ex) EE : $u_a = K_E \cdot u_c$

PT : $\frac{L_a}{R_a} \frac{di_a}{dt} + i_a = \frac{1}{R_a}(u_a - e_\omega)$

$e_\omega = k_e \omega \quad , T_a = \frac{L_a}{R_a}$

$m_a = K_m i_a \quad , m_f = k_f \omega \quad , k_f \approx 0$

$J \frac{d\omega}{dt} = m_a - m_f - m_s$

EM : $u_\omega = K_{M\omega} \omega \quad , u_i = K_{Mi} i_a$

$$\begin{cases} x' = A \cdot x + B \cdot u \\ y = C^T \cdot x \end{cases}$$

- intrare : u_c, m_s $\rightarrow u = \begin{bmatrix} u_c \\ m_s \end{bmatrix}$
- stare : ω, i_a $\quad x = \begin{bmatrix} i_a \\ \omega \end{bmatrix}$ $\quad y = u_\omega$
- iesire : u_ω

- aplic scheletul :

$$\begin{bmatrix} i_a' \\ \omega' \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{k_e}{L_a} \\ \frac{k_m}{J} & -\frac{k_f}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{k_e}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} u_c \\ m_s \end{bmatrix}$$

$$u_\omega = \begin{bmatrix} 0 & K_{M\omega} \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix}$$

$$\bullet \frac{L_a}{R_a} i_a' + i_a = \frac{1}{R_a}(u_a - e_\omega)$$

$$\Rightarrow i_a' = \left(\frac{u_a}{R_a} - \frac{e_\omega}{R_a} - i_a\right) \cdot \frac{R_a}{L_a}$$

$$i_a' = \frac{u_a \cdot \cancel{R_a}}{\cancel{R_a} \cdot L_a} - \frac{e_\omega \cdot \cancel{R_a}}{\cancel{R_a} \cdot L_a} - \frac{i_a R_a}{L_a}$$

$$i_a' = \frac{u_a}{L_a} - \frac{e_\omega}{L_a} - \frac{i_a R_a}{L_a}$$

$$i_a' = \frac{K_E \cdot u_c}{L_a} - \frac{K_e \omega}{L_a} - \frac{i_a R_a}{L_a}$$

$$\bullet J \frac{d\omega}{dt} = m_a - m_f - m_s$$

$$\Rightarrow \omega' = \frac{m_a}{J} - \frac{m_f}{J} - \frac{m_s}{J}$$

$$= \frac{K_m \cdot i_a}{J} - \frac{K_f \omega}{J} - \frac{m_s}{J}$$

$$\bullet u_\omega = K_{T\omega}\, \omega$$

Ex intrare – ieșire

EE: $u_a(t) = K_A u_c$, $\omega = K_m \cdot u_a$ $\quad q_a = K_p \omega$

PT: $h' = \frac{1}{a}(q_a - q_e)$

$$q_e = K_{Re} \cdot v \sqrt{2 \cdot g \cdot h}$$

$u = \begin{bmatrix} u_c \\ v \end{bmatrix}$, $y = h$

$$h'(t) = \frac{1}{A} K_p K_m \cdot K_A \cdot u_c(t) - \frac{1}{A} K_{Re} \sqrt{2gh(t)} \cdot v(t)$$

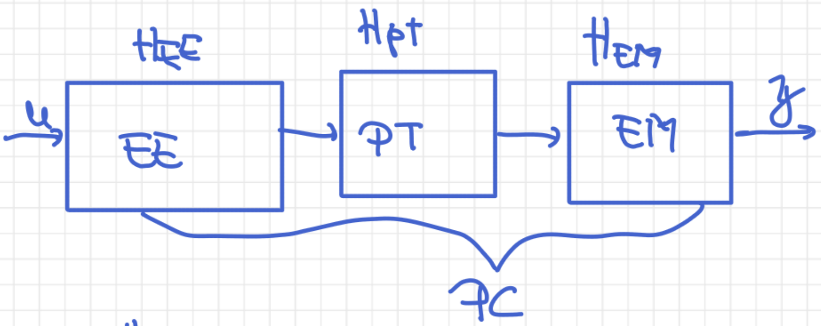
Funcția de transfer

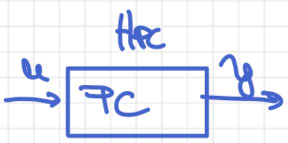
Forma generală a modelului matematic:

$$\begin{cases} x'(t) = A \cdot x(t) + B \cdot u(t) \\ y(t) = C^T \cdot x(t) \end{cases}$$

$\Downarrow$ T. lui Laplace: $x^{(n)}(t) = S^n \cdot x(S)$

$$\begin{cases} S \cdot x(S) = A \cdot x(S) + B \cdot u(S) \\ y(S) = C^T \cdot x(S) \end{cases}$$





Din ecuații: $S \cdot x(S) - A\,x(S) = B \cdot u(S)$

$$S \cdot I \cdot x(S) - A \cdot x(S) = B \cdot u(S)$$

matr. unitate

$$(SI - A) \cdot x(S) = B \cdot u(S)$$

$$\Rightarrow x(S) = (SI - A)^{-1} \cdot B \cdot u(S)$$

$$\left.\begin{aligned} &\Rightarrow y(S) = \underbrace{C^T \cdot (SI - A)^{-1} \cdot B}_{H_{PC}} \cdot u(S) \quad \text{funcția de transfer} \\ &y(S) = H_{PC} \cdot u(S) \end{aligned}\right\} \Rightarrow H_{PC} = \frac{y(S)}{u(S)}$$

$$\Rightarrow \boxed{H_{PC} = C^T \cdot (SI - A)^{-1} \cdot B}$$

$$\Rightarrow H_{PC} = \begin{pmatrix} 0 & K_m \end{pmatrix} \left[\begin{pmatrix} S + \frac{k_p}{C_p} & -\frac{k_p}{C_p} \\ -\frac{k_p + k_c}{C_p} & S + \frac{k_p}{C_p} \end{pmatrix}^{-1} \right] \cdot \begin{pmatrix} \frac{k_e}{C_p} & 0 \\ 0 & -\frac{k_c}{C_p} \end{pmatrix}$$

$$= \begin{bmatrix} 0 & K_m \end{bmatrix} \cdot \frac{1}{\left(S + \frac{k_p}{C_p}\right)^2 - \frac{k_p(k_p + k_c)}{C_p^2}} \begin{bmatrix} S + \frac{k_p}{C_p} & \frac{k_p}{C_p} \\ \frac{k_p + k_c}{C_p} & S + \frac{k_p}{C_p} \end{bmatrix} \cdot \begin{bmatrix} \frac{k_e}{C_p} & 0 \\ 0 & -\frac{k_c}{C_p} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & K_m \end{bmatrix} \cdot \frac{C_p^2}{S^2 C_p^2 + 2 \cdot S \cdot k_p \cdot C_p + \cancel{k_p^2} - \cancel{k_p^2} - k_c \cdot k_p} \begin{bmatrix} S + \frac{k_p}{C_p} & \frac{k_p}{C_p} \\ \frac{k_p + k_c}{C_p} & S + \frac{k_p}{C_p} \end{bmatrix} \cdot \begin{bmatrix} \frac{k_e}{C_p} & 0 \\ 0 & -\frac{k_c}{C_p} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & K_m \end{bmatrix} \cdot \frac{1}{S^2 C_p^2 + k_p(2 S C_p - k_c)} \cdot \begin{bmatrix} S C_p + k_p & k_p \\ k_p + k_c & S C_p + k_p \end{bmatrix} \begin{bmatrix} K_e & 0 \\ 0 & -K_c \end{bmatrix}$$

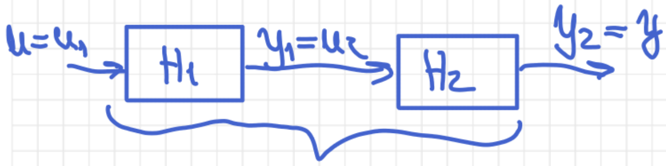
$$= \frac{1}{S^2 C_p^2 + k_p(2 S C_p - k_c)} \begin{bmatrix} K_m k_p + K_m K_c & K_m S C_p + K_m K_p \end{bmatrix} \begin{bmatrix} k_e & 0 \\ 0 & -K_c \end{bmatrix}$$

$$= \frac{1}{S^2 C_p^2 + k_p(2 S C_p - k_c)} \begin{bmatrix} k_e K_m k_p + k_e K_m K_c & -K_m K_c S C_p - k_c K_m k_p \end{bmatrix}$$

$$= \begin{bmatrix} \frac{k_e K_m \cdot k_p + k_e K_m K_c}{S^2 C_p^2 + k_p(2 S C_p - k_c)} & \frac{-S K_m K_c \cdot C_p - K_m K_p K_c}{S^2 C_p^2 + k_p(2 S C_p - k_c)} \end{bmatrix}$$

Conexiuni

CONEXIUNEA SERIE



$y_1 = H_1 \cdot u_1$

$y_2 = H_2 \cdot u_2$

$u = u_1 \Longrightarrow y_1 = H_1 \cdot u \;\Big\}\Rightarrow u_2 = H_1 \cdot u$

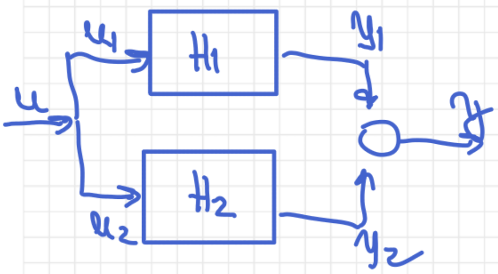
$u_2 = y_1$

$y = y_2$

$y_2 = H_2 \cdot H_1 \cdot u \Rightarrow y = \underbrace{H_1 \cdot H_2}_{H} \cdot u$

$\Longrightarrow \boxed{H = H_1 \cdot H_2 \text{ în serie}}$

CONEXIUNEA PARALEL



$u = u_1 = u_2$

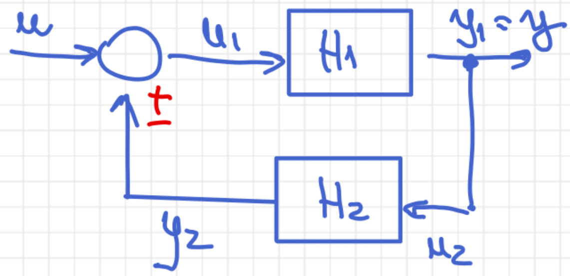
$y = y_1 + y_2 = H_1 \cdot u_1 + H_2 \cdot u_2$

$y = (H_1 + H_2) \cdot u$

$\Downarrow$

$\boxed{H = H_1 + H_2 \text{ în paralel}}$

CONEXIUNEA CU REACȚIE



$$y = y_1$$

$$y_1 = H_1 \cdot u_1$$

$$H_1 \cdot u_1 = H_1(u \pm y_2) = H_1 \cdot (u \pm H_2 \cdot u_2) = H_1(u \pm H_2 y)$$

$$\Rightarrow y = H_1 u \pm H_1 H_2 y$$

$$\Rightarrow y(1 \mp H_1 H_2) = H_1 \cdot u$$

$$\Rightarrow y = \frac{H_1}{1 \mp H_1 H_2} u \quad \Rightarrow \boxed{H = \frac{H_1}{1 \mp H_1 H_2}}$$

calea directă

calea cu reacție