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Bayesian Networks for Reliability and Risk Analysis

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Probability

Definition: Probability P is a function that maps all events A onto real numbers and satisfies the following three axioms:

1. If S is the set of all possible outcomes, then $P(S) = 1$
2. $0 \leq P(A) \leq 1$
3. If A and B are mutually exclusive ($A \cap B = \emptyset$) then
$$P(A \cup B) = P(A) + P(B)$$

From the three axioms it follows that:

I. $P(\emptyset) = 0$

II. If $A \subset B$, then $P(A) \leq P(B)$

III. $P(\bar{A}) = 1 - P(A)$

IV. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional probability

Definition of independence: Two events A and B are independent if

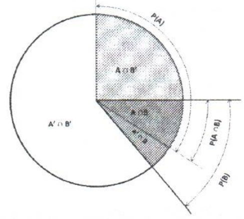
$$P(A \cap B) = P(A)P(B)$$

Conditional probability $P(A|B)$ of A given that B has occurred is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Note: If A and B are independent, the probability of A (B) does not depend on whether B (A) has occurred or not:

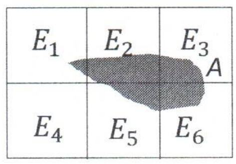
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,P(B)}{P(B)} = P(A)$$



Law of total probability

If $E_1, \ldots, E_n$ are mutually exclusive and collectively exhaustive events, then

$$P(A) = P(A|E_1)P(E_1) + \cdots + P(A|E_n)P(E_n)$$



Most frequent use of this law:

– Events A and B are mutually exclusive and collectively exhaustive

– Probabilities $P(A|B)$, $P(A|\bar{B})$, and $P(B)$ are known

– These can be used to compute

$$P(A) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B})$$

Bayes' rule

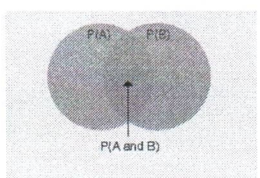
Bayes' rule: $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

It follows from:

- Definition of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)};\ P(B|A) = \frac{P(B \cap A)}{P(A)}.$$

- Commutative laws: $P(B \cap A) = P(A \cap B)$.



Bayes' rule

Example:

- The probability of a fire in a certain building is 1/10000 any given day.
- An alarm is activated whenever there is an actual fire, but also once in every 200 days for no reason (false alarm).
- Suppose the alarm is activated. What is the probability that there is a fire?

Solution:

$F = Fire,\ \bar{F} = No\ fire,\ A = Alarm,\ \bar{A} = No\ alarm$

$P(F) = 0.0001,\ P(\bar{F}) = 0.9999,\ P(A|F) = 1,\ P(A|\bar{F}) = 0.005$

Bayes: $P(F|A)\ = \frac{P(A|F)P(F)}{P(A)}\ = \frac{1 \cdot 0.0001}{0.0051} \approx\ 2\%$

Law of total probability: $P(A) = P(A|F)P(F) + P(A|\bar{F})\ P(\bar{F}) = 0.0051$

Exercise

A test for diagnosing a particular degradation mechanism is known to be 95% accurate.

The test is performed on a component and the result is positive.

Suppose the component comes from a fleet of 100'000, where 2000 suffer from this degradation.

What is the probability that the component is affected by the considered degradation mechanism?

D= degradated , P= positive

$\mathbb{P}(D) = 0.02 \qquad \mathbb{P}(P|D) = 0.95\ , \qquad \mathbb{P}(\bar{P}|D) = 0.05$

$\mathbb{P}(\bar{D}) = 0.98 \qquad \mathbb{P}(\bar{P}|\bar{D}) = 0.95\ , \qquad \mathbb{P}(P|\bar{D}) = 0.05$

$$\mathbb{P}(D|P) = \frac{\mathbb{P}(P|D)\ \mathbb{P}(D)}{\mathbb{P}(P)} = \frac{\mathbb{P}(P|D)\mathbb{P}(D)}{\mathbb{P}(P|D)\mathbb{P}(D) + \mathbb{P}(P|\bar{D})\ \mathbb{P}(\bar{D})} = \frac{(0.95)(0.02)}{(0.95)(0.02) + (0.05)(0.98)} = \frac{0.019}{0.068}$$

$= 27,94\%$

- Basics
- Bayesian Networks: key concepts
- Bayesian Networks for Reliability and Risk Analysis
- Applications
- Enhancements
- Conclusions

Bayesian Networks: what?

Bayesian Network (BN): is a directed acyclic graph consisting of:

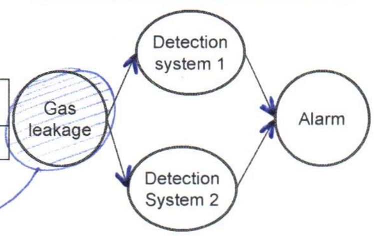
- **Nodes** $V = \{1, \dots, N\}$, shown as circles, represent the random events whose combination can lead to system failure.
- **Directed arcs** $E \subseteq \{(i,j) | i,j \in V, i \neq j\}$ indicate conditional dependencies among nodes. Specifically, the arc $(i,j) \in E$ which connects node $j \in V$ to node $i \in V$ shows that the event at node j is conditionally dependent to the event at node i.

Bayesian Network are also called **Bayesian Belief Networks (BBNs)**

Bayesian Network Example

Leakage	DS1: Activated	DS1: Not Activated
Yes	0.95	0.05
No	0.05	0.95

Leakage yes	Leakage no
0.01	0.99



Detection System 1	Detection System 2	Alarm: yes	Alarm: no
Activated	Activated	0.99	0.01
Activated	Not Activated	0.95	0.05
Not Activated	Activated	0.95	0.05
Not Activated	Not Activated	0.01	0.99

LEAF NODE : (independent) we start from here

Leakage	DS2: Activated	DS2: Not Activated
Yes	0.95	0.05
No	0.05	0.95

Acyclic… what?

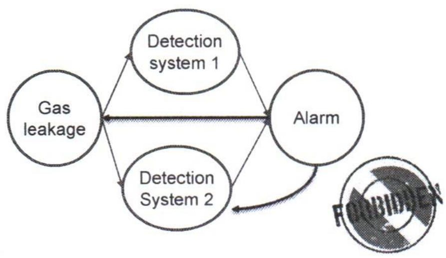
A **path** is a sequence of nodes $(i_1, i_2, \dots, i_\eta)$, $\eta > 1$ such that

$$(i_j, i_{j+1}) \in E;\ j < \eta$$

BBN **acyclic** → there is no path $(i_1, i_2, \dots, i_\eta)$, $\eta > 1$ such that

$$(i_j, i_{j+1}) \in E, j < \eta \text{ and } i_1 = i_\eta$$

we are **not** allowed to link in a cyclic way two nodes



Bayesian Networks, why?

Bayesian networks are **probabilistic graphical models**, which offer a convenient and efficient way of generating joint distribution of all its events.

- **Convenient**: causal relationships between events are easy to model.
- **Efficient**: no redundancies in terms of graphical modelling and probability computations.
- **Flexible**: capable of handling imprecise information by capturing quantitative and qualitative data.

"Microsoft's competitive advantage lies in its expertise in Bayesian Networks"
-- Bill Gates, quoted in LA Times, 1996

Solving Bayesian Networks

We define:

- **follower** nodes of $i \in V$: $V_+^i = \{j | (i,j) \in E\}$
- **predecessor** nodes (parent) of $i \in V$: $V_-^i = \{j | (j,i) \in E\}$

All nodes can be partitioned into

- **Leaf nodes** $V^L = \{i \in V | V_-^i = \emptyset\}$
- **Dependent nodes** $V^D = V \setminus V^L = \{i \in V | V_-^i \neq \emptyset\}$

The **depth** of node $i \in V$ in the network can be calculated recursively by

$$d^i = \begin{cases} 0 & V_-^i = \emptyset \\ 1 + \max_{j \in V_-^i} d^j & V_-^i \neq \emptyset \end{cases}$$

in the previous example:
depth(alarm) = 2, depth(detection) = 1, depth(leakage) = 0

Solving Bayesian Networks

$\boldsymbol{X}^i$ = random variable representing the uncertainty in the state of event at node $i \in V$.

The realization s of $\boldsymbol{X}^i$ belongs to the set of states $S^i = \{0, \ldots, |S^i|\}$

In the case of gas leakage we have 2 state: true/false. If the states are more than 2 and we cannot represent a variable as boolean, the resulting bayesian network is called: MULTI-STATE BAYESIAN NETWORK

$\boldsymbol{X} = [\boldsymbol{X}^1, \ldots, \boldsymbol{X}^N]$= BN state vector (where N is the number of node)

probability that each node is found in a specific state

$$P(\boldsymbol{X}) = \prod_{i=1}^{N} P(\boldsymbol{X}^i | \boldsymbol{X}^j, j \in V_-^i)$$

The BN can be solved by propagating the uncertainty from leaf nodes V^L to nodes with the largest depth

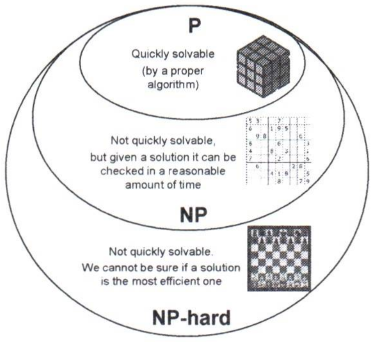
Computational issues

Theorem: Computing event probabilities in a Bayesian network is **NP-hard**.

problems that cannot be solved by a machine in a polynomial time
⟹ very difficult problems (computationally expensive)

NP-hard: complexity class of problems which cannot be solved by a **N**ondeterministic (ideal) machine in **P**olynomial time (i.e., necessary number of steps upper bounded by a polynomial function of the number of inputs)

That means that there is no general way to solve a NP-hard problem!



Problem complexity classification

Computational issues

Theorem: Computing event probabilities in a Bayesian network is **NP-hard**

Hardness does not mean it is impossible to perform inference, but:

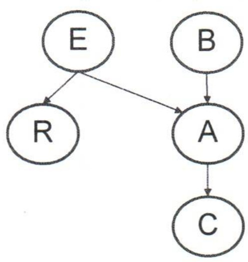
- There is no general procedure that works efficiently for all networks
- For particular families of networks, there are proved efficient procedures
- Different algorithms are developed for inferences in Bayesian networks

There are available software that efficiently perform Bayesian Network inference through a library of functions for several popular algorithms, among those:

- **GeNle Modeler**: https://www.bayesfusion.com/genie-modeler
- **HUGIN Expert**: https://www.hugin.com/

Computational issues

Factored representation may have exponentially fewer parameters than full joint $P(\boldsymbol{X})$



probability of finding each one of these events in a particular state →

$$P(E,B,R,A,C)$$
$$= P(E)P(B|E)P(R|B,E)P(A|R,B,E)P(C|A,R,B,E)$$
$$= P(E)P(B)P(R|E)P(A|B,E)P(C|A)$$

If $|S^i|=2$ for every i, the number of parameters reduces from $2^5 - 1 = 31$ to $1+1+2+4+2=10$

Computational issues

Factored representation may have exponentially fewer parameters than full joint $P(\boldsymbol{X})$

A real case study: Monitoring Intensive-Care Patients

- 37 variables
- 509 parameters, instead of 2^{37}

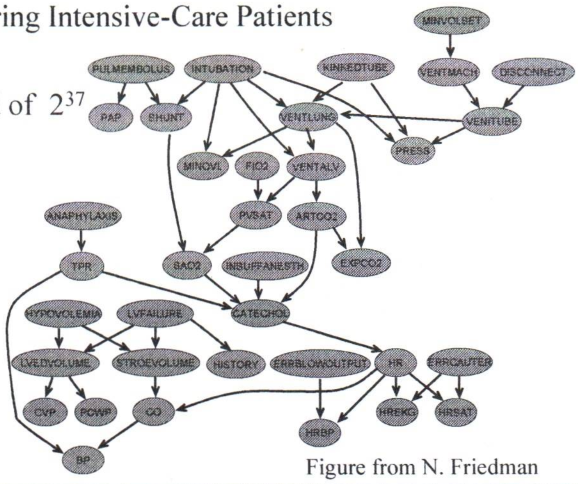


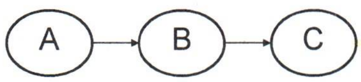
Figure from N. Friedman

Inference

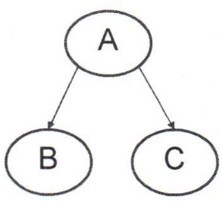
Assume to collect some observation (**evidence**) from the system. In this case the state is said "instantiated" (since we've providing some particular states)

How would this evidence impact the probabilities of the events?

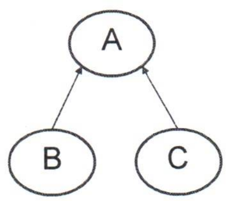
Consider the following three schemes:



Serial connection



Diverging connection



Converging connection

Serial Connection (1.)

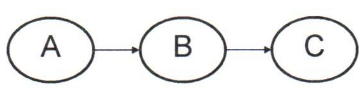
- Evidence on A influences successor nodes
- Evidence on C influences predecessor nodes
- Evidence on B makes A and C independent (!)

Example

A: Failure Event (yes/no)

B: Alarm Activation (yes/no)

C: Plant Evacuation (yes/no)



If we have evidence of a Failure Event→ we change the probability of Alarm Activation and, then, the probability of having a Plant Evacuation.

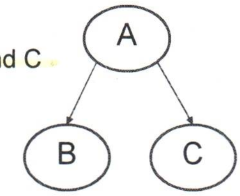
If we have evidence of Plant Evacuation → we change the probability of Alarm Activation and, then, the probability of having a failure event.

If we know that the alarm is activated (or not), we cut the communication between Failure Event and Plant Evacuation

! If we know that the alarm is activated we recalculate the probability of evacuation, however we're not capable of linking the evacuation with the fact that something has occurred in the plant

Diverging Connection (2.)

- Evidence on A influences nodes B and C
- Evidence on C influences A and, then, B
- Evidence on A cuts the communication between B and C



Diverging connection

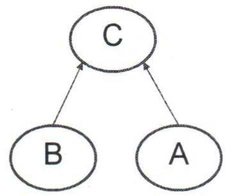
Example
A: Failure Event (yes/no)
B: Detection System 1 (activated/not activated)
C: Detection System 2 (activated/not activated)

If we have evidence of the activation of Detection System 1 → we change the probability of failure event and, then, the probability of activation of Detection System 2.

If we know that the failure event is occurrred (or not), we know the probability of activation of Detection System 1 and Detection System 2, which do not influence each other

Converging Connection (3.)

- Evidence on C influences nodes B and A
- Evidence on B influences node C,only
- Evidence on C and B, influences node A differently from influence of node C only



Example
A: Fuel level in a car (empty/full)
B: Spark plugs (working/failed)
C: Start (Yes/No)

If we have evidence that the car cannot start→ we change the probability that the fuel level is empty and sparks are failed.

If we have evidence that the car cannot start and that the fuel is full→ we change the probability that sparks are failed

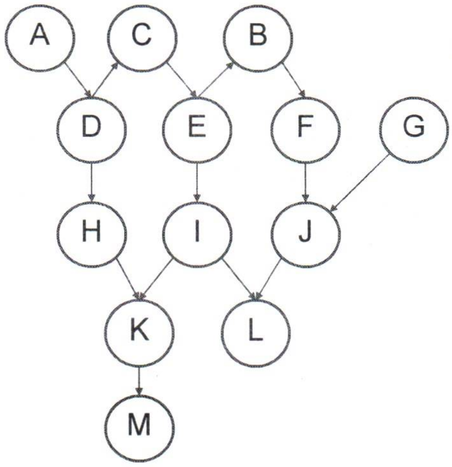
d-separation

In a Bayesian Network, two nodes A and B are d(irectional)-separated if for all indirected paths between A and B there is a node C such that at least one of the following conditions holds:

- The connection A-B is serial or diverging and C is instantiated
- The connection A-B contains converging structure and neither C nor any of C's successors are instantiated

If events A and B are d-separeted, evidence on A does not influence B

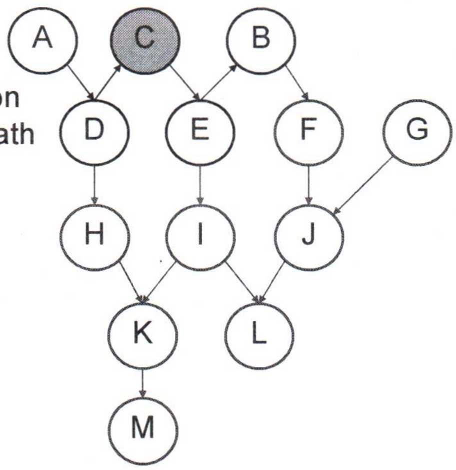
Example



Are A and B d-separated?

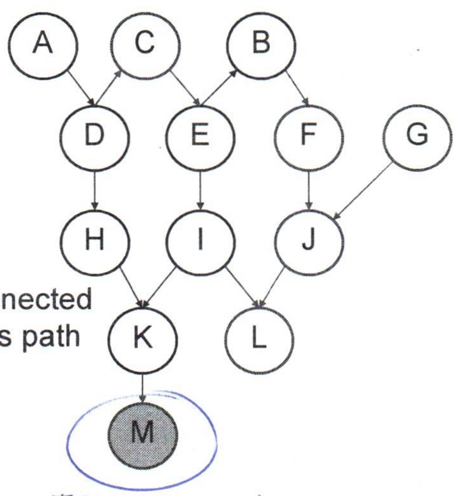
Example

Yes, on this path



Are A and B d-separated?

Example



d-connected by this path

Are A and B d-separated?

This evidence will influence both **A** and **B**

= calculating the probability tables

Inference on Bayesian Networks

Question: assume to collect some observations (**evidence**) from the system; how would this evidence impact the probabilities of the events?

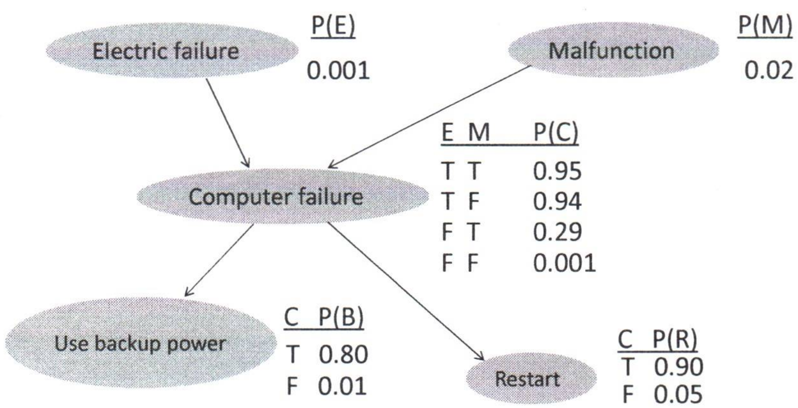
The conditional probability of a random event given the evidence is known as **a posteriori belief**, useful in case of:

- **Prediction**: computing the probability of an outcome event given the starting condition → Target is a descendent of the evidence!
- **Diagnosis**: computing the probability of disease/fault given symptoms → Target is an ancestor of the evidence!

Note: probabilistic inference can propagate and combine evidences from all parts of the network (the directions of arcs do not limit the directions of the queries)

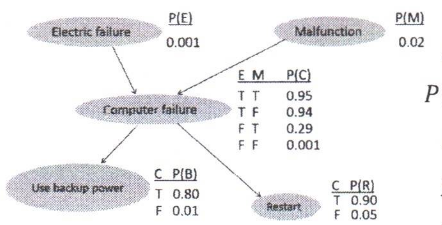
The conditional probability tables are defined as probability tables conditioned on the predecessor events. If we collect an evidence, however, also the probabilities of the generating events will be changed.

Computer example



Examples of inference

What it is the probability that the backup power is working given an electrical failure?



$$P(B|E) = P(B|C)P(C|E) + P(B|\bar{C})P(\bar{C}|E)$$

$$P(C|E) = P(C|\bar{M},E)P(\bar{M}) + P(C|M,E)P(M)$$

$$P(C|E) = 0.98 \cdot 0.94 + 0.02 \cdot 0.95 = 0.94$$

$$P(B|E) = P(C|E) \cdot 0.8 + P(\bar{C}|E) \cdot 0.01$$

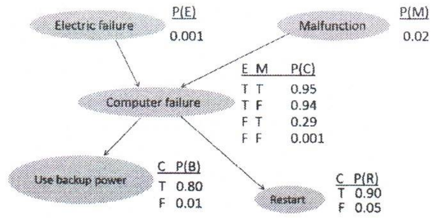
$$P(B|E) = 0.94 \cdot 0.8 + 0.06 \cdot 0.01 = 0.75$$

This is a case of PREDICTION: given a cause and an evidence we compute the probability of an outcome

Exercise on inference

What it is the probability that the electricity is working given a backup power failure?

This is a case of **DIAGNOSIS**: given a failure we want to calculate the probability of the functionality of a leaf component



$$\mathbb{P}(E|B) = \frac{\mathbb{P}(B|E)\,\mathbb{P}(E)}{\mathbb{P}(B)} = \frac{\mathbb{P}(B|E)\,\mathbb{P}(E)}{\mathbb{P}(B|C)\mathbb{P}(C) + \mathbb{P}(B|\bar{C})\mathbb{P}(\bar{C})}$$

$$\begin{aligned}\mathbb{P}(C) = {} & \mathbb{P}(C|E,M)\mathbb{P}(E)\mathbb{P}(M) + && (0.95)(0.001)(0.02)\\ & + \mathbb{P}(C|E,\bar{M})\mathbb{P}(E)\mathbb{P}(\bar{M}) + && (0.94)(0.001)(0.98)\\ & + \mathbb{P}(C|\bar{E},M)\mathbb{P}(\bar{E})\mathbb{P}(M) + && (0.29)(0.999)(0.02)\\ & + \mathbb{P}(C|\bar{E},\bar{M})\mathbb{P}(\bar{E})\mathbb{P}(\bar{M}) && (0.001)(0.999)(0.98)\\ = {} & 0.0077\end{aligned}$$

$$\Rightarrow \mathbb{P}(E|B) = \frac{(0.75)(0.001)}{(0.80)(0.0077) + (0.01)(1-0.0077)} = \frac{0.00075}{0.016} = 0.047$$

- Basics
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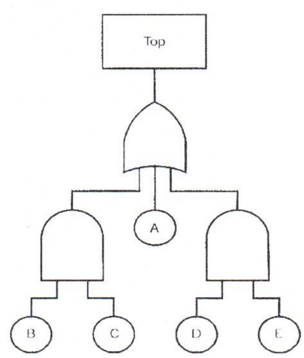
BNs for Risk and Reliability Analysis

Scenario modeling and quantification are pursued through:

FAULT TREE ANALYSIS (FTA)

⇩

1. Events are binary events (operating/not-operating);
2. Events are statistically independent;
3. Relationships between events and causes are represented by logical gates (e.g., AND and OR for coherent FT);
4. The undesirable event, called Top Event, is postulated and the possible ways for the occurrence of this event are systematically deduced.



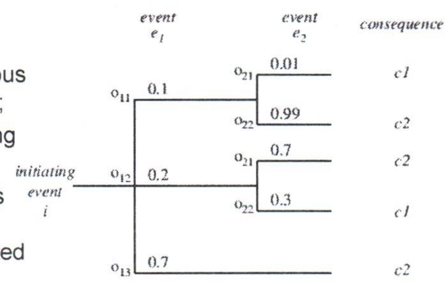
BNs for Risk and Reliability Analysis

Scenario modeling and quantification are pursued through:

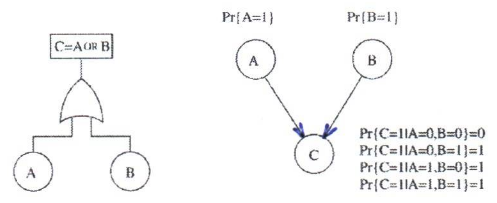
EVENT TREE ANALYSIS (ETA)

⇩

1. System evolution following the hazardous occurrence is divided into discrete events;
2. System evolution starts from an initiating event;
3. Each event has a finite set of outcomes (commonly there are two outcomes: occurring event or not occurring) associated with the occurrence probabilities;
4. The leafs of the event tree represent the event scenarios to be analyzed.



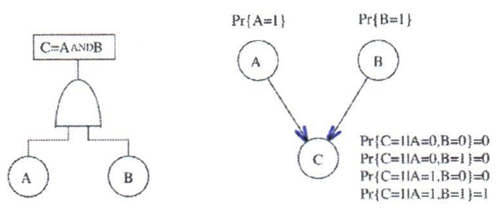
Mapping FT into Bayesian Network



FAULT - TREE: OR Gate

BAYESIAN NETWORK: OR Node

Switching from binary logic to probabilistic logic!



FAULT - TREE: AND Gate

BAYESIAN NETWORK: AND Node

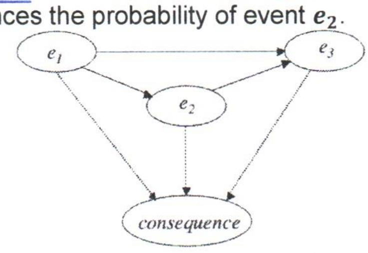
***Reference**: Bobbio A., Portinale L., Minichino M., Ciancamerla E., "Improving the analysis of dependable systems by mapping fault trees into Bayesian networks", Reliability Engineering and System Safety 71, pp. 249–260 (2001).*

Mapping ET into Bayesian Network

Any event tree with three events e_1, e_2, and e_3 can be represented by the BN shown below.

Two types of directed arc complete the network:

- Consequence arcs (shown as dotted lines) connect each event node to the consequence node. This relationship is deterministic: the probability table for the consequence node encodes the logical relationship between the events and the consequences.
- Causal arcs (shown as solid lines) connect each event node to all events later in time. For instance, event e_1 is a causal factor for event e_2, thus it influences the probability of event e_2.



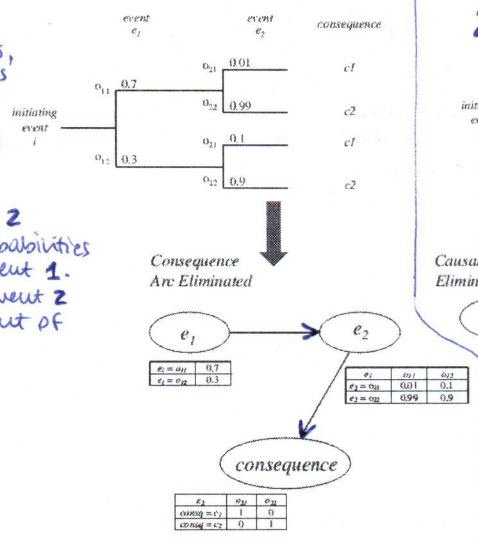
***Reference**: Bearfield G., Marsh W., "Generalizing Event Trees Using Bayesian Networks with a Case Study of Train Derailment", Computer Safety, Reliability, and Security (2005).*

Mapping ET into Bayesian Network (alternatives)

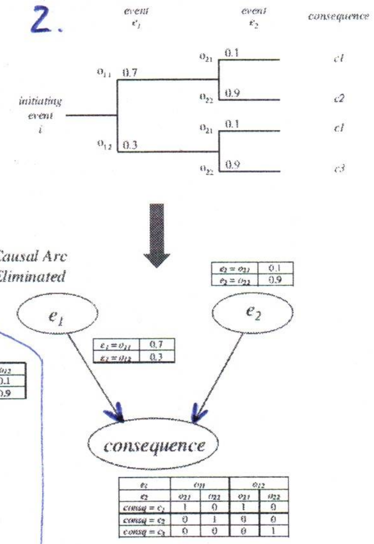
1.

Two events, the combinations may lead only to two consequences (c1, c2)

Here the event 2 changes its probabilities based on the event 1. This leads to event 2 being dependent of event 1.



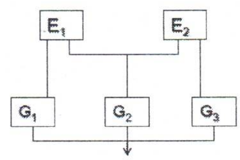
2.



Here event 2 doesn't change based on event 1, we can conclude that the two events are ⫫

Reliability Modeling through BN: example

Electrical generating system

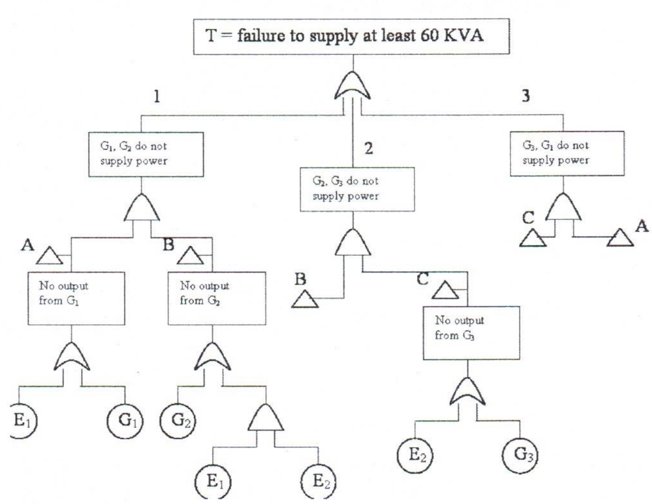


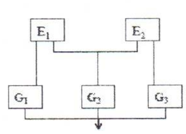
E1, E2 = engines

G1, G2, G3 = generators, each one is rated at 30 KVA

The generation system fails when the delivered power is under 60KVA

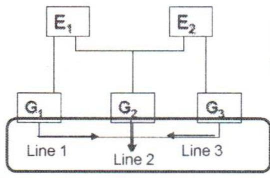
Fault Tree





Reliability Modeling through BN: example

Electrical generating system



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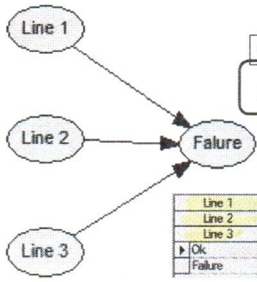
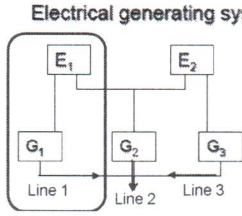


Line 1	PowerOk				PowerKo			
Line 2	PowerOk		PowerKo		PowerOk		PowerKo	
Line 3	PowerOk	PowerKo	PowerOk	PowerKo	PowerOk	PowerKo	PowerOk	PowerKo
Ok	1	1	1	0	1	0	0	0
Failure	0	0	0	1	0	1	1	1

Reliability Modeling through BN: example

Electrical generating system



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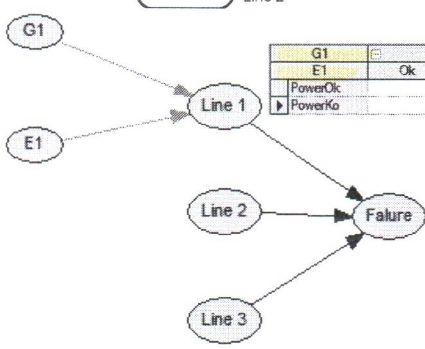
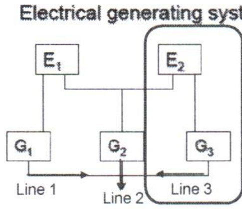


G1	Ok		Failed	
E1	Ok	Failed	Ok	Failed
PowerOk	1	0	0	0
PowerKo	0	1	1	1

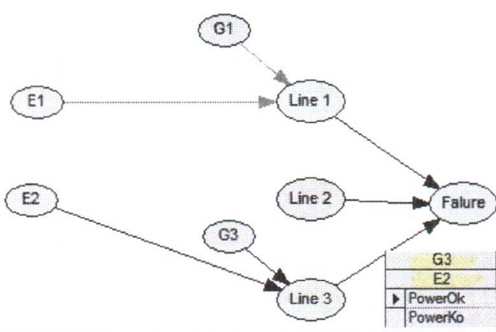
Reliability Modeling through BN: example

Electrical generating system



E1, E2 = engines

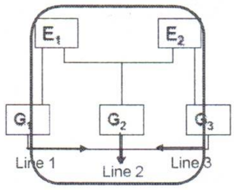
G1, G2, G3 = generators, each one is rated at 30 KVA



G3	Ok		Failed	
E2	Ok	Failed	Ok	Failed
PowerOk	1	0	0	0
PowerKo	0	1	1	1

Reliability Modeling through BN: example

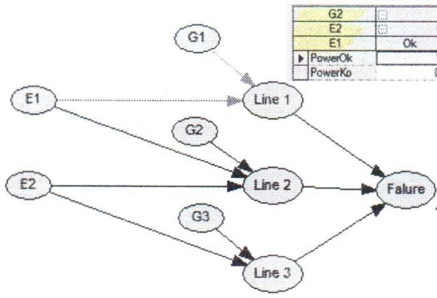
Electrical generating system



E1, E2 = engines

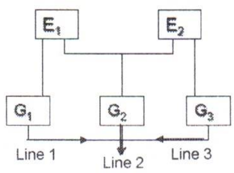
G1, G2, G3 = generators, each one is rated at 30 KVA

G2	Ok				Failed			
E2	Ok		Failed		Ok		Failed	
E1	Ok	Failed	Ok	Failed	Ok	Failed	Ok	Failed
PowerOk	1	1	1	0	0	0	0	0
PowerKo	0	0	0	1	1	1	1	1



Reliability Modeling through BN: example

Electrical generating system



E1, E2 = engines

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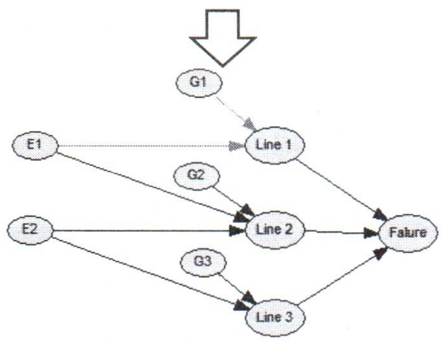
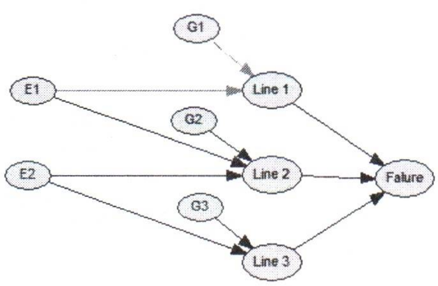


Advantages of BN model

- Multi-state modeling

A great advantage is that Bayesian Networks are not constrained to the boolean logic!

G1	Ok			Degraded			Failed		
E1	Ok	Degraded	Failed	Ok	Degraded	Failed	Ok	Degraded	Failed
PowerOk	1	0	0	0	0	0	0	0	0
PowerDegraded	0	1	0	1	1	0	0	0	0
PowerKo	0	0	1	0	0	1	1	1	1



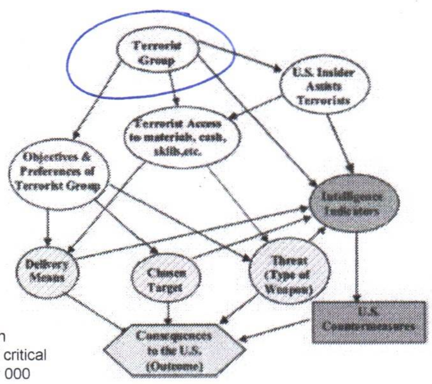
BN applications

- Medical diagnosis
- Genetic pedigree analysis
- Speech recognition (HMMs)
- Gene sequence/expression analysis
- Microsoft Answer Wizards, (printer) troubleshooters 
- …

BNs applications to Risk and Reliability Engineering

BNs have been applied to different contexts in Reliability and Risk Engineering

- Risk & Vulnerability analysis



A. Misuri, N. Khakzad, G. Reniers, V. Cozzani, A Bayesian network methodology for optimal security management of critical infrastructures, Reliability Engineering and System Safety 000 (2018) 1–14

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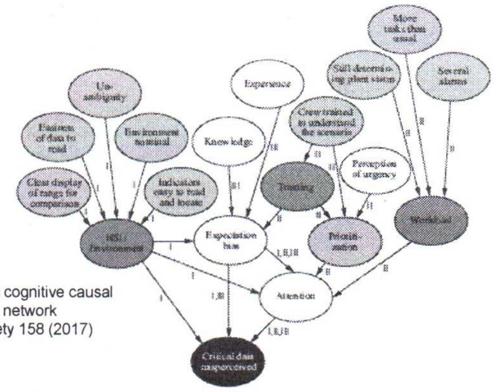
- Human Reliability Analysis



K. Zwirglmaier, D. Straub, K.M. Groth, Capturing cognitive causal paths in human reliability analysis with Bayesian network models, Reliability Engineering and System Safety 158 (2017) 117–129

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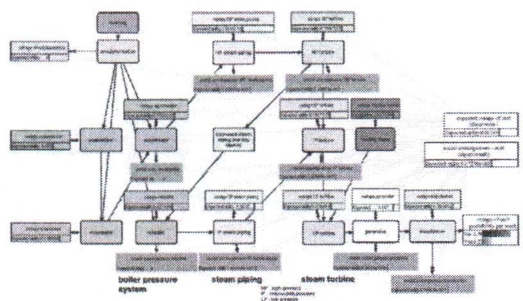
- Human Reliability Analysis
- Risk assessment of complex systems considering multiple objectives (e.g., availability, safety, etc.)



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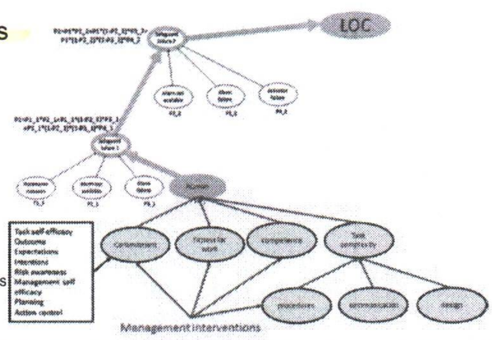
- Risk modeling of process plants



B. Ale, C. van Gulijk, A. Hanea, D. Hanea, P. Hudson, P.-H. Lin, S. Sillem, Towards BBN based risk modelling of process plants, Safety Science, 69, pp. 48-56, 2014.

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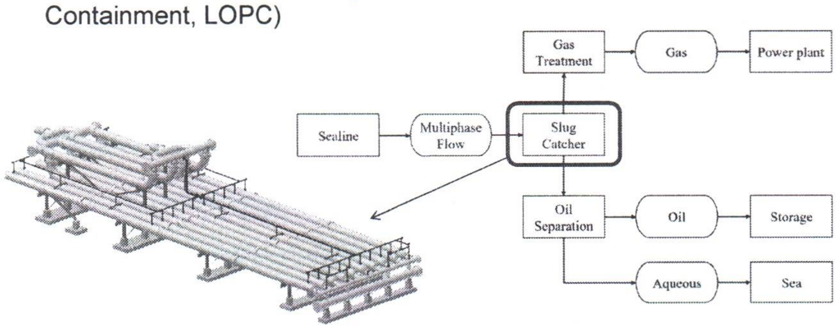
- Risk modeling of process plants
- ...

BNs application for Risk Assessment in Oil & Gas Industry
Slug catcher Risk Assessment

Unit considered: slug catcher of an oil & gas onshore plant

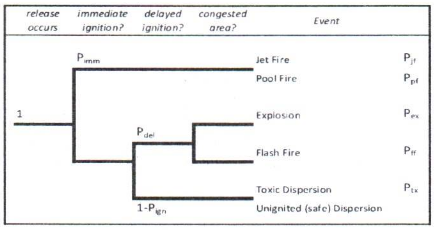
Unit function: preliminary phase separation of slugs from the multiphase flow collected from offshore plants

Failure mode: release of dangerous flammable material (Loss Of Primary Containment, LOPC)



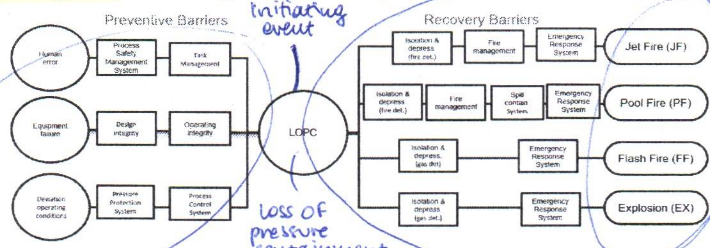
Bow-Tie for slug catcher LOPC

Fire escalation event tree:



EVENT TREE

Bow-Tie diagram (considering safety barriers):



initiating event

loss of pressure containment

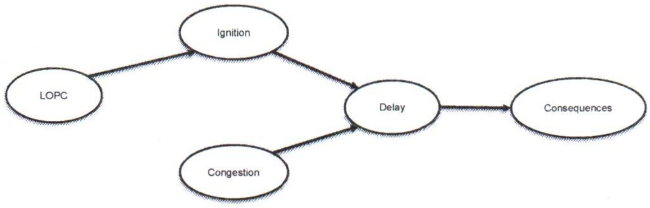
FAULT TREE

we can design some safety barriers to prevent these (actually: safety barriers can be PREVENTING or MITIGATING barriers)

Mapped BN without safety barriers

BN converted from the Bow-Tie (without considering safety barriers):

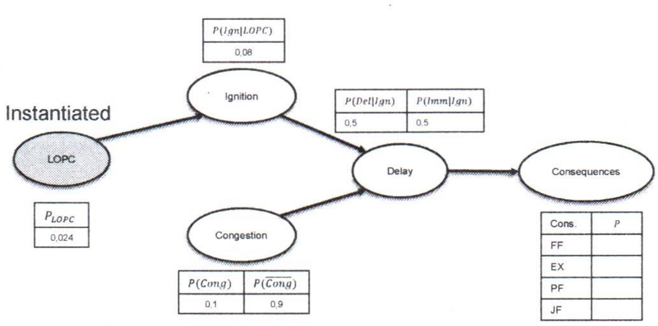
we can map the Bow-Tie into a Bayesian Network (since it's a composition of fault tree and event tree and we know that we can map those in BN)



Mapped BN without safety barriers

BN converted from the Bow-Tie (without considering safety barriers):

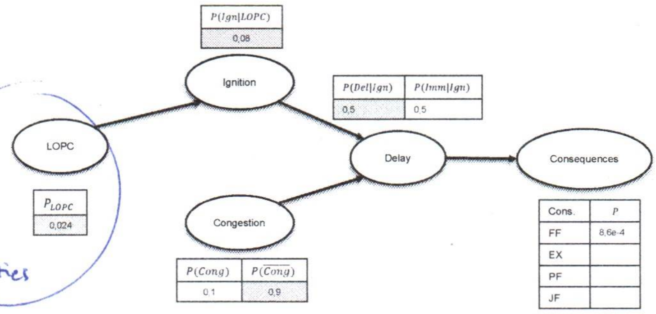
we can assign probabilities to every node



For the P_{LOPC} node, evidence is available for the case without implemented barrier

BNs application for Risk Assessment in Oil & Gas Industry
Mapped BN without safety barriers

BN converted from the Bow-Tie (without considering safety barriers):



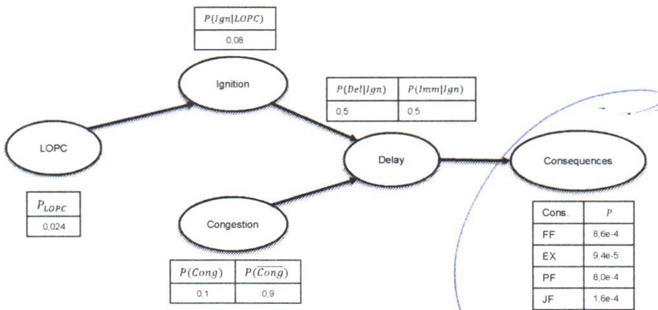
If we instantiate this node we can use inference to calculate the probabilities of the consequences

Consequence evaluation:

$P(FF) = P_{LOPC} \cdot P(Ign|LOPC) \cdot P(\overline{Cong}) \cdot P(Del|Ign) = 0{,}024 * 0{,}08 * 0{,}9 * 0{,}5 = 0{,}00086$

BNs application for Risk Assessment in Oil & Gas Industry
Mapped BN without safety barriers

BN converted from the Bow-Tie (without considering safety barriers):



Consequence evaluation:

$P(FF) = P_{LOPC} \cdot P(Ign|LOPC) \cdot P(\overline{Cong}) \cdot P(Del|Ign) = 0{,}024 * 0{,}08 * 0{,}9 * 0{,}5 = 0{,}00086$

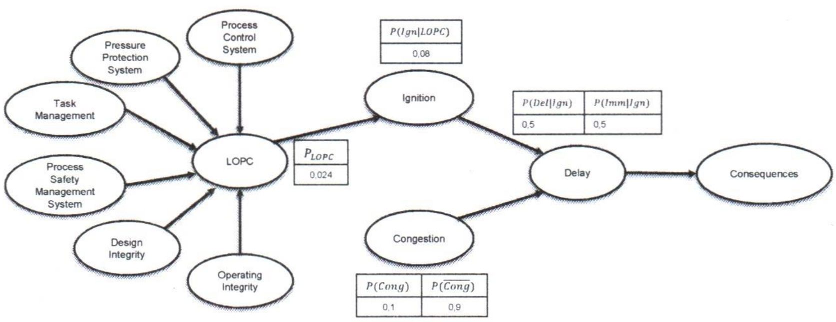
$P(EX) = P_{LOPC} \cdot P(Ign|LOPC) \cdot P(Cong) \cdot P(Del|Ign) = 0{,}024 * 0{,}08 * 0{,}1 * 0{,}5 = 0{,}00094$

$P(JF) = P_{LOPC} \cdot P(Ign|LOPC) \cdot P(\overline{Cong}) \cdot P(Del|Ign) = 0{,}024 * 0{,}08 * 0{,}5 * 5/6 = 0{,}00080$

$P(PF) = P_{LOPC} \cdot P(Ign|LOPC) \cdot P(\overline{Cong}) \cdot P(Del|Ign) = 0{,}024 * 0{,}08 * 0{,}5 * 1/6 = 0{,}00016$

BNs application for Risk Assessment in Oil & Gas Industry
BN with Preventive Safety Barriers

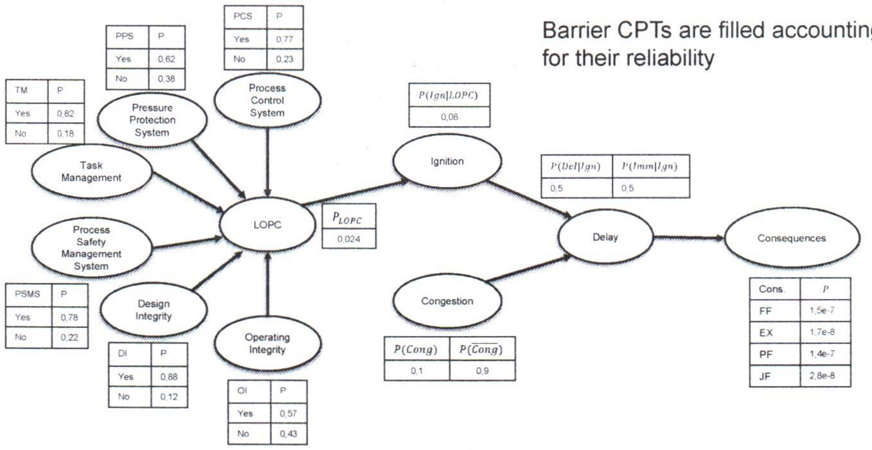
BN converted from the Bow-Tie (with preventive safety barriers):



BNs application for Risk Assessment in Oil & Gas Industry
Binary barrier characterization

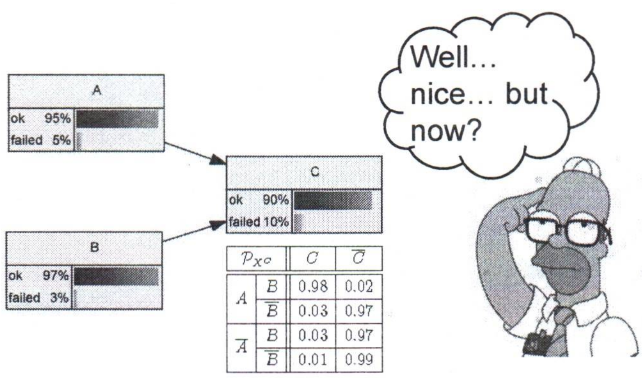
BN converted from the Bow-Tie (with preventive safety barriers):

Barrier CPTs are filled accounting for their reliability



Influence Diagrams

Influence Diagrams (IDs) extend BNs to support decision makers to identify the optimal decision policy

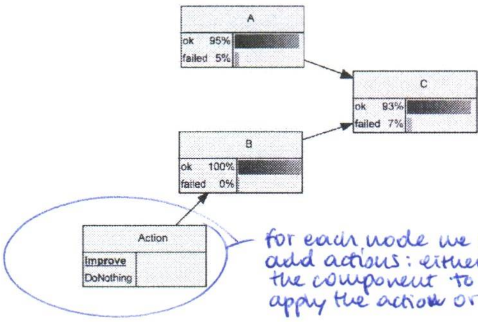


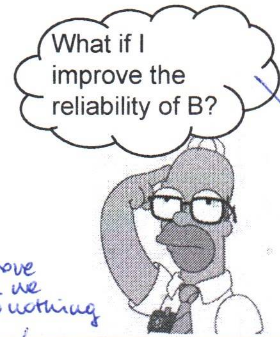
From Bayesian Networks to Influence Diagrams

It is possible to apply actions on nodes $V^A \subseteq V$ → the probability distribution of follower nodes is modified $P(\boldsymbol{X}^i) \rightarrow P(\boldsymbol{X}_a^i)$

Nodes V^A are usually indicated by squares (instead of circles)

The set of alternative actions at node $i \in V^A$ is $A^i = \{1, \dots, |A^i|\}$.





for each node we can add actions: either improve the component to which we apply the action or we do nothing

changing the action we influence everything that depends, in this case, on B (hence, changing actions on B will reflect changings also on C)

From Bayesian Networks to Influence Diagrams

It is possible to apply actions on nodes $V^A \subseteq V$ → the probability distribution of follower nodes is modified $P(\boldsymbol{X}^i) \rightarrow P(\boldsymbol{X}_a^i)$

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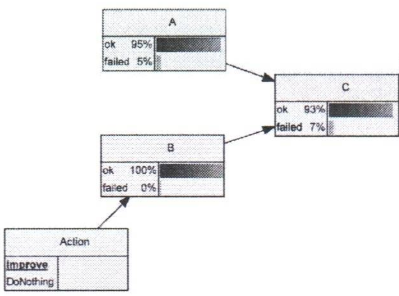
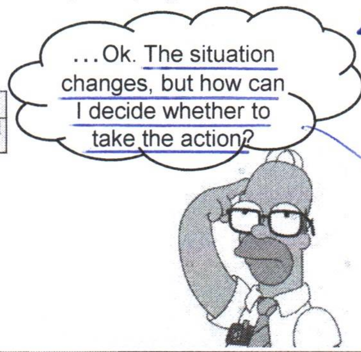




The effects of the action nodes can be quantified by adding another node to the network that is called UTILITY NODE.

The utility node quantifies the benefits of changing the probability tables.

From Bayesian Networks to Influence Diagrams

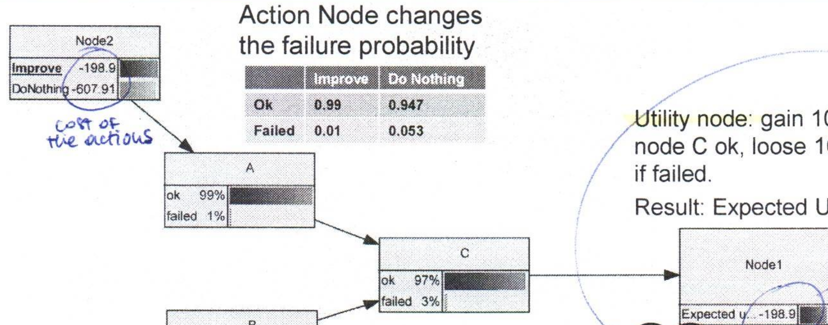
It is possible also to consider a utility node

- All decision problems rely on preferences, i.e., the ordering of alternatives based on their relative utility→ utility is a measure of preference
- Utility function maps on the set of real numbers the outcomes of a decision process, which can concern both objective quantities (material usage, factory output, financial gain, etc.) and quantities with no obvious numerical measure (e.g., health state, customer satisfaction, etc.)
- Utility functions are obtained from a decision maker through utility elicitation (subjective)
- Utility is determined up to a linear transformation: a decision maker preference over different alternatives is invariant to multiplying the utility by a non-negative number and adding a constant → utility has neither a meaningful zero point, nor a meaningful scale

From Bayesian Networks to Influence Diagrams

Action Node changes the failure probability

	Improve	Do Nothing
Ok	0.99	0.947
Failed	0.01	0.053



Utility node: gain 100 if node C ok, loose 10000 if failed.

Result: Expected Utility

cost of finding C in failed/working condition according to the cost that we assigned to the component

The utility node calculates what is the expected utility when we change some settings of the original leaf nodes

Action node: in this case two alternatives (i.e., Improve, Do Nothing)

Application of Influence Diagrams: Value of Information

VOI_i = value of knowing what is the state of the event i

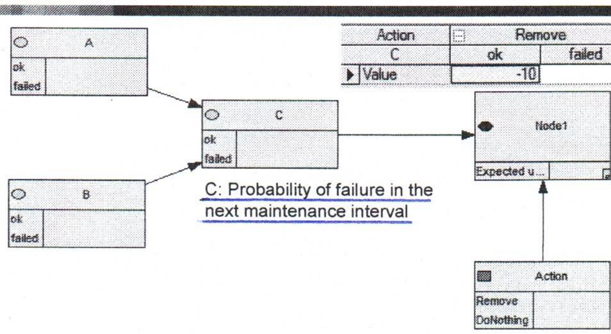
VOI for node i is the expected difference in expected utility (EU) for the two situations:

- Node i is observed,
- Node i is unobserved.

Rougly speaking: VOI is the price that one would be willing to pay in order to gain access to perfect information

Application of Influence Diagrams: Value of Information

Action	Remove		DoNothing	
C	ok	failed	ok	failed
Value	-10	0	100	-1000



C: Probability of failure in the next maintenance interval

we can add to C the cost on the fact that we can monitor/not monitor C

Expected monetary value: The maximum utility if choosing without knowing the value of node C

$$EMV = \max_a \sum_{s_j} p_{s_j} R_{a,s_j} = \max_a(0.93 \cdot (-10) + 0.07 \cdot 0; 0.93 \cdot 100 + 0.07 \cdot (-1000)) = 23$$

we're considering probabilities and costs of the slide 69

The expected value given perfect information of node C (here we're monitoring comp. C)

$$EVPI = \sum_{s_j} p_{s_j} \max_a(R_{a,s_j}) = (0.93 \cdot 100 + 0.07 \cdot 0) = 93$$

We perform VOI for every node, we rank all the actions and consequences based on VOI and based on this ranking we'll decide which actions to take.

cost that we're willing to pay to perfectly know the state of C

$$VOI = \underbrace{EVPI}_{\text{monitoring}} - \underbrace{EMV}_{\text{not monitoring}} = 70$$ The value of having a prognostic system!

Basics

Bayesian Networks: key concepts

Bayesian Network for Reliability and Risk Analysis

Applications

Enhancements

- BNs for Decisions: Influence Diagrams
- Multistate BN for Risk Assessment in Oil & Gas industry

not necessarily boolean

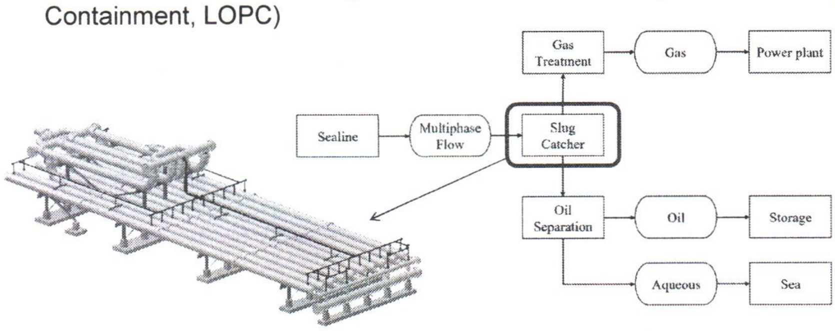
Conclusions

Multistate BN for Risk Assessment in Oil & Gas industry
A Typical Oil & Gas Onshore Plant Case (Reprise)

Unit considered: slug catcher of an oil & gas onshore plant

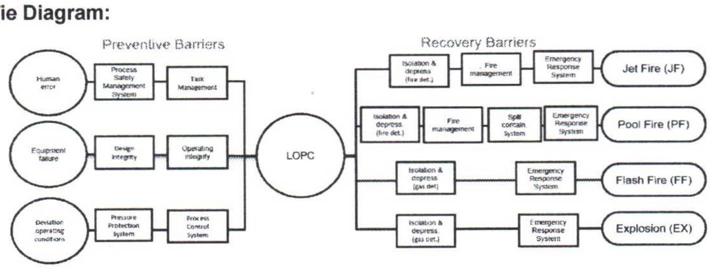
Unit function: preliminary phase separation of slugs from the multiphase flow collected from offshore plants

Failure mode: release of dangerous flammable material (Loss Of Primary Containment, LOPC)

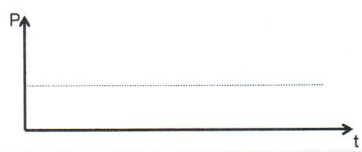


Multistate BN for Risk Assessment in Oil & Gas industry
Bow-Tie Limitation

Bow-Tie Diagram:



STATIC RISK ASSESSMENT



Bow-Tie Limitation

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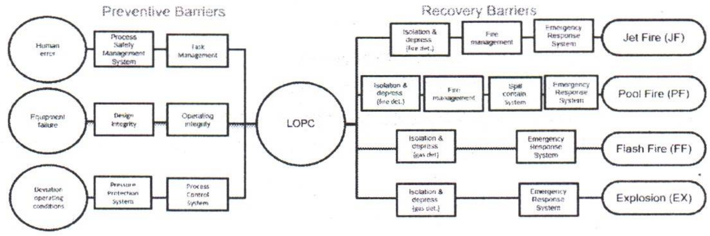
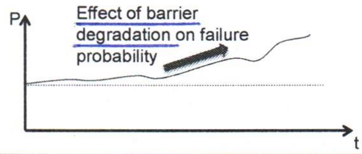


STATIC RISK ASSESSMENT → **BARRIER DEGRADATION IS NOT CONSIDERED**



In the boolean case a degradated barrier is a non-functioning barrier

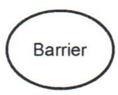
Multistate Barrier Characterization

Health State (HS)

Condition of a barrier, described as a **multistate variable**, whose states are:

- High (H)
- Medium (M)
- Low (L)

A probability of realization is associated to each state:



HS	P(HS)
H	P(H)
M	P(M)
L	P(L)

Failure Probability (FP)

Probability that a barrier in a certain HS will not perform its task:

- FP_H
- FP_M
- FP_L

These will influence the LOPC probability table

Safety barrier effect on CPT

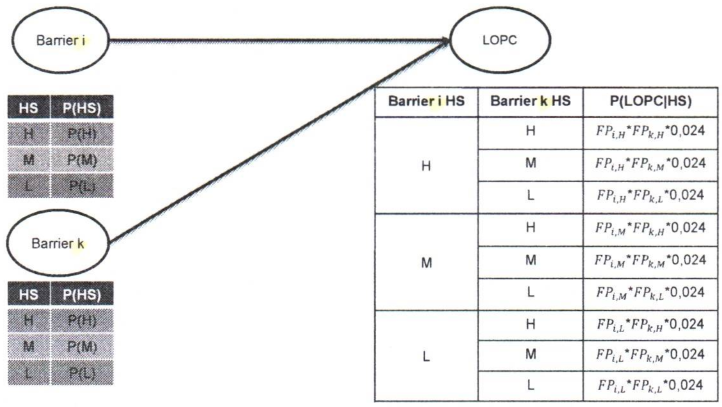
HS	P(HS)
H	P(H)
M	P(M)
L	P(L)

it's not a boolean barrier anymore, now it's multi-state

Barrier i HS	P(LOPC|HS)
H	$FP_{i,H}$*0,024
M	$FP_{i,M}$*0,024
L	$FP_{i,L}$*0,024

P_{LOPC} is influenced by the failure probability corresponding to each barrier health state

Safety barrier effect on CPT

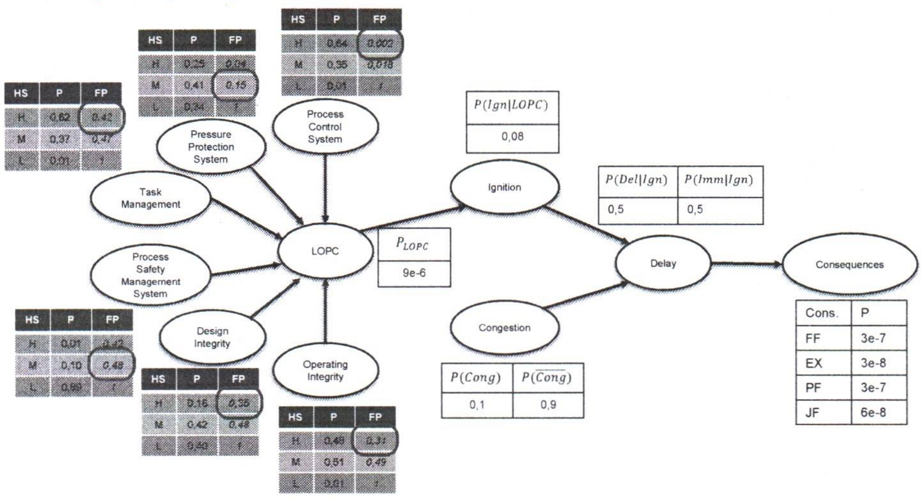


Barrier i HS	Barrier k HS	P(LOPC|HS)
H	H	$FP_{i,H}*FP_{k,H}$*0,024
	M	$FP_{i,H}*FP_{k,M}$*0,024
	L	$FP_{i,H}*FP_{k,L}$*0,024
M	H	$FP_{i,M}*FP_{k,H}$*0,024
	M	$FP_{i,M}*FP_{k,M}$*0,024
	L	$FP_{i,M}*FP_{k,L}$*0,024
L	H	$FP_{i,L}*FP_{k,H}$*0,024
	M	$FP_{i,L}*FP_{k,M}$*0,024
	L	$FP_{i,L}*FP_{k,L}$*0,024

The CPT accounts for every combination of the HSs of the barriers, growing quickly

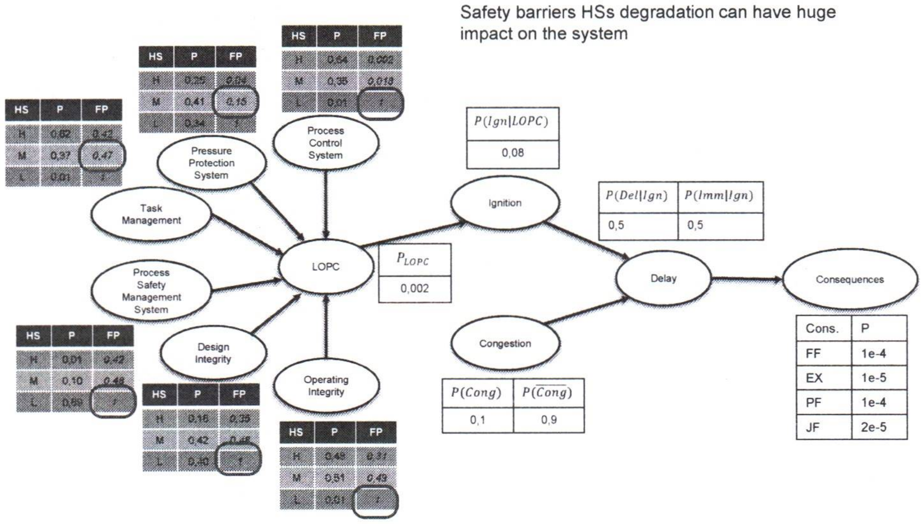
Degradation impact

In this case we have 6 barriers, each one defined on 3 states:



Degradation impact

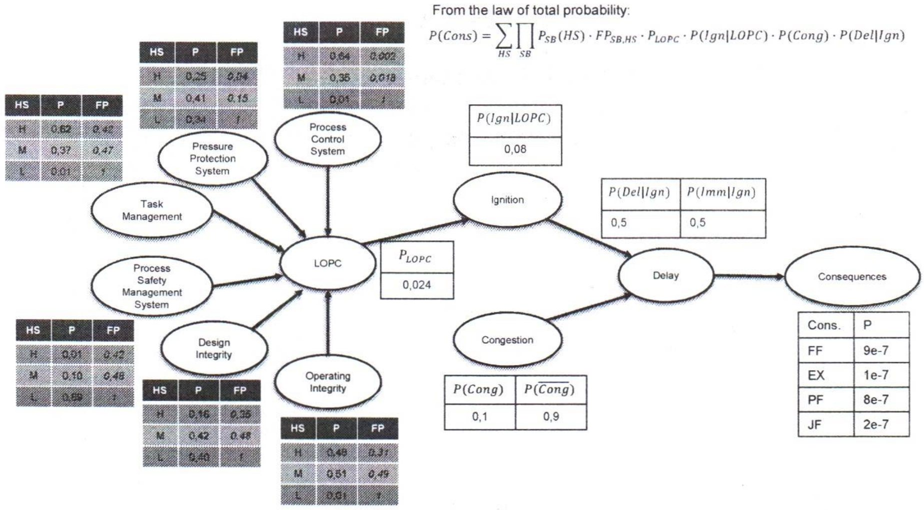
Safety barriers HSs degradation can have huge impact on the system



Multistate BN for Risk Assessment in Oil & Gas industry
Consequence evaluation

From the law of total probability:

$$P(Cons) = \sum_{HS} \prod_{SB} P_{SB}(HS) \cdot FP_{SB,HS} \cdot P_{LOPC} \cdot P(Ign|LOPC) \cdot P(Cong) \cdot P(Del|Ign)$$



HS	P	FP
H	0,62	0,42
M	0,37	0,47
L	0,01	1

HS	P	FP
H	0,25	0,04
M	0,41	0,15
L	0,34	1

HS	P	FP
H	0,64	0,002
M	0,35	0,018
L	0,01	1

HS	P	FP
H	0,01	0,42
M	0,10	0,48
L	0,89	1

HS	P	FP
H	0,16	0,35
M	0,42	0,48
L	0,40	1

HS	P	FP
H	0,48	0,31
M	0,51	0,49
L	0,01	1

P_{LOPC}
0,024

$P(Ign\|LOPC)$
0,08

$P(Del\|Ign)$	$P(Imm\|Ign)$
0,5	0,5

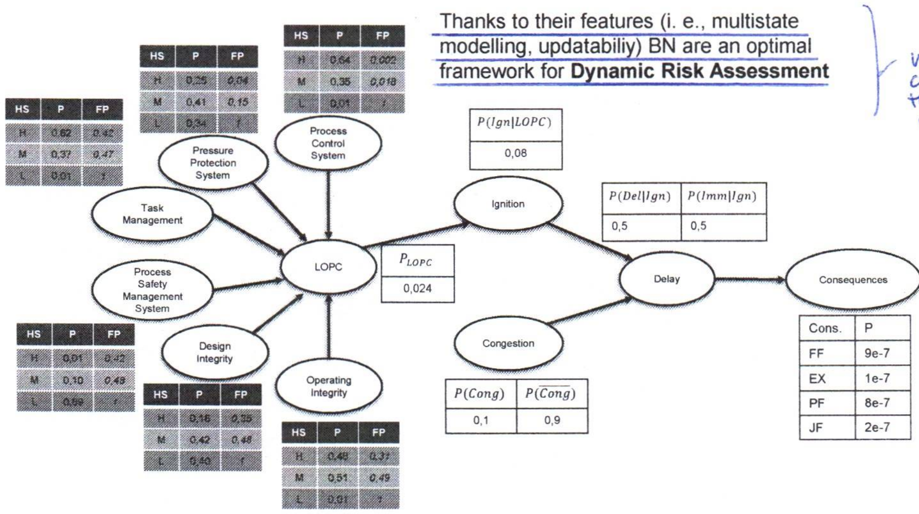
$P(Cong)$	$P(\overline{Cong})$
0,1	0,9

Cons.	P
FF	9e-7
EX	1e-7
PF	8e-7
JF	2e-7

Multistate BN for Risk Assessment in Oil & Gas industry
Dynamic Risk Assessment

Thanks to their features (i. e., multistate modelling, updatabiliy) BN are an optimal framework for **Dynamic Risk Assessment**

we can use the BN to continuously update (feed) the nodes (leaf nodes) with fresh informations


Pressure Protection System
Process Control System
Task Management
Process Safety Management System
Design Integrity
Operating Integrity
LOPC
Ignition
Congestion
Delay
Consequences

HS	P	FP
H	0,62	0,42
M	0,37	0,47
L	0,01	1

HS	P	FP
H	0,25	0,04
M	0,41	0,15
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P_{LOPC}
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$P(Ign\|LOPC)$
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$P(Del\|Ign)$	$P(Imm\|Ign)$
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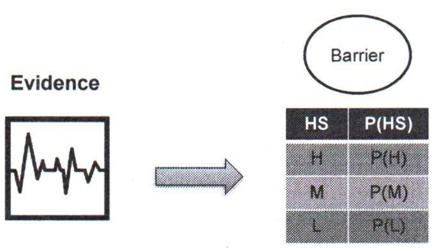
$P(Cong)$	$P(\overline{Cong})$
0,1	0,9

Cons.	P
FF	9e-7
EX	1e-7
PF	8e-7
JF	2e-7

Multistate BN for Risk Assessment in Oil & Gas industry
HS updating

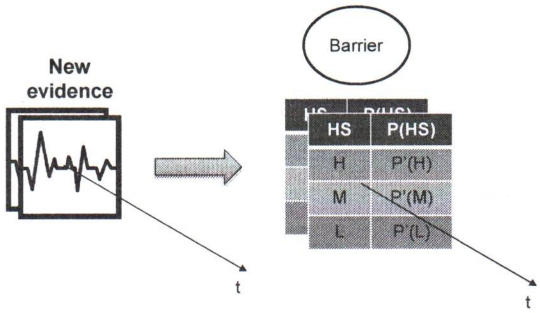
HS probability distribution is evaluated collecting evidence from the online plant by monitoring and inspecting



HS	P(HS)
H	P(H)
M	P(M)
L	P(L)

HS probability distribution is evaluated collecting evidence from the online plant by monitoring and inspecting

With each new piece of information, knowledge and data, we are able to update over time the HS probability distribution for each barrier



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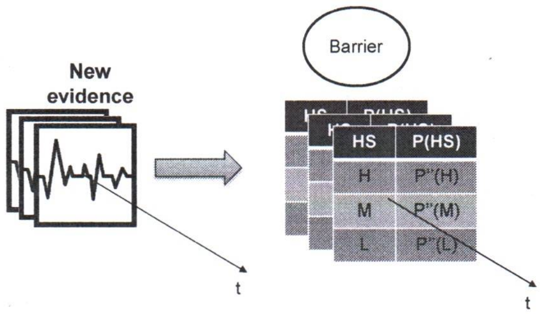
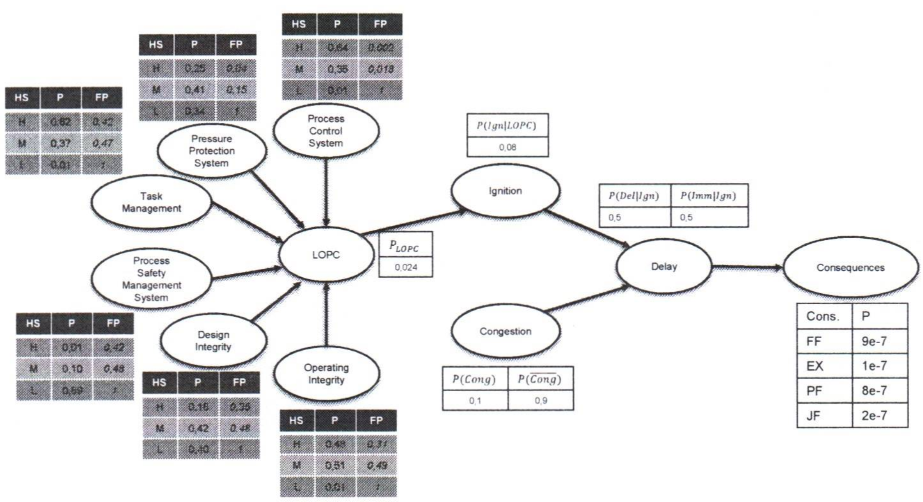
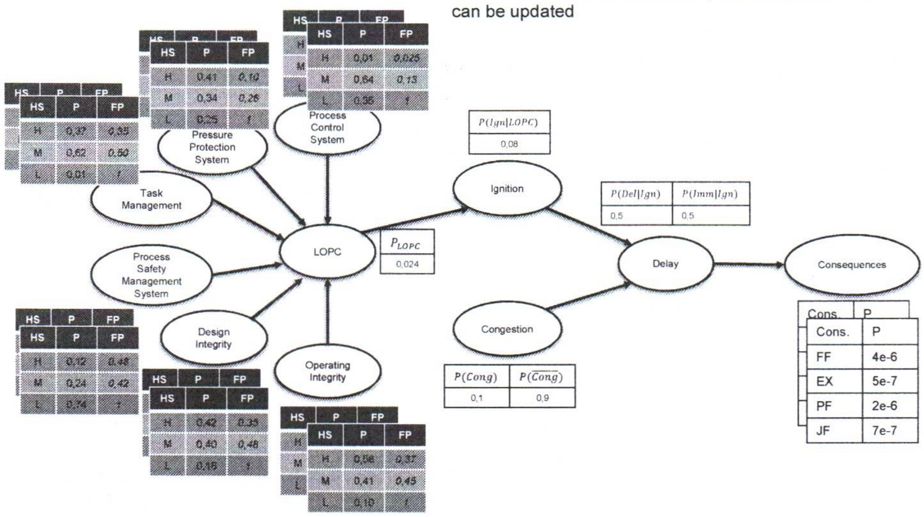




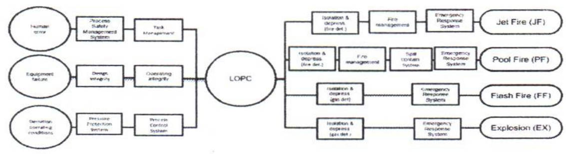
Multistate BN for Risk Assessment in Oil & Gas industry
Risk Assessment Update

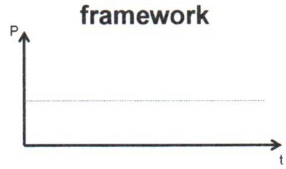
When new evidence is collected, the risk assessment can be updated



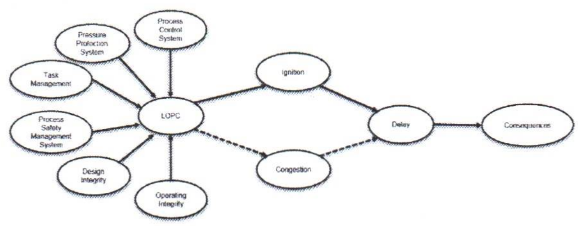
Multistate BN for Risk Assessment in Oil & Gas industry
Bow-Tie vs BN

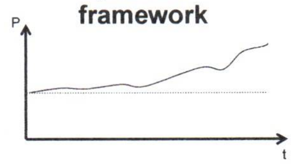
Static framework





Dynamic framework





Bayesian Networks are powerful tools able to:

- **Adapt** to several frameworks and applications
- **Incorporate new evidence** from different sources
- **Update** probability assessments (dynamic applications)
- **Handle multistate variables** (realistic assessments)

it's very easy to add nodes and update the probabilities (no need to re-structure everything from scratch)

But:

- Require innovative solving approaches

(the conditional probability tables can become huge. moreover we have some computational problems when we have many nodes)