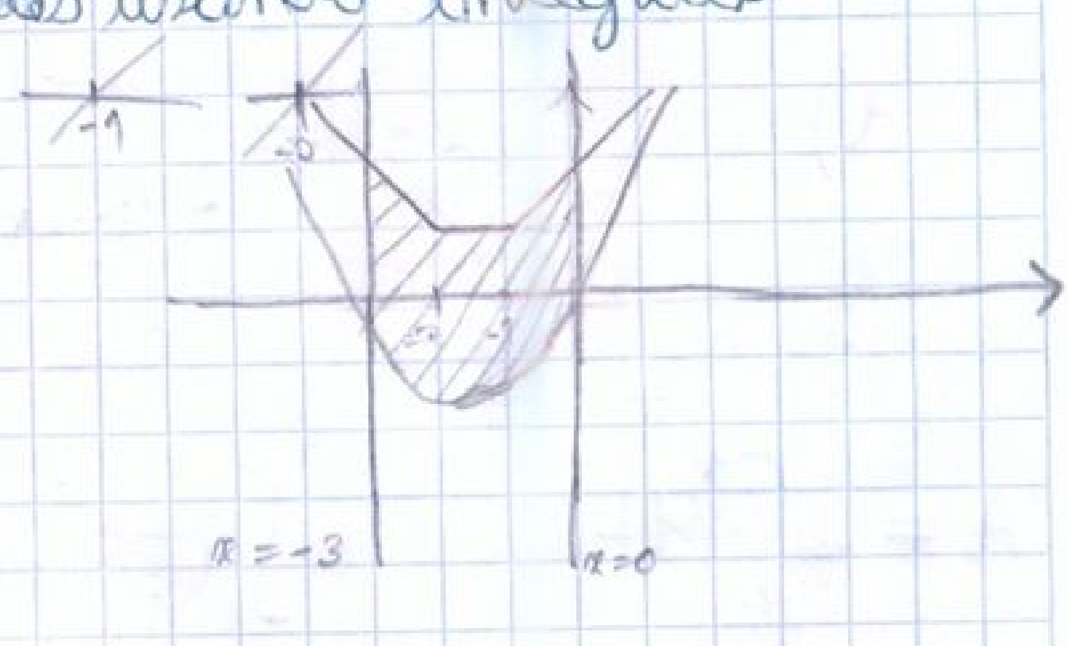
3. Cálculo de áreas usando integrais

$x+1=0 \Leftrightarrow x=-1$

$x+2=0 \Leftrightarrow x=-2$

$-x-1 \quad -x-2$

1. $|x+1|+|x+2|$



$$f(x)=\begin{cases} -x-1-x-2, & x<-2 \Leftrightarrow -2x-3 \\ -x-1+x+2, & -2<x<-1 \Leftrightarrow 1 \\ x+1+x+2, & x>-1 \Leftrightarrow 2x+3 \end{cases}$$

$x^2+3x=0 \Leftrightarrow x(x+3)=0 \Leftrightarrow x=0 \vee x=-3$

$$A_1=\int_{-3}^{-2}(-2x-3)\,dx=\Big[-x^2-3x\Big]_{-3}^{-2}=-4+6-(-9+9)=2-0=2$$

$$A_2=\int_{-2}^{-1}1\,dx=\Big[x\Big]_{-2}^{-1}=-1-(-2)=-1+2=1$$

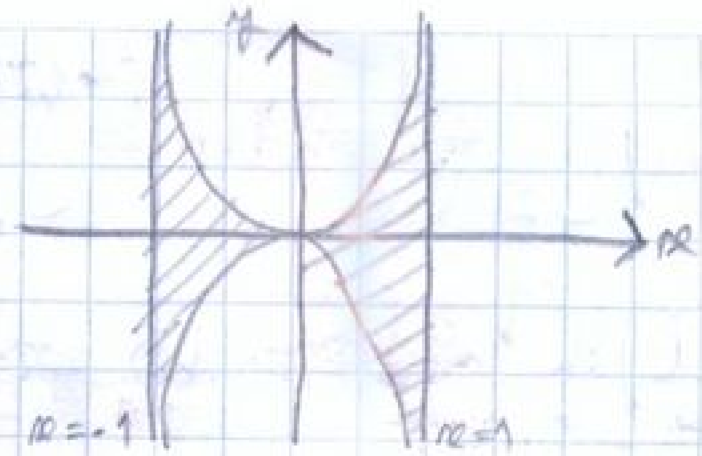
$$A_3=\int_{-1}^{0}2x+3\,dx=\Big[x^2+3x\Big]_{-1}^{0}=0-(1-3)=0+2=2$$

$$A_4=\int_{-3}^{0}x^2+3x\,dx=\Big[\frac{1}{3}x^3+\frac{3}{2}x^2\Big]_{-3}^{0}=0-\left(-9+\frac{27}{2}\right)=\left|\frac{18}{2}-\frac{27}{2}\right|=\left|-\frac{9}{2}\right|=\frac{9}{2}$$

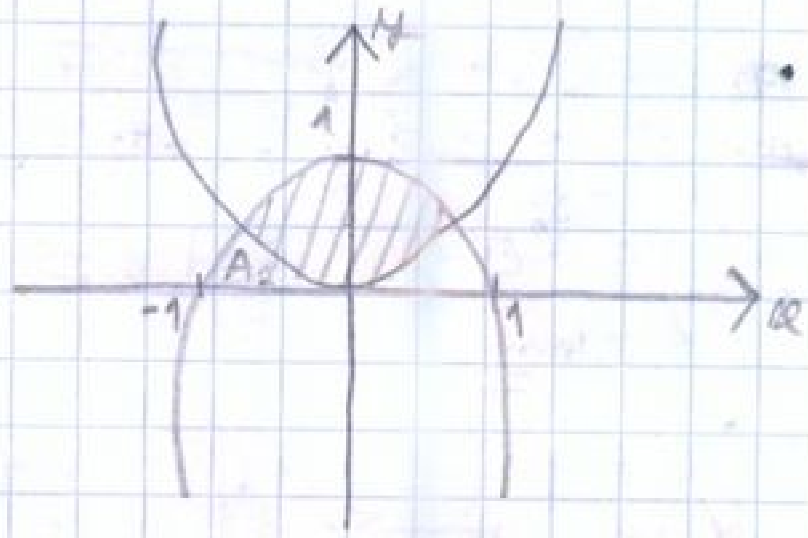
$$A_{total}=2+2+1+\frac{9}{2}=\frac{19}{2}$$

2 a) $A = 4\int_{-1}^{0} x^2\,dx = 4\left[\frac{1}{3}x^3\right]_{-1}^{0} =$

$= 4\left(0 - \left(-\frac{1}{3}\right)\right) = 4 \times \frac{1}{3} = \frac{4}{3}$



b) $A = A_1 - 2A_2$ $\quad 1 - x^2 = 0 \Leftrightarrow x = -1 \vee x = 1$



$x^2 = 1 - x^2 \Leftrightarrow 2x^2 = 1 \Leftrightarrow x = \pm\sqrt{\frac{1}{2}} \Leftrightarrow$

$\Leftrightarrow x = -\frac{\sqrt{2}}{2} \vee x = \frac{\sqrt{2}}{2}$

$A = \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} (1 - x^2)\,dx - \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} x^2\,dx =$

$= \left[x - \frac{1}{3}x^3\right]_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} - \left[\frac{1}{3}x^3\right]_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2} - \frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)^3 - \left[-\frac{\sqrt{2}}{2} - \frac{1}{3}\times\left(-\frac{\sqrt{2}}{2}\right)^3\right] - \left[\frac{1}{3}\times\left(\frac{\sqrt{2}}{2}\right)^3 - \frac{1}{3}\times\left(-\frac{\sqrt{2}}{2}\right)^3\right] =$

$= \left|\frac{\sqrt{2}}{2} - \frac{2\sqrt{2}}{24} + \frac{\sqrt{2}}{2} - \frac{2\sqrt{2}}{24} - \frac{2\sqrt{2}}{24} - \frac{2\sqrt{2}}{24}\right| = \frac{24\sqrt{2}}{24} - \frac{8\sqrt{2}}{24} = \frac{16\sqrt{2}}{24} = \frac{2\sqrt{2}}{3}$

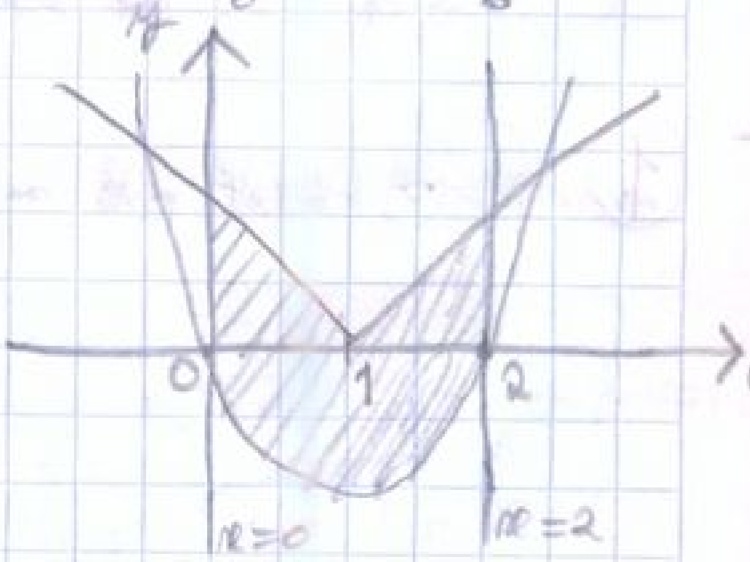
c) $A = \int_0^1 x\,dx + \int_1^2 \sqrt{x}\,dx = \left|\int_0^2 x = 2\right.$

$= \left[\frac{1}{2}x^2\right]_0^1 + \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^2 = \frac{1}{2}\times 1^2 - 0 + \frac{2}{3}\times 2^{\frac{3}{2}} - \frac{2}{3}\times 1^{\frac{3}{2}} =$

$= \frac{1}{2} + \frac{4\sqrt{2}}{3} - \frac{2}{3} = \frac{3}{6} + \frac{8\sqrt{2}}{6} - \frac{4}{6} = \frac{8\sqrt{2} - 1}{6}$

$A_{total} = 2 - \frac{8\sqrt{2} - 1}{6} = \frac{13 - 8\sqrt{2}}{6}$

d) $f(x) = |x - 1| = \begin{cases} x - 1 & \text{se } x \geq 1 \\ 1 - x & \text{se } x < 1 \end{cases}$



$x^2 - 2x = 0 \Leftrightarrow x(x - 2) = 0 \Leftrightarrow x = 0 \vee x = 2$

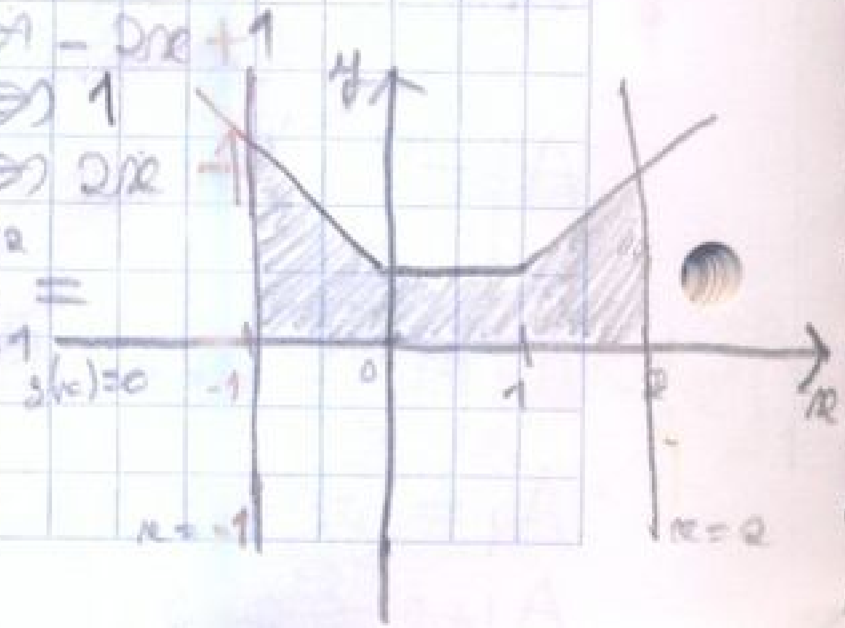
$A = 2\int_0^1 (1 - x)\,dx + \int_0^2 (x^2 - 2x)\,dx =$

$= 2\left[x - \frac{1}{2}x^2\right]_0^1 + \left[\frac{1}{3}x^3 - x^2\right]_0^2 = \left|2\left(1 - \frac{1}{2}\right)\right| + \left|\frac{1}{3}\times 2^3 - 2^2\right| = 1 + \frac{4}{3} = \frac{7}{3}$

e) $f(x) = |x| + |x - 1| = \begin{cases} -x - x + 1 & \text{se } x \leq 0 \Leftrightarrow -2x + 1 \\ x - x + 1 & \text{se } 0 < x < 1 \Leftrightarrow 1 \\ x + x - 1 & \text{se } x \geq 1 \Leftrightarrow 2x \end{cases}$

$A = \int_{-1}^{0} (-2x + 1)\,dx + \int_0^1 1\,dx + \int_1^2 (2x - 1)\,dx = \left[-x^2 + x\right]_{-1}^{0} + \left[x\right]_0^1 + \left[x^2 - x\right]_1^2 =$

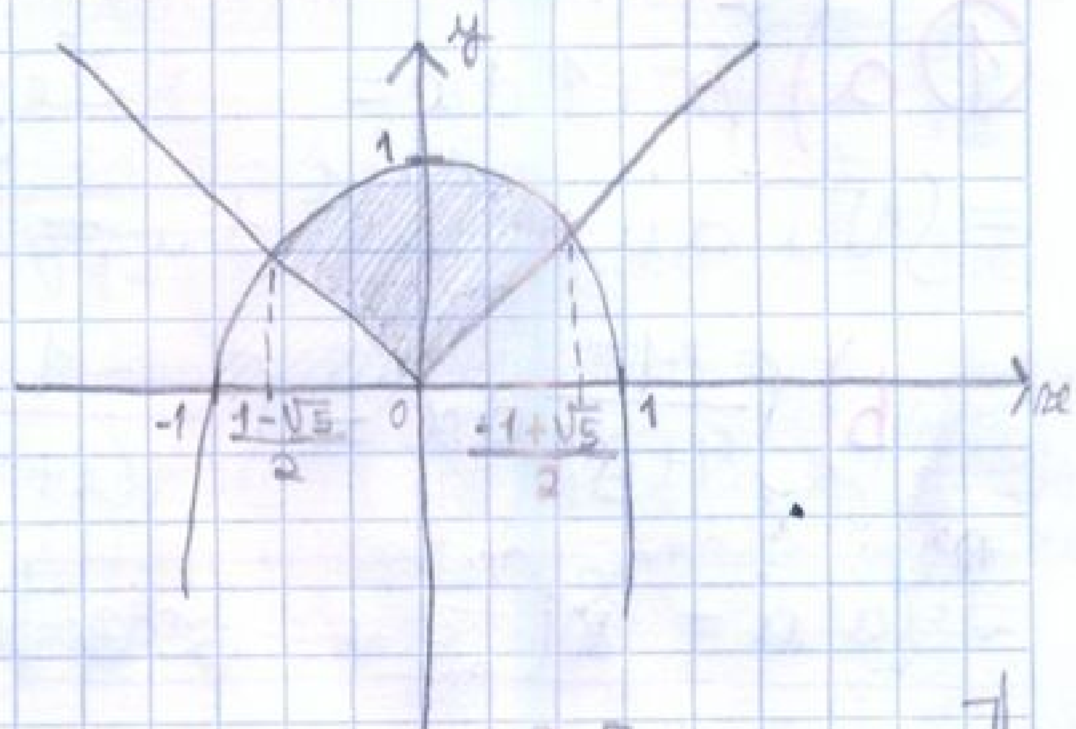
$= |-(-1 - 1)| + |1| + |4 - 2 - 1 + 1| = 2 + 1 + 2 = 5 \quad A = 5$



f) $f(x)=|x|=\begin{cases}-x & \text{se } x\leq 0\\ x & \text{se } x>0\end{cases}$ $\quad 1-x^2=0 \Leftrightarrow x^2=1 \Leftrightarrow x=-1 \vee x=1$

$-x=1-x^2 \Leftrightarrow x^2-x-1=0 \Leftrightarrow x=\frac{1\pm\sqrt{1+4}}{2} \Leftrightarrow$
$\Leftrightarrow \cancel{x=\frac{1+\sqrt{5}}{2}} \vee x=\frac{1-\sqrt{5}}{2} \leftarrow x<0$

$x=1-x^2 \Leftrightarrow x^2+x-1=0 \Leftrightarrow x=\frac{-1\pm\sqrt{1+4}}{2} \Leftrightarrow$
$\Leftrightarrow \cancel{x=\frac{-1-\sqrt{5}}{2}} \vee x=\frac{-1+\sqrt{5}}{2} \leftarrow x>0$



$$A=\int_{\frac{1-\sqrt{5}}{2}}^{\frac{-1+\sqrt{5}}{2}}(1-x^2)\,dx-2\int_{\frac{1-\sqrt{5}}{2}}^{0}(-x)\,dx=$$

$$=\left[x-\frac{1}{3}x^3\right]_{\frac{1-\sqrt{5}}{2}}^{\frac{-1+\sqrt{5}}{2}}-2\left[-\frac{1}{2}x^2\right]_{\frac{1-\sqrt{5}}{2}}^{0}=\left|\frac{-1+\sqrt{5}}{2}-\frac{1}{3}\times\left(\frac{-1+\sqrt{5}}{2}\right)^3-\left[\frac{1-\sqrt{5}}{2}-\frac{1}{3}\times\left(\frac{1-\sqrt{5}}{2}\right)^3\right]\right|-2\left(\frac{1}{2}\times\left(\frac{1-\sqrt{5}}{2}\right)^2\right)$$

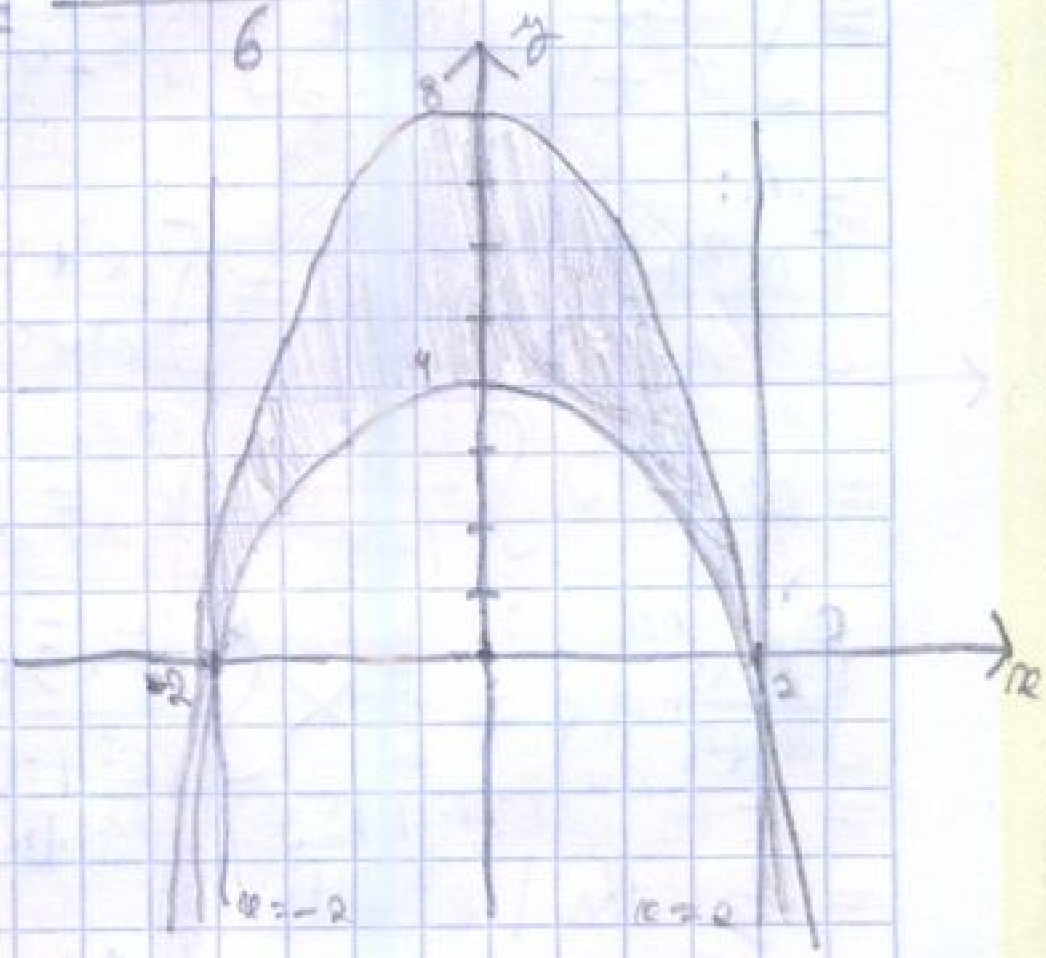
$$=\left|\frac{-1+\sqrt{5}}{2}-\frac{1}{3}\times\left(\frac{(-1+\sqrt{5})(1-2\sqrt{5}+5)}{8}\right)-\left[\frac{1-\sqrt{5}}{2}-\frac{1}{3}\times\frac{(1-\sqrt{5})(1-2\sqrt{5}+5)}{8}\right]\right|-2\left|\left(\frac{1}{2}\times\frac{1-2\sqrt{5}+5}{4}\right)\right|=$$

$$=\left|\frac{-12+12\sqrt{5}}{24}-\frac{-1+2\sqrt{5}-5+\sqrt{5}-10+5\sqrt{5}}{24}-\frac{12-12\sqrt{5}}{24}+\frac{1-2\sqrt{5}+5-\sqrt{5}+10-5\sqrt{5}}{24}\right|-2\left|\frac{1-2\sqrt{5}+5}{8}\right|=$$

$$=\left|\frac{-24+24\sqrt{5}+2-4\sqrt{5}+10-2\sqrt{5}+20-10\sqrt{5}}{24}\right|-\frac{1-2\sqrt{5}+5}{4}=$$

$$=\frac{8+8\sqrt{5}}{24}-\frac{6-12\sqrt{5}+30}{24}=\frac{-28+20\sqrt{5}}{24}=\frac{-7+5\sqrt{5}}{6}$$

g) $4-x^2=0 \Leftrightarrow x^2=4 \Leftrightarrow x=-2 \vee x=2$
$8-2x^2=0 \Leftrightarrow 2x^2=8 \Leftrightarrow x^2=4 \Leftrightarrow x=-2 \vee x=2$



$$A=\int_{-2}^{2}(8-2x^2)\,dx-\int_{-2}^{2}(4-x^2)\,dx=$$

$$=\left[8x-\frac{2}{3}x^3\right]_{-2}^{2}-\left[4x-\frac{1}{3}x^3\right]_{-2}^{2}=$$

$$=\left|16-\frac{16}{3}-\left(-16+\frac{16}{3}\right)\right|-\left|\left[8-\frac{8}{3}-\left(-8+\frac{8}{3}\right)\right]\right|=$$

$$=32-\frac{32}{3}-\left(16-\frac{16}{3}\right)=16-\frac{16}{3}=\frac{48}{3}-\frac{16}{3}=\frac{32}{3}$$