

1.3. Speeding up multiplication with the higher radix

1.3. 1 Redundant digit sets

radix-2 $\{0, 1\}$ redundant radix 2 - $\{\bar{1}, 0, \bar{1}\}$

$$10\bar{1}1 = 2^3 + (-1)2^1 + 2^0 = 7 = 111_2$$

$\begin{array}{c} 1 \\ \{ \quad 1 \\ 0 \end{array}$ $\begin{array}{c} 0 \\ \{ \quad 0 \\ 1 \end{array}$ $\begin{array}{c} \bar{1} \\ \{ \quad 0 \\ 1 \end{array}$

radix-4 $\{0, 1, 2, 3\}$

redundant radix 3 $\{\bar{2}, \bar{1}, 0, 1, 2\}$

$$\begin{aligned} 10\bar{2}2 &= 4^3 - 2 \cdot 4^1 + 2 \\ &= 64 - 8 + 2 = 58 \end{aligned}$$

$\begin{array}{c} 2 \\ \{ \quad 1 \quad 0 \\ 0 \quad 0 \end{array}$ $\begin{array}{c} 1 \\ \{ \quad 0 \quad 1 \\ 0 \quad 0 \end{array}$ $\begin{array}{c} 0 \\ \{ \quad 0 \quad 0 \\ 0 \quad 0 \end{array}$

$\begin{array}{c} \bar{2} \\ \{ \quad 0 \quad 0 \\ 1 \quad 0 \end{array}$ $\begin{array}{c} \bar{1} \\ \{ \quad 0 \quad 2 \\ 0 \quad 1 \end{array}$

$$10\bar{2}2 \rightarrow 01 \quad 00 \quad 00 \quad 10 \quad -$$

$$00 \quad 00 \quad 10 \quad 00$$

$$\overbrace{\quad\quad\quad\quad\quad}^{00111010}$$

1.3.2.

Radix - 4 Booth

x_{i+1}	x_i	x_{i-1}	Operation
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	2
1	0	0	<u>2</u>
1	0	1	<u>1</u>
1	1	0	<u>1</u>
1	1	1	0

$$\begin{aligned}
 & 0 \times 2^{i+1} + 0 \times 2^i = 0 \times 2^i \\
 & 1 \times 2^i + 0 \times 2^{i+1} = 1 \cdot 2^i \\
 & \overline{1} \times 2^i + 1 \times 2^{i+1} = \\
 & = 2^i (-1 + 2) = 2^i \\
 & 0 \times 2^i + 1 \times 2^{i+1} = 2 \times 2^i \\
 & 0 \times 2^i - 1 \times 2^{i+1} = -2 \times 2^i \\
 & 1 \times 2^i - 1 \times 2^{i+1} = -2^i \\
 & -1 \times 2^i + 0 \cdot 2^{i+1} = -1 \cdot 2^i
 \end{aligned}$$

Însumătare temporară!! de la 8 clk
la 4 clk

pt $2, \bar{2}$ → adunare / scădere $2^i \neq Y$
+ 2 shiftări (orică cat)

$$M = 110000101 \quad \begin{array}{l} \text{(pe 9 biti)} \\ \text{cu 1 bit rez.)} \end{array}$$

$$2M = 10000101 \underline{P}$$

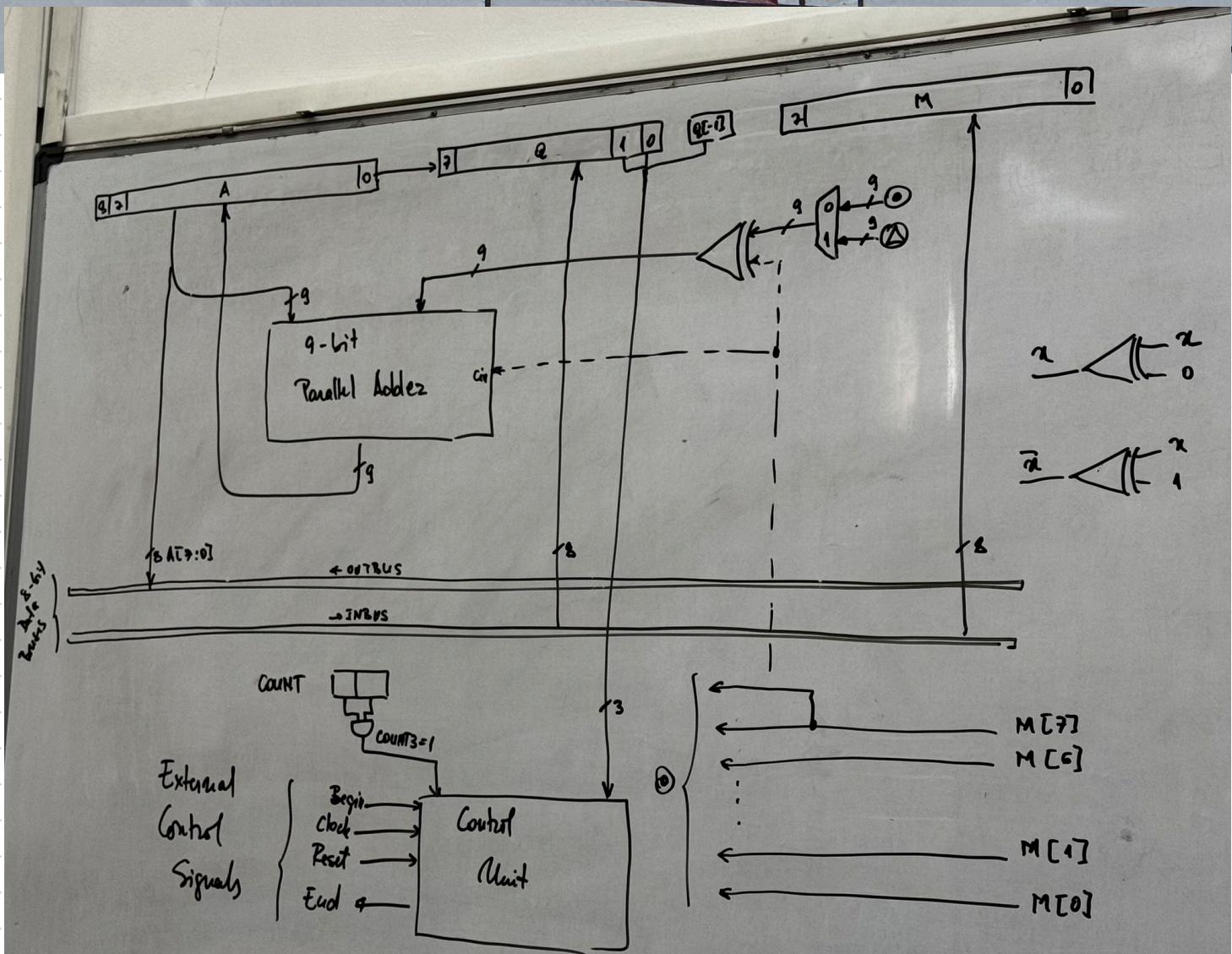
$$-M = \dots \dots$$

$$-2M = \dots \dots \dots$$

$$1011 \ 1001 = -128 + 32 + 16 + 8 + 1 = -71$$

$$1000 \ 0101 = -123$$

COUNT	A	Q	M
00	$0 \ 0000 \ 0000$ $+ 1 \ 1000 \ 0101$ $\hline 1 \ 1000 \ 0101$ $1 \ 1110 \ 0001$	$1011 \ 1001 \ 0$	$1000 \ 0101$ +M
01	$0 \ 1111 \ 0110$ $- 0 \ 1101 \ 0111$ $\hline 0 \ 0011 \ 0101$	$0110 \ 1110 \ 0$	-2M
10	$0 \ 0000 \ 1101$	$1101 \ 1011 \ 1$	OM
11	$+ 0 \ 0111 \ 1011$ $\hline 0 \ 1000 \ 1000$	$0111 \ 0110 \ 1$	-M
	$0 \ 0010001000011101$		1



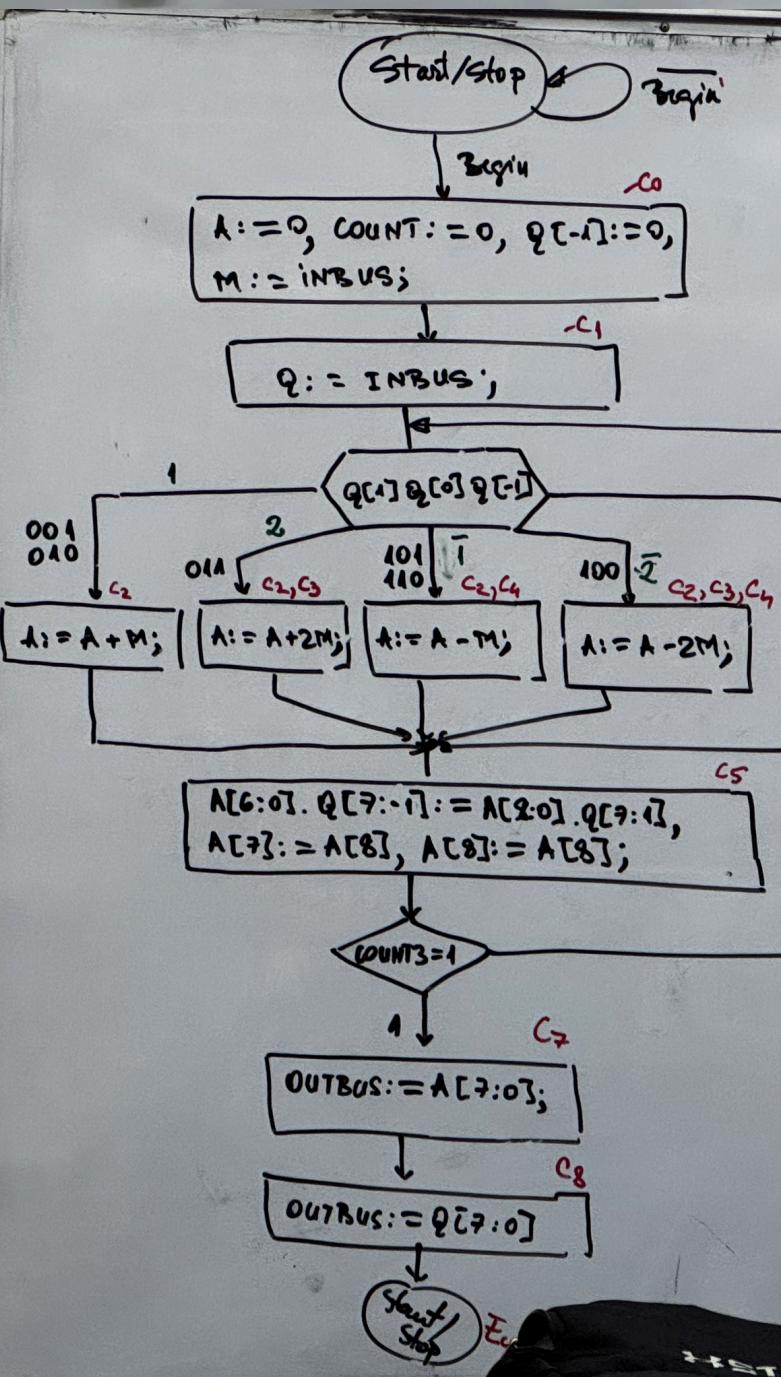
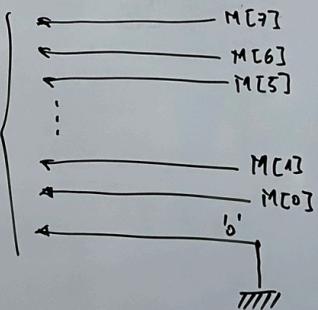
1.3.2 Radix-4 Booth

$x_{i+1} x_i x_{i-1}$	$0 \times 2^{i+1}$	0×2^i	Operation
0 0 0	0	0×2^i	$0 \times 2^{i+1} + 0 \times 2^i = 0 \times 2^i$
0 0 1	1	1×2^i	$1 \times 2^i + 0 \times 2^{i+1} = 1 \times 2^i$
0 1 0	1	$1 \times 2^i + 1 \times 2^{i+1} = 2^i(-1+2) = 1 \times 2^i$	$1 \times 2^i + 1 \times 2^{i+1} = 2^i(-1+2) = 1 \times 2^i$
0 1 1	2	$0 \times 2^i + 1 \times 2^{i+1} = 2 \times 2^i$	$0 \times 2^i + 1 \times 2^{i+1} = 2 \times 2^i$
1 0 0	2	$0 \times 2^i - 1 \times 2^{i+1} = -2 \times 2^i$	$0 \times 2^i - 1 \times 2^{i+1} = -2 \times 2^i$
1 0 1	1	$1 \times 2^i - 1 \times 2^{i+1} = 2^i(1-2) = -1 \times 2^i$	$1 \times 2^i - 1 \times 2^{i+1} = 2^i(1-2) = -1 \times 2^i$
1 1 0	1	$-1 \times 2^i + 0 \times 2^{i+1} = -1 \times 2^i$	$-1 \times 2^i + 0 \times 2^{i+1} = -1 \times 2^i$
1 1 1	0	$0 \times 2^i + 0 \times 2^{i+1} = 0 \times 2^i$	$0 \times 2^i + 0 \times 2^{i+1} = 0 \times 2^i$

$$\begin{array}{r} -123 \\ -71 \\ \hline 123 \\ 861 \\ \hline 8733 \end{array}$$

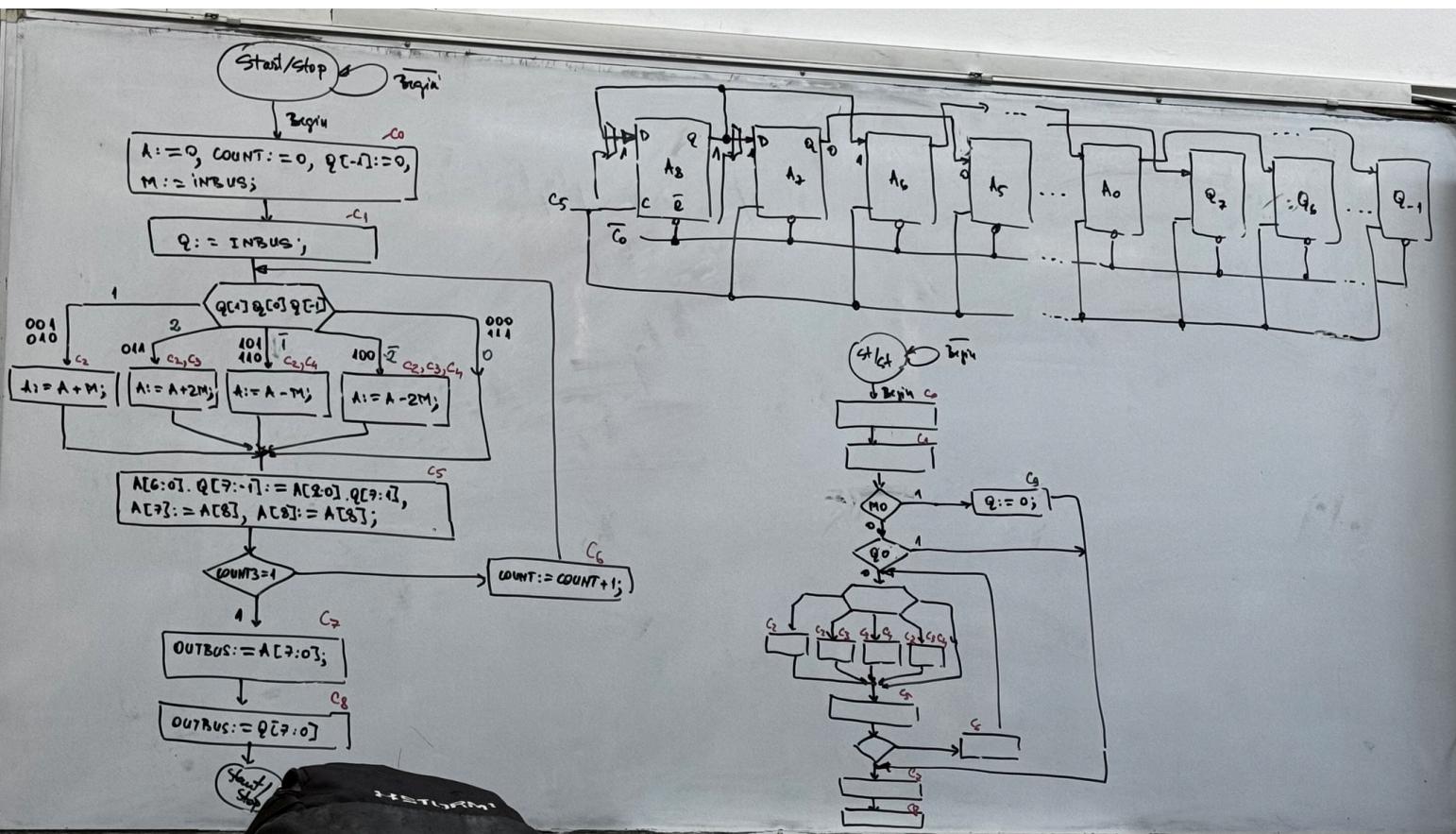
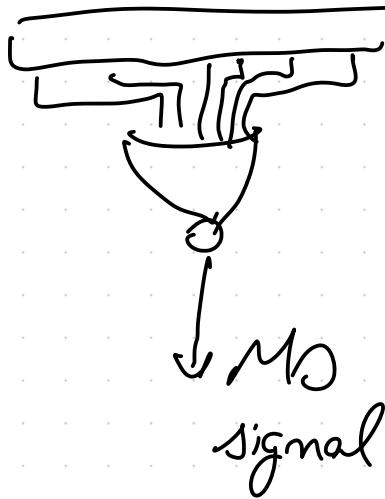
$$\begin{aligned} M &= 110000101 \\ 2M &= 100001010 \\ -M &= 001111011 \\ -2M &= 011110110 \end{aligned}$$

COUNT	A	Q	Q[7:1]	M
00	0 0000 0000 + 1 1000 0101 \textcircled{A} 1 0000 0104 1 1110 0001	1011 1001 0	1000 0101	+M
01	0 1111 0110 0 1101 0111 0 0011 0101	0110 1110 0	0110 1110 0	-2M
10	0 0000 1101	1101 1011 1	1101 1011 1	0M
11	+ 0011110111 0 1000 1000 0 0010001000	0111 1011 0	0111 1011 0	-M



O Checking

$\rightarrow \text{NOR pe } M$



1.3.3 Radix-8 Booth

x_{i+2}	x_{i+1}	x_i	x_{i-1}	OP	2^i
0	0	0	1	0	0
0	0	1	0	1	1
0	0	1	1	2	2
0	1	0	0	2	3
0	1	0	1	3	3
0	1	1	0	3	4
0	1	1	1	4	5
1	0	0	0	5	3
1	0	0	1	3	3
1	0	1	0	2	2
1	0	1	1	2	1
1	1	0	0	1	1
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	0	0

$-2^{i+2} \cdot x_i$

Accumulator per 10 bit; ($8 + 2$ pt shift)

$Q[8] Q[-1]$

1.3.3 Radix-2 Booth

x_{i+2}	x_i	x_{i-1}	OP
0	0	0	0
0	0	0	1
0	0	1	1
1	0	1	2
0	1	0	2
1	1	0	3
0	1	1	3
1	1	1	4

$$\begin{array}{l}
 +M = 0000101 \\
 -M = 1111011 \\
 2M = 00001010 \\
 \hline
 3M = 10001111
 \end{array}$$

$$\begin{aligned}
 & +2^i \\
 & -2^i + 2^{i+1} = 2^i(2-1) = 2^i \\
 & 0 \times 2^i + 2^{i+1} = 2 \cdot 2^i \\
 & -2^{i+1} + 2^{i+2} = -(2+4)2^i = -2 \cdot 2^i \\
 & -2^i + 2^{i+2} = 2^i(4-1) = 3 \cdot 2^i \\
 & -2^i + 2^{i+2} = 2^i(-1+4) = 3 \cdot 2^i \\
 & +2^{i+2} = 4 \cdot 2^i \\
 & -2^{i+2} = -4 \cdot 2^i \\
 & +2^i - 2^{i+2} = 2^i(1-4) = -3 \cdot 2^i \\
 & +2^i - 2^{i+2} = -3 \cdot 2^i \\
 & 2^{i+1} - 2^{i+2} = 2^i(2-4) = -2 \cdot 2^i \\
 & -2^{i+1} = -2 \cdot 2^i \\
 & -1 \times 2^i + 0 = -1 \cdot 2^i \\
 & -1 \times 2^i
 \end{aligned}$$

$$\begin{array}{l}
 11100001010 \\
 1100001010 \\
 \hline
 1010001111
 \end{array}$$

COUNT	A	Q	QC-3	11
00	0000000000	1001100010	01	1000010101
+ 1110000101	1111110000	1011110110	-M	
01*	0001111011	0111010111		
+ 0000001101	0000000000	0111011101	-M	
10*	0001111011	0010001000		
+ 0000010001	0000001000	0000011101	1	

1.4. Division

1.4 Division

$$\begin{array}{r} \text{dividend} \\ 5763 \\ \hline \text{divisor} \\ 131 \\ \hline \end{array}$$
$$\begin{array}{r} 524 \\ \hline = 533 \\ 393 \\ \hline 130 \end{array}$$

$$\begin{array}{r} 1101 \quad 0101 \\ -1001 \quad | \quad | \quad | \\ \hline 1000 \quad | \quad | \quad | \\ -1001 \quad | \quad | \quad | \\ \hline 10000 \quad | \quad | \quad | \\ -1001 \quad | \quad | \quad | \\ \hline 001111 \quad | \quad | \quad | \\ -1001 \quad | \quad | \quad | \\ \hline 0110 \end{array}$$

quotient
 23_{10}

dividend

$$\begin{array}{r} 21+ \\ 64 \\ \hline 128 \\ \hline \end{array}$$
$$\begin{array}{r} 213 | 9 \\ 18 | 23 \\ \hline = 33 \\ 27 \\ \hline = 6 \\ \hline \end{array}$$

divisor

quotient

remainder

n -bit divisor and
 n -bit quotient and remainder

$\Rightarrow (2^{n-1})$ -bit dividend

1.4.1 Restoring Division