Álgebra

Exercício de inversão de matrizes quadrados:

$$\underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 0 \\ 3 & 0 & 1 \end{bmatrix}}_{A} \times \begin{bmatrix} A^{-1} \\ 3\times 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|ccc} 6 & 0 & -6 & 6 & 0 & 0 \\ 6 & -3 & 0 & 0 & 3 & 0 \\ 6 & 0 & 2 & 0 & 0 & 2 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{ccc|ccc} 6 & 0 & -6 & 6 & 0 & 0 \\ 0 & -3 & 6 & -6 & 3 & 0 \\ 0 & 0 & 8 & -6 & 0 & 2 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|ccc} 24 & 0 & -24 & 24 & 0 & 0 \\ 0 & 12 & -24 & 24 & -12 & 0 \\ 0 & 0 & 24 & -18 & 0 & 6 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{ccc|ccc} 24 & 0 & 0 & 6 & 0 & 6 \\ 0 & 12 & 0 & 6 & -12 & 6 \\ 0 & 0 & 24 & -18 & 0 & 6 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{2}{4} & -\frac{4}{4} & \frac{2}{4} \\ 0 & 0 & 1 & -\frac{3}{4} & 0 & \frac{1}{4} \end{array}\right]$$

$$I \qquad A^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 0 & 1 \\ 2 & -4 & 2 \\ -3 & 0 & 1 \end{bmatrix}$$

Exercícios resolvidos 1:

1 a) $\begin{cases} 2x + y + 3z = 8 \\ 4x + 2y + 2z = 4 \\ 2x + 5y + 3z = -12 \end{cases}$ $\quad \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 2 \\ 2 & 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -12 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 8 \\ 4 & 2 & 2 & 4 \\ 2 & 5 & 3 & -12 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 4 & 2 & 6 & 16 \\ 4 & 2 & 2 & 4 \\ 4 & 10 & 6 & -24 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 4 & 2 & 6 & 16 \\ 0 & 0 & -4 & -12 \\ 0 & 8 & 0 & -40 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 4 & 2 & 6 & 16 \\ 0 & 8 & 0 & -40 \\ 0 & 0 & -4 & -12 \end{array}\right]$$

$-4z = -12 \Leftrightarrow z = 3 \qquad 8y = -40 \Leftrightarrow y = -5$

$4x + 2\times(-5) + 6\times 3 = 16 \Leftrightarrow x = 2$

Sistema possível e determinado. Solução: $(x, y, z) = (2, -5, 3)$

1 c) $\begin{cases} 2x - y + 2z = -2 \\ x + 3y - z = -2 \\ -4x - y = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 2 & -1 & 2 & -2 \\ 1 & 3 & -1 & -2 \\ -4 & -1 & 0 & 6 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 4 & -2 & 4 & -4 \\ 4 & 12 & -4 & -8 \\ -4 & -1 & 0 & 6 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 4 & -2 & 4 & -4 \\ 0 & 14 & -8 & -4 \\ 0 & -3 & 4 & 2 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 4 & -2 & 4 & -4 \\ 0 & 7 & -4 & -2 \\ 0 & -3 & 4 & 2 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 2 & -1 & 2 & -2 \\ 0 & 21 & -12 & -6 \\ 0 & -21 & 28 & 14 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 2 & -1 & 2 & -2 \\ 0 & 21 & -12 & -6 \\ 0 & 0 & 16 & 8 \end{array}\right]$$

$16z = 8 \Leftrightarrow z = \frac{1}{2}$ $\quad 21y - 12 \times \left(\frac{1}{2}\right) = -6 \Leftrightarrow y = 0$ $\quad 2x - 0 + 2 \times \frac{1}{2} = -2 \Leftrightarrow x = -\frac{3}{2}$

Sistema possível e determinado $\quad$ C.S: $(x, y, z) = \left(-\frac{3}{2}, 0, \frac{1}{2}\right)$

1 b) $\begin{cases} y + z = 3 \\ x + 2y - z = 1 \\ x + y + z = 4 \end{cases}$

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 3 \\ 1 & 2 & -1 & 1 \\ 1 & 1 & 1 & 4 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 3 \\ 1 & 1 & 1 & 4 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & -1 & 2 & 3 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

$3z = 6 \Leftrightarrow z = 2$ $\quad y + (2) = 3 \Leftrightarrow y = 1$ $\quad x + 2(1) - 2 = 1 \Leftrightarrow x = 1$

Sistema possível e determinado. C.S: $(x, y, z) = (1, 1, 2)$

3 a) $\begin{cases} 2x - 3y - z = -4 \\ -x + y + z = 1 \end{cases}$

$$\left[\begin{array}{ccc|c} 2 & -3 & -1 & -4 \\ -1 & 1 & 1 & 1 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 2 & -3 & -1 & -4 \\ -2 & 2 & 2 & 2 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 2 & -3 & -1 & -4 \\ 0 & -1 & 1 & -2 \end{array}\right]$$

$-y + z = -2 \Leftrightarrow z = y - 2$ $\quad 2x - 3y - y + 2 = -4 \Leftrightarrow$

$\Leftrightarrow 2x - 4y = -6 \Leftrightarrow x = \dfrac{4y - 6}{2} \Leftrightarrow x = 2y - 3$

Sistema possível e simplesmente indeterminado.

Solução: $(x, y, z) = (2y - 3, y, y - 2)$

③ b) $\begin{cases} x - y + 4z = 1 \\ 2x - y - 2z = 0 \\ 3x - 2y + 2z = 1 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 2 & -1 & -2 & 0 \\ 3 & -2 & 2 & 1 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 6 & -6 & 24 & 6 \\ 6 & -3 & -6 & 0 \\ 6 & -4 & 4 & 2 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 6 & -6 & 24 & 6 \\ 0 & 3 & -30 & -6 \\ 0 & 2 & -20 & -4 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 6 & -6 & 24 & 6 \\ 0 & 6 & -60 & -12 \\ 0 & 6 & -60 & -12 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 0 & 6 & -60 & -12 \\ 0 & 0 & 0 & 0 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 0 & 1 & -10 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x - y + 4z = 1 \Leftrightarrow x = 6z - 1$
$y - 10z = -2 \Leftrightarrow y = 10z - 2$

Sistema possível e simplesmente indeterminado.

$(x, y, z) = (6k - 1, 10k - 2, k), k \in \mathbb{R} = (-1, -2, 0) + k(6, 10, 1), k \in \mathbb{R}$

③ e) $\begin{cases} 2x + 4y = 16 \\ 5x - 2y = 4 \\ 3x + y = 9 \\ 4x - 5y = -7 \end{cases}$

$$\left[\begin{array}{cc|c} 2 & 4 & 16 \\ 5 & -2 & 4 \\ 3 & 1 & 9 \\ 4 & -5 & -7 \end{array}\right] \longleftrightarrow \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 5 & -2 & 4 \\ 3 & 1 & 9 \\ 4 & -5 & -7 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 0 & -12 & -36 \\ 0 & -5 & -15 \\ 0 & -13 & -39 \end{array}\right] \longleftrightarrow \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{array}\right] \longleftrightarrow \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$y = 3 \qquad x + 2y = 8 \Leftrightarrow x + 2 \times 3 = 8 \Leftrightarrow x = 8 - 6 \Leftrightarrow x = 2$

Sistema possível e determinado. $(x, y) = (2, 3)$

② Determine o valor do escalar real α, tal que o sistema de equações seja possível e determinado.

$\begin{cases} 5x + 3z + 2 = 0 \\ 2y - z + 1 = 0 \\ \alpha x + 6y + \alpha = 0 \end{cases}$

$$\left[\begin{array}{ccc|c} 5 & 0 & 3 & -2 \\ 0 & 2 & -1 & -1 \\ \alpha & 6 & 0 & -\alpha \end{array}\right] \underset{l_3 - \frac{\alpha}{5} l_1}{\longleftrightarrow} \left[\begin{array}{ccc|c} 5 & 0 & 3 & -2 \\ 0 & 2 & -1 & -1 \\ 0 & 6 & -\frac{3\alpha}{5} & -\frac{3\alpha}{5} \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 5 & 0 & 3 & -2 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 3-\frac{3\alpha}{5} & 3-\frac{3\alpha}{5} \end{array}\right]$$

① - Se $3 - \frac{3\alpha}{5} \neq 0$ ($\alpha \neq 5$), o sistema tem solução única.

Sistema possível e determinado.

② - Se $3 - \frac{3\alpha}{5} = 0$ ($\alpha = 5$):

$$\left[\begin{array}{ccc|c} 5 & 0 & 3 & -2 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$5x + 3z = -2 \Leftrightarrow x = \frac{-3z-2}{5}$

$2y - z = -1 \Leftrightarrow y = \frac{z-1}{2}$ $\qquad (x, y, z) = \left(\frac{-3z-2}{5}, \frac{z-1}{2}, z\right)$

Sistema possível e simplesmente indeterminado.

Nota: O sistema nunca é impossível neste caso.

(5) → Estude a influência do real β na solução do sistema de equações:

$$\begin{cases} \beta z + 6w = 0 \\ y + 7z + 8w = 1 \\ x + 2y + 3z + 4w = 0 \\ 5z + 6w = 0 \end{cases}$$

$x + 2y + 3z + 4w = 0$ → colocar esta em 1.º

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 7 & 8 & 1 \\ 0 & 0 & 5 & 6 & 0 \\ 0 & 0 & \beta & 6 & 0 \end{array}\right] \longleftrightarrow \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 7 & 8 & 1 \\ 0 & 0 & 5 & 6 & 0 \\ 0 & 0 & 0 & 6-\frac{6\beta}{5} & 0 \end{array}\right]$$

1 – Se $6 - \frac{6\beta}{5} \neq 0$ $(\beta \neq 5)$:

Solução para $\beta \neq 5$, por ex. $\beta = 0$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 7 & 8 & 1 \\ 0 & 0 & 5 & 6 & 0 \\ 0 & 0 & 0 & 6 & 0 \end{array}\right] \begin{array}{l} \\ \\ \to z = 0 \\ \to w = 0 \end{array}$$

Sistema possível e determinado.

$y + 7(0) + 8(0) = 1 \Leftrightarrow y = 1$

$x + 2(1) + 3(0) + 4(0) = 0 \Leftrightarrow x = -2$

2 – Se $6 - \frac{6\beta}{5} = 0$ $(\beta = 5)$:

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 7 & 8 & 1 \\ 0 & 0 & 5 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$5z + 6w = 0 \Leftrightarrow z = -\frac{6w}{5}$

$y + 7\left(-\frac{6w}{5}\right) + 8w = 1 \Leftrightarrow y = \frac{42w}{5} - 8w + 1 = \frac{2w}{5} + 1$

$x + 2\left(\frac{2w}{5} + 1\right) + 3\left(-\frac{6w}{5}\right) + 4w = 0 \Leftrightarrow x = -\frac{4w}{5} - 2 + \frac{18w}{5} - 4w \Leftrightarrow$

$\Leftrightarrow x = -\frac{6w}{5} - 2$ $\quad (x, y, z, w) = \left(-\frac{6w}{5} - 2, \frac{2w}{5} + 1, -\frac{6w}{5}, w\right)$

Sistema possível e simplesmente indeterminado.

(9) a) $\begin{cases} x + ay + z = 2 \\ -x - ay + 2z = b \\ 2x + az = 3 \end{cases}$ $(a, b \in \mathbb{R})$

$$\left[\begin{array}{ccc|c} 1 & a & 1 & 2 \\ -1 & -a & 2 & b \\ 2 & 0 & a & 3 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 1 & a & 1 & 2 \\ 0 & 0 & 3 & b+2 \\ 0 & -2a & a-2 & -1 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & a & 1 & 2 \\ 0 & -2a & a-2 & -1 \\ 0 & 0 & 3 & b+2 \end{array}\right]$$

Se $a = 0$: $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & 3 & b+2 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 0 & -6 & -3 \\ 0 & 0 & 6 & 2b+4 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 0 & -6 & -3 \\ 0 & 0 & 0 & 2b+1 \end{array}\right]$

Conclusão: Com $a = 0 \wedge 2b + 1 = 0$ → possível e simplesmente indeterminado.
Com $a = 0 \wedge 2b + 1 \neq 0$ → impossível.

Se $a = 2$: $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & 3 & b+2 \end{array}\right]$ → qualquer que seja o valor de b, a solução é sempre única.

Sistema possível e determinado.

Dado ainda, para $a \neq 0 \wedge b \in \mathbb{R}$, o sistema é sempre possível e determinado.

Introdução aos sistemas de equações lineares (Exercícios propostos)

4 a) $\begin{cases} 2x+4y=16 \\ 5x-2y=4 \\ 10x-4y=3 \end{cases}$ $\quad \left[\begin{array}{cc|c} 2 & 4 & 16 \\ 5 & -2 & 4 \\ 10 & -4 & 3 \end{array}\right] \longleftrightarrow$

$$\longleftrightarrow \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 5 & -2 & 4 \\ 10 & -4 & 3 \end{array}\right] \longleftrightarrow \left[\begin{array}{cc|c} 10 & 20 & 80 \\ 10 & -4 & 8 \\ 10 & -4 & 3 \end{array}\right] \longleftrightarrow \left[\begin{array}{cc|c} 10 & 20 & 80 \\ 0 & -24 & -72 \\ 0 & -24 & -77 \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{cc|c} 10 & 20 & 80 \\ 0 & -24 & -72 \\ 0 & 0 & -5 \end{array}\right] \quad 0=-5 \longrightarrow \text{Sistema impossível.}$$

b) $\begin{cases} x-y+z=2 \\ 3x+2y-2z=6 \\ -x-4y+4z=1 \end{cases}$ $\quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 3 & 2 & -2 & 6 \\ -1 & -4 & 4 & 1 \end{array}\right] \longleftrightarrow$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 5 & -5 & 0 \\ 0 & -5 & 5 & 3 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 5 & -5 & 0 \\ 0 & 0 & 0 & 3 \end{array}\right] \quad 0=3 \rightarrow \text{Sistema impossível.}$$

6 $\begin{cases} x+2y-z+u=0 \\ 3t-2u=0 \\ -x-2y+z+3t-3u=0 \end{cases}$ $\quad \left[\begin{array}{ccccc|c} -1 & -2 & 1 & 3 & -3 & 0 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & -2 & 0 \end{array}\right] \longleftrightarrow$

$$\longleftrightarrow \left[\begin{array}{ccccc|c} -1 & -2 & 1 & 3 & -3 & 0 \\ 0 & 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 3 & -2 & 0 \end{array}\right] \longleftarrow \left[\begin{array}{ccccc|c} -1 & -2 & 1 & 3 & -3 & 0 \\ 0 & 0 & 0 & 3 & -2 & 0 \end{array}\right]$$

$3t-2u=0 \Leftrightarrow t=\frac{2u}{3} \quad -x-2y+z+3\left(\frac{2u}{3}\right)-3u=0 \Leftrightarrow$

$\Leftrightarrow x=-2y+z-u \quad$ Solução geral: $\left(-2y+z-u, y, z, \frac{2u}{3}, u\right), y, z, u \in \mathbb{R}$

Solução particular: $(-2, 1, 0, 0, 0, 0)$ (para $y=1$, $z=0$ e $u=0$).

7 a) $\begin{cases} x+y+z+t=0 \\ x+\alpha y+2z+3t=0 \\ x+y+\alpha z+4t=0 \\ x+y+z+\alpha t=0 \end{cases}$ $\quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & \alpha & 2 & 3 & 0 \\ 1 & 1 & \alpha & 4 & 0 \\ 1 & 1 & 1 & \alpha & 0 \end{array}\right] \longleftrightarrow$

(com solução nula)

$$\longleftrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & \alpha-1 & 1 & 2 & 0 \\ 0 & 0 & \alpha-1 & 3 & 0 \\ 0 & 0 & 0 & \alpha-1 & 0 \end{array}\right]$$

Para que seja possível e determinado:

$\alpha-1 \neq 0 \Leftrightarrow \alpha \neq 1$

R: $\alpha \neq 1$

$(\alpha-1)t=0 \Leftrightarrow t=0$

$(\alpha-1)z+3(0)=0 \Leftrightarrow z=0$

$(\alpha-1)y+1(0)+2(0)=0 \Leftrightarrow y=0$

$x+1(0)+1(0)+1(0)=0 \Leftrightarrow x=0$

b) Para que seja possível e simplesmente indeterminado: $\alpha=1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$3t=0 \Leftrightarrow t=0 \qquad z+2(0)=0 \Leftrightarrow z=0$

$x+y+1(0)+1(0)=0 \Leftrightarrow x+y=0 \Leftrightarrow y=-x$

R: $(x,y,z,t)=(x,-x,0,0)$

8) $\begin{cases} x - y + 2z + 2t = b \\ x - y + a t = 5 \\ x - y + 3z - t = -1 \end{cases}$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 2 & b \\ 1 & -1 & 0 & a & 5 \\ 1 & -1 & 3 & -1 & -1 \end{array}\right] \longleftrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & 2 & b \\ 0 & 0 & -2 & a-2 & 5-b \\ 0 & 0 & 1 & -3 & -1-b \end{array}\right] \longleftrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & 2 & b \\ 0 & 0 & -2 & a-2 & 5-b \\ 0 & 0 & 2 & -6 & -2-2b \end{array}\right] \longleftrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & 2 & b \\ 0 & 0 & 2 & -6 & -2-2b \\ 0 & 0 & -2 & a-2 & 5-b \end{array}\right] \longleftrightarrow$$

$$\longleftrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & 2 & b \\ 0 & 0 & 2 & -6 & -2-2b \\ 0 & 0 & 0 & a-8 & 3-3b \end{array}\right] \longleftrightarrow \left[\begin{array}{cccc|c} 1 & -1 & 2 & 2 & b \\ 0 & 0 & 1 & -3 & -1-b \\ 0 & 0 & 0 & a-8 & 3-3b \end{array}\right]$$

Para que o sistema seja duplamente indeterminado:

$\begin{cases} a - 8 = 0 \\ 3 - 3b = 0 \end{cases} \Leftrightarrow \begin{cases} a = 8 \\ b = 1 \end{cases}$ $\left[\begin{array}{cccc|c} 1 & -1 & 2 & 2 & 1 \\ 0 & 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$z - 3t = -2 \Leftrightarrow z = 3t - 2$ $x - y + 6t - 4 + 2t = 1 \Leftrightarrow$

$\Leftrightarrow x = y - 8t + 5$ R: $a = 8$ e $b = 1$; $(x, y, z, t) = (y - 8t + 5, y, 3t - 2t)$

9) b) $\begin{cases} 2x - y + z = 1 \\ -2y + z = 0 \\ 4x + ay + 2z = 2 \\ 6x - 3y + 3z = b \end{cases}$ $\left[\begin{array}{ccc|c} 2 & -1 & 1 & 1 \\ 0 & -2 & 1 & 0 \\ 4 & a & 2 & 2 \\ 6 & -3 & 3 & b \end{array}\right] \longleftrightarrow$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 24 & -12 & 12 & 12 \\ 0 & -2 & 1 & 0 \\ 24 & 6a & 12 & 12 \\ 24 & -12 & 12 & 4b \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 24 & -12 & 12 & 12 \\ 0 & -2 & 1 & 0 \\ 0 & 6a+12 & 0 & 0 \\ 0 & 0 & 0 & 4b-12 \end{array}\right]$$

$6a + 12 = 0 \Leftrightarrow a = -2$

$4b - 12 = 0 \Leftrightarrow b = 3$

Sistema possível e determinado: $a \neq -2$ e $b = 3$
Sistema possível e simplesmente indeterminado: $a = -2$ e $b = 3$
Sistema impossível: $b \neq 3 \wedge a \in \mathbb{R}$.

c) $\begin{cases} x + y = 0 \\ -2x + y + az = 1 \\ ax + y = a - 1 \\ -2x + 2az = 2 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ -2 & 1 & a & 1 \\ a & 1 & 0 & a-1 \\ -2 & 0 & 2a & 2 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 3 & a & 1 \\ 0 & 1-a & 0 & a-1 \\ 0 & 2 & 2a & 2 \end{array}\right] \longleftrightarrow$

$$\longleftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 3 & a & 1 \\ 0 & 0 & 0 & a-1 \\ 0 & 0 & 2a & 2 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 3 & a & 1 \\ 0 & 0 & 2a & 2 \\ 0 & 0 & 0 & a-1 \end{array}\right]$$

$a - 1 = 0 \Leftrightarrow a = 1$

$2az = 2 \Leftrightarrow az = 0 \Leftrightarrow z = 0$

$3y = 1 \Leftrightarrow y = \frac{1}{3}$

$x + \frac{1}{3} = 0 \Leftrightarrow x = -\frac{1}{3}$

Sistema possível e determinado se $a = 1$

Sistema impossível se $a \neq 1$.

d) $\begin{cases} x + ay + 2z = 0 \\ -x + 2y + z = b \\ y + az = 1 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & a & 2 & 0 \\ -1 & 2 & 1 & b \\ 0 & 1 & a & 1 \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & a & 2 & 0 \\ 0 & a+2 & 3 & b \\ 0 & 1 & a & 1 \end{array}\right] \longleftrightarrow$

$\longleftrightarrow \left[\begin{array}{ccc|c} 1 & a & 2 & 0 \\ 0 & 1 & a & 1 \\ 0 & a+2 & 3 & b \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & a & 2 & 0 \\ 0 & 1 & a & 1 \\ 0 & 0 & a^2+2a-3 & a+2-b \end{array}\right]$

$a^2 + 2a - 3 = 0 \Leftrightarrow a = \frac{-2 \pm \sqrt{4+12}}{2} \Leftrightarrow a = \frac{-2 \pm 4}{2} \Leftrightarrow a = -3 \vee a = 1$

$a + 2 - b = 0 \Leftrightarrow b = a + 2 \begin{cases} b = -1 \text{ se } a = -3 \\ b = 3 \text{ se } a = 1 \end{cases}$

Sistema impossível se $(a = -3 \wedge b \neq -1)$ ou $(a = 1 \wedge b \neq 3)$.

Se $a = -3 \wedge b = -1$:

$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $x - 3y + 2 = 0 \Leftrightarrow x - 9z - 3 + 2 = 0 \Leftrightarrow x = 9z + 1$

$y - 3z = 1 \Leftrightarrow y = 3z + 1 \quad (x, y, z) = (9z+1, 3z+1, z)$

Se $a = 1 \wedge b = 3$:

$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $x + y + 2 = 0 \Leftrightarrow x + 1 - z + 2 = 0 \Leftrightarrow x = z - 3$

$y + z = 1 \Leftrightarrow y = 1 - z \quad (x, y, z) = (z-3, 1-z, z)$

Sistema possível e simplesmente indeterminado se $(a = -3 \wedge b = -1)$ ou $(a = 1 \wedge b = 3)$

Sistema possível e determinado se $a \neq -3 \wedge a \neq 1 \wedge b \in \mathbb{R}$

e) $\begin{cases} x + a^2 y + az = ab \\ x + y + z = b \\ x + a^2 y + a^2 z = ab \end{cases}$ $\left[\begin{array}{ccc|c} 1 & a^2 & a & ab \\ 1 & 1 & 1 & b \\ 1 & a^2 & a^2 & ab \end{array}\right] \longleftrightarrow \left[\begin{array}{ccc|c} 1 & a^2 & a & ab \\ 0 & 1-a^2 & 1-a & b-ab \\ 0 & 0 & a^2-a & 0 \end{array}\right]$

$1 - a = 0 \Leftrightarrow a = 1$

$a^2 - a = 0 \Leftrightarrow a(a-1) = 0 \Leftrightarrow a = 0 \vee a = 1 \qquad 1 - a^2 = 0 \Leftrightarrow a^2 = 1 \Leftrightarrow a = -1 \vee a = 1$

Se $a = 0$: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & b \\ 0 & 0 & 0 & 0 \end{array}\right]$ $x = 0$

$y + z = b$

Se $a = -1$: $\left[\begin{array}{ccc|c} 1 & 1 & -1 & -b \\ 0 & 0 & 2 & 2b \\ 0 & 0 & 2 & 0 \end{array}\right]$ $x + y - 1(0) = 0 \Leftrightarrow x + y = 0 \Leftrightarrow x = -y$

$2(0) = 2b \Leftrightarrow b = 0$

$z = 0$

Se $a = 1$: $\left[\begin{array}{ccc|c} 1 & 1 & 1 & b \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $x + y + z = b \Leftrightarrow x = -y - z + b$

Sistema possível e determinado se $(a \in \mathbb{R} \setminus \{-1, 0, 1\} \wedge b \in \mathbb{R})$

Sistema possível e simplesmente indeterminado se $(a = 0 \wedge b \in \mathbb{R})$ ou $(a = -1 \wedge b = 0)$

Sistema possível e duplamente indeterminado se $(a = 1 \wedge b \in \mathbb{R})$

Sistema impossível se $(a = -1 \wedge b \neq 0)$