

# Python ICPC Cheatsheet

Comprehensive Reference for Competitive Programming

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# 1 Input/Output

**Description:** Efficient input/output is crucial in competitive programming, especially for problems with large datasets. Using `sys.stdin.readline` is significantly faster than the default `input()` function.

```

1 # Fast I/O - Essential for large inputs
2 import sys
3 input = sys.stdin.readline
4
5 # Read single integer
6 n = int(input())
7
8 # Read multiple integers on one line
9 a, b = map(int, input().split())
10
11 # Read array of integers
12 arr = list(map(int, input().split()))
13
14 # Read strings (strip to remove trailing newline)
15 s = input().strip()
16 words = input().split()
17
18 # Multiple test cases pattern
19 t = int(input())
20 for _ in range(t):
21     # process each test case
22
23 # Print without newline
24 print(x, end=' ')
25
26 # Formatted output with precision
27 print(f"{x:.6f}") # 6 decimal places

```

## 2 Basic Data Structures

### 2.1 List Operations

**Description:** Python lists are dynamic arrays with  $O(1)$  amortized append and  $O(n)$  insert/delete at arbitrary positions.

```

1 # Initialize lists
2 arr = [0] * n # n zeros
3 matrix = [[0] * m for _ in range(n)] # Correct way!
4
5 # List comprehension - concise and efficient
6 squares = [x*x2 for x in range(n)]
7 evens = [x for x in arr if x % 2 == 0]
8
9 # Sorting -  $O(n \log n)$ 
10 arr.sort() # in-place, modifies arr
11 arr.sort(reverse=True) # descending
12 arr.sort(key=lambda x: (x[0], -x[1])) # custom
13 sorted_arr = sorted(arr) # returns new list
14
15 # Binary search in sorted array
16 from bisect import bisect_left, bisect_right
17 idx = bisect_left(arr, x) # leftmost position
18 idx = bisect_right(arr, x) # rightmost position
19
20 # Common operations
21 arr.append(x) #  $O(1)$  amortized
22 arr.pop() #  $O(1)$  - remove last
23 arr.pop(0) #  $O(n)$  - remove first (slow!)
24 arr.reverse() #  $O(n)$  - in-place
25 arr.count(x) #  $O(n)$  - count occurrences
26 arr.index(x) #  $O(n)$  - first occurrence

```

### 2.2 Deque (Double-ended Queue)

**Description:** Deque (pronounced "deck") provides  $O(1)$  append and pop operations from both ends, unlike lists which have  $O(n)$  for operations at the front. Essential for BFS, sliding window problems, implementing efficient queues/stacks, and maintaining monotonic queues. Use when you need fast insertions/deletions at both ends.

```
1 from collections import deque
```

```

2 dq = deque()
3
4 #  $O(1)$  operations on both ends
5 dq.append(x) # add to right
6 dq.appendleft(x) # add to left
7 dq.pop() # remove from right
8 dq.popleft() # remove from left
9
10 # Sliding window maximum -  $O(n)$ 
11 # Maintains decreasing order of elements
12 def sliding_max(arr, k):
13     dq = deque() # stores indices
14     result = []
15
16     for i in range(len(arr)):
17         # Remove indices outside window
18         while dq and dq[0] < i - k + 1:
19             dq.popleft()
20
21         # Remove smaller elements (not useful)
22         while dq and arr[dq[-1]] < arr[i]:
23             dq.pop()
24
25         dq.append(i)
26         if i >= k - 1:
27             result.append(arr[dq[0]])
28
29 return result

```

### 2.3 Heap (Priority Queue)

**Description:** Python's `heapq` module implements a min-heap (smallest element always at index 0). Provides  $O(\log n)$  insert and extract-min operations,  $O(n)$  heapify, and  $O(1)$  peek. For max-heap, negate values before insertion. Critical for Dijkstra's algorithm, Prim's MST, k-th largest/smallest problems, merge k sorted lists, and any problem requiring repeated access to minimum/maximum elements. More efficient than sorting when you only need partial ordering.

```

1 import heapq
2
3 # Min heap (default)
4 heap = []
5 heapq.heappush(heap, x) #  $O(\log n)$ 
6 min_val = heapq.heappop(heap) #  $O(\log n)$ 
7 min_val = heap[0] #  $O(1)$  peek
8
9 # Max heap - negate values
10 heapq.heappush(heap, -x)
11 max_val = -heapq.heappop(heap)
12
13 # Convert list to heap in-place -  $O(n)$ 
14 heapq.heapify(arr)
15
16 # K largest/smallest -  $O(n \log k)$ 
17 k_largest = heapq.nlargest(k, arr)
18 k_smallest = heapq.nsmallest(k, arr)
19
20 # Custom comparator using tuples
21 # Compares first element, then second, etc.
22 heapq.heappush(heap, (priority, item))

```

### 2.4 Dictionary & Counter

**Description:** Hash maps with  $O(1)$  average case insert/lookup. Counter is specialized for counting occurrences.

```

1 from collections import defaultdict, Counter
2
3 # defaultdict - provides default value
4 graph = defaultdict(list) # empty list default
5 count = defaultdict(int) # 0 default
6
7 # Counter - count elements efficiently
8 cnt = Counter(arr)
9 cnt['x'] += 1
10 most_common = cnt.most_common(k) # k most frequent
11
12 # Dictionary operations

```

```

13 d = {}
14 d.get(key, default_val)
15 d.setdefault(key, default_val)
16 for k, v in d.items():
17     pass

```

## 2.5 Set Operations

**Description:** Hash sets provide  $O(1)$  average-case membership testing, insertion, and deletion. Unlike lists, sets store only unique elements (no duplicates) and are unordered. Essential for removing duplicates, fast membership queries, and mathematical set operations (union, intersection, difference). Use when element uniqueness matters or you need fast lookups without caring about order. For sorted sets, consider using sorted containers or maintaining a sorted list separately.

```

1 s = set()
2 s.add(x)           # O(1)
3 s.remove(x)        # O(1), KeyError if not exists
4 s.discard(x)      # O(1), no error if not exists
5
6 # Set operations - all O(n)
7 a | b # union
8 a & b # intersection
9 a - b # difference
10 a ^ b # symmetric difference
11
12 # Ordered set workaround
13 from collections import OrderedDict
14 oset = OrderedDict.fromkeys([])

```

## 2.6 Skip List

**Description:** Skip lists are probabilistic balanced data structures that support  $O(\log n)$  expected time for search, insert, and delete. They are an alternative to balanced BSTs and are simple to implement. Good when you want ordered set/map operations with expected logarithmic time.

**How it works:** A skip list is composed of multiple levels of sorted linked lists; level 0 contains all elements and higher levels act as “express lanes” with a subset of elements (each element appears in a random number of levels, forming a “tower”). To search, start at the head on the highest level, move right while the next key is less than the target, and drop down one level when you can no longer move right; repeat until level 0. New node levels are chosen randomly with probability  $p$  (common choice  $p = 0.5$ ). For contests use  $p = 0.5$  and  $L_{\max} \approx \lceil \log_2 N \rceil + 2$  for safety.

```

1 import random
2
3 class SkipNode:
4     def __init__(self, key, value=None, level=0):
5         self.key = key
6         self.value = value
7         # forward pointers for levels 0..level
8         self.forward = [None] * (level + 1)
9
10 class SkipList:
11     def __init__(self, max_level=16, p=0.5):
12         self.max_level = max_level
13         self.p = p
14         self.level = 0
15         self.head = SkipNode(None, level=max_level)
16
17     def random_level(self):
18         lvl = 0
19         while random.random() < self.p and lvl < self.max_level:
20             lvl += 1
21         return lvl
22
23     def search(self, key):
24         node = self.head
25         # start from highest level
26         for i in range(self.level, -1, -1):
27             while node.forward[i] and node.forward[i].key < key:
28                 node = node.forward[i]

```

```

29         node = node.forward[0]
30         if node and node.key == key:
31             return node.value
32         return None
33
34     def insert(self, key, value=None):
35         update = [None] * (self.max_level + 1)
36         node = self.head
37         for i in range(self.level, -1, -1):
38             while node.forward[i] and node.forward[i].key < key:
39                 node = node.forward[i]
40             update[i] = node
41         node = node.forward[0]
42
43         # If key exists, update value
44         if node and node.key == key:
45             node.value = value
46             return
47
48         lvl = self.random_level()
49         if lvl > self.level:
50             for i in range(self.level + 1, lvl + 1):
51                 update[i] = self.head
52             self.level = lvl
53
54         new_node = SkipNode(key, value, lvl)
55         for i in range(lvl + 1):
56             new_node.forward[i] = update[i].forward[i]
57             update[i].forward[i] = new_node
58
59     def delete(self, key):
60         update = [None] * (self.max_level + 1)
61         node = self.head
62         for i in range(self.level, -1, -1):
63             while node.forward[i] and node.forward[i].key < key:
64                 node = node.forward[i]
65             update[i] = node
66         node = node.forward[0]
67
68         if not node or node.key != key:
69             return False # not found
70
71         for i in range(self.level + 1):
72             if update[i].forward[i] != node:
73                 break
74             update[i].forward[i] = node.forward[i]
75
76         # adjust current level
77         while self.level > 0 and self.head.forward[self.level] is None:
78             self.level -= 1
79         return True
80
81 # Example usage:
82 # s = SkipList()
83 # s.insert(10, 'ten')
84 # print(s.search(10))
85 # s.delete(10)

```

## 3 String Operations

**Description:** Strings in Python are immutable. For building strings, use list and join for  $O(n)$  complexity instead of repeated concatenation which is  $O(n^2)$ .

```

1 # Common string methods
2 s.lower(), s.upper()
3 s.strip() # remove whitespace both ends
4 s.lstrip() # remove left whitespace
5 s.rstrip() # remove right whitespace
6 s.split(delimiter)
7 delimiter.join(list)
8 s.replace(old, new)
9 s.startswith(prefix)
10 s.endswith(suffix)
11 s.isdigit(), s.isalpha(), s.isalnum()
12
13 # String building - EFFICIENT O(n)
14 result = []
15 for x in data:

```

```

16     result.append(str(x))
17 s = ''.join(result)
18
19 # String concatenation - SLOW O(n^2)
20 # s = ""
21 # for x in data:
22 #     s += str(x) # Don't do this!
23
24 # ASCII values
25 ord('a') # 97
26 chr(97) # 'a'
27
28 # String to character array (for mutations)
29 chars = list(s)
30 chars[0] = 'x'
31 s = ''.join(chars)

```

### 3.1 KMP Pattern Matching

**Description:** Find all occurrences of pattern in text. Time:  $O(n+m)$ .

```

1 def kmp_search(text, pattern):
2     # Build LPS (Longest Proper Prefix which is Suffix)
3     def build_lps(pattern):
4         m = len(pattern)
5         lps = [0] * m
6         length = 0 # Length of previous longest prefix
7         i = 1
8
9         while i < m:
10            if pattern[i] == pattern[length]:
11                length += 1
12                lps[i] = length
13                i += 1
14            else:
15                if length != 0:
16                    length = lps[length - 1]
17                else:
18                    lps[i] = 0
19                    i += 1
20
21     return lps
22
23 n, m = len(text), len(pattern)
24 lps = build_lps(pattern)
25
26 matches = []
27 i = j = 0 # Indices for text and pattern
28
29 while i < n:
30     if text[i] == pattern[j]:
31         i += 1
32         j += 1
33
34     if j == m:
35         matches.append(i - j)
36         j = lps[j - 1]
37     elif i < n and text[i] != pattern[j]:
38         if j != 0:
39             j = lps[j - 1]
40         else:
41             i += 1
42
43 return matches

```

### 3.2 Z-Algorithm

**Description:** Compute Z-array where  $Z[i] =$  length of longest substring starting from  $i$  that matches prefix. Time:  $O(n)$ .

```

1 def z_algorithm(s):
2     n = len(s)
3     z = [0] * n
4     l, r = 0, 0
5
6     for i in range(1, n):
7         if i <= r:
8             z[i] = min(r - i + 1, z[i - 1])
9
10    while i + z[i] < n and s[z[i]] == s[i + z[i]]:

```

```

11        z[i] += 1
12
13        if i + z[i] - 1 > r:
14            l, r = i, i + z[i] - 1
15
16    return z
17
18 # Pattern matching using Z-algorithm
19 def z_search(text, pattern):
20     # Concatenate pattern + $ + text
21     s = pattern + '$' + text
22     z = z_algorithm(s)
23
24     matches = []
25     m = len(pattern)
26
27     for i in range(m + 1, len(s)):
28         if z[i] == m:
29             matches.append(i - m - 1)
30
31     return matches

```

### 3.3 Rabin-Karp (Rolling Hash)

**Description:** Fast pattern matching using hashing. Average:  $O(n+m)$ , Worst:  $O(nm)$ .

```

1 def rabin_karp(text, pattern):
2     MOD = 10**9 + 7
3     BASE = 31 # Prime base for hashing
4
5     n, m = len(text), len(pattern)
6     if m > n:
7         return []
8
9     # Compute hash of pattern
10    pattern_hash = 0
11    power = 1
12    for i in range(m):
13        pattern_hash = (pattern_hash * BASE +
14                         ord(pattern[i])) % MOD
15        if i < m - 1:
16            power = (power * BASE) % MOD
17
18    # Rolling hash
19    text_hash = 0
20    matches = []
21
22    for i in range(n):
23        # Add new character
24        text_hash = (text_hash * BASE +
25                     ord(text[i])) % MOD
26
27        # Remove old character if window full
28        if i >= m:
29            text_hash = (text_hash -
30                         ord(text[i - m]) * power) % MOD
31            text_hash = (text_hash + MOD) % MOD
32
33        # Check match
34        if i >= m - 1 and text_hash == pattern_hash:
35            # Verify actual match (avoid hash collision)
36            if text[i - m + 1:i + 1] == pattern:
37                matches.append(i - m + 1)
38
39    return matches

```

## 4 Mathematics

### 4.1 Basic Math Operations

```

1 import math
2
3 # Common functions
4 math.ceil(x), math.floor(x)
5 math.gcd(a, b) # Greatest common divisor
6 math.lcm(a, b) # Python 3.9+
7 math.sqrt(x)
8 math.log(x), math.log2(x), math.log10(x)
9

```

```

10 # Powers
11 x ** y
12 pow(x, y, mod) #  $(x^y) \% \text{mod}$  - efficient modular exp
13
14 # Infinity
15 float('inf'), float('-inf')
16
17 # Custom GCD using Euclidean algorithm -  $O(\log \min(a,b))$ 
18 def gcd(a, b):
19     while b:
20         a, b = b, a % b
21     return a
22
23 def lcm(a, b):
24     return a * b // gcd(a, b)

```

## 4.2 Combinatorics

**Description:** Compute combinations and permutations. For modular arithmetic, compute factorial arrays and use modular inverse.

```

1 from math import factorial, comb, perm
2
3 # nCr (combinations) - "n choose r"
4 comb(n, r) # Built-in Python 3.8+
5
6 # nPr (permutations)
7 perm(n, r) # Built-in Python 3.8+
8
9 # Manual nCr implementation
10 def ncr(n, r):
11     if r > n: return 0
12     r = min(r, n - r) # Optimization:  $C(n,r) = C(n,n-r)$ 
13     num = den = 1
14     for i in range(r):
15         num *= (n - i)
16         den *= (i + 1)
17     return num // den
18
19 # Precompute factorials with modulo
20 MOD = 10**9 + 7
21 def modfact(n):
22     fact = [1] * (n + 1)
23     for i in range(1, n + 1):
24         fact[i] = fact[i-1] * i % MOD
25     return fact
26
27 # Modular combination using precomputed factorials
28 # First precompute inverse factorials
29 def compute_inv_factorials(n, mod):
30     fact = modfact(n)
31     inv_fact = [1] * (n + 1)
32     inv_fact[n] = pow(fact[n], mod - 2, mod)
33     for i in range(n - 1, -1, -1):
34         inv_fact[i] = inv_fact[i + 1] * (i + 1) % mod
35     return fact, inv_fact
36
37 def modcomb(n, r, fact, inv_fact, mod):
38     if r > n or r < 0: return 0
39     return fact[n] * inv_fact[r] % mod * inv_fact[n-r] % mod

```

## 5 Number Theory

**Description:** Essential algorithms for problems involving primes, modular arithmetic, and divisibility.

### 5.1 Modular Arithmetic

```

1 # Modular inverse using Fermat's Little Theorem
2 # Only works when mod is prime
3 #  $a^{-1} = a^{(mod-2)} \pmod{p}$ 
4 def modinv(a, mod):
5     return pow(a, mod - 2, mod)
6
7 # Extended Euclidean Algorithm
8 # Returns (gcd, x, y) where  $ax + by = \text{gcd}(a,b)$ 
9 # Can find modular inverse for any coprime a, mod
10 def extgcd(a, b):
11     if b == 0:

```

```

12         return a, 1, 0
13     g, x1, y1 = extgcd(b, a % b)
14     x = y1
15     y = x1 - (a // b) * y1
16     return g, x, y

```

### 5.2 Sieve of Eratosthenes

**Description:** Find all primes up to n in  $O(n \log \log n)$  time. Memory:  $O(n)$ .

```

1 def sieve(n):
2     is_prime = [True] * (n + 1)
3     is_prime[0] = is_prime[1] = False
4
5     for i in range(2, int(n**0.5) + 1):
6         if is_prime[i]:
7             # Mark multiples as composite
8             for j in range(i*i, n + 1, i):
9                 is_prime[j] = False
10
11     return is_prime
12
13 # Get list of primes
14 primes = [i for i in range(n+1) if is_prime[i]]

```

### 5.3 Prime Factorization

**Description:** Decompose n into prime factors in  $O(\sqrt{n})$  time.

```

1 def factorize(n):
2     factors = []
3     d = 2
4
5     # Check divisors up to  $\sqrt{n}$ 
6     while d * d <= n:
7         while n % d == 0:
8             factors.append(d)
9             n //= d
10        d += 1
11
12    # If n > 1, it's a prime factor
13    if n > 1:
14        factors.append(n)
15
16    return factors
17
18 # Get prime factors with counts
19 from collections import Counter
20 def prime_factor_counts(n):
21     return Counter(factorize(n))
22
23 # Count divisors
24 def count_divisors(n):
25     count = 0
26     i = 1
27     while i * i <= n:
28         if n % i == 0:
29             count += 1 if i * i == n else 2
30             i += 1
31     return count
32
33 # Sum of divisors
34 def sum_divisors(n):
35     total = 0
36     i = 1
37     while i * i <= n:
38         if n % i == 0:
39             total += i
40             if i != n // i:
41                 total += n // i
42             i += 1
43     return total

```

### 5.4 Chinese Remainder Theorem

**Description:** Solve system of congruences  $x \equiv a_1 \pmod{m_1}$ ,  $x \equiv a_2 \pmod{m_2}$ , ... Time:  $O(n \log M)$  where M is product of moduli.

```

1 def chinese_remainder(remainders, moduli):
2     # Solve  $x = remainders[i] \pmod{moduli[i]}$ 

```

```

3 # Assumes moduli are pairwise coprime
4
5 def extgcd(a, b):
6     if b == 0:
7         return a, 1, 0
8     g, x1, y1 = extgcd(b, a % b)
9     return g, y1, x1 - (a // b) * y1
10
11 total = 0
12 prod = 1
13 for m in moduli:
14     prod *= m
15
16 for r, m in zip(remainders, moduli):
17     p = prod // m
18     g, inv, _ = extgcd(p, m)
19     # inv may be negative, normalize it
20     inv = (inv % m + m) % m
21     total += r * inv * p
22
23 return total % prod

```

## 5.5 Euler's Totient Function

**Description:**  $\phi(n)$  = count of numbers  $\leq n$  coprime to n. Time:  $O(\sqrt{n})$ .

```

1 def euler_phi(n):
2     result = n
3     p = 2
4
5     while p * p <= n:
6         if n % p == 0:
7             # Remove factor p
8             while n % p == 0:
9                 n //= p
10            # Multiply by  $(1 - 1/p)$ 
11            result -= result // p
12        p += 1
13
14    if n > 1:
15        result -= result // n
16
17    return result
18
19 # Phi for range [1, n] using sieve
20 def phi_sieve(n):
21     phi = list(range(n + 1)) # phi[i] = i initially
22
23     for i in range(2, n + 1):
24         if phi[i] == i: # i is prime
25             for j in range(i, n + 1, i):
26                 phi[j] = phi[j] // i * (i - 1)
27
28     return phi

```

## 5.6 Fast Exponentiation with Matrix

**Description:** Already covered in matrix section, but useful pattern.

```

1 # Modular exponentiation
2 def mod_exp(base, exp, mod):
3     result = 1
4     base %= mod
5
6     while exp > 0:
7         if exp & 1:
8             result = (result * base) % mod
9             base = (base * base) % mod
10            exp >>= 1
11
12    return result

```

# 6 Graph Algorithms

## 6.1 Graph Representation

**Description:** Adjacency list is most common for sparse graphs. Use defaultdict for convenience.

```

1 from collections import defaultdict, deque
2
3 # Unweighted graph
4 graph = defaultdict(list)
5 for _ in range(m):
6     u, v = map(int, input().split())
7     graph[u].append(v)
8     graph[v].append(u) # for undirected
9
10 # Weighted graph - store (neighbor, weight) tuples
11 graph[u].append((v, weight))

```

## 6.2 BFS (Breadth-First Search)

**Description:** Explores graph level by level. Finds shortest path in unweighted graphs. Time:  $O(V+E)$ , Space:  $O(V)$ .

```

1 def bfs(graph, start):
2     visited = set([start])
3     queue = deque([start])
4     dist = {start: 0}
5
6     while queue:
7         node = queue.popleft()
8
9         for neighbor in graph[node]:
10            if neighbor not in visited:
11                visited.add(neighbor)
12                queue.append(neighbor)
13                dist[neighbor] = dist[node] + 1
14
15    return dist
16
17 # Grid BFS - common in maze/path problems
18 def grid_bfs(grid, start):
19     n, m = len(grid), len(grid[0])
20     visited = [[False] * m for _ in range(n)]
21     queue = deque([start])
22     visited[start[0]][start[1]] = True
23
24     # 4 directions: right, down, left, up
25     dirs = [(0,1), (1,0), (0,-1), (-1,0)]
26
27     while queue:
28         x, y = queue.popleft()
29
30         for dx, dy in dirs:
31             nx, ny = x + dx, y + dy
32
33             # Check bounds and validity
34             if (0 <= nx < n and 0 <= ny < m
35                 and not visited[nx][ny]
36                 and grid[nx][ny] != '#'):
37
38                 visited[nx][ny] = True
39                 queue.append((nx, ny))

```

## 6.3 DFS (Depth-First Search)

**Description:** Explores as far as possible along each branch. Used for connectivity, cycles, topological sort. Time:  $O(V+E)$ , Space:  $O(V)$ .

```

1 # Recursive DFS
2 def dfs(graph, node, visited):
3     visited.add(node)
4
5     for neighbor in graph[node]:
6         if neighbor not in visited:
7             dfs(graph, neighbor, visited)
8
9     # Iterative DFS using stack
10 def dfs_iterative(graph, start):
11     visited = set()
12     stack = [start]
13
14     while stack:
15         node = stack.pop()
16
17         if node not in visited:
18             visited.add(node)

```

```

19         for neighbor in graph[node]:
20             if neighbor not in visited:
21                 stack.append(neighbor)
22
23 # Cycle detection in undirected graph
24 def has_cycle(graph, n):
25     visited = [False] * n
26
27     def dfs(node, parent):
28         visited[node] = True
29
30         for neighbor in graph[node]:
31             if not visited[neighbor]:
32                 if dfs(neighbor, node):
33                     return True
34
35             # Back edge to non-parent = cycle
36             elif neighbor != parent:
37                 return True
38
39     return False
40
41 # Check all components
42 for i in range(n):
43     if not visited[i]:
44         if dfs(i, -1):
45             return True
46
47 return False
48
49 # Cycle detection in directed graph
50 def has_cycle_directed(graph, n):
51     WHITE, GRAY, BLACK = 0, 1, 2
52     color = [WHITE] * n
53
54     def dfs(node):
55         color[node] = GRAY
56
57         for neighbor in graph[node]:
58             if color[neighbor] == GRAY:
59                 return True # Back edge = cycle
60             if color[neighbor] == WHITE:
61                 if dfs(neighbor):
62                     return True
63
64         color[node] = BLACK
65     return False
66
67 for i in range(n):
68     if color[i] == WHITE:
69         if dfs(i):
70             return True
71 return False
72
73 # Connected components count
74 def count_components(graph, n):
75     visited = [False] * n
76     count = 0
77
78     def dfs(node):
79         visited[node] = True
80         for neighbor in graph[node]:
81             if not visited[neighbor]:
82                 dfs(neighbor)
83
84     for i in range(n):
85         if not visited[i]:
86             dfs(i)
87             count += 1
88
89     return count
90
91 # Bipartite check (2-coloring)
92 def is_bipartite(graph, n):
93     color = [-1] * n
94
95     def bfs(start):
96         from collections import deque
97         queue = deque([start])
98         color[start] = 0
99

```

```

100    while queue:
101        node = queue.popleft()
102
103        for neighbor in graph[node]:
104            if color[neighbor] == -1:
105                color[neighbor] = 1 - color[node]
106                queue.append(neighbor)
107            elif color[neighbor] == color[node]:
108                return False
109
110    return True
111
112    for i in range(n):
113        if color[i] == -1:
114            if not bfs(i):
115                return False
116
117    return True

```

## 6.4 Strongly Connected Components (SCC)

**Description:** Find all SCCs in directed graph using Tarjan's algorithm. Time:  $O(V+E)$ .

```

1 def tarjan_scc(graph, n):
2     index_counter = [0]
3     stack = []
4     lowlink = [0] * n
5     index = [0] * n
6     on_stack = [False] * n
7     index_initialized = [False] * n
8     sccs = []
9
10    def strongconnect(v):
11        index[v] = index_counter[0]
12        lowlink[v] = index_counter[0]
13        index_counter[0] += 1
14        index_initialized[v] = True
15        stack.append(v)
16        on_stack[v] = True
17
18        for w in graph[v]:
19            if not index_initialized[w]:
20                strongconnect(w)
21                lowlink[v] = min(lowlink[v], lowlink[w])
22            elif on_stack[w]:
23                lowlink[v] = min(lowlink[v], index[w])
24
25            if lowlink[v] == index[v]:
26                scc = []
27                while True:
28                    w = stack.pop()
29                    on_stack[w] = False
30                    scc.append(w)
31                    if w == v:
32                        break
33                sccs.append(scc)
34
35        for v in range(n):
36            if not index_initialized[v]:
37                strongconnect(v)
38
39    return sccs

```

## 6.5 Bridges and Articulation Points

**Description:** Find critical edges (bridges) and vertices (articulation points). Time:  $O(V+E)$ .

```

1 def find_bridges(graph, n):
2     visited = [False] * n
3     disc = [0] * n
4     low = [0] * n
5     parent = [-1] * n
6     time = [0]
7     bridges = []
8
9     def dfs(u):
10        visited[u] = True
11        disc[u] = low[u] = time[0]
12        time[0] += 1

```

```

13
14     for v in graph[u]:
15         if not visited[v]:
16             parent[v] = u
17             dfs(v)
18             low[u] = min(low[u], low[v])
19
20             # Bridge condition
21             if low[v] > disc[u]:
22                 bridges.append((u, v))
23             elif v != parent[u]:
24                 low[u] = min(low[u], disc[v])
25
26     for i in range(n):
27         if not visited[i]:
28             dfs(i)
29
30     return bridges
31
32 def find_articulation_points(graph, n):
33     visited = [False] * n
34     disc = [0] * n
35     low = [0] * n
36     parent = [-1] * n
37     time = [0]
38     ap = set()
39
40     def dfs(u):
41         children = 0
42         visited[u] = True
43         disc[u] = low[u] = time[0]
44         time[0] += 1
45
46         for v in graph[u]:
47             if not visited[v]:
48                 children += 1
49                 parent[v] = u
50                 dfs(v)
51                 low[u] = min(low[u], low[v])
52
53                 # Articulation point conditions
54                 if parent[u] == -1 and children > 1:
55                     ap.add(u)
56                 if parent[u] != -1 and low[v] >= disc[u]:
57                     ap.add(u)
58             elif v != parent[u]:
59                 low[u] = min(low[u], disc[v])
60
61     for i in range(n):
62         if not visited[i]:
63             dfs(i)
64
65     return list(ap)

```

```

23     for j in range(1, self.LOG):
24         for i in range(n):
25             if self.parent[i][j-1] != -1:
26                 self.parent[i][j] = self.parent[
27                     self.parent[i][j-1]][j-1]
28
29     def lca(self, u, v):
30         # Make u deeper
31         if self.depth[u] < self.depth[v]:
32             u, v = v, u
33
34         # Bring u to same level as v
35         diff = self.depth[u] - self.depth[v]
36         for i in range(self.LOG):
37             if (diff >> i) & 1:
38                 u = self.parent[u][i]
39
40             if u == v:
41                 return u
42
43         # Binary search for LCA
44         for i in range(self.LOG - 1, -1, -1):
45             if self.parent[u][i] != self.parent[v][i]:
46                 u = self.parent[u][i]
47                 v = self.parent[v][i]
48
49         return self.parent[u][0]
50
51     def dist(self, u, v):
52         # Distance between two nodes
53         l = self.lca(u, v)
54         return self.depth[u] + self.depth[v] - 2 * self.depth[l]

```

## 7 Shortest Path Algorithms

### 7.1 Dijkstra's Algorithm

**Description:** Finds shortest paths from a source to all vertices in weighted graphs with non-negative edges. Time:  $O((V+E) \log V)$  with heap.

```

1 import heapq
2
3 def dijkstra(graph, start, n):
4     # Initialize distances to infinity
5     dist = [float('inf')] * n
6     dist[start] = 0
7
8     # Min heap: (distance, node)
9     heap = [(0, start)]
10
11    while heap:
12        d, node = heapq.heappop(heap)
13
14        # Skip if already processed with better distance
15        if d > dist[node]:
16            continue
17
18        # Relax edges
19        for neighbor, weight in graph[node]:
20            new_dist = dist[node] + weight
21
22            if new_dist < dist[neighbor]:
23                dist[neighbor] = new_dist
24                heapq.heappush(heap, (new_dist, neighbor))
25
26    return dist
27
28 # Path reconstruction
29 def dijkstra_with_path(graph, start, n):
30     dist = [float('inf')] * n
31     parent = [-1] * n
32     dist[start] = 0
33     heap = [(0, start)]
34
35    while heap:
36        d, node = heapq.heappop(heap)
37        if d > dist[node]:
38            continue

```

## 6.6 Lowest Common Ancestor (LCA)

**Description:** Find LCA of two nodes in a tree. Binary lifting preprocessing:  $O(n \log n)$ , Query:  $O(\log n)$ .

```

1 class LCA:
2     def __init__(self, graph, root, n):
3         self.n = n
4         self.LOG = 20 # log2(n) + 1
5         self.parent = [[-1] * self.LOG for _ in range(n)]
6         self.depth = [0] * n
7
8         # DFS to set parent and depth
9         visited = [False] * n
10
11        def dfs(node, par, d):
12            visited[node] = True
13            self.parent[node][0] = par
14            self.depth[node] = d
15
16            for neighbor in graph[node]:
17                if not visited[neighbor]:
18                    dfs(neighbor, node, d + 1)
19
20        dfs(root, -1, 0)
21
22        # Binary lifting preprocessing

```

```

40     for neighbor, weight in graph[node]:
41         new_dist = dist[node] + weight
42         if new_dist < dist[neighbor]:
43             dist[neighbor] = new_dist
44             parent[neighbor] = node
45             heapq.heappush(heap, (new_dist, neighbor))
46
47     return dist, parent
48
49 def reconstruct_path(parent, target):
50     path = []
51     while target != -1:
52         path.append(target)
53         target = parent[target]
54     return path[::-1]

```

## 7.2 Bellman-Ford Algorithm

**Description:** Finds shortest paths with negative edges. Detects negative cycles. Time:  $O(VE)$ .

```

1 def bellman_ford(edges, n, start):
2     # edges = [(u, v, weight), ...]
3     dist = [float('inf')] * n
4     dist[start] = 0
5
6     # Relax edges n-1 times
7     for _ in range(n - 1):
8         for u, v, w in edges:
9             if dist[u] != float('inf') and \
10                dist[u] + w < dist[v]:
11                 dist[v] = dist[u] + w
12
13     # Check for negative cycles
14     for u, v, w in edges:
15         if dist[u] != float('inf') and \
16            dist[u] + w < dist[v]:
17             return None # Negative cycle exists
18
19     return dist

```

## 7.3 Floyd-Warshall Algorithm

**Description:** All-pairs shortest paths. Works with negative edges (no negative cycles). Time:  $O(V^3)$ .

```

1 def floyd_marshall(n, edges):
2     # Initialize distance matrix
3     dist = [[float('inf')]] * n for _ in range(n)]
4
5     for i in range(n):
6         dist[i][i] = 0
7
8     for u, v, w in edges:
9         dist[u][v] = min(dist[u][v], w)
10
11    # Dynamic programming
12    for k in range(n): # Intermediate vertex
13        for i in range(n):
14            for j in range(n):
15                dist[i][j] = min(dist[i][j],
16                                  dist[i][k] + dist[k][j])
17
18    return dist
19
20 # Check for negative cycle
21 def has_negative_cycle(dist, n):
22     for i in range(n):
23         if dist[i][i] < 0:
24             return True
25     return False

```

## 7.4 Minimum Spanning Tree

### 7.4.1 Kruskal's Algorithm

**Description:** MST using Union-Find. Sort edges by weight. Time:  $O(E \log E)$ .

```

1 def kruskal(n, edges):
2     # edges = [(weight, u, v), ...]

```

```

3     edges.sort() # Sort by weight
4
5     uf = UnionFind(n)
6     mst_weight = 0
7     mst_edges = []
8
9     for weight, u, v in edges:
10        if uf.union(u, v):
11            mst_weight += weight
12            mst_edges.append((u, v, weight))
13
14     return mst_weight, mst_edges
15
16 class UnionFind:
17     def __init__(self, n):
18         self.parent = list(range(n))
19         self.rank = [0] * n
20
21     def find(self, x):
22         if self.parent[x] != x:
23             self.parent[x] = self.find(self.parent[x])
24         return self.parent[x]
25
26     def union(self, x, y):
27         px, py = self.find(x), self.find(y)
28         if px == py:
29             return False
30         if self.rank[px] < self.rank[py]:
31             px, py = py, px
32             self.parent[py] = px
33         if self.rank[px] == self.rank[py]:
34             self.rank[px] += 1
35         return True

```

### 7.4.2 Prim's Algorithm

**Description:** MST using heap. Good for dense graphs. Time:  $O(E \log V)$ .

```

1 import heapq
2
3 def prim(graph, n):
4     # graph[u] = [(v, weight), ...]
5     visited = [False] * n
6     min_heap = [(0, 0)] # (weight, node)
7     mst_weight = 0
8
9     while min_heap:
10        weight, u = heapq.heappop(min_heap)
11
12        if visited[u]:
13            continue
14
15        visited[u] = True
16        mst_weight += weight
17
18        for v, w in graph[u]:
19            if not visited[v]:
20                heapq.heappush(min_heap, (w, v))
21
22    return mst_weight

```

## 8 Topological Sort

**Description:** Linear ordering of vertices in a DAG (Directed Acyclic Graph) such that for every edge  $u \rightarrow v$ ,  $u$  comes before  $v$ . Used for task scheduling, course prerequisites, build systems. Time:  $O(V+E)$ .

### 8.1 Kahn's Algorithm (BFS-based)

**Advantages:** Detects cycles, can process nodes level by level.

```

1 from collections import deque
2
3 def topo_sort(graph, n):
4     # Count incoming edges for each node
5     indegree = [0] * n
6     for u in range(n):

```

```

7     for v in graph[u]:
8         indegree[v] += 1
9
10    # Start with nodes having no dependencies
11    queue = deque([i for i in range(n)
12                  if indegree[i] == 0])
13    result = []
14
15    while queue:
16        node = queue.popleft()
17        result.append(node)
18
19        # Remove this node from graph
20        for neighbor in graph[node]:
21            indegree[neighbor] -= 1
22
23        # If neighbor has no more dependencies
24        if indegree[neighbor] == 0:
25            queue.append(neighbor)
26
27    # If not all nodes processed, cycle exists
28    return result if len(result) == n else []

```

## 8.2 DFS-based Topological Sort

**Advantages:** Simpler code, uses less space.

```

1 def topo_dfs(graph, n):
2     visited = [False] * n
3     stack = []
4
5     def dfs(node):
6         visited[node] = True
7
8         # Visit all neighbors first
9         for neighbor in graph[node]:
10            if not visited[neighbor]:
11                dfs(neighbor)
12
13         # Add to stack after visiting all descendants
14         stack.append(node)
15
16     # Process all components
17     for i in range(n):
18         if not visited[i]:
19             dfs(i)
20
21     # Reverse stack gives topological order
22     return stack[::-1]

```

## 9 Union-Find (Disjoint Set Union)

**Description:** Efficiently tracks disjoint sets and supports union and find operations. Used for Kruskal's MST, connected components, cycle detection. Time:  $O(\alpha(n)) \approx O(1)$  per operation with path compression and union by rank.

### Applications:

- Kruskal's minimum spanning tree
- Detecting cycles in undirected graphs
- Finding connected components
- Network connectivity problems

```

1 class UnionFind:
2     def __init__(self, n):
3         # Each node is its own parent initially
4         self.parent = list(range(n))
5         # Rank for union by rank optimization
6         self.rank = [0] * n
7
8     def find(self, x):
9         # Path compression: point directly to root
10        if self.parent[x] != x:
11            self.parent[x] = self.find(self.parent[x])
12        return self.parent[x]
13
14    def union(self, x, y):
15        # Find roots
16        px, py = self.find(x), self.find(y)

```

```

18        # Already in same set
19        if px == py:
20            return False
21
22        # Union by rank: attach smaller tree under larger
23        if self.rank[px] < self.rank[py]:
24            px, py = py, px
25
26        self.parent[py] = px
27
28        # Increase rank if trees had equal rank
29        if self.rank[px] == self.rank[py]:
30            self.rank[px] += 1
31
32        return True
33
34    def connected(self, x, y):
35        return self.find(x) == self.find(y)
36
37    # Count number of disjoint sets
38    def count_sets(self):
39        return len(set(self.find(i)
40                    for i in range(len(self.parent))))
41
42    # Example: Detect cycle in undirected graph
43    def has_cycle_uf(edges, n):
44        uf = UnionFind(n)
45        for u, v in edges:
46            if uf.connected(u, v):
47                return True # Cycle found
48            uf.union(u, v)
49        return False

```

## 10 Binary Search

**Description:** Search in  $O(\log n)$  time. Works on monotonic functions where condition changes from false to true (or vice versa) exactly once.

### 10.1 Basic Templates

```

1 # Standard binary search (find exact element)
2 def binary_search(arr, target):
3     left, right = 0, len(arr) - 1
4
5     while left <= right:
6         mid = (left + right) // 2
7
8         if arr[mid] == target:
9             return mid
10        elif arr[mid] < target:
11            left = mid + 1
12        else:
13            right = mid - 1
14
15    return -1 # Not found
16
17 # Find FIRST position where condition is True
18 def lower_bound(arr, target):
19     left, right = 0, len(arr)
20
21     while left < right:
22         mid = (left + right) // 2
23
24         if arr[mid] < target:
25             left = mid + 1
26         else:
27             right = mid
28
29     return left
30
31 # Find LAST position where condition is True + 1
32 def upper_bound(arr, target):
33     left, right = 0, len(arr)
34
35     while left < right:
36         mid = (left + right) // 2
37
38         if arr[mid] <= target:
39             left = mid + 1

```

```

40     else:
41         right = mid
42
43     return left

```

## 10.2 Advanced Templates

```

1 # Find first True in boolean array
2 def find_first_true(check_func, left, right):
3     while left < right:
4         mid = (left + right) // 2
5
6         if check_func(mid):
7             right = mid # Could be answer, search left
8         else:
9             left = mid + 1 # Not answer, search right
10
11    return left
12
13 # Find last True in boolean array
14 def find_last_true(check_func, left, right):
15     while left < right:
16         mid = (left + right + 1) // 2 # Round up!
17
18         if check_func(mid):
19             left = mid # Could be answer, search right
20         else:
21             right = mid - 1 # Not answer, search left
22
23    return left
24
25 # Binary search on floating point
26 def binary_search_float(check_func, left, right, eps=1e-9):
27     while right - left > eps:
28         mid = (left + right) / 2
29
30         if check_func(mid):
31             right = mid
32         else:
33             left = mid
34
35    return left

```

## 10.3 Common Use Cases

```

1 # 1. Find insertion point
2 def search_insert_position(nums, target):
3     return lower_bound(nums, target)
4
5 # 2. Find range of target in sorted array
6 def search_range(nums, target):
7     left = lower_bound(nums, target)
8     if left >= len(nums) or nums[left] != target:
9         return [-1, -1]
10    right = upper_bound(nums, target) - 1
11    return [left, right]
12
13 # 3. Peak element (not strictly increasing/decreasing)
14 def find_peak_element(nums):
15     left, right = 0, len(nums) - 1
16
17     while left < right:
18         mid = (left + right) // 2
19
20         if nums[mid] > nums[mid + 1]:
21             right = mid # Peak is on left or mid
22         else:
23             left = mid + 1 # Peak is on right
24
25    return left
26
27 # 4. Rotated sorted array
28 def search_rotated(nums, target):
29     left, right = 0, len(nums) - 1
30
31     while left <= right:
32         mid = (left + right) // 2
33
34         if nums[mid] == target:
35             return mid

```

```

37     # Left half is sorted
38     if nums[left] <= nums[mid]:
39         if nums[left] <= target < nums[mid]:
40             right = mid - 1
41         else:
42             left = mid + 1
43     # Right half is sorted
44     else:
45         if nums[mid] < target <= nums[right]:
46             left = mid + 1
47         else:
48             right = mid - 1
49
50     return -1
51
52 # 5. Square root (integer)
53 def sqrt_int(x):
54     if x < 2:
55         return x
56
57     left, right = 1, x // 2
58
59     while left <= right:
60         mid = (left + right) // 2
61         square = mid * mid
62
63         if square == x:
64             return mid
65         elif square < x:
66             left = mid + 1
67         else:
68             right = mid - 1
69
70     return right # Largest integer whose square <= x

```

## 10.4 Binary Search on Answer

```

1 # Template for optimization problems
2 def minimize_max_distance(stations, k):
3     # Can we place k stations with max distance <= max_dist?
4     def possible(max_dist):
5         stations_needed = 0
6         for i in range(len(stations) - 1):
7             gap = stations[i + 1] - stations[i]
8             stations_needed += int(gap / max_dist)
9
10        return stations_needed <= k
11
12    left, right = 0.0, stations[-1] - stations[0]
13
14    while right - left > 1e-6:
15        mid = (left + right) / 2
16
17        if possible(mid):
18            right = mid
19        else:
20            left = mid
21
22    return left
23
24 # Capacity allocation problem
25 def split_array_largest_sum(nums, m):
26     # Can we split into m subarrays with max sum <= max_sum?
27     def can_split(max_sum):
28         count = 1
29         current_sum = 0
30
31         for num in nums:
32             if current_sum + num > max_sum:
33                 count += 1
34                 current_sum = num
35                 if count > m:
36                     return False
37             else:
38                 current_sum += num
39
40         return True
41
42     left, right = max(nums), sum(nums)
43
44     while left < right:
45         mid = (left + right) // 2

```

```

45
46     if can_split(mid):
47         right = mid
48     else:
49         left = mid + 1
50
51     return left
52
53 # Resource allocation with greedy check
54 def minimum_speed_to_eat_bananas(piles, h):
55     # Can we eat all bananas in h hours at speed k?
56     def can_finish(k):
57         return sum((pile + k - 1) // k for pile in piles) <= h
58
59     left, right = 1, max(piles)
60
61     while left < right:
62         mid = (left + right) // 2
63
64         if can_finish(mid):
65             right = mid
66         else:
67             left = mid + 1
68
69     return left

```

**Key Points:**

- Always clarify what you're searching for (first/last occurrence, exact match, etc.)
- For "find first True": use `right = mid`
- For "find last True": use `left = mid` and `mid = (left + right + 1) // 2`
- Binary search on answer: define a monotonic check function
- Handle edge cases: empty arrays, single elements, all same elements

## 10.5 Common Pitfalls

**Watch out for:**

- **Infinite loops:** Wrong midpoint calculation for "find last" (missing +1)
- **Off-by-one errors:** Mixing `len(arr)` vs `len(arr)-1` in initial bounds
- **Integer overflow:** Use `left + (right - left) // 2` instead of `(left + right) // 2`
- **Wrong condition:** `<` vs `<=` in loop condition (`left < right` vs `left <= right`)
- **Non-monotonic function:** Binary search only works if condition changes at most once
- **Empty result handling:** Check bounds before accessing `arr[result]`
- **Floating point precision:** Use appropriate epsilon for convergence
- **Wrong search space:** Ensure your `left` and `right` bounds contain the answer

# 11 Dynamic Programming

**Description:** Solve problems by breaking them into overlapping subproblems. Store results to avoid recomputation.

## 11.1 Longest Increasing Subsequence

**Description:** Find length of longest strictly increasing subsequence. Time:  $O(n \log n)$  using binary search.

```

1 def lis(arr):
2     from bisect import bisect_left
3
4     # dp[i] = smallest ending value of LIS of length i+1
5     dp = []
6
7     for x in arr:
8         # Find position to place x
9         idx = bisect_left(dp, x)

```

```

10
11     if idx == len(dp):
12         dp.append(x) # Extend LIS
13     else:
14         dp[idx] = x # Better ending for this length
15
16     return len(dp)
17
18 # LIS with actual sequence
19 def lis_with_sequence(arr):
20     from bisect import bisect_left
21
22     n = len(arr)
23     dp = []
24     parent = [-1] * n
25     dp_idx = [] # indices in dp
26
27     for i, x in enumerate(arr):
28         idx = bisect_left(dp, x)
29
30         if idx == len(dp):
31             dp.append(x)
32             dp_idx.append(i)
33         else:
34             dp[idx] = x
35             dp_idx[idx] = i
36
37         if idx > 0:
38             parent[i] = dp_idx[idx - 1]
39
40     # Reconstruct sequence
41     result = []
42     idx = dp_idx[-1]
43     while idx != -1:
44         result.append(arr[idx])
45         idx = parent[idx]
46
47     return result[::-1]

```

## 11.2 0/1 Knapsack

**Description:** Maximum value with weight capacity. Each item can be taken 0 or 1 time. Time:  $O(n \times \text{capacity})$ , Space:  $O(n \times \text{capacity})$ .

```

1 def knapsack(weights, values, capacity):
2     n = len(weights)
3     # dp[i][w] = max value using first i items,
4     #           weight <= w
5     dp = [[0] * (capacity + 1) for _ in range(n + 1)]
6
7     for i in range(1, n + 1):
8         for w in range(capacity + 1):
9             # Don't take item i-1
10            dp[i][w] = dp[i-1][w]
11
12            # Take item i-1 if it fits
13            if weights[i-1] <= w:
14                dp[i][w] = max(
15                    dp[i][w],
16                    dp[i-1][w - weights[i-1]] + values[i-1]
17                )
18
19    return dp[n][capacity]
20
21 # Space-optimized O(capacity)
22 def knapsack_optimized(weights, values, capacity):
23     dp = [0] * (capacity + 1)
24
25     for i in range(len(weights)):
26         # Iterate backwards to avoid using updated values
27         for w in range(capacity, weights[i] - 1, -1):
28             dp[w] = max(dp[w],
29                         dp[w - weights[i]] + values[i])
30
31     return dp[capacity]

```

### 11.3 Edit Distance (Levenshtein Distance)

**Description:** Minimum operations (insert, delete, replace) to transform s1 to s2. Time:  $O(m \times n)$ , Space:  $O(m \times n)$ .

```

1 def edit_dist(s1, s2):
2     m, n = len(s1), len(s2)
3     # dp[i][j] = edit distance of s1[:i] and s2[:j]
4     dp = [[0] * (n + 1) for _ in range(m + 1)]
5
6     # Base cases: empty string transformations
7     for i in range(m + 1):
8         dp[i][0] = i # Delete all
9     for j in range(n + 1):
10        dp[0][j] = j # Insert all
11
12    for i in range(1, m + 1):
13        for j in range(1, n + 1):
14            if s1[i-1] == s2[j-1]:
15                # Characters match, no operation needed
16                dp[i][j] = dp[i-1][j-1]
17            else:
18                dp[i][j] = 1 + min(
19                    dp[i-1][j], # Delete from s1
20                    dp[i][j-1], # Insert into s1
21                    dp[i-1][j-1] # Replace in s1
22                )
23
24    return dp[m][n]
```

### 11.4 Longest Common Subsequence (LCS)

**Description:** Longest subsequence common to two sequences. Time:  $O(m \times n)$ .

```

1 def lcs(s1, s2):
2     m, n = len(s1), len(s2)
3     dp = [[0] * (n + 1) for _ in range(m + 1)]
4
5     for i in range(1, m + 1):
6         for j in range(1, n + 1):
7             if s1[i-1] == s2[j-1]:
8                 dp[i][j] = dp[i-1][j-1] + 1
9             else:
10                dp[i][j] = max(dp[i-1][j], dp[i][j-1])
11
12    return dp[m][n]
13
14 # Reconstruct LCS
15 def lcs_string(s1, s2):
16     m, n = len(s1), len(s2)
17     dp = [[0] * (n + 1) for _ in range(m + 1)]
18
19     for i in range(1, m + 1):
20         for j in range(1, n + 1):
21             if s1[i-1] == s2[j-1]:
22                 dp[i][j] = dp[i-1][j-1] + 1
23             else:
24                 dp[i][j] = max(dp[i-1][j], dp[i][j-1])
25
26 # Backtrack
27 result = []
28 i, j = m, n
29 while i > 0 and j > 0:
30     if s1[i-1] == s2[j-1]:
31         result.append(s1[i-1])
32         i -= 1
33         j -= 1
34     elif dp[i-1][j] > dp[i][j-1]:
35         i -= 1
36     else:
37         j -= 1
38
39 return ''.join(reversed(result))
```

### 11.5 Coin Change

**Description:** Minimum coins to make amount, or count ways. Time:  $O(n \times \text{amount})$ .

```

1 # Minimum coins
2 def coin_change_min(coins, amount):
```

```

3     dp = [float('inf')] * (amount + 1)
4     dp[0] = 0
5
6     for coin in coins:
7         for i in range(coin, amount + 1):
8             dp[i] = min(dp[i], dp[i - coin] + 1)
9
10    return dp[amount] if dp[amount] != float('inf') else -1
11
12 # Count ways
13 def coin_change_ways(coins, amount):
14     dp = [0] * (amount + 1)
15     dp[0] = 1
16
17     for coin in coins:
18         for i in range(coin, amount + 1):
19             dp[i] += dp[i - coin]
20
21    return dp[amount]
```

### 11.6 Palindrome Partitioning

**Description:** Minimum cuts to partition string into palindromes. Time:  $O(n^2)$ .

```

1 def min_palindrome_partition(s):
2     n = len(s)
3
4     # is_pal[i][j] = True if s[i:j+1] is palindrome
5     is_pal = [[False] * n for _ in range(n)]
6
7     # Every single character is palindrome
8     for i in range(n):
9         is_pal[i][i] = True
10
11    # Check all substrings
12    for length in range(2, n + 1):
13        for i in range(n - length + 1):
14            j = i + length - 1
15            if s[i] == s[j]:
16                is_pal[i][j] = (length == 2 or
17                                is_pal[i+1][j-1])
18
19    # dp[i] = min cuts for s[0:i+1]
20    dp = [float('inf')] * n
21
22    for i in range(n):
23        if is_pal[0][i]:
24            dp[i] = 0
25        else:
26            for j in range(i):
27                if is_pal[j+1][i]:
28                    dp[i] = min(dp[i], dp[j] + 1)
29
30    return dp[n-1]
```

### 11.7 Subset Sum

**Description:** Check if subset sums to target. Time:  $O(n \times \text{sum})$ .

```

1 def subset_sum(arr, target):
2     n = len(arr)
3     dp = [[False] * (target + 1) for _ in range(n + 1)]
4
5     # Base case: sum 0 is always achievable
6     for i in range(n + 1):
7         dp[i][0] = True
8
9     for i in range(1, n + 1):
10        for s in range(target + 1):
11            # Don't take arr[i-1]
12            dp[i][s] = dp[i-1][s]
13
14            # Take arr[i-1] if possible
15            if s >= arr[i-1]:
16                dp[i][s] = dp[i][s] or dp[i-1][s - arr[i-1]]
17
18    return dp[n][target]
19
20 # Space optimized
21 def subset_sum_optimized(arr, target):
22     dp = [False] * (target + 1)
```

```

23     dp[0] = True
24
25     for num in arr:
26         for s in range(target, num - 1, -1):
27             dp[s] = dp[s] or dp[s - num]
28
29     return dp[target]

```

## 12 Array Techniques

### 12.1 Prefix Sum

**Description:** Precompute cumulative sums for O(1) range queries. Time: O(n) preprocessing, O(1) query.

```

1 # 1D prefix sum
2 prefix = [0] * (n + 1)
3 for i in range(n):
4     prefix[i + 1] = prefix[i] + arr[i]
5
6 # Range sum query [l, r] inclusive
7 range_sum = prefix[r + 1] - prefix[l]
8
9 # 2D prefix sum - for rectangle sum queries
10 def build_2d_prefix(matrix):
11     n, m = len(matrix), len(matrix[0])
12     prefix = [[0] * (m + 1) for _ in range(n + 1)]
13
14     for i in range(1, n + 1):
15         for j in range(1, m + 1):
16             prefix[i][j] = (matrix[i-1][j-1] +
17                             prefix[i-1][j] +
18                             prefix[i][j-1] -
19                             prefix[i-1][j-1])
20
21     return prefix
22
23 # Rectangle sum from (x1,y1) to (x2,y2) inclusive
24 def rect_sum(prefix, x1, y1, x2, y2):
25     return (prefix[x2+1][y2+1] -
26             prefix[x1][y2+1] -
27             prefix[x2+1][y1] +
28             prefix[x1][y1])

```

### 12.2 Difference Array

**Description:** Efficiently perform range updates. O(1) per update, O(n) to reconstruct.

```

1 # Initialize difference array
2 diff = [0] * (n + 1)
3
4 # Add 'val' to range [l, r]
5 def range_update(diff, l, r, val):
6     diff[l] += val
7     diff[r + 1] -= val
8
9 # After all updates, reconstruct array
10 def reconstruct(diff):
11     result = []
12     current = 0
13     for i in range(len(diff) - 1):
14         current += diff[i]
15         result.append(current)
16     return result
17
18 # Example: Multiple range updates
19 diff = [0] * (n + 1)
20 for l, r, val in updates:
21     range_update(diff, l, r, val)
22 final_array = reconstruct(diff)

```

### 12.3 Sliding Window

**Description:** Maintain a window of elements while traversing. Time: O(n).

```

1 # Fixed size window
2 def max_sum_window(arr, k):
3     window_sum = sum(arr[:k])
4     max_sum = window_sum

```

```

5
6     # Slide window: add right, remove left
7     for i in range(k, len(arr)):
8         window_sum += arr[i] - arr[i - k]
9         max_sum = max(max_sum, window_sum)
10
11     return max_sum
12
13     # Variable size window - two pointers
14     def min_subarray_sum_geq_target(arr, target):
15         left = 0
16         current_sum = 0
17         min_len = float('inf')
18
19         for right in range(len(arr)):
20             current_sum += arr[right]
21
22             # Shrink window while condition holds
23             while current_sum >= target:
24                 min_len = min(min_len, right - left + 1)
25                 current_sum -= arr[left]
26                 left += 1
27
28         return min_len if min_len != float('inf') else 0
29
30     # Longest substring with at most k distinct chars
31     def longest_k_distinct(s, k):
32         from collections import defaultdict
33
34         left = 0
35         char_count = defaultdict(int)
36         max_len = 0
37
38         for right in range(len(s)):
39             char_count[s[right]] += 1
40
41             # Shrink if too many distinct
42             while len(char_count) > k:
43                 char_count[s[left]] -= 1
44                 if char_count[s[left]] == 0:
45                     del char_count[s[left]]
46                 left += 1
47
48             max_len = max(max_len, right - left + 1)
49
50     return max_len

```

## 13 Advanced Data Structures

### 13.1 Segment Tree

**Description:** Supports range queries and point updates in O(log n). Can be modified for range updates with lazy propagation.

```

1 class SegmentTree:
2     def __init__(self, arr):
3         self.n = len(arr)
4
5         # Tree size: 4n is safe upper bound
6         self.tree = [0] * (4 * self.n)
7         self.build(arr, 0, 0, self.n - 1)
8
9     def build(self, arr, node, start, end):
10        if start == end:
11            # Leaf node
12            self.tree[node] = arr[start]
13        else:
14            mid = (start + end) // 2
15            # Build left and right subtrees
16            self.build(arr, 2*node+1, start, mid)
17            self.build(arr, 2*node+2, mid+1, end)
18            # Combine results (sum in this case)
19            self.tree[node] = (self.tree[2*node+1] +
20                               self.tree[2*node+2]))
21
22    def update(self, node, start, end, idx, val):
23        if start == end:
24            # Leaf node - update value
25            self.tree[node] = val
26        else:
27            mid = (start + end) // 2
28            if start < idx <= mid:
29                self.update(2*node+1, start, mid, idx, val)
30            else:
31                self.update(2*node+2, mid+1, end, idx, val)
32            self.tree[node] = (self.tree[2*node+1] +
33                               self.tree[2*node+2]))

```

```

27     if idx <= mid:
28         # Update left subtree
29         self.update(2*node+1, start, mid, idx, val)
30     else:
31         # Update right subtree
32         self.update(2*node+2, mid+1, end, idx, val)
33     # Recompute parent
34     self.tree[node] = (self.tree[2*node+1] +
35                         self.tree[2*node+2])
36
37 def query(self, node, start, end, l, r):
38     # No overlap
39     if r < start or end < l:
40         return 0
41
42     # Complete overlap
43     if l <= start and end <= r:
44         return self.tree[node]
45
46     # Partial overlap
47     mid = (start + end) // 2
48     left_sum = self.query(2*node+1, start, mid, l, r)
49     right_sum = self.query(2*node+2, mid+1, end, l, r)
50     return left_sum + right_sum
51
52 # Public interface
53 def update_val(self, idx, val):
54     self.update(0, 0, self.n-1, idx, val)
55
56 def range_sum(self, l, r):
57     return self.query(0, 0, self.n-1, l, r)

```

```

6 class Trie:
7     def __init__(self):
8         self.root = TrieNode()
9
10    def insert(self, word):
11        # Insert word - O(len(word))
12        node = self.root
13        for char in word:
14            if char not in node.children:
15                node.children[char] = TrieNode()
16            node = node.children[char]
17        node.is_end = True
18
19    def search(self, word):
20        # Exact word search - O(len(word))
21        node = self.root
22        for char in word:
23            if char not in node.children:
24                return False
25            node = node.children[char]
26        return node.is_end
27
28    def starts_with(self, prefix):
29        # Prefix search - O(len(prefix))
30        node = self.root
31        for char in prefix:
32            if char not in node.children:
33                return False
34            node = node.children[char]
35        return True
36
37 # Find all words with given prefix
38 def words_with_prefix(self, prefix):
39     node = self.root
40
41     for char in prefix:
42         if char not in node.children:
43             return []
44         node = node.children[char]
45
46     # DFS to collect all words
47     words = []
48     def dfs(n, path):
49         if n.is_end:
50             words.append(prefix + path)
51         for char, child in n.children.items():
52             dfs(child, path + char)
53
54     dfs(node, "")
55     return words

```

## 13.2 Fenwick Tree (Binary Indexed Tree)

**Description:** Simpler than segment tree, supports prefix sum and point updates in  $O(\log n)$ . More space efficient.

```

1 class FenwickTree:
2     def __init__(self, n):
3         self.n = n
4         # 1-indexed for easier implementation
5         self.tree = [0] * (n + 1)
6
7     def update(self, i, delta):
8         # Add delta to position i (1-indexed)
9         while i <= self.n:
10             self.tree[i] += delta
11             # Move to next node: add LSB
12             i += i & (-i)
13
14     def query(self, i):
15         # Get prefix sum up to i (1-indexed)
16         s = 0
17         while i > 0:
18             s += self.tree[i]
19             # Move to parent: remove LSB
20             i -= i & (-i)
21         return s
22
23     def range_query(self, l, r):
24         # Sum from l to r (1-indexed)
25         return self.query(r) - self.query(l - 1)
26
27 # Usage example
28 bit = FenwickTree(n)
29 for i, val in enumerate(arr, 1):
30     bit.update(i, val)
31
32 # Range sum [l, r] (1-indexed)
33 result = bit.range_query(l, r)

```

## 13.3 Trie (Prefix Tree)

**Description:** Tree for storing strings, enables fast prefix searches. Time:  $O(m)$  for operations where  $m$  is string length.

```

1 class TrieNode:
2     def __init__(self):
3         self.children = {} # char -> TrieNode
4         self.is_end = False # End of word marker
5

```

```

1 import random
2
3 class TreapNode:
4     __slots__ = ("key", "prio", "left", "right", "size")
5     def __init__(self, key):
6         self.key = key
7         self.prio = random.randint(1, 1 << 30)
8         self.left = None
9         self.right = None
10        self.size = 1
11
12    def _sz(t):
13        return t.size if t else 0
14
15    def _upd(t):
16        if t:
17            t.size = 1 + _sz(t.left) + _sz(t.right)
18
19    def _merge(a, b):
20        # assumes all keys in a < all keys in b
21        if not a or not b:
22            return a or b
23        if a.prio > b.prio:
24            a.right = _merge(a.right, b)

```

```

25     _upd(a)
26     return a
27 else:
28     b.left = _merge(a, b.left)
29     _upd(b)
30     return b
31
32 def _split(t, key):
33     # returns (l, r): l has keys < key, r has keys >= key
34     if not t:
35         return (None, None)
36     if key == t.key:
37         l, t.left = _split(t.left, key)
38         _upd(t)
39         return (l, t)
40     else:
41         t.right, r = _split(t.right, key)
42         _upd(t)
43         return (t, r)
44
45 def _erase(t, key):
46     if not t:
47         return None
48     if key == t.key:
49         return _merge(t.left, t.right)
50     if key < t.key:
51         t.left = _erase(t.left, key)
52     else:
53         t.right = _erase(t.right, key)
54     _upd(t)
55     return t
56
57 class Treap:
58     def __init__(self):
59         self.root = None
60
61     def __len__(self):
62         return _sz(self.root)
63
64     def contains(self, key):
65         t = self.root
66         while t:
67             if key == t.key:
68                 return True
69             t = t.left if key < t.key else t.right
70         return False
71
72     def insert(self, key):
73         if self.contains(key):
74             return
75         node = TreapNode(key)
76         l, r = _split(self.root, key)
77         self.root = _merge(_merge(l, node), r)
78
79     def remove(self, key):
80         self.root = _erase(self.root, key)
81
82     def kth_smallest(self, k):
83         # 0-indexed k
84         t = self.root
85         while t:
86             ls = _sz(t.left)
87             if k < ls:
88                 t = t.left
89             elif k == ls:
90                 return t.key
91             else:
92                 k -= ls + 1
93                 t = t.right
94         return None # k out of range
95
96     def count_less_than(self, key):
97         # number of keys < key
98         t, cnt = self.root, 0
99         while t:
100            if key <= t.key:
101                t = t.left
102            else:
103                cnt += 1 + _sz(t.left)
104                t = t.right
105        return cnt

```

```

106
107     def lower_bound(self, key):
108         # smallest key >= key; returns None if none
109         t, ans = self.root, None
110         while t:
111             if t.key >= key:
112                 ans = t.key
113                 t = t.left
114             else:
115                 t = t.right
116         return ans
117
118 # Usage example
119 T = Treap()
120 for x in [5, 1, 7, 3]:
121     T.insert(x)
122 T.contains(3)           # True
123 T.kth_smallest(1)       # 3 (0-indexed)
124 T.count_less_than(6)    # 3 (1,3,5)
125 T.remove(5)
126 len(T)                  # 3

```

## 14 Bit Manipulation

**Description:** Efficient operations using bitwise operators. Useful for sets, flags, and optimization.

```

1 # Check if i-th bit (0-indexed) is set
2 is_set = (n >> i) & 1
3
4 # Set i-th bit to 1
5 n |= (1 << i)
6
7 # Clear i-th bit (set to 0)
8 n &= ~(1 << i)
9
10 # Toggle i-th bit
11 n ^= (1 << i)
12
13 # Count set bits (popcount)
14 count = bin(n).count('1')
15 count = n.bit_count() # Python 3.10+
16
17 # Get lowest set bit
18 lsb = n & -n # Also n & (~n + 1)
19
20 # Remove lowest set bit
21 n &= (n - 1)
22
23 # Check if power of 2
24 is_pow2 = n > 0 and (n & (n - 1)) == 0
25
26 # Check if power of 4
27 is_pow4 = n > 0 and (n & (n-1)) == 0 and (n & 0x55555555) != 0
28
29 # Iterate over all subsets of set represented by mask
30 mask = (1 << n) - 1 # All bits set
31 submask = mask
32 while submask > 0:
33     # Process submask
34     submask = (submask - 1) & mask
35
36 # Iterate through all k-bit masks
37 def iterate_k_bits(n, k):
38     mask = (1 << k) - 1
39     while mask < (1 << n):
40         # Process mask
41         yield mask
42         # Gosper's hack
43         c = mask & ~mask
44         r = mask + c
45         mask = (((r ^ mask) >> 2) // c) | r
46
47 # XOR properties
48 # a ^ a = 0 (number XOR itself is 0)
49 # a ^ 0 = a (number XOR 0 is itself)
50 # XOR is commutative and associative
51 # Find unique element when all others appear twice:
52 def find_unique(arr):
53     result = 0

```

```

54     for x in arr:
55         result ^= x
56     return result
57
58 # Subset enumeration
59 n = 5 # Number of elements
60 for mask in range(1 << n):
61     subset = [i for i in range(n) if mask & (1 << i)]
62     # Process subset
63
64 # Check parity (odd/even number of 1s)
65 def parity(n):
66     count = 0
67     while n:
68         count ^= 1
69         n &= n - 1
70     return count # 1 if odd, 0 if even
71
72 # Swap two numbers without temp variable
73 a, b = 5, 10
74 a ^= b
75 b ^= a
76 a ^= b
77 # Now a=10, b=5

```

## 15 Matrix Operations

**Description:** Matrix operations for DP optimization, graph algorithms, and recurrence relations.

### 15.1 Matrix Multiplication

```

1 # Standard matrix multiplication - O(n^3)
2 def matmul(A, B):
3     n, m, p = len(A), len(A[0]), len(B[0])
4     C = [[0] * p for _ in range(n)]
5
6     for i in range(n):
7         for j in range(p):
8             for k in range(m):
9                 C[i][j] += A[i][k] * B[k][j]
10
11    return C
12
13 # With modulo
14 def matmul_mod(A, B, mod):
15     n = len(A)
16     C = [[0] * n for _ in range(n)]
17
18     for i in range(n):
19         for j in range(n):
20             for k in range(n):
21                 C[i][j] = (C[i][j] +
22                             A[i][k] * B[k][j]) % mod
23
24    return C

```

### 15.2 Matrix Exponentiation

**Description:** Compute  $M^n$  in  $O(k^3 \log n)$  where  $k$  is matrix dimension. Used for solving linear recurrences efficiently.

```

1 def matpow(M, n, mod):
2     size = len(M)
3
4     # Identity matrix
5     result = [[1 if i==j else 0
6               for j in range(size)]
7               for i in range(size)]
8
9     # Binary exponentiation
10    while n > 0:
11        if n & 1:
12            result = matmul_mod(result, M, mod)
13        M = matmul_mod(M, M, mod)
14        n >>= 1
15
16    return result
17
18 # Example: Fibonacci using matrix exponentiation

```

```

19 # F(n) = [[1, 1], [1, 0]]^n
20 def fibonacci(n, mod):
21     if n == 0: return 0
22     if n == 1: return 1
23
24     M = [[1, 1], [1, 0]]
25     result = matpow(M, n - 1, mod)
26     return result[0][0]
27
28 # Linear recurrence: a(n) = c1*a(n-1) + c2*a(n-2) + ...
29 # Build transition matrix and use matrix exponentiation
30 def linear_recurrence(coeffs, init, n, mod):
31     k = len(coeffs)
32
33     if n < k:
34         return init[n]
35
36     # Transition matrix
37     # [a(n), a(n-1), ..., a(n-k+1)]
38     M = [[0] * k for _ in range(k)]
39     M[0] = coeffs # First row
40     for i in range(1, k):
41         M[i][i-1] = 1 # Identity for shifting
42
43     # Initial state vector [a(k-1), a(k-2), ..., a(0)]
44     state = init[k-1::-1]
45
46     # M^(n-k+1)
47     result_matrix = matpow(M, n - k + 1, mod)
48
49     # Multiply with initial state
50     result = 0
51     for i in range(k):
52         result = (result + result_matrix[0][i] * state[i]) % mod
53
54     return result
55
56 # Example: Tribonacci T(n) = T(n-1) + T(n-2) + T(n-3)
57 def tribonacci(n, mod):
58     if n == 0: return 0
59     if n == 1 or n == 2: return 1
60
61     coeffs = [1, 1, 1]
62     init = [0, 1, 1]
63     return linear_recurrence(coeffs, init, n, mod)

```

## 16 Miscellaneous Tips

### 16.1 Python-Specific Optimizations

```

1 # Fast input for large datasets
2 import sys
3 input = sys.stdin.readline
4
5 # Increase recursion limit for deep DFS/DP
6 sys.setrecursionlimit(10**6)
7
8 # Threading for higher stack limit (CAUTION: use carefully)
9 import threading
10 threading.stack_size(2**26) # 64MB
11 sys.setrecursionlimit(2**20)
12
13 # Deep copy (be careful with performance)
14 from copy import deepcopy
15 new_list = deepcopy(old_list)
16
17 # Fast output (for printing large results)
18 import sys
19 print = sys.stdout.write # Only use for string output

```

### 16.2 Useful Libraries

```

1 # Iterator tools - powerful combinations
2 from itertools import *
3
4 # permutations(iterable, r) - all r-length permutations
5 perms = list(permutations([1,2,3], 2))
6 # [(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)]
7
8 # combinations(iterable, r) - r-length combinations

```

```

9 combs = list(combinations([1,2,3], 2))
10 # [(1,2), (1,3), (2,3)]
11
12 # product - cartesian product
13 prod = list(product([1,2], ['a','b']))
14 # [(1,'a'), (1,'b'), (2,'a'), (2,'b')]
15
16 # accumulate - running totals
17 acc = list(accumulate([1,2,3,4]))
18 # [1, 3, 6, 10]
19
20 # chain - flatten iterables
21 chained = list(chain([1,2], [3,4]))
22 # [1, 2, 3, 4]

```

## 16.3 Common Patterns

```

1 # Lambda sorting with multiple keys
2 arr.sort(key=lambda x: (-x[0], x[1]))
3 # Sort by first desc, then second asc
4
5 # All/Any - short-circuit evaluation
6 all(x > 0 for x in arr) # True if all positive
7 any(x > 0 for x in arr) # True if any positive
8
9 # Zip - parallel iteration
10 for a, b in zip(list1, list2):
11     pass
12
13 # Enumerate - index and value
14 for i, val in enumerate(arr):
15     print(f"arr[{i}] = {val}")
16
17 # Custom comparison function
18 from functools import cmp_to_key
19
20 def compare(a, b):
21     # Return -1 if a < b, 0 if equal, 1 if a > b
22     if a + b > b + a:
23         return -1
24     return 1
25
26 arr.sort(key=cmp_to_key(compare))
27
28 # DefaultDict with lambda
29 from collections import defaultdict
30 d = defaultdict(lambda: float('inf'))
31
32 # Multiple assignment
33 a, b = b, a # Swap
34 a, *rest, b = [1,2,3,4,5] # a=1, rest=[2,3,4], b=5

```

## 16.4 Common Pitfalls

```

1 # Integer division - floors toward negative infinity
2 print(7 // 3) # 2
3 print(-7 // 3) # -3 (not -2!)
4
5 # For ceiling division toward zero:
6 def div.ceil(a, b):
7     return -(a // b)
8
9 # Modulo with negative numbers
10 print((-5) % 3) # 1 (not -2!)
11 print(5 % -3) # -1
12
13 # List multiplication creates references!
14 matrix = [[0] * m] * n # WRONG! All rows same object
15 matrix[0][0] = 1 # Changes all rows!
16
17 # Correct way
18 matrix = [[0] * m for _ in range(n)]
19
20 # Float comparison - don't use ==
21 a, b = 0.1 + 0.2, 0.3
22 print(a == b) # False!
23
24 # Use epsilon comparison
25 eps = 1e-9
26 print(abs(a - b) < eps) # True
27

```

```

28 # String immutability
29 s = "abc"
30 # s[0] = 'd' # ERROR!
31 s = 'd' + s[1:] # OK
32
33 # For many string mutations, use list
34 chars = list(s)
35 chars[0] = 'd'
36 s = ''.join(chars)
37
38 # Mutable default arguments - dangerous!
39 def func(arr=[]): # WRONG!
40     arr.append(1)
41     return arr
42
43 # Each call modifies same list
44 print(func()) # [1]
45 print(func()) # [1, 1]
46
47 # Correct way
48 def func(arr=None):
49     if arr is None:
50         arr = []
51     arr.append(1)
52     return arr
53
54 # Generator expressions save memory
55 sum(x*x for x in range(10**6)) # Memory efficient
56 # vs
57 sum([x*x for x in range(10**6)]) # Creates full list
58
59 # Ternary operator
60 x = a if condition else b
61
62 # Dictionary get with default
63 count = d.get(key, 0) + 1
64
65 # Matrix rotation 90 degrees clockwise
66 def rotate_90(matrix):
67     return [list(row) for row in zip(*matrix[::-1])]
68
69 # Matrix transpose
70 def transpose(matrix):
71     return [list(row) for row in zip(*matrix)]

```

## 16.5 Time Complexity Reference

Common time complexities (Python, rough guides for 1–2s limits):

- $O(1)$ ,  $O(\log n)$ : instant
- $O(n)$ : usually fine up to  $\sim 10^7$  operations ( $\sim 1\text{s}$ )
- $O(n \log n)$ : OK for  $n$  up to several  $10^5$  depending on constants
- $O(n\sqrt{n})$ : risky in Python (may be OK for  $n$  up to a few  $10^4$  with low constants)
- $O(n^2)$ : often TLE for  $n > 10^4$
- $O(2^n)$ : TLE for  $n > 20$  (unless heavy pruning/memoization)
- $O(n!)$ : TLE for  $n > 11$

Input size guidelines (Python-focused):

- $n \leq 12$ :  $O(n!)$  (brute-force permutations)
- $n \leq 20$ :  $O(2^n)$  (subset DP / bitmask DP)
- $n \leq 500$ :  $O(n^3)$  may sometimes pass for small constants
- $n \leq 5000$ :  $O(n^2)$  borderline; optimize heavily
- $n \leq 10^6$ :  $O(n \log n)$  common;  $O(n)$  preferred when possible
- $n \leq 10^7$ :  $O(n)$  may be OK for tight loops
- $n > 10^7$ : aim for  $O(n)$  with very low constants, or  $O(\log n)/O(1)$

### Complexity examples (Python implementations)

- $O(1)$ : array access, dictionary lookup, push/pop from list end.
- $O(\log n)$ : binary search (bisect), heap push/pop (heappq), operations in sortedcontainers.
- $O(n)$ : single-pass scans, two-pointers, prefix sums, counting frequencies (Counter).
- $O(n \log n)$ : sorting (Timsort via sorted()/list.sort()), heap construction, divide-and-conquer merges.
- $O(n\sqrt{n})$ : sqrt-decomposition queries, some Mo's algorithm vari-



```

        inside = not inside

    p1x, p1y = p2x, p2y

    return inside

```

```

21     # Reconstruct path
22     path = []
23     while parent[u] is not None:
24         path.append((parent[u], u))
25         u = parent[u]
26     return path[::-1]
27
28     for v in range(n):
29         if v not in visited and residual[u][v] > 0:
30             visited.add(v)
31             parent[v] = u
32             queue.append(v)
33
34     return None
35
36 max_flow_value = 0
37
38 # Find augmenting paths
39 while True:
40     path = bfs_path()
41     if path is None:
42         break
43
44     # Find minimum capacity along path
45     flow = min(residual[u][v] for u, v in path)
46
47     # Update residual graph
48     for u, v in path:
49         residual[u][v] -= flow
50         residual[v][u] += flow
51
52     max_flow_value += flow
53
54 return max_flow_value
55
56 # Example usage
57 # graph[u][v] = capacity
58 graph = defaultdict(lambda: defaultdict(int))
59 graph[0][1] = 10
60 graph[0][2] = 10
61 graph[1][3] = 4
62 graph[1][4] = 8
63 graph[2][4] = 9
64 graph[3][5] = 10
65 graph[4][3] = 6
66 graph[4][5] = 10
67
68 n = 6 # Number of nodes
69 result = max_flow(graph, 0, 5, n)

```

18 Network Flow

## 18.1 Maximum Flow - Edmonds-Karp (BF based Ford-Fulkerson)

**Description:** Find maximum flow from source to sink. Time  $O(VE^2)$ .

```
from collections import deque, defaultdict

def max_flow(graph, source, sink, n):
    # graph[u][v] = capacity from u to v
    # Build residual graph
    residual = defaultdict(lambda: defaultdict(int))
    for u in graph:
        for v in graph[u]:
            residual[u][v] = graph[u][v]

    def bfs_path():
        # Find augmenting path using BFS
        parent = {source: None}
        visited = {source}
        queue = deque([source])

        while queue:
            u = queue.popleft()

            if u == sink:
                return parent

            for v in graph[u]:
                if v not in visited:
                    parent[v] = u
                    visited.add(v)
                    queue.append(v)

    path = bfs_path()
    flow = 0
    while path:
        current = sink
        previous = None
        while current != source:
            previous = current
            current = parent[current]
            if current is None:
                break
        else:
            flow += 1
            current = previous
            while current != source:
                parent[current][previous] -= 1
                parent[previous][current] += 1
                current = parent[current]
    return flow
```

## 18.2 Dinic's Algorithm (Faster)

**Description:** Faster max flow using level graph and blocking flow.  
**Time:**  $O(V^2E)$ .

```
from collections import deque, defaultdict

class Dinic:
    def __init__(self, n):
        self.n = n
        self.graph = defaultdict(lambda: defaultdict(int))

    def add_edge(self, u, v, cap):
        self.graph[u][v] += cap

    def bfs(self, source, sink):
        # Build level graph
        level = [-1] * self.n
        level[source] = 0
        queue = deque([source])

        while queue:
            u = queue.popleft()

            for v in range(self.n):
                if level[v] == -1 and self.graph[u][v] > 0:
                    level[v] = level[u] + 1
                    queue.append(v)

    return level[sink] if level[sink] != -1 else None
```

```

27 def dfs(self, u, sink, pushed, level, start):
28     if u == sink:
29         return pushed
30
31     while start[u] < self.n:
32         v = start[u]
33
34         if (level[v] == level[u] + 1 and
35             self.graph[u][v] > 0):
36
37             flow = self.dfs(v, sink,
38                               min(pushed, self.graph[u][v]),
39                               level, start)
40
41         if flow > 0:
42             self.graph[u][v] -= flow
43             self.graph[v][u] += flow
44             return flow
45
46         start[u] += 1
47
48     return 0
49
50 def max_flow(self, source, sink):
51     flow = 0
52
53     while True:
54         level = self.bfs(source, sink)
55         if level is None:
56             break
57
58         start = [0] * self.n
59
60         while True:
61             pushed = self.dfs(source, sink, float('inf'),
62                                 level, start)
63             if pushed == 0:
64                 break
65             flow += pushed
66
67     return flow

```

### 18.3 Min Cut

**Description:** Find minimum cut after computing max flow.

```

1 def min_cut(graph, source, n, residual):
2     # After running max_flow, residual graph is available
3     # Min cut = set of reachable nodes from source
4     visited = [False] * n
5     queue = deque([source])
6     visited[source] = True
7
8     while queue:
9         u = queue.popleft()
10        for v in range(n):
11            if not visited[v] and residual[u][v] > 0:
12                visited[v] = True
13                queue.append(v)
14
15    # Cut edges
16    cut_edges = []
17    for u in range(n):
18        if visited[u]:
19            for v in range(n):
20                if not visited[v] and graph[u][v] > 0:
21                    cut_edges.append((u, v))
22
23    return cut_edges

```

### 18.4 Bipartite Matching

**Description:** Maximum matching in bipartite graph using flow.

```

1 def max_bipartite_matching(left_size, right_size, edges):
2     # edges = [(left_node, right_node), ...]
3     # Add source (0) and sink (left_size + right_size + 1)
4
5     n = left_size + right_size + 2
6     source = 0
7     sink = n - 1

```

```

9     graph = defaultdict(lambda: defaultdict(int))
10
11     # Source to left nodes
12     for i in range(1, left_size + 1):
13         graph[source][i] = 1
14
15     # Left to right edges
16     for l, r in edges:
17         graph[l + 1][left_size + r + 1] = 1
18
19     # Right nodes to sink
20     for i in range(1, right_size + 1):
21         graph[left_size + i][sink] = 1
22
23     return max_flow(graph, source, sink, n)

```

## 19 Advanced Python Syntax

**Description:** Beyond basic loops - powerful Python constructs for competitive programming.

### 19.1 List Comprehensions and Generators

```

1 # Basic list comprehension
2 squares = [x**2 for x in range(10)]
3 evens = [x for x in range(20) if x % 2 == 0]
4
5 # Nested comprehensions
6 matrix = [[i*j for j in range(5)] for i in range(3)]
7 flattened = [x for row in matrix for x in row]
8
9 # Dictionary comprehensions
10 char_count = {char: text.count(char) for char in set(text)}
11 squares_dict = {x: x**2 for x in range(5)}
12
13 # Set comprehensions
14 unique_lengths = {len(word) for word in words}
15
16 # Generator expressions (memory efficient)
17 sum_squares = sum(x**2 for x in range(1000000))
18 any_even = any(x % 2 == 0 for x in numbers)
19
20 # Conditional expressions in comprehensions
21 processed = [x if x > 0 else 0 for x in numbers]
22 filtered = [x for x in numbers if x > 0 and x < 100]

```

### 19.2 Advanced Iteration Patterns

```

1 # Zip - parallel iteration
2 names = ['Alice', 'Bob', 'Charlie']
3 scores = [85, 92, 78]
4 for name, score in zip(names, scores):
5     print(f"{name}: {score}")
6
7 # Enumerate with custom start
8 for i, item in enumerate(items, 1): # Start from 1
9     print(f"Item {i}: {item}")
10
11 # Zip longest (from itertools)
12 from itertools import zip_longest
13 for a, b in zip_longest(list1, list2, fillvalue=0):
14     # Continues until longest list is exhausted
15
16 # Unpacking with star operator
17 first, *middle, last = [1, 2, 3, 4, 5]
18 # first=1, middle=[2,3,4], last=5
19
20 # Multiple assignment
21 a, b = b, a # Swap variables
22 x, y, z = input().split() # Parse multiple inputs
23
24 # Walking with indices and slicing
25 for i in range(len(arr) - 1):
26     current, next_item = arr[i], arr[i + 1]
27
28 # Sliding window with enumerate
29 for i, val in enumerate(arr[:-k+1]):
30     window = arr[i:i+k] # k-length sliding window

```

## 19.3 Advanced Data Structure Operations

```

1 # Dictionary operations
2 from collections import defaultdict, Counter
3
4 # DefaultDict with different types
5 adj_list = defaultdict(list) # Adjacency list
6 counts = defaultdict(int) # Frequency counter
7 groups = defaultdict(set) # Group sets
8
9 # Counter - frequency counting
10 text = "hello world"
11 freq = Counter(text)
12 print(freq['l']) # 3
13 most_common = freq.most_common(3) # Top 3 frequent
14
15 # Dictionary merging (Python 3.9+)
16 dict1 = {'a': 1, 'b': 2}
17 dict2 = {'c': 3, 'd': 4}
18 merged = dict1 | dict2
19
20 # Dictionary comprehension with conditions
21 filtered_dict = {k: v for k, v in original.items() if v > 0}
22
23 # Nested dictionary access
24 from collections import defaultdict
25 nested = defaultdict(lambda: defaultdict(int))
26 nested[key1][key2] += 1
27
28 # Set operations
29 set1 & set2 # Intersection
30 set1 | set2 # Union
31 set1 - set2 # Difference
32 set1 ^ set2 # Symmetric difference
33
34 # List slicing tricks
35 arr[::-1] # Reverse
36 arr[::2] # Every 2nd element
37 arr[1::2] # Every 2nd starting from index 1
38 arr[-3:] # Last 3 elements
39 arr[:-2] # All except last 2

```

## 19.4 Functional Programming Features

```

1 # Map, filter, reduce
2 numbers = [1, 2, 3, 4, 5]
3
4 # Map - apply function to each element
5 doubled = list(map(lambda x: x * 2, numbers))
6 strings = list(map(str, numbers))
7
8 # Filter - select elements meeting condition
9 evens = list(filter(lambda x: x % 2 == 0, numbers))
10
11 # Reduce - accumulate values (from functools)
12 from functools import reduce
13 product = reduce(lambda x, y: x * y, numbers)
14 max_val = reduce(max, numbers)
15
16 # Lambda functions for sorting
17 points = [(1, 3), (2, 1), (0, 5)]
18 sorted_by_y = sorted(points, key=lambda p: p[1])
19 sorted_by_dist = sorted(points, key=lambda p: p[0]**2 + p[1]**2)
20
21 # Multiple sort keys
22 students = [('Alice', 85, 'A'), ('Bob', 92, 'B'), ('Charlie', 85, 'A')]
23 # Sort by grade (desc), then score (desc), then name (asc)
24 sorted_students = sorted(students,
25                         key=lambda x: (-ord(x[2]), -x[1], x[0]))

```

## 19.5 Advanced String Operations

```

1 # String formatting
2 name, score = "Alice", 95
3 formatted = f"{name} scored {score:>5} points" # Right align
4 binary = f"{42:08b}" # 00101010 (8-digit binary)
5 hex_val = f"{255:02x}" # ff (2-digit hex)
6
7 # String methods for parsing
8 text = " hello,world "

```

```

9 words = text.strip().split(',')
10 joined = ' | '.join(words)
11
12 # String manipulation
13 s = "programming"
14 s.startswith("prog") # True
15 s.endswith("ming") # True
16 s.find("gram") # 3 (index, -1 if not found)
17 s.replace("ram", "ROM") # "progROMMing"
18
19 # Character operations
20 char = 'A'
21 ord(char) - ord('A') # 0 (offset from 'A')
22 chr(ord('a') + 1) # 'b'
23
24 # Regular expressions (when needed)
25 import re
26 pattern = r'\d+' # One or more digits
27 matches = re.findall(pattern, "abc123def456") # ['123', '456']

```

## 19.6 Control Flow and Error Handling

```

1 # Try-except for parsing
2 def safe_int(s):
3     try:
4         return int(s)
5     except ValueError:
6         return 0
7
8 # Else clause with loops
9 for item in items:
10     if condition(item):
11         break
12 else:
13     # Executed if loop completed without break
14     print("No item found")
15
16 # While-else similar pattern
17 while condition:
18     if found:
19         break
20 else:
21     print("Condition became false, not broken")
22
23 # Ternary operator (conditional expression)
24 result = "positive" if x > 0 else "non-positive"
25 max_val = a if a > b else b
26
27 # Chained comparisons
28 if 0 <= x < len(arr): # Bounds check
29     if a < b < c: # Three-way comparison
30
31 # Walrus operator (Python 3.8+) - assignment in expression
32 while (line := input()) != "END":
33     process(line)
34
35 if (n := len(items)) > 10:
36     print(f"Many items: {n}")

```

## 19.7 Advanced Input/Output Patterns

```

1 # Multiple assignment from input
2 a, b, c = map(int, input().split())
3 arr = list(map(int, input().split()))
4
5 # Reading matrix
6 n, m = map(int, input().split())
7 matrix = []
8 for _ in range(n):
9     row = list(map(int, input().split()))
10    matrix.append(row)
11
12 # Or with comprehension
13 matrix = [list(map(int, input().split())) for _ in range(n)]
14
15 # Reading until EOF
16 import sys
17 lines = sys.stdin.read().strip().split('\n')
18
19 # Fast I/O for large inputs
20 import sys

```

```

21 input = sys.stdin.readline
22 print = sys.stdout.write # Only for string output
23
24 # Multiple test cases pattern
25 t = int(input())
26 for _ in range(t):
27     # Process each test case
28     pass
29
30 # Output formatting
31 print(*arr)                # Space-separated
32 print(*arr, sep='\n')        # Newline-separated
33 print(f"{x:.6f}")           # 6 decimal places

```

## 19.8 Type conversions and casting

```

1 # Common conversions
2 s = str(123)                 # '123'
3 i = int('42')                 # 42
4 f = float('3.14')              # 3.14
5 b = bool(0)                   # False
6
7 # Container conversions
8 lst = list((1,2))             # [1,2]
9 tup = tuple([1,2])             # (1,2)
10 st = set([1,2,2])             # {1,2}
11 d = dict([('a',1), ('b',2)])  # {'a':1, 'b':2}
12
13 # Characters and bytes
14 c = chr(65)                  # 'A'
15 o = ord('A')                  # 65
16 by = bytes('abc', 'utf-8')     # b'abc'
17 s2 = by.decode('utf-8')         # 'abc'
18
19 # Number formatting to hex/bin/oct
20 h = hex(255)                  # '0xff'
21 bstr = bin(10)                 # '0b1010'
22 octr = oct(8)                  # '0o10'
23 # Formatting without prefixes
24 h2 = format(255, 'x')          # 'ff'
25 b2 = format(10, 'b')           # '1010'
26
27 # Safe literal evaluation (use instead of eval when parsing
28 # literals)
29 import ast
30 data = ast.literal_eval("[1, 2, {'a':3}]")
31
32 # Common contest patterns
33 # Convert split input to ints
34 arr = list(map(int, input().split()))
35 # Join values back into a string
36 s = ' '.join(map(str, arr))

```

## ASCII conversions (quick reference)

Char	Code	Char	Code
'A'	65	'a'	97
'Z'	90	'z'	122
'0'	48	'9'	57
' '	(space)	'\n'	10
'!'	33	'?'	63

Use `ord(char)` to get the numeric code and `chr(code)` to convert back.

### Key Benefits:

- **Readability:** More concise and expressive code
- **Performance:** Built-in functions are often faster than manual loops
- **Memory:** Generators save memory for large datasets
- **Debugging:** Functional style often easier to debug
- **Contest speed:** Write solutions faster with fewer lines

## 20 Problem procedure and contest workflow

This section summarizes a concise, repeatable procedure to approach each contest problem efficiently.

1. **Quick read (1–2 minutes):** Read statement carefully. Identify input/output formats and required output exactly.
2. **Classify and estimate (1–3 minutes):** Determine problem type (graphs, math, DP, greedy, flow, geometry, data structures). Estimate difficulty and a rough solution approach and complexity.
3. **Decide whether to attempt now**
  - Solve immediately if it looks easy or you already know the pattern.
  - Skip and mark for later if it looks hard or needs long derivation.
4. **Concrete examples and edge cases (2–5 minutes):** Create small examples, corner cases, and verify understanding (empty inputs, single element, max/min values).
5. **Design outline (5–15 minutes):** Write a clear plan, data structures, and algorithm steps. Record complexity and memory. If proof/argument needed, sketch correctness and invariants.
6. **Implement carefully**
  - Start from a short, tested template (fast I/O, common helpers).
  - Implement core logic in readable chunks and name variables clearly.
  - Avoid clever one-liners that are hard to debug under time pressure.
7. **Test locally (2–8 minutes):** Run sample tests, your hand-crafted examples, and stress small random cases if possible. Check edge cases (overflows, boundary indices, integer division, floating precision, modulo behavior).
8. **Optimize only if necessary**
  - If complexity is borderline, identify hotspots and optimize (use faster data structures, avoid repeated work, precompute).
  - Prefer simple, robust fixes (change algorithmic approach if needed).
9. **Verify and submit**
  - Re-check input parsing and exact output formatting.
  - Comment out debug prints and run final tests.
  - Submit when confident; if WA, read feedback and debug quickly (reproduce failing case locally if possible).
10. **Time management and team strategy**
  - Track time per problem; don't spend excessive time on a single problem early.
  - Coordinate with teammates (divide/parallelize work, let one implement while another tests or reads other problems).
  - Keep a visible board of which problems are attempted, solved, or to revisit.

### Quick checklist before submitting

- Correct handling of limits and types (int vs float, long long, big integers).
- Off-by-one and index orientation.
- Exact output formatting (extra spaces/newlines).
- Performance within time/memory limits.
- No remaining debug prints.

### Common patterns and hints

- Try greedy first for constructive problems; if it fails, look for DP/graph framing.

- Reduce to shortest paths / flow for routing and matching problems.
- For counting/number theory, consider modular arithmetic and combinatorics shortcuts.
- Use Union-Find for connectivity under merges; use binary search on answer when monotone.
- When stuck, simplify constraints or solve a special case to gain insight.