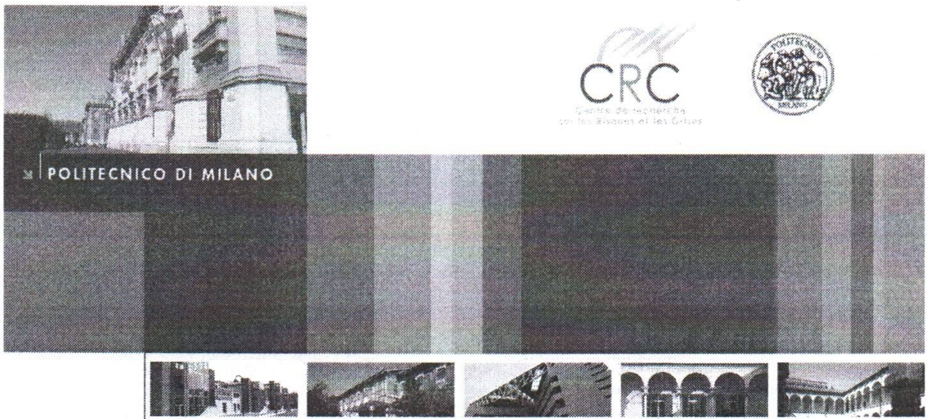


Importance Measures

— we want to identify the critical components in a system w.r.t. risk /reliability /contribution to the failure of the system
→ based on this we design the system

Introduction

- **Objective**: Importance Measures (IMs) aim at quantifying the contribution of components or basic events to the considered measure of system performance
- **Examples**:
  - **Nuclear**: risk measure (Core Damage Frequency, Large Early Release Frequency)
  - **Aerospace**: unreliability
  - **Power generation**: unavailability
- **Why**: great practical aid to system designers and managers: trace system bottlenecks and provides guidelines for effective system improvement

Introduction (continued)

there are 2 points of view of contribution of a component in a system :

- **How**: *ranking* and *categorization* with respect to
  - *risk-significance*: if the component failure or unavailability contributes significantly to system risk measure } how much a component is relevant for causing a failure
  - *safety-significance*: if the component plays an important role in the **prevention** of system undesired states (in system success) } how much a component is relevant for a system operation
- **Hypotheses**:
  - *n* binary components (0 or 1) functioning/failed
  - risk measures adopted: system reliability $R(t)$ (from the safety point of view) and failure probability $F(t) = 1 - R(t)$ (from the risk point of view)
  - $\underline{r}(t) = (r_1(t), r_2(t), \ldots, r_n(t))$ vector of the reliabilities at time t of the individual components
  - $R(\underline{r}(t))$ system reliability (depending on the reliability of the components and on the structure of the system)

1. Birnbaum's measure

- Definition

$$I_j^B(t) = \frac{\partial R(\underline{r}(t))}{\partial r_j(t)}$$

it tries to capture (locally) how much a deviation of the reliability of the system is due to a small change in the j-th component reliability

- It measures how much a change in the system reliability is due to a change in component j's reliability
- Properties:
  - $0 \le I_j^B(t) \le 1$
  - When $R(\underline{r}(t))$ is a linear function of $\underline{r}(t)$ and if all components are independent, then $I_j^B(t)$ does not depend on $r_j(t)$, $j = 1,2,\ldots,n$ — because $r_j(t)$ will disappear when taking the derivative

Birnbaum's example

- Consider various configurations of 3 binary components with r_1=0.98, r_2=0.96, r_3=0.94 (component 1 is the most reliable)

Configuration #	System configuration (system components)	R	I_1^B	I_2^B	I_3^B
I	Series (1-2)	0.9408	r_2=0.96	r_1=0.98	/
II	Parallel (1-2)	0.9992	1-r_2=0.04	1-r_1=0.02	/
III	2-out-of-3 (1-2-3)	0.9957	0.0952	0.0776	0.0584

$R(\underline{r}(t)) = r_1 \cdot r_2$, $I_1^B = r_2$, $I_2^B = r_1$

$R(\underline{r}(t)) = 1-(1-r_1)(1-r_2)$

$I_2^B > I_1^B \Rightarrow$ 2 is more important — and this is rational because the 1st component is more reliable and they're in series (it's enough if one breaks for the system to break) *

$I_1^B > I_2^B \Rightarrow$ 1 is more important — rational because it's a parallel system, both components can do the same thing (and the 1st is more reliable) *

* Series system? all components have to work, the more important is the less reliable (we have to pay more attention to it)
Parallel system? every component can perform the same action, the more important is the more reliable

- In a series $I_j^B(t)$ prioritizes components according to *increasing* reliability (it ranks from the lowest rel. to the highest)
- In a parallel $I_j^B(t)$ prioritizes components according to *decreasing* reliability (it ranks from the highest rel. the lowest)

Birnbaum's and Structure Function

Relation with the **structure function** (function that determines the state of the system).
Let's rewrite it extracting the indicator variable of the component j of which we want to calculate the importance measure by Birnbaum

- $$X_T = \Phi(X_1, X_2, \ldots, X_n) = \Phi(\underline{X})$$

$$\Phi[\underline{X}(t)] = X_j(t)\Phi[\underline{X}(t), X_j = 1] + (1 - X_j(t))\Phi[\underline{X}(t), X_j = 0]$$
$$= X_j(t)\{\Phi[\underline{X}(t), X_j = 1] - \Phi[\underline{X}(t), X_j = 0]\} + \Phi[\underline{X}(t), X_j = 0]$$

Hp: component independence

$R(\underline{r}(t)) = \mathbb{E}[\Phi(\underline{X}(t))]$
$r_j(t) = \mathbb{E}[X_j(t)]$
(because $X_j \in \{0/1\}$)

$$R(\underline{r}(t)) = r_j(t)\cdot\{E[\Phi[\underline{X}(t), X_j = 1]] - E[\Phi[\underline{X}(t), X_j = 0]]\} + E[\Phi[\underline{X}(t), X_j = 0]] =$$
$$= r_j(t)\cdot\{R(r_j = 1, \underline{r}(t)) - R(r_j = 0, \underline{r}(t))\} + R(r_j = 0, \underline{r}(t)) =$$
$$= r_j(t)\cdot\{R_j^+(t) - R_j^-(t)\} + R_j^-(t)$$

now the reliability of the system is linear on the reliability of the component j

- $R_j^+(t) = R(r_j = 1, \underline{r}(t)) = E[\Phi[\underline{X}(t), X_j = 1]]$ — the reliability of j is perfect, it never fails (highest reliability we can get w.r.t. j, j unbreakable)

  $R_j^-(t) = R(r_j = 0, \underline{r}(t)) = E[\Phi[\underline{X}(t), X_j = 0]]$ — the reliability of j is always 0, it always fails

Birnbaum's and Structure Function (2)

- $$I_j^B(t)=\frac{\partial R(\underline{r}(t))}{\partial r_j(t)}=R\left[r_j=1,\underline{r}(t)\right]-R\left[r_j=0,\underline{r}(t)\right]=R_j^+(t)-R_j^-(t)$$

it measures the difference in the system reliability considering the case in which component j can never fail and the system in which the component j is useless (never works) (it considers the two extreme cases w.r.t. component j)

$\Phi\left[\underline{X}(t),X_j=1\right]-\Phi\left[\underline{X}(t),X_j=0\right]=\{0,1\}$

$$I_j^B(t)=E\left\{\left[\Phi\left[\underline{X}(t),X_j=1\right]-\Phi\left[\underline{X}(t),X_j=0\right]\right]\right\}$$
$$=P\left\{\Phi\left[\underline{X}(t),X_j=1\right]-\Phi\left[\underline{X}(t),X_j=0\right]=1\right\}$$

! it's the probability that the difference between the state of the system when j functions and the state of the system when j doesn't function is equal to 1 → it gives the criticality of the component j

- $I_j^B(t)$ is the probability that $(\underline{X}(t),X_j=1)$ is a **critical** path vector, i.e. **the system is in such a state that it functions only when *j* functions**

Birnbaum's and Structure Function (3)

We can do the dual w.r.t. risk (we looked at reliability for safety, now we look at the failure prob. of the system)

- Let us denote the component's unreliability

$$q_j(t)=1-r_j(t)$$

$$I_j^B(t)=\frac{\partial R(\underline{r}(t))}{\partial r_j(t)}=R\left[r_j=1,\underline{r}(t)\right]-R\left[r_j=0,\underline{r}(t)\right]=\frac{\partial F(\underline{q}(t))}{\partial q_j(t)}=$$
$$=F\left[q_j=1,\underline{q}(t)\right]-F\left[q_j=0,\underline{q}(t)\right]=F_j^+(t)-F_j^-(t)$$

- $$F_j^+(t)=F\left[q_j=1,\underline{q}(t)\right]=P\left\{\Phi\left[\underline{X}(t),X_j=0\right]=0\right\}$$
$$F_j^-(t)=F\left[q_j=0,\underline{q}(t)\right]=P\left\{\Phi\left[\underline{X}(t),X_j=1\right]=0\right\}$$

≠ $R_j^+(t)$ which was for $X_j=1$

now the "+" is when the component gives the maximum contribution to the indicator which now is the failure

2.

Criticality measure

- Let $C\left[\underline{X}(t),X_j=1\right]$ be the event such that *j* is critical & work

$$P\left\{C\left[\underline{X}(t),X_j=1\right]\right\}=I_j^B(t)$$

- Probability that *j* is critical and failed at time *t* is

$$P\left\{C\left[\underline{X}(t),X_j=1\right]\cap\left[X_j(t)=0\right]\right\}=I_j^B(t)\cdot\left[1-r_j(t)\right]$$

- Definition: probability conditioned on system failure

CRITICALITY MEASURE
probability that j is critical and it's failed, conditioned to the fact that the system is failed ($\Phi[\underline{X}(t)]=0$)

$$I_j^{cr}(t)=P\left\{C\left[\underline{X}(t),X_j=1\right]\cap\left[X_j(t)=0\right]\middle|\Phi\left[\underline{X}(t)\right]=0\right\}$$
$$=\frac{P\left\{C\left[\underline{X}(t),X_j=1\right]\cap\left[X_j(t)=0\right]\right\}}{P\left\{\Phi\left[\underline{X}(t)\right]=0\right\}}=\frac{I_j^B(t)\cdot\left[1-r_j(t)\right]}{1-R(\underline{r}(t))}=\frac{I_j^B(t)\cdot q_j(t)}{1-R(\underline{r}(t))}$$

criticality of j measuring its contribution to the failure of the system by considering situations in which j is critical and it's failed and the system is failed

- Probability *j* caused system failure, given system is failed !!

Criticality example

- Consider various configurations of 3 binary components with r_1=0.98, r_2=0.96, r_3=0.94

Configuration #	*System configuration (system components)*	I_1^{cr}	I_2^{cr}	I_3^{cr}
I	Series (1-2)	$\frac{I_1^B(1-r_1)}{1-r_1r_2}=0.3243$	$\frac{I_2^B(1-r_2)}{1-r_1r_2}=0.662$	/
II	Parallel (1-2)	$\frac{I_1^B(1-r_1)}{1-r_1-r_2+r_1r_2}=1$	1	/
III	2-out-of-3 (1-2-3)	0.4428	0.7219	0.8149

- In a series system, the most important component according to I^{cr} is the least reliable one (critical) (because it becomes the cause of failure)
- In a simple parallel, is the same value (both/+ are critical in the same way)
- In a general parallel, I^{cr} increases with decreasing component reliability, opposite to Birnbaum's measure (the smaller the reliability of a component, the more important it is the component

3.

Fussel-Vesely importance measure

- A component may contribute to the system risk measure without being critical
- The component contributes to system failure when a minimal cut set (mcs), containing the component, occurs
- $I_j^{FV}(t)=$ probability that at least one mcs containing j is verified at time t, given that the system is failed at t
- Let:
  - m_j = number of mcs containing component j, j = 1, 2, …, n
  - $M_{jh}(t)$ = h-th mcs among those containing component j, verified at time t
  - $D_j(t)$ = event that at least one mcs that contains component j is verified at time t = $(M_{j1}(t) \cup M_{j2}(t) \cup \ldots \cup M_{jm_j})$

Fussel-Vesely importance measure (2)

probability that one of the minimal cut sets containing j occurs given that the system is failed

- $$I_j^{FV}(t)=P\{D_j(t)\,|\,\Phi[\underline{X}(t)]=0\}=\frac{P\{D_j(t)\cap\Phi[\underline{X}(t)]=0\}}{P\{\Phi[\underline{X}(t)]=0\}}=\frac{P\{D_j(t)\}}{P\{\Phi[\underline{X}(t)]=0\}}$$

- **Hypotheses**:
  - Independent components $\longrightarrow$ $P\{M_{jh}(t)\}=\prod_{l\in M_{jh}}(1-r_l(t))$ failure of all the components of the h-th minimal cutset containing the component j
  - Independence of cut-sets containing j $\rightarrow$ $P\{D_j(t)\}\cong 1-\prod_{h=1}^{m_j}[1-P\{M_{jh}(t)\}]$ IP(at least one minimal cutset containing j occurs) = 1 − IP(no one) = 1 − π IP(not this one) = 1 − π[1 − IP(this one)]

$$I_j^{FV}(t)\cong\frac{1-\prod_{h=1}^{m_j}[1-P\{M_{jh}(t)\}]}{1-R(\underline{r}(t))}$$

  - Rare-event approximation

components failures are rare (because the goal is to make them reliable), probabilities are small, and so: $1-\pi[1-IP(\cdot\cdot)] \approx \sum IP(\cdot\cdot)$

$$I_j^{FV}(t)\cong\frac{\sum_{h=1}^{m_j}P\{M_{jh}(t)\}}{1-R(\underline{r}(t))}=\frac{\sum_{h=1}^{m_j}P\{M_{jh}(t)\}}{F(t)}=\frac{F(t)-F_j^-(t)}{F(t)}$$

$= \dfrac{\text{IP(failure of the system)} - \text{IP(failure of the system when component j is failed)}}{\text{IP(failure of the system)}}$

Fussel-Vesely example

- Consider various configurations of 3 binary components with r_1=0.98, r_2=0.96, r_3=0.94

Configuration #	System configuration (system components)	I_1^{FV}	I_2^{FV}	I_3^{FV}
I	Series (1-2)	$\frac{1-r_1}{1-r_1 r_2}=0.3378$	$\frac{1-r_2}{1-r_1 r_2}=0.6757$	/
II	Parallel (1-2)	1	1	/
III	2-out-of-3 (1-2-3)	0.4651	0.7442	0.8372

- Value similar to I^{cr}: both aim at quantifying the contribution of a component to the system failure probability
- In a series the difference is evident!
- In the **parallel** system configuration II, the system itself constitutes a minimal cut set → $I_1^{FV}=I_2^{FV}=1$.

4. →

RAW Risk Achievement Worth

very important importance measures from the point of view of risk

- Definition:

$$RAW_j(t)=\frac{F\left[q_j=1,\underline{q}(t)\right]}{F(t)}=\frac{F_j^+(t)}{F(t)}$$

how much we gain if we don't let the component fail

worst case for the component j: $F_j^+(t)$ (the component is always failed) over the actual value of risk $F(t)$

- Ratio of the risk when component *j* is considered always failed in (0,*t*) (q_j = 1, X_j = 0) to the actual value of the risk
- It highlights the importance of maintaining the current level of reliability with respect to the basic failure event
- Appropriate for **temporary** changes, may be misleading for **permanent** changes (often not complete unavailability)

The highest the $RAW_j(t)$, the more critical the component is
→ the highest the $RAW_j(t)$, the more the component j needs for an intervention (to make it more reliable)

5. →

RRW Risk Reduction Worth

- Definition:

$$RRW_j(t)=\frac{F(t)}{F\left[q_j(t)=0,\underline{q}(t)\right]}=\frac{F(t)}{F_j^-(t)}$$

true unreliability of the system over the best scenario possible: $F_j^-(t)$ (smallest probability of failure of the system w.r.t. the contribution of j)

- Ratio of the actual value of the risk to the risk when component *j* is always available in (0,*t*) (q_j = 0, X_j = 1)
- It measures the potential of component *j* in reducing the risk
- Useful for identifying **improvements** towards risk reduction

The highest $RRW_j(t)$ the highest the gain in reliability if we work on (improve) the component j
→ high RRW_j → work on j to reduce the true failure probability (because we gain the most)

RAW & RRW Example

- Consider various configurations of 3 binary components with r_1=0.98, r_2=0.96, r_3=0.94

Configuration #	*System configuration (system components)*	RAW_1	RAW_2	RAW_3
I	Series (1-2)	$\frac{1}{q_1+q_2-q_1q_2}=16.89$	$\frac{1}{q_1+q_2-q_1q_2}=16.89$	/
II	Parallel (1-2)	$\frac{q_2}{q_1q_2}=\frac{1}{q_1}=50$	$\frac{1}{q_2}=25$	/
III	2-out-of-3 (1-2-3)	22.67	18.31	13.75

Configuration #	*System configuration (system components)*	RRW_1	RRW_2	RRW_3
I	Series (1-2)	$\frac{q_1+q_2-q_1q_2}{q_2}=1.48$	$\frac{q_1+q_2-q_1q_2}{q_1}=2.96$	/
II	Parallel (1-2)	$\frac{q_1q_2}{0}=\infty$	$\frac{q_1q_2}{0}=\infty$	/
III	2-out-of-3 (1-2-3)	1.79	3.58	5.38

RAW & RRW Example (2)

- In a series components have the same RAW and are ranked by RRW in increasing order of failure probability
- In a parallel components have the same RRW and are ranked by RAW in decreasing order of failure probability
- In a parallel the achievement in risk (RAW) is highest if the most reliable component is taken out of service (failed)
- In a series the reduction in risk (RRW) achievable by improving the component to perfection is highest for the components which contribute most to the system failure, i.e, the least reliable

Comments

system failure probability

- Let us write the risk metric F

$$F = \alpha_j \cdot q_j + \beta_j$$

- q_j = unavailability of j

  $\alpha_j = F_j^+ - F_j^-$ = coefficient of q_j in the risk equation

  $\beta_j = F_j^-$ = collection of all the other terms of F with $q_j = 0$

important measures

- Which IMs are more appropriate to rank or categorize components by risk-significance or by safety-significance?

Comments (2)

- **Fussel-Vesely** → risk significance

$$I_j^{FV} = \frac{\alpha_j q_j + \beta_j - \beta_j}{\alpha_j q_j + \beta_j} = \frac{\alpha_j q_j}{\alpha_j q_j + \beta_j} \approx \frac{\alpha_j q_j}{\beta_j}$$

the Fussel-Vesely importance of component j is proportional to the probability of component j failure (direct representation of the contribution of j to the risk metric)

  - The assumption $\alpha_j q_j << \beta_j$ is verified in high-risk installations
  - I_j^{FV} is proportional to the unavailability of component j and represents the contribution of j to the risk metric F
  - I_j^{FV} represents the risk-significance of component j

- **RAW** → typically safety significance

$$RAW_j = \frac{\alpha_j + \beta_j}{\alpha_j q_j + \beta_j} \approx \frac{\alpha_j}{\beta_j} + 1$$

it doesn't depend on q_j → safety significance (how much it's worth to keep the good reliability that the component has for the reliability of the system)

  - For $\alpha_j q_j << \beta_j$ RAW_j is independent on q_j → degree of defence against failure provided by the rest of the installation
  - A high RAW_j means component j is highly safety-significant since the risk increase due to the unavailability of the component is high → **prevention**!

Comments (3)

used to decide which component should be prioritized in the maintenence

- **Birnbaum** → appropriate for establishing test and maintenance programs
- **Fussel-Vesely** and **Birnbaum** → appropriate for the system design phase:
  - I^{FV} is used for selecting the components candidate for improvement because most contributing to the risk
  - I^B allows identifying for which components the improvements are more effective
- **Fussel-Vesely** → appropriate for identifying components most probably causing system failure (help set up a repair priority checklist)

Drawbacks

1. IMs deal with changes in risk only **at the extremes (0,1)** of the defined ranges of probability

always functioning/ always failed

2. IMs rank only individual components or basic events whereas they are not directly applicable to combinations or groups (e.g. change in technical specifications or component's failures made up of more basic events)

the role of one component is never isolated w.r.t. a group

3. IMs do not typically consider the credible uncertainty range of components' unavailabilities
4. IMs have been mainly applied to systems made up of binary components (i.e. functioning or faulty)

good way to smoothly see the importance of investing for improving component j w.r.t. its impact on the probability of failure of the system

1. Generalized risk importance measure

partial change of the unreliability of the system (it's not from 0 to 1, but partially: from the value q_j to another value: q_j^n) (q_j new)

- Consider the following relative change in risk due to a change in the probability of the basic failure event j from the value q_j to the value q_j^n

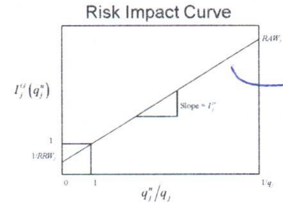
$$\frac{\Delta F_j}{F} = \frac{F_j^n - F}{F} = \left\{F\left[q_j = 1, \underline{q}(t)\right] - F\left[q_j = 0, \underline{q}(t)\right]\right\}\left(\frac{q_j^n - q_j}{F}\right)$$

$$I_j^G\left(q_j^n\right) = \frac{F_j^n}{F} = \left\{F\left[q_j = 1, \underline{q}(t)\right] - F\left[q_j = 0, \underline{q}(t)\right]\right\}\left(\frac{q_j^n - q_j}{F}\right) + 1$$

importance measure of component j when is taken at a level of unreliability q_j^n

$$I_j^G\left(q_j^n\right) = \frac{F_j^n}{F} = I_j^{cr}\left(\frac{q_j^n}{q_j}\right) + \frac{1}{RRW_j}$$

Risk Impact Curve



linear in q_j^n

2. Importance measures for multiple basic events

- No simple relation between the importance measures of single components and a group
- Let us consider the indicator variable T of the top event

top event structure function (in function of the states of the components ($X_A, X_B, \ldots$)) (note: C_1, C_2, C_3 are a type of component)

$$T = AB(C_1 + C_3) + DE(C_2 + C_4) + F(C_1 + C_3)(C_2 + C_4) + GH$$

(we write A instead of X_A)

- **RAW**:

$$RAW(C_1) = \frac{F_j^+}{F} = \frac{AB(1 + C_3) + DE(C_2 + C_4) + F(1 + C_3)(C_2 + C_4) + GH}{AB(C_1 + C_3) + DE(C_2 + C_4) + F(C_1 + C_3)(C_2 + C_4) + GH} \quad \boxed{C_I = 1}$$

  - Substitute C_i = 1? **NO**: 2 appears in structure function
  - Adding RAWs for basic events? **NO**
  - Substitute C_i = C and Boolean reduction and C = 1? **YES**

we put C_1 and C_3 as one component, C_2 and C_4 as one component

$$T = ABC + DEC + FC + GH$$

$$RAW(C) = \frac{AB + DE + F + GH}{AB(C_1 + C_3) + DE(C_2 + C_4) + F(C_1 + C_3)(C_2 + C_4) + GH}$$

2. Importance measures for multiple basic events (2)

- **Birnbaum**:

$$I_j^B = F\left[q_j = 1, \underline{q}(t)\right] - F\left[q_j = 0, \underline{q}(t)\right]$$

  - Substitute C_i = 1 with **no** Boolean reduction?

$$I_C^B = 2AB + 2DE + 4F$$

  - Substitute C_i = C and Boolean reduction and C = 1?

$$I_C^B = AB + DE + F$$

  - Both are **unappropriate!**

- **Fussel-Vesely**:

$$I_C^{FV}(C) = \frac{AB(C_1 + C_3) + DE(C_2 + C_4) + F(C_1 + C_3)(C_2 + C_4)}{AB(C_1 + C_3) + DE(C_2 + C_4) + F(C_1 + C_3)(C_2 + C_4) + GH}$$

  - Any cutset with a contribution from any C_k of the group is included!
  - Already appropriate measure of group importance!
  - No additivity

2. Importance measures for multiple basic events (3)

- **Differential Importance measure (DIM)** :
  - Sensitivity measure that ranks the parameters of the risk model according to the fraction of the total change in the risk that is due to a small change in the parameters' values, taken one at a time
  - Additivity!

$$F = F(p_1, p_2, \ldots, p_{Np})$$

$$dF = \frac{\partial F}{\partial p_1} \cdot dp_1 + \frac{\partial F}{\partial p_2} \cdot dp_2 + \ldots + \frac{\partial F}{\partial p_{N_p}} \cdot dp_{N_p}$$

$$\mathrm{DIM}(p_i) = \frac{dF_{p_i}}{dF} = \frac{\frac{\partial F}{\partial p_i} \cdot dp_i}{\frac{\partial F}{\partial p_1} \cdot dp_1 + \frac{\partial F}{\partial p_2} \cdot dp_2 + \ldots + \frac{\partial F}{\partial p_{N_p}} \cdot dp_{N_p}}$$

3. IMs uncertainty

- F and F_j- are to be considered as random variables characterized by given probability distributions
- IMs is a random variable for which specific statistics can be calculated!

4. Binary components

- Extensions to **multi-state** systems! – a component can be failed / be functioning but can also be degraded (work partially) (functioning at given %, not only 0%/100%)