MAY 2021

ILLIQUIDITY AND ASSET PRICES

MASTER PROJECT - MASTER 2 FINANCIAL MARKETS

EDHEC BUSINESS SCHOOL

PAULINE COHEN
JEANNE ROMULUS
ALEXIA DEFORGE

TABLE OF CONTENTS

Introduction

The liquidity of the markets is a worldwide and perennial issue for investors. According to Market Liquidity, Theory, Evidence and Policy written by Marco Pagano, illiquidity results from the fact that not every player is active simultaneously in all the financial markets, and that not all of those active players have the same level of information about the security's fundamentals. In his article Efficient Capital Markets : a Review of Theory and Empirical Works, Eugene Fama explains that “in an efficient market, at any point in time, the actual price of a security will be a good estimate of its intrinsic value”. However, the amount of flows does not depend only on the amount information available but are also executed in reaction to speculative noise. Marco Panago comes to the conclusion that when prices are pressure by trading forces rather than through a public announcement, liquidity suffers.

The goal of our Master Project is to assess there is a premium offered by illiquid securities. Indeed, illiquid stocks are harder to trade and should offer higher returns to their owners to compensate for the liquidity risk. To introduce our subject, we first decided to plot BlackRock indexes to represent the returns of micro-cap, small-cap and large-cap since 2005 as the market capitalization is very correlated to the liquidity.

Figure 1:

Source: Python

We can see that since 2005, the indexes Micro-Cap and Small-Cap do not always outperform the Large Cap index. Indeed, illiquid stocks should offer higher returns but are also highly volatile. The creation of illiquid against liquid portfolios captures more efficiently the illiquid premium. We first calculated the illiquidity Amihud’s measure and tried to measure the ratio differences between stocks, sectors, countries, and periods. Indeed, the first goal of our thesis is to understand which industries are the most illiquid, to identify the period when the market (both liquid and illiquid securities) encountered liquidity issues and assess the correlation of illiquid periods and a high volatility of financial markets. Then, we tried and demonstrated that illiquid portfolios on average (against an individual stock perspective) did offer higher returns through the Fama-French regressions from 2010 to 2018.

We wanted to capture the illiquidity premium through the traditional Amihud’s measure but we also created an alternative measure of liquidity based upon the bid-ask spread theoretical approach to track once again the premium of our portfolios. In the end, we also built trading strategies to capture the illiquidity premium on short term periods at time when the market is Fama-French efficient.

Question 1

Amihud's measure and statistics

In this question, we are looking to evaluate the illiquidity of stocks using the Amihud's illiquidity measure:

$$Illiq_{i,t} = \frac{1}{n}\sum_{d}\frac{|R_{d,i}|}{VOLD_{d,i}}$$

Stocks such as Apple have a ratio around 10^{-12} whereas illiquid stocks are around 10^{-8} which represents a ratio difference of 10^{-9} between both securities.

Figure 2:

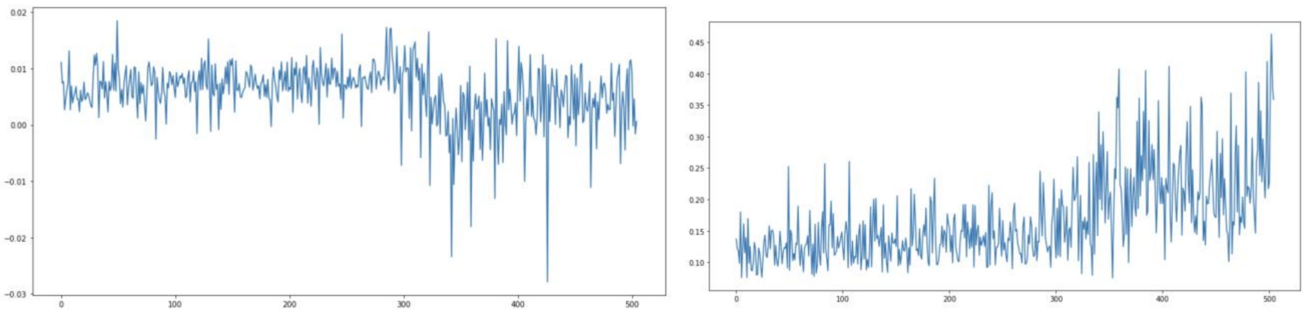
mean	2.10309E-08
std	9.8529E-08
min	2.0526E-12
max	1.70618E-06

Source: Python

On the whole period, we can see that we have on average there is a difference of almost 10^{-6}. The difference appears very small but shows a deep difference of liquidities. The difference between ratios is very small, as illiquid stocks are still traded on the market (against private equity). We can already assess some limits to Amihud's illiquidity measure: if a time t, a stock's volume is 0, the Amihud measure become inefficient (is not defined), so cannot always measure the illiquidity of a company.

Relationship between the Amihud's measure and expected returns

Figure 3 and 4: Ranking of the average of the expected and the absolute log-returns against illiquidity



Source: Python

We ranked all the stocks according to their illiquidity measure and plot their returns to see if we could already see a rising trend in the returns of the most illiquid stocks. However in this first graph, we cannot see a rising trend as we move from liquid to illiquid securities. Most notably, we see that for relatively liquid securities ranging from 0 to 350, the volatilities of the returns are fairly small compared to the more illiquid securities. Nonetheless, starting 350, there is a slight uptake in the returns which is a first interesting result. It means that to capture the liquidity premium of stocks, securities need to be highly illiquid. In the second graph, we also plotted the absolute returns because we figured that a first strategy based upon the liquidity of securities could capture illiquid premium if it was allowed to short the securities with negative returns.

Liquidity comparison between stocks and sectors

We evaluated the illiquidity of securities using the Amihud's liquidity measure. We need to do some comparisons to understand the meaning of this measure.

Between stocks

Figure 5: Apple and Xoma Amihud's illiquidity ratio

AAPL	2.08E-12
XOMA	2.70E-08

Source: Python

The first thing to look at is the ratio difference between a stock with a big market capitalization against a lower one. For example, when we compute the Amihud's illiquidity measure we obtained $2E^{-12}$ for Apple, (Market Cap = 2.23 Bn) in 2018 and $3E^{-8}$ for Xoma Corporation (Market Cap = 400M). Smaller companies tend to be more illiquid.

Between sectors

Figure 6: Ranking of the Illiquidity ratio average by sectors

	Utilities	**Materials**	**Energy**	**Real Estate**	**Health Care**	**Communication**	**Information Technology**	**Industrials**	**Financials**	**Consumer Discretionary**	**Consumer Staples**
Ratio	3.1E-09	3.76E-09	4E-09	4.41E-09	4.61E-09	5.13E-09	5.74E-09	6.52E-09	1.38E-08	1.55E-08	1.82E-08

Source: Python

We also computed the illiquidity ratio for sectors. It is interesting to see that the Real Estate industry is in the middle. Indeed, the Real Estate is generally considered as one of the most illiquid sectors. Nonetheless, the Real Estate public stocks has to be traded even more to compensate its core illiquidity and that is why the Real Estate sector remains in the middle ranking. It should also be noted that the Information Technology sector is highly illiquid. At first, the tech industries could have been considered as very liquid (Apple, Facebook, Netflix etc.). But as many illiquid stocks are in tech companies (Zhone Technologies, NXP Semiconductors), they are exerting upward pressure on the Amihud's liquidity measure. All things considered, the illiquidity ratios remain very similar.

Liquidity comparison between asset classes

It is also interesting to see the difference in illiquidity between two different asset classes. We wanted to analyze the difference of liquidity ratios between the US 10-year Treasury bond and the Real Estate. As mentioned above, the Real estate sector is one of the most illiquid sectors. But in the public market, Real Estate has to be traded more which exert downward pressure on its Amihud's liquidity measure. So we cannot compare the difference between ratios but we still wanted to mention this qualitative argument that liquidity can also differ between asset classes.

In The Big Short Michael Lewis describes several of the key players in the creation of the credit default swap (CDS) market, who sought to bet against collateralized debt obligations (CDO) on Real Estate and therefore who ended up profiting from the financial crisis of 2007-2010. To swap the risk of default, the lender buys a CDS from another investor who agrees to reimburse the lender in the case the borrower defaults. Nonetheless, when the Real Estate bubble actually burst and the Real Estate Market defaulted, the buyers of CDS on Real Estate almost did not obtain their reimbursement because the CDS market was highly illiquid: those key players represented a very small number of participants in the market.

Liquidity comparison between countries

Figure 7: Banks' illiquidity ratios from different countries

JP Morgan	1.11E-11
BNP Paribas	1.03E-10
BBVA Argentina	3.19E-08
Standard Bank	1.94E-07

Source: Python

Here we decided to plot the ratio illiquidity difference banks from the United States, France, Argentina and Africa : JP Morgan, BNP Paribas, Banco BBVA Argentina and Standard Bank Group Limited. The difference between illiquidity ratios is also very clear. There are more active players in the US Market of French Market than in Argentina and in South Africa.

Liquidity comparison according to the period with the VIX

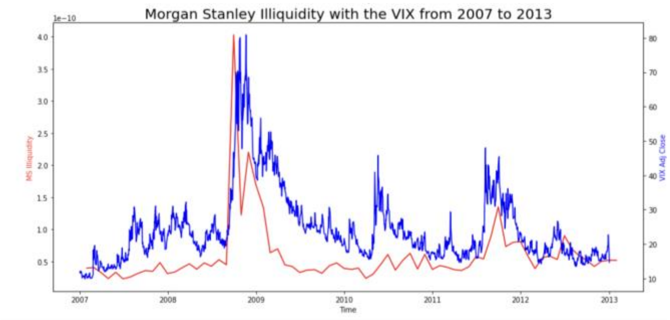
Finally, major events happening in the world from the Subprime Crisis to the ongoing health crisis, can poses a serious systemic threat to the liquidity of companies. Strictly speaking, a financial crisis is triggered by a sudden drop in the value of assets, sufficiently important to pose a risk of liquidity or even solvency on financial institutions, households, and even States. In that case, Bagehot in 1873 in *Lombard Street* encouraged the Central Bank to intervene to facilitate access to liquidity. Central banks can use interest rates, to support the economy as it did between the e-crash and 2002: the Fed's key rate was then dropped from 6% to 1% to promote the resumption of activity. Between September 2007 and January 2008, the US key rate was lowered by 2.25 basic points. Nonetheless, in reaction to the current crisis, central banks were stuck in what John Maynard Keynes called the "Liquidity Trap" as interest rates were already very low in the United States and became negative in Europe in 2019. Given that interest rates were bumping up against the zero lower bound, there was limited scope for further stimulus if the economy falters again.

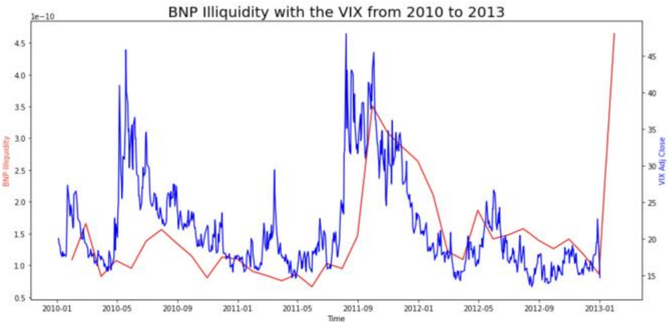
However, there is a risk, highlighted by former Treasury Secretary and Harvard Professor Larry Summers, that the fiscal stimulus of the Biden Administration could reignite inflation and exert upward pressure on US yields. The rise in yields poses a serious threat to the current excessive valuation of tech stocks, and could unveil a financial bubble. To quote Warren Buffet, "only when the tide goes out do you discover who's been swimming naked". According to Shiller and Akerlof, "Animal Spirits" on the financial markets can lead to major illiquidity issues. This will surely be one of the critical issues of the coming months.

Illiquidity of the Financial Markets during crisis

It is interesting to plot multiple time periods to build an analysis. For example, if we take the illiquidity ratio of Morgan Stanley during the Subprime crisis and plot it with the VIX, we can see that there is an almost perfect correlation. We also thought it would be interesting to plot the illiquidity ratio of BNP Paribas from 2010 to 2013, highly exposed to the Greek debt (3.2 billion euros). Indeed, BNP Paribas faced illiquidity issues during the Greek Crisis.

Figure 8 and 9 : Correlation = 0.53 (8) & Correlation = 0.7 (9)

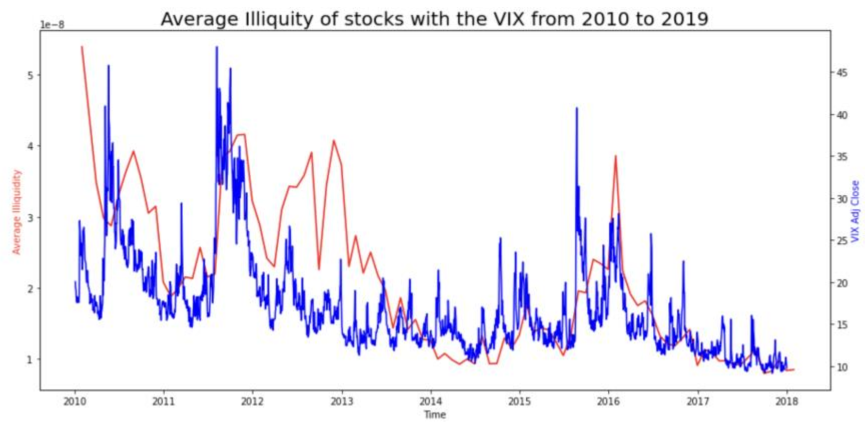




Source : Python

Illiquidity of the Financial Markets during the overall period

Figure 10 - Mean of illiquidity ratios of all stocks plotted with the VIX: Correlation = 0.68



Source: Python

In both cases, we see that the correlation is high between 0.5 and 0.7. This is accurate as the more volatile the market is, the more illiquidity issues companies can encounter. A crisis can create a serious liquidity issue for companies even liquid ones. The VIX and the illiquidity measure are highly correlated during those periods.

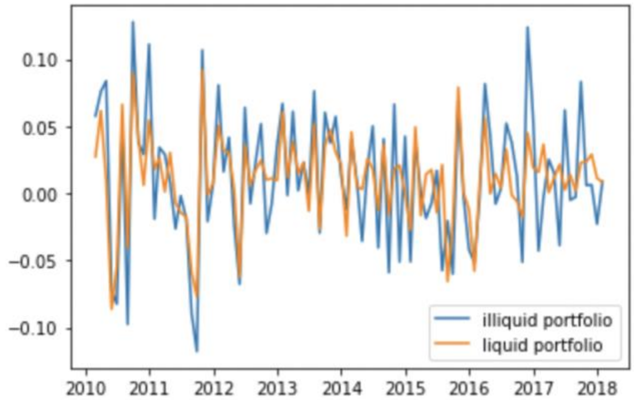
Question 2

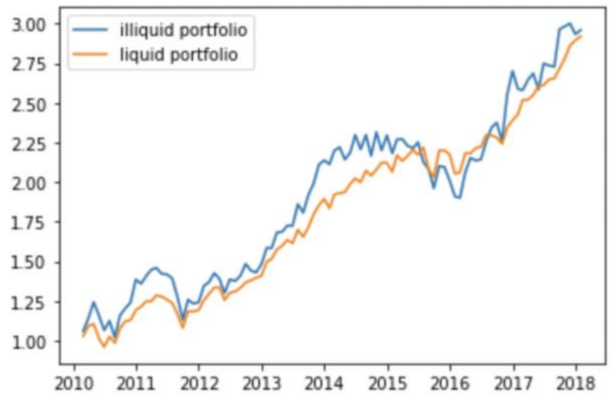
After deeply defining the notion of liquidity in question one, the next step of our analysis is to demonstrate the positive relationship between illiquidity and returns. We first built five different portfolios, to rank stocks from most illiquid to most liquid: the first portfolio containing the most illiquid stocks and the fifth portfolio including the most liquid stocks. We then used the Fama-French Model to predict the returns of our portfolios. We used the Fama-French regression to try and demonstrate that our illiquid portfolios can offer higher returns than our liquid portfolios.

Construction of portfolios

As explained above, we constructed five portfolios with portfolio 1 including very illiquid stocks and Portfolio 5 with only very liquid stocks.

Figure 11 and 12: Average Returns and Cumulative Performance of portfolio 1 and portfolio 5





Source: Python

After building our two portfolios (1 and 5), we can see that both curves are very similar. We obtain very similar average returns for illiquid and liquid portfolios, with also very comparable performances. However, our results assess that illiquid portfolios do offer upper returns! The difference in returns is notable but still remains quite small, probably due to the selection or our period, perhaps smaller than in the actual Fama-French model and not 100% efficient.

Figure 13 : Long Short Portfolio

Source: Python

Indeed, when looking at our long-short portfolio, we can see that we have on average positive returns especially from 2011 to 2014 likely because it was the beginning of the recovery of the 2008 crisis in the United States. However, the returns are decreasing from 2015 to 2016 and are increasing again after 2016. Indeed between mid 2014 and 2015, US stocks all went down due to a slowing growth in China and were also impacted by the Greek default in Europe (mentioned above). Coupled with less Quantitative Easings (due to the recovery) and the Brexit vote which created a lot of uncertainties, returns all went down at a similar pace which exerted downward pressure on returns.

Figure 14 : Average returns of our portfolios

Portfolio 1	0.012
Portfolio 2	0.0022
Portfolio 3	0.015
Portfolio 4	0.012
Portfolio 5	0.011

Returns vary according to the period. We can see that our portfolio 1 offer upper returns than our portfolio 5 but are still very similar. Portfolio 4 also gives greater returns than our portfolio 5 (4 is more illiquid than 5). Portfolio 3 is the portfolio that offers the highest returns. It is interesting and it is due to the fact that portfolio 3 contains both growth stocks and value stocks, and can benefit from both premiums. According to the long-short portfolio, illiquid portfolios performed better from 2011 to 2015 and 2016 to 2018. Nonetheless, we decided to stop in 2018 because in 2019, rates were very low (even negative in Europe) and offered a lot of support to liquid stocks (especially tech stocks) with longer maturities and higher sensitivity to change in yields, which would have created a noise in our analysis. We also did not take into account 2020, to build an analysis prior to the health crisis.

Fama-French regressions results

Figure 15: Regressions

	All returns	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	L-S Portfolio
R-squared	0.978	0.912	0.926	0.901	0.934	0.953	0.736
Prob (F statistic)	4.8E-76	2.45E-48	8.03E-52	3.81E-46	3.41E-54	4.26E-61	1.62E-26
Alpha	-0.0011	0.0007	-0.0105	0.0034	0.0012	-0.0005	0.001
Beta_MKR	0.929	0.8649	1.0015	0.9488	0.9089	0.921	-0.057
Beta_SMB	0.4357	0.9843	0.9573	0.1928	0.1067	-0.0626	1.0481
Beta_HML	0.1154	0.5066	0.1204	-0.0339	-0.028	0.0118	0.4953
p_value (alpha)	0.078	0.692	0	0.011	0.219	0.546	0.575
p-value (Beta_MKR)	0	0	0	0	0	0	0.266
p-value (Beta_SMB)	0	0	0	0.002	0.019	0.088	0
p-value (Beta_HML)	0	0	0.074	0.555	0.516	0.735	0

Source: Python

Regression for All returns

We first plotted the Fama-French Regression with all our stocks in one portfolio. We just wanted to check if the results were significant. They are almost all significant, and the factors are all positive as they all represent a premium in the returns of the stocks, which is in line with Fama's results. The HML factor is not significant, probably because our database contains a lot of very high market capitalization (Apple, FB, Amazon) which is making our HML factor less significant. Now we are going to do the regression of portfolio 1,2,3,4 and 5 and comment our results.

Overall model

First of all, the R^2 is impressive between around 95% for all portfolios and around 75% for the Long-Short Portfolio. R^2 gives us how much of the excess returns are explained by the FAMA-French factors. Indeed, the coefficient of determination of the multiple linear regression model (with a constant term) is the ratio of the total variance explained by model to the total variance of the data. The model is highly significant. The lower R^2 (around 75%) is for the portfolio Long-Short, as the excess returns to the market portfolio (MKR) do not explain the variable. It makes sense as our portfolio 6 is a difference between excess returns, so cannot be explained by excess returns themselves.
We also showed the probability of the global F-test to test the significance of all the explicative variables. The p-value associated to the F-stat is very small for all portfolios which insist again on the significance of the overall model.

On the six portfolios

Significance of the variables (p-values)

The model is also significant as p-values remain very low for the coefficient MKT, SMB and HML for almost all portfolios. The p-value for each independent variable associated to the t-stat (the Student test) tests the null hypothesis that the variable has no correlation with the dependent variable. The lower the p-value, the greater the statistical significance of the observed difference. We highlighted in green each p-value above 10%, to see which factors have a 90% level of significance in the model.

- For the portfolios 1 and 2, the p values are all close to 0. We can reject H0 at a 99% level than any of the variable is equal to 0. For the most illiquid portfolios, all Fama-French variables are significant to explain the excess returns! As all the variables are positive in the portfolio 1 and 2, we can see that all the Fama-French factors generate returns. We now need to compare with other portfolios to see if those portfolios do offer higher returns.
- For the portfolio 3, the p-values are all close to 0 except for the HML parameter which means we cannot reject H0 at a 90% level that HML is equal to 0 (not significant in the model). Indeed, HML is significant when the price to book value is extreme, which is true in more illiquid portfolios. However, as portfolio 3 is in the middle, the book value and the market value must be very similar so it is harder to evaluate the significance of the HML parameter in the model.
- For the portfolios 4 and 5 we follow the same demonstration: since we have a mix of value and growth stocks, the HML is not significant. Indeed in portfolio 4, market capitalization can be excessive in comparison to their revenues.
- The last portfolio is accurate as all the variables are significant except MKR. Indeed, as we are doing a long-short strategy, the MKR variable cannot be significant in this portfolio.

Explanation of coefficients associated to Fama variables (MKR, SMB and HML)

- <u>The MKR factor associated to the excess returns of the market :</u> Portfolios 1 to 5 are positively correlated to the MKR. This is coherent as the sign of the beta should be positive (as it is in the CAPM model). The only exception is the portfolio 6, not significant. Again this is accurate, as the portfolio 6 represents a difference between excess returns.
- <u>The SMB factor associated to the size</u> : In the long run, small companies should outperform long companies. From portfolio 1 to 5, we added a color scale to show that the SMB is decreasing with the liquidity of the stock. In portfolio 5, the SMB factor is even negative! what we can deduce is that the size is more impactful for illiquid stocks. The results are very coherent with the theoretical approach as the SMB coefficient represents the premium that is paid by illiquid companies.
- <u>The HML factor associated to the value</u>: The HML factor should states that in the long-term, value stocks (high book-to-market ratio) enjoy higher returns than growth stocks (low book-to-market ratio). Portfolio 1, 2 and 3 are significant (as explained above) and the betas are almost decreasing from illiquid stocks to less illiquid stocks. This is accurate as illiquid portfolios contain mostly growth stocks (significant) whereas liquid stocks can contain both value stocks and growth stocks (not significant).

Conclusion

The Fama French regression is actually very significant in our model especially for illiquid portfolios. In the first part, we saw that between 2010 and 2018, illiquid stocks did not always offer higher returns than liquid stocks. We still saw a slightly increase in returns for very illiquid stocks, which explains why portfolio 1 and 2 are the most significant. The results represent good empirical evidence to assess that illiquid securities do generate an illiquidity premium with positive Fama-French coefficients.

Another methodology

Another methodology would have been to reunite stocks all together in one portfolio and to create 5 other variables d1, d2, d3, d4 and d5. d1 would have taken the value 1 if the stock i was in portfolio 1 or 0 if not. d2 would have taken the value 2 if the stock if was in portfolio 2 and 0 if not. The same applies for d3, d4 and d5. We would then added in our regression the factor SMB * d1, SMB * d2 etc. and would have seen that SMB * d1 factor was highly significant (a DEMI measure), as the SMB is very correlated to the illiquidity of the stocks (as we have already sene in the current regression).

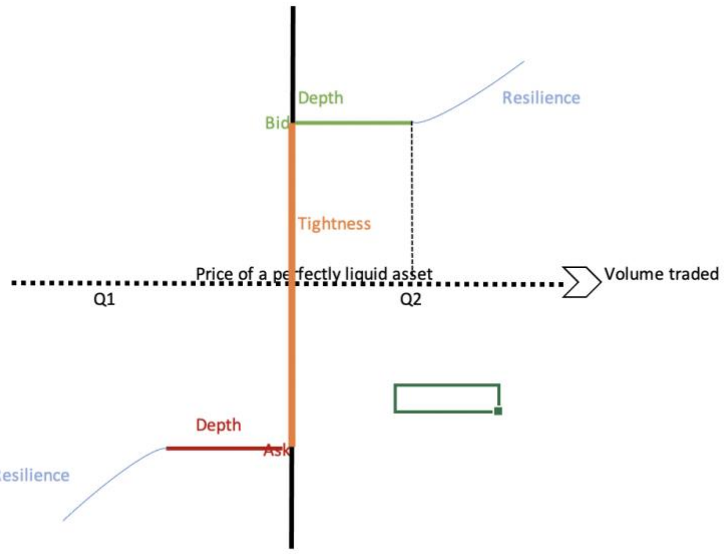
Question 3

Trading liquidity varies between different assets and asset classes owing to a number of market frictions. To capture different dimensions of liquidity, such as the breadth and depth of markets, the costs of transactions, the speed which transactions can occur, and the resilience of prices to trading activity, several types of measures can be defined to quantify liquidity.

Volume-based measures distinguish liquid markets by the volume of transactions to understand breadth and depth of a market asset. The Amihud liquidity measure belongs to this category and its popularity derives from empirical evidence that more active markets tend to be more liquid. Furthermore, it is widely available as volume figures are regularly reported for most assets.

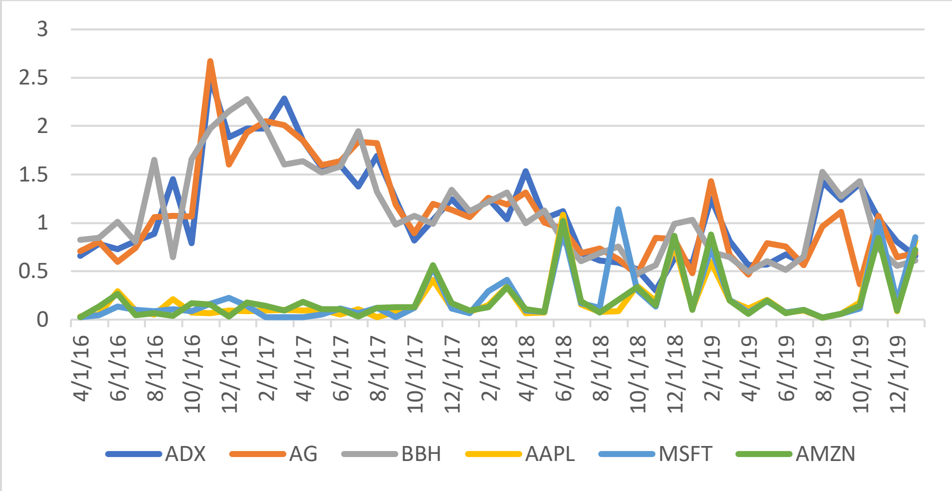
Spread-related measures: the bid-ask spread is the difference between the quoted sale price (bid) and the quoted purchase price (ask) of an asset. Indeed, even with a minimum amount of transacted volumes, the buyer must pay the bid price to enter the transaction. The bid price is normally above the fundamental price for a perfectly liquid asset (which price is not related to trading volume). On the contrary, the seller must accept to receive the ask price which is normally below the fundamental price of a perfectly liquid asset. The discount represents the **illiquidity cost to the seller.**
Indeed, where two assets have similar cash flows but vary in terms of liquidity, investors typically require a premium to invest in the asset with lower liquidity, thus leading to a reduction in its price. This is because lack of liquidity presents a risk of being unable to sell or having to sell at a discount at the specific time when the investor needs to exit.
The difference between the bid and the ask (bid-ask spread) is the **tightness of the traded asset** (orange line on our graph). The larger the spread, the more illiquid the asset.
If the buyer increases the order flow, the marginal impact of such a change is initially zero and the length of the horizontal green line defines the **market depth of the asset** (the longer the line, the deeper the market). However, after a certain threshold of transacted volumes Q2 in our graph, the marginal impact of trading an additional unit of volume increases and the speed of this continuous increase defines the **resiliency** of such a market. The same effect is observable from the seller perspective.

<u>Dimensions of market liquidity</u>



The relative spread represents the most extensively used measure of illiquidity since it allows comparison between stocks with different stock prices. It can be computed as a percentage of the middle price (average of bid and ask prices) = **(bid-ask)/midpoint.**
We can notice however that the most important problem faced in spread related illiquidity measure is the difficulty to find spread related data for stocks. Thanks to the BDH function on Excel, we managed however to extract from Bloomberg monthly average bid-ask spreads in % for every stock of our coverage from 2016 to 2020. This average bid-ask spread can be used as an approximation of the relative spread.

Figure: Relative-spread comparison between stocks of different market capitalizations



Source: Excel

On the first sight, it is easy to notice that relative bid-ask spreads are much higher for lower capitalization stocks like ADX, AG or BBH (that have been randomly selected among small capitalization stocks) than for large market capitalization stocks like AAPL, MSFT or AMZN. This seems logic as the higher the market capitalization the higher the number of shares outstanding and therefore the lower the bid-ask spread as explained before.

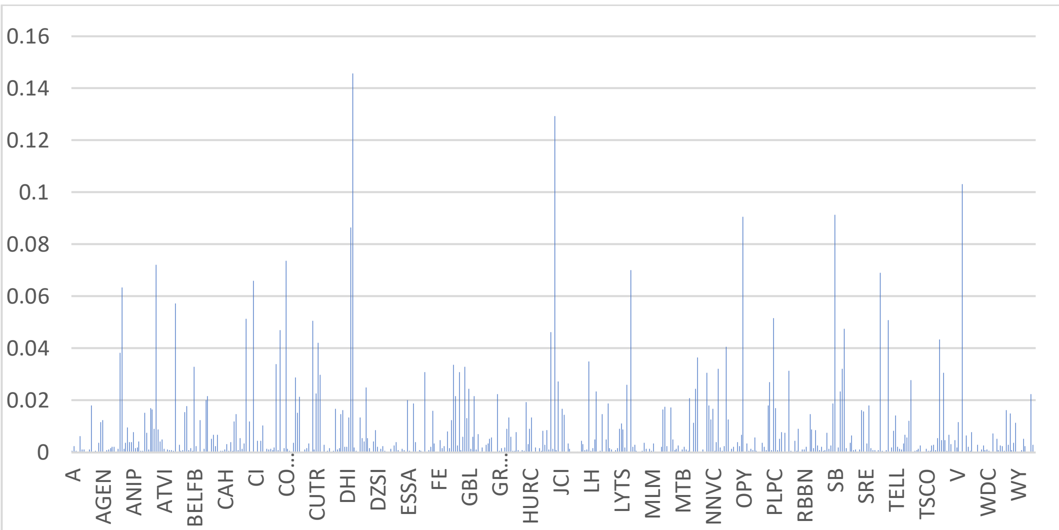
To compare the relative-spread with the Amihud volume-based measure, we can therefore define the following spread-related illiquidity measure:

$$spread - related\ Illiqu\ (i,t) = abs\big(R(i,m)\big) * relative\ spread(i,m)$$

Where Spread-related Illiqu measures the illiquidity of stock i in month t, R is the return of stock i in month m, relative spread is the average bid-ask spread of stock i in month m.

We decided to multiply the absolute monthly returns by the monthly relative spread (while in the Amihud measure, absolute returns are divided by the volume) because relative spreads and volumes are inversely correlated. The higher the traded volume the lower the bid-ask spread indeed.

Figure: Mean of our spread-related illiquidity measure from 2016 to 2020 per stock

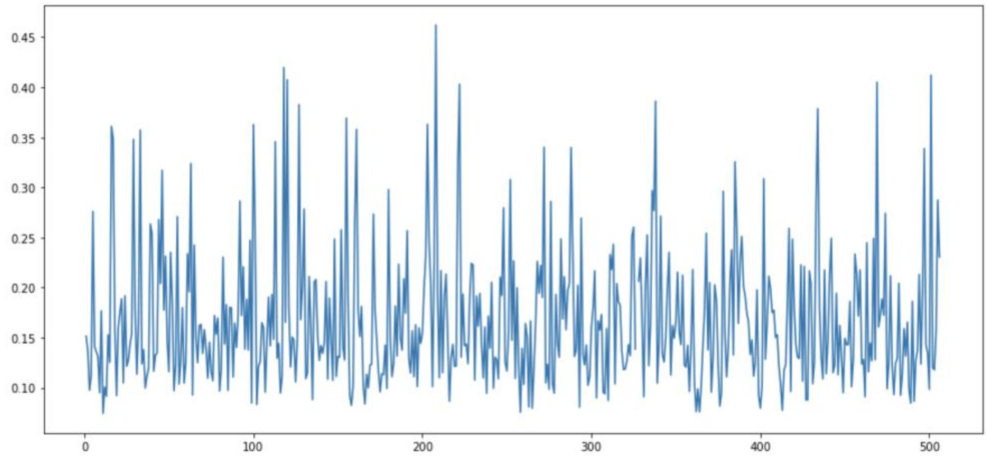


Source: Excel

We computed this ratio on Python and extracted it on Excel. On the Figure above, we can easily notice that the stocks that have the higher spread-related illiquidity measure are DHR, DHX, ITI, MCF, POY, SB or VHC (all are small capitalization stocks) while big capitalization stocks like AAPL or MSFT exhibit very low measures. We can therefore believe that our hypothesis according to which the higher the relative spread the lower the liquidity of the stock is verified.

However, plotting monthly returns against the ranking of the most to less liquid stocks according to our ratio on the below graph, we cannot see a rising trend as we move from liquid to illiquid securities.

Figure: monthly returns against ranking of the most to less liquid stocks according to our spread-related illiquidity measure ratio



To conclude, our spread-related measure seems not to be relevant on a global view to prove a clear relationship between returns and illiquidity of stocks. This absence of trend is weird as bid-ask spreads are the best measure of liquidity for an asset even if the measure is hard to find. However, we found an explication to this phenomenon: a highly illiquid stock with a total number of trades of 0 in one month will have a bid-ask spread of 0 (so lower than highly liquid stocks that are traded millions of times every month). And so according to our formula, stocks that are not traded are the most liquid ones, which is in reality the contrary. Our reasoning becomes therefore more complicated to implement to obtain relevant results.

Question 4

Forecasting the direction of future asset prices is a widely studied topic in many fields including trading, finance, statistics and computer science. The motivation for which is naturally to predict the direction of future prices such that assets can be bought and sold at profitable positions.

Most financial firms use therefore algorithms to buy and sell financial assets. It is possible for amateur investors with programming knowledge or vice-versa, to implement algorithms and improve their strategies. That is why it is important to find the best strategy. Thus, in this part, we implement and compare 4 algorithmic trading strategies: 3 based on signals created through the use technical indicators (SMA, Momentum Strategy and Mean-reversion Strategy) and 1 based on the application of machine learning methodologies through the OLS linear regression.

Our objective will be to optimise the portfolios 1 and 5 previously built: the most liquid one and the most illiquid one. The goal is also to compare these strategies and to define which one will be the most efficient for us to optimise our portfolios.

To do so, each strategy is properly backtested and we deliver analysis of the performance, strengths and weaknesses of the trading strategies.

In order to implement the different strategies, we have to build a table with prices of our portfolios. Thus, we calculate the adjusted close prices on a 100 basis. Then, we create a table with prices for Portfolio 1 (illiquid) and Portfolio 5 (liquid).

Simple Moving Average Strategy

The moving average is a technique used in technical analysis that smooths price histories by averaging monthly prices over some period of time. A person would typically use historical data like the last 3, 6 or 9 months with technical indicators to predict a stock price. A simple moving average is a technical indicator that can aid in determining if an asset price will continue or if it will reverse a bull or bear trend. A simple moving average (SMA) is an arithmetic moving average calculated by adding recent prices and then dividing that figure by the number of time periods in the calculation average. For example, one could add the closing price of a security for a number of time periods and then divide this total by that same number of periods.
Logically, short-term averages respond quickly to changes in the price of the underlying security, while long-term averages are slower to react.

First, we find the optimal window for the SMA strategy using an algorithmic code. The optimal window (in months) to implement the SMA strategy on both portfolios are:

	Short Window SMA1	Long Window SMA2
Liquid Portfolio	2	11
Illiquid Portfolio	3	10

Therefore, we are implementing now the SMA strategy on each portfolio, where the trading rules are:

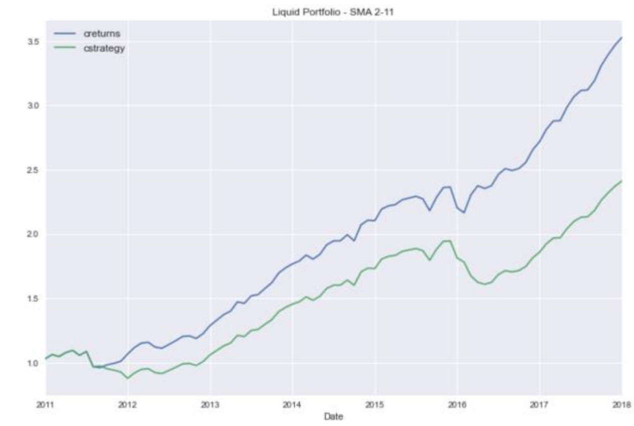
- Go LONG (= +1) when the SMA1 is above the SMA2
- Go SHORT (= -1) when the SMA1 is below the SMA2

(For a Long Only strategy, we would use +1 for a long position and 0 for a neutral one)

Then, we implement the Vectorized Strategy Backtesting. It follows the different steps below:

- Calculate the log-returns.
- Multiply the position values, shifted by one month, by the log-returns (the shift is required to avoid a foresight bias): the basic idea is that the algorithm can only set a position in the portfolio given today's market data and the position then earns next month's return.
- Calculate the value of one invested dollar in both portfolio over time using the strategy and not using it in order to compare the strategy to historical returns.

	SMA Outperformance
Liquid Portfolio	-1.11
Illiquid Portfolio	0.06



We can see that the strategy underperforms in general for the Liquid Portfolio and it is more and more the case as time goes by. This is supported by the last calculation of outperformance which is negative (-1.11). Regarding the Illiquid Portfolio, the strategy outperforms since mid-2015 but otherwise it underperforms between 2012 and 2015 which means that returns are higher than those implementing the strategy. However, in general the strategy outperforms as the last calculation of outperformance is positive (0.06). Thus, the SMA strategy performs better on the Illiquid Portfolio than on the Liquid Portfolio. This can be explained by the volatility of the Illiquid Portfolio: as its assets are more volatile, they also offer bigger return implementing a strategy. Notice that transaction costs (fixed fees, bid-ask spreads, lending costs...) are not included. This might be justifiable for a trading strategy that leads to a few trades only over multiple years, like this example. It is also assumed that all trades take place at the end-of-month closing prices for the portfolios. A more realistic backtesting approach would take these and other (market microstructure) elements into account.

Momentum Strategy

There are two basic types of momentum strategies. The first type is cross-sectional_ momentum strategies. Selecting from a larger pool of instruments, these strategies buy those instruments that have recently outperformed relative to their peers (or a benchmark) and sell those instruments that have underperformed. The basic idea is that the instruments continue to outperform or underperform, respectively - at least for a certain period of time.

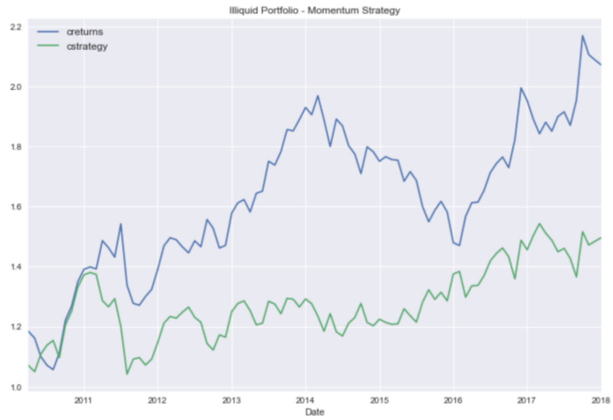
Jegadeesh and Titman (1993, 2001) and Chan et al. (1996) study these types of trading strategies and their potential sources of profit. Cross-sectional momentum strategies have traditionally performed quite well. Jegadeesh and Titman (1993) wrote: "This paper documents that strategies which buy stocks that have performed well in the past and sell stocks that have performed poorly in the past generate significant positive returns over 2- to 12-month holding periods."

The second type is time series momentum strategies. These strategies buy those instruments that have recently performed well and sell those instruments that have recently performed poorly. In this case, the benchmark is the past returns of the instrument itself. Moskowitz et al. (2012) analyze this type of momentum strategy in detail across a wide range of markets: "Rather than focus on the relative returns of securities in the cross-section, time series momentum focuses purely on a security's own past... Our finding on time series momentum in virtually every instrument we examine seems to challenge the 'random walk' hypothesis, which in its most basic form implies that knowing whether a price went up or down in the past should not be informative about whether it will go up or down in the future."

The simplest time series momentum strategy is to buy the portfolio if the last return was positive and to sell it if it was negative. We just take the sign of the last available return as the market position.

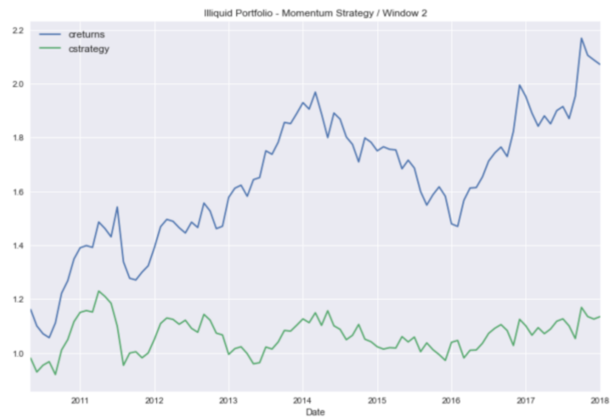
Then, we implement the Vectorized Strategy Backtesting:

	Momentum Outperformance
Liquid Portfolio	-2.25
Illiquid Portfolio	-0.58



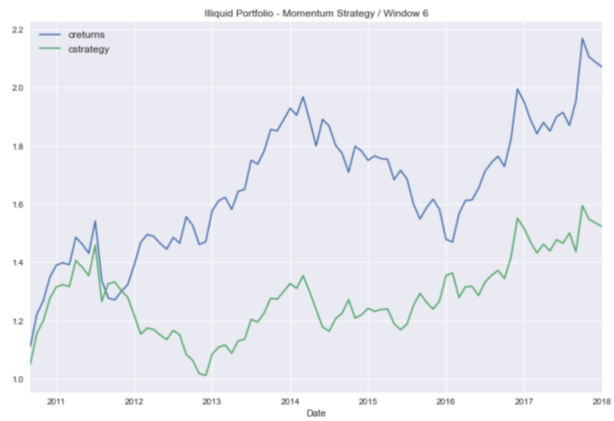
The strategy does significantly underperform the base instrument. Using a rolling time window, the time series momentum strategy can be generalized to more than just the last return. For example, the average of the last 2 returns can be used to generate the signal for the positioning.

	Momentum Outperformance (window 2 months)
Liquid Portfolio	-1.21
Illiquid Portfolio	-0.94



As previously, the strategy underperforms for both portfolios, but it is a little bit better with a rolling window of 2 months for the Liquid Portfolio. In fact, we can see that the strategy outperforms between mid-2011 and 2013. Outperformance is also better for the Liquid Portfolio with a window of 2 months. However, the performance is quite sensitive to the time window parameter. Choosing the last 6 returns instead of 2 leads to a different performance.

	Momentum Outperformance (window 6 months)
Liquid Portfolio	-1.77
Illiquid Portfolio	-0.55

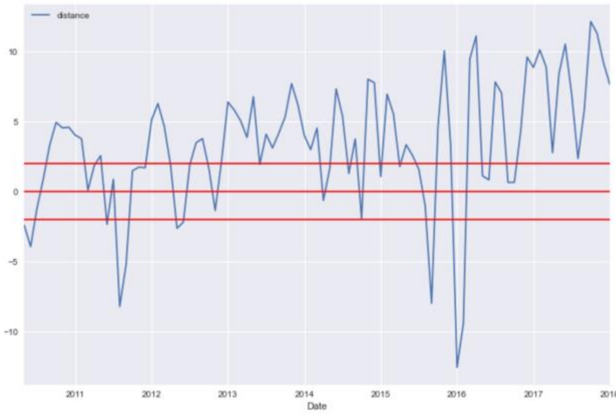


Here, the strategy is still underperforming for both portfolios, but it is a little bit better with a rolling window of 6 months for the Illiquid Portfolio. In fact, we can see that the strategy outperforms between mid-2011 and 2012. Outperformance is also better for the Liquid Portfolio with a window of 6 months. Thus, the performance is quite sensitive to the time window parameter. More generally, we can see that the SMA strategy performs better than the Momentum strategy for both portfolios.

Mean-Reversion Strategy

Roughly speaking, mean-reversion strategies rely on a reasoning that is the opposite of momentum strategies. If a financial instrument has performed "too well" relative to its trend, it is shorted, and vice versa. To put it differently, while (time series) momentum strategies assume a positive correlation between returns, mean-reversion strategies assume a negative correlation. Balvers et al. (2000) write: "Mean reversion refers to a tendency of asset prices to return to a trend path". Working with a simple moving average (SMA) as a proxy for a "trend path", a mean-reversion strategy says the portfolio can be backtested in a similar fashion as the ones of the SMA and momentum strategy. The idea is to define a threshold for the distance between the current price and the SMA, which signals a long or short position.

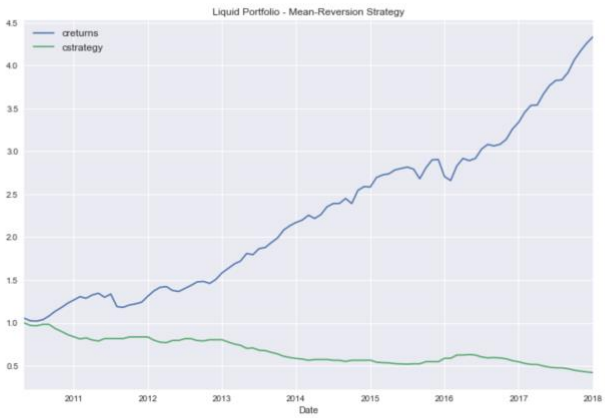
We implement a mean-reversion strategy on the basis of an SMA of 3 months and a threshold value of 2 for the absolute deviation of the current price to deviate from the SMA to signal a positioning.
First, we calculate the distance for every single point in time to the threshold.

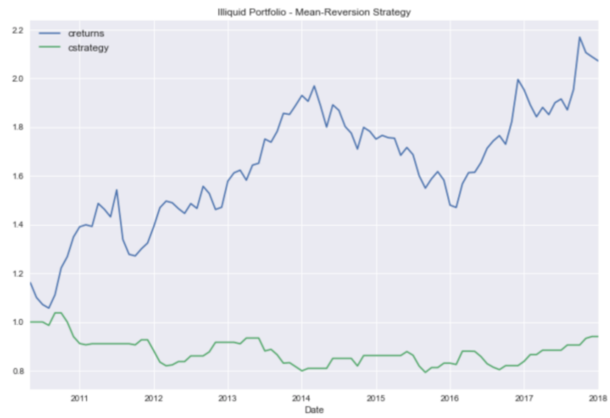


Based on the differences and the fixed threshold values, positionings can again be derived in vectorized fashion:
-1 when the distance is above the threshold and 1 when the distance is below -threshold.
If there is a change in the sign of the distance value, go market neutral (set 0), otherwise, keep the column position unchanged. Forward fill all NaN positions with the previous values, replace all remaining NaN vaues by 0.

Then, we implement the Vectorized Strategy Backtesting:

	Mean-Reversion Outperformance
Liquid Portfolio	-3.91
Illiquid Portfolio	-1.13





The Mean-Reversion Strategy underperforms for both portfolio with negative outperformances: -3.91 for the Liquid Portfolio and -1.13 for the Illiquid Portfolio. However, the strategy works better for the Illiquid Portfolio than for the Liquid Portfolio. The difference of performance between the 2 portfolios is due (again) to the volatility of the Illiquid Portfolio (higher returns). More generally, we can see that the Mean-Reversion Strategy is even worse than the Momentum Strategy for both portfolio and thus, the best performing strategy for the moment is still the SMA Strategy.

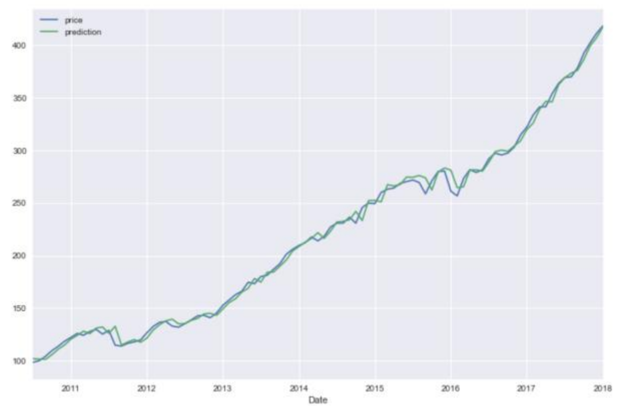
Machine Learning Strategy - Linear Regression

Machine learning is a data analysis technique that learns from experience using computational data to learn information directly from data without relying on a predetermined equation.
The linear regression analyzes two separate variables in order to define a single relationship and is a useful measure for technical and quantitative analysis in financial markets. It returns an equation that determines the relationship between the independent variables and the dependent variable. When used in machine learning, linear regression is a simple technique that is based on supervised learning. This is mostly used for finding out the relationship between variables and forecasting and more specifically between variables and future prices.
The Ordinary Least Squares (OLS) procedure seeks to minimize the sum of the squared residuals. This means that given a regression line through the data we calculate the distance from each data point to the regression line, square it, and sum all of the squared errors together. This is the quantity that ordinary least squares seek to minimize.

The random walk theory of asset prices has been popularized by figures such as Eugene Fama and Burton Malkiel. In its simplest form, the theory states that asset prices cannot be predicted because changes in prices are random.
The theory presupposes that financial markets are efficient. That is, all publicly available information is incorporated in the stock's current price due to market participants being rational profit-maximizers. If any anomaly existed, it would quickly be exploited and removed, leading to a more efficient state. New news is the only thing that can change the price of a stock, and since the news-cycle is unpredictable, asset prices, therefore, move randomly.
Because stock prices follow a random walk, according to the random walk theory, an asset's price today is the best predictor for its price tomorrow (here next month because we use monthly data). To test this claim, we can compare lagged prices of both portfolio during a various interval, to their most recent price in order to determine whether they are indicative of today's price. If that's the case, then each portfolio is priced accordingly, and would therefore move randomly. On the other hand, if the lagged prices bear little to no relationship with today's price, then today's price is not the best predictor of portfolios price and therefore the market is not efficient.

Here, we use 5 lags. We calculate the optimal parameters to illustrate the random walk hypothesis:
Liquid: [1.12752061, -0.31193819, 0.15322502, 0.18073172, -0.13305291]
Illiquid: [1.0649785, -0.22643667, 0.20443831, -0.06887635, 0.03203124]

The first one measures the proportion of next month's price explained by today's price. The second one measures the proportion of next month's price explained by the last month's price. And so on. Therefore, with our liquid and illiquid portfolio, the hypothesis which states that assets follow a random walk and therefore that today's price is the best predictor for next month price is verified. Indeed, next month's price is totally explained by today's price (more than 1). The four other values have hardly any weight assigned.



The above graphics show the price and the prediction price for the Liquid and Illiquid Portfolios since 2011. For both portfolios, the two curves are difficult to distinguish so that prove on the first sight that linear regression is an efficient way to predict prices. Indeed, the two curves follow exactly the same shape which proves that the linear regression is a very efficient way to predict prices on a daily basis.
The previous graphics show is that even if the two curves follow the same shapes, the OLS method has some bias especially for volatile assets (Illiquid Portfolio). Indeed, when the price actually increases and sharply decreases right after to increase again right after, the OLS estimator becomes less efficient to predict movements as it has what we can call a "time for reaction to the price movement". This time for reaction is acceptable when the direction of the asset price does not change frequently but becomes nonacceptable if the direction of the asset's price changes too often.

However, on this particular example using linear regression, we can see that Illiquid Portfolio's lagged prices have an extremely strong relationship with today's price. In this particular example, lagged prices are indicative of today's price.

Moreover, this method refers to a month-to-month strategy as next month's price is explained by today's price. The buyer of the strategy must adapt his position on a month-to-month basis which can be costly.

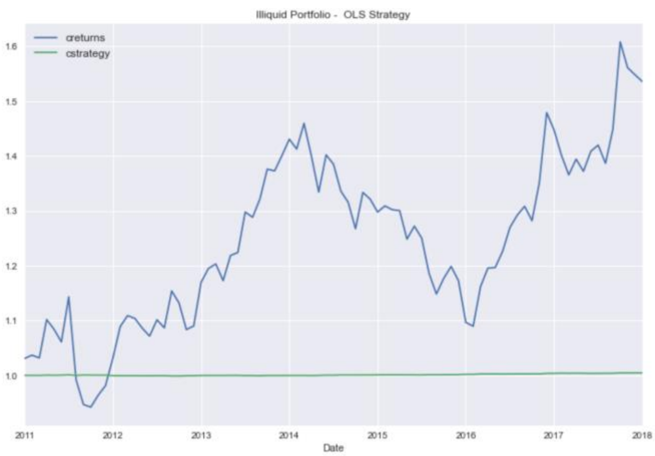
So far, our analysis was based on prices. But it can also be relevant to put emphasis on log-returns that might be a better choice for such statistical applications. Indeed, returns have interesting characteristics that prices don't have (for instance stationarity). We calculate the optimal parameters:
Liquid: [0.18314781, 0.03136646, 0.08107883, 0.12295007, 0.1847042]
Illiquid: [0.05001294, -0.15505699, 0.07696091, 0.02538912, 0.08488789]

Here, we notice that today's return is not anymore, the relevant predictor for next month's return. Neither are the other parameters individually. They are all explaining a part of next month's return and are therefore are important to combine to predict future returns.

The hit Ratio is the number of times that the correct prediction was made in ratio to the number of total predictions. The hit ratio of our strategy on the Liquid Portfolio is 77.6% and 57.6% on the Illiquid Portfolio. The hit ratio is greater for the Liquid Portfolio than for the Illiquid Portfolio. This can be explained by the volatility of the Illiquid Portfolio, which make it more difficult to make the correct prediction. However, we can notice that the hit ratio has serious flaws because it does not take into account the proportion of right values obtained by chance.

Then, we implement the Vectorized Strategy Backtesting:

	OLS Outperformance
Liquid Portfolio	-3.91
Illiquid Portfolio	-1.13



We can see that the strategy only outperforms between mid-2011 and 2012 for both portfolios. Otherwise, it underperforms for the 2 portfolios. This is supported by the last calculations of outperformances that are negative: -2.51 for the Liquid Portfolio and -0.53 for the Illiquid Portfolio. Here again, the strategy performs better on the Illiquid portfolio than on the Liquid Portfolio. This is also due to the volatility of illiquid assets.
Thus, the OLS strategy is better than the mean-reversion one but better than the momentum and SMA.

To conclude on the OLS tool to predict future prices, we can notice that one implication from the RWH is that traditional methods employed to pick assets, such as technical and fundamental analysis, are of little use. Both techniques imply that investors can leverage these methods to develop profitable trading strategies, but the RWH thinks this is typically self-defeating because traders will exploit and therefore neutralize these anomalies, making the market efficient.

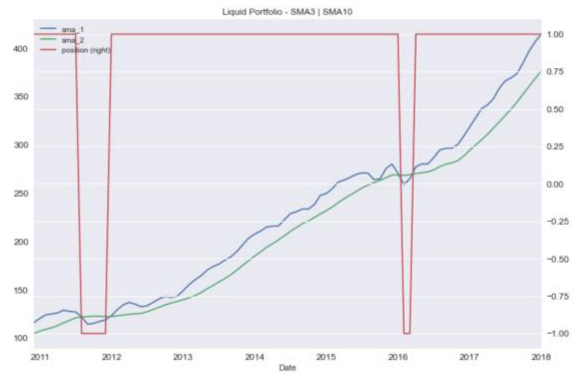
Strengths and Weaknesses of each algorithmic trading strategy

We can conclude that the best strategy for both the Liquid and the Illiquid Portfolio is the SMA. This method works better than the other ones for our portfolios with higher outperformances and positive one for the Illiquid Portfolio.
Unfortunately, it doesn't perform well for the Liquid Portfolio. This is not surprising as we have studied before that returns are higher for illiquid assets than for liquid assets, implying excess returns. Thus, the strategy enables to profit from the excess returns of illiquidity while hedging the market exposure.

The strength of SMA is that it helps cut down the amount of "noise" on a price chart by smoothing the data and thus providing a clearer visual picture of the current trend. SMA strategy also gives relevant signals that can give a precise answer as to what the trend is. However, the main weakness of Movering Average (MA) is that it is calculated based on historical data, and nothing about the calculation is predictive in nature. Therefore, results using moving averages (MA) can be random. At times, the market seems to respect MA support/resistance and trade signals, and at other times, it shows these indicators no respect.
The second "best" strategy is the OLS one. However, the problem of Linear Regression is that it is very sensitive to outliers. So, outliers should be analyzed and removed before applying Linear Regression to the dataset.

Conclusion

When we implemented the SMA strategy, we have plotted the position advised by the strategy for both the Liquid and Illiquid Portfolio.



We can see that the main difference of position is between 2015 and 2016: the strategy advice to short the Illiquid Portfolio from 2015 to 2016 whereas it advises to short the Liquid Portfolio only during a quick slot at the beginning of 2016. After that, studying the graphics of the value of one invested dollar in both portfolio over time, we can see that the strategy on the Illiquid Portfolio is outperforming between 2015 and 2016 and the strategy enables to have increasing returns whereas returns without the strategy were decreasing in this period. Moreover, the strategy is underperforming at this period for the Liquid Portfolio, having not advised to be short, long time before early 2015.

To conclude, it shows how the SMA strategy enabled to profit from the excess returns of illiquidity while hedging the market exposure (especially between 2015 and 2016).