POLITECNICO DI MILANO

CRC
Centre de recherche sur les Risques et les Crises

Availability of Systems

Prof. Enrico Zio

Politecnico di Milano
Dipartimento di Energia

Introduction: reliability and availability

- **Reliability** and **availability**: important performance parameters of a system, with respect to its ability to fulfill the required mission in a given period of time
- Two different system types:
  - Systems which must satisfy a specified mission within an assigned period of time: **reliability** quantifies the ability to achieve the desired objective without failures
  - Systems maintained: **availability** quantifies the ability to fulfill the assigned mission at **any** specific moment of the life time

Now we're here. We're saying: the system can fail but can also be repaired. This is important not only for systems that allow downtime of service but in particular for SAFETY SYSTEMS. Safety systems are not in operation during the normal operationing of the system. They intervene when it's needed. They're in standby

Availability definition (1)

Availability is a concept that applies to situations in which failures are repaired.

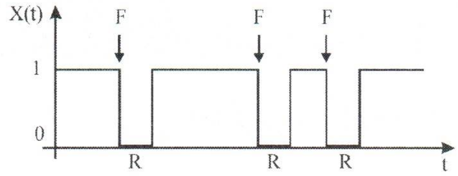
Availability is a measure of the degree to which an item is in an operable state when called upon to perform.

Availability is expressed as a probability.

Availability definition (2)

— it describes in which state we find the system along time

- $X(t)$ = indicator variable such that:
  - ➢ $X(t) = 1$, system is operating at time t
  - ➢ $X(t) = 0$, system is failed at time t



F = Failed
R = under Repair

- Instantaneous **availability** $p(t)$ and **unavailability** $q(t)$

$$p(t) = P[X(t)=1] = \underbrace{E[X(t)]}$$ $$q(t) = P[X(t)=0] = 1 - p(t)$$

$$\mathbb{E}[X(t)] = 1 \cdot \mathbb{P}(X(t)=1) + 0 \cdot \mathbb{P}(X(t)=0)$$

Contributions to Unavailability

- **Unrevealed failure**

  A stand-by component fails unnoticed. The system goes on without noticing the component failure until a test on the component is made or the component is demanded to function

An important topic is the optimization of testing the emergency system. If we perform the testing once a year and a system breaks the day after a testing then we will not know until the year after.

- **Testing / preventive maintenance**

  A component is removed from the system because it has to be tested or must undergo preventive maintenance

- **Repair**

  A component is unavailable because under repair

Average availability descriptors

- How to compare different maintenance strategies?

- We need to define quantities for an **average** description of its probabilistic behavior:
  - ➢ Components under corrective maintenance (stochastic repair time):

It means that we wait for failures, correct the failure by repair

$$p = \lim_{t \to \infty} p(t)$$: value at which the availability curve tends in time

  - ➢ Components under periodic maintenance:

Average time the system is functioning (UP) within T_M

$$p_T = \underbrace{\frac{1}{T}\int_0^T p(t)dt} = \frac{\overline{UPtime}}{T} = \frac{\overline{T_U}}{T}$$

average factor of utilization

(if a certain plant has 78% factor of operation then it means that on average 78% of the times is operating)

average: sum of the availabilities over time divided by the total time of observations

Simplest case: when istantaneous availability corresponds to the reliability (case in which we consider the availability of a component which is used and thrown away, no repair)

Availability of an unattended component (no repair allowed)

- The probability $q(t)$ that at time t the component is not functioning is **equal** to the probability that it failed before t

$$q(t) \equiv F(t)$$ = probability that the component failed up to time t

$$p(t) = 1 - q(t) \equiv R(t)$$

where $F(t)$ is the cumulative failure probability and $R(t)$ is the reliability

→ Reliability is a particular case of availability (when we don't have preventive maintenance / inspection and we don't even have repairs)

(Since we don't allow repairs) The availability at time t ($p(t)$) is the probability of all scenarios that allow the component to be available at time t, but if we don't have repairs the only scenario possible is that the component never failed before t. In this case availability coincides with reliability and unavailability coincides with the failure probability.

Availability of a continuously monitored component (1)

component continuously monitored = component that we're using

When the component fails we start repairing it and the repair is not immediate. The duration of the repair is stochastic: we need to have a distribution of the repair time.

We have:
- a distribution of the failure in time
- a distribution of the repairing time

- **Objective**:
  - Computation of the availability $p(t)$
- **Hypotheses**:
  - N = number of identical components at time $t = 0$
  - Restoration starts immediately after the component failure
  - Probability density function of the random time duration T_R of the repair process = $g(t)$

we assume also an EXPONENTIAL distr. for the FAILURE TIMES

Balance equation between time t and time $t+\Delta t$

We want to have an expected number of components available at time $t + \Delta t$ given the number of available components (expected) at time t

to do so: balance equation

Availability of a continuously monitored component (2)

$$\underbrace{N \cdot p(t+\Delta t)}_{(1)} = \underbrace{N \cdot p(t)}_{(2)} - \underbrace{N \cdot p(t) \cdot \lambda \cdot \Delta t}_{(3)} + \underbrace{\int_0^t N \cdot p(\tau) \cdot \lambda \cdot \Delta\tau \cdot g(t-\tau) \cdot \Delta t}_{(4)}$$

(1) expected number of available components at time $t + \Delta t$
(2) expected number at time t
(3) number of those who failed *
(4) number of those who finished the repair in Δt *

1) Number of items UP at time $t+\Delta t$
2) Number of items UP at time t
3) Number of items failing during the interval Δt : * $\begin{cases} \lambda \cdot \Delta t = \text{probability of failure in } \Delta t \\ N \cdot p(t) = \text{available at time } t \text{ (expected number of av. comp)} \end{cases}$
4) Number of items that had failed in $(\tau, \tau+\Delta\tau)$ and whose restoration terminates in $(t, t+\Delta t)$;

* $N \cdot p(\tau) \cdot \lambda \cdot \Delta\tau$ = components (number) that failed at time τ
$g(t-\tau)\,\Delta t$ = probability of being repaired from τ (probability that if it brokes at τ the repairing ends in Δt)

→ since τ can be anything → ∞ possibilities → integral

Starting from the previous balance equation we eliminate N (simplicity), divide by Δt and do the limit $\Delta t \to 0$. We obtain →

Availability of a continuously monitored component (3)

- The integral-differential form of the balance

$$\frac{dp(t)}{dt} = -\lambda \cdot p(t) + \int_0^t \lambda \cdot p(\tau) \cdot g(t-\tau) \cdot d\tau \qquad p(0)=1$$

This means that when we buy a new component we assume it to be available to start.

This is a convolutional integral

- The solution can be obtained introducing the Laplace transforms

$$f(x) \quad \to \quad L[f(x)] = \tilde{f}(s) = \int_0^\infty e^{-s \cdot x} f(x) dx$$

$$\frac{df(x)}{dx} \quad \to \quad L\left[\frac{df(x)}{dx}\right] = s \cdot \tilde{f}(s) - f(0)$$

Availability of a continuously monitored component (4)

- Applying the Laplace transform we obtain:

$$s \cdot \tilde{p}(s) - 1 = -\lambda \cdot \tilde{p}(s) + \lambda \cdot \tilde{p}(s) \cdot \tilde{g}(s)$$

which yields:

$$\tilde{p}(s) = \frac{1}{s + \lambda \cdot (1 - \tilde{g}(s))}$$

result of the availability (istantaneous) in the Laplace domain

- Inverse Laplace transform → *p(t)*
- Limiting availability: (for components with corrective maintenance)

Zero-value theorem of Laplace

CONSTANT

$$p_\infty = \lim_{t\to\infty} p(t) = \lim_{s\to 0}[s \cdot \tilde{p}(s)] = \lim_{s\to 0}\left[\frac{s}{s + \lambda \cdot (1 - \tilde{g}(s))}\right]$$

The value of this limit depends on $\tilde{g}(s)$

asymptotic availability (after an initial transient time, p_∞ gives the steady state value of availability for any time after the transient time)

Availability of a continuously monitored component (5)

- As $s \to 0$, the first order approximation of $\tilde{g}(s)$ is the following:

$\int_0^\infty g(\tau)d\tau = 1$ since $g(\tau)$ is a density and $[0,+\infty)$ is its domain

$$\tilde{g}(s) = \int_0^\infty e^{-s\tau} g(\tau) d\tau = \int_0^\infty (1 - s \cdot \tau + \dots) g(\tau) d\tau \cong 1 - s \cdot \int_0^\infty \tau g(\tau) d\tau = 1 - s \cdot \bar{\tau}_R$$

$$\bar{\tau}_R = E_g[T_R]$$

expected value of the random variable repair-time

MTTF → MTBF

Mean Time To Failure — Mean Time Between Failures (since we accept repairs)

$$p_\infty = \lim_{s\to 0} \frac{s}{s + \lambda \cdot s \cdot \bar{\tau}_R} = \frac{1}{1 + \lambda \cdot \bar{\tau}_R} = \frac{1/\lambda}{1/\lambda + \bar{\tau}_R} = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} = \frac{\text{average time the component is UP}}{\text{average time of a failure/repair "cycle"}}$$

= ratio between the UP time and the whole period

Mean Time To Repair

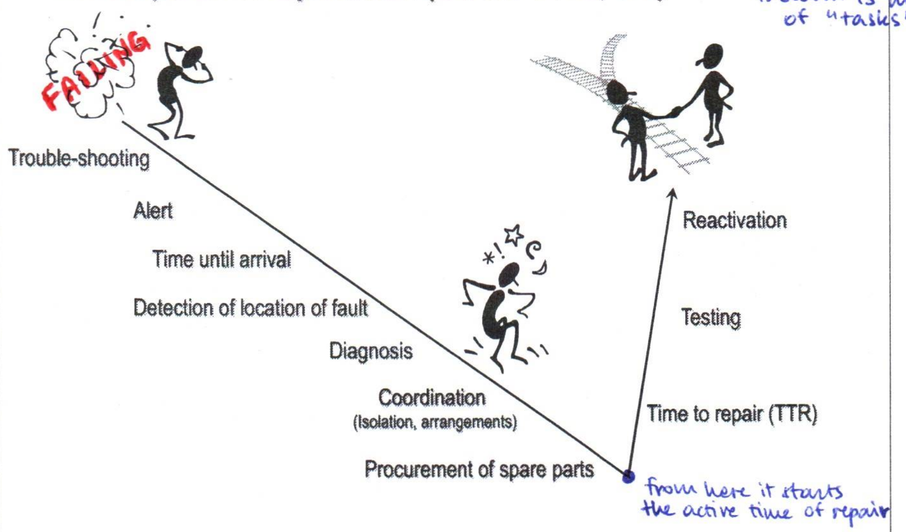
It depends on λ since we assumed the exponential and it depends on the mean time of repair-time.

General result !

Availability of a continuously monitored component (5)

Interruption of Operation (Down Time, DT)

– The time in which a component is down is made by a number of "tasks".



Availability of a component under periodic maintenance (1)

- Safety systems are generally in standby until accident and their components must be periodically tested
- The instantaneous unavailability is a periodic function of time
- The performance indicator used is the average unavailability (which is not a probability !)

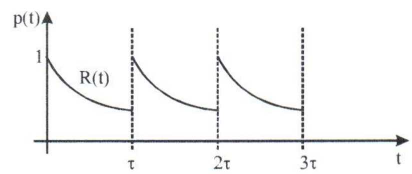
$$q_T = \frac{1}{T}\cdot\int_0^T q(t)dt = \frac{\overline{DOWNtime}}{T} = \frac{\overline{T_D}}{T}$$

average downtime over a given period

Availability of a component under periodic maintenance (2)

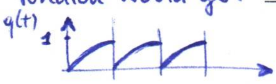
- Suppose the unavailability is due to unrevealed random failures, e.g. with constant rate λ
- Assume also instantaneous and perfect testing and maintenance procedures
- The instantaneous availability within a period τ coincides with the reliability

We perform maintenence every τ (quantity of time).



– The function of istantaneous availability goes like this under the assumption that:
- the testing period is negligible w.r.t. the period between successive tests
- the maintenance is "perfect" and so the component is brought back to perfect availability

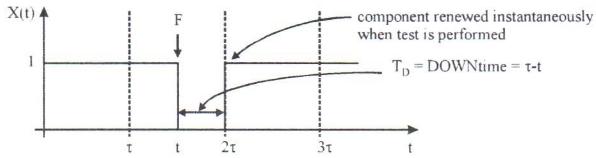
A component is available at time t if it never failed since the last inspection. The function between successive inspection is the **RELIABILITY FUNCTION**. The UNAVAILABILITY function would go:



At every maintainence it goes to zero.

Availability of a component under periodic maintenance (3)

- **Objective**: computation of the average unavailability



- The average unavailability within one period τ is:

$$q_\tau = \frac{mean\ \ DOWNtime}{\tau} = \frac{\overline{T}_D}{\tau}$$

$$\overline{T}_D = \int_0^\tau (\tau - t) f_T(t)dt = \int_0^\tau (\tau - t)dF_T =$$

$$= (\tau - t)F_T(t)\Big|_0^\tau + \int_0^\tau F_T(t)dt = \int_0^\tau F_T(t)dt$$

Availability of a component under periodic maintenance (4)

- The average unavailability and availability are then:

the availability in an interval τ coincides with the reliability

$$q_\tau = \frac{\overline{T}_D}{\tau} = \frac{\int_0^\tau F_T(t)dt}{\tau} \qquad p_\tau = \frac{\overline{T}_U}{\tau} = \frac{\int_0^\tau R(t)dt}{\tau}$$

average unavailability over a period τ

average availability in a period

collection of uptimes divided by the period

collection of downtimes divided by the period

For different systems, we can compute p_τ e q_τ by first computing their failure probability distribution $F_T(t)$ and reliability $R(t)$ and then applying the above expressions

!

What if we cannot assume negligible times for the repair?

Availability of a component under periodic maintenance (5)

- Assume a finite repair time τ_R

that's another time of unavailability

- The average unavailability and availability over the complete maintenance cycle period $\tau + \tau_R$ are:

We can again calculate it as down time over the whole time period (that this time includes the repair time)

the duration of the down time due to repair is always a lot smaller than the period between two sequential maintenances

$$\overline{q}_{\tau+\tau_R} = \frac{\tau_R + \int_0^\tau F_T(t)dt}{\tau + \tau_R} \approx [\tau_R << \tau] \approx \frac{\tau_R + \int_0^\tau F_T(t)dt}{\tau}$$

we cannot neglect this because unlike the denominator, the numerator is small in both its components (τ_R is small w.r.t. τ, so it's negligible in $\tau + \tau_R$, however is comparable with $\int_0^\tau F_T(t)dt$, so it's not negligible in the numerator)

$$\overline{p}_{\tau+\tau_R} = \frac{\int_0^\tau R(t)dt}{\tau + \tau_R} \approx [\tau_R << \tau] \approx \frac{\int_0^\tau R(t)dt}{\tau}$$

EXAMPLE

Availability of a continuously monitored component

- Find instantaneous and limiting availability for an exponential component whose restoration probability density is

$$g(t) = \mu \cdot e^{-\mu \cdot t}$$

- The Laplace transform of the restoration density is

$$\tilde{g}(s) = L[g(t)] = \frac{\mu}{s+\mu}$$

$$\tilde{p}(s) = \frac{1}{s+\lambda\cdot\frac{s}{s+\mu}} = \frac{s+\mu}{s\cdot(s+\mu+\lambda)} \longrightarrow \begin{cases} p(t) = L^{-1}[\tilde{p}(s)] = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda}\cdot e^{-(\mu+\lambda)t} \\ p_\infty = \frac{\mu}{\mu+\lambda} \end{cases}$$

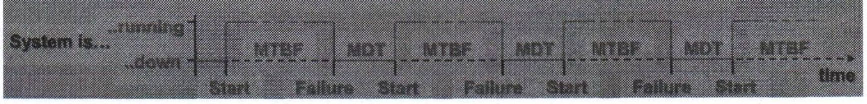
$t \to \infty$

EXAMPLE

Availability of a continuously monitored system

Traffic signalling by a system with field replaceable units :

A field replaceable unit follows on average he following schedule:



Availability A:

$$A = \frac{MTBF}{MTBF + MDT}$$

uptime divided by the period

Unavailability N (=q) :

$$N = 1 - A = \frac{MDT}{MTBF+MDT}$$

MTBF: Mean Time between Failure

MDT: Mean Down Time (~MTTR)

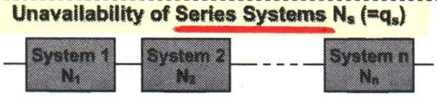
MDT-figures includes travelling-, administrative-, fault detection- + active repair times (MTTR), assuming 24h/d readiness of maintenance staff and availability of sufficient spares.

Repair or replace of	MDT (≈MTTR)
Unit	4 h
Subrack	6 ... 8 h

Let's do a series (parallel) system, not just one component:

Availability of a continuously monitored system

Unavailability of Series Systems N_s (=q_s)



$$N_s \approx N_1 + N_2 + \cdots + N_n = \sum_{i=1}^{n} N_i$$

The unavailability of the system is the sum of the unavailability of the components (we neglect all the intersections)

It's larger than the single one: we have more than one component and it takes only one of it to fail to fail all the system

In series:

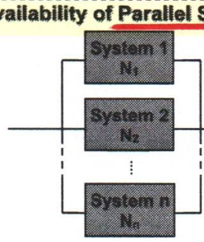
$N_{syst} > N_{single}$

Example: System 1 = System 2
MTBF = 10yrs = 87 600h; MDT = 4h :

$$N_{1/2} = \frac{MDT}{MTBF+MDT} = \frac{4h}{87600h+4h} \approx 4{,}6\cdot10^{-5}$$

$$N_{s\_tot} = N_1 + N_2 = 2\cdot N_{1/2} = 9{,}2\cdot10^{-5} \approx 10^{-4}$$

Unavailability of Parallel Systems N_p (=q_p)



$$N_p = N_1 \bullet N_2 \bullet \cdots \bullet N_n = \prod_{i=1}^{n} N_i$$

The system fails if all the subsystems fail.

In parallel:

$N_{syst} < N_{single}$

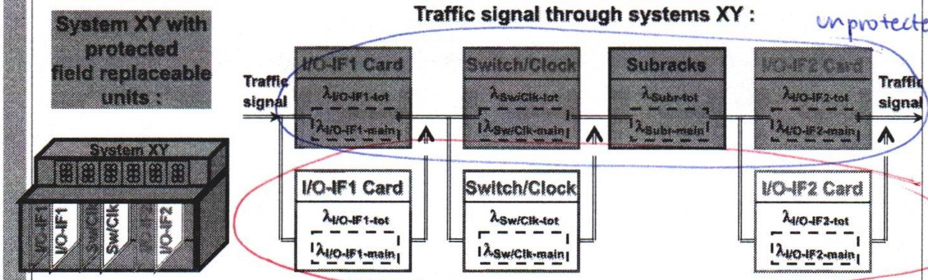
Example: System 1 = System 2
MTBF = 5yrs = 43 800h; MDT = 5h :

$$N_{1/2} = \frac{MDT}{MTBF+MDT} = \frac{5h}{43800h+4h} \approx 1{,}14\cdot10^{-4}$$

$$N_{tot} = N_1 \cdot N_2 = N_{1/2}^2 = 1{,}3\cdot10^{-8}$$

EXAMPLE

Availability of a continuously monitored system



unprotected → in series *

we protect the unprotected with a redoundancy
protected → stand by *

Unit in Operation

Unit In Stand-by Mode

* Unavailability of a system with unprotected units only:

$$N_{unprot} = N_{I/O\text{-}IF1} + N_{Sw/Clk} + N_{Subr} + N_{I/O\text{-}IF2}$$

+ Unavailability of a system with possible protected units:

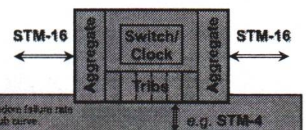
$$N_{prot} = N^2_{I/O\text{-}IF1} + N^2_{Sw/Clk1} + \underline{N_{Subr}} + N^2_{I/O\text{-}IF2} \approx N_{Subr}$$

the subtrack does not have the protection (redoundancy)

in concrete values :

Availability of a continuously monitored system

Example: Availability Calculation for traffic signal of a transport system with List of Material (LoM) (no current values):



SIEMENS

The FIT-values are statistical values and based only upon the random failure rate corresponding to the "flat" component of the typical bathtub curve. The calculation method is in acc. with IEC 61709!

		total		Main path				Quantity of items STM-16 and given ports: a) unprotected				b) Trib→Aggreg→Switch-prot.			
Item-No	Name	λ FIT	MTBF (year)	λ_{main} FIT	$MTBF_{main}$ (year)	MDT (h)	Non-Availab.	Total	STM1e	STM4	STM16	Total	STM1e	STM4	STM16
SM10-1.11	Subrack	2.334	48,9	540	211,4	4	2,16E-6	1	1	1	1	1	1	1	1
SM10-1.12	SCOH	4.694	24,3	0	infinite	4	0	1				1			
SM10-5.1	CLU	2.771	41,2	2.024	56,4	4	8,10E-6	1	1	1	1	2	1+1	1+1	1+1
SM10-8.1	SWITCH FABRIC VC-4	2.986	38,3	2.302	49,6	4	9,21E-6	1	1	1	1	2	1+1	1+1	1+1
SM10-26.1	STM-16 BOARD	5.133	22,2	3.242	35,2	4	1,30E-5	1	1	1	2	2	1+1	1+1	1+1
SM10-26.32	STM-16 MODULE L-16.2/3	500	228,3	500	228,3	4	2,00E-6	4	1	1	2	8	1+1	1+1	1+1
SM10-23.1	STM-4 BOARD	2.931	38,9	1.665	67,7	4	6,74E-6	1		1		2		1+1	
SM10-23.11	STM-4 MODULE L-4.2/3	500	228,3	500	228,3	4	2,00E-6	2		1		4		1+1	
SM10-27.1	STM-1 CARD, 8 X STM-1e	3.489	32,7	2.288	49,9	4	9,15E-6	1	1			2	1+1		
SM10-6.1	SWITCH FABRIC VC-12	4.866	23,5	4.014	28,4	4	1,61E-5	1				2			
	Failure rates in FIT (1FIT=1Failure/10^9h)			Total				37.563				66.102			
	MTBF in years			Total				3,04				1,73			
	Failure rates in FIT (1FIT=1Failure/10^9h)			Main path					10.886	10.793	12.360		540	540	540
	MTBF in years			Main path					10,48	10,58	9,24		211,4	211,4	211,4
	Non-availability			Main path					4,36E-5	4,32E-5	4,94E-5		2,16E-6	2,16E-6	2,16E-6
	Failure intensity in failures per year			Main path					22,91	22,69	25,96		1,14	1,14	1,14
	Non-availability in min/year			Main path					0,095	0,095	0,108		0,005	0,005	0,005
	Availability in %			Main path					99,996	99,996	99,995		99,9998	99,9998	99,9998

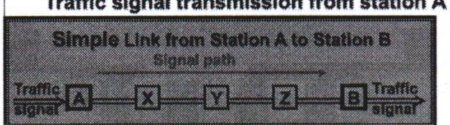
Availability/Reliability Table Page 1 - 1

We can create different networks:

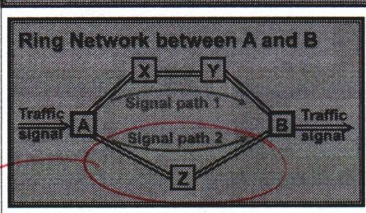
Availability of a continuously monitored network system

we have to take into account all the possible paths (we sum each path's unavailability)

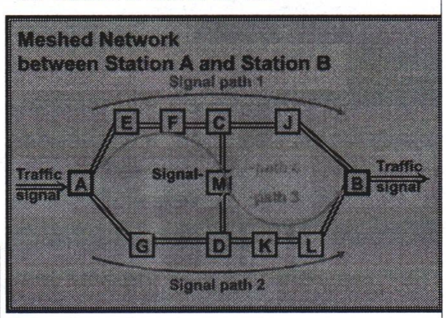
Traffic signal transmission from station A to station B for several types of Networks

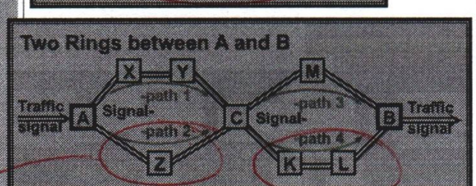


Which signal paths are possible to come from A to B?



redoundant paths





multiple redoundant paths

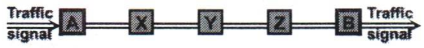
Traffic signal can be e.g. PDH/SDH-signal and/or fibre channel

Cable

A Station A

Availability of a continuously monitored network system

Link from Station A to Station B



The stations contain the transport system only

What about cables between the stations?

The cable is a very important function block
in the consideration of system availability

Table: Failure rates of several cable types (approx. values)

Types of (duct-)cable	Failure/100km/year	Failure rate / km	MTBF * km	MDT *) (incl. MTTR,etc.)
Terrestrial cable City	10 ... 25	11.500 ... 28.600 FIT/km	4 ... 10 yr*km	12 ... 14 h
Terrestrial cable Country	0,1 ... 0,6	115 ... 700 FIT/km	160 ... 1.000 yr*km	12 ... 14 h
Sea cable in depth of < 1.000 km	0,2 ... 0,4	230 ... 450 FIT/km	250 ... 500 yr*km	10 ... 30 d
Sea cable in depth of > 1.000 km	0,1... 0,2	115 ... 230 FIT/km	500 ... 230 yr*km	10 ... 30 d

*) fault detection- + active repair times (MTTR), assuming 24h/d readiness of maintenance staff

(Duct-) Cable   A Station A

Availability of a continuously monitored network system

Example: Availability of a simple link:
(no current values)

SIEMENS

The FIT-values are statistical values and based only upon the random failure rate corresponding to the "flat" component of the typical bathtub curve.
The calculation method is in acc.with IEC 61709!

Item-No	Name	Main transmission path (Main Path) λ_{main} FIT	MTBF$_{main}$ (year)	MDT (h)	Non-Availab.	Quantity of items for following items: STM-16 Link from A to B — with unprotected parts	with protected parts
SM10-1.11	Subrack	540	211,4	4	2,16E-6	5	5
SM10-1.12	SCOH	0	infinite	4	0		
SM10-5.1	CLU	2,024	56,4	4	8,10E-6	5	1+1
SM10-8.1	SWITCH FABRIC VC-4	2,302	49,6	4	9,21E-6	5	1+1
SM10-20.1	STM-16 BOARD	3,242	35,2	4	1,30E-5	5	1+1
SM10-20.32	STM-16 MODULE L-16.2/3	500	228,3	4	2,00E-6	8	1+1
SM10-6.1	SWITCH FABRIC VC-12	4,014	28,4	4	1,61E-5	5	1+1
SM10-25.2	IF2M CARD 63*E1 120ohm)	1,894	161,9	4	2,82E-6	2	1:n
SM10-25.12	LSU CARD 63*E1 120ohm)	705	228,3	4	6,74E-6	2	2
0,3 Failure intensity of fibre (failures(100km/year):							
	Fibre per km	342	333,3	12	4,11E-6	200 km	200 km
	Failure rates in FIT (1FIT=1Failure/10⁹h)		Main path			138,301	72,603
	MTBF in years		Main path			0,83	1,57
	Non-availability of unprotected parts					1,10E-3	8,38E-4
	Non-availability of paths 1 & 2						
	Non-availability of 1+1 protected parts						
	Total Non-availability					1,10E-3	8,38E-4
	Total Non-availability in min/year					578,76	440,64
	Total Availability in %					[illegible]	99,9162

Computation rules: The signal flow through equipment is marked by the figures in the relevant columns.
Remark: The results for the main transmission path are related to one bi-directional signal/channel.

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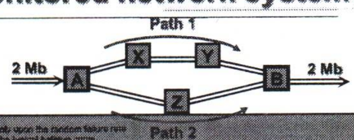
Availability of a continuously monitored network system

Example: Availability of a Ring network:
(no current values)



SIEMENS

The FIT-values are statistical values and based only upon the random failure rate corresponding to the "flat" component of the typical bathtub curve.
The calculation method is in acc.with IEC 61709!

Item-No	Name	Main transmission path (Main Path) λ_{main} FIT	MTBF$_{main}$ (year)	MDT (h)	Non-Availab.	Quantity of items for following items: Ring-Protection from A to B — unprotected parts	STM-16 Ring Path 1	STM-16 Ring Path 2
SM10-1.11	Subrack	540	211,4	4	2,16E-6	2	2	1
SM10-1.12	SCOH	0	infinite	4	0			
SM10-5.1	CLU	2,024	56,4	4	8,10E-6	1+1	1+1	1+1
SM10-8.1	SWITCH FABRIC VC-4	2,302	49,6	4	9,21E-6	1+1	1+1	1+1
SM10-20.1	STM-16 BOARD	3,242	35,2	4	1,30E-5		4	3
SM10-20.32	STM-16 MODULE L-16.2/3	500	228,3	4	2,00E-6		6	4
SM10-6.1	SWITCH FABRIC VC-12	4,014	28,4	4	1,61E-5	1+1		
SM10-25.2	IF2M CARD 63*E1 120ohm)	1,894	161,9	4	2,82E-6	1+n		
SM10-25.12	LSU CARD 63*E1 120ohm)	705	228,3	4	6,74E-6	2		
0,3 Failure intensity of fibre (failures(100km/year):								
	Fibre per km	342	333,3	12	4,11E-6	0 km	150 km	100 km
	Failure rates in FIT (1FIT=1Failure/10⁹h)		Main path			2,490	68,418	45,513
	MTBF in years		Main path			45,85	1,67	2,45
	Non-availability of unprotected parts					9,99E-6		
	Non-availability of paths 1 & 2						6,85E-4	4,60E-4
	Non-availability of 1+1 protected parts						3,15E-7	
	Total Non-availability						1,03E-5	
	Total Non-availability in min/year						5,40	
	Total Availability in %						[illegible]	

Computation rules: The signal flow through equipment is marked by the figures in the relevant columns.
Remark: The results for the main transmission path are related to one bi-directional signal/channel.

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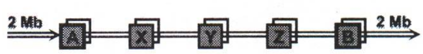
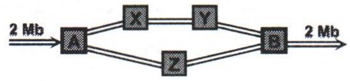
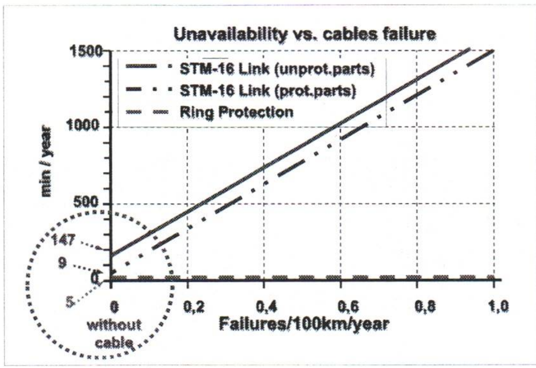
Availability of a continuously monitored network system

Unavailability Diagram for Link- and Ring Network

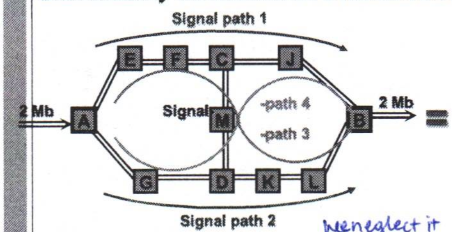


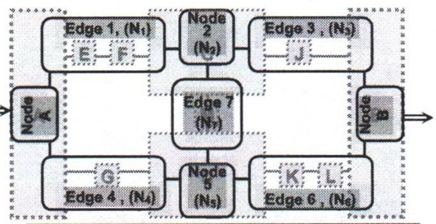




Meshed Network Structure

Unavailability Calculation for a meshed network:





we neglect it

Theorem of total probability (addition rule):

$$\text{Prob}\{a \cup b\} = \text{Prob}\{a\} + \text{Prob}\{b\} - \text{Prob}\{a \cap b\}$$

Possible paths from A to B

Path 1: Edge1+Node2+Edge3

Path 2: Edge4+Node5+Edge6

Path 3: Edge1+Node2+Edge7+Node5+Edge6

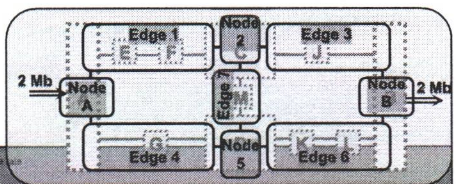
Path 4: Edge4+Node5+Edge7+Node2+Edge3

Unavailability N_{tot} for this meshed network, for $N_i << 1$:

$$\begin{aligned} N_{tot} \approx N_{A+B} &+ N_1\left[N_4(1-N_2) + N_5(1-N_2-N_3-N_4) + (N_6 \cdot N_7)\right] \\ &+ N_2\left[N_4(1-N_6) + N_5(1-N_3-N_4-N_6) + N_6(1-N_3)\right] \\ &+ N_3\left[N_5 + N_6 + N_4 \cdot N_7 - N_5 \cdot N_6\right] \end{aligned}$$

Example: Availability for a meshed network

(no current values)



SIEMENS

The FIT-values are statistical values and based only upon the random failures corresponding to the "flat" component of the typical bathtub curve. The calculation method is in acc. with IEC 61709!

Item-No	Name	λ_{main} FIT	$MTBF_{main}$ (year)	MDT (h)	Non-Availab.	Unprotect. Parts A,B	Edge 1 A,E,F,C	Node 2 C	Edge 3 C,J,B	Edge 4 A,G,D	Node 5 D	Edge 6 D,K,L,B	Edge 7 C,M,D
Main transmission path (Main Path)						Quantity of items for following items:							
SM10-1,21	Subrack DC	268	466,0	4	1,07E-6	2	2	1	1	1	1	2	2
SM10-5.1	CLU	2,024	56,4	4	8,10E-6	1+1	1+1	1+1	1+1	1+1	1+1	1+1	1+1
SM10-8.1	SWITCH FABRIC VC-4	2,302	49,6	4	9,21E-6	1+1	1+1	1+1	1+1	1+1	1+1	1+1	1+1
SM10-20.1	STM-16 BOARD	3,242	35,2	4	1,30E-5		4		3	3		4	3
SM10-20.32	STM-16 MODULE L-16.2/3	500	228,3	4	2,00E-6		6		4	4		6	4
SM10-6.1	SWITCH FABRIC VC-12	4,014	28,4	4	1,61E-5	1+1	1+1	1+1	1+1	1+1	1+1	1+1	1+1
SM10-25.2	IF2M CARD 63*E1 120ohm)	1,894	181,9	4	2,82E-6	1+1							
SM10-25.12	LSU CARD 63*E1 120ohm)	705	228,3	4	5,74E-6	2							
0,3 Failure intensity of fibre (failures(100km/year):													
	Fibre per km	342	333,3	12	4,11E-6		249 km		161 km	174 km		231 km	157 km
Non-availability of unprotected parts at Node A and Node B						7,78E-6							
Non-availability of each Edges/Nodes 1 ... 7							1,09E-3	1,07E-6	7,10E-4	7,63E-4	1,07E-6	1,02E-3	6,93E-4
Non-availability over all Edges/Nodes 1 ... 7										1,55E-6		w/o Edge 7	
Total Non-availability related to 2Mb end to end										9,341E-6		1,094E-5	
Total Non-availability in min/year related to 2Mb end to end										4,909		5,750	
Total Availability in % related to 2Mb end to end										[illegible]		[illegible]	

Computation rules: The signal flow through equipment is marked by the figures in the relevant columns.

Remark: The results for the main transmission path are related to one bi-directional signal/channel.

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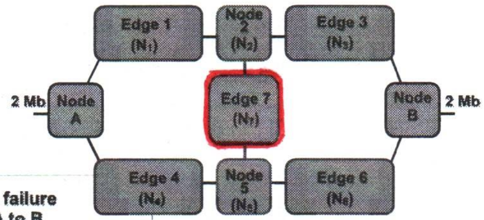
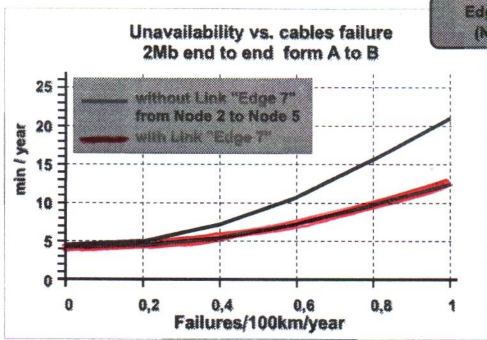
Availability of a continuously monitored network system

Meshed Network Non-availability Diagram

Example: Unavailability Diagram for a meshed Network





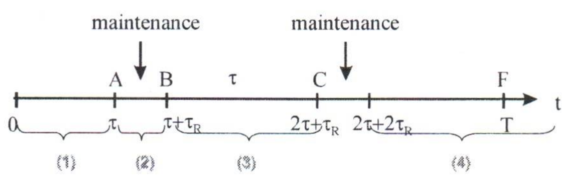
Availability of a component under periodic maintenance

- **Objective**: computation of the average unavailability over the lifetime $[0, T_M]$

$$\overline{q_{0T_M}} = \frac{\overline{T_D}}{T_M}$$

- **Hypoteses**:
  - The component is initially working: $q(0) = 0$; $p(0) = 1$
  - Failure causes:
    1. random failure at any time $T \sim F_T(t)$
    2. on-line switching failure on demand $\sim Q_0$ — it fails on demand: when we demand it, it fails
    3. maintenance disables the component $\sim \gamma_0$ (human error during inspection, testing or repair)

Component under periodic maintenance: a more realistic case



we have to calculate the istantaneous unavailability along the whole period [0, T]

1) The probability of finding the component DOWN at the generic time t is due either to the fact that it was demanded to start but failed or to the fact that it failed unrevealed randomly before t. The average DOWNtime is:

the switch works $(1-Q_0)$ however the component has failed before (cumulative distribution of the failure)

$$q_{0A}(t) = \underbrace{Q_0}_{\text{probability of failure on demand}} + (1-Q_0)F_T(t)$$

$$\underbrace{\overline{T}_{D(0A)}}_{\text{(mean) downtime in the part (0, A)}} = \int_0^\tau q_{0A}(t)dt = \int_0^\tau [Q_0 + (1-Q_0)\cdot F_T(t)]dt = Q_0\cdot\tau + (1-Q_0)\cdot\int_0^\tau F_T(t)dt$$

2) During the maintenance period the component remains disconnected and the average DOWNtime is the whole maintenance time:

$$\overline{T}_{D(AB)} = \tau_R$$

3) The component can be found failed because, by error, it remained disabled from the previous maintenance or because it failed on demand or randomly before *t*. The average DOWNtime is:

How can the component be unavailable?
- disabled by maintainence (γ_0)
- not disabled by maintainence ($1-\gamma_0$) but the switch is not working (Q_0)
- not disabled by maintainence ($1-\gamma_0$), the switch worked ($1-Q_0$) but the component has previously failed ($F_T(t)$)

not disabled by maintainence (1- prob. of making an error (γ_0))

$$q_{BC}(t) = \underbrace{\gamma_0}_{\text{disabled by error from the previous maintainence}} + (1-\gamma_0)\cdot\left[\underbrace{Q_0}_{\text{the switch is not working}} + \underbrace{(1-Q_0)\cdot F_T(t)}_{\text{the switch worked but the component is failed}}\right]$$

$$\underbrace{\overline{T}_{D(BC)}}_{\text{downtime in (BC)}} = \int_0^\tau q_{BC}(t)dt = \gamma_0\cdot\tau + (1-\gamma_0)\cdot\left[Q_0\cdot\tau + (1-Q_0)\cdot\int_0^\tau F_T(t)dt\right]$$

4) The normal maintenance cycle is repeated throughout the component lifetime T_M. The number of repetitions, i.e. the number of AB-BC maintenance cycles, is:

$$k = \frac{T_M}{\tau + \tau_R}$$

- The total expected DOWNtime is:

$$\overline{T}_D = \underbrace{Q_0\tau + (1-Q_0)\cdot\int_0^\tau F_T(t)dt}_{\text{first period}} + \underbrace{\frac{T_M}{\tau+\tau_R}}_{k}\cdot\left\{\underbrace{\tau_R}_{\text{unavailability due to the repair}} + \underbrace{\gamma_0\tau + (1-\gamma_0)\cdot\left[Q_0\cdot\tau + (1-Q_0)\cdot\int_0^\tau F_T(t)dt\right]}_{\text{possible unavailability in the period following an inspection}}\right\}$$

!!

$$\overline{q}_{T_M} = \frac{\overline{T}_{D(0T_M)}}{T_M} = \frac{Q_0\tau}{T_M} + \frac{1-Q_0}{T_M}\cdot\int_0^\tau F_T(t)dt + \frac{1}{\tau+\tau_R}\cdot\left\{\tau_R + \gamma_0\tau + (1-\gamma_0)\cdot\left[Q_0\cdot\tau + (1-Q_0)\cdot\int_0^\tau F_T(t)dt\right]\right\}$$

!!

average unavailability over the mission time (total downtime over all the mission time)

- Q_0 and $F_T(t)$ are generally small, and since typically $\tau_R \ll \tau$ and $\tau \ll T_M$, the average unavailability can be simplified to:

$$\overline{q}_{T_M} \cong \frac{\tau_R}{\tau} + \gamma_0 + (1-\gamma_0)\cdot\left[Q_0 + \frac{1-Q_0}{\tau}\cdot\int_0^\tau F_T(t)dt\right]$$

- Consider an exponential component with small, constant failure rate $\lambda \Rightarrow F_T(t) = 1 - e^{-\lambda t} \cong \lambda\cdot t$
- Since typically $\gamma_0 \ll 1$, $Q_0 \ll 1$, the average unavailability reads:

ratio of the downtime due to repair and the period

$$\boxed{\overline{q}_{0T_M} \cong \frac{\tau_R}{\tau} + \gamma_0 + Q_0 + \frac{1}{2}\cdot\lambda\cdot\tau}$$

average downtime within τ due to the fails of λ

Maintenance

Error after test

Switching failure on demand

Random, unrevealed failures between tests

average unavailability of a component undergoing periodic inspectional maintainence every τ with a duration of inspection: τ_R, and probability of failure on demand Q_0, a failure rate λ and a probability of error at test γ_0.