

School of Electrical and Electronic Engineering

**EEEN 40010**

**Control Theory**

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| Date: | 02/12/2013 |
| Assignment Title: | Digital Control |

# Objectives:

To investigate the design of digital controllers in correspondence with a plant with input and output that is modelled by the following state-space equation:

,

With matrices

# Problem

## Characteristics of plant (Continuous-time)

The continuous time step response of the plant can be seen in figure 1.



Figure 1: Continuous-time step response of plant

From figure 1 it can be seen that the a unit step input to the plant yields a percentage overshoot of , a rise-time of , a settling time of and a steady-state error of **.**

## Sampling period

In order to accurately convert from continuous-time system to discrete-time system a suitable sampling period must be chosen. This was chosen by taking a time less than of the fastest time constant of the open-loop system. Since the fastest time constant of this system is , I chose the sampling period to be .

## Poles of Plant

The poles of the continuous-time plant are and .

The poles of the discrete-time plant are and .

These are related by the equation .

## Controller Design

A discrete-time PID controller is to be designed so as the system will meet the following specification:

(i) Zero steady-state error to step input;

(ii) Percentage overshoot not exceeding 30%

(iii) 2% settling time not exceeding 3.5 sec.

From the specification the desired damping ratio and **closed-loop** system poles can be found. The damping ratio is found by the equation: this gives a desired damping ratio of .

From the settling time () it was found that the controller poles () would like to be place to the left of . This was found from the equation: . Relating this to a discrete system by the equation .

And finally in the controller needs an open loop pole at one, i.e an I term to ensure zero steady-state error.

Designing a controller of the form:

I chose now using the drawing rules and employing and I can chose a gain value . Figure 2 shows the root locus plot and the gain values:



Figure 2: shows use of rlocfind to place gain value chose relatively close to 0.3 damping ratio and to the left of 0.9719

This give a gain value of , thus the controller was

Applying negative feedback this yielded closed-loop step response in figure 3:



Figure 3: Closed loop step response of system with PI controller

It is clear that the system now meets the specification with a settling time of , and zero steady-state error.

It can be confirmed that therequired speed of response is much greater than the system without control. The system without control had a response time of and the controlled systems speed of response is required to be . **IS THIS REASONABLE?**

# Linear state feedback controller

First we see if the system is completely controllable and observable.



From the above equations we can confirm that the system is completely controllable and observable as the of and both return the value , which is equal to the order of the system.

The system can be described by the following equations:

Plant:

Observer:

Feedback:

These equations can be represented as

,

From the previous part of the question the dominant pole will be placed at . The dominated pole will be placed at a value less than this, it will be placed at . These poles were placed using the MATLAB command . These poles were successfully placed by using gains of values and .

The preamplifier was found by equation:

This gives

The resulting closed-loop step response can be seen in figure 4



Figure 4: Closed loop step response of system with full system linear feedback with observer

# Sensitivity

## Pole Placement

The closed-loop poles of the system depend on the eigenvalues of both the matrix and the matrix. From changing the gain values of we can see that the poles are sensitive to the gains. We find that their sensitivity to change is approximately proportional to change in the gains. for example an approximate change in the gain values leads to an approximate change in the pole placement.

Changing the gain values of by approximately yields a massive change in the the eigenvalues of the component. When the gain of the dominant pole is increased by the pole is placed at ; and the dominated pole is placed at . This shows that the dominated pole is extremely sensitive to an increase in gain and while the dominated pole is changed, it is nowhere near as drastic as the dominant pole. A decrease in the gains shows the poles being placed at and respectively. This now shows a large change in the dominant pole and an extremely large change in the dominated pole. These eigenvalues are very sensitive to gains.

## Closed-loop step response

The change in the gains as described above do affect the closed-loop step response somewhat. This only applies to the poles relating to the as the observer does not affect the step response at all. An increase in the gains slows the step response of the system but not drastically. However, a decrease in the gains does speed up the step-response of the system to a . A large change in these gains will cause the system to become unstable. Below shows a table which displays the percentage change in gains and the resulting change in poles and closed-loop step response.

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From the above table it can be seen that a small change in the gain does result in a significant in the , in some cases causing the system to not meet the specification of even arguably over meeting the specification, which may cause the system to react to unwanted disturbances. The change in gains does not affect the percentage overshoot or the steady-state error. Although extreme gain changes not recorded here as they are likely to never occur, will cause the system will become unstable.