1.2 Accelerating multiplication

1.2.1 Multiplier performance

Ex.1

$X = 1001\ 1101;0$ (x_{-1})

$X' = \bar{1}010\ 0\bar{1}1\bar{1}$

Robertson → 5 OP.

Booth → 5 OP.

Booth encoding

x_i	x_{i-1}	OP
0	0	0
0	1	1
1	0	$\bar{1}$
1	1	0

Ex.2

$X = 1110\ 0111;0$

$X' = 00\bar{1}0\ 10\bar{1}$

Robertson → 5 OP.

Booth → 3 OP.

Ex.3

$X = 0101\ 0101;0$

$X' = 1\bar{1}1\bar{1}\ 1\bar{1}1\bar{1}$

Robertson → 4 OP

Booth → 8 OP.

1.2.2 Modified Booth

- Runs of 0s, runs of 1s

$x_{i+1}\ x_i\ x_{i-1}$

isolated 1: 001 → $-2^i \cdot y + 2^{i+1} \cdot y = (-1+2) \cdot 2^i \cdot y = 2^i \cdot y$

isolated 0: 110 → $+2^i \cdot y - 2^{i+1} \cdot y = (1-2) \cdot 2^i \cdot y = -2^i \cdot y$

x_{i+1}	x_i	x_{i-1}	R	OP	R^*
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	0	1	$\bar{1}$	1	1
1	1	0	$\bar{1}$	1	1
1	1	1	0	1	1

Ex 1'

$X = 1;1001\ 1101$

$X'' = \bar{1}010\ 0\bar{1}01$

$R = 1001\ 1100\ 0$

MBooth → 4OP

Ex 2'

$X = 1;1110\ 01\text{-}1$

$X'' = 00\bar{1}0\ 10\bar{1}$

$R = 1110\ 0110$

MBooth → 3OP

Ex 3'

$X = 0;0101\ 0101$

$X'' = 0101\ 0101$

$R = 0000\ 00000$

MBooth → 4OP

R^* ($x_{i+1}\ x_i$)

R \	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$R^* = x_{i+1} \cdot x_i + R \cdot x_i + R \cdot x_{i+1}$

Example $X = -105$, $Y = -123$

$X = 1110\ 1001_{SM} = 1001\ 0111_{C_2}$

$Y = 1111\ 1011_{SM} = 1000\ 0101_{C_2}$

105 - 64 - 32 - 9

123 - 64 - 59 - 32 - 27 - 16 - 11

$P = 315 + 210 + 105 + 12915$

$M = 1000\ 0101$

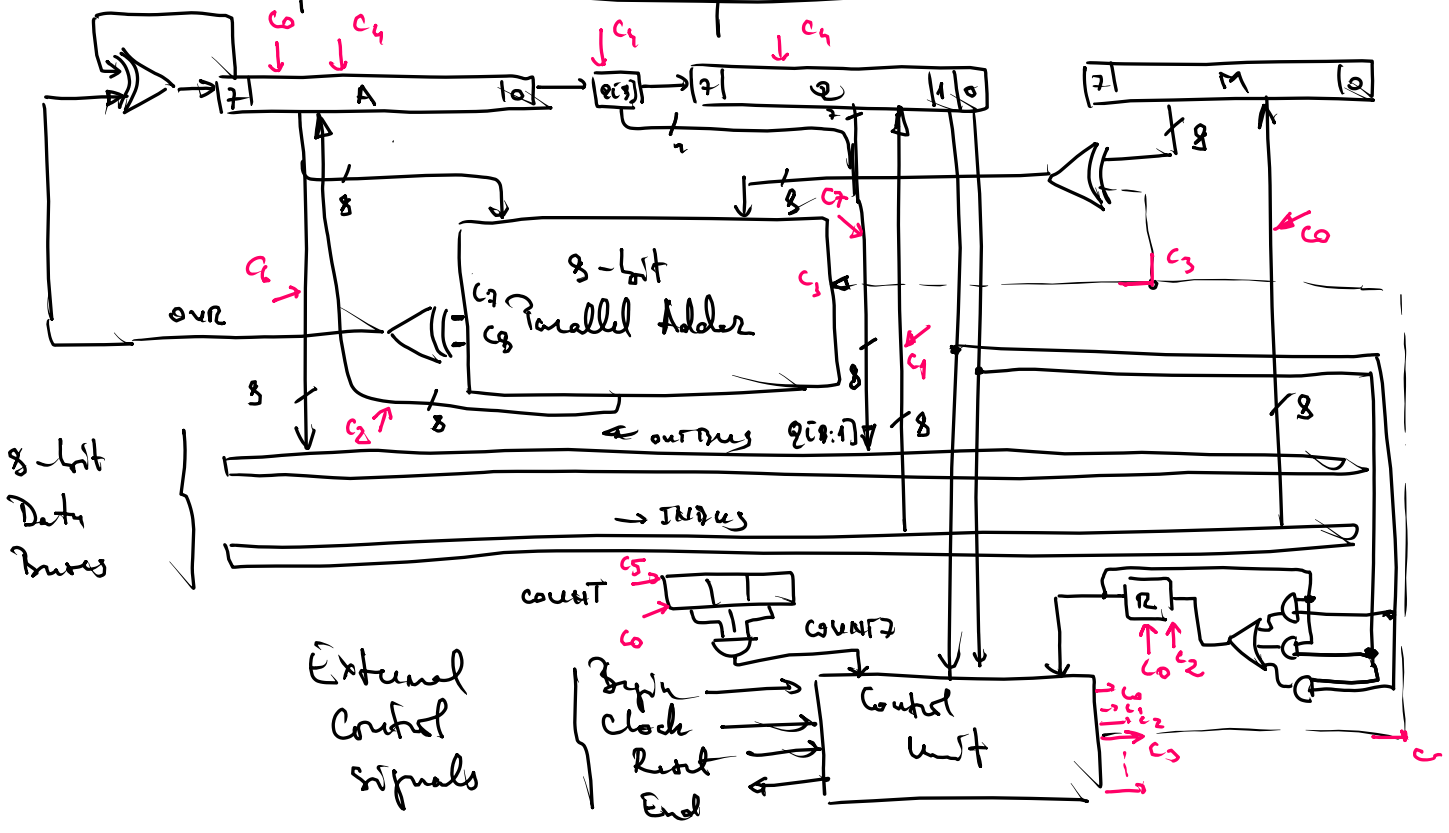
$-M = 0111\ 1011$

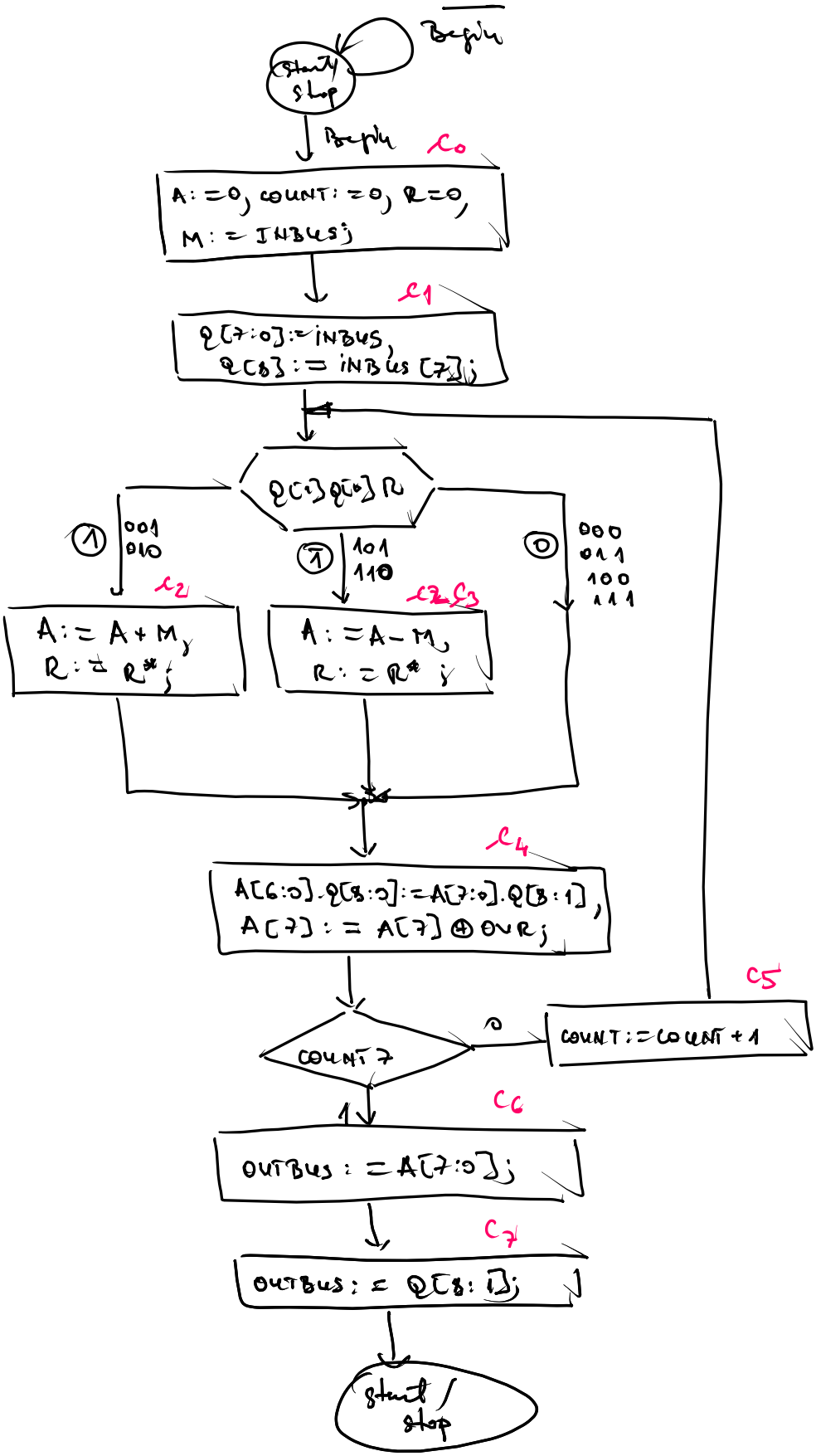
COUNT	OVR	A	Q[0]	Q	R	M
000	0	0000 0000	1	1001 0111	0	1000 0101
	+	0111 1011				
	0	0111 1011			1	
	0	0011 1101	1	1100 1011		
001	0	0001 1110	1	1110 0101	1	
010	0	0000 1111	0	1111 0010	1	
011	+	0111 1011				
	1	1000 1010				
	1	0100 0101	0	0111 1001	1	
100	0	0010 0010	1	0011 1100	1	
101	+	1000 0101				
	0	1010 0111				
	1	1010 0111	1	1001 1110	0	
110	0	1101 0011	1	1100 1111	0	
111	+	0111 1011		Q[8:1]		
	0	0110 0100				
	0	0011 0010	0	1110 0111	1	

$A'[7] = \overline{OVR} \cdot A[7] + OVR \cdot \overline{A[7]} = OVR \oplus A[7]$

19 + 32 + 64 + 512 + 4096 + 8192 = 12915

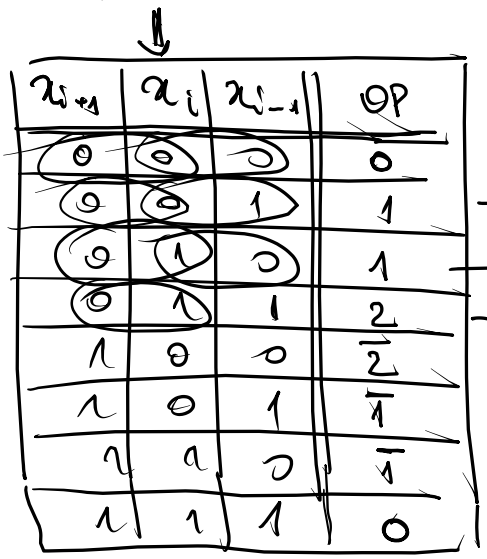
1.2.3 Modified Booth HW implementation




Begin
Start / Stop
Begin
C0
A:=0, COUNT:=0, R=0,
M:=INBUS;
C1
Q[7:0]:=INBUS,
Q[8]:=INBUS[7];
Q[1]Q[0]R
001
010
101
110
000
011
100
111
C2
A:=A+M,
R:=R*;
C3
A:=A-M,
R:=R*;
C4
A[6:0].Q[8:1]:=A[7:0].Q[8:1],
A[7]:=A[7]⊕OVR;
C5
COUNT:=COUNT+1
COUNT 7
0
1
C6
OUTBUS:=A[7:0];
C7
OUTBUS:=Q[8:1];
Start / Stop

1.3 Speeding multiplication with the higher radix

Radix-4



x_{i+1}	x_i	x_{i-1}	OP
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	2
1	0	0	$\bar{2}$
1	0	1	$\bar{1}$
1	1	0	$\bar{1}$
1	1	1	0

$1 \rightarrow +y$
$\bar{1} \rightarrow -y$
$2 \rightarrow +2y$
$\bar{2} \rightarrow -2y$

$\rightarrow y \cdot 2^i + 0 = \underline{y \cdot 2^i}$

$-y \cdot 2^i + y \cdot 2^{i+1} = y \cdot 2^i$

$0 + y \cdot 2^{i+1} = +2y \cdot 2^i$

$\Rightarrow \quad 1 \quad 1 \quad 1 \quad 1$

4 iterations