1}{8}$

$+\frac{1}{2}$ if $x_0 == 1$

$-\frac{1}{2}$ if $x_3 == 0$

$$\varepsilon_{medie} = \frac{0 - \frac{1}{8} - \frac{1}{4} - \frac{3}{8} + \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \frac{1}{8}}{8} = \frac{\frac{1}{2}}{8} = \frac{1}{16}$$

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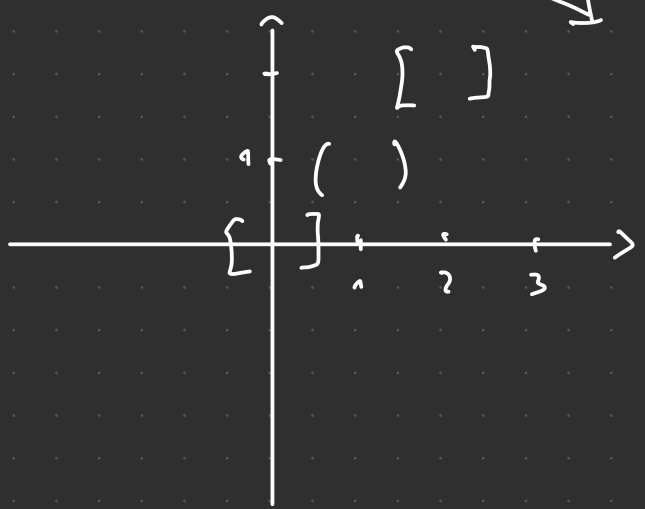
① rtne round to nearest even

5 → $-\frac{1}{2}$ $x_0 == 0$

5 → $+\frac{1}{2}$ $x_0 == 1$



Reguli de normalizare și rotunjirea unui rezultat f.p.

$x_S = 1.\, x_{m-2}\, x_{m-3} \dots x_1\, x_0$

$y_S = 1.\, y_{m-2}\, y_{m-3} \dots y_1\, y_0$

if $x_E \geq y_E$

sticky bits -> se păstrează precizia full

$z_S = x_S + y_S$ d. =>

$z_S = z_m\, z_{m-1}.\, z_{m-2}\, z_{m-3} \dots z_1 \boxed{g\,1\,1}$

$x_S = 1.\,0\,0\,0\,0\,1$

$y_{S\,d}$

$z_{S\,normalizat} = 1.\, z_{m-2_n}\, z_{m-3_n} \dots z_{1_n}\, z_{0_n} \,|\, RS$

rezultat FP (contin.)

$Z_{M_n} =$

Caz 1) $z_m = 1$	1. z_{m-2_n} z_{m-3_n} ... z_{1_n} z_{0_n}	R S	
	1. z_{m-1} z_{m-2} ... z_2 z_1	z_0	(g OR r OR s)
⇒ depl. dreapta 1 bit, Z_E++			
Caz 2) $z_m = 0$, $z_{m-1} = 1$	1. z_{m-2} z_{m-3} ... z_1 z_0	g	(r OR s)
⇒ Z_M este deja normalizat			
Caz 3) $z_m = 0$, $z_{m-1} = 0$, $z_{m-2} = 1$	1. z_{m-3} z_{m-4} ... z_0 g	r	s
⇒ depl. stânga 1 bit, Z_E--			
Caz 4) $z_m = 0$, $z_{m-1} = 0$, $z_{m-2} = 0$, $z_{m-3} = 1$	1. z_{m-4} z_{m-5} ... g 0	0	0
⇒ depl. stânga 1 bit, $Z_E- = 2$			

Rounding mode	$z_{s_n} \geq 0$
truncate	ignore R, S
down	
up	if (R or S) then $z_{s_n} + 1$
nearest even	if (R and (S or z_{0_n})) then $z_{s_n} + 1$

Rounding mode	$z_{s_n} < 0$
truncate	
down	if (R or S) then $z_{s_n} - 1$
up	
nearest even	if (R and (S or z_{0_n})) then $z_{s_n} - 1$

Adunarea / Scăderea FP

Format FP simplificat / redus

1 bit semn
3 exponent ($e=3$) → bias $2^{3-1}-1=3$
4 significand

$x = +6.5 = 110.1 \cdot 2^0 = 1.101 \cdot 2^2$

$y = -7 = -111.0 \cdot 2^0 = 1.110 \cdot 2^2$

x: 2+3=5

x					y				
0	1	0	1	1.101	1	1	0	1	1.110

Pasul (2) $d = x_E - y_E$

$d < 0 \rightarrow |x| < |y|$ $z_E = y_E$

$d \geq 0 \rightarrow z_E = x_E$

$d = 0$

$\rightarrow z_E = x_E = 5$

(3) if $sign(x) \neq sign(y)$ → complementare y_S

$y_{S_{C_2}} = 0.010$

④ Aliniere

$-x+y$

$-(x-y)$

|d|; if ③ compl. → introducere biți de 1

g, r, s

⑤ Adunare semnificand

x_s 1.101

y_s 0.010 | 0 0 0

$z_s = 0$ 1.111 | 0 0 0

negativ

g 2 s

$z_{s_{c_2}}$ 0.001 | 0 0 0

z r s

$= 1.000 | 0\ 0$

⑥ $z_{s\ c_2\ norm.}$

$z_E - = 3$

3 shiftări la dr

→ $z_E = 2$

⑦

⑧ Rotunjirea $z_{m_n} \rightarrow z^*_m$

$R \cdot (S + z_{0_m}) = 0(0+0) = 0$

⑨