

Round to Zero

Significant x_m

1.

! m. fano wenn

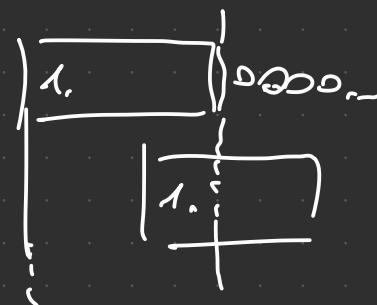
y_m abrät

$$x+y = \left[x_m + y_m \cdot 2^{\frac{(y_e - x_m)}{2}} \right] \cdot 2^{x_e}, \text{ if } x_e \geq y_e$$

$$\sum_m \cdot 2^{x_e}$$

$$x_m$$

y_m abrät



$$\sum_m$$



$$\bar{x} = +3.625$$

$$x = 0011.101$$

SM

$$\underline{x^* = 0011 = 3 \quad (\text{trunc})}$$

$$\underline{x = -5.625}$$

$$x = 1101.101 \text{ SM}$$

$$\underline{x^* = 1101_{\text{SM}} = -5 \quad (\text{fanc})}$$

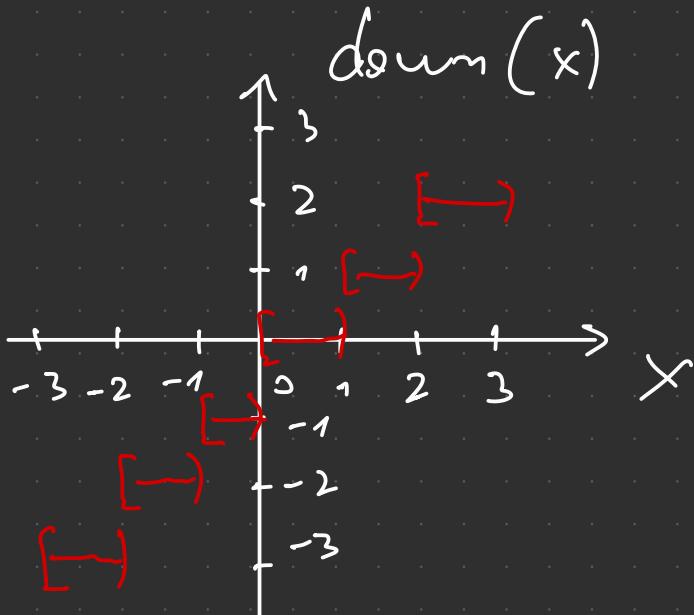
$$|x^*| \leq |x|$$

$$\begin{aligned} 3.625 &\rightarrow 3 \\ -5.625 &\rightarrow -5 \end{aligned}$$

B

$$x^* \leq x \quad \text{fuer}$$

$$x \leq x^* \quad \text{ceil}$$



$$-1.625 \rightarrow -2$$

\curvearrowleft 1.625 $\rightarrow 1$
in C_2 !!

$$\begin{aligned} x = 3.625 &= 0011.101_{C_2} \\ x^* = 3 &= 0011_{C_2} \end{aligned}$$

$$x = 5.625 = 10101.101_{C_2}$$

$$x = -5.625 = 1010.011_{C_2}$$

$$x^* = 1010_{C_2} \rightarrow x^* = -6$$

in SM

$$x \leq 0 \quad x^* \left\{ \begin{array}{l} x_{n-1} x_{n-2} \dots x_0 \quad \text{if} \\ 0 x_{n-1} x_{n-2} \dots x_{n-m}=0 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_{n-1} x_{n-2} \dots x_0 \quad \text{if} \\ -x_{n-1} x_{n-2} \dots x_1 x_0 = 0 \end{array} \right.$$

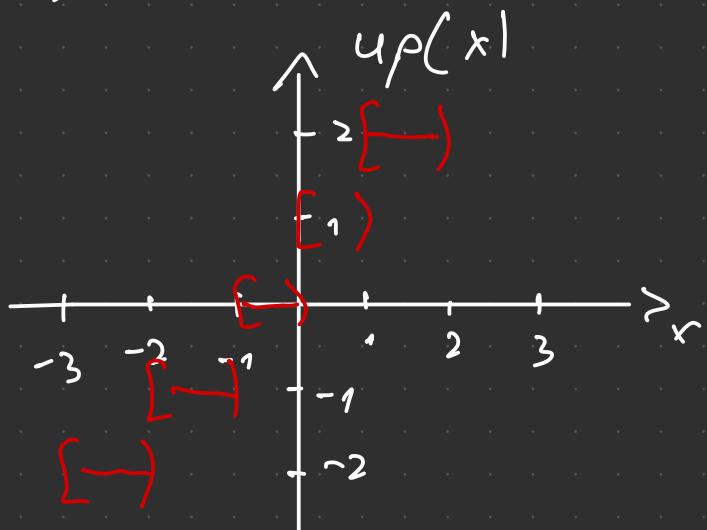
$$x \geq 0 : x_{n-1} x_{n-2} \dots x_1 x_0$$

(C)

$$x^* \geq x$$

$$+3.625 \rightarrow 4$$

$$-5.625 \rightarrow -5$$



~~x*~~
 if $x \leq 0$
 if $x > 0$

$$\left. \begin{array}{l} x^* = x_{n-1} x_{n-2} \dots x_1 x_0 \\ x^* = x_{n-1} x_{n-2} \dots x_1 x_0 \text{ if } \\ \quad x_1 x_2 \dots = 0 \\ x^* = x_{n-1} x_{n-2} \dots x_1 x_0 + 1 \text{ if } \\ \quad -x_{-1} x_2 \dots = 1 = 0 \end{array} \right\}$$

(D)

notn

round to nearest

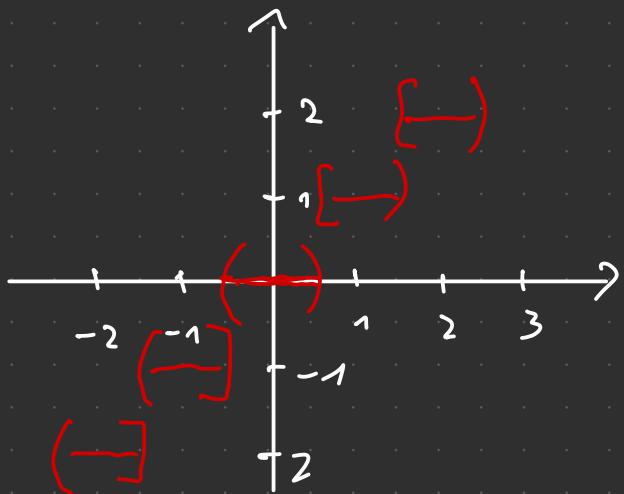
$$+2.2 \rightarrow 2$$

$$+2.99 \rightarrow 3$$

$$+2.5 \rightarrow 3$$

$$+4.5 \rightarrow 5$$

$$x^* = \begin{cases} x_{m-1} & \text{if } x_{m-1} - x_0 < \frac{1}{2} \\ x_{m-1} + 1 & \text{if } x_{m-1} - x_0 \geq \frac{1}{2} \end{cases}$$



$$x = x_{m-1} - x_0 \neq x_{-1} - x_{-2}$$

$$\begin{matrix} 2^{-1} & 2^{-2} \\ x_{-1} & x_{-2} \end{matrix} \quad x^*$$

$$\mathcal{E} = x^\infty - x$$

$$\begin{matrix} 0_{(10)} & 0 & 0 & x_{m-1} - x_0 & 0 \end{matrix}$$

$$\begin{matrix} 25_{(10)} & 0 & 1 & x_{m-1} - x_0 & -\frac{1}{4} \end{matrix}$$

$$\begin{matrix} 5_{(10)} & 1 & 0 & x_{m-1} - x_0 + 1 & +\frac{1}{2} \end{matrix}$$

$$\begin{matrix} 75_{(10)} & 1 & 1 & x_{m-1} - x_0 + 1 & +\frac{1}{4} \end{matrix}$$

$$\mathcal{E}_{\text{medie}} = \frac{\rho - \frac{1}{4} + \frac{1}{2} + \frac{1}{4}}{4} = \frac{1}{8}$$

x_{-1}	x_{-2}	x_{-3}	ϵ
0	0	0	0
0	0	1	$-\frac{1}{8}$
0	1	0	$-\frac{1}{4}$
0	1	1	$-\frac{3}{8}$
1	0	0	$+\frac{1}{2}$
1	0	1	$+\frac{3}{8}$
1	1	0	$+\frac{1}{4}$
1	1	1	$+\frac{1}{8}$

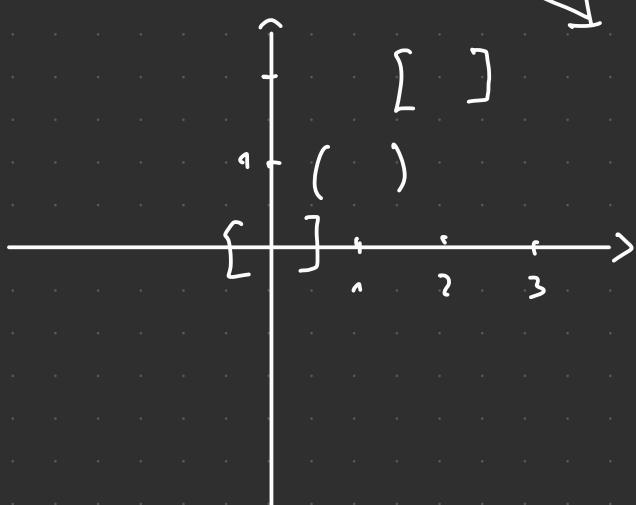
$\leftarrow +\frac{1}{2} \quad \text{if } x_0 = 1$
 $\leftarrow -\frac{1}{2} \quad \text{if } x_0 = 0$

$$\epsilon_{\text{media}} = \frac{0 - \cancel{\frac{1}{8}} - \cancel{\frac{1}{2}} - \cancel{\frac{3}{8}} + \cancel{\frac{1}{2}} + \cancel{\frac{3}{8}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{8}}}{8} = \frac{1}{2} - \frac{1}{8} = \frac{1}{16}$$

(1) note round to nearest even

$$0.5 \quad \xrightarrow{-\frac{1}{2}} \quad x_0 = 0$$

$$0.5 \quad \xrightarrow{+\frac{1}{2}} \quad x_0 = 1$$



Reguli de normalizare și rotunjirea unui rezultat f.p.

$$x_s = 1. \ x_{m-2} \ x_{m-3} \dots \ x_1 \ x_0$$

$$y_s = 1. \ y_{m-2} \ y_{m-3} \dots \ y_1 \ y_0$$

$$\text{f } x_E \geq y_E$$



sticky bits \rightarrow se păstrează precizia full

$$z_s = x_s + y_s \text{ al.} \Rightarrow$$

$$z_s = z_m z_{m-1} \dots z_2 z_1 z_0$$

$$x_s = 1. \ 0000 \ 1$$

$$y_s =$$

$$z_s \text{ normalize} = 1. \ z_{m-2} z_{m-3} \dots z_1 z_0 | RS$$

$Z_{M_n} =$	Rezultat FP (contin.)					
Caz 1) $z_m = 1$	1. z_{m-2n} z_{m-3n} ... z_{1n} z_{0n} R S					
\Rightarrow depl. dreapta 1 bit, $Z_E ++$	1. z_{m-1} z_{m-2} ... z_2 z_1 z_0 (g OR r OR s)					
Caz 2) $z_m = 0, z_{m-1} = 1$	1. z_{m-2} z_{m-3} ... z_1 z_0 g (r OR s)					
$\Rightarrow Z_M$ este deja normalizat						
Caz 3) $z_m = 0, z_{m-1} = 0, z_{m-2} = 1$	1. z_{m-3} z_{m-4} ... z_0 g r s					
\Rightarrow depl. stânga 1 bit, $Z_E --$						
$z_m = 0, z_{m-1} = 0, z_{m-2} = 0, z_{m-3} = 1$	1. z_{m-4} z_{m-5} ... g 0 0 0					
\Rightarrow depl. stânga 1 bit, $Z_E - = 2$						

Rounding mode

truncate	if $z_{sm} \geq 0$
down	ignore R, S
up	if (R or S) then $z_{sm} + 1$
nearest even	if (R and (S or z_{0n})) then $z_{sm} + 1$

Rounding mode

truncate	if $z_{sm} < 0$
down	if (R or S) then $z_{sm} - 1$
up	if (R and (S or z_{0n})) then $z_{sm} - 1$
nearest even	

Adunarea / Scăderea FP

Format FP simplificat / redus

1 bit semn

3 exponent ($e=3$) \rightarrow bias $2^{3-1} = 3$

4 significand

$$x = +6.5 = 110.1 \cdot 2^0 = 1.101 \cdot 2^2$$

$$y = -7 = -111.0 \cdot 2^0 = 1.110 \cdot 2^2$$

x y

$0 101 1101$	$1 101 1110$
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Pasul ②

$$d = X_E - Y_E$$

$$d < 0 \rightarrow |x| < |y| \quad Z_E = Y_E$$

$$d \geq 0 \rightarrow Z_E = X_E$$

$$d = 0$$

$$\rightarrow Z_E = X_E = 5$$

③ if $\text{sign}(x) \neq \text{sign}(y) \rightarrow$ complementare Y_S

$$Y_{SC_2} = 0.010$$

(2) Adunarea

| d | ; if (3) compl. \rightarrow introducere biti de 1

$$\begin{array}{r} -x + y \\ -(x - y) \end{array}$$

f_1, f_2, s

(5) Adunare significand

$$x_s \quad 1.101$$

$$y_s \quad 0.010 \quad 0 \quad 0$$

$$z_s = 0 \underbrace{1.111}_{\text{negative}} \quad 0 \quad 0$$

$$z_{sc_2} \quad \begin{matrix} & f & 2 & s \\ 0.001 & | & 0 & 0 & 0 \end{matrix}$$

$$z_s = 1.000 \quad 0 \quad 0$$

c_2 norm.

$$z_E = 3$$

3 shifteaza la

$$\rightarrow \boxed{z_E = 2}$$

(6)

Rezumarea $z_m \rightarrow z^*_m$

$$R \cdot (S + z_{om}) = 0(0+0) = 0$$

(7)

⑨

Verifcare: $Z = \underline{\text{1010100000}}$

$$Z = (-1)^1 \cdot 2^{2-3} \cdot 1.000 = -2^1 \cdot 1.0 = -1 = 0.5$$

$$\left. \begin{array}{l} Z_{2m} = z_2 \cdot l_{10} + z_1 \cdot l_1 + z_0 \cdot l_0 + g \cdot l_3 + z_3 \cdot r_1 \\ Z_{1m} = z_1 \cdot l_{10} + z_0 \cdot l_1 + g \cdot l_2 + z_2 \cdot r_1 \\ Z_{0m} = z_0 \cdot l_{10} + g \cdot l_1 + z_1 \cdot r_1 \\ R = g \cdot l_{10} + r \cdot l_1 + (g \oplus r \oplus d) \cdot r_1 \\ S = (r \oplus d) \cdot l_{10} + d \cdot l_1 \end{array} \right\}$$

Dacă una este activă la un moment dat

$\rightarrow \cancel{\text{max}}$ Priority Encoder

encoded $S_2 \quad S_1 \quad S_0$

⑤	$Z_3 = Z_4 \cdot Z_5$	Z_2	Z_1	Z_0	g	R	S
Q1	$1) Z_3 = 1 \Rightarrow 1$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$(g \oplus r \oplus d)$
Z_4	$2) Z_4 = 0, Z_5 = 1$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$(r \oplus s)$
Z_5	$3) Z_4 = 0, Z_5 = 0$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	d
l_1	$z_2 = 1 \ll 1$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$(r \oplus s)$
l_2	$4) Z_4 = 0, Z_5 = 1 \ll 2$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	d
l_3	$5) Z_4 = 0, Z_5 = 0, Z_2 = 0, Z_1 = 0, Z_0 = 1 \ll 3$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	0

Mutually
exclusive

↑
encoded

on S_2, S_1, S_0

r_1	l_3	l_2	l_1	l/l_{10}	S_2	S_1	S_0
0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	1	0	0	0	0	1	1
1	0	0	0	0	1	0/d	0/d

Capitolul 4

Analiza și sinteza dispozitivelor
de înmulțire binară

$$X * Y = X_m * Y_m * 2^{(X_E + Y_E)}$$

$$X_m = (1 \text{ } \underline{\text{hidden bit}} \text{ } 1 \text{ } 0 \text{ } 1 \dots)$$

Metode de înmulțire

$$P = X * Y, \text{ de înmulțit}$$

immulțitor

$$X = 15_{(10)} = 1111_{(2)}$$

$$Y = 22_{(10)} = 10110_{(2)}$$

a) Paper & pencil

$$\begin{array}{r} 10110 \\ - \quad - \quad - \quad - \quad - \quad Y \\ 1111 \quad x_3 x_2 x_1 x_0 \quad - \quad - \quad X \\ \hline \end{array}$$

$$\begin{array}{r} 10110 \quad - \quad - \quad - \quad - \quad x_0 \cdot y \cdot 2^0 \\ 10110 \quad - \quad - \quad - \quad - \quad x_1 \cdot y \cdot 2^1 \\ 10110 \quad - \quad - \quad - \quad - \quad x_2 \cdot y \cdot 2^2 \\ 10110 \quad - \quad - \quad - \quad - \quad x_3 \cdot y \cdot 2^3 \\ \hline \end{array}$$

$$P = \sum_{i=0}^3 x_i y \cdot 2^i$$

$$\begin{array}{r} 101001010 \\ \curvearrowleft +^2 +^1 +^2 +^1 +^1 \\ \hline 330 \rightarrow 22 \cdot 15 \end{array}$$

$$n + r = 9 \text{ bits}$$

Max Product bits $\rightarrow m + n$

b) keep the partial products fixed

$$P_{j+n} = P_j + x_i y \cdot 2^i \quad i \geq 0, P_0 = 0$$

\rightarrow Summator per $(m \times n)$ bit

c) keep the 1 bit products fixed



1 0 1 1 0

1 1 1 1

$$\begin{array}{r}
 & 1 0 1 1 0 \\
 + & 1 1 1 1 \\
 \hline
 & 0 0 0 0 0 \\
 & 1 0 1 1 0
 \end{array}
 \quad P_0 = 0$$

$$P_0 = 0$$

$$x_0 \cdot y$$

$$\begin{array}{r}
 & 1 0 1 1 0 \\
 + & \boxed{0} \overrightarrow{1 0 1 1 0} \\
 \hline
 & 1 0 1 1 0
 \end{array}
 \quad P_{0,0} = P_0 + x_0 \cdot y$$

$$P_1 = P_0 \times 2^{-1}$$

$$x_1 \cdot y$$

$$1 0 0 0 0 1 0 \quad P_1 = P_1 + x_1 \cdot y$$

$$\begin{array}{r}
 + 1 0 0 0 0 1 0 \\
 \hline
 & 1 0 1 1 0
 \end{array}
 \quad P_2 = P_1 \times 2^{-1}$$

$$x_2 \cdot y$$

$$1 0 0 1 1 0 1 0 \quad P_2 = P_2 + x_2 \cdot y$$

$$\begin{array}{r}
 + 1 0 0 1 1 0 1 0 \\
 \hline
 & 1 0 1 1 0
 \end{array}
 \quad P_3 = P_2 \times 2^{-1}$$

$$x_3 \cdot y$$

$$1 0 1 0 0 1 0 1 0 \quad P_3 = P_3 + x_3 \cdot y$$

$$P_4 = P_3 \times 2^{-1} = P$$

1 0 1 0 0 1 0 1 0

Suma totală va trebui să fie pe lungimea lui Y
 $\rightarrow M_r$ iteratii X

Sumă diferențială binară secvențială pentru SM

x, y în SM 8 Bits

$$x = \underbrace{x_7 \cdot x_6 \cdots x_0}_{\text{sign}} + \underbrace{2^{-1} \cdot 2^{-2} \cdots 2^{-7}}_{\text{weights}}$$

($7 + 7 = 14$)

$$P = P_{15} \cdot P_{14} \cdots P_0$$

$\underbrace{\phantom{P_{15} \cdot P_{14} \cdots P_0}}_{X \oplus Y}$

\downarrow

0

15 biti produs
adăugat 0 la final
pf. a fi 16