

1.1.1 Example - integer numbers (Booth)

$$X = -101 ; Y = -53$$

$$P = \begin{array}{r} 101 \times \\ 53 \\ \hline 303 \\ 505 \\ \hline 5353 \end{array}$$

$$\begin{array}{r} 101- \\ 64 \\ \hline = 37- \\ 32 \\ \hline = 5 \\ 53- \\ 32 \\ \hline 21- \\ 16 \\ \hline = 5 \end{array}$$

$$X = 11100101_{sm} = 10011011_{c2}$$

$$Y = 10110101_{sm} = 11001011_{c2}$$

COUNT	A	Q	Q[-1]	M
000 +	$\begin{array}{r} 00000000 \\ 00110101 \\ \hline 00110101 \\ 00011010 \end{array}$	10011011	0	11001011
001	00001101	01100110	1	
010 +	$\begin{array}{r} 11001011 \\ 11011000 \\ 11101100 \end{array}$	00110011	0	
011 +	$\begin{array}{r} 00110101 \\ 00100001 \\ 00010000 \end{array}$	10011001	1	
100	00001000	01001100	1	
101 +	$\begin{array}{r} 11001011 \\ 11010011 \\ 11101001 \end{array}$	10100110	0	
110	11110100	11010011	0	
111 +	$\begin{array}{r} 00110101 \\ 00101001 \\ 00010100 \end{array}$	11101001	1	

$$M = 11001011$$

$$-M = 00110101$$

$$\begin{array}{r} 256 | 7 \\ \hline 0256 \times \end{array}$$

0.25

$$\begin{array}{r} 2567 \\ \downarrow \downarrow \\ 2560 \end{array}$$

$$\begin{array}{r} 41+ \\ 64 \\ 128 \\ 1024 \\ 4096 \end{array}$$

$$+5353$$

1.1.2 Booth example - fractional numbers in C2

$$X = -\frac{107}{128} \quad ; \quad Y = +\frac{9}{64}$$

$$\begin{array}{r} 107- \\ 64 \\ \hline 43- \\ 32 \\ \hline 11 \end{array}$$

$$X = 1 \underset{\Delta}{1} 10 \ 1011_{SM} \\ = 1001 \ 0101_{C2}$$

$$Y = \boxed{0 \ 001001 \underset{\Delta}{0}}_{C2}$$

$$\begin{array}{l} 0.256 \\ 0.2560 \end{array}$$

$$Z = +\frac{963}{2^{13}} = -\frac{963 \cdot 4}{2^{15}} = -\frac{3852}{2^{15}}$$

COUNT	A	Q	Q[-1]	M
000 +	$\begin{array}{r} 0000 \ 0000 \\ 1110 \ 1110 \\ \hline 1110 \ 1110 \\ 1111 \ 0111 \end{array}$	$\begin{array}{r} 1001 \ 0101 \\ \hline 0100 \ 1010 \end{array}$	0	0001 0010
001 +	$\begin{array}{r} 0001 \ 0010 \\ 0000 \ 1001 \\ 0000 \ 0100 \end{array}$	$\begin{array}{r} 1010 \ 0101 \\ \hline 0101 \ 0010 \end{array}$	1	
010 +	$\begin{array}{r} 1110 \ 1110 \\ 1111 \ 0010 \\ 1111 \ 1001 \end{array}$	$\begin{array}{r} 0101 \ 0010 \\ \hline 1101 \ 0100 \end{array}$	1	
011 +	$\begin{array}{r} 0001 \ 0010 \\ 0000 \ 1011 \\ 0000 \ 0101 \end{array}$	$\begin{array}{r} 1010 \ 1001 \\ \hline 1110 \ 1010 \end{array}$	0	
100 +	$\begin{array}{r} 1110 \ 1110 \\ 1111 \ 0011 \\ 1111 \ 1001 \end{array}$	$\begin{array}{r} 1101 \ 0100 \\ \hline 1110 \ 1010 \end{array}$	1	
101 +	$\begin{array}{r} 0001 \ 0010 \\ 0000 \ 1011 \\ 0000 \ 0101 \end{array}$	$\begin{array}{r} 1110 \ 1010 \\ \hline 1111 \ 0101 \end{array}$	0	
110	00000010	11110101	0	
111 +	$\begin{array}{r} 1110 \ 1110 \\ 1111 \ 0000 \end{array}$	11110100		

$$M = 00010010 \\ -M = 11101110$$

$$\begin{array}{r} 256 \\ 12 \\ \hline 268 \end{array}$$

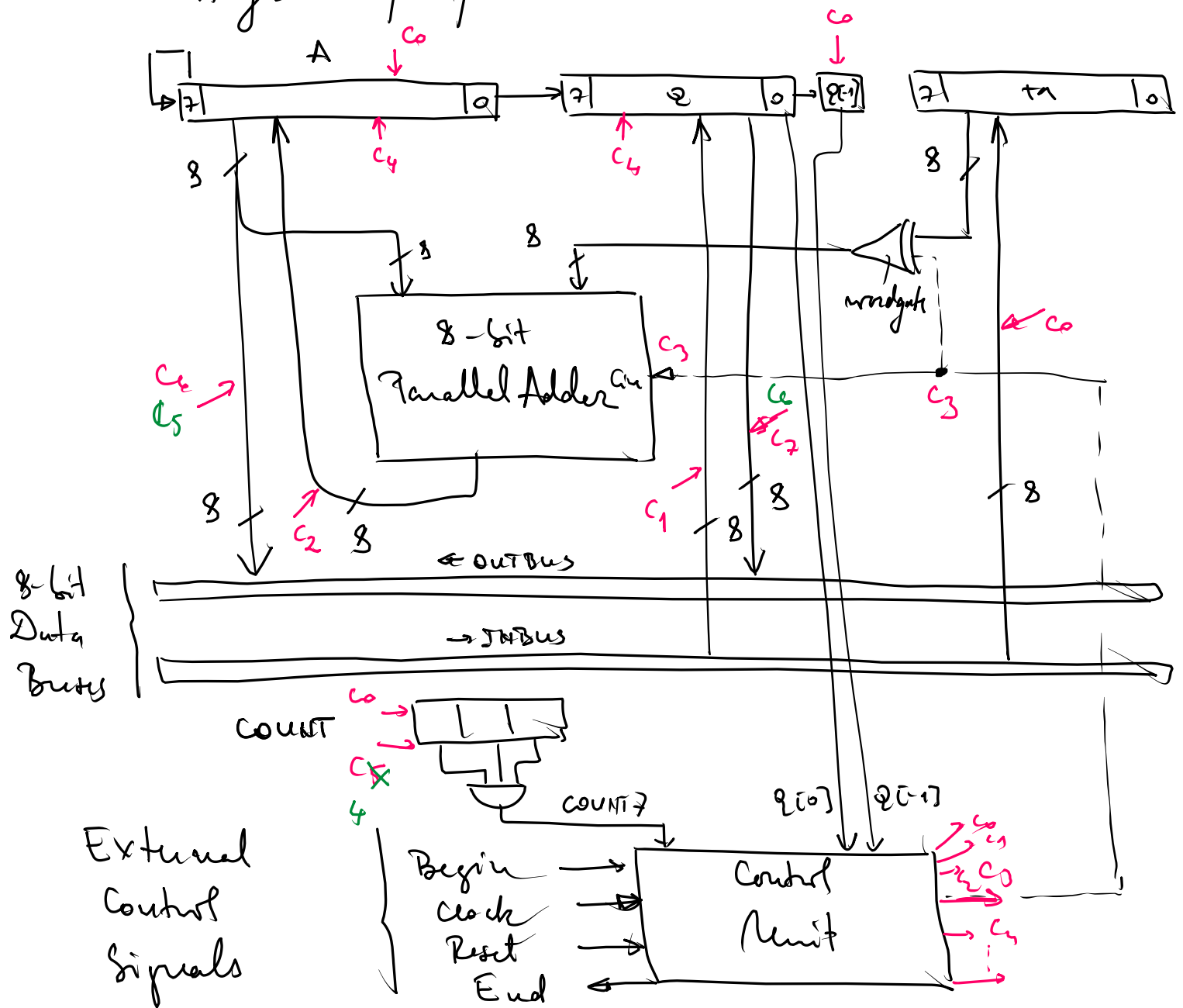
$$\begin{array}{l} 256 \\ 2560 \end{array}$$

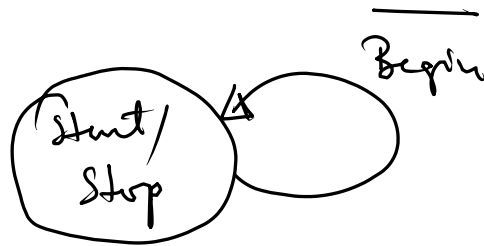
$$\begin{array}{l} 1111 \ 0000 \ 1111 \ 0100 \\ \downarrow \\ 1111 \ 0000 \ 1111 \ 0011 \\ \downarrow \\ 1000 \ 1111 \ 0000 \ 1100 \end{array}$$

$$\begin{array}{r} 268 + \\ 512 \\ 1024 \\ 2048 \\ \hline 3852 \end{array}$$

1.1.3 Booth - HW implementation

- Hayes HW platform



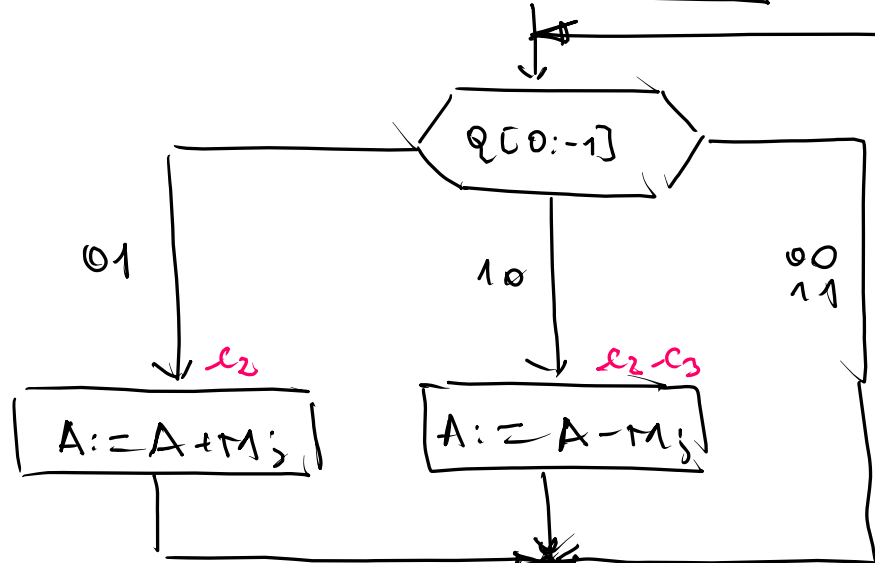


Begin c_0

$A := 0, COUNT := 0,$
 $Q[-1] := 0, M := INBUS;$

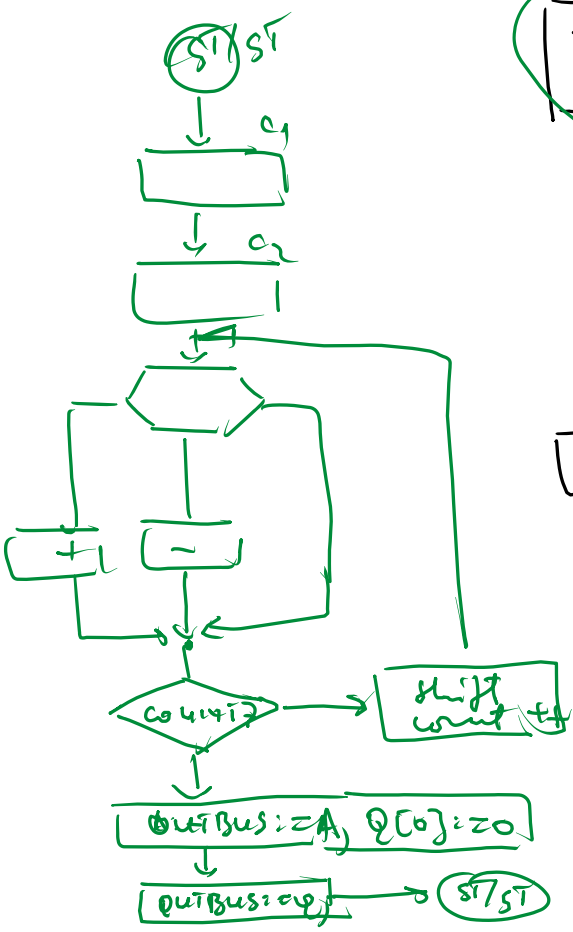
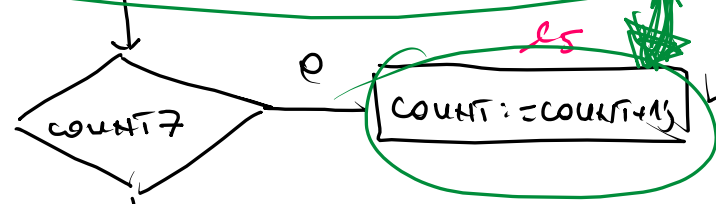
c_1

$Q[7:0] := INBUS;$



c_4

$A[6:0], Q[7:-1] := A[7:0], Q[7:0];$
 $A[7] := A[7];$



1 COMPUTER ARITHMETIC

1.1 Recap

Booth's algorithm : 2s complement multiplier

$$X = (\overline{x_{n-1}})x_{n-2} \dots x_i \dots x_1 x_0 ; x_i \in \mathbb{B} = \{0, 1\}, i = \overline{0, n-1}$$

$$X_{ten} = -x_{n-1} \times 2^{n-1} + \sum_{i=0}^{n-2} x_i \times 2^i$$

2s complement numbers
Robertson's formula

$$\begin{array}{c} 123 \\ \swarrow \quad \downarrow \quad \searrow \\ 10^2 \quad 10^1 \quad 10^0 \end{array}$$

step i

x_i	x_{i-1}	OP	
0	0	0	0
0	1	1	+y
1	0	1	-y
1	1	0	

$$Y \times X = Y \times (\overline{x_{n-1}} x_{n-2} \dots x_i \dots x_1 x_0)$$

multiplicand multiplier

$$(\overline{x_{i-1}} - x_i) \times 2^i \times Y$$

added to the partial product in step i

$$x_i \times 2^{i+1} - x_i \times 2^i = x_i \times 2^i (2-1) = \overline{x_i} \times 2^i$$

step 0 $(\overline{x_{n-1}} - x_0) \times 2^0 \times Y +$

step 1 $(\overline{x_0} - x_1) \times 2^1 \times Y +$

step 2 $(\overline{x_1} - x_2) \times 2^2 \times Y +$

step i-1 $(\overline{x_{i-2}} - x_{i-1}) \times 2^{i-1} \times Y +$

step i $(\overline{x_{i-1}} - x_i) \times 2^i \times Y +$

step n-2 $(\overline{x_{n-3}} - x_{n-2}) \times 2^{n-2} \times Y +$

step n-1 $(\overline{x_{n-2}} - x_{n-1}) \times 2^{n-1} \times Y$

$$P = Y \times \left(-\overline{x_{n-1}} \times 2^{n-1} + \sum_{i=0}^{n-2} x_i \times 2^i \right)$$

P = ?