1.3 Speeding up multiplication with the higher radix

x_{i+1}	x_i	x_{i-1}	OP
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	2
1	0	0	$\bar{2}$
1	0	1	$\bar{1}$
1	1	0	$\bar{1}$
1	1	1	0

$i+1$ i

$+y \cdot 2^i$

$-y \cdot 2^i + y \cdot 2^{i+1} = (-1+2)\, y \cdot 2^i = y \cdot 2^i$

$0 + y \cdot 2^{i+1} = 2y \cdot 2^i$

$0 - y \cdot 2^{i+1} = -2y \cdot 2^i$

Redundant digit set

Radix 2 $\longrightarrow \{\bar{1}, 0, 1\}$

normal $\{0, 1\}$

$$\begin{array}{cccc} 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 0 & \bar{1} & 1 \end{array} = 1 \cdot 2^0 - 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 =$$

$$= 1 - 2 + 8 = 7_{ten}$$

$0111 = 7_{ten}$

$100\bar{1} = 7_{ten}$

Radix 4 $\longrightarrow \{0, 1, 2, 3\}$ normal

redundant $\{\bar{2}, \bar{1}, 0, 1, 2\}$

$79 - 64 = 15$

$105 - 64 = 41$, $41 - 32 = 9$

1.3.1 Radix-4 Booth's algorithm

$X = -105$; $Y = -79$

$X = 1110\,1001_{SM} = \boxed{1001\,0111}_{C2}$

$Y = 1100\,1111_{SM} = \boxed{1011\,0001}_{C2}$

$-105 \times$
-79
945
735
$+8295$

$M = 11011\,0001$

$-M = 00100\,1111$

$2M = 10110\,0010$

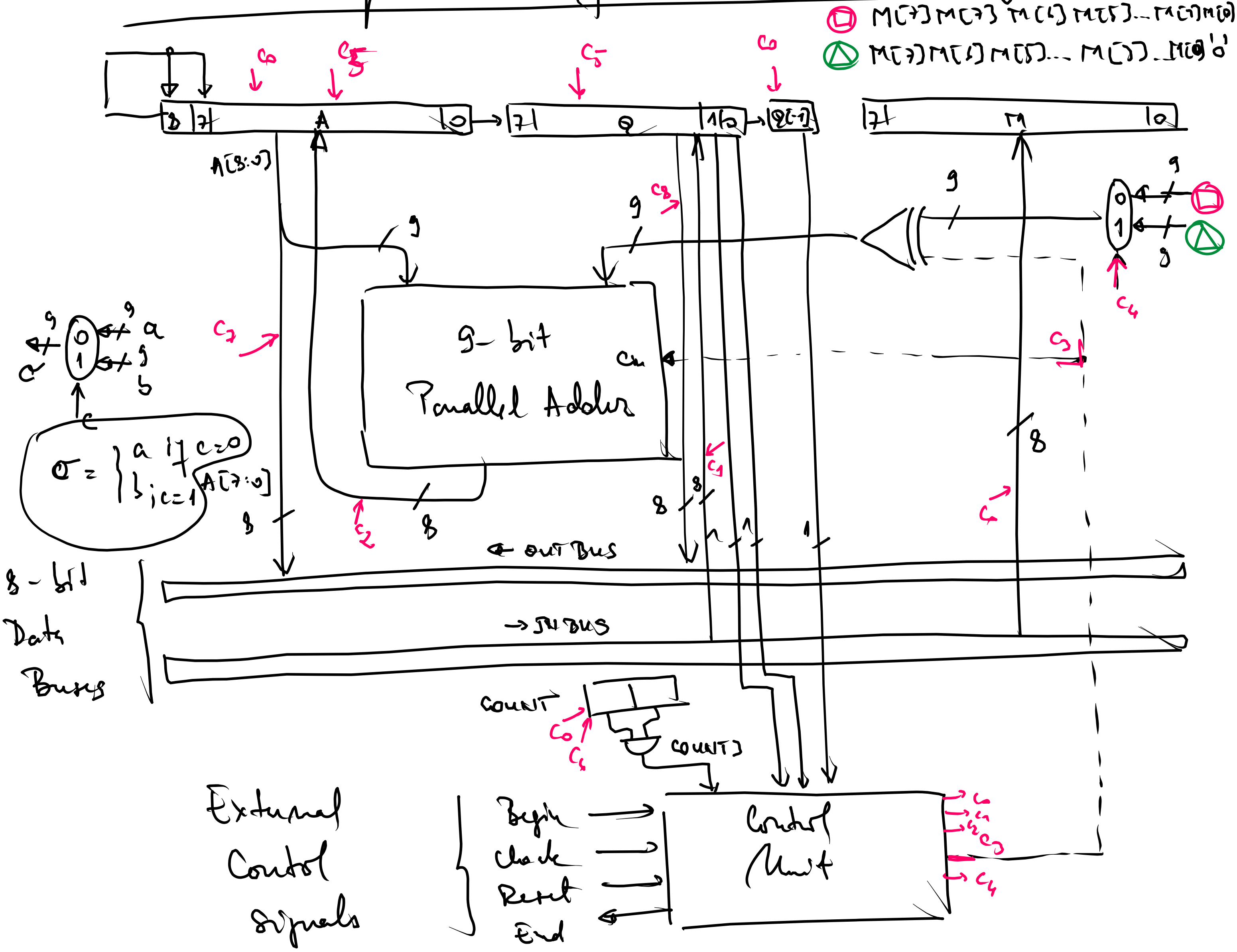
$-2M = 01001\,1110$

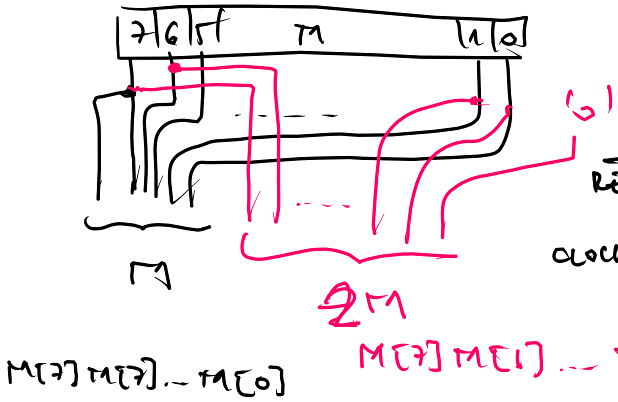
$39 +$
64
8192
$+8295$

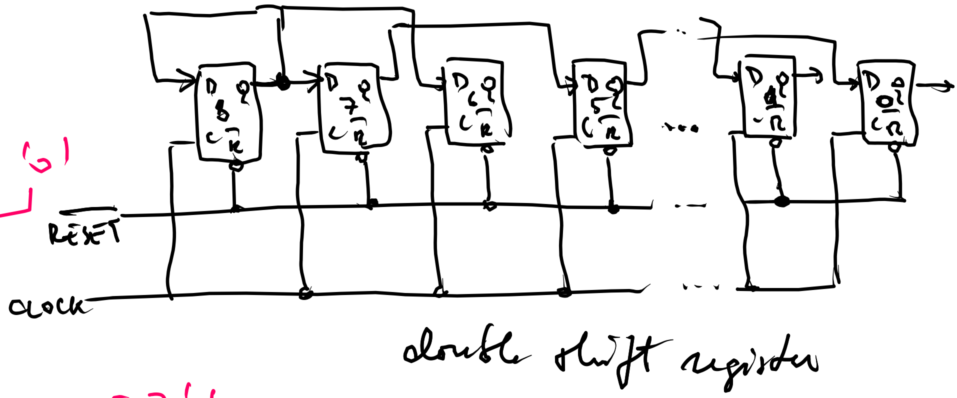
4096

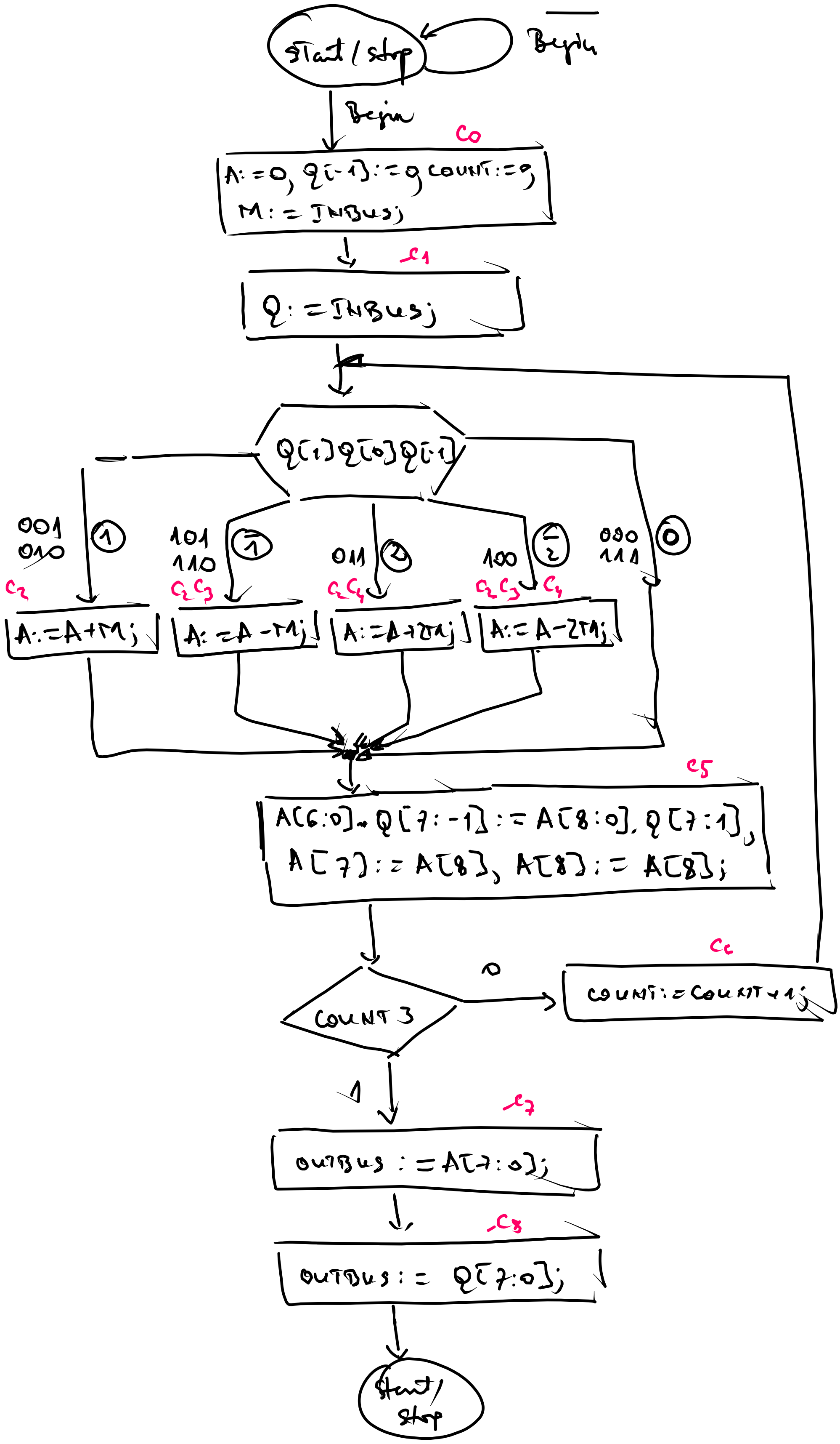
COUNT	A	Q	Q[-1]	M
00 +	0 0000 0000 0 0100 1111 0 0100 1111 0 0000 10011	1 0010111 1110 0101	0 $-M$ 1	1011 0001
01 +	10 110 0010 10 111 0101 11 10 111 01	 0111 1001	$+2M$ 0	
10 +	11011 0001 11 000 1110 11 11 000 11	 10 01 1110	$+M$ 0	
11 +	01 00 1 1110 01 0000 0001 $\boxed{00\ 0100\ 0000\ \vert\ 01\ 10\ 01\ 11}$		$-2M$ 1	

1.3.2 HW Implementation of the Radix-4 Booth algorithm








Start / stop
Begin
Begin
C0
A:=0, Q[-1]:=0, COUNT:=0,
M:=INBUS;
C1
Q:=INBUS;
Q[1]Q[0]Q[-1]
001
010
1
C2
A:=A+M;
101
110
-1
C2 C3
A:=A-M;
011
2
C2 C4
A:=A+2M;
100
-2
C2 C3 C4
A:=A-2M;
000
111
0
C5
A[6:0].Q[7:-1]:=A[8:0].Q[7:1],
A[7]:=A[8], A[8]:=A[8];
COUNT 3
0
C6
COUNT:=COUNT+1;
1
C7
OUTBUS:=A[7:0];
C8
OUTBUS:=Q[7:0];
Start/
Stop

1.3.3 Increasing the radix (radix-8)

1 1001 0111 | 0

x_{i+2}	x_{i+1}	x_i	x_{i-1}	OP
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	2
0	1	0	0	2
0	1	0	1	3
0	1	1	0	3
0	1	1	1	4
1	0	0	0	$\bar{4}$
1	0	0	1	$\bar{3}$
1	0	1	0	$\bar{3}$
1	0	1	1	$\bar{2}$
1	1	0	0	$\bar{2}$
1	1	0	1	$\bar{1}$
1	1	1	0	$\bar{1}$
1	1	1	1	0

$$+y\cdot 2^i - y\cdot 2^{i+1} + y\cdot 2^{i+2} = (1-2+4)\cdot y\cdot 2^i = 3y\cdot 2^i$$

$$-y\cdot 2^i + y\cdot 2^{i+2} = (-1+4)\cdot y\cdot 2^i = 3y\cdot 2^i$$

Q[-1]

COUNT	A	Q		M
00	00 0000 0000	1 1001 0111	0	1011110001
+	0001 00 1111			
	0001 00 1111			
	00 0000 1001	1111 1 0000	1	
10 +	11 0001 0011	+3M		
	11 000 11100			
	111 1 1 000 11	10011 1110	0	
10 +	001 00 11110			
	0010 000001			
	000 0010000	001100111	1	

M = 11 1011 0001	−M = 0001001111
2M = 1101100010	−2M = 001001 1110
3M = 1100010011	−3M = 001110 1101
4M = 1011000100	−4M = 010011 1100