1. Computer arithmetic

1.1 Recap

Booth's multiplier → 2s compliment operands

multiplier

$Y \times X$, where $X, Y \in \mathbb{Z}$

multiplicand

$d_3 d_2 d_1 d_0 = 7543 \qquad d_i \in \{0, 1 \dots 9\}$

$10^3\ 10^2\ 10^1\ 10^0$

x_{-1}

$X = x_{n-1}\, x_{n-2} \dots x_i \dots x_1\, x_0\, 0; \quad x_i \in B = \{0, 1\}$

$i = \overline{0, n-1}$

Unsigned integer $\quad X = \sum_{i=0}^{n-1} x_i \times 2^i$

2s complement

$$\boxed{X = \sum_{i=0}^{n-2} x_i \times 2^i - x_{n-1} \times 2^{n-1}}$$

x_i	x_{i-1}	Op	
0	0	0	
0	1	1	$+Y$
1	0	$\bar{1}$	$-Y$
1	1	0	

Booth's algorithm

$(x_{i-1} - x_i) \times 2^i \times Y$

for each i

step 0 $\quad (x_{-1} - x_0) \times 2^0 \times y \; +$

step 1 $\quad (x_0 - x_1) \times 2^1 \times y \; +$

step 2 $\quad (x_1 - x_2) \times 2^2 \times y \; +$

$\vdots$

step i-1 $\quad (x_{i-2} - x_{i-1}) \times 2^{i-1} \times y \; +$

step i $\quad (x_{i-1} - x_i) \times 2^i \times y \; +$

step i+1 $\quad (x_i - x_{i+1}) \times 2^{i+1} \times y \; +$

$\vdots$

step n-2 $\quad (x_{n-3} - x_{n-2}) \times 2^{n-2} \times y \; +$

step n-1 $\quad (x_{n-2} - x_{n-1}) \times 2^{n-1} \times y$

$x_{-1} \to 0$

$$\boxed{\begin{aligned} -x_i \times 2^i + x_i \times 2^{i+1} &= \\ = 2^i(-x_i + 2x_i) &= \\ = 2^i \cdot x_i & \end{aligned}}$$

$$P = \underbrace{\left(\sum_{i=0}^{n-2} x_i \times 2^i - x_{n-1} \times 2^{n-1} \right) \times y}_{X_{C_2} \times Y_{C_2}}$$

Robertson & Booth are equivalent

1.1.1 Example – integer operands

$$\boxed{\begin{aligned} X &= -103 \\ Y &= -41 \end{aligned}}$$

$X = 11100111_{SM}$

$= \boxed{10011001}_{C_2}$

$Y = 10101001_{SM}$

$= \boxed{11010111}_{C_2}$

$103 - 64 = 39$, $39 - 32 = 7$

$41 - 32 = 9$

P

$$\begin{array}{r} 103 \times \\ 41 \\ \hline 103 \\ 412 \\ \hline + \boxed{4223} \end{array}$$ ✓

COUNT	A	Q	Q[-1]	M
000	0000 0000	1001 1001	0	1101 0111
+	0010 1001			
	0010 1001			
	0001 0100	1100 1100	1	
001 +	1101 0111			
	1110 1011			
	1111 0101	1110 0110	0	
010	1111 1010	1111 0011	0	
011 +	0010 1001			
	0010 0011			
	0001 0001	1111 1001	1	
100	0000 1000	1111 1100	1	
101 +	1101 0111			
	1101 1111			
	1110 1111	1111 1110	0	
110	1111 0111	1111 1111	0	
111 +	0010 1001			
	0010 0000	1111 1111	0	
	0001 0000	0111 1111	1	

$-M = 0010\,1001$

S M M ... M
↓
1   7

14 bits +
1 bit sign
15 bits

Operands → n bits
Product → 2n−1 bits

$2^{10} = 1024$
$2^{11} = 2048$
$2^{12} = 4096$

135 $\boxed{7}$
0 135

127 +
4096
$\boxed{4223}$

0.130
135 → 0135

$$X = -\frac{85}{128} = 1_\Delta 1010101_{\Delta SM} = 1010\ 1011_{C2}$$

85 −
64
21 −
16
5

$$Y = +\frac{11}{32} = 0_\Delta 01011_\Delta 00_{SM} = 0010\ 1100_{C2}$$

$$= \frac{+44}{128}$$

85 ×
44
340
340
−3740

2

x.xx...x (15)

$$x \cdot y = -\frac{3740}{2^{14}} = -\frac{7480}{2^{15}}$$

COUNT	A	Q	Q[-1]	M
000 +	0000 0000 1101 0100 1101 0100 1110 1010	1010 1011 0101 0101	0 1	0010 1100 1101 0011 + 1 1101 0100
001	1111 0101	0010 1010	1	
010	+ 0010 1100 0010 0001 0001 0000	 1001 0101	 0	
011 +	1101 0100 1110 0100 1111 0010	 0100 1010	 1	
100 +	0010 1100 0001 1110 0000 1111	 0010 0101	 0	
101 +	1101 0100 1110 0011 1111 0001	 1001 0010	 1	
110 +	0010 1100 0001 1101 0000 1110	 1100 1001	 0	
111 +	1101 0100 1110 0010 1110 0010	 1100 1001 1100 1000	 0 	

−M = 11010100

C_2

1110 0010 1100 0111 C_1

1 00 11101 0011 1000 SM

$+\frac{7480}{2^{15}}$ ✓

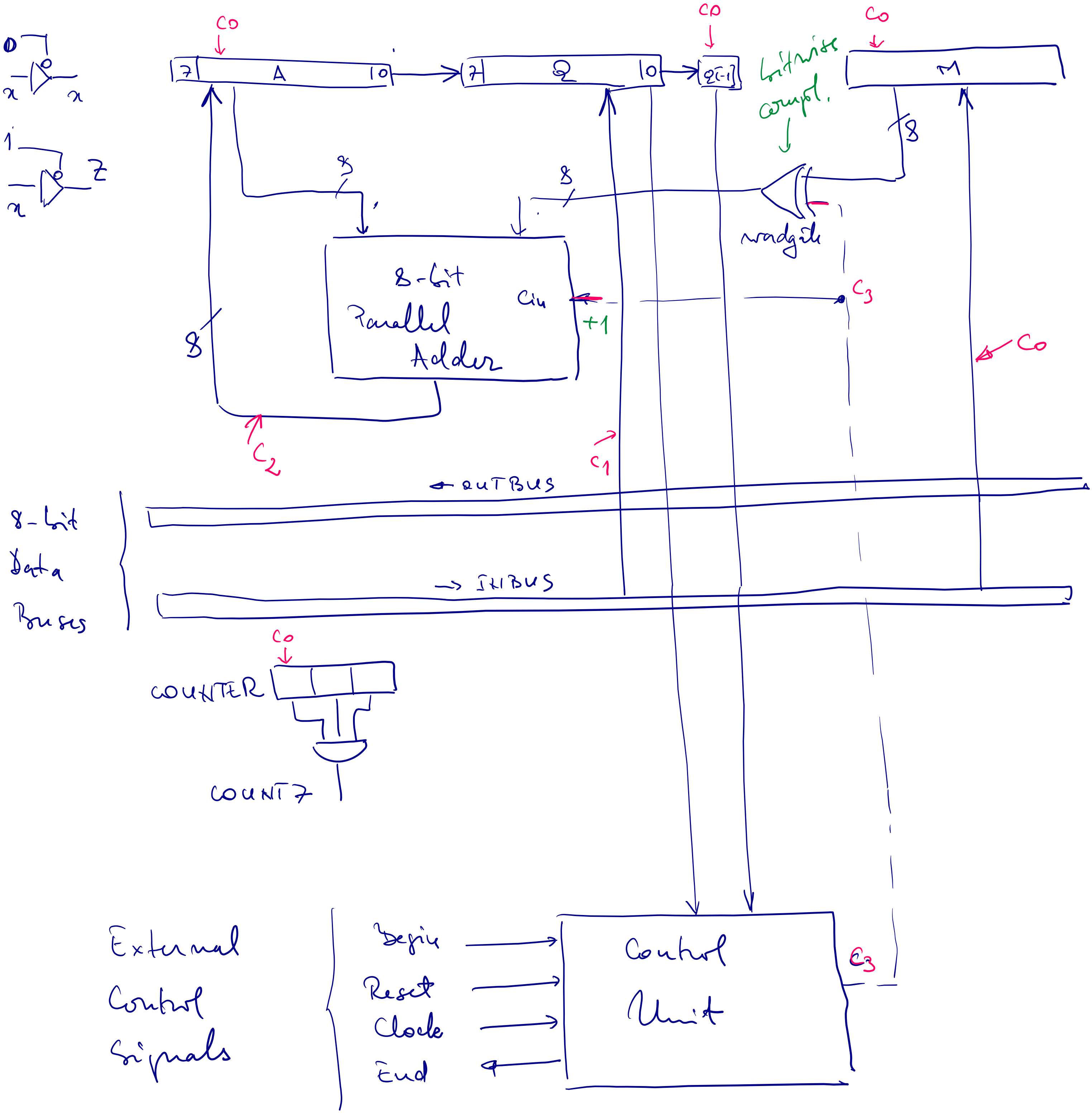
0.1250

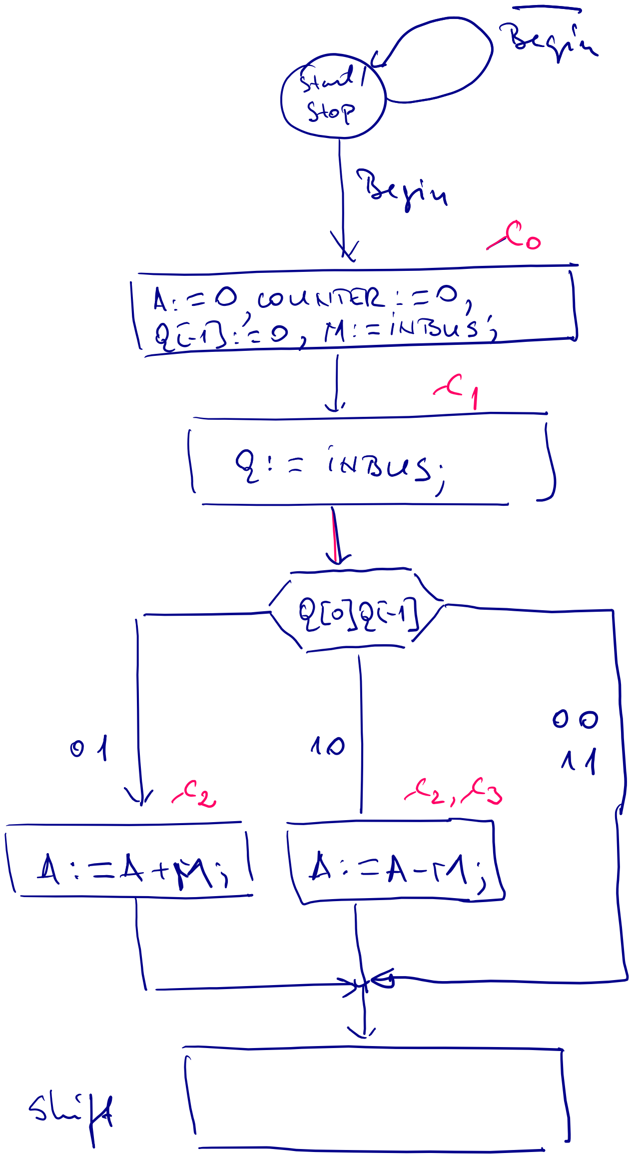
0.125(7)4

0.01257

24
32
56 +
256
1024
2048
4096
7480

1.1.2 Hardware implementation





Begin
Start / Stop
Begin
c_0
A:=0, COUNTER:=0,
Q[-1]:=0, M:=INBUS;
c_1
Q:=INBUS;
Q[0]Q[-1]
01
10
00
11
c_2
c_2, c_3
A:=A+M;
A:=A-M;
shift