1.1.1 Example – integer numbers (Booth)

$X = -101$; $Y = -53$

$P = 101 \times 53$

```
  101×
   53
 ----
  303
 505
 ----
 5353
```

$X = 1110\ 0101_{SM}$

$= \boxed{1001\ 1011_{C2}}$

$Y = 1011\ 0101_{SM}$

$= \boxed{1100\ 1011_{C2}}$

101−
64
= 37−
32
= 5

53−
32
= 21−
16
= 5

$M = 1100\ 1011$

$-M = 0011\ 0101$

COUNT	A	Q	Q[-1]	M
000	0000 0000	1001 1011	0	1100 1011
+	0011 0101			
	0011 0101			
	0001 1010	1100 1101	1	
001	0000 1101	0110 0110	1	
010+	1100 1011			
	1101 1000			
	1110 1100	0011 0011	0	
011+	0011 0101			
	0010 0001			
	0001 0000	1001 1001	1	
100	0000 1000	0100 1100	1	
101+	1100 1011			
	1101 0011			
	1110 1001	1010 0110	0	
110	1111 0100	1101 0011	0	
111+	0011 0101			
	0010 1001			
	$\boxed{0001\ 0100\ 1110\ 1001}$		1	

256|7
$\boxed{0\ 2\ 5\ 6}$ ×

0.256

~~0~~.2567
↓ ↓
.2560

41+
64
128
1024
4096

$\boxed{+5353}$

1.1.2 Booth example – fractional numbers in C2

$$X = -\frac{107}{128} \quad ; \quad Y = +\frac{9}{64}$$

$107 - 64 = 43$; $43 - 32 = 11$

$X = 1_\Delta 110\ 1011_{SM}$

$= \boxed{1001\ 0101}_{C2}$

$Y = \boxed{0\,0010010}$ C2

0.256

0.2560

$$P = -\frac{963}{2^{13}} = -\frac{963 \cdot 4}{2^{15}} = -\frac{3852}{2^{15}}$$

COUNT	A	Q	Q[-1]	M
000	0000 0000	1001 0101	0	0001 0010
+	1110 1110			
	1110 1110			
	1111 0111	0100 1010	1	
001	+0001 0010			
	0000 1001			
	0000 0100	1010 0101	0	
010	+1110 1110			
	1111 0010			
	1111 1001	0101 0010	1	
011+	0001 0010			
	0000 1011			
	0000 0101	1010 1001	0	
100+	1110 1110			
	1111 0011			
	1111 1001	1101 0100	1	
101+	0001 0010			
	0000 1011			
	0000 0101	1110 1010	0	
110	0000 0010	1111 0101	0	
111+	1110 1110			
	1111 0000	1111 0100		

M = 00010010

−M = 11101110

256 + 12 = 268

.256

.2560

1111 0000 1111 0100 C2

1111 0000 1111 0011

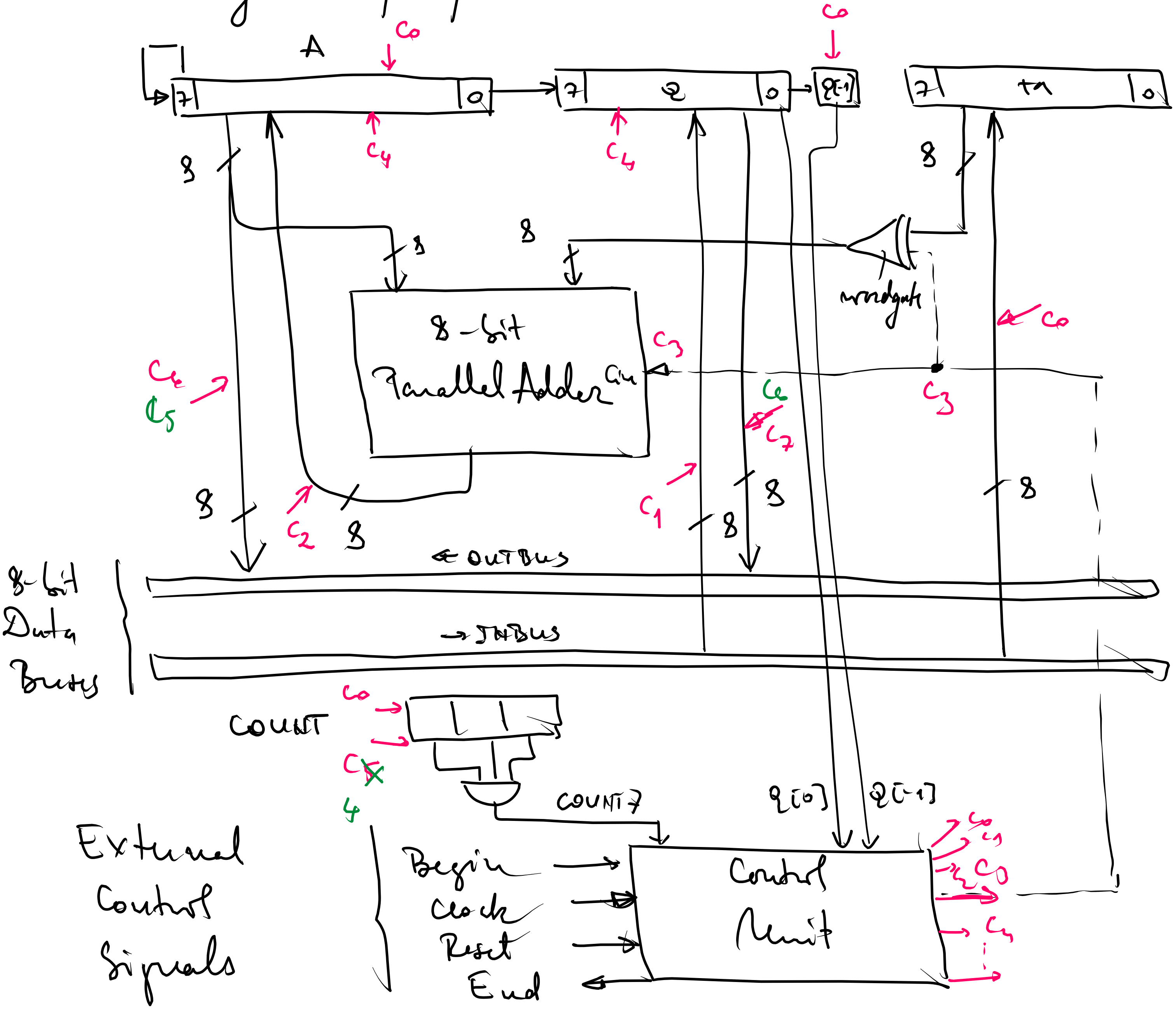
↓

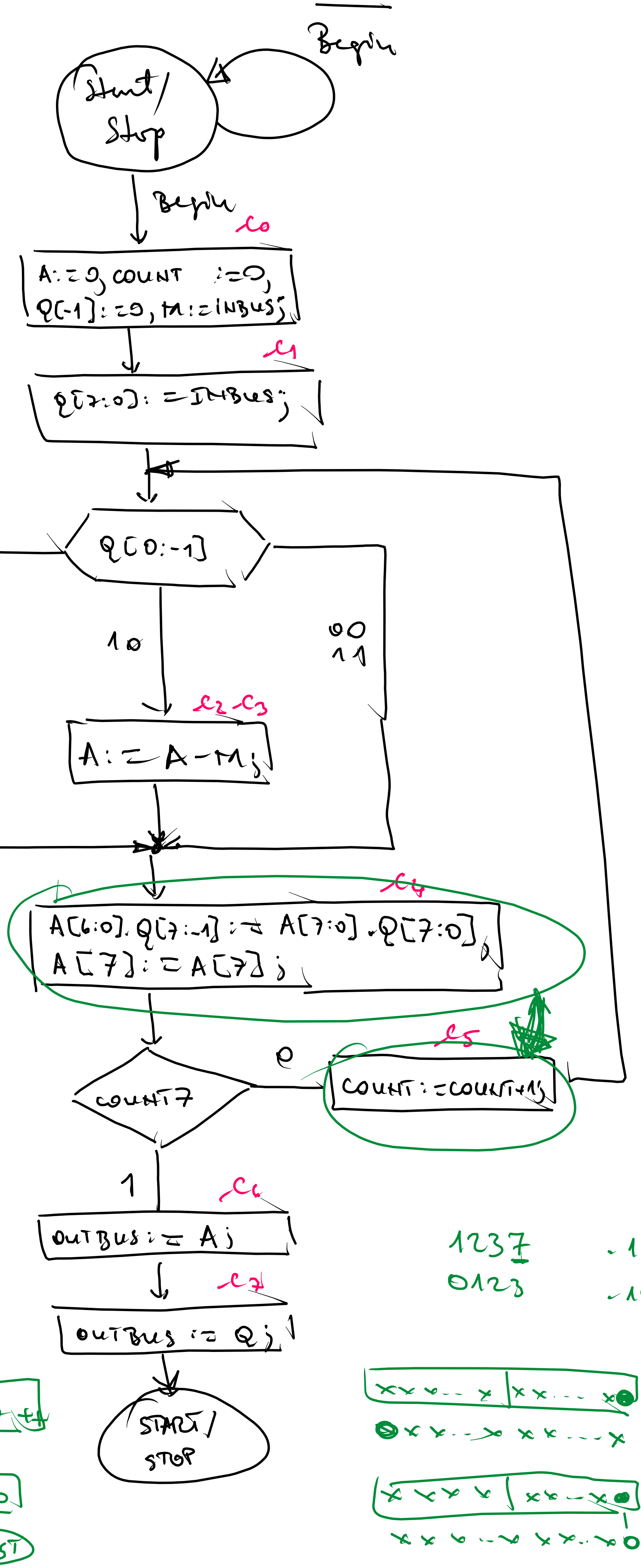
1000 1111 0000 1100 SM

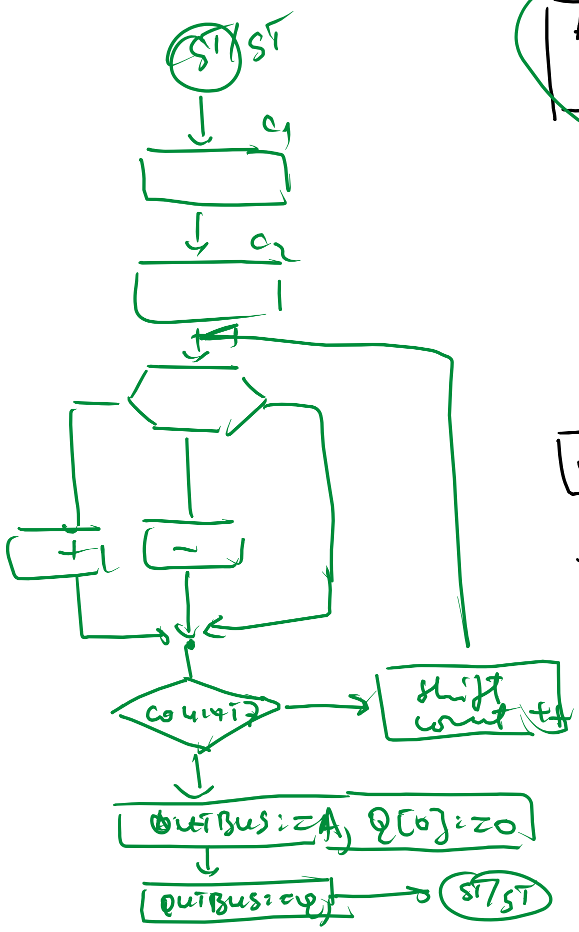
268 + 512 + 1024 + 2048 = −3852 ✓

1.1.3 Booth – HW implementation

– Hayes HW platform







1237 -123

0123 -1230

1 COMPUTER ARITHMETIC

1.1 Recap

Booth's algorithm : 2s complement multiplier

$X = x_{n-1} x_{n-2} \dots x_i \dots x_1 x_0$; $x_i \in \mathbb{B} = \{0,1\}$, $i = \overline{0, n-1}$

$$\boxed{X_{ten} = -x_{n-1}\times 2^{n-1} + \sum_{i=0}^{n-2} x_i \times 2^i}$$

2s complement numbers

Robertson's formula

123

10^2 10^1 10^0

step $i=0$

x_{-1}

$$Y \times X = Y \times (x_{n-1} x_{n-2} \dots x_i \dots x_1 x_0)0$$

$x_{-1}=0$

multiplicand multiplier

$$\boxed{(x_{i-1} - x_i)\times 2^i \times Y}$$

added to the partial product in step i

step i

x_0

x_i	x_{i-1}	Op	
0	0	0	0
0	1	1	$+Y$
1	0	$\bar{1}$	$-Y$
1	1	0	

$$x_i \times 2^{i+1} - x_i 2^i = x_i \cdot 2^i (2-1) = x_i 2^i$$

step 0 $(x_{-1} - x_0) \times 2^0 \times Y +$

step 1 $(x_0 - x_1) \times 2^1 \times Y +$

step 2 $(x_1 - x_2) \times 2^2 \times Y +$

...

step $i-1$ $(x_{i-2} - x_{i-1}) \times 2^{i-1} \times Y +$

step i $(x_{i-1} - x_i) \times 2^i \times Y +$

...

step $n-2$ $(x_{n-3} - x_{n-2}) \times 2^{n-2} \times Y +$

step $n-1$ $(x_{n-2} - x_{n-1}) \times 2^{n-1} \times Y$

$$\boxed{P = Y \times \left(-x_{n-1}\times 2^{n-1} + \sum_{i=0}^{n-2} x_i \cdot 2^i\right)}$$

$P = ?$