Astro 270 – Astrophysical Dynamics – The Orbits of Stars

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1. Orbits in static spherical potentials

Our fundamental equation in this chapters are the equations of motion which are derived from the Lagrangian

$$L = r^2\dot{\phi} \tag{1}$$

Our equation of motion is:

$$u^2 + \frac{2[\Phi(1/u) - E]}{L^2} = 0 \tag{2}$$

where u is 1/r

The time that it takes a star to go from apocenter to pericenter back to apocenter is defined as the **??**. Since this is a spherical symmetric potential, pericenter and apocenter are always the same for any orbit, though it may precess. The azimuthal period is the amount of time a particle will take to complete one orbit (traverse 2π of ϕ if it travelled at the average speed. This will only be a rational number if the orbit is closed, i.e. the beginning of an orbit ends at the same position. There are two cases where all bound orbits are closed. Note that *all* potentials have at least one orbit that is bound - that of a circular orbit.

1.1 Examples of Spherical Potentials

1.1.1 Spherical Harmonic Oscillator

In this case, the radial period is π/Ω. In order, the particle only needs to traverse half of one orbit to reach back to the same distance. This is because all of the orbits are ellipses centered on the center of the potential. This is in contrast to the Kepler potential which has radial period of $2\pi/\Omega$, where the ellipse has its *focus* at the center of the potential

1.1.2 Kepler Potential

1.1.3 Isochrone Potential

1.1.4 Hyperbolic Encounters

1.2 Constants and integrals of motion

A **constant of motion** is a quantity that

$$C(\mathbf{x}(t_1)\mathbf{v}(t_1);t_1) = C(\mathbf{x}(t_2)\mathbf{v}(t_2);t_2) \tag{3}$$

This is different from an integral of motion because you need an initial condition in order to calculate one of these quantities, i.e. you need to know the value at some particular time in order to be able to get it at a different time. Generally any initial conditions or boundary conditions can be considered constants of motion. Therefore, any orbit, which has six coordinates in phase space has six constants of motion. From the current time and the six coordinates and phase space it is possible to determine the position in phase space.

Integrals of motion are determined solely by the potential, and are invariant in time. They limit the amount of phase space that a particle can inhabit. Thus the number of integrals of motion describe how many coordinates are required to specify the location in phase space. In other words, having n integrals of motion means that any particle's orbit will lie on a 6-n dimensional subspace of x. A non-isolating integral is one that does not limit the number of dimensions that the orbit can lie on. They're fucking useless.

2. Orbits in Axisymmetric Potentials

Effective potential

$$\Phi_{eff} = \Phi(R,z) + \frac{L_z^2}{2R^2} \tag{4}$$