Exercise 1

(a)

Entropy of 1 attribute: $E(S) = \sum_{i=1}^{c} -p_i \log_2 p_i$

Wait	
Yes	No
9	6

Table 1: Frequency Table, 1 attribute, Wait, E = 0.97

$E(Wait) = E(9, 6) = E(0.6, 0.4) = 0.97$

Entropy of 2 attributes: $E(T, X) = \sum_{c \in X} P(c)E(c)$

Gain: $E(T) - E(T, X)$

		Wait		
		Yes	No	
Type	Burger	1	4	5
	Pizza	0	2	2
	Seafood	8	0	8

Table 2: Frequency Table, 2 attributes, Wait and Type, Gain = 0.15

$E(Wait, Type) = \frac{5}{15}E(1, 4) + \frac{2}{15}E(0, 2) + \frac{8}{15}E(8, 0) = 0.72$

		Wait		
		Yes	No	
Cost	$	2	1	3
	$$	4	2	6
	$$$	3	3	6

Table 3: Frequency Table, 2 attributes, Wait and Cost, Gain = 0.0192

$E(Wait, Cost) = \frac{1}{5}E(2, 1) + \frac{2}{5}E(4, 2) + \frac{2}{5}E(3, 3) = 0.9508$

		Wait		
		Yes	No	
Hunger	Yes	7	3	10
	No	2	3	5

Table 4: Frequency Table, 2 attributes, Wait and Hunger, Gain = 0.059

$E(Wait, Hunger) = \frac{2}{3}E(7, 3) + \frac{1}{3}E(2, 3) = 0.911$

The largest gain is from Type, so we split from there. As the entropy of Pizza and Seafood is 0 for each, we set those as leaf nodes with the decision being No and Yes, respectively. The entropy of Burger is 0.72, so we further split this branch.

		Wait		
		Yes	No	
Cost	$	0	1	1
	$$	1	1	2
	$$$	0	2	2

Table 5: Frequency Table, 2 attributes, Wait and Cost, Gain = 0.32

$E(Wait, Cost) = \frac{1}{5}E(0,1) + \frac{2}{5}E(1,1) + \frac{2}{5}E(0,2) = 0.4$

		Wait		
		Yes	No	
Hunger	Yes	1	1	2
	No	0	3	3

Table 6: Frequency Table, 2 attributes, Wait and Hunger, Gain = 0.32

$E(Wait, Hunger) = \frac{2}{5}E(1,1) + \frac{3}{5}E(0,3) = 0.4$

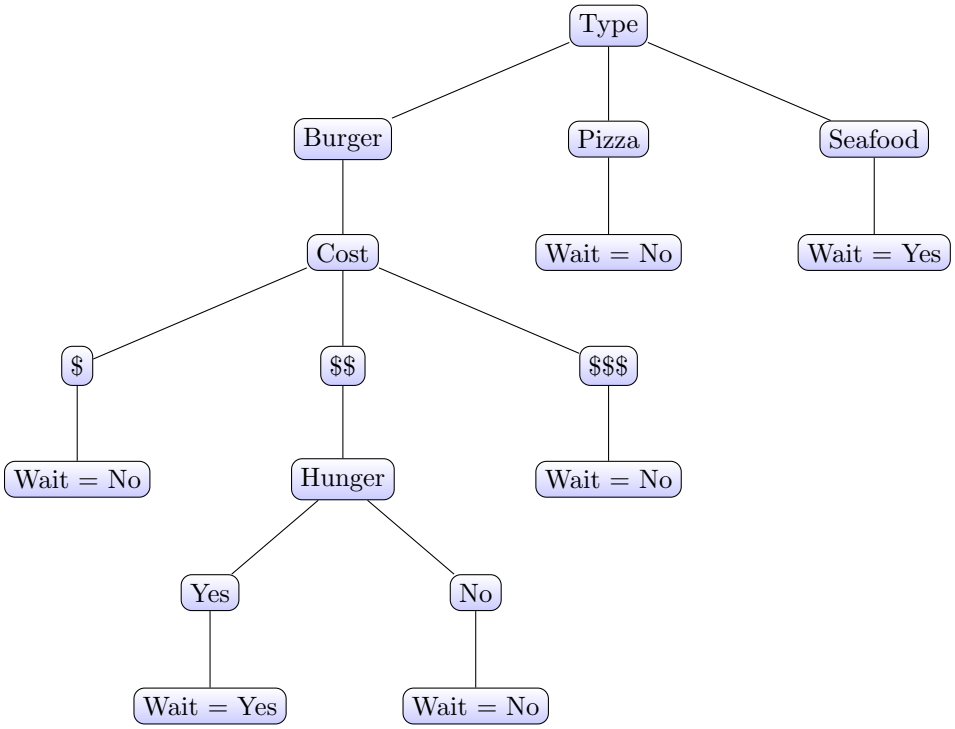
Here the gain is the same for both, so we choose to split from Cost arbitrarily. Here both the $ and $$$ costs have 0 entropy, so those are set as leaf nodes with values No for both. The entropy of $$ is 0.4, so we split this branch.

		Wait		
		Yes	No	
Hunger	Yes	1	0	1
	No	0	1	1

Table 7: Frequency Table, 2 attributes, Wait and Hunger, Gain = 0.4

As both values have entropy = 0, we can set the value Yes to Yes and the value No to No.

Here is the final decision tree:



(b)

Seeing as the Type is Pizza, we follow that path on the decision tree and find we will not wait.