CS170–Fall 2013 — Solutions to Homework 1

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January 19, 2014

1.

(a) $f(n) = \Omega(g(n))$
Both are power functions. The bigger the power is, the faster it grows.

(b) $f(n) = \Theta(g(n))$
Both are linear function add polylog term.

(c) $f(n) = \Theta(g(n))$
ignore the constant terms to get two same power functions.

(d) $f(n) = \Theta(g(n))$
since $\log 500n = \log 500 + \log n$, and $\log 8n = \log 8 + \log n$ so ignore the constant terms to get two same log functions.

(e) $f(n) = \Theta(g(n))$
since $\log n^7 = 7 \log n$, both log functions have the same growth rate.

(f) $f(n) = \Theta(g(n))$
$5n \log 5n = 5n(\log 5 + \log n) = 5n \log 5 + 5n \log n$ ignore the lower term.

(g) $f(n) = O(g(n))$
limit of g(n)/f(n) goes to infinite as n goes to infinite.

(h) $f(n) = O(g(n))$
log function grows lower than power function with power greater than zero.

(i) $f(n) = O(g(n))$
same as (h)

(j) $f(n) = \Omega(g(n))$
same reason as (h).

(k) $f(n) = O(g(n))$
$f(n) = O(n^2)$, and $n^k = O(g(n))$.

(l) $f(n) = O(g(n))$
$n^(1/3) = O(a^n)$, and $a > 1$

(m) $f(n) = \Omega(g(n))$
divide both functions by 4^n, become $f(n) = (\frac{5}{4})^n$.

(n) $f(n) = O(g(n))$

(o) $f(n) = \Theta(g(n))$

(p) $f(n) = O(g(n))$

(q) $f(n) = O(g(n))$

2.

(1) To show $\sum_{i=1}^{n} i^k = O(n^{k+1})$:

$$\sum_{i=1}^{n} i^k = 1^k + 2^k + ... + n^k, and \sum_{i=1}^{n} n^k = n^{k+1}$$

Apparently, $\sum_{i=1}^{n} i^k \leq \sum_{i=1}^{n} n^k = n^{k+1}$
To show $\sum_{i=1}^{n} i^k = \Omega(n^{k+1})$:

$$\sum_{i=n/2}^{n} (\frac{n}{2})^k = (\frac{n}{2})(\frac{n}{2})^k = \frac{n^{k+1}}{2^{k+1}}$$

Apparently, $\sum_{i=1}^{n} i^k \geq \sum_{i=n/2}^{n} i^k = \frac{n^{k+1}}{2^{k+1}}$
Therefore, $\sum_{i=1}^{n} i^k = \Theta(n^{k+1})$.

(2)

(3)

3.

(a)

(b)

(c)

4.

(a)

(b)

(c)

(d)

5.

6.

(a)

(b)

(c)

(d)

(e)