MAY 2021

ILLIQUIDITY AND ASSET PRICES – IAP4

MASTER PROJECT - MASTER 2 FINANCIAL MARKETS

EDHEC BUSINESS SCHOOL

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Abstract

The liquidity of the markets is a worldwide and perennial issue for investors. According to Market Liquidity, Theory, Evidence and Policy by Marco Pagano, illiquidity results from the fact that not every player is active simultaneously in all financial markets, and that not all of those active players have the same level of information about the security's fundamentals. In his article Efficient Capital Markets : a Review of Theory and Empirical Works, Eugene Fama explains that "*in an efficient market, at any point in time, the actual price of a security will be a good estimate of its intrinsic value*". However, the amount of trade does not depend only on the amount of information available but also upon those executed in reaction to speculative noise. Marco Panago comes to the conclusion that when prices are pressured by trading forces rather than through a public announcement, liquidity suffers.

The goal of our Master Project is to analyze the premium offered by illiquid securities. We first calculated the illiquidity Amihud's measure and tried to measure the liquidity differences between stocks, sectors, countries, and periods. Indeed, the first goal of our thesis is to understand which industries are the most illiquid, to identify the period when the market encountered liquidity issues and assess the correlation of illiquid periods and a high volatility of financial markets. Then, we demonstrate that illiquid portfolios on average (against an individual stock perspective) did offer higher returns through the Fama-French regressions from 2010 to 2018. Using the Fama-French framework, we demonstrate that the SMB factor capturing the small-cap premium is larger for illiquid portfolios than for the more liquid portfolios. We take this result as evidence that illiquidity matters.

We first capture the illiquidity premium through the traditional Amihud's measure but we also created an alternative measure of liquidity based upon the bid-ask spread approach to track the premium of our portfolios. In the end, we also build trading strategies to capture the illiquidity premium on short term periods at time when the market is Fama-French efficient.

Question 1

Amihud's measure and statistics

We are looking to evaluate the illiquidity of stocks using the Amihud's illiquidity measure:

$$Illiq_{i,t} = \frac{1}{n}\sum_{d}\frac{|R_{d,i}|}{VOLD_{d,i}}$$

Figure 1: features of the Amihud's illiquidity measure on our sample of stocks

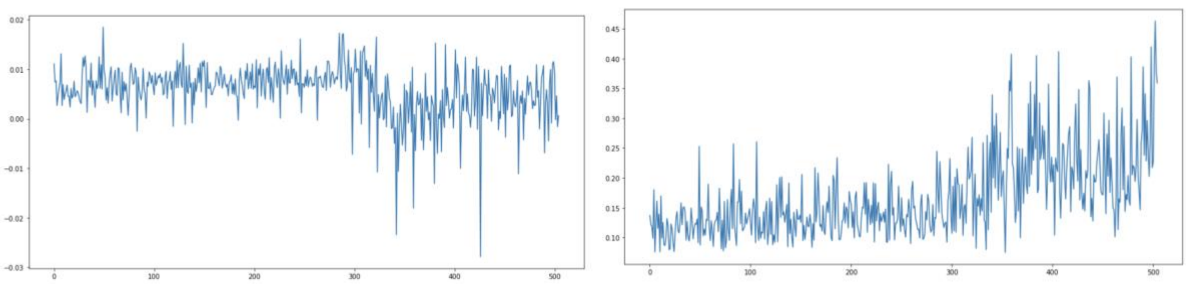
mean	**2.10309E-08**
std	9.8529E-08
min	2.0526E-12
max	1.70618E-06

Source: Python

On the whole period, we can see that we have on average a max/min difference of almost 10^{-6} compared to a mean of $2*10^{-8}$. The difference is very small. We can already assess some limits to Amihud's illiquidity measure: if at time *t*, a stock's volume is 0, the Amihud measure becomes inefficient (is not defined), and cannot always measure the illiquidity of a company.

Relationship between the Amihud's measure and expected returns

Figure 2 & 3: Ranking of the average of the expected and the absolute log-returns against illiquidity



Source: Python

We ranked all the stocks according to their illiquidity measure and plot their returns to see if we could already see a rising trend in the returns along with illiquidity. However in this first graph, we cannot find a positive slope as we move from liquid to illiquid securities. Most notably, we see that for relatively liquid securities ranging from 0 to 350, the volatilities of the returns are fairly small compared to the more illiquid securities. Nonetheless, starting 350, there is a slight uptake in the returns which is a first interesting result. It means that to capture the liquidity premium of stocks, securities need to be highly illiquid. In the second graph, we also plotted the absolute returns because we figured that a first strategy based upon the liquidity of securities could capture illiquid premium if it was allowed to short the securities with negative returns.

Liquidity comparison between stocks and sectors

We now evaluate the illiquidity of securities using the Amihud's liquidity measure. We need to do some comparisons to understand the meaning of this measure.

- *Between stocks*

Figure 4: Apple and Xoma Amihud's illiquidity ratio

AAPL	**2.08E-12**
XOMA	2.70E-08

Source: Python

The first thing to look at is the ratio difference between a stock with a big market capitalization against a lower one. For example, when we compute the Amihud's illiquidity measure we obtained 210^{-12} for Apple, (Market Cap = 2.23 Bn) in 2018 and 310^{-8} for Xoma Corporation (Market Cap = 400M). Smaller companies tend to be more illiquid.

- *Between sectors*

Figure 5: Ranking of the Illiquidity ratio average by sectors

	Utilities	**Materials**	**Energy**	**Real Estate**	**Health Care**	**Communication**	**Information Technology**	**Industrials**	**Financials**	**Consumer Discretionary**	**Consumer Staples**
Ratio	3.1E-09	3.76E-09	4E-09	4.41E-09	4.61E-09	5.13E-09	5.74E-09	6.52E-09	1.38E-08	1.55E-08	1.82E-08

Source: Python

We also computed the illiquidity ratio for sectors. Information Technology sector is highly illiquid, although intuitively, tech industries could have been considered as very liquid (Apple, Facebook, Netflix etc.). But as many illiquid stocks are in tech companies (Zhone Technologies, NXP Semiconductors), they are exerting upward pressure on the Amihud's liquidity measure.

Liquidity comparison between asset classes

It is also interesting to see the difference in illiquidity between two different asset classes. Even if we cannot properly compute the difference between asset classes (not all the data are public) we still wanted to mention this qualitative argument that liquidity can also differ between asset classes. For example, in The Big Short Michael Lewis describes several of the key players in the creation of the credit default swap (CDS) market, who sought to bet against collateralized debt obligations (CDO) on Real Estate and ended up profiting from the financial crisis of 2008. Nonetheless, when the Real Estate bubble actually burst and the Real Estate Market defaulted, the buyers of CDS on Real Estate did not obtain immediately their reimbursement because the CDS market was highly illiquid: those key players represented a small number of participants in the market.

Liquidity comparison between countries

Figure 6: Banks' illiquidity ratios from different countries

JP Morgan	1.11E-11
BNP Paribas	1.03E-10
BBVA Argentina	3.19E-08
Standard Bank	1.94E-07

Source: Python

Here we plot the ratio illiquidity difference banks from the United States, France, Argentina and Africa : JP Morgan, BNP Paribas, Banco BBVA Argentina and Standard Bank Group Limited. The difference between illiquidity ratios is also very clear. There are more active players in the US Market of French Market than in Argentina and in South Africa.

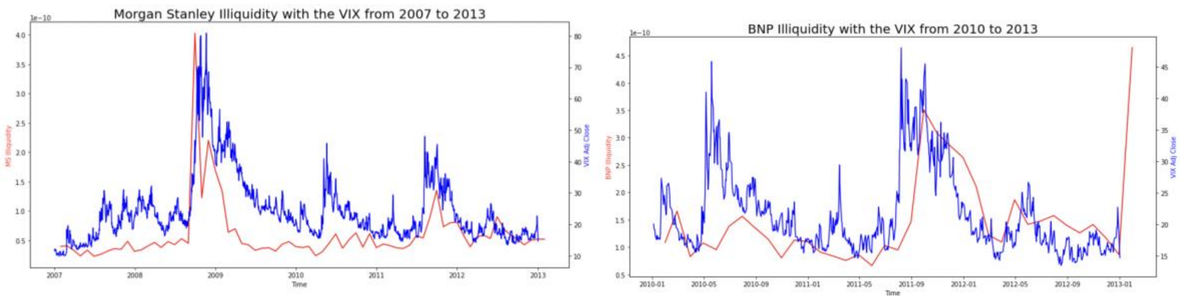
<u>Liquidity comparisons across periods with VIX</u>

Finally, major events happening in the world from the Subprime Crisis to the ongoing health crisis, have created a serious systemic threat to the liquidity of companies. Central banks can use interest rates to support the economy as they did after the e-crash and the Subprime Crisis. Nonetheless, in reaction to the current crisis, central banks were stuck in what John Maynard Keynes called the "Liquidity Trap" as interest rates were already very low in the United States and became negative in Europe in 2019. Given that interest rates were hitting the zero lower bound, there was limited scope for further stimulus if the economy faltered again. Today however, there is a risk, highlighted by former Treasury Secretary and Harvard Professor Larry Summers in Bloomberg interviews, that the fiscal stimulus of the Biden Administration could reignite inflation and exert upward pressure on US yields. The rise in yields poses a serious threat to the current excessive valuation of tech stocks, and could unveil a financial bubble. This will surely be one of the critical issues of the coming months.

- *<u>Illiquidity of the Financial Markets during crisis</u>*

It is interesting to plot multiple time periods to build our analysis. For example, if we take the illiquidity ratio of Morgan Stanley during the Subprime crisis and plot it with the VIX, we can see that there is an almost perfect correlation. We also thought it would be interesting to plot the illiquidity ratio of BNP Paribas from 2010 to 2013, highly exposed to the Greek debt (3.2 billion euros).

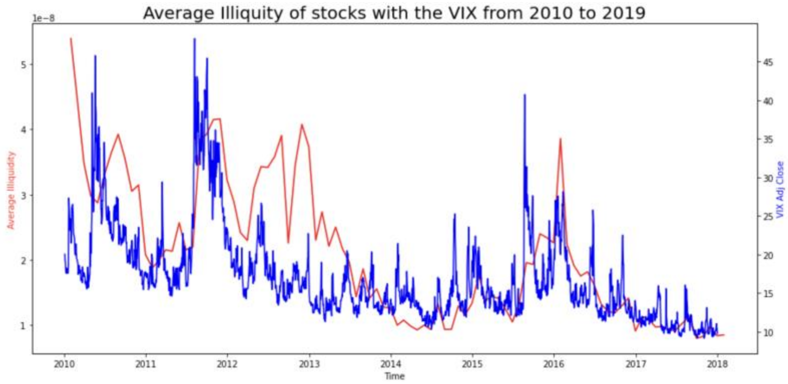
<u>Figure 7 & 8: Correlation = 0.53 (8) & Correlation = 0.7 (9)</u>



Source: Python

- *Illiquidity of the Financial Markets during the overall period*

Figure 9: Mean of illiquidity ratios of all stocks plotted with the VIX: Correlation = 0.68



Source: Python

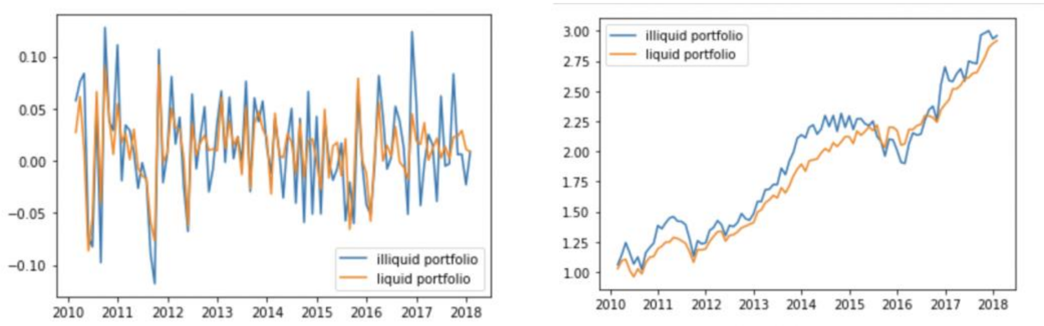
In both cases, we see that the correlation is high between 0.5 and 0.7. This is accurate as the more volatile the market is, the more illiquidity constraints companies can encounter. A crisis can create a serious liquidity issue for companies even liquid ones. The VIX and the illiquidity measure are highly correlated during those periods.

Question 2

After having defined the notion of liquidity in question one, the next step of our analysis is to demonstrate the positive relationship between illiquidity and returns. We first build five different portfolios, to rank stocks from most illiquid to most liquid: the first portfolio containing the most illiquid stocks and the fifth portfolio including the most liquid stocks. We then used the Fama-French Model to predict the returns of our portfolios and demonstrate that our illiquid portfolios offer higher returns than our liquid portfolios.

Construction of portfolios

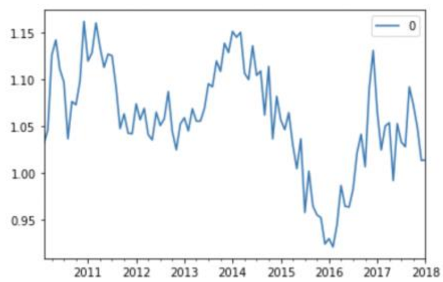
Figure 10 & 11: Average Returns and Cumulative Performance of portfolio 1 and portfolio 5



Source: Python

After building our two portfolios (1 and 5), we can see that annual returns look quite similar. We obtain similar average returns for illiquid and liquid portfolios, with also very comparable performances. However, looking more systematically to cumulative performance, our results show that illiquid portfolios do offer upper returns. The difference in returns although notable still remains quite small, probably due to the selection of our period, shorter than in Fama-French analysis.

Figure 12: Long Short Portfolio



Source: Python

Indeed, when looking at our long-short portfolio, we can see that we have on average positive returns especially from 2011 to 2014 likely because it was the beginning of the recovery of the 2008 crisis in the United States. However, the returns are decreasing from 2015 to 2016 and are increasing again after 2016. Indeed between mid 2014 and 2015, US stocks all went down due to a slowing growth in China and were also impacted by the Greek default in Europe (mentioned above). Coupled with less Quantitative Easings (due to the recovery) and the Brexit vote which created a lot of uncertainties, returns all went down at a similar pace.

Figure 13: Average returns of our portfolios

Portfolio 1	0.012
Portfolio 2	0.0022
Portfolio 3	0.015
Portfolio 4	0.012
Portfolio 5	0.011

Source: Python

Returns vary according to the period. We can see that our portfolio 1 offer upper returns than our portfolio 5 but are still very similar. Portfolio 4 also gives greater returns than our portfolio 5 (4 is more illiquid than 5). Portfolio 3 is the portfolio that offers the highest returns. It is interesting and it is due to the fact that portfolio 3 contains both growth stocks and value stocks.

Fama-French regressions results

Figure 14: Regressions

	All returns	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	L-S Portfolio
R-squared	0.978	0.912	0.926	0.901	0.934	0.953	0.736
Prob (F statistic)	4.8E-76	2.45E-48	8.03E-52	3.81E-46	3.41E-54	4.26E-61	1.62E-26
Alpha	-0.0011	0.0007	-0.0105	0.0034	0.0012	-0.0005	0.001
Beta_MKR	0.929	0.8649	1.0015	0.9488	0.9089	0.921	-0.057
Beta_SMB	0.4357	0.9843	0.9573	0.1928	0.1067	-0.0626	1.0481
Beta_HML	0.1154	0.5066	0.1204	-0.0339	-0.028	0.0118	0.4953
p_value (alpha)	0.078	0.692	0	0.011	0.219	0.546	0.575
p-value (Beta_MKR)	0	0	0	0	0	0	0.266
p-value (Beta_SMB)	0	0	0	0.002	0.019	0.088	0
p-value (Beta_HML)	0	0	0.074	0.555	0.516	0.735	0

Source: Python

- *Regression for All returns*

We first plotted the Fama-French Regression with all our stocks in one portfolio. We just wanted to check if the results were significant. They are almost all significant, and the factors are all positive as they all represent a premium in the returns of the stocks, which is in line with Fama-French's results. The HML factor is not significant, probably because our database contains a lot of very high market capitalization (Apple, FB, Amazon) which is making our HML factor less significant. Now we are going to do the same regressions for portfolios 1,2,3,4 and 5 and comment our results.

- *Overall model*

First of all, the R^2 is impressive between around 95% for all portfolios and around 75% for the Long-Short Portfolio. R^2 gives us how much of the excess returns are explained by the Fama-French factors. Indeed, the coefficient of determination of the multiple linear regression model (with a constant term) is the ratio of the total variance explained by model to the total variance of the data. The model is highly significant. The lower R^2 (around 75%) is for the portfolio Long-Short, as the excess returns to the market portfolio (MKR) do not explain the variable. It makes sense as our portfolio 6 is a difference between excess returns, so cannot be explained by excess returns themselves.

We also showed the probability of the global F-test to test the significance of all the explicative variables. The p-value associated to the F-stat is very small for all portfolios which insist again on the significance of the overall model.

- *On the six portfolios*

Significance of the variables (p-values)

The model is also significant as p-values remain very low for the coefficient MKT, SMB and HML for almost all portfolios. The p-value for each independent variable associated to the t-stat (the Student test) tests the null hypothesis that the variable has no correlation with the dependent variable. The lower the p-value, the greater the statistical significance of the observed difference. We highlighted in green each p-value above 10%, to see which factors have a 90% level of significance in the model.

- For the portfolios 1 and 2, the p values are all close to 0. We can reject H0 at a 99% level that any of the variable is equal to 0. For the most illiquid portfolios, all Fama-French variables are significant to explain the excess returns. As all the variables are positive in the portfolio 1 and 2, we can see that all the Fama-French factors generate returns. We now need to compare with other portfolios to see if those portfolios do offer higher returns.
- For the portfolio 3, the p-values are all close to 0 except for the HML parameter which means we cannot reject H0 at a 90% level that HML is equal to 0 (not significant in the model). Indeed, HML is significant when the price to book value is extreme, which is true in more illiquid portfolios. However, as portfolio 3 is in the middle, the book value and the market value must be very similar so it is harder to evaluate the significance of the HML parameter in the model.
- For the portfolios 4 and 5 we follow the same demonstration: since we have a mix of value and growth stocks, the HML is not significant. Indeed in portfolio 4, market capitalization can be excessive in comparison to their revenues.
- The last portfolio is accurate as all the variables are significant except MKR. Indeed, as we are doing a long-short strategy, the MKR variable cannot be significant in this portfolio.

Explanation of coefficients associated to Fama-French variables (MKR, SMB and HML)

- <u>The MKR factor associated to the excess returns of the market:</u> Portfolios 1 to 5 are positively correlated to the MKR. This is coherent as the sign of the beta should be positive (as it is in the CAPM model). The only exception is the portfolio 6, not significant. Again this is accurate, as the portfolio 6 represents a difference between excess returns.

- The SMB factor associated to the size: In the long run, small companies should outperform long companies. From portfolio 1 to 5, we added a color scale to show that the SMB is decreasing with the liquidity of the stock. In portfolio 5, the SMB factor is even negative! What we can deduce is that the size is more impactful for illiquid stocks. The results are very coherent with the theoretical approach as the SMB coefficient represents the premium that is paid by small illiquid companies.
- The HML factor associated to the value: The HML factor should states that in the long-term, value stocks (high book-to-market ratio) enjoy higher returns than growth stocks (low book-to-market ratio). Portfolio 1, 2 and 3 are significant (as explained above) and the betas are almost decreasing from illiquid stocks to less illiquid stocks. This is accurate as illiquid portfolios contain mostly growth stocks (significant) whereas liquid stocks can contain both value stocks and growth stocks (not significant).

The Fama French regression allows us to dig deeper into the analysis. It is very powerful and highly significant for illiquid portfolios (especially the SMB size factor).

Another methodology

Another methodology would have been to reunite stocks all together in one portfolio and to create 5 other variables d_i. d_1 would have taken the value 1 if the stock i was in portfolio 1 or 0 if not. The same applies for d2, d3, d4 and d5. Adding this into our regession, we would have seen that SMB * d_1 factor was highly significant (a DEMI measure), as the SMB is very correlated to the illiquidity of the stocks.

Question 3

Trading liquidity varies between different assets and asset classes owing to a number of market frictions. To capture different dimensions of liquidity, such as the breadth and depth of markets, the costs of transactions, the speed which transactions can occur, and the resilience of prices to trading activity, several types of measures can be defined to quantify liquidity.

Volume-based measures

Volume-based measures distinguish liquid markets by the volume of transactions to understand breadth and depth of a market asset. The Amihud liquidity measure belongs to this category and its popularity derives from empirical evidence that more active markets tend to be more liquid. Furthermore, it is widely available as volume figures are regularly reported for most assets.

Spread-related measures

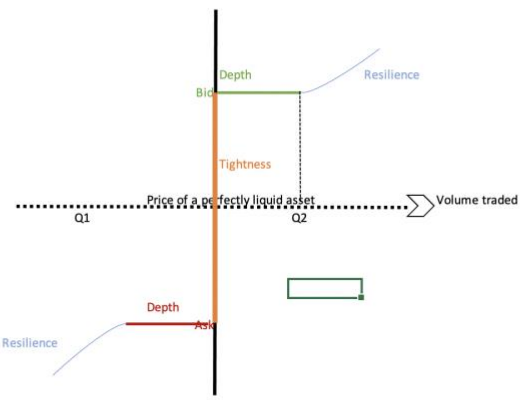
The bid-ask spread is the difference between the quoted sale price (bid) and the quoted purchase price (ask) of an asset. Indeed, even with a minimum amount of transacted volumes, the buyer must pay the bid price to enter the transaction. The bid price is normally above the fundamental price for a perfectly liquid asset (which price is not related to trading volume). On the contrary, the seller must accept to receive the ask price which is normally below the fundamental price of a perfectly liquid asset. The discount represents the illiquidity cost to the seller.

Indeed, where two assets have similar cash flows but vary in terms of liquidity, investors typically require a premium to invest in the asset with lower liquidity, thus leading to a reduction in its price. This is because lack of liquidity presents a risk of being unable to sell or having to sell at a discount at the specific time when the investor needs to exit.

The difference between the bid and the ask (bid-ask spread) is the tightness of the traded asset (orange line on our graph). The larger the spread, the more illiquid the asset.

If the buyer increases the order flow, the marginal impact of such a change is initially zero and the length of the horizontal green line defines the market depth of the asset (the longer the line, the deeper the market). However, after a certain threshold of transacted volumes Q2 in our graph, the marginal impact of trading an additional unit of volume increases and the speed of this continuous increase defines the resiliency of such a market. The same effect is observable from the seller perspective.

Figure 15: Dimensions of market liquidity

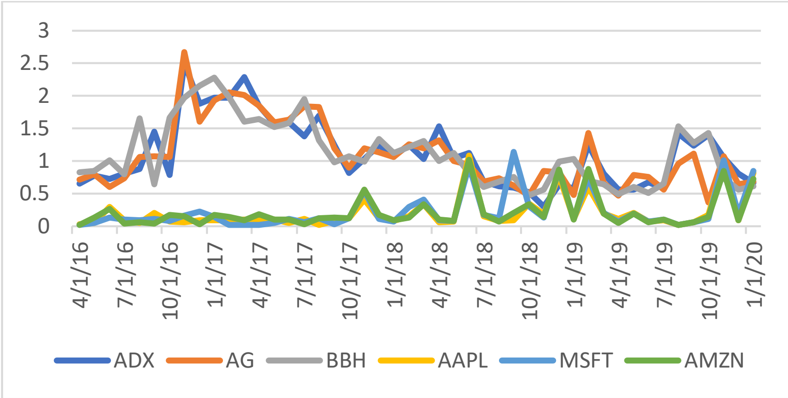


Source: Excel

The relative spread represents the most extensively used measure of illiquidity since it allows comparison between stocks with different stock prices. It can be computed as a percentage of the middle price (average of bid and ask prices) = (bid-ask)/midpoint.

We can notice however that the most important problem faced in spread related illiquidity measure is the difficulty to find spread related data for stocks. Thanks to the BDH function on Excel, we managed however to extract from Bloomberg monthly average bid-ask spreads in % for every stock of our coverage from 2016 to 2020. This average bid-ask spread can be used as an approximation of the relative spread.

Figure 16: Relative-spread comparison between stocks of different market capitalizations



Source: Excel

On the first sight, it is easy to notice that relative bid-ask spreads are much higher for lower capitalization stocks like ADX, AG or BBH (that have been randomly selected among small capitalization stocks) than for large market capitalization stocks like AAPL, MSFT or AMZN. This seems logic as the higher the market capitalization the higher the number of shares outstanding and therefore the lower the bid-ask spread as explained before.

Our spread-related illiquidity measure

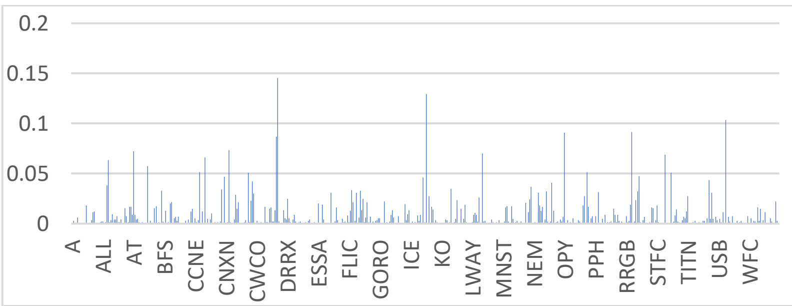
To compare the relative-spread with the Amihud volume-based measure, we can therefore define the following spread-related illiquidity measure:

$$spread - related\ Illiqu\ (i,t) = abs\big(R(i,m)\big) * relative\ spread(i,m)$$

Where Spread-related Illiqu measures the illiquidity of stock i in month t, R is the return of stock i in month m, relative spread is the average bid-ask spread of stock i in month m.

We decided to multiply the absolute monthly returns by the monthly relative spread because relative spreads and volumes are inversely correlated. The higher the traded volume the lower the bid-ask spread indeed.

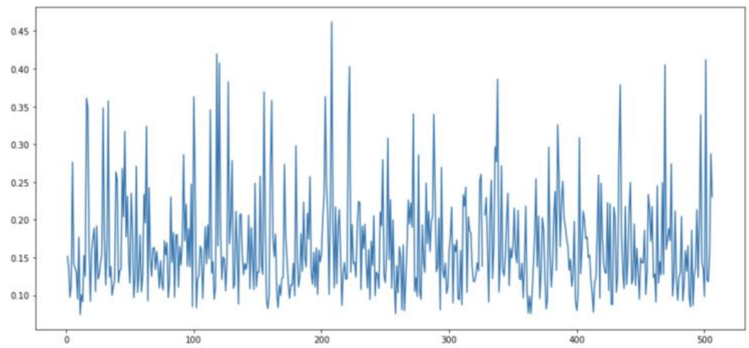
Figure 17: Mean of our spread-related illiquidity measure from 2016 to 2020 per stock



Source: Python

We computed this ratio on Python and extracted it on Excel. On the Figure above, we can easily notice that the stocks that have the higher spread-related illiquidity measure are DHR, DHX, ITI, MCF, POY, SB or VHC (all are small capitalization stocks) while big capitalization stocks like AAPL or MSFT exhibit very low measures. We can therefore believe that our hypothesis according to which the higher the relative spread the lower the liquidity of the stock is verified. However, plotting monthly returns against the ranking of the most to less liquid stocks according to our ratio on the below graph, we cannot see a rising trend as we move from liquid to illiquid securities.

Figure 18: monthly returns against ranking of the most to less liquid stocks



Source: Python

To conclude, our spread-related measure seems not to be relevant on a global view to prove a clear relationship between returns and illiquidity of stocks. This absence of trend is weird as bid-ask spreads are the best measure of liquidity for an asset even if the measure is hard to find. However, we found an explication to this phenomenon: a highly illiquid stock with a total number of trades of 0 in one month will have a bid-ask spread of 0 (so lower than highly liquid stocks that are traded millions of times every month). And so according to our formula, stocks that are not traded are the most liquid ones, which is in reality the contrary. Our reasoning becomes therefore more complicated to implement to obtain relevant results.

Question 4

Our goal in this question is to predict future prices to profit from the excess returns of illiquid stocks demonstrated in question 2 while hedging the market exposure. Thus, in this part, we implement and compare 2 algorithmic trading strategies: 1 based on signals created through the use technical indicators (the SMA), and 1 based on the application of machine learning methodologies through the OLS linear regression. Our objective is to optimize the portfolios 1 and 5 previously built (the most liquid one and the most illiquid one) to compare previous mentioned strategies and to define which one will be the most efficient for us to achieve our initial goal.

Simple Moving Average Strategy

The moving average is a technique used in technical analysis that smooths price histories by averaging monthly prices over some period of time. Logically, short-term averages respond quickly to changes in the price of the underlying security, while long-term averages are slower to react.

The optimal window (in months) to implement the SMA strategy on both portfolios are:

Figure 19: optimal windows to implement SMA strategy on the most liquid and the most illiquid portfolios

	Short Window SMA1	**Long Window SMA2**
Liquid Portfolio	2	11
Illiquid Portfolio	3	10

Source: Python

Therefore, we implement now the SMA strategy on each portfolio, where the trading rules are:

- Go LONG (= +1) when the SMA1 is above the SMA2
- Go SHORT (= -1) when the SMA1 is below the SMA2

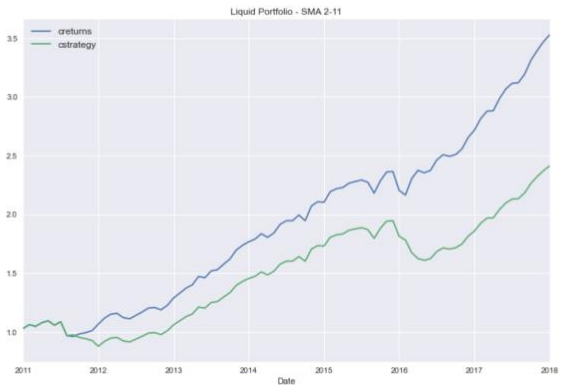
Then, we implement the Vectorized Strategy Backtesting after implemented the same method for log-returns, multiplying the position values (shifted by 1 month) by the log-returns and therefore compute the value of one invested dollar in both portfolio over time using the strategy vs not using it.

Figure 20: SMA outperformance

	SMA Outperformance
Liquid Portfolio	-1.11
Illiquid Portfolio	0.06

Source: Python

Figure 21 & 22: SMA plot for liquid and illiquid portfolios



Source: Python

We can see that the SMA strategy underperforms in general for the Liquid Portfolio and it is more and more the case as time goes by. This is supported by the last calculation of outperformance which is negative (-1.11). Regarding the Illiquid Portfolio, the strategy outperforms since mid-2015 but otherwise it underperforms between 2012 and 2015 which means that returns are higher than those implementing the strategy. However, in general the strategy outperforms as the last calculation of outperformance is positive (0.06). Thus, the SMA strategy performs better on the Illiquid Portfolio than on the Liquid Portfolio. This can be explained by the volatility of the Illiquid Portfolio: as its assets are more volatile, they also offer bigger return implementing a strategy. Notice that transaction costs (fixed fees, bid-ask spreads, lending costs...) are not included. This might be justifiable for a trading strategy that leads to a few trades only over multiple years, like this example. It is also assumed that all trades take place at the end-of-month closing prices for the portfolios. A more realistic backtesting approach would take these and other (market microstructure) elements into account.

Machine Learning Strategy - Linear Regression

The Ordinary Least Squares (OLS) procedure seeks to minimize the sum of the squared residuals. This means that given a regression line through the data we calculate the distance from each data point to the regression line, square it, and sum all of the squared errors together. According to the random walk theory, an asset's price today is the best predictor for its price tomorrow (here next month because we use monthly data). To test this claim, we can compare lagged prices of both portfolio during a various interval to their most recent price in order to determine whether they are indicative of today's price. If that's the case, then each portfolio is priced accordingly, and would therefore move randomly. On the other hand, if the lagged prices bear little to no relationship with today's price, then today's price is not the best predictor of portfolios price and therefore the market is not efficient.

Here, we use 5 lags. We calculate the optimal parameters to illustrate the random walk hypothesis:

Liquid: [1.12752061, -0.31193819, 0.15322502, 0.18073172, -0.13305291]

Illiquid: [1.0649785, -0.22643667, 0.20443831, -0.06887635, 0.03203124]

The first parameter the proportion of next month's price explained by today's price. Therefore, with our liquid and illiquid portfolio, the hypothesis which states that assets follow a random walk and therefore that today's price is the best predictor for next month price is verified. Indeed, next month's price is totally explained by today's price (more than 1). The four other values have hardly any weight assigned.

Figure 22 & 23: Price prediction for both portfolios

Source: Python

The above graphics show the price and the prediction price for the Liquid and Illiquid Portfolios since 2011. For both portfolios, the two curves are difficult to distinguish so that prove on the first sight that linear regression is an efficient way to predict prices. On this particular example using linear regression, we can see that Illiquid Portfolio's lagged prices have an extremely strong relationship with today's price.

However, this method refers to a month-to-month strategy as next month's price is explained by today's price. The buyer of the strategy must adapt his position on a month-to-month basis which can be costly.

Then, we implement the Vectorized Strategy Backtesting:

	OLS Outperformance
Liquid Portfolio	-3.91
Illiquid Portfolio	-1.13

Source: Python

Figure 25 & 26: OLS Strategy on liquid (25) and illiquid portfolio (26)

We can see that the strategy only outperforms between mid-2011 and 2012 for both portfolios. Otherwise, it underperforms for the 2 portfolios. This is supported by the last calculations of outperformances that are negative: -2.51 for the Liquid Portfolio and -0.53 for the Illiquid Portfolio. Here again, the strategy performs better on the Illiquid portfolio than on the Liquid Portfolio. This is also due to the volatility of illiquid assets.

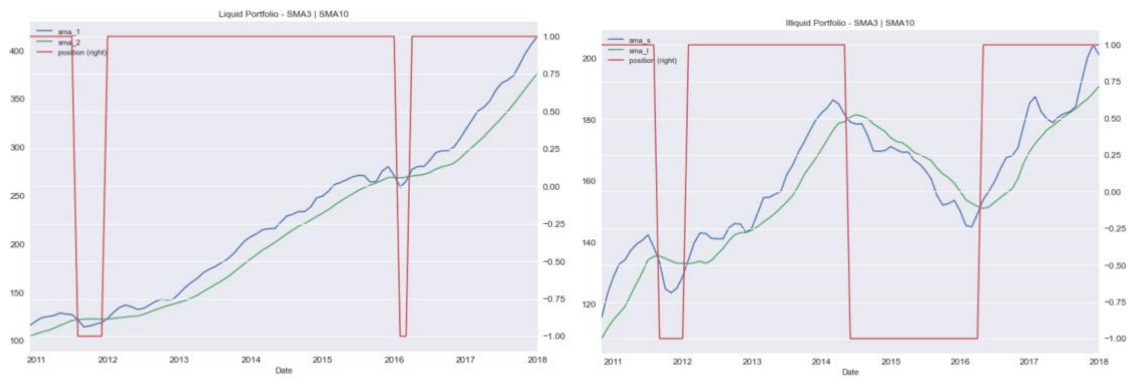
Thus, the OLS strategy is better than the mean-reversion one but better than the momentum and SMA.

Challenges of each algorithmic trading strategy

We can conclude that the best strategy for both the Liquid and the Illiquid Portfolio is the SMA. This method works better than the other ones for our portfolios with higher outperformances and positive one for the Illiquid Portfolio. Unfortunately, it doesn't perform well for the Liquid Portfolio. This is not surprising as we have studied before that returns are higher for illiquid assets than for liquid assets, implying excess returns. Thus, the strategy enables to profit from the excess returns of illiquidity while hedging the market exposure.

We plotted below the position advised by the strategy for both the Liquid and Illiquid Portfolio.

Figure 27 & 28: recommended position for both portfolios



Source: Python

We can see that the main difference of position is between 2015 and 2016: the strategy advice to short the Illiquid Portfolio from 2015 to 2016 whereas it advises to short the Liquid Portfolio only during a quick slot at the beginning of 2016. After that, studying the graphics of the value of one invested dollar in both portfolio over time, we can see that the strategy on the Illiquid Portfolio is outperforming between 2015 and 2016 and the strategy enables to have increasing returns whereas returns without the strategy were decreasing in this period. Moreover, the strategy is underperforming at this period for the Liquid Portfolio, having not advised to be short, long time before early 2015.

However, the main weakness of Movering Average (MA) is that it is calculated based on historical data, and nothing about the calculation is predictive in nature. Therefore, results using moving averages (MA) can be random. At times, the market seems to respect MA support/resistance and trade signals, and at other times, it shows these indicators no respect.

Concerning the OLS strategy, the problem of Linear Regression is that it is very sensitive to outliers. So, outliers should be analyzed and removed before applying Linear Regression to the dataset.

Finally, all this shows how the SMA strategy enabled to profit from the excess returns of illiquidity while hedging the market exposure (especially between 2015 and 2016).

Conclusion

In the first part, we saw that between 2010 and 2018, illiquid stocks did not always offer higher returns than liquid stocks. We still saw a slightly increase in returns for very illiquid stocks, which explains why portfolio 1 and 2 are the most significant. The Fama French regression allows us to dig deeper into the analysis. It is very powerful and especially for illiquid portfolios. We found strong empirical evidence that illiquid securities do generate an illiquidity premium associated with the FMB factor. We then define another measure of liquidity, the bid ask spread, to try and demonstrate similar results. Finally, we built trading strategies to benefit from the illiquidity premium while hedging the market exposure.