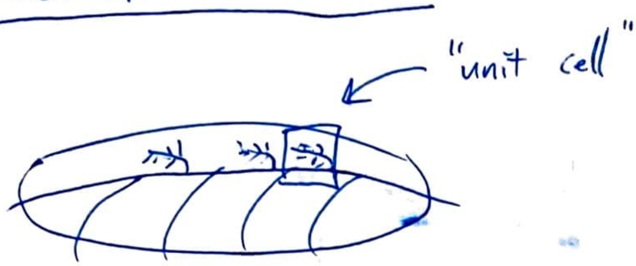
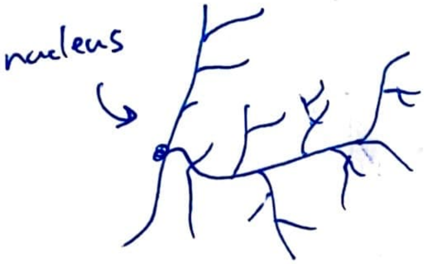
Network Perfusion



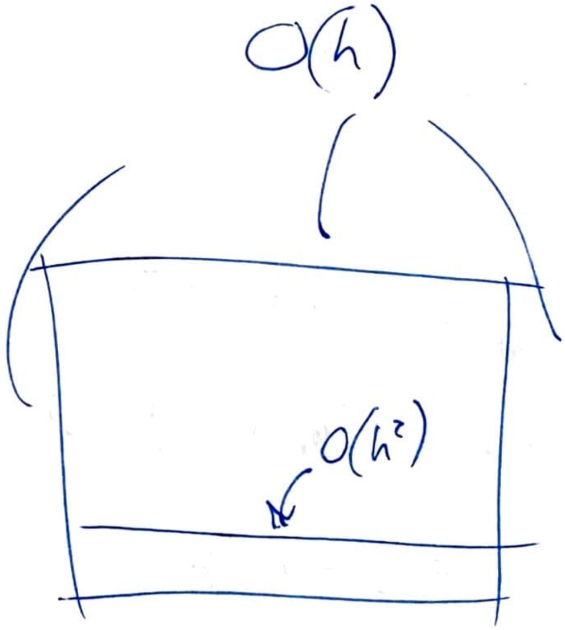


$$\Delta u - k^2 u$$

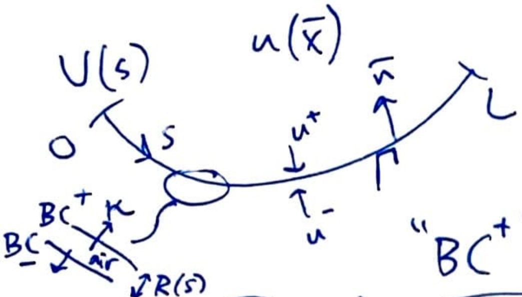
"Yakawa"

- Ensemble of trees from experiments
  ↳ for each ensemble, compute functional from steady-state solution
  ↳ can perform shape optimization from forward solves.

lumen = interior of air channel.



- CO_2 absorption
- Entrance & exit open and close.



$\left(u_n = \bar{n} \cdot \nabla u \right.$ lim on $\pm$ side $\left. \right)$

PDE bulk: $(\Delta - \phi^2) u = 0, \quad \phi^2 = \frac{\nu}{D_2}$

"BC^+": $u^+(s) - \ell u_n^+(s) = U(s)$ on Γ

$\ell = \frac{\gamma}{\phi^2}, \quad \gamma \sim \frac{1}{\kappa}$

"BC^-": $u^-(s) + \ell u_n^- = U(s)$ on Γ

1D conservation law: $\underbrace{-D_1 (RU')'}_{\text{div } j} + D_2 (u_n^- - u_n^+) = 0$

flux from both sides

Fick: flux $j = -D_1 R(s) U'(s)$

i.e. $(RU')' = -\alpha (u_n^+ - u_n^-)$

$D_2 / D_1 \ll 1$

$U(0) = 1$

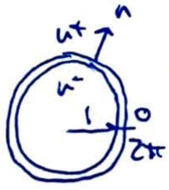
$U'(L) = 0$

$BC^+ + BC^-$: $\frac{u^+ + u^-}{2} - U = \ell (u_n^+ - u_n^-)$

$BC^+ - BC^-$: $u^+ - u^- - \ell (u_n^+ + u_n^-) = 0$ "Robin"

BIE: Ansatz: $u = S_\phi \sigma + D_\phi \tau$

(e.g. $(S\sigma)(x) := \int_\Gamma K_0(\phi |x-y|) \sigma(y) \, ds_y$



fix $U(s) \equiv c \;\; \forall s$

Does $u^+ = u^-$?

Solve a, b s.t. $\frac{u^+ + u^-}{2} - U = \ell (u_n^+ - u_n^-)$

$u^+ - u^- = \ell (u_n^+ + u_n^-)$

1D cons: $1 + (RU')' = -\alpha (u_n^+ - u_n^-)$

$u(r) = \begin{cases} a K_0(\phi r), & r > 1 \\ b I_0(\phi r), & r < 1 \end{cases}$

$K_0'(z) = -K_1(z)$ $\quad\quad$ $\partial_r K_0(\phi r) = -\phi K_1(\phi r)$

$I_0'(z) = I_1(z)$

$$u_n^+ = -a \overbrace{\phi K_1(\phi)}^{K'}$$

$$u_n^- = b \underbrace{\phi I_1(\phi)}_{I'}$$

"cons": $\frac{-1}{\alpha} = -K'a - I'b$

"sum": $\frac{aK + bI}{2} - c = \ell \underbrace{(u_n^+ - u_n^-)}_{-1/\alpha} = \frac{-\ell}{\alpha}$

"diff": $Ka - Ib = \ell(-K'a + I'b)$

BIE: sum / diff

$$\begin{cases} \begin{bmatrix} \ell I + S & D \\ D & \frac{I}{2\ell} - T \end{bmatrix} \begin{bmatrix} \sigma \\ \tau \end{bmatrix} = \begin{bmatrix} U \\ 0 \end{bmatrix} & \text{on } \Gamma \\ (RU')' - \alpha\sigma = 0 & \text{on } \Gamma \end{cases}$$

hypersingular

$\sigma = \ell_{\text{jant}}$

Eliminate U?