1.3. Speeding up multiplication with the higher radix

1.3.1 Redundant digit sets

radix-2 $\{0,1\}$ redundant radix 2 - $\{\bar{1},0,1\}$

$$10\bar{1}1 = 2^3 + (-1)2^1 + 2^0 = 7 = 111_2$$

$$1\begin{cases}1\\0\end{cases} \qquad 0\begin{cases}0\\0\end{cases} \qquad \bar{1}\begin{cases}0\\1\end{cases}$$

radix-4 $\{0,1,2,3\}$

redundant radix 4 $\{\bar{2},\bar{1},0,1,2\}$

$$\begin{array}{cccc} 4^3 & 4^2 & 4^1 & 4^0 \\ 1 & 0 & \bar{2} & 2 \end{array} = 4^3 - 2\cdot 4^1 + 2$$

$$= 64 - 8 + 2 = 58$$

$$2\begin{cases}1\,0\\0\,0\end{cases} \qquad 1\begin{cases}0\,1\\0\,0\end{cases} \qquad 0\begin{cases}0\,0\\0\,0\end{cases}$$

$$\bar{2}\begin{cases}0\,0\\1\,0\end{cases} \qquad \bar{1}\begin{cases}0\,0\\0\,1\end{cases}$$

$$1\,0\,\bar{2}\,2 \rightarrow \begin{array}{r} 01\ 00\ 00\ 10 \\ -\ 00\ 00\ 10\ 00 \\ \hline 00\ 11\ 10\ 10 \end{array}$$

1.3.2. Radix – 4 Booth

x_{i+1}	x_i	x_{i-1}	operation
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	2
1	0	0	$\bar{2}$
1	0	1	$\bar{1}$
1	1	0	$\bar{1}$
1	1	1	0

$0 \times 2^{i+1} + 0 \times 2^i = 0 \times 2^i$

$1 \times 2^i + 0 \times 2^{i+1} = 1 \cdot 2^i$

$\bar{1} \times 2^i + 1 \times 2^{i+1} =$
$= 2^i(-1+2) = 2^i$

$0 \times 2^i + 1 \times 2^{i+1} = 2 \times 2^i$

$0 \times 2^i - 1 \times 2^{i+1} = -2 \times 2^i$

$1 \times 2^i - 1 \times 2^{i+1} = -2^i$

$-1 \times 2^i + 0 \cdot 2^{i+1} = -1 \cdot 2^i$

Înjumătățire timp!! de la 8 clk
la 4 clk

pt $2, \bar{2}$ → adunare / scădere $2 \times Y$
+ 2 shiftări (orice caz)

$M = 1|1\,000\,0101$ (pe 9 biți, dublat bit semn)

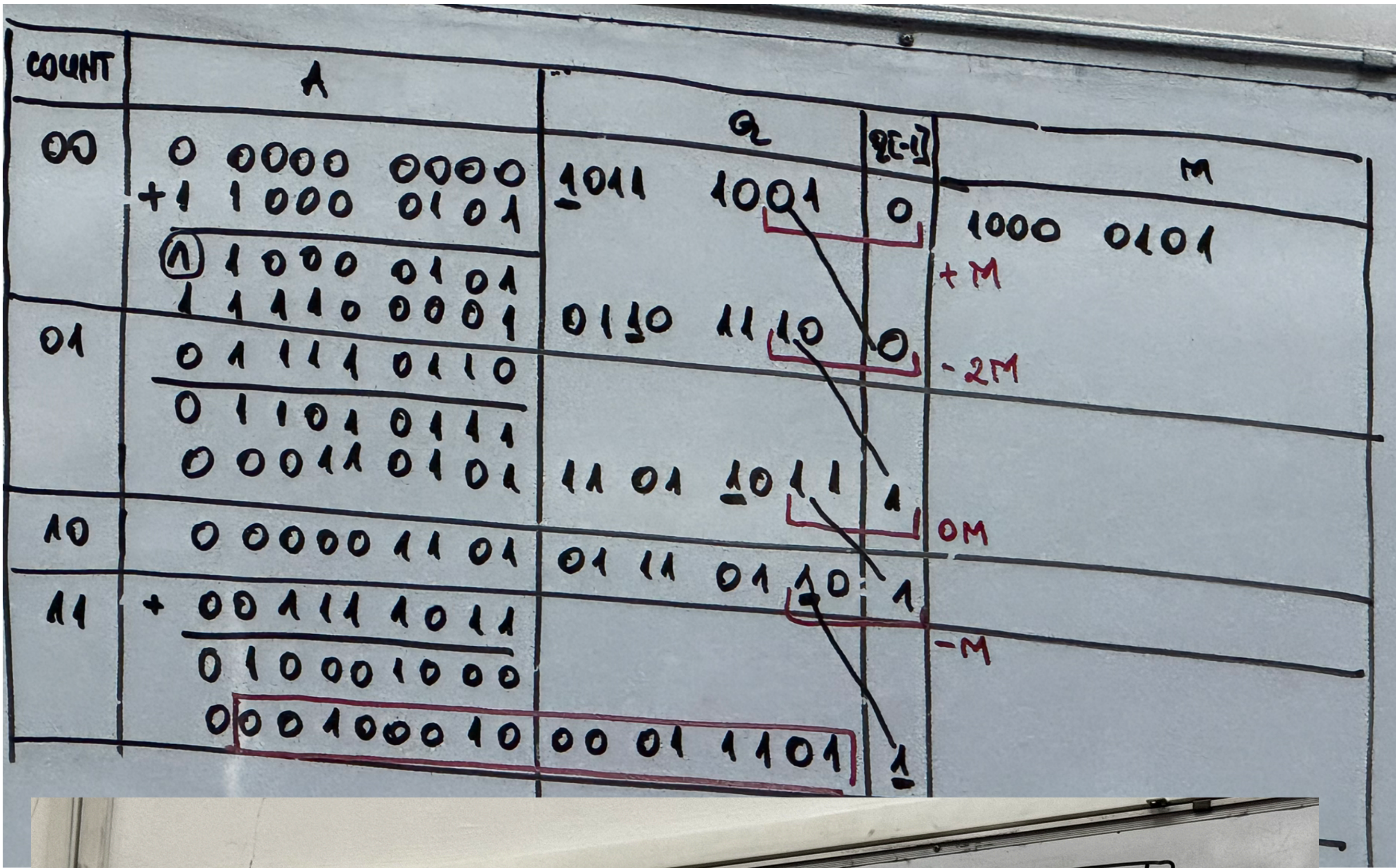
$2M = 1000\,0101\,0$

$-M = \_\ \_\_\ \_$

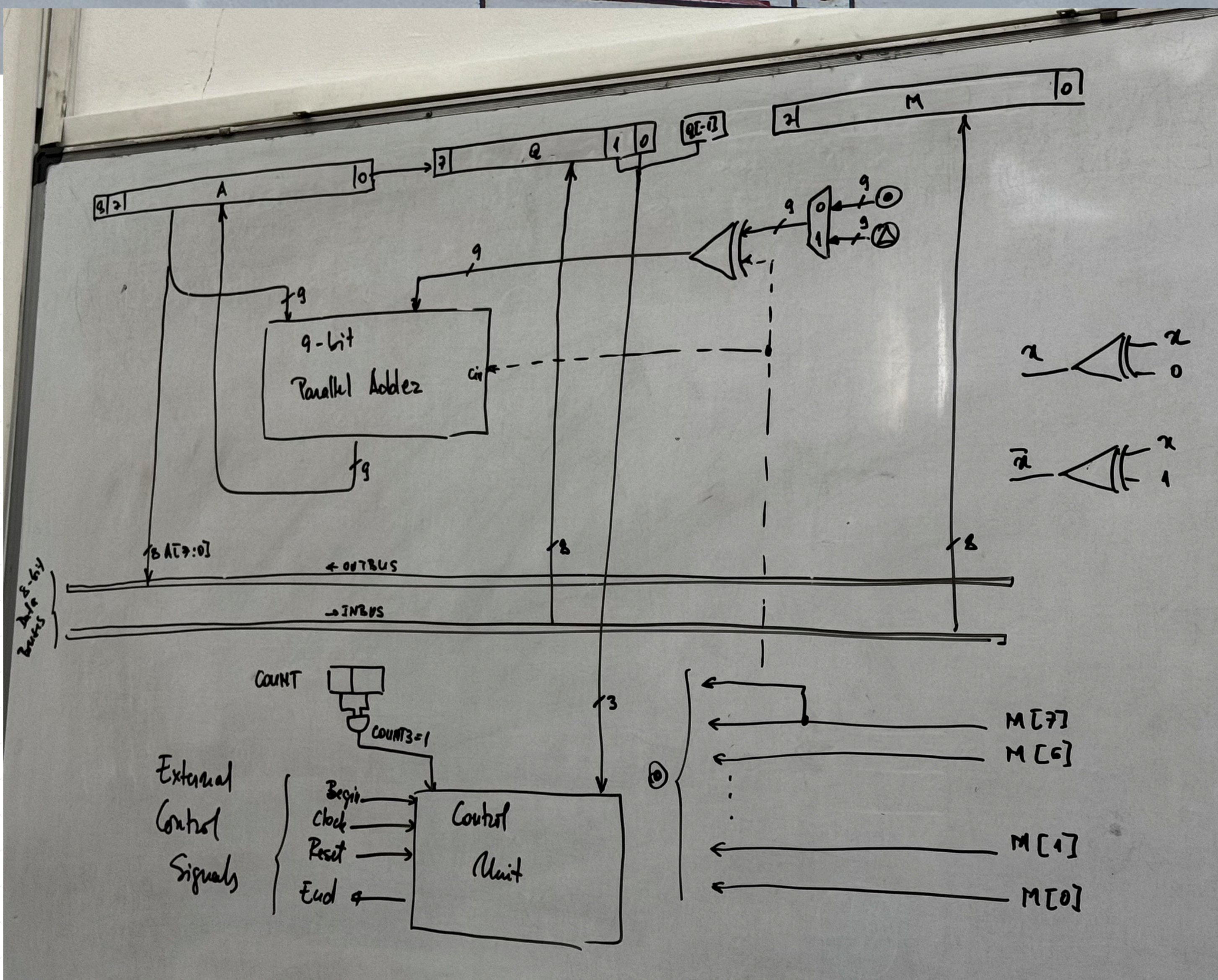
$-2M = \_\_\ \_\_\ \_\_\ \_\_$

1011 1001 = -128 + 32 + 16 + 8 + 1 = -71

1000 0101 = -123



COUNT	A	Q	Q[-1]	M
00	0 0000 0000	1011 1001	0	1000 0101
	+1 1000 0101			+M
	1 1000 0101			
	1 1100 0001	0110 1110	0	
01	0 1111 0110			-2M
	0 1101 0111			
	0 0011 0101	1101 1011	1	
10	0 0000 1101	0111 0110	1	0M
11	+ 0 0111 1011			-M
	0 1000 1000			
	0 0010 0010	0001 1101	1	



1.3.2 Radix-4 Booth

$0 \times 2^{i+1}$ 0×2^{i}

x_{i+1}	x_i	x_{i-1}	Operation
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	2
1	0	0	$\bar{2}$
1	0	1	$\bar{1}$
1	1	0	$\bar{1}$
1	1	1	0

$0 \times 2^{i+1} + 0 \times 2^{i} = 0 \times 2^{i}$

$1 \times 2^{i} + 0 \times 2^{i+1} = 1 \times 2^{i}$

$\bar{1} \times 2^{i} + 1 \times 2^{i+1} = 2^{i}(-1+2) = 1 \times 2^{i}$

$0 \times 2^{i} + 1 \times 2^{i+1} = 2 \times 2^{i}$

$0 \times 2^{i} - 1 \times 2^{i+1} = -2 \times 2^{i}$

$1 \times 2^{i} - 1 \times 2^{i+1} = 2^{i}(1-2) = -1 \times 2^{i}$

$-1 \times 2^{i} + 0 \times 2^{i+1} = -1 \times 2^{i}$

$0 \times 2^{i} + 0 \times 2^{i+1} = 0 \times 2^{i}$

COUNT	A	Q	Q[-1]	M	
00	0 0000 0000 +1 1000 0101 (1) 1000 0101 1 1110 0001	1011 1001 0110 1110	0 0	1000 0101	+M -2M
01	0 1111 0110 0 1101 0111 0 0011 0101	 11 01 1011	 1		 0M
10	0 0000 1101	01 11 0110	1		
11	+ 00111 1011 010001000 000100010	 00 01 1101	 1		-M

```
  - 123 x
  -  71
  ------
    123
   861
  ------
 + 8733
```

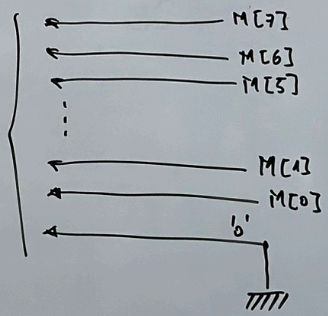
M = 110000101

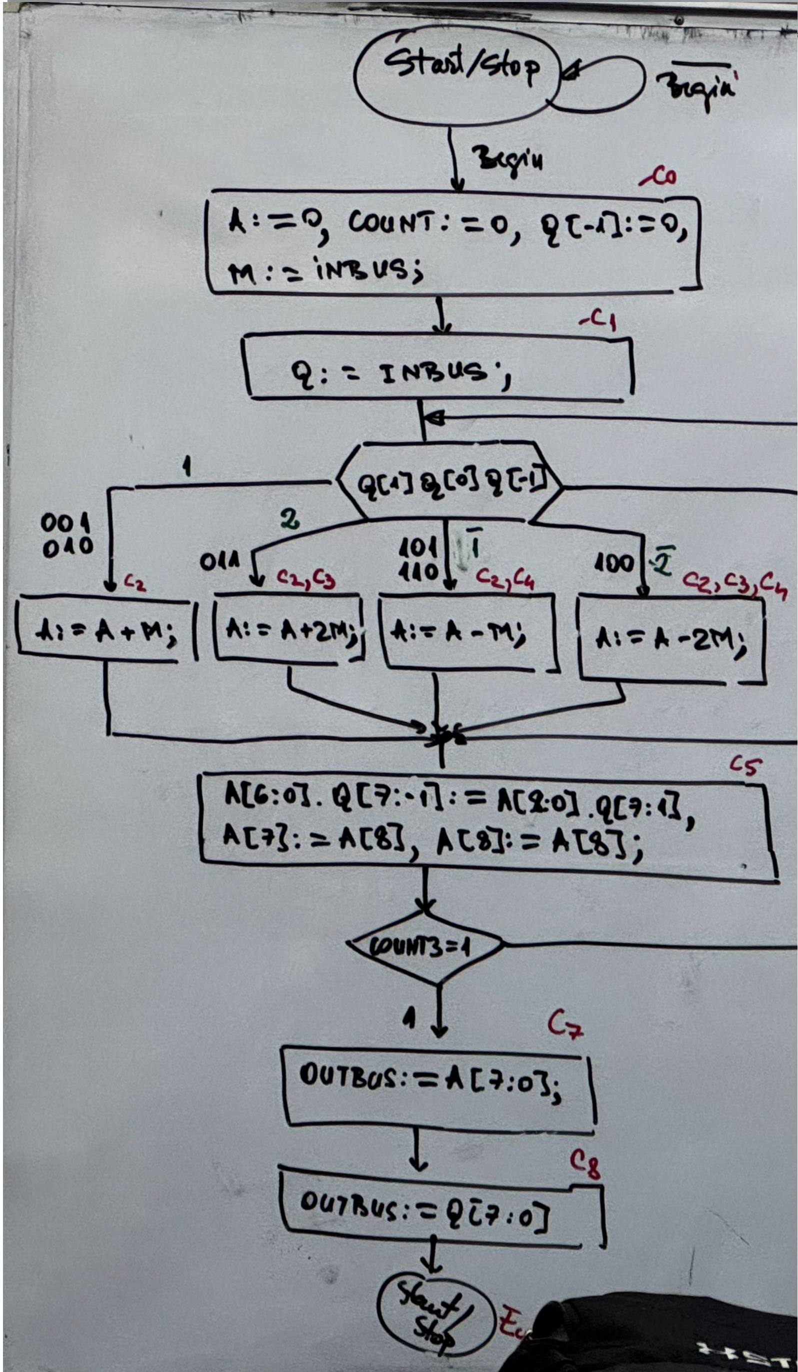
2M = 100001010

−M = 001111011

−2M = 011110110

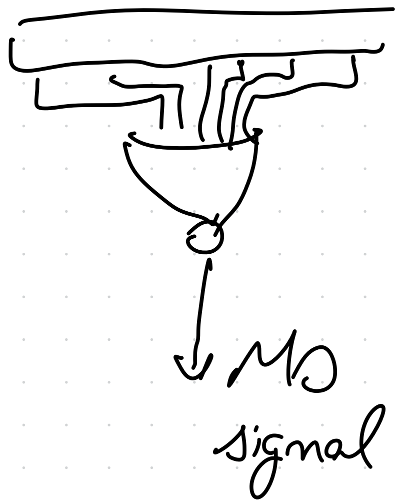
(A)

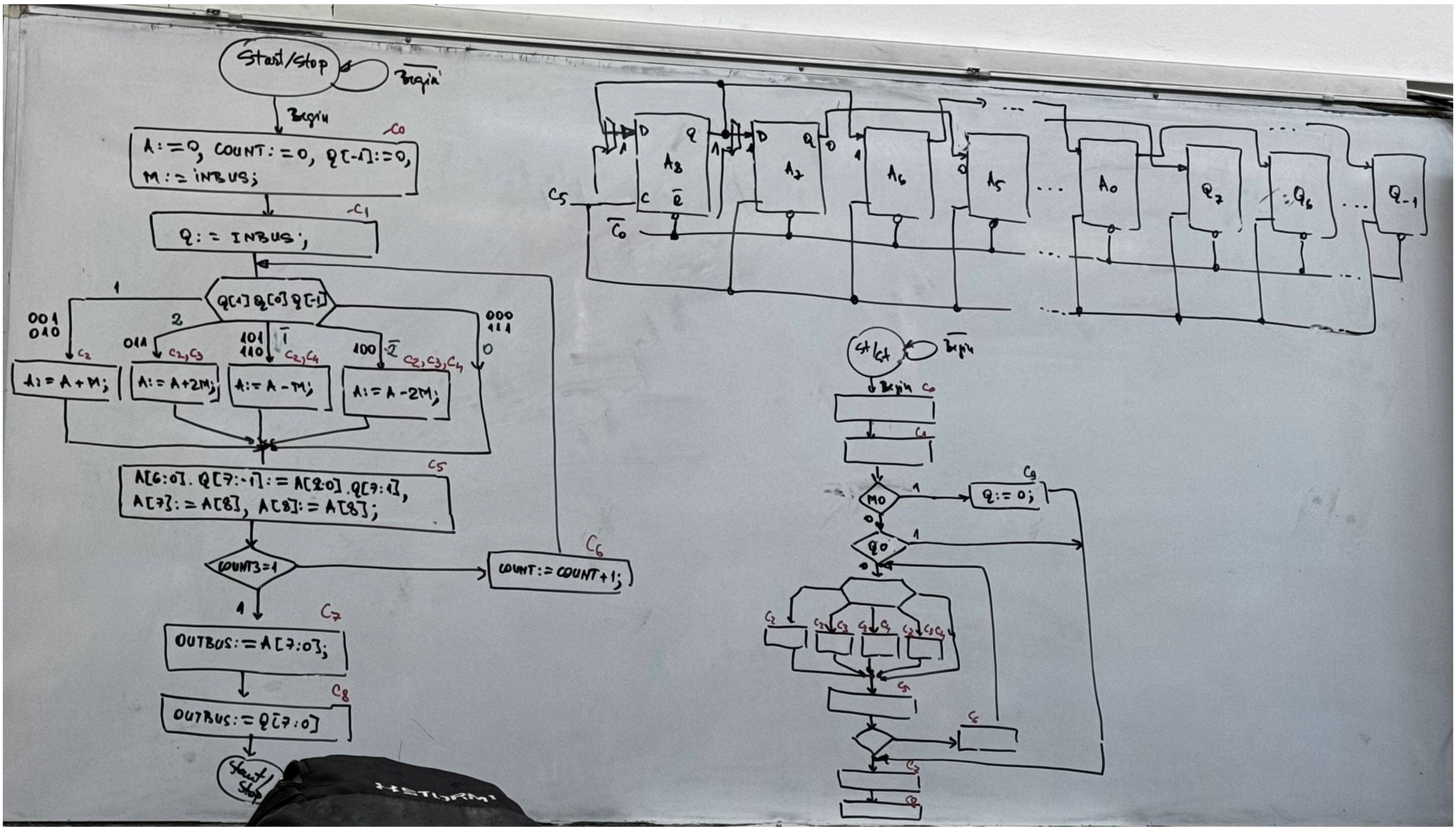




0 Checking

-> NOR pe M





1.3.3 Radix-8 Booth

x_{i+2}	x_{i+1}	x_i	x_{i-1}	op
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	2
0	1	0	0	2
0	1	0	1	3
0	1	1	0	3
0	1	1	1	4
1	0	0	0	$\overline{4}$
1	0	0	1	$\overline{3}$
1	0	1	0	$\overline{3}$
1	0	1	1	$\overline{2}$
1	1	0	0	$\overline{2}$
1	1	0	1	$\overline{1}$
1	1	1	0	$\overline{1}$
1	1	1	1	0

2^{i+2} 2^{i+1} 2^i

$-2^i + 2^{i+1}$

Acumulator pe 10 biți (8 + 2 pt shift)

Q[8] Q[-1]

1.3.3 Radix-8 Booth

2^{i+2}, 2^{i+1}, 2^i

x_{i+2}	x_{i+1}	x_i	x_{i-1}	OP	
0	0	0	0	0	
0	0	0	1	1	$+2^i$
0	0	1	0	1	$-2^i+2^{i+1} = 2^i(2-1) = 2^i$
0	[illegible]	[illegible]	1	2	$0\times 2^i + 2^{i+1} = 2\cdot 2^i$
[illegible]	[illegible]	[illegible]	0	2	$-2^{i+1}+2^{i+2} = (-2+4)2^i = 2\cdot 2^i$
[illegible]	[illegible]	[illegible]	1	3	$-2^i + 2^{i+2} = 2^i(4-1) = 3\cdot 2^i$
[illegible]	[illegible]	[illegible]	0	3	$-2^i + 2^{i+2} = 2^i(-1+4) = 3\cdot 2^i$
[illegible]	[illegible]	[illegible]	1	4	$+2^{i+2} = 4\cdot 2^i$
1	[illegible]	[illegible]	[illegible]	$\bar{4}$	$-2^{i+2} = -4\cdot 2^i$
1	0	0	[illegible]	$\bar{3}$	$+2^i - 2^{i+2} = 2^i(1-4) = -3\times 2^i$
1	0	1	[illegible]	$\bar{3}$	$+2^i - 2^{i+2} = -3\times 2^i$
1	0	[illegible]	[illegible]	$\bar{2}$	$2^{i+1} - 2^{i+2} = 2^i(2-4) = -2\times 2^i$
1	1	[illegible]	0	$\bar{2}$	$-2^{i+1} = -2\times 2^i$
1	1	[illegible]	1	$\bar{1}$	$-1\times 2^i + 0 = -1\times 2^i$
1	1	[illegible]	0	$\bar{1}$	-1×2^i
1	1	[illegible]	1	0	

+M = [illegible]000 0101

−M = [illegible]111 1011

2M = [illegible]000 01010

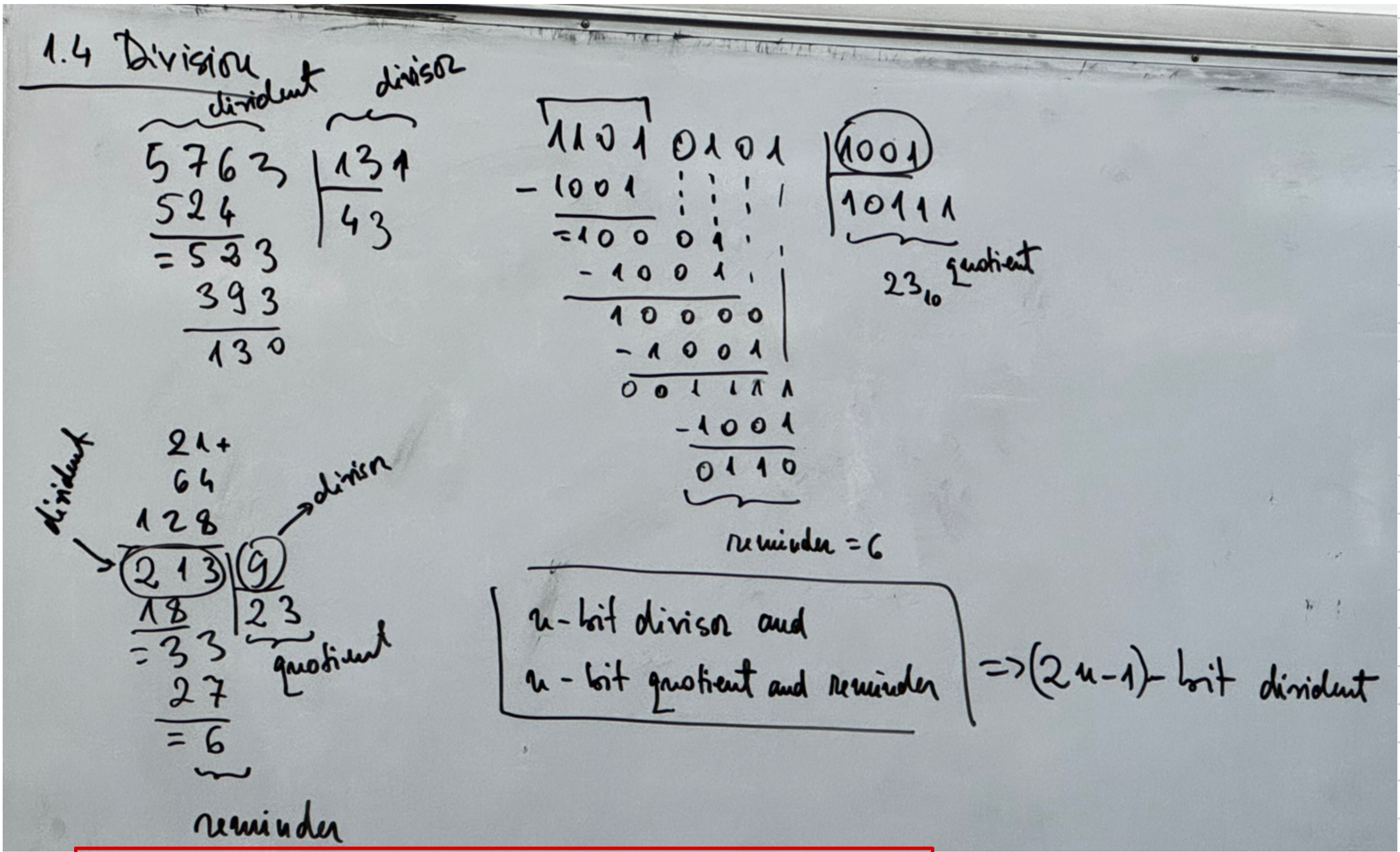
3M = [illegible]1000 1111

```
  111 000 0101
  110 000 1010
  101 000 1111
```

COUNT	A	Q	Q[-1]	M
	000000 0000	1 1011 1 001	0	1000 0101
00 +	111000 0101			
	111000 0101			
	111111 0000	10111 0 111	0	−M
01 +	000111 1011			
	000110 1011			
	000000 1101	01110 1 110	1	−M
10 +	000111 1011			
	001000 1000			
	000001 000 1	00001 1101	1	

+M

1.4. Division



1.4.1. Restoring Division