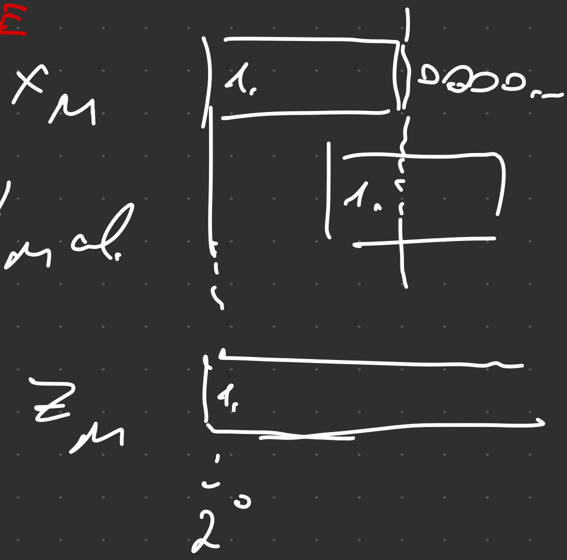
Round to Zero

Significand x_M $\boxed{1.\qquad\qquad}$

!nr. fără semn

$$x+y = \underbrace{\Big[x_M + \overbrace{y_M \cdot 2^{(y_E - x_E)}}^{y_M \text{ aliniat}}\Big] \cdot 2^{x_E}}_{Z_M \cdot 2^{x_E}}, \text{ if } x_E \geq y_E$$



Ex. $x = +3.625$

$x = 0011.1\cancel{01}_{SM}$

$x^* = 0011 = 3$ (trunc)

$x = -5.625$

$x = 1101.101_{SM}$

$x^* = 1101_{SM} = -5$ (trunc)

$$|x^*| \le |x|$$

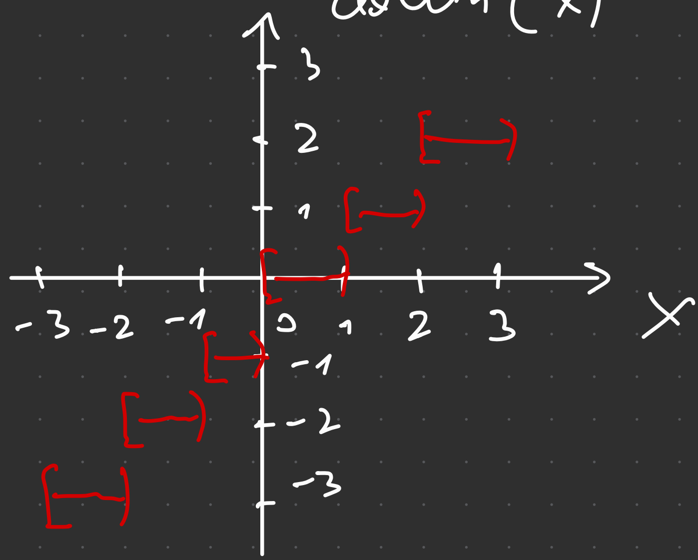
$3.625 \to 3$

$-5.625 \to -5$

(B) $x^* \le x$ floor rotunjire în jos

$x \le x^*$ ceil

down(x)



$-1.625 \to -2$

$1.625 \to 1$

în C_2 !!!

$x = 3.625 = 0011.101_{C_2}$

$x^* = 3 = 0011_{C_2}$

$x = 5.625 = 0101.101_{C_2}$

$x = -5.625 = 1010.\cancel{011}_{C_2}$

$x^* = 1010_{C_2} \to \boxed{x^* = -6}$

în SM if $x \le 0$

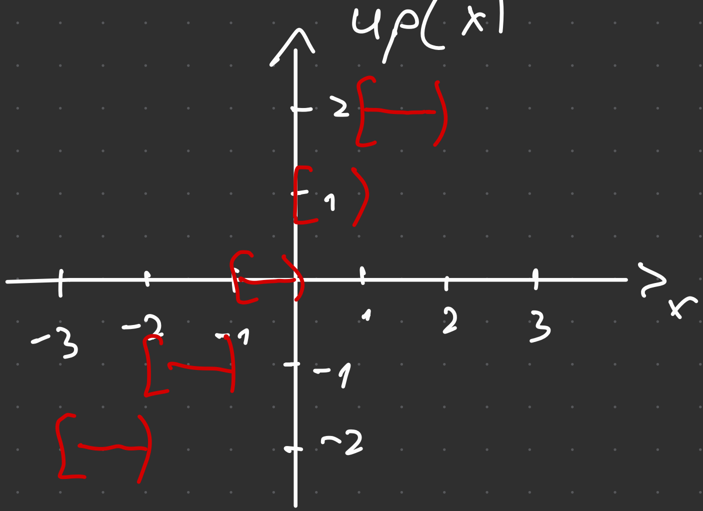
$$x^* = \begin{cases} x_{n-1} x_{n-2} \dots x_0 & \text{if } .x_{-1} x_{-2} \dots x_{-m} == 0 \\ x_{n-1} x_{n-2} \dots x_0 - 1 & \text{if } .x_{-1} x_{-2} \dots x_{-m} != 0 \end{cases}$$

if $x \ge 0$: $x_{n-1} x_{n-2} \dots x_1 x_0$

(c) $x^* \geq x$

$+3.625 \to 4$

$-5.625 \to -5$



$$x^* \begin{cases} \text{if } x \leq 0 \\ \text{if } x > 0 \end{cases}$$

$$x^* = x_{n-1}\, x_{n-2} \dots x_1\, x_0$$

$$\begin{cases} x^* = x_{n-1}\, x_{n-2} \dots x_1\, x_0 \text{ if } \\ .x_{-1}\, x_{-2} \dots == 0 \\ x^* = x_{n-1}\, x_{n-2} \dots x_1\, x_0 + 1 \text{ if } \\ .x_{-1}\, x_{-2} \dots \neq 0 \end{cases}$$

(d) rtn

round to nearest

$+2.2 \to 2$

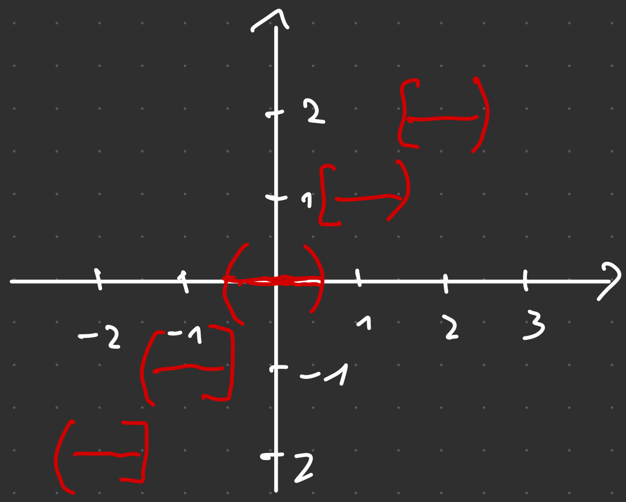
$+2.99 \to 3$

$+2.5 \to 3$

$+4.5 \to 5$

if $x \geq 0$

$$x^* = \begin{cases} x_{n-1}\, x_{n-2} \dots x_0, & \text{if } .x_{-1}\, x_{-2} \dots x_{-m} < \frac{1}{2} \\ x_{n-1} \dots x_0 + 1, & \text{if } .x_{-1}\, x_{-2} \dots \geq \frac{1}{2} \end{cases}$$



$$x = x_{n-1} \dots x_1\, x_0 \,.\, x_{-1}\, x_{-2}$$

$.x_{-1}$ (2^{-1})	x_{-2} (2^{-2})	x^*	$\varepsilon = x^* - x$
.0(10) 0	0	$x_{n-1} \dots x_1 x_0$,	0
.25(10) 0	1	$x_{n-1} \dots x_0$.	$-\frac{1}{4}$
.5(10) 1	0	$x_{n-1} \dots x_0 + 1$	$+\frac{1}{2}$
.75(10) 1	1	$x_{n-1} \dots x_0 + 1$	$+\frac{1}{4}$

$$\varepsilon_{medie} = \frac{0 - \cancel{\frac{1}{4}} + \frac{1}{2} + \cancel{\frac{1}{4}}}{4} = \frac{1}{8}$$

x_{-1}	x_{-2}	x_{-3}	ε
0	0	0	0
0	0	1	$-\frac{1}{8}$
0	1	0	$-\frac{1}{4}$
0	1	1	$-\frac{3}{8}$
1	0	0	$+\frac{1}{2}$
1	0	1	$+\frac{3}{8}$
1	1	0	$+\frac{1}{4}$
1	1	1	$+\frac{1}{8}$

$+\frac{1}{2}$ if $x_0 == 1$

$-\frac{1}{2}$ if $x_0 == 0$

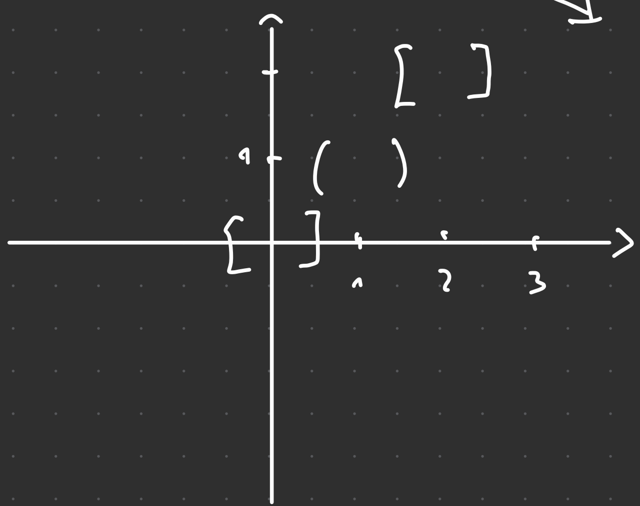
$$\varepsilon_{medie} = \frac{0 \cancel{-\frac{1}{8}} \cancel{-\frac{1}{4}} \cancel{-\frac{3}{8}} + \frac{1}{2} \cancel{+\frac{3}{8}} \cancel{+\frac{1}{4}} \cancel{+\frac{1}{8}}}{8} = \frac{\frac{1}{2}}{8} = \frac{1}{16}$$

---

(5) rtn e round to nearest even

$.5 \to -\frac{1}{2}$ $x_0 == 0$

$.5 \to +\frac{1}{2}$ $x_0 == 1$

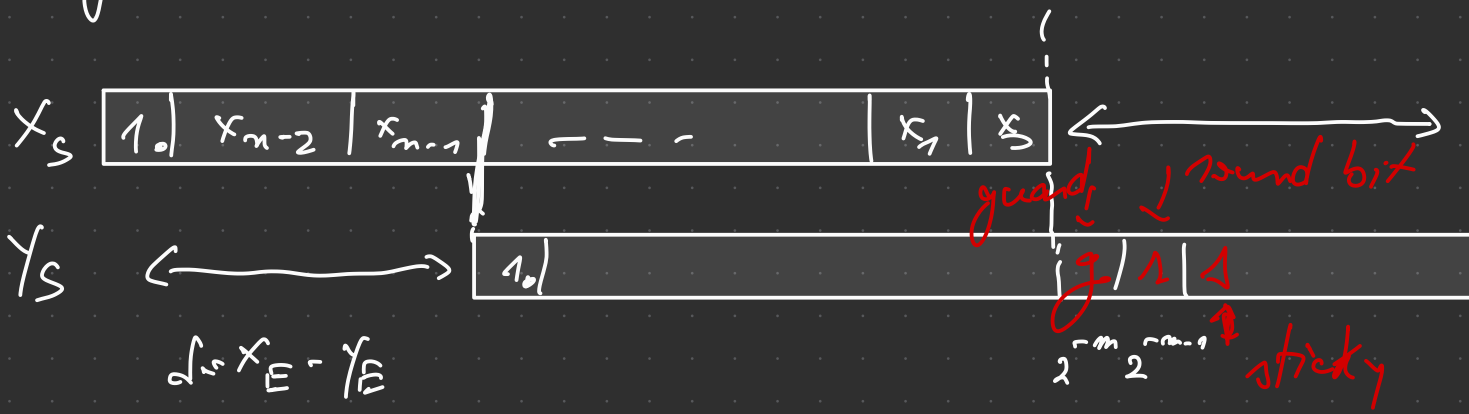


Reguli de normalizare și rotunjirea unui rezultat f.p.

$x_S = 1.\ x_{n-2}\ x_{n-3} \dots x_1\ x_0$

$y_S = 1.\ y_{n-2}\ y_{n-3} \dots y_1\ y_0$

if $\quad x_E \geq y_E$



sticky bits $\rightarrow$ se păstrează precizie full

$Z_S = X_S + Y_{S\,d.} \Rightarrow$

$Z_S = z_n\ z_{n-1} . z_{n-2}\ z_{n-3} \dots z_1 \boxed{g\ r\ s}$

$x_S = 1.00001$

$Y_{Sd} =$

$Z_{S\,normalizat} = 1.\ z_{n-2_n}\ z_{n-3_n} \dots z_{1_n}\ z_{0_n} \mid RS$

...are: ...ezultat FP (contin.)

$Z_{M_n} =$	$1.\ z_{m-2_n}\ z_{m-3_n}\ \dots\ z_{1_n}\ z_{0_n} \mid R\ \ S$
Caz 1) $z_m = 1$	$1.\ z_{m-1}\ z_{m-2}\ \dots\ z_2\ z_1 \mid z_0$ (g OR r OR s)
⇒ depl. dreapta 1 bit, Z_E++	
Caz 2) $z_m = 0$, $z_{m-1} = 1$	$1.\ z_{m-2}\ z_{m-3}\ \dots\ z_1\ z_0 \mid g$ (r OR s)
⇒ Z_M este deja normalizat	
Caz 3) $z_m = 0$, $z_{m-1} = 0$, $z_{m-2} = 1$	$1.\ z_{m-3}\ z_{m-4}\ \dots\ z_0\ g \mid r\ \ s$
⇒ depl. stânga 1 bit, Z_E--	
Caz 4) $z_m = 0$, $z_{m-1} = 0$, $z_{m-2} = 0$, $z_{m-3} = 1$	$1.\ z_{m-4}\ z_{m-5}\ \dots\ g\ 0 \mid 0\ \ 0$
⇒ depl. stânga 1 bit, $Z_E- = 2$	

Rounding mode	$Z_{S_n} \geq 0$
truncate	ignore R, S
down	—
up	if (R or S) then $Z_{S_n} + 1$
nearest even	if (R and (S or z_{0_n})) then $Z_{M_n} + 1$

Rounding mode	$Z_{S_n} < 0$
truncate	—
down	if (R or S) then $Z_{M_n} - 1$
up	—
nearest even	if (R and (S or z_{0_n})) then $Z_{M_n} - 1$

Adunarea / Scăderea FP

Format FP simplificat / redus

1 bit semn
3 exponent $(e=3) \to$ bias $2^{3-1}-1=3$
4 significand

$x = +6.5 = 110.1 \cdot 2^0 = 1.101 \cdot 2^2$

$y = -7 = -111.0 \cdot 2^0 = 1.110 \cdot 2^2$

$2+3=5$

x: | 0 | 101 | 1.101 |

y: | 1 | 101 | 1.110 |

Pasul (2) $d = x_E - y_E$

$d < 0 \to |x| < |y| \quad z_E = y_E$

$d \geq 0 \to z_E = x_E$

$d = 0$

$\to z_E = x_E = 5$

(3) if $\text{sign}(x) \neq \text{sign}(y) \to$ complementare y_S

$y_{S_{C_2}} = 0.010$

(4) Alinierea

$|d|$; if (3) compl. $\rightarrow$ introducere biți de 1

$$\sim x + y$$
$$-(x - y)$$

g, r, s

(5) Adunare significand

$$X_S \quad 1.101$$
$$Y_S \quad 0.010 \mid 0 \; 0 \; 0$$
$$Z_S = 0 \; 1.111 \mid 0 \; 0 \; 0$$

negativ

$$\quad g \; r \; s$$
$$Z_{S_{C_2}} \quad 0.001 \mid 0 \; 0 \; 0$$

(6) $Z_{S_{C_2}}$ norm. $= 1.000 \mid 0 \; 0$

$Z_E - = 3$

3 shiftări la st

$\rightarrow \boxed{Z_E = 2}$

(7)

(8) Rotunjirea $Z_{M_n} \rightarrow Z^*_M$

$$R \cdot (S + Z_{0_M}) = 0(0+0) = 0$$

(9)

9

Y/S

01110

10010

Verificare: z: $\boxed{1|0|1|0|1|0|0|0}$

exponent, bias $\underbrace{}_{=2_{(10)}}$

$$z = (-1)^1 \cdot 2^{2-3} \cdot 1.000 = -2^{-1} \cdot 1.0 = -\frac{1}{2} = -0.5$$

$$\begin{cases} z_{2m} = z_2 \cdot ll/r_0 + z_1 \cdot l_1 + z_0 \cdot l_2 + g \cdot l_3 + z_3 \cdot r_1 \\ z_{1m} = z_1 \cdot ll/r_0 + z_0 \cdot l_1 + g \cdot l_2 + z_2 \cdot r_1 \\ z_{0m} = z_0 \cdot ll/r_0 + g \cdot l_1 + z_1 \cdot r_1 \\ R = g \cdot ll/r_0 + r \cdot l_1 + z_0 \cdot r_1 \\ S = (r \text{ or } s) \cdot ll/r_0 + s \cdot l_1 + (g \text{ or } R \text{ or } S) \cdot r_1 \end{cases}$$

Doar una este activă la un moment dat

→ ~~Mux~~ Priority Encoder

encoded s_2 s_1 s_0

5

$z_5 = z_4 z_3 .\ z_2\ z_1\ z_0\ g\ R\ S$

		$z_{5n} =$					
		$1.$	z_{2m}	z_{1m}	z_{0m}	R	S
r_1	1) $z_4 = 1$ >>1	$1^{z_4}.$	z_3	z_2	z_1	z_0	$(g \text{ or } R \text{ or } S)$
ll/r_0	2) $z_4=0, z_3=1$	$1^{z_3}.$	z_2	z_1	z_0	g	$(r \text{ or } s)$
l_1	3) $z_4=0, z_3=0, z_2=1$ <<1	$1^{z_2}.$	z_1	z_0	g	r	s
l_2	4) $z_4=0, z_3=0, z_2=0, z_1=1$ <<2	$1^{z_1}.$	z_0	g	0	0	0
l_3	5) $z_4=0, z_3=0, z_2=0, z_1=0, z_0=1$ <<3	$1^{z_0}.$	g	0	0	0	0

↑ Mutually exclusive ⇓ encoded cu s_2, s_1, s_0

r_1	l_3	l_2	l_1	ll/r_0	s_2	s_1	s_0
0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	1	0	0	0	0	1	1
1	0	0	0	0	1	0/d	0/d

Capitolul 4

Analiza și sinteza dispozitivelor de înmulțire binară

$$X * Y = X_M * Y_M * 2^{(X_E + Y_E)}$$

$x_M = (1)\ 1\ 0\ 1 \ldots$

hidden bit

Metode de înmulțire

$P = X \cdot Y$ deînmulțit

înmulțitor

$X = 15_{(10)} = 1111_{(2)}$

$Y = 22_{(10)} = 10110_{(2)}$

a) Paper & pencil

$$\begin{array}{rl}
10110 & Y \\
1111 & X \quad (x_3\, x_2\, x_1\, x_0) \\
\hline
10110 & x_0 \cdot Y \cdot 2^0 \\
10110 & x_1 \cdot Y \cdot 2^1 \\
10110 & x_2 \cdot Y \cdot 2^2 \\
10110 & x_3 \cdot Y \cdot 2^3 \\
\hline
101001010 &
\end{array}$$

+2 +1 +2 +1 +1

$$P = \sum_{i=0}^{3} x_i \, Y \cdot 2^i$$

330 = 22 · 15

4 + 5 = 9 biti

Max Produs biti -> $m + n$

b) keep the partial products fixed

$$P_{i+1} = P_i + x_i \, Y \cdot 2^i \qquad i \geq 0, \; P_0 = 0$$

-> Sumator pe $(m+n)$ biti

c) keep the 1 bit products fixed

right shift register

bit now

```
          1 0 1 1 0
            1 1 1 1
-------------------------------------
  +  |  0 0 0 0 0          P_0 = 0
     |  1 0 1 1 0          x_0 · y
-------------------------------------
        1 0 1 1 0          P_0 = P_0 + x_0 · y
shift | [0] 1 0 1 1 0      P_1 = P_0 * 2^-1
  +   | 1 0 1 1 0          x_1 · y
-------------------------------------
     1 0 0 0 0 1 0         P_1 = P_1 + x_1 · y
  +  | 1 0 0 0 0 1 0       P_2 = P_1 * 2^-1
     | 1 0 1 1 0           x_2 · y
-------------------------------------
     1 0 0 1 1 0 1 0       P_2 = P_2 + x_2 · y
  +  | 1 0 0 1 1 0 1 0     P_3 = P_2 * 2^-1
     | 1 0 1 1 0           x_3 · y
-------------------------------------
     1 0 1 0 0 1 0 1 0     P_3 = P_3 + x_3 · y
     [1 0 1 0 0 1 0 1 0]   P_4 = P_3 * 2^-1    = P
```

Sumatorul va trebui să fie pe lungimea lui Y

$\rightarrow$ nr. iterații X

Înmulțire binară secvențială pentru SM

x, y în SM 8 bits

$$x = \underbrace{x_7}_{\text{sign}} . \underbrace{x_6 \ldots x_0}_{\underbrace{2^{-1} \; 2^{-2} \; \ldots \; 2^{-7}}_{\text{weights}}}$$

$$P = \underbrace{P_{15}}_{x \oplus y} . P_{14} \ldots P_0$$

$P_0 \rightarrow 0$

(7 + 7 = 14)

15 biți produs

atașez 0 la final

pt. a fi 16