

**Syllabus**

- A.E. Eiben & J.E. Smith "Introduction to Evolutionary Computing" (Natural Computing Series) 2nd ed. 2015
- Sean Luke "Essentials of Metaheuristics" <https://cs.gmu.edu/~sean/book/metaheuristics/>
- David E. Goldberg "The Design of Innovation" Springer 2002
- Martin Pelikan "Hierarchical Bayesian optimization algorithm: Toward a new generation of evolutionary algorithms", Springer 2005
- Zbigniew Michalewicz "Genetic Algorithms + Data Structures = Evolution Programs" Springer 1998

What's the problem we are trying to solve?

Given a certain problem and a target function over the domain find the maximum/minimum  
(we're not looking for the prediction)

e.g. in linear regression with Lasso we want to find the best  $\alpha$  parameter, or we want to find the best set of initial centroids which leads to the best clustering results, or we want to find the best subset of original variables which allows to have the highest performance with the least number of variables

If we knew the analytic form, ...

**Algorithm 1** Gradient Ascent

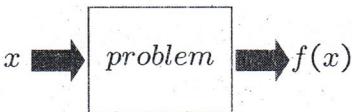
- 1:  $\vec{x} \leftarrow$  random initial vector
- 2: **repeat**
- 3:      $\vec{x} \leftarrow \vec{x} + \alpha \nabla f(\vec{x})$
- 4: **until**  $\vec{x}$  is the ideal solution or we have run out of time
- 5: **return**  $\vec{x}$

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If we do not know the analytic form, ...



we give some inputs to the problem and the problem gives us a result

trial and error: best  $x$ ? we try several  $x$ 's and we take the best based on the performance of the model (black-box procedure)

we try some values at random and we keep the best (w.r.t. performance)

we have a value and we try similar ones to see if they lead to an improvement

#### Basic Black Box Optimization

- Random search
- Hill Climbing
- Tabu Search
- Simulated Annealing
- ...

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#### Genetic Algorithms

##### Basic Idea

Evolve a population (multiset) of candidate solutions using the concepts of survival of the fitness, variation, and inheritance

##### Genetic Algorithm

Generate an initial population

Repeat

Select promising solutions from the population

Create new solutions by applying variation

Incorporate new solutions into original population

Until stop criterion met

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#### A Simple Genetic Algorithm for Binary Strings

```
Genetic algorithm(n,N,f,pm,pc)
P = generate(n,N);
while (!done())
    fv = evaluate(P,f,n,N);
    S = selection(P,fv,n,N);
    O = variation(S,n,N,pm,pc);
    P = replacement(O,P,n,N);
```

##### Parameters

- n The number of bits  
N The population size  
f The objective function  
pm Probability of mutation  
pc Probability of crossover
- (number of variables on which the solutions depend)

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#### Common Terminology in Genetic Algorithms

- |                           |                             |
|---------------------------|-----------------------------|
| • Solution String         | • Individual, chromosome    |
| • String position         | • Locus                     |
| • Bit, feature value      | • Allele                    |
| • Objective Function      | • Fitness function, fitness |
| • Selected solutions      | • Parents                   |
| • New candidate solutions | • Offspring                 |
| • Iteration               | • Generation                |
| • Structure               | • Genotype                  |
| • Decoded structure       | • Phenotype                 |
| • Nonlinearity            | • Epistasis                 |

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Operators	
Initialization	Randomly generates the initial population of strings.
Evaluation	Evaluates the population of strings using the given fitness function.
Selection	Selects promising solutions from the current population by making more copies of better solutions at the expense of the worse ones.
Variation	Processes selected solutions to generate new candidate solutions that share similarities with selected solutions but are novel in some way.
Replacement	Incorporates new candidate solutions into the original population.

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Generating the Initial Population	
<pre>generate(n,N) P = new population of size N; for i=1 to N     P[i] = generate_random(n); return P;</pre>	$n = \# \text{ parameters (variables)}$ $N = \text{population size}$

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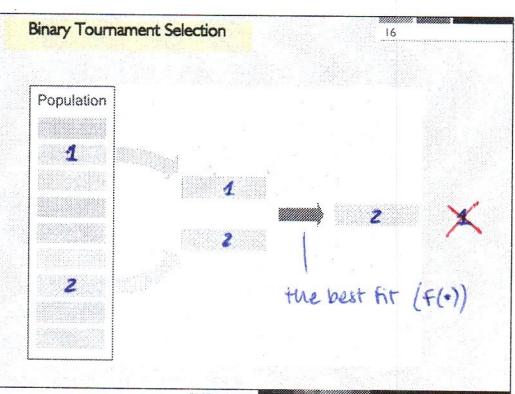
Evaluation	
<pre>evaluate(P,f,n,N) fv = new array of N real numbers; for i=1 to N     fv[i]=f(P[i],n); return fv;</pre>	$f = \text{fitness function}$

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How do we select the best ones?

Tournament selection	
Parameters	Tournament size k
Procedure	To select each candidate solution, <u>select a random subset of k solutions from the original population and then select the best solution out of this subset.</u>
Comments	<ul style="list-style-type: none"> <li>Tournament selection is among the most popular selection methods in genetic algorithms.</li> <li>Binary tournament selection (<math>k = 2</math>) is probably most popular.</li> </ul>

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## Binary Tournament Selection

```
binary_tournament_selection(P,fv,n,N)
S = new population of size N;
for i=1 to n
    a=rand(1,n);
    b=rand(1,n-i);
    if (b>=a) b++;
    if (fv[a]>fv[b])
        S[i]=P[a];
    else
        S[i]=P[b];
return S;
```

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## Variation

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### Purpose

- Process selected promising solutions.
- Create new solutions that share features with selected solutions but are new in some way.
- Two basic principles:
  - variation (introducing novelty), and
  - inheritance (reusing the old).

### Variation in genetic algorithms

#### Two components

- Crossover: Combines bits and pieces of promising solutions.
- Mutation: Makes small perturbations to promising solutions.

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## Variation: Basic Procedure

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```
variation(S,n,N,pm,pc)
O = new population of size N;
shuffle(S,n,N);
for i=1 to N with step 2
    O[i] = mutation(S[i],n,pm);
    O[i+1] = mutation(S[i+1],n,pm);
    if (rand01()<pc)
        crossover(O[i],O[i+1],n);
return O;
```

### Comments

- The call `shuffle(S,n,N)` randomly reorders strings in the population P

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## One-point Crossover

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### Basic idea

- Randomly select one string position called crossing point.
- Exchange all bits after this position.



### Pseudocode

```
onepoint_crossover(x,y,n)
    cp=random(2,n);
    for i=cp to n
        exchange(x[i],y[i]);
```

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## Two-point Crossover

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### Basic idea

- Exchange all bits between two randomly chosen points



### Pseudocode

```
twopoint_crossover(x,y,n)
    cp1=random(1,n);
    cp2=random(1,n);
    if (cp1>cp2)
        exchange(cp1,cp2);
    for i=cp1 to cp2
        exchange(x[i],y[i]);
```

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## Uniform Crossover

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- Basic idea  
• Exchange every bit with probability 0.5



### Pseudocode

```
uniform_crossover(x,y,n)
for i=1 to n
    if (rand01()<0.5)
        exchange(x[i],y[i]);
```

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## Bit-Flip Mutation

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- Basic idea  
• Flip every bit with a specified probability  
• Usually one or only few bits should be mutated  
• Typically  $pm = 1/n$



### Pseudocode

```
mutation(x,n)
y=new binary string of n bits;
for i=1 to n
    if (rand01()<pm)
        y[i]=1-x[i];
    else
        y[i]=x[i];
```

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## Replace All

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- Basic idea  
• Assume that the offspring population is of the same size as the original population.  
• Replace the entire original population with offspring.

### Pseudocode

```
replace_all(O,P,n,N)
new_P = new population of N strings;
for i=1 to N
    new_P[i]=O[i];
return new_P;
```

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## Replace Worst (Elitism)

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- Basic idea  
• Assume that the offspring population is smaller than the original population.  
• Replace worst strings in the original population by new offspring.

### Elitism

Elitist genetic algorithms preserve best solutions found so far.  
This is one of elitist schemes.

Real-Coded Genetic Algorithms

- Candidate solutions are vectors of real values.
- Random initialization generates a random number from an interval for each variable.
- Operators must deal with real-valued parameters.
- There are many ways to do this.
- We only look at some simple methods.

- Two real-valued parents
  - $x = (x_1, \dots, x_n)$
  - $y = (y_1, \dots, y_n)$
- Choose random variable  $i \in \{1, 2, \dots, n\}$ .
- Randomly generate  $\alpha \in [0, 1]$ .
- The first child is
  - $o_1 = (x_1, \dots, x_{i-1}, \alpha y_i + (1-\alpha)x_i, x_{i+1}, \dots, x_n)$
- The second child is
  - $o_2 = (y_1, \dots, y_{i-1}, \alpha x_i + (1-\alpha)y_i, y_{i+1}, \dots, y_n)$

numerical  
combinations  
between the  
parents

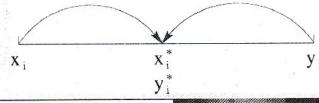
- Example 1

$$\alpha=0.25$$



- Example 2

$$\alpha=0.5$$



- Simple mutation

- Change each variable with a fixed probability.
- To change a variable, start by generating a random number from  $[-\delta, \delta]$  where  $\delta > 0$  is a small number.
- Add the generated number to the variable.

- Gaussian mutation

- The change is generated according to the Gaussian distribution  $N(0, \sigma^2)$  where  $\sigma^2$  is the variance of the mutation steps, which is a small number.

- Another idea

- Choose a random variable for each solution and mutate that one.