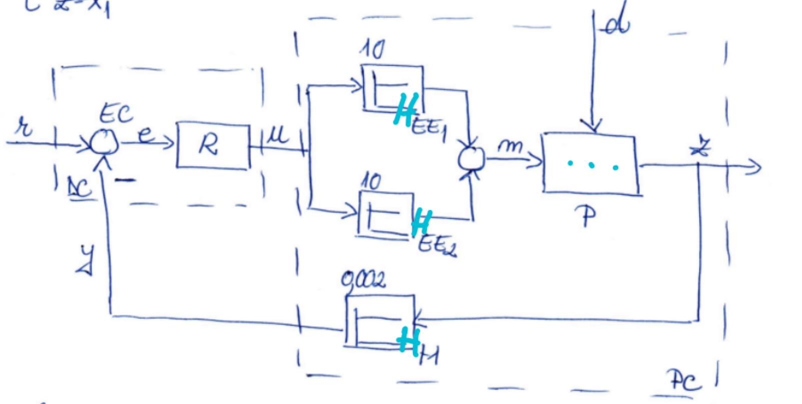
Se consideră sistemul de reglare automată cu schema bloc prezentată în figură, în care r(t) este referința, e(t) este eroarea de reglare și modelul de stare (MM-ISI) al blocului P este:

$$\begin{cases} \dot{x}_1 = -2x_1 + 2x_2 + 40d \\ \dot{x}_2 = -0{,}5x_2 + 12{,}5m \\ z = x_1 \end{cases}$$



– trebuie găsite matricile A, B și C pentru a se găsi matricea de transfer $H(s) = C\cdot(sI-A)^{-1}\cdot B = C\cdot M^{-1}\cdot B$

$$\begin{cases} \dot{x}(t) = A\cdot x(t) + B\cdot u(t) \\ y(t) = c^T\cdot x(t) \end{cases}$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

$$\begin{cases} \dot{x}_1 = -2x_1 + 2x_2 + 0\cdot m + 40d \\ \dot{x}_2 = 0\cdot x_1 - 0{,}5x_2 + 12{,}5m \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 2 \\ 0 & -0{,}5 \end{bmatrix}}_{A} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 40 \\ 12{,}5 & 0 \end{bmatrix}}_{B} \cdot \begin{bmatrix} m \\ d \end{bmatrix}$$

← variabile de stare

$$z = 1\cdot x_1 + 0\cdot x_2 \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$H(s) = C\cdot(\underbrace{sI - A}_{M})^{-1}\cdot B$$

$$M = s\cdot\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 2 \\ 0 & -0{,}5 \end{bmatrix} = \begin{bmatrix} s+2 & -2 \\ 0 & s+0{,}5 \end{bmatrix}$$

$$M^T = \begin{bmatrix} s+2 & 0 \\ -2 & s+0{,}5 \end{bmatrix}; \quad M^{-1} = \frac{1}{\det M}\cdot M^*$$

$$\det M = (n+2)(n+0{,}5) = 2(0{,}5n+1)\cdot 0{,}5(1+2n) =$$
$$= (0{,}5n+1)(1+2n)$$

$$M^{*} = \begin{bmatrix} (-1)^{1+1}\cdot(n+0{,}5) & (-1)^{1+2}\cdot(-2) \\ (-1)^{2+1}\cdot 0 & (-1)^{2+2}\cdot(n+2) \end{bmatrix} =$$

$$= \begin{bmatrix} n+0{,}5 & 2 \\ 0 & n+2 \end{bmatrix}$$

$$M^{-1} = \frac{1}{(0{,}5n+1)(1+2n)} \begin{bmatrix} n+0{,}5 & 2 \\ 0 & n+2 \end{bmatrix}$$

$$H(n) = C \cdot M^{-1} \cdot B$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \frac{1}{(0{,}5n+1)(1+2n)} \begin{bmatrix} n+0{,}5 & 2 \\ 0 & n+2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 40 \\ 12{,}5 & 0 \end{bmatrix}$$

$$= \frac{1}{(0{,}5n+1)(1+2n)} \cdot \begin{bmatrix} n+0{,}5 & 2 \end{bmatrix} \begin{bmatrix} 0 & 40 \\ 12{,}5 & 0 \end{bmatrix} =$$

$$= \frac{1}{(n+2)(n+0{,}5)} \begin{bmatrix} 25 & 40(n+0{,}5) \end{bmatrix} =$$

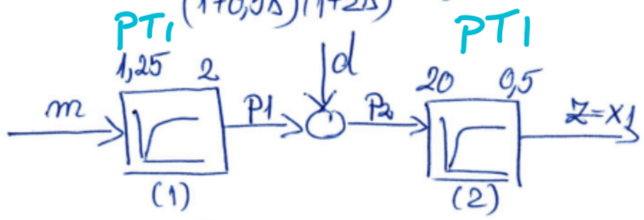
d

$$= \begin{bmatrix} \dfrac{25}{(n+2)(n+0{,}5)} & \dfrac{40}{n+2} \end{bmatrix}$$

$$H_{zm}(s)\Big|_{d=0} = \frac{25}{(s+2)(s+0,5)}$$

$$H_{zd}(s)\Big|_{m=0} = \frac{40}{s+2}$$

$$H_{zm}(s) = \frac{25}{(1+0,5s)(1+2s)} \quad \text{și} \quad H_{zd}(s) = \frac{20}{1+0,5s}$$



=> verificăm să vedem dacă obținem HM-ISI-ul.

$p_1(s) = \frac{1,25}{1+2s} m(s) \Rightarrow p_1(s) + 2s\,p_1(s) = 1,25\,m(s) \Rightarrow$ trebuie să avem $12,5\,m(s) \Rightarrow$ despărțim

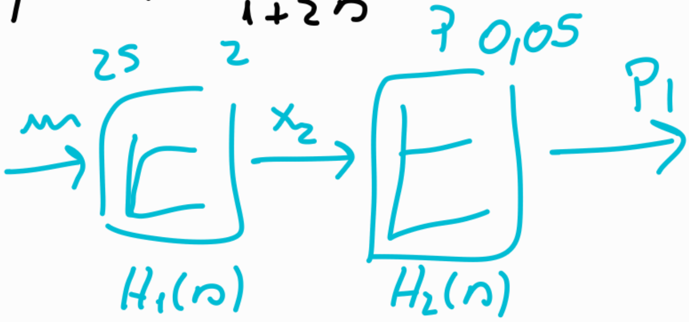
$$H(s) = \frac{K}{sT+1}$$

$$H_{zm}(s) = H \cdot H_{zd}$$

$$\frac{25}{(1+0,5s)(1+2s)} = H \cdot \frac{20}{1+0,5s}$$

$$H = \frac{25}{\cancel{(1+0,5s)}(1+2s)} \cdot \frac{\cancel{1+0,5s}}{20} = \frac{1,25}{1+2s}$$

$$p_1(s) = \frac{1,25}{1+2s} m(s) \Rightarrow p_1(s) + 2s\,p_1(s) = 1,25\,m(s)$$



$$H_1(s) \cdot H_2(s) = \frac{1,25}{1+2s}$$

$$\frac{25}{1+2s} \cdot H_2(s)$$

$$x_2(s) = H_1(s) \cdot m(s)$$

$$x_2(s) = \frac{25}{1+2s} \cdot m(s) \Rightarrow x_2(s) + 2s\,x_2(s) = 25\,m(s) \Big| :2$$

$$0,5\,x_2(s) + s\,x_2(s) = 12,5\,m(s)$$

$$0,5\,x_2(t) + \dot{x}_2(t) = 12,5\,m(t)$$

$$\dot{x}_2 = -0,5\,x_2 + 12,5\,m$$

$$p_2(s) = p_1(s) + d(s) \Rightarrow \boxed{p_2 = p_1 + d}$$

$$\underset{\overset{\|}{x_1}}{z}(s) = H_{zd}(s)\cdot p_2(s) = \frac{20}{1+0,5s}\,p_2(s)$$

$$x_1(s)\left[1 + 0,5s\right] = 20\,p_2(s)$$

$$x_1(s) + 0,5\,s\,x_1(s) = 20\,p_2(s) \quad \Big|$$ Transformăm în domeniu timp

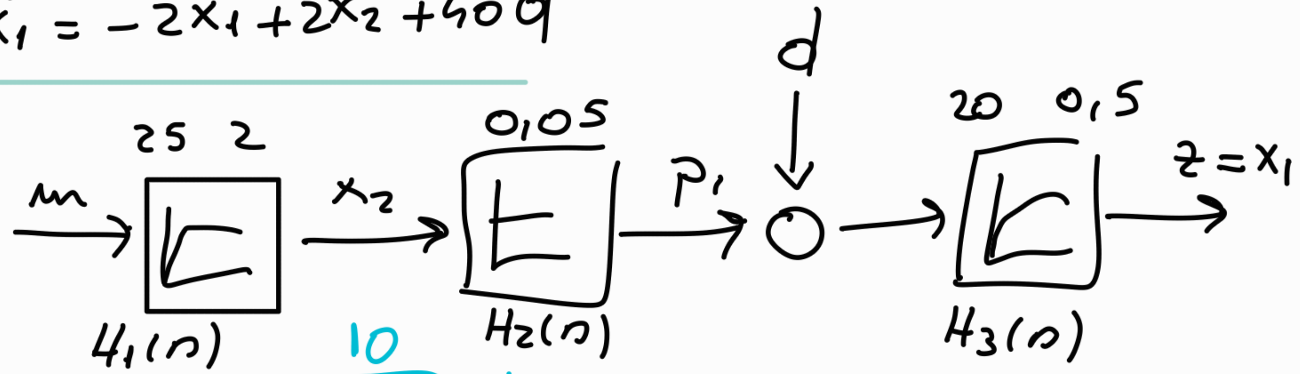
$$\left.\begin{aligned} x_1 + 0,5\,\dot{x}_1 &= 20\,p_2 \\ p_2 &= p_1 + d \end{aligned}\right\} \Rightarrow x_1 + 0,5\,\dot{x}_1 = 20(p_1 + d)$$

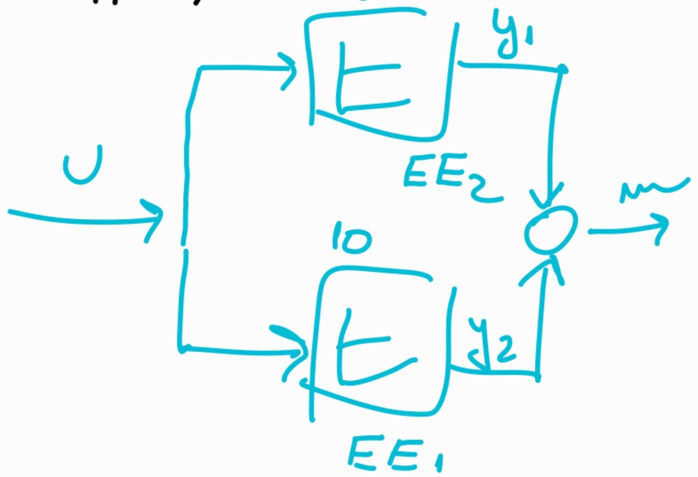
$$0,5\,\dot{x}_1 = -x_1 + 20p_1 + 20d$$

$$\boxed{p_1 = 0,05\,x_2}$$

$$0,5\,\dot{x}_1 = -x_1 + x_2 + 20d \quad /:0,5$$

$$\dot{x}_1 = -2x_1 + 2x_2 + 40d$$





$H_{EE_1}(s) = 10$ (ET-P)

$H_{EE_2}(s) = 10$ (ET-P)

〃

element de transfer

H_{EE_1}, H_{EE_2} – paralel $\Rightarrow H_{EE}(s) = H_{EE_1}(s) + H_{EE_2}(s)$

$$H_{EE}(s) = 20 \quad (P)$$

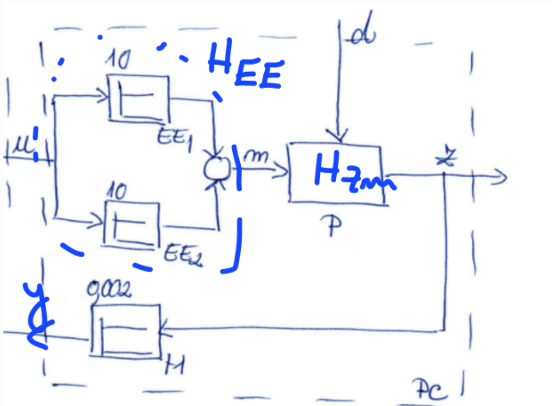
$$H_1(s) = \frac{25}{1+2s} \quad (PT1)$$

$$H_2(s) = 0{,}05 \quad (P)$$

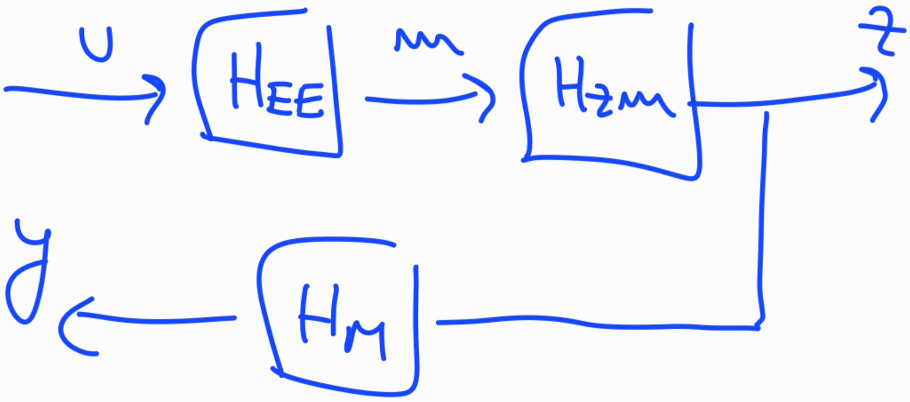
$$H_3(s) = \frac{20}{1+0{,}5s} \quad (PT1)$$

$$H_M(s) = 0{,}002 \quad (P)$$

① $H_{y,r}(s)$, $H_{y,d}(s)$, $H_0(s)$



$= H_{yu}$

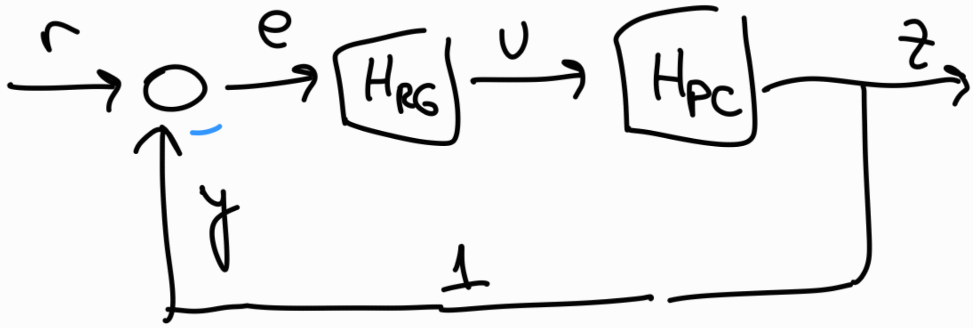


$$H_{yu} = H_{EE} \cdot H_{zm} \cdot H_M = 20 \cdot \frac{25}{(s+2)(s+0{,}5)} \cdot 0{,}002$$

$$H_{yu}(s)\Big|_{d=0} = \frac{1}{(s+2)(s+0{,}5)}$$

pt. R_1: $T_i = 2{,}5$ sec.

R_2: $T_d = 2{,}5$ sec; $T_z = 0{,}1$ sec



H_R, H_{PC} – serie: $H_\alpha = H_R \cdot H_{PC}$

H_α, 1 – reacție (unitară)

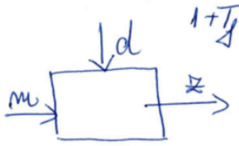
$$H_{y,r} = \frac{H_\alpha}{1 + H_\alpha \cdot 1} = \frac{\frac{k_R(1+2{,}5s)}{2{,}5s} \cdot \frac{1}{(s+2)(s+0{,}5)}}{1 + \frac{k_R(1+2{,}5s)}{2{,}5s} \cdot \frac{1}{(s+2)(s+0{,}5)}}$$

$$H_\alpha = \frac{k_R(1+2{,}5s)}{2{,}5s} \cdot \frac{1}{(s+2)(s+0{,}5)}$$

Sunt considerate 2 variante de regulatoare (R) cu f.d.t.:

R_1: $H_R(s) = k_R\left(1 + \frac{1}{T_i s}\right) = \frac{k_R(1+sT_i)}{sT_i}$ (ET-PI)

R_2: $H_R(s) = \frac{k_R(1+T_d s)}{1+T_1 s}$ (ET-PDT1)

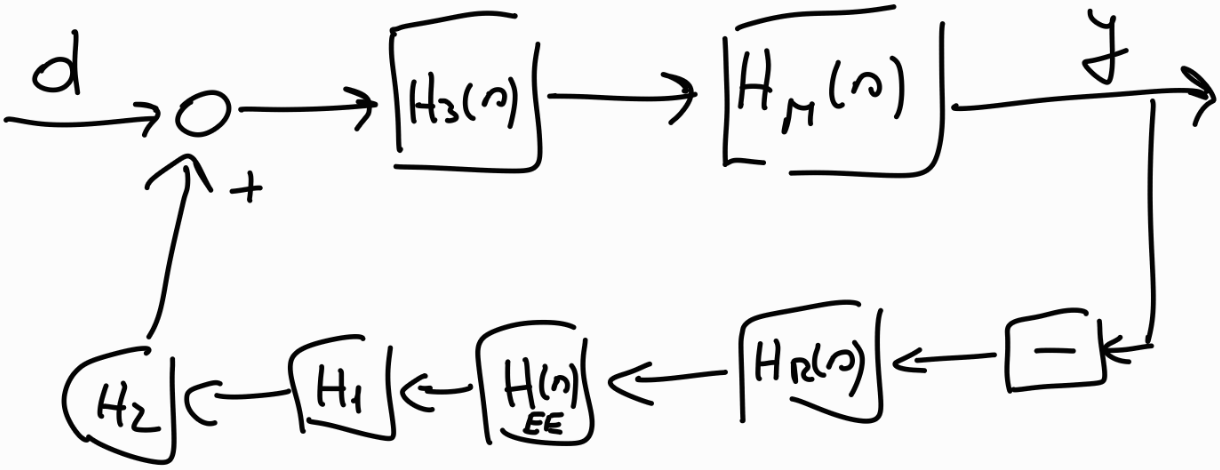


$\Rightarrow H_{zm}(s) = \frac{Z(s)}{m(s)}\Big|_{d=0}$ și $H_{zd}(s) = \frac{Z(s)}{d(s)}\Big|_{m=0}$

$$H_{y,r}(s) = \frac{k_R(1+2{,}5s)}{2{,}5s(s+2)(s+0{,}5) + k_R(1+2{,}5s)} =$$

$$= \frac{k_R(1+2{,}5s)}{2{,}5s^3 + 6{,}25s^2 + 2{,}5(1+k_R)s + k_R}$$

$$H_{y,d}(s)\Big|_{r=0} = \frac{\frac{20}{1+0{,}5s} \cdot 0{,}002}{1 + \frac{25}{1+2s} \cdot 0{,}05 \cdot \frac{20}{1+0{,}5s} \cdot 0{,}002 \cdot 20 \cdot \frac{k_R(1+2{,}5s)}{2{,}5s}}$$



$$H_{yd} = \frac{H_3 \cdot H_M}{1 - H_1 \cdot H_2 \cdot H_3 \cdot H_M \cdot H_{EE} \cdot H_R \cdot (-1)}$$

$$= \frac{\dfrac{0{,}04}{1+0{,}5s}}{1 + \dfrac{K_R(1+2{,}5s)}{2{,}5s(1+0{,}5s)(1+2s)}} =$$

$$= \frac{0{,}04}{\cancel{1+0{,}5s}} \cdot \frac{2{,}5s(\cancel{1+0{,}5s})(1+2s)}{2{,}5s^3+6{,}25s^2+2{,}5(1+K_R)s+K_R} =$$

$$= \frac{0{,}1s(1+2s)}{2{,}5s^3+6{,}25s^2+2{,}5(1+K_R)s+K_R}$$

$$H_0(s) = H_{RG}(s) \cdot H_{PC}(s) =$$

$$= \frac{K_R(1+2{,}5s)}{2{,}5s} \cdot \frac{1}{(1+0{,}5s)(1+2s)} = \frac{K_R(1+2{,}5s)}{2{,}5s(1+0{,}5s)(1+2s)}$$

② Det. val. param. $K_R > 0$ pt. SRA stabil.

$$\Delta(s) = 1 + H_0(s) = 2{,}5s^3+6{,}25s^2+2{,}5(1+K_R)s+K_R$$

$$a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0$$

$a_3 = 2,5 > 0$

$a_2 = 6,25 > 0$

$a_1 = 1 + k_R > 0 \Rightarrow k_R > -1 \Rightarrow k_R \in (-1; +\infty)$

$a_0 = k_R > 0 \Rightarrow k_R \in (0, +\infty)$

$\Rightarrow k_R \in (0; +\infty)$

$$n = 3 \Rightarrow H = \begin{bmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{bmatrix} = \begin{bmatrix} 6,25 & k_R & 0 \\ 2,5 & 1+k_R & 0 \\ 0 & 6,25 & k_R \end{bmatrix}$$

$\det(H_1) = 6,25 > 0$

$\det(H_2) = 6,25(1+k_R) - 2,5k_R = 3,75 k_R + 6,25 > 0$

$k_R > -\frac{6,25}{3,75}$

$\det(H_3) = a_0 \cdot \det(H_2) = k_R \cdot \ldots > 0$

$$\Rightarrow \boxed{k_R \in (0, +\infty)}$$

3 STATISMUL NATURAL

$$\gamma_n = \left.\frac{y_\infty}{v_\infty}\right|_{w_\infty = 0} = \frac{k_N}{1 + k_0} \quad \text{sau}$$

$$\gamma_n = \left.\frac{z_\infty}{v_\infty}\right|_{w_\infty = 0} = \frac{k_N}{1 + k_0}$$

- I, Pi, PiD $\longrightarrow e_\infty = 0$; $\gamma_n = 0$ (fără statism, ASTATIC)
- P, PT_1, PDT_1 $\longrightarrow e_\infty \neq 0$ și $\gamma_n \neq 0$ (cu statism)

$k_0 = k_R \cdot k_{PC}$

PDT_1: $H_R(s) = \frac{k_R(1 + T_d s)}{1 + T_j s}$

$\boxed{R_1}$: Regulatorul este de tip PI $\Rightarrow \tilde{\gamma}_M(y) = 0$

$$\tilde{\gamma}_M(y)^{\%} = \tilde{\gamma}_M(y) \cdot \frac{d_M}{y_M} \cdot 100\% \Rightarrow \tilde{\gamma}_M(y)^{\%} = 0$$

(L4, pag. 3, relația 28)

$\boxed{R_2}$: $\gamma_M(y) = \left.\frac{y_\infty}{d_\infty}\right|_{r_\infty = 0}$ sau $\tilde{\gamma}_M(y) = \frac{k_N(y)}{1+k_0}$

$d \to y$: $k_N = 20 \cdot 0{,}002 = 0{,}04$

$$\left.\begin{array}{l} k_0 = k_R \cdot k_{PC} \\ k_R = 3 \text{ (îl alegem din interval)} \\ u \to y: k_{PC} = 20 \cdot 25 \cdot 0{,}05 \cdot 20 \cdot 0{,}002 = 1 \end{array}\right\} \Rightarrow k_0 = 3.$$

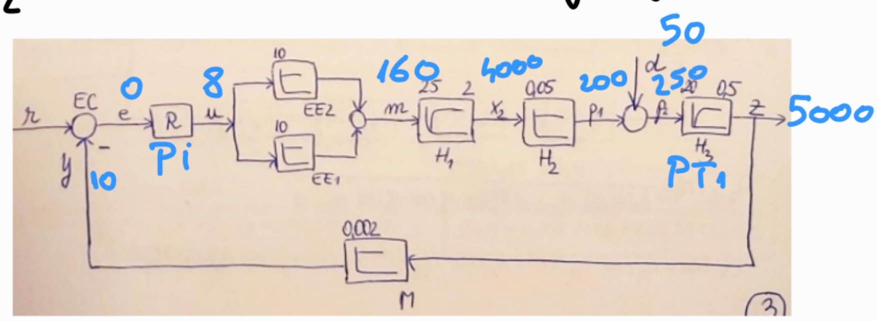
$$\tilde{\gamma}_M(y) = \frac{0{,}04}{1+3} \Rightarrow \tilde{\gamma}_M(y) = 0{,}01$$

$$\tilde{\gamma}_M(y)^{\%} = 0{,}01 \cdot \frac{50}{100} \cdot 100\% = 0{,}5$$

(4) $k_R > 0$, $d_\infty = 50$, $z_\infty = 5000$

VRSC$\{z_\infty, e_\infty, u_\infty, m_\infty, y_\infty\}$

$\boxed{R_1}$:



P, PT1, PDT1	D, PD, DT1	I, PI, PID
$y_\infty = k \cdot u_\infty$	$u_\infty = \text{const}$	$y_\infty = \text{const}$
$u_\infty = \text{const.}$	$y_\infty = (u_\infty)' = 0$	$u_\infty = (y_\infty)' = 0$
$y_\infty = \text{const.}$		

$u_\infty = const$

$e_\infty = 0$

$y_\infty = 0,002 \cdot z_\infty = 0,002 \cdot 5000 = 10$

$r_\infty - y_\infty = e_\infty$

$r_\infty - 10 = 0 \Rightarrow r_\infty = 10$

$z_\infty = 20 \cdot p_{2\infty}$

$$p_{2\infty} = \frac{z_\infty}{20} = \frac{5000}{20} = 250$$

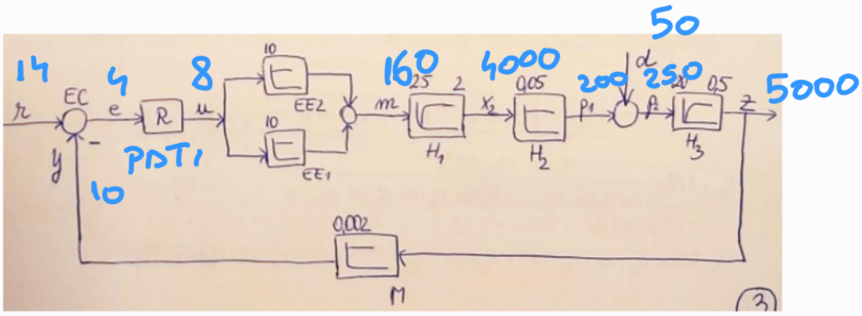
$p_{2\infty} = p_{1\infty} + d_\infty \Rightarrow p_{1\infty} = 250 - 50 = 200.$

$$p_{1\infty} = 0,05 \cdot x_{2\infty} \Rightarrow x_{2\infty} = \frac{200}{0,05} = 4000$$

$$x_{2\infty} = 25 \cdot m_\infty \Rightarrow m_\infty = \frac{4000}{25} = 160$$

$$m_\infty = 20 \cdot u_\infty \Rightarrow u_\infty = \frac{m_\infty}{20} = \frac{160}{20} = 8.$$

$\boxed{R_2}$:



$y_\infty = 0,002 \cdot z_\infty = 10$

$$u_\infty = k_R \cdot e_\infty = 2 \cdot e_\infty \Rightarrow e_\infty = \frac{u_\infty}{2} = 4.$$

$r_\infty - y_\infty = e_\infty \Rightarrow r_\infty = e_\infty + y_\infty = 10+4 = 14$

5 $$H(z) = \frac{3z^2-4z+1}{z^3-2z^2+(c+1,3)z-0,1}$$

$\Delta(z) = z^3 - 2z^2 + (c+1,3)z - 0,1$ > $n = 3$, $a_3 = 1 > 0$

$$\Delta(1) = 1^3 - 2\cdot 1^2 + (c+1,3)\cdot 1 - 0,1 =$$

$$= 1 - 2 + c + 1,3 - 0,1 =$$

$$= c + 0,2 > 0$$

$$\Rightarrow c > -0,2 \; : \; \boxed{c \in (-0,2\,;\,+\infty)}$$

$\Delta(-1) < 0$ (n - impar)

$$\Delta(-1) = (-1)^3 - 2\cdot(-1)^2 + (c+1,3)\cdot(-1) - 0,1 =$$

$$= -1 - 2 - c - 1,3 - 0,1 =$$

$$= -c - 4,4 < 0$$

$$c + 4,4 > 0 \Rightarrow c > -4,4 \; : \; \boxed{c \in (-4,4\,;\,+\infty)}$$

$$|a_0| = |-0,1| = 0,1 < 1 \quad \checkmark$$

$$b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}$$

$$b_0 = \begin{vmatrix} a_0 & a_3 \\ a_3 & a_0 \end{vmatrix} = a_0^2 - a_3^2 = $$

$$= 0,1^2 - 1^2 =$$

$$= -0,99$$

$$b_1 = \begin{vmatrix} a_0 & a_2 \\ a_3 & a_1 \end{vmatrix} = a_0\cdot a_1 - a_2\cdot a_3 =$$

$$= -0,1\cdot(c+1,3) - (-2)\cdot 1 =$$

$$= -0,1c - 0,13 + 2 =$$

$$= -0,1c + 1,87$$

$$b_2 = \begin{vmatrix} a_0 & a_1 \\ a_3 & a_2 \end{vmatrix} = a_0\cdot a_2 - a_1\cdot a_3 =$$

$$= -0,1\cdot(-2) - (c+1,3)\cdot 1 =$$

$$= 0,2 - c - 1,3 =$$

$$= -c - 1,1$$

Linie	z^0	z^1	z^2	z^3
1	$-0,1$ (a_0)	$c+1,3$ (a_1)	-2 (a_2)	1 (a_3)
2	1 (a_3)	(a_2)	(a_1)	(a_0)
3	$-0,99$ (b_0)	$1,87-0,1c$ (b_1)	$-(c+1,1)$ (b_2)	—
4	$-(c+1,1)$ (b_2)	$1,87-0,1c$ (b_1)	$-0,99$ (b_0)	—

$$\left.\begin{aligned} |b_0| &= 0,99 \\ |b_2| &= |-(c+1,1)| = c+1,1 \end{aligned}\right\} \Rightarrow |b_0| > |b_2|$$

$$\Rightarrow 0,99 > c+1,1 \Rightarrow c < -0,11 \Rightarrow c \in (-\infty; -0,11)$$

$$\left.\begin{aligned} &\cdot \quad c \in (0,2; +\infty) \\ &\cdot\cdot \quad c \in (-4,4; +\infty) \\ &\cdot\cdot\cdot \quad c \in (-\infty; -0,11) \end{aligned}\right\} \Rightarrow c \in (-0,11; 0,2)$$