Phys 220A – Classical Mechanics – Lec02

UCLA, Fall 2014

Tuesday 7$^{\text{th}}$ October, 2014

1 Introduction

A couple of general comments about Newton's laws.

1.1 Something funny about Lorentz force and magnetic fields

Consider the Lorentz force between two charged particles, given by $\mathbf{F}_{12} = q\mathbf{v}_1 \times \mathbf{B}_2$. Consider particle 1 to lie on the positive y axis with velocity in the positive y direction and particle 2 to lie on the positive z axis with velocity in the positive x axis. Then we find that $\mathbf{F}_{12} \neq 0$ points in the positive x direction yet $\mathbf{F}_{21} = 0$.

What's going on here? Is this a case of violation of angular momentum conservation? No, in fact the EM field itself carries angular momentum.

1.2 Virial Theorem (useful in stat. mech)

Take some function f and write the time average

$$\langle f \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{t_0}^{t_0+T} f(t)dt. \tag{1}$$

Note that there is no dependence on t_0 for a bounded quantity,

$$\frac{\partial}{\partial t_0} \langle f_0 \rangle = \lim_{T\to\infty} \frac{f(t_0+T) - f(t_0)}{T} = 0 \tag{2}$$

which vanishes because the numerator in the limit is bounded.

Next, we hope to obtain an expression for $\langle T \rangle$ for conservative forces. Notice that

$$\sum_i m_i \ddot{\mathbf{x}}_i \cdot \mathbf{x} = \sum_i \mathbf{F}_i \cdot \mathbf{x}_i, \tag{3}$$

which we can rewrite as

$$\frac{d}{dt}\left(\sum_i m_i \dot{\mathbf{x}}_i \cdot \mathbf{x}\right) - \underbrace{\sum_i m_i \dot{\mathbf{x}}_i^2}_{2T} = -\sum_i \boldsymbol{\nabla}_i V \cdot \mathbf{x}_i. \tag{4}$$

If the motion is bounded then the derivative goes away when we average, so averaging over time we find

$$2\langle T\rangle = \left\langle \sum_i \boldsymbol{\nabla}_i V \cdot \mathbf{x}_i \right\rangle . \tag{5}$$

This is the Virial theorem. It is especially useful if V is homogeneous, i.e.

$$V(\lambda\mathbf{x}_1, \dots \lambda\mathbf{x}_n) = \lambda^k V(\mathbf{x}_1, \dots, \mathbf{x}_n) \tag{6}$$

for some constant k and any value of λ. Taking the derivative of this equation w.r.t. λ we have

$$\sum_i \boldsymbol{\nabla}_i V(\lambda\mathbf{x}_1, \dots, \lambda\mathbf{x}_n) \cdot \mathbf{x}_i = k\lambda^{k-1} V(\mathbf{x}_1, \dots, \mathbf{x}_n) \tag{7}$$

and setting $\lambda = 1$ we find $\sum_i \boldsymbol{\nabla} V \cdot \mathbf{x}_i = kV$. Thus the Virial theorem for a homogeneous potential gives us

$$2\langle T\rangle = k\langle V\rangle . \tag{8}$$

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