E. Outros exercícios de aplicações de derivadas

a) $f'(x) = \left(\frac{x^2}{x^2+1}\right)' = \frac{2x(x^2+1) - 2x(x^2)}{(x^2+1)^2} = \frac{2x^3+2x-2x^3}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$

$f'(x) = 0 \Leftrightarrow \frac{2x}{(x^2+1)^2} = 0 \Leftrightarrow 2x = 0 \wedge (x^2+1)^2 \neq 0 \Leftrightarrow x = 0 \quad f(0) = \frac{0^2}{0^2+1} = \frac{0}{1} = 0$

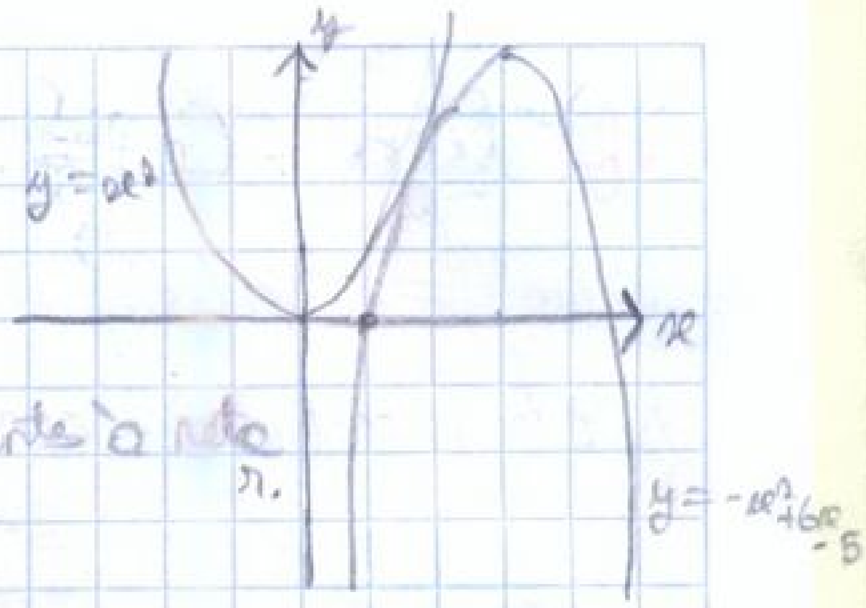
R: No ponto $(0,0)$

b) $P'(x) = (50\sqrt{x} - 0{,}5x - 500)' = 25 \times \frac{1}{\sqrt{x}} - 0{,}5 = \frac{25\sqrt{x}}{x} - 0{,}5$

$P'(900) = \frac{25\sqrt{900}}{900} - 0{,}5 = 0{,}33 \qquad P'(1600) = \frac{25\sqrt{1600}}{1600} - 0{,}5 = 0{,}125$

c) Seja $f(x)=x^2$ e $g(x)=-x^2+6x-5$

$f'(x)=2x$ $\quad g'(x)=-2x+6$ $\quad m_r=f'(a)=g'(b)$



Sejam $A(a,f(a))$ e $B(b;g(b))$ dois pontos pertencentes à reta r.

$f'(a)=g'(b) \Leftrightarrow 2a=-2b+6 \Leftrightarrow a=-b+3$

$A(-b+3,(-b+3)^2)$ e $B(b,-b^2+6b-5)$

$$m_r=\frac{(-b+3)^2-(-b^2+6b-5)}{-b+3-b}=\frac{b^2-6b+9+b^2-6b+5}{-2b+3}=\frac{2b^2-12b+14}{-2b+3}$$

$$g'(b)=\frac{2b^2-12b+14}{-2b+3} \Leftrightarrow -2b+6=\frac{2b^2-12b+14}{-2b+3} \Leftrightarrow 4b^2-6b-12b+18=2b^2-12b+14 \Leftrightarrow$$

$$\Leftrightarrow 2b^2-6b+4=0 \Leftrightarrow b=\frac{6\pm\sqrt{36-32}}{4} \Leftrightarrow b=\frac{6\pm2}{4} \Leftrightarrow b=2 \vee b=1 \Leftrightarrow a=1 \vee a=2$$

$g'(1)=-2+6=4$ $\quad g'(2)=-4+6=2$

$g(1)=-1+6-5=0$ $\quad g(2)=-4+12-5=3$

r_1	r_2
$m_r=2$	$m_r=4$
$y=2x+b \Leftrightarrow$	$y=4x+b \Leftrightarrow$
$\Leftrightarrow 0=2\times1+b \Leftrightarrow$	$\Leftrightarrow 3=4\times2+b \Leftrightarrow$
$\Leftrightarrow b=-2$	$\Leftrightarrow b=-5$

R: As duas retas são $y=2x-2$ e $y=4x-5$

d) $x+2y-6=0 \Leftrightarrow 2y=-x+6 \Leftrightarrow y=-\frac{1}{2}x+3$ $\quad m=-\frac{1}{2}$

Se é paralela, $m_r=m=-\frac{1}{2}$ $\quad$ Seja $f(x)=\frac{1}{\sqrt{x}}$

$$f'(x)=(x^{-\frac{1}{2}})'=-\frac{1}{2}x^{-\frac{3}{2}}=-\frac{1}{2x\sqrt{x}}$$

$$\cancel{-}\frac{1}{2x\sqrt{x}}=\cancel{-}\frac{1}{2} \Leftrightarrow 2x\sqrt{x}=2 \Leftrightarrow x\sqrt{x}=1 \Leftrightarrow x^{\frac{3}{2}}=1 \Leftrightarrow \sqrt{x^3}=1 \Leftrightarrow$$

$\Leftrightarrow x^3=1 \Leftrightarrow x=1$ $\quad f(1)=\frac{1}{\sqrt{1}}=1$ $\quad y-1=-\frac{1}{2}(x-1) \Leftrightarrow$

$\Leftrightarrow y=-\frac{1}{2}x+\frac{3}{2}$

e) $(y-5)=m(x-2) \Leftrightarrow y=mx-2m+5$ $\quad \begin{cases} y=4x-x^2 \\ y=mx-2m+5 \end{cases}$

$4x-x^2=mx-2m+5 \Leftrightarrow x^2+(-4+m)x-2m+5=0 \Leftrightarrow$

Como tem que intersetar $y=4x-x^2$ num único ponto:

$$\sqrt{(-4+m)^2-4(-2m+5)}=0 \Leftrightarrow (-4+m)^2+8m-20=0 \Leftrightarrow m^2-8m+16+8m-20=0 \Leftrightarrow$$

$\Leftrightarrow m^2-4=0 \Leftrightarrow m^2=4 \Leftrightarrow m=-2 \vee m=2$

$y-5=-2(x-2) \Leftrightarrow y=-2x+9$

$y-5=2(x-2) \Leftrightarrow y=2x+1$

f) Seja $f(x) = sec(x)$ e $g(x) = cosec(x)$

$f'(x) = (sec(x))' = sec(x)\tan(x)$ $\quad g'(x) = (cosec(x))' = -cosec(x)cotan(x)$

$sec(x)\tan(x) = -cosec(x)\,cotan(x) \Leftrightarrow \frac{1}{\cos x} \times \frac{\sin x}{\cos x} = -\frac{1}{\sin x} \times \frac{\cos x}{\sin x} \Leftrightarrow$

$\Leftrightarrow \frac{\sin x}{\cos^2 x} = -\frac{\cos x}{\sin^2 x} \Leftrightarrow \frac{\sin^3 x}{\cos^3 x} = -1 \Leftrightarrow \tan^3 x = -1 \Leftrightarrow \tan x = -1 \Leftrightarrow$

$\Leftrightarrow x = -\frac{\pi}{4} + k\pi, k \in \mathbb{Z} \Leftrightarrow x = \frac{3\pi}{4} \vee x = \frac{7\pi}{4}$, pois $x \in [0, 2\pi[$

g) $y' = \left(\frac{1}{1+x+\ln x}\right)' = \frac{-(1+\frac{1}{x})}{(1+x+\ln x)^2} = -\frac{\frac{x+1}{x}}{(1+x+\ln x)^2} = -\frac{x+1}{x(1+x+\ln x)^2}$

$x y' = y(y \ln x - 1) \Leftrightarrow x \times -\frac{x+1}{x(1+x+\ln x)^2} = \frac{1}{1+x+\ln x}\left(\frac{1}{1+x+\ln x} \times \ln x - 1\right) \Leftrightarrow$

$\Leftrightarrow -\frac{x+1}{(1+x+\ln x)^2} = \frac{1}{1+x+\ln x}\left(\frac{\ln x - (1+x+\ln x)}{1+x+\ln x}\right) \Leftrightarrow -\frac{x+1}{(1+x+\ln x)^2} = -\frac{x+1}{(1+x+\ln x)^2}$ c.q.p.

h) $x^2 = x^3 \Leftrightarrow x^2 - x^3 = 0 \Leftrightarrow x^2(1-x) = 0 \Leftrightarrow x = 0 \vee x = 1$ $\quad m = \tan\theta$

$P(0, y_0)$ $\quad Q(1, y_1)$ $\quad y_0 = 0^2 = 0$ $\quad y_1 = 1^2 = 1$ $\quad P(0,0)$ $\quad Q(1,1)$

$f'(x) = (x^2)' = 2x$ $\quad g'(x) = (x^3)' = 3x^2$ $\quad f'(0) = 2\times 0 = 0$ $\quad g'(0) = 3\times 0^2 = 0$

$y = b \Leftrightarrow b = 0$ $\quad y = 0$ $\quad m = \tan\theta \Leftrightarrow \tan\theta = 0 \Leftrightarrow \theta = 0°$

$y = b \Leftrightarrow b = 0$ $\quad y = 0$ $\quad m = \tan\theta \Leftrightarrow \tan\theta = 0 \Leftrightarrow \theta = 0°$

$f'(1) = 2\times 1 = 2$ $\quad y = 2x + b \Leftrightarrow b = 1 - 2\times 1 \Leftrightarrow b = -1$ $\quad y = 2x - 1$

$g'(1) = 3\times 1^2 = 3$ $\quad y = 3x + b \Leftrightarrow b = 1 - 3\times 1 \Leftrightarrow b = -2$ $\quad y = 3x - 2$

$m = \tan\theta \Leftrightarrow 2 = \tan\theta \Leftrightarrow \theta = \arctan 2 \Leftrightarrow \theta = 63{,}435°$

$m = \tan\theta \Leftrightarrow 3 = \tan\theta \Leftrightarrow \theta = \arctan 3 \Leftrightarrow \theta = 71{,}565°$

No ponto $P(0,0)$, o ângulo entre as curvas é dado por:

$0 - 0 = 0°$

No ponto $Q(1,1)$, o ângulo entre as curvas é dado por:

$71{,}565 - 63{,}435 = 8{,}13°$

Z