$$\int \frac{u'}{1+u^2} = \arctan(u)$$

4. Primitivação por substituição

① a) $\int \sqrt{x+1}\, dx =$ Seja $u = x+1$

$= \int \sqrt{u}\, du = \int u^{\frac{1}{2}}\, du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2u\sqrt{u}}{3} + c = \frac{2(x+1)\sqrt{x+1}}{3} + c, c \in \mathbb{R}$

b) $\int \frac{1}{4+x^2}\, dx = \int \frac{1}{4(1+\frac{1}{4}x^2)} dx = \int \frac{1}{4} \times \frac{1}{1+\frac{x^2}{4}}\, dx = \frac{1}{2} \int \frac{\frac{1}{2}}{1+\left(\frac{x}{2}\right)^2}\, dx =$

$\boxed{\text{Seja } u = \frac{x}{2} \quad du = \frac{1}{2} dx}$ $= \frac{1}{2} \int \frac{du}{1+u^2} = \frac{1}{2} \arctan(u) = \frac{1}{2} \arctan\left(\frac{x}{2}\right)$

c) $\int \frac{e^x}{\sqrt[3]{1+2e^x}}\, dx$ Seja $u = 1+2e^x$, $du = 2e^x\, dx$

$\int \frac{e^x}{\sqrt[3]{1+2e^x}}\, dx = \frac{1}{2} \int \frac{2e^x}{\sqrt[3]{1+2e^x}}\, dx = \frac{1}{2} \int \frac{1}{\sqrt[3]{u}}\, du = \frac{1}{2} \int u^{-\frac{1}{3}}\, du =$

$= \frac{1}{2} \times \frac{3}{2} u^{\frac{2}{3}} + c = \frac{3}{4} u^{\frac{2}{3}} + c = \frac{3}{4}(1+2e^x)^{\frac{2}{3}} + c$

d) $\int \frac{x^2+5x+6}{x^2+4}\, dx = \int \left(\frac{x^2}{x^2+4} + \frac{5x}{x^2+4} + \frac{6}{x^2+4}\right) dx =$

$= \int \frac{x^2}{x^2+4}\, dx + \int \frac{5x}{x^2+4}\, dx + \int \frac{6}{x^2+4}\, dx = (1)$

Seja $u = \frac{x}{2}$, $du = \frac{1}{2} dx$

C.A.:

$\int \frac{x^2+4-4}{x^2+4}\, dx = \int \frac{x^2+4}{x^2+4}\, dx - \int \frac{4}{x^2+4}\, dx = \int 1\, dx - \int \frac{1}{1+\left(\frac{x}{2}\right)^2}\, dx$

$= x - 2 \int \frac{\frac{1}{2}}{1+\left(\frac{x}{2}\right)^2}\, dx = x - 2 \int \frac{1}{1+u^2}\, du = x - 2\arctan(u) = x - 2\arctan\left(\frac{x}{2}\right)$

Seja $u = x^2+4$ $du = 2x\, dx$

$\int \frac{5x}{x^2+4}\, dx = \frac{5}{2} \times \int \frac{\frac{2}{5} \times 5x}{x^2+4}\, dx = \frac{5}{2} \int \frac{2x}{x^2+4}\, dx = \frac{5}{2} \int \frac{1}{u}\, du = \frac{5}{2} \times \ln(|u|)$

$= \frac{5}{2} \times \ln|x^2+4|$

Seja $u = \frac{x}{2}$ $du = \frac{1}{2} dx$

$\int \frac{6}{x^2+4}\, dx = 6 \int \frac{1}{x^2+4}\, dx = 6 \times \int \frac{\frac{1}{4}}{1+\left(\frac{x}{2}\right)^2}\, dx = 6 \times \frac{1}{2} \times \int \frac{\frac{1}{2}}{1+\left(\frac{x}{2}\right)^2}\, dx =$

$= 3 \int \frac{1}{1+u^2}\, du = 3\arctan(u) = 3\arctan\left(\frac{x}{2}\right)$

$(1) = x - 2\arctan\left(\frac{x}{2}\right) + \frac{5}{2} \ln|x^2+4| + 3\arctan\left(\frac{x}{2}\right) + c =$

$= x + \arctan\left(\frac{x}{2}\right) + \frac{5}{3} \ln(x^2+4) + c, c \in \mathbb{R}$

$\int a^u du = \frac{a^u}{\ln(a)}+c$ | $\int \frac{1}{\sqrt{1-u^2}} du = \operatorname{arcsin}(u)+c$ | $\int \sec^2 u = \tan u$ | $\operatorname{cotan}(x) = \frac{\cos x}{\sin x}$, $\sec(x) = \frac{1}{\cos x}$, $\operatorname{cosec}(u) = \frac{1}{\sin x}$

Seja $u = x$ $du = 1dx$

e) $\int a^x dx = \int a^u du = \frac{a^u}{\ln(a)} + c = \frac{a^x}{\ln(a)} + c, \; c \in \mathbb{R}$

Seja $u = \frac{x}{\sqrt{2}}$ $du = \frac{1}{\sqrt{2}} dx$

f) $\int \frac{1}{\sqrt{2-x^2}} dx = \int \frac{\frac{1}{\sqrt{2}}}{\sqrt{1-\frac{x^2}{2}}} dx = \int \frac{\frac{1}{\sqrt{2}}}{\sqrt{1-\left(\frac{x}{\sqrt{2}}\right)^2}} dx =$

$= \int \frac{1}{\sqrt{1-u^2}} du = \operatorname{arcsin}(u) + c = \operatorname{arcsin}\left(\frac{x}{\sqrt{2}}\right) + c = \operatorname{arcsin}\left(\frac{x\sqrt{2}}{2}\right) + c, \; c \in \mathbb{R}$

Seja $u = 3x$ $du = 3\,dx$

g) $\int 2^{3x} dx = \frac{1}{3}\int 3 \times 2^{3x} dx = \frac{1}{3}\int 2^u du = \frac{1}{3} \times \frac{2^u}{\ln(2)} + c = \frac{2^{3x}}{3\ln(2)} + c, \; c \in \mathbb{R}$

Seja $u = x^2$ $du = 2x\,dx$

h) $\int x \sec^2(x^2) dx = \frac{1}{2}\int 2x \sec^2(x^2) dx = \frac{1}{2}\int \sec^2(u) du =$

$= \frac{1}{2}\tan(u) + c = \frac{1}{2}\tan(x^2) + c, \; c \in \mathbb{R}$

i) $\int \frac{\ln x}{x} dx =$ Seja $u = \ln x$ $du = \frac{1}{x} dx$

$= \int u \cdot du = \frac{1}{2}u^2 + c = \frac{1}{2}\ln^2(x) + c, \; c \in \mathbb{R}$

j) $\int \operatorname{cotan} x \, dx = \int \frac{\cos x}{\sin x} dx =$ Seja $u = \sin x$ $du = \cos x\, dx$

$= \int \frac{1}{u} \cdot du = \ln(u) + c = \ln(\sin x) + c, \; c \in \mathbb{R}$

k) $\int \frac{4}{(1+2x)^3} dx = 2\int \frac{2}{(1+2x)^3} dx =$ Seja $u = 1+2x$ $du = 2\,dx$

$= 2\int \frac{1}{u^3} du = 2\int u^{-3} du = 2 \times \left(-\frac{1}{2}u^{-2}\right) + c = -u^{-2} + c =$

$= -(1+2x)^{-2} + c = -\frac{1}{(1+2x)^2} + c, \; c \in \mathbb{R}$

l) $\int \cos x \sin^2 x \, dx =$ Seja $u = \sin x$ $du = \cos x\, dx$

$= \int u^2 du = \frac{1}{3}u^3 + c = \frac{1}{3}\sin^3 x + c, \; c \in \mathbb{R}$

Z

m) $\int \frac{2a}{(a-x)^2}\,dx =$ Seja $u = a-x \quad du = -1\,dx$

$= \int \frac{2a}{u^2} \times (-1)\,dt = \int -\frac{2a}{u^2}\,dt = -2a \int \frac{1}{u^2}\,dt = -2a \int u^{-2}\,dt =$

$= -2a \times (-u^{-1}) + c = 2a u^{-1} + c = \frac{2a}{u} + c = \frac{2a}{a-x} + c,\ c \in \mathbb{R}$

n) $\int \frac{x e^{x^2-1}}{e^{x^2-1}-1}\,dx = \frac{1}{2}\int \frac{2x e^{x^2-1}}{e^{x^2-1}-1}\,dx =$ Seja $u = e^{x^2-1}-1 \quad du = 2x e^{x^2-1}\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + c = \frac{1}{2}\ln|e^{x^2-1}-1| + c,\ c \in \mathbb{R}$

o) $\int \frac{1}{a^2-x^2}\,dx = \int \frac{1}{(a-x)(a+x)}\,dx = \frac{1}{2a}\int \frac{2a}{(a-x)(a+x)} =$ $\quad a-x+a+x = 2a$

$= \frac{1}{2a}\int \left(\frac{1}{a-x} + \frac{1}{a+x}\right) dx = \frac{1}{2a}\left[\int \frac{1}{a-x}\,dx + \int \frac{1}{a+x}\,dx\right] =$

Seja $u = a-x$
$du = -1\,dx$
Seja $t = a+x$
$dt = 1\,dx$

$= \frac{1}{2a}\left[-\int \frac{1}{u}\,du + \int \frac{1}{t}\,dt\right] = \frac{1}{2a}\left[-\ln(u) + \ln(t)\right] + c =$

$= \frac{1}{2a}\left(-\ln(a-x) + \ln(a+x)\right) + c = \frac{1}{2a}\left(\ln(a-x)^{-1} + \ln(a+x)\right) + c =$

$= \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c,\ c \in \mathbb{R}$

Seja $u = x^3 \quad du = 3x^2$

p) $\int x^2 \cos x^3\,dx = \frac{1}{3}\int 3x^2 \cos(x^3)\,dx = \frac{1}{3}\int \cos(u)\,du = \frac{1}{3}\sin(u) + c =$

$= \frac{1}{3}\sin(x^3) + c,\ c \in \mathbb{R}.$

Seja $u = x^4 + a^4 \quad du = 4x^3$

q) $\int \frac{x^3}{x^4+a^4}\,dx = \frac{1}{4}\int \frac{4x^3}{x^4+a^4} = \frac{1}{4}\int \frac{1}{u}\,du = \frac{1}{4}\ln|u| + c =$

$= \frac{1}{4}\ln|x^4+a^4| + c,\ c \in \mathbb{R}$

Seja $u = 2x \quad du = 2\,dx$

r) $\int \sec 2x \tan 2x\,dx = \frac{1}{2}\int 2\sec 2x \tan 2x\,dx = \frac{1}{2}\int \sec(u)\tan(u)\,du =$

$= \frac{1}{2} \times \sec(u) + c = \frac{1}{2} \times \sec(2x) + c,\ c \in \mathbb{R}$

s) $\int \frac{x}{a+bx}\,dx =$ Seja $u = a+bx \Leftrightarrow x = \frac{u-a}{b}$ $\quad du = b\,dx \Leftrightarrow dx = \frac{du}{b}$

$= \int \frac{\frac{u-a}{b}}{bu}\,du = \int \frac{u-a}{b^2 u}\,du = \frac{1}{b^2}\int \frac{u-a}{u}\,du = \frac{1}{b^2}\int\left(1-\frac{a}{u}\right)du =$

$= \frac{1}{b^2}\left[\int 1\,du - \int \frac{a}{u}\,du\right] = \frac{1}{b^2}\left[u - a\ln|u|\right] + c = \frac{a+bx-a\ln|a+bx|}{b^2} + c, c\in\mathbb{R}$

Seja $u = x$ $\quad du = 1\,dx$

t) $\int \cosh x\,dx = \int \cosh(u)\,du = \sinh(u)+c = \sinh(x)+c, c\in\mathbb{R}$

u) $\int \frac{x^2+1}{x-1}\,dx = \int\left(\frac{x^2}{x-1} + \frac{1}{x-1}\right)dx = \int \frac{x^2}{x-1}\,dx + \int \frac{1}{x-1}\,dx =$

Fazer $u = x-1 \Leftrightarrow x = u+1$ $\quad du = 1\,dx$ $\quad$ Fazer $t = x-1$ $\quad dt = 1\,dx$

$= \int \frac{(u+1)^2}{u}\,du + \int \frac{1}{t}\,dt = \int \frac{u^2+2u+1}{u}\,du + \ln|t| + c =$

$= \int\left(u+2+\frac{1}{u}\right)du + \ln|t| + c = \int u\,du + \int 2\,du + \int \frac{1}{u}\,du + \ln|t| + c =$

$= \frac{1}{2}u^2 + 2u + \ln|u| + \ln|t| + c = \frac{(x-1)^2}{2} + 2x - 2 + \ln|x-1| + \ln|x-1| + c =$

$= \frac{x^2-2x+1}{2} + 2x - 2 + 2\ln|x-1| + c = \frac{x^2+2x-3}{2} + 2\ln|x-1| + c, c\in\mathbb{R}$

v) $\int \cos x \sin x\, e^{\cos^2 x}\,dx =$ Seja $u = \cos x$ $\quad du = -\sin x$

Seja $t = u^2$ $\quad dt = 2u\,du$

$= -\int u\,e^{u^2}\,du = -\frac{1}{2}\int 2u\,e^{u^2}\,du = -\frac{1}{2}\int e^t\,dt = -\frac{1}{2}e^t + c =$

$= -\frac{1}{2}e^{u^2} + c = -\frac{1}{2}e^{\cos^2 x} + c, c\in\mathbb{R}$

x) $\int \frac{x}{(x+1)^2}\,dx =$ Seja $u = x+1 \Leftrightarrow x = u-1$ $\quad du = 1\,dx$

$= \int \frac{u-1}{u^2}\,du = \int\left(\frac{1}{u} - \frac{1}{u^2}\right)du = \int \frac{1}{u}\,du - \int u^{-2}\,du =$

$= \ln|u| + c - (-u^{-1}) + c = \ln|u| + \frac{1}{u} + c = \ln|x+1| + \frac{1}{x+1} + c, c\in\mathbb{R}$