Celula Sumatoare (FAC)

$$\begin{cases} z_i = x_i \oplus y_i \oplus c_i \\ c_{i+1} = x_i y_i + x_i c_i + y_i c_i \end{cases}$$

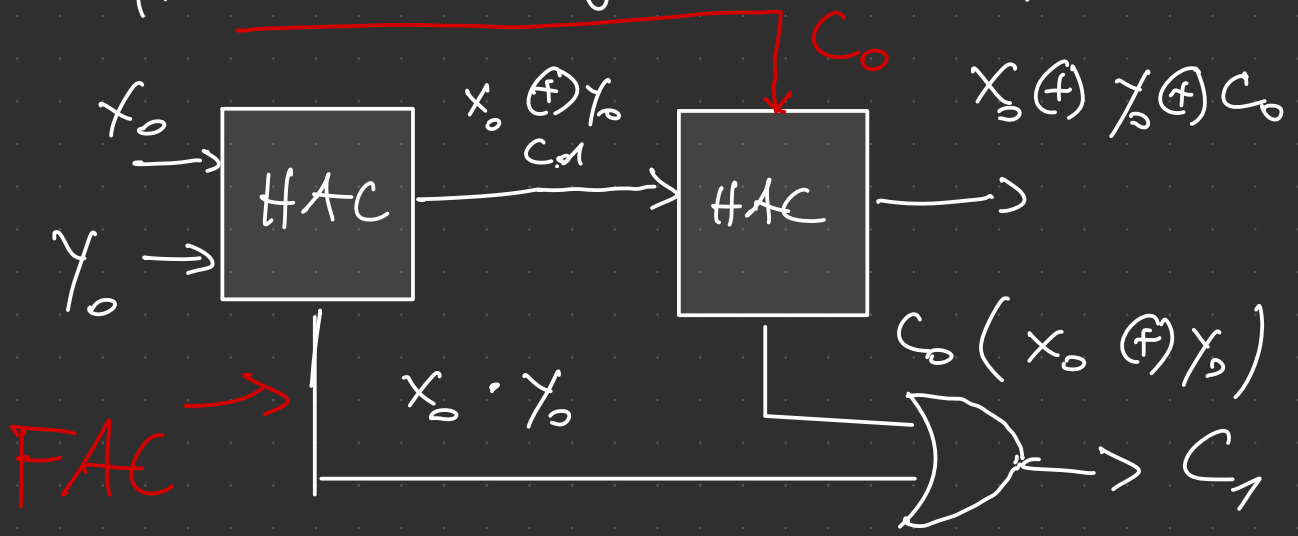
$$a \oplus b = a\bar{b} + \bar{a}b$$

$\oplus$ → doar 2 intrări

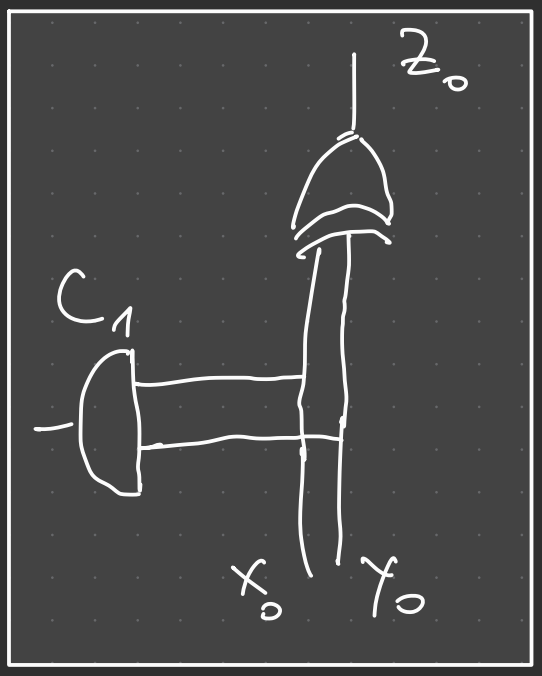
$$c_{in} = c_0 = 0$$

FAC: $\begin{cases} z_0 = x_0 \oplus y_0 \oplus 0 = x_0 \oplus y_0 \\ c_1 = x_0 y_0 + x_0 0 + y_0 0 = x_0 y_0 \end{cases}$

HAC (Half-Adder Cell)



① EXOR i ANI



② NOR

$$z_0 = x_0 \oplus y_0 = x_0 \overline{y_0} + \overline{y_0} x_0$$

$$= x_0 (\overline{x_0} + \overline{y_0}) + (\overline{x_0} + \overline{y_0}) y_0 =$$

$$= \overline{\overline{x_0 + y_0} + \overline{x_0 + y_0}}$$

$$C_1 = \overline{x_0 + y_0}$$

---

③ NANI

$$z_0 = x_0 \oplus y_0 = x_0 \overline{y_0} + \overline{x_0} y_0$$

$$= x_0 (\overline{x_0} + \overline{y_0}) + y_0 (\overline{x_0} + \overline{y_0})$$

$$= y_0 \overline{x_0 \cdot y_0} + y_0 \overline{x_0 y_0} =$$

$$= \overline{x_0 \overline{x_0 y_0}} \cdot \overline{y_0 \overline{x_0 y_0}}$$

$$C_1 = \overline{x_0 y_0}$$

CMOS

NAND → cea mai eficientă + viteză
↓
NOR

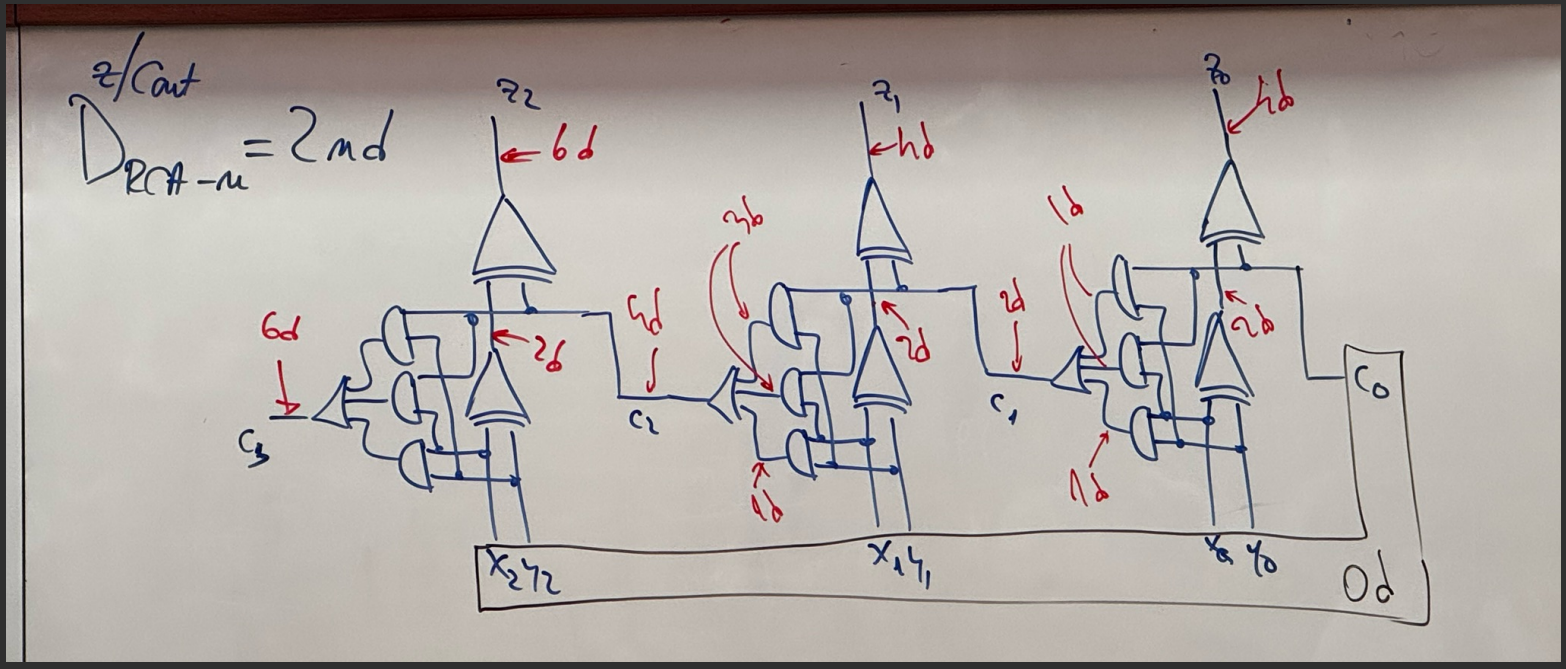
Calea critică

→ întârzierea maximă pe o cale
→ orice poartă primitivă → întârziere 1d

AND, NAND
OR, NOR } → 1d

NOT → 0d

EXOR → 2d



→ carry from MSB
→ overflow V
→ zero
→ negative

$x = x_{n-1}\ x_{n-2} \cdots x_1\ x_0$

$y = y_{n-1}\ y_{n-2} \cdots y_1\ y_0$

$z = z_{n-1}\ z_{n-2} \cdots z_1\ z_0$

---

x_i	y_i	c_i	z_i	V (overflow)
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	0
1	0	1	0	0
1	1	0	0	1
1	1	1	1	0

$$V = \overline{X_{n-1}}\ \overline{Y_{n-1}}\ C_{n-1} + X_{n-1} Y_{n-1} \overline{C_{n-1}}$$

$$I_1 = (A \oplus B) \cdot C = AC \oplus BC$$

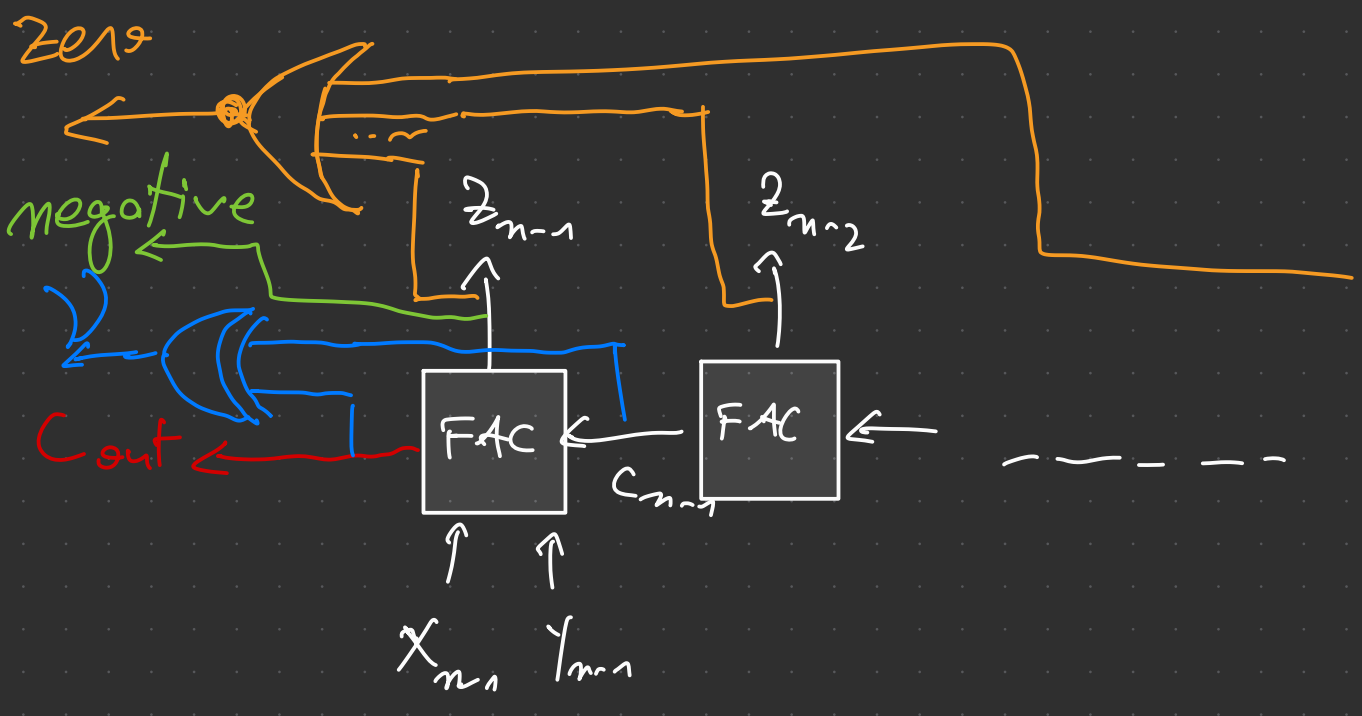
$$I_2 = A \oplus B = (A + B) \oplus AB$$

$$I_2' = A + B = A \oplus B \oplus AB$$

$$X \oplus 1 = \overline{X}$$

$$V = \overline{X_{n-1}}\ \overline{Y_{n-1}}\ C_{n-1} + X_{n-1} Y_{n-1} C_{n-1}$$

$$\boxed{V = C_n \oplus C_{n-1}}$$



Adunarea cu o constantă

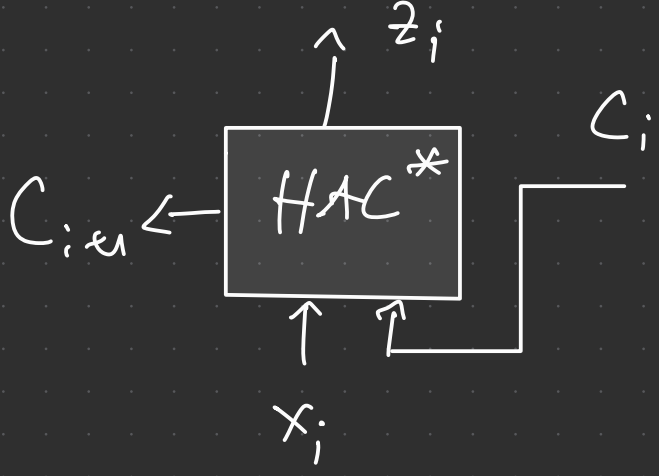
- se consideră constanta impară

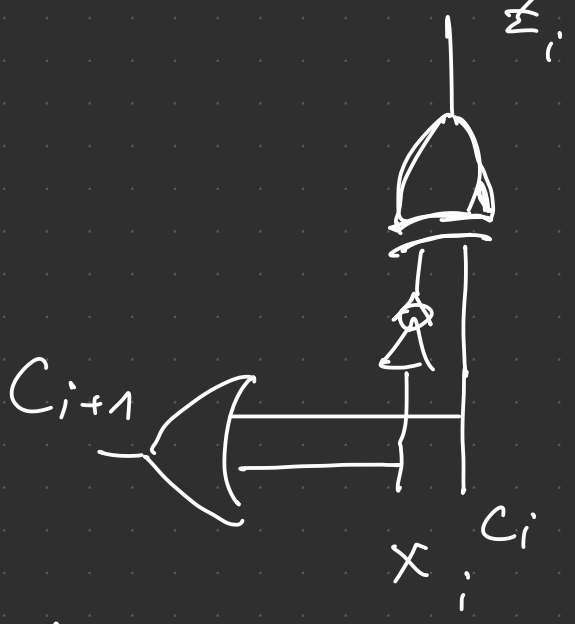
x, y pe n biți $\quad z = x + y$

$y \rightarrow$ constant

$$y_i = 0 \begin{cases} z_i = x_i \oplus c_i \\ c_{i+1} = x_i c_i \end{cases}$$

HAC





$k_i = 0 \rightarrow$ HAC

$k_i = 1 \rightarrow$ HAC*

$$\begin{cases} z_i = \overline{x_i} \oplus c_i \\ c_{i+1} = x_i + c_i \end{cases}$$

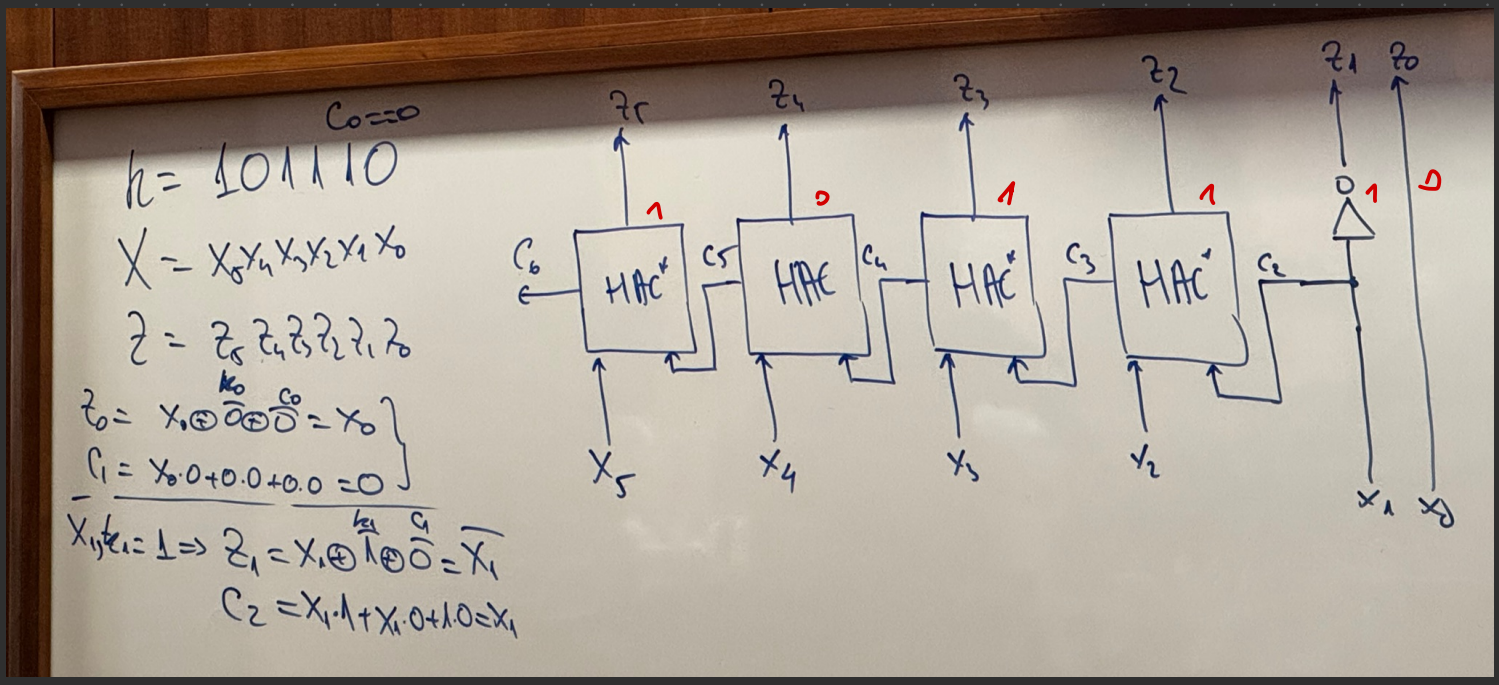
$k = 1\ 0\ 1\ 1\ 1\ 0$

$x = x_5\ x_4\ x_3\ x_2\ x_1\ x_0$

$z = z_5\ z_4\ z_3\ z_2\ z_1\ z_0$

$z_0 = x_0 \oplus 0 \oplus 0 = x_0$

$c_1 = x_0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 = 0$



Adunare BCD

$x_i + y_i <$ z_i cifră sumă
c_{i+1} carry

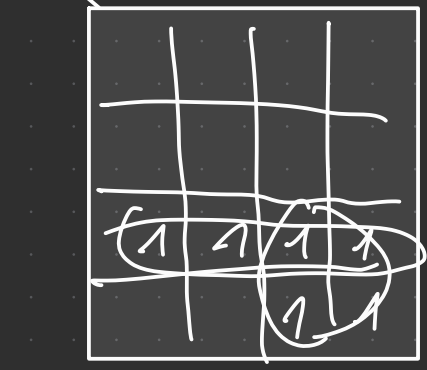
- $x_i + y_i < 10$ → $z_i = x_i + y_i$, $c_{i+1} = 0$
- $x_i + y_i \geq 10$ → $z_i = x_i + y_i - 10$, $c_{i+1} = 1$

-10 → correction

$x_i + y_i \geq 10$ $\quad C^* z_3^* z_2^* z_1^* z_0^* \geq 10$

$\begin{cases} 10 \leq \dots < 16 \\ 16 \leq \dots \end{cases}$, (C1) OR (C2)

(C1) $C^* == 0$; $10 \leq z_3 z_2 z_1 z_0 < 16$



$C^* (z_3^* z_2^* + z_3^* z_1^*)$

(C2) $C^* == 1$ $\quad x_i + y_i \geq 10$

$= C^* + z_3^* \cdot z_2^* + z_3^* z_1^*$

$x_i + y_i - 10$ -> cifră între 0 și 9 (19-10 = 9)

$(x_i + y_i - 10) \bmod 2^4$ (4 biți)

$+16 - 10 = +6$

13 - 1101 — 13+
10 1010 6
3 0011 19 %16 = 3
3

1101
0110
✗0011

19 - 10011 25 %16 = 9 0011+
10 1010 (19+6) 0110
9 ✗1101 1001
9

* → rez. intermediar al sumei

$$C^* + z_3^* \cdot z_2^* + z_3^* \cdot z_1^*$$

$x_i + y_i \geq 10$

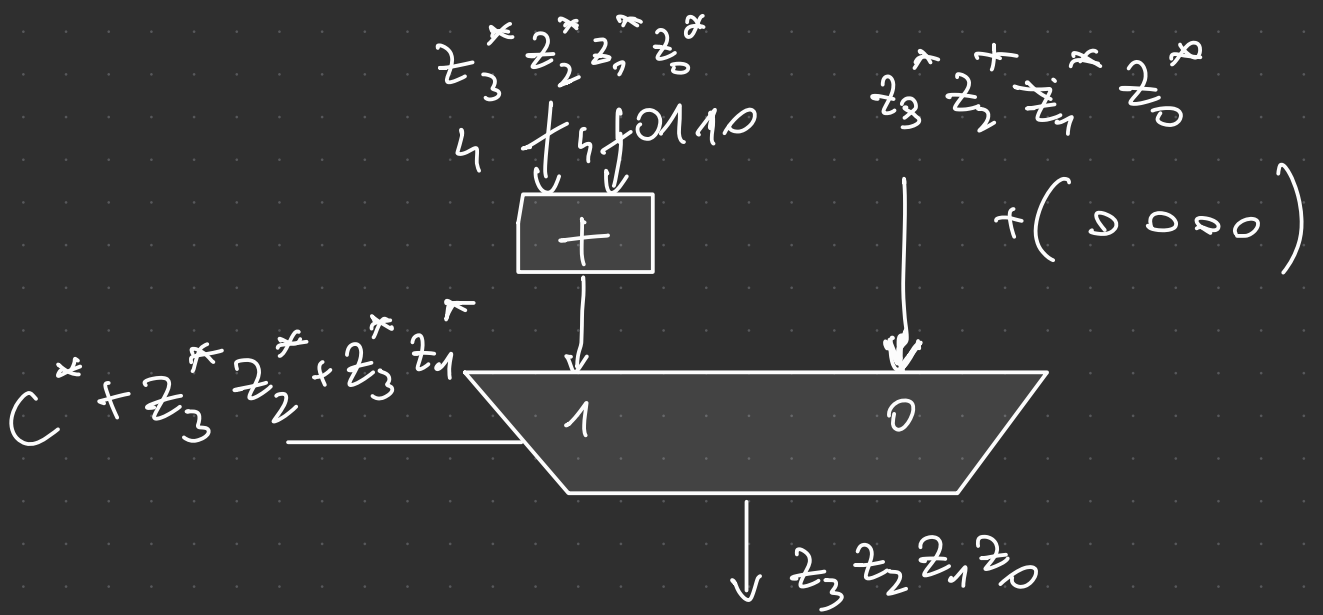
$z_i = z_3^* \; z_2^0 \; z_1^* \; z_0^* +$
$\quad\quad 0 \quad 1 \quad 1 \quad 0$

1 ⇒ $C_{i+1} = 1$

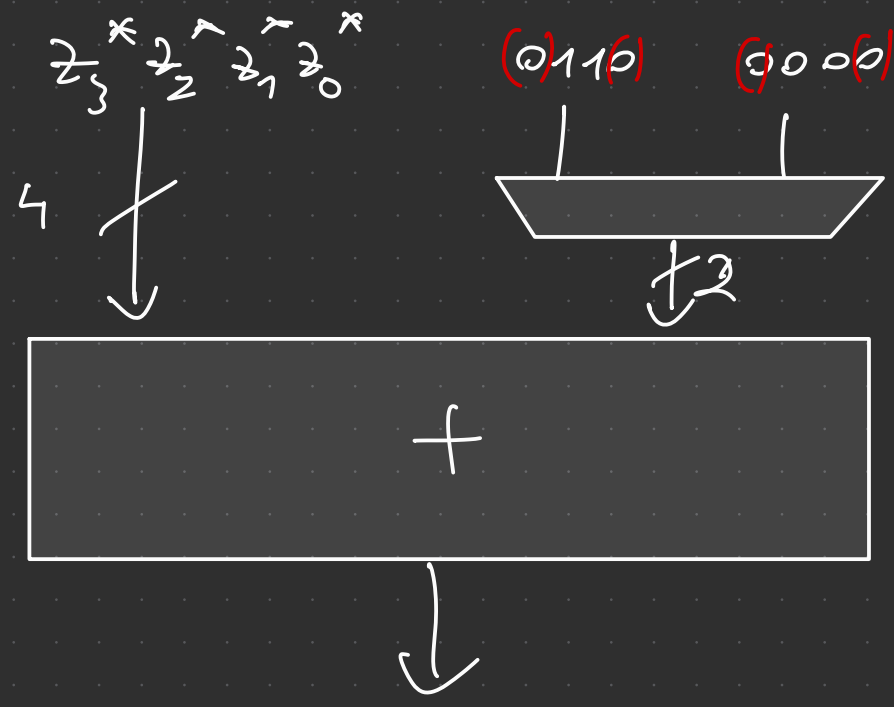
$z_i = z_3^* \; z_2^* \; z_1^* \; z_0^*$

$x_i + y_i < 10$

0 ⇒ $C_{i+1} = 0$



→ mux pt. suma pe care o vom aduna la z_3^* ---

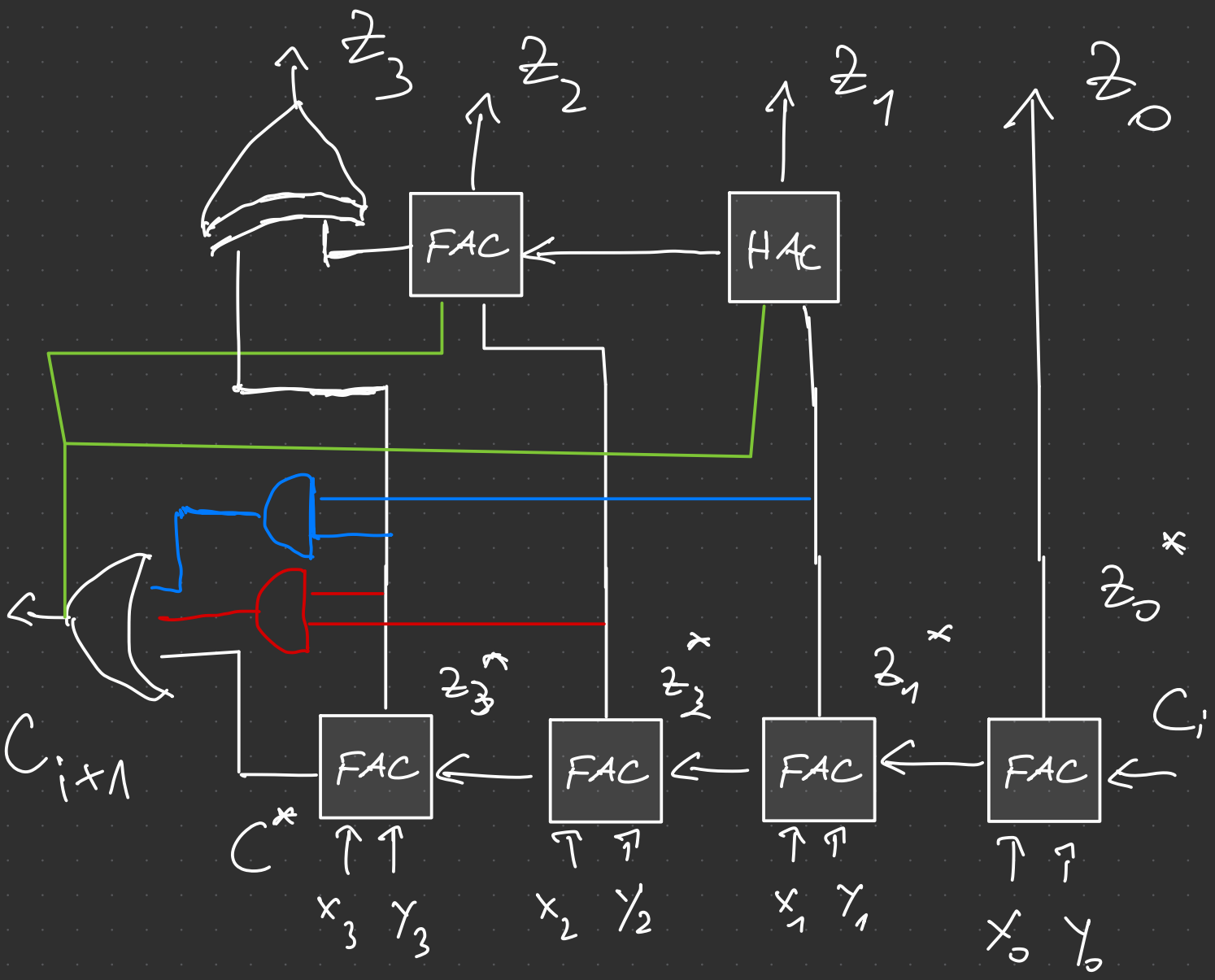


Etajul de corecție la însumarea a două cifre BCD:

$C_{i+1} = C_i^* + z_3^* \cdot z_2^* + z_3^* \cdot z_1^*$

z_i	=	z_3^*	z_2^*	z_1^*	z_0^*
		0	C_{i+1}	C_{i+1}	0

1 BİT BCD ADDER



$$
\begin{array}{r}
683+ \\
197 \\
\hline
880
\end{array}
\qquad
\begin{array}{r}
0110 \\
0001 \\
\hline
0111
\end{array}
\qquad
\begin{array}{r}
1000 \\
1001 \\
\hline
10010 \\
0110 \\
\hline
1000
\end{array}
\qquad
\begin{array}{r}
0011 \\
0111 \\
\hline
1010 \\
0110 \\
\hline
0000
\end{array}
$$

$+1$ $+1$ $+$ $+2_{10}$ >16

8_{BCD} $\longleftrightarrow$ 8_{BCD} $\longleftrightarrow$ 0_{BCD}

Exces de 3

$Z_{i\,E3}$

$x_i = BCD$

$x_{i\,E3} = \text{Exces de 3}$

$x_{i\,E_3} = x_i + 3$

$x_i + y_i \geq 10 \Rightarrow \begin{cases} z_i = x_i + y_i - 10 \\ C_{i+1} = 1 \end{cases}$

$z_{iE_3} = x_{iE_3} + y_{iE_3} - 13$

$(+3)\ \%16$

$x_i + y_i < 10 \Rightarrow \begin{cases} z_i = x_i + y_i \\ C_{i+1} = 0 \end{cases}$

$z_{iE_3} = x_{iE_3} + y_{iE_3} - 3$

$= (+13)\ \%16$

$x_i + y_i \geq 10 \ /+6 \qquad x_{iE_3} + y_{iE_3} \geq 16$

$x_{iE_3} + y_{iE_3} = C'' z_3'' z_2'' z_1'' z_0'' \geq 16 = C''$

$x_i + y_i \geq 10 \Rightarrow C'' = 1 \Rightarrow \begin{cases} z_{iE_3} = z_3'' z_2'' z_1'' z_0'' + \\ 0\ 0\ 1\ 1 \\ C_{i+1} = 1 \end{cases}$

$x_i + y_i < 10 \Rightarrow C'' = 0 \Rightarrow \begin{cases} z_{iE_3} = z_3'' z_2'' z_1'' z_0'' + \\ 1\ 1\ 0\ 1 \\ C_{i+1} = 0 \end{cases}$

$z_{iE_3} = \overline{C_{i+1}}\, z_3'' z_2'' z_1'' z_0'' + C_{i+1}\, 1$