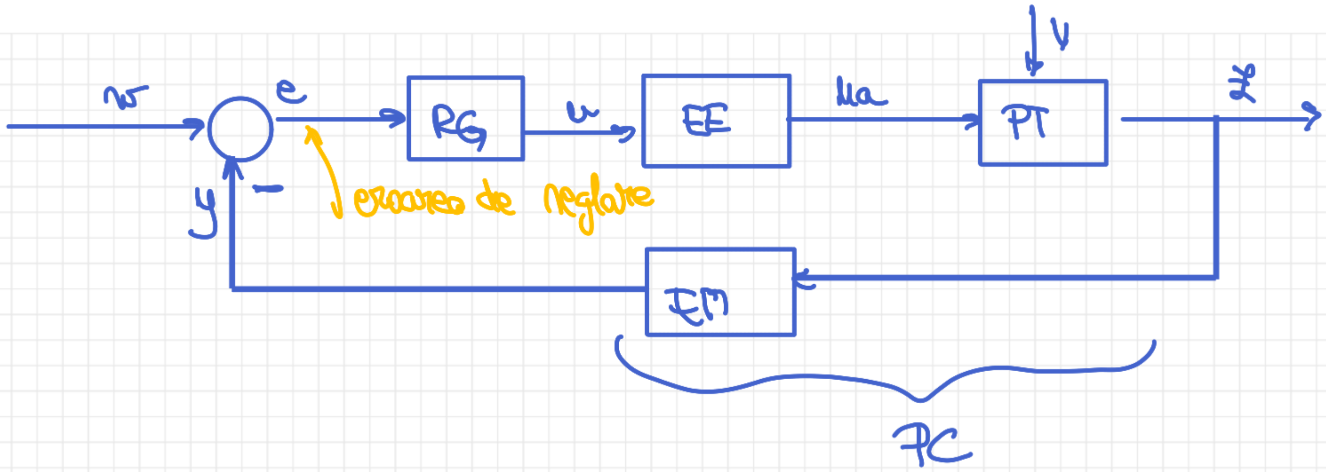
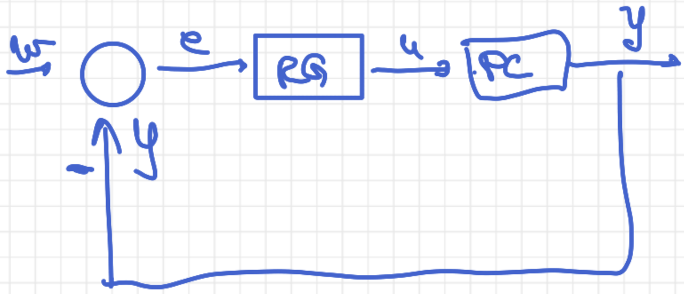


• analiza stabilității: determin H din



$$\rightarrow H_w = \frac{H_{RG}\cdot H_{PC}}{1+H_{RG}\cdot H_{PC}} \qquad \Delta(s)$$

$$\Delta(s) = 1+\underbrace{H_{RG}\cdot H_{PC}}_{H_D} = 0$$

$$\Delta(s) = a_n s^n + a_{n-1}s^{n-1}+\dots+a_1 s + a_0 = 0$$

• primul pas: $a_i > 0, \forall i = \overline{0,n}$

• 2: matricea lui Hurwitz ?

• constr.: $$\mathcal{H} = \begin{bmatrix} a_{n-1} & a_{n-3} & a_{n-5} & \dots & 0 \\ a_n & a_{n-2} & a_{n-4} & \dots & 0 \\ 0 & a_{n-1} & a_{n-3} & \dots & 0 \\ 0 & a_n & a_{n-2} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & a_0 \end{bmatrix}$$

• 3: $\delta_1 = |a_{n-1}| > 0$

$$\left.\begin{array}{l} \delta_1 = |a_{n-1}| > 0 \\ \delta_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} > 0 \\ \det \mathcal{H} > 0 \end{array}\right\} \Rightarrow \text{sistemul (RG + PC) este stabil}$$

ex: $H_{IW} = \dfrac{100}{0{,}025 S^4 + 0{,}55 S^3 + 1{,}32 S^2 + 0{,}64 S}$

SC1

$\Rightarrow \Delta_S = 0{,}025 S^4 + 0{,}55 S^3 + 1{,}32 S^2 + 0{,}64 S + 0 = 0$

4 $0{,}025 \geq 0$

3 $0{,}55 \geq 0$

2 $1{,}32 \geq 0$

1 $1{,}64 \geq 0$

0 $2 \geq 0$

$$\frac{100}{0{,}025 S^4 + 0{,}55 S^3 + 1{,}32 S^2 + 1{,}64 S + 2}$$

$\Rightarrow$ constr. matricea Hurwitz:

$$\mathcal{H} = \begin{bmatrix} 0{,}55 & 1{,}64 & 0 & 0 \\ 0{,}025 & 1{,}32 & 2 & 0 \\ 0 & 0{,}55 & 1{,}64 & 0 \\ 0 & 0{,}025 & 1{,}32 & 2 \end{bmatrix} \leftarrow a_0$$

→ de aceeași ordin ca ec în S

$\delta_1 = |0{,}55| = 0{,}55$

$\delta_2 = \begin{vmatrix} 0{,}55 & 1{,}64 \\ 0{,}025 & 1{,}32 \end{vmatrix} = 0{,}726 - 0{,}041 = 0{,}685$

$\delta_3 = \begin{vmatrix} 0{,}55 & 1{,}64 & 0 \\ 0{,}025 & 1{,}32 & 0 \\ 0 & 0{,}55 & 0{,}64 \end{vmatrix} = 0{,}64 \begin{vmatrix} 0{,}55 & 1{,}64 \\ 0{,}025 & 1{,}32 \end{vmatrix} = 0{,}4384$

$\delta_4 = 2 \cdot \delta_3 = 0{,}8768$

(ex) Să se det. dom de variație a lui k a.î. sist.

$\Delta(s) = s^3 + 3ks^2 + (k+2)s + 4$ este stabil

① $\left.\begin{array}{l} 1>0 \\ 3k>0 \\ k+2>0 \\ 4>0 \end{array}\right\} \Rightarrow k>0. \quad (1)$

② $\mathcal{H} = \begin{bmatrix} 3k & 4 & 0 \\ 1 & k+2 & 0 \\ 0 & 3k & 4 \end{bmatrix}$

$\delta_1 = 3k > 0 \Rightarrow k>0 \quad (2)$

$\delta_2 = 3k(k+2) - 4 = 3k^2 - 6k - 4 > 0$

$\text{delta} = 36 - 4 \cdot 3(-4) = 36+48 = 84$

$k_{1,2} = \frac{6 \pm \sqrt{84}}{6} = 1 \pm \frac{\sqrt{84}}{6}$

$\Rightarrow k \in \left(-\infty; 1 - \frac{\sqrt{84}}{6}\right] \cup \left(1 + \frac{\sqrt{84}}{6}; \infty\right) \quad (3)$

$\delta_3 = 4 \cdot \delta_2 > 0 \overset{\delta_2}{\Rightarrow} k \in \left(-\infty; 1 - \frac{\sqrt{84}}{6}\right] \cup \left(1 + \frac{\sqrt{84}}{6}; \infty\right) \quad (4)$

Din $(1) \div (4) \Rightarrow k \in \left(1 + \frac{\sqrt{21}}{3}; \infty\right)$

$k \in (2,52; \infty)$

Ex $H_{PC}(s) = \dfrac{2}{s^3+4s^2+5s+2}$

a) $H_{REG}(s) = K$

$$\Rightarrow H_0 = \frac{\dfrac{2K}{s^3+4s^2+5s+2}}{1+\dfrac{2K}{s^3+4s^2+5s+2}} = \frac{2K}{s^3+4s^2+5s+2+2K}$$

$$\Rightarrow s^3+4s^2+5s+(2K+2) = 0$$

① $\left.\begin{array}{l} 1>0 \\ 4>0 \\ 5>0 \\ 2K+2>0 \end{array}\right\} \Rightarrow K > -1. \quad (1)$

② $\mathcal{H} = \begin{pmatrix} 4 & 2K+2 & 0 \\ 1 & 5 & 0 \\ 0 & 4 & 2K+2 \end{pmatrix}$

$\delta_1 = 4 > 0$

$\delta_2 = 20-(2K+2) = 18-2K > 0 \Rightarrow K < 9 \quad (2)$

$\delta_3 = (2K+2)\cdot\delta_2 = (2K+2)(18-2K) = 36K - 4K^2 + 36 - 4K =$

$= -4K^2+32K+36 = 0$

$\Rightarrow K^2 - 8K - 9 = 0.$

$(K+1)(K-9) = 0 \begin{array}{l} \nearrow K=-1 \\ \searrow K=9 \end{array}$

$\Rightarrow K \in (-1;9) \quad (3)$

Din (1), (2), (3) $\Rightarrow K \in (0,9)$

b) $H_{RG}(s) = k_p + \frac{k_i}{s}$

$$\Rightarrow H_w = \frac{\frac{2}{s^3+4s^2+5s+2} \cdot \left(k_p + \frac{k_i}{s}\right)}{1 + \frac{2}{s^3+4s^2+5s+2} \cdot \left(k_p + \frac{k_i}{s}\right)} =$$

$$= \frac{2\left(k_p + \frac{k_i}{s}\right)}{s^3+4s^2+5s+2+2\left(k_p + \frac{k_i}{s}\right)} =$$

$$= \frac{2(sk_p + k_i)}{s^4+4s^3+5s^2+2s+2sk_p+2k_i} =$$

$$= \frac{2(sk_p+k_i)}{s^4+4s^3+5s^2+s(2+2k_p)+2k_i}$$

$$\Rightarrow \quad s^4+4s^3+5s^2+s(2+2k_p)+2k_i = 0$$

① $1>0$

$4>0$ $\quad \Rightarrow k_p > -1$

$5>0$ $\quad\quad k_i > 0$ $\quad$ (1)

$2+2k_p > 0$

$2k_i > 0$

② $$\mathcal{H} = \begin{pmatrix} 4 & 2+2k_p & 0 & 0 \\ 1 & 5 & 2k_i & 0 \\ 0 & 4 & 2+2k_p & 0 \\ 0 & 1 & 5 & 2k_i \end{pmatrix}$$

$\delta_1 = 4 > 0$

$\delta_2 = 20 - 2 - 2k_p = 18 - 2k_p > 0 \Rightarrow k_p < 9$ $\quad$ (2)

$\delta_3 = 20(2+2k_p) - (2+2k_p)^2 - 32k_i > 0$

$$= \underset{5}{40 + 40k_p} - 4 - \underset{5}{4k_p^2} - \underset{5}{8k_p} - \underset{5}{32k_i} > 0.$$

$= -4k_p^2 + 32k_p - 32k_i + 36 > 0$

$-k_p^2 + 8k_p + 9 - 8k_i > 0$

$\Rightarrow -(k_p+1)(k_p-9) > 8k_i$

$\Rightarrow \underbrace{(k_p+1)(k_p-\overset{<0}{9})}_{>0} < -8\underbrace{k_i}_{>0} \Rightarrow$ fals $\Rightarrow$

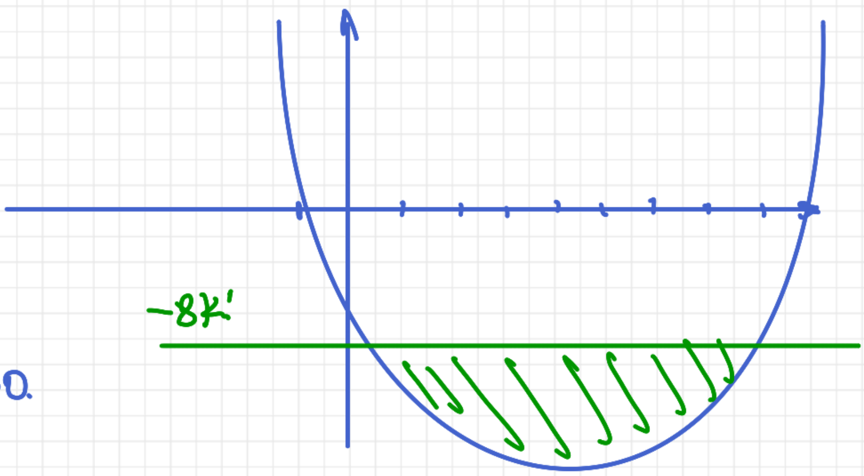
I. pentru $k \in (-\infty; -1] \cup [9; \infty) \Rightarrow \underbrace{(k_p+1)(k_p-\overset{0}{9})}_{>0} < -8\underbrace{k_i}_{>0} \Rightarrow$ fals $\Rightarrow$

$\Downarrow$

$k \in [9, \infty)$

$\Rightarrow$ sistemul e instabil

II. pentru $k_p \in (-1; 9) \Rightarrow (k_p+1)(k_p-9) < -8k_i$



$\delta_4 = 2k_i \cdot \delta_3 > 0 \Rightarrow k_i > 0$

pentru $k_p \in (-1; 9)$, $k_i \in (0, \infty)$, sist. e stabil