2. Integral definido

① a) $\int_1^2 (5x^4-1)\,dx = \left[x^5 - x\right]_1^2 = 2^5 - 2 - (1^5 - 1) = 30 - 0 = 30$

b) $\int_0^2 (5x^3 - 3x + 6)\,dx = \left[\frac{5}{4}x^4 - \frac{3}{2}x^2 + 6x\right]_0^2 = \frac{5}{4}\times 2^4 - \frac{3}{2}\times 2^2 + 6\times 2 - 0 = 26$

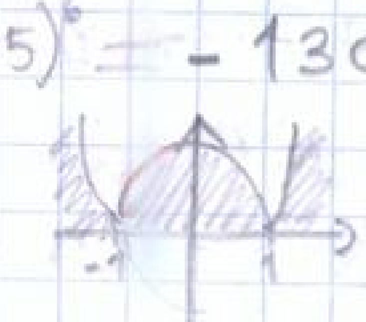
c) $\int_{-1}^0 (x+1)^2\,dx = \left[\frac{1}{3}(x+1)^3\right]_{-1}^0 = \frac{1}{3} - 0 = \frac{1}{3}$

d) $\int_{-1}^4 (1-t)(t-2)\,dt = \int_{-1}^4 (-t^2+3t-2)\,dt = \left[-\frac{1}{3}t^3 + \frac{3}{2}t^2 - 2t\right]_{-1}^4 =$

$= -\frac{1}{3}\times 4^3 + \frac{3}{2}\times 4^2 - 2\times 4 - \left(-\frac{1}{3}\times(-1)^3 + \frac{3}{2}\times(-1)^2 - 2\times(-1)\right) = -\frac{55}{6}$

e) $\int_0^3 (2x-5)^5\,dx = \left[\frac{1}{12}(2x-5)^6\right]_0^3 = \frac{1}{12}(6-5)^6 - \frac{1}{12}(-5)^6 = -1302$

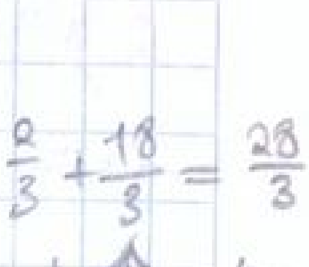
f) $\int_{-2}^3 |x^2-1|\,dx =$ $\quad x^2-1=0 \Leftrightarrow x=-1 \vee x=1$



$= \int_{-2}^{-1} (x^2-1)\,dx + \int_{-1}^1 (1-x^2)\,dx + \int_1^3 (x^2-1)\,dx =$

$= \left[\frac{1}{3}x^3 - x\right]_{-2}^{-1} + \left[x - \frac{1}{3}x^3\right]_{-1}^1 + \left[\frac{1}{3}x^3 - x\right]_1^3 = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{18}{3} = \frac{28}{3}$

g) $\int_{-1}^3 |x(1-x)|\,dx = \int_{-1}^3 |x - x^2|\,dx$ $\quad x - x^2 = 0 \Leftrightarrow x = 0 \vee x = 1$



$= \int_{-1}^0 x^2 - x + \int_0^1 x - x^2 + \int_1^3 x^2 - x = \left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_{-1}^0 + \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1 + \left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_1^3 =$

$= 0 - \left(-\frac{5}{6}\right) + \frac{1}{6} - 0 + \frac{9}{2} - \left(-\frac{1}{6}\right) = \frac{5}{6} + \frac{1}{6} + \frac{27}{6} + \frac{1}{6} = \frac{34}{6} = \frac{17}{3}$

h) $\int_2^5 \frac{1}{\sqrt{x}}\,dx = \int_2^5 x^{-\frac{1}{2}}\,dx = \left[2x^{\frac{1}{2}}\right]_2^5 = 2\sqrt{5} - 2\sqrt{2} = 2(\sqrt{5} - \sqrt{2})$

i) $\int_4^1 \sqrt[5]{5x}\,dx = \int_4^1 (5x)^{\frac{1}{5}}\,dx = \left[\frac{1}{6}(5x)^{\frac{6}{5}}\right]_4^1 = \frac{1}{6} \times 5^{\frac{6}{5}} - \frac{1}{6} \times 20^{\frac{6}{5}} =$

$= \frac{\sqrt[5]{5^6}}{6} - \frac{\sqrt[5]{20^6}}{6} = \frac{5\sqrt[5]{5}}{6} - \frac{20\sqrt[5]{20}}{6} = \frac{5\sqrt[5]{5}}{6} - \frac{10\sqrt[5]{20}}{3}$

j) $\int_1^8 4\sqrt[3]{x-1}\,dx = \int_1^8 4(x-1)^{\frac{1}{3}} = \left[3(x-1)^{\frac{4}{3}}\right]_1^8 = 3 \times 7^{\frac{4}{3}} = 3\sqrt[3]{7^4} = 21\sqrt[3]{7}$

k) $\int_1^4 \frac{t-3}{\sqrt{t}}\,dt = \int_1^4 \frac{t}{t^{\frac{1}{2}}} - \frac{3}{t^{\frac{1}{2}}} = \int_1^4 \sqrt{t} - 3t^{-\frac{1}{2}} = \left[\frac{2t\sqrt{t}}{3} - 6\sqrt{t}\right]_1^4 =$

$= \frac{2 \times 4\sqrt{4}}{3} - 6\sqrt{4} - \left(\frac{2}{3} - 6\right) = \frac{16}{3} - 12 - \frac{2}{3} + 6 = \frac{14}{3} - 6 = -\frac{4}{3}$

l) $\int_1^3 \left(x^2 + \frac{1}{x^2}\right)dx = \int_1^3 (x^2 + x^{-2}) = \left[\frac{1}{3}x^3 - x^{-1}\right]_1^3 = \frac{1}{3} \times 3^3 - 3^{-1} - \left(\frac{1}{3} - 1\right) =$

$= 9 - \frac{1}{3} - \frac{1}{3} + 1 = 10 - \frac{2}{3} = \frac{28}{3}$

2 a) $\int_0^2 f(x)\,dx = \int_0^1 x^2\,dx + \int_1^2 (2-x)\,dx = \left[\frac{1}{3}x^3\right]_0^1 + \left[2x - \frac{1}{2}x^2\right]_1^2 =$

$= \frac{1}{3} - 0 + 2 - \frac{3}{2} = \frac{2}{6} + \frac{12}{6} - \frac{9}{6} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$