1.A. Regras de derivação

① $f'(x) = \left(\sqrt[5]{\frac{1}{x}} + \sqrt[3]{3x^7}\right)' = \left(x^{-\frac{1}{5}} + \sqrt[3]{3}\,x^{\frac{7}{3}}\right)' = -\frac{1}{5}x^{-\frac{6}{5}} + \frac{7\sqrt[3]{3}}{3}x^{\frac{4}{3}} =$

$= -\frac{1}{5x^{\frac{6}{5}}} + \frac{7}{3}\times\sqrt[3]{3}\times\sqrt[3]{x^4} = -\frac{1}{5\sqrt[5]{x^6}} + \frac{7}{3}\times\sqrt[3]{3}\times x\sqrt[3]{x} = -\frac{1}{5x\sqrt[5]{x}} + \frac{7x}{3}\sqrt[3]{3x}$

② $f'(x) = \left(\frac{5\pi}{x^2+x+3}\right)' = -\frac{(2x+1)5\pi}{(x^2+x+3)^2} = -\frac{10\pi x+5\pi}{(x^2+x+3)^2}$

③ $f'(x) = \left(\frac{x^5}{x^2+1}+\sqrt{x}\right)' = \frac{5x^4(x^2+1)-(2x)x^5}{(x^2+1)^2} + \frac{1}{2\sqrt{x}} =$

$= \frac{5x^6+5x^4-2x^6}{(x^2+1)^2}+\frac{1}{2\sqrt{x}} = \frac{3x^6+5x^4}{(x^2+1)^2}+\frac{1}{2\sqrt{x}}$

④ $f'(x) = \left(x^{\frac{3}{2}}+\frac{5}{2}x^2\right)' = \frac{3}{2}x^{\frac{1}{2}}+5x = \frac{3}{2}\sqrt{x}+5x$

⑤ $f'(x) = (x^2e^{-x})' = 2xe^{-x} - e^{-x}x^2 = e^{-x}(2x-x^2)$

⑥ $f'(x) = \left(\frac{e^x-x-1}{x^2}\right)' = \frac{(e^x-1)x^2-2x(e^x-x-1)}{x^4} = \frac{e^x x^2 - x^2 - 2xe^x+2x^2+2x}{x^4} =$

$= \frac{e^x x^2 + x^2 - 2xe^x + 2x}{x^4} = \frac{e^x x + x - 2e^x + 2}{x^3}$

⑦ $f'(x) = e^{x^2+\frac{1}{x}+3} = \left(2x-\frac{1}{x^2}\right)e^{x^2+\frac{1}{x}+3}$

⑧ $f'(x) = (3x\ln(x))' = 3\ln(x)+\frac{3x}{x} = 3\ln(x)+3$

⑨ $f'(x) = \left(\frac{\ln(x)}{2x}\right)' = \frac{\frac{2x}{x}-2\ln(x)}{(2x)^2} = \frac{2-2\ln(x)}{4x^2} = \frac{1-\ln(x)}{2x^2}$

⑩ $f'(x) = (3x\ln(3x^2+1))' = 3\ln(3x^2+1)+\frac{6x}{3x^2+1}\times 3x =$

$= 3\ln(3x^2+1)+\frac{18x^2}{3x^2+1}$

⑪ $f'(x) = \left(\frac{x^{\frac{3}{2}}}{1+\ln(x)}\right)' = \frac{\frac{3}{2}\sqrt{x}(1+\ln(x)) - \frac{x^{\frac{3}{2}}}{x}}{(1+\ln(x))^2} = \frac{\frac{3}{2}\sqrt{x}(1+\ln(x))-\sqrt{x}}{(1+\ln(x))^2} =$

$= \frac{3\sqrt{x}(1+\ln(x))-2\sqrt{x}}{2(1+\ln(x))^2} = \frac{3\sqrt{x}+3\sqrt{x}\ln(x)-2\sqrt{x}}{2(1+\ln(x))^2} = \frac{\sqrt{x}+3\sqrt{x}\ln(x)}{2(1+\ln(x))^2}$

12) $$f'(x) = \left(\ln(\ln(x+1))\right)' = \frac{\frac{1}{x+1}}{\ln(x+1)} = \frac{1}{(x+1)(\ln(x+1))}$$

13) $$f'(x) = \left(\ln\left(\frac{x+1}{x+2}\right)\right)' = \frac{\frac{(x+2)-(x+1)}{(x+2)^2}}{\frac{x+1}{x+2}} = \frac{\frac{1}{(x+2)^2}}{\frac{x+1}{(x+2)}} = \frac{1}{(x+1)(x+2)}$$

14) $$f'(x) = \left(e^{\frac{1}{x}}\ln(x)\right)' = -\frac{1}{x^2}e^{\frac{1}{x}}\ln(x) + \frac{e^{\frac{1}{x}}}{x} = -\frac{e^{\frac{1}{x}}\ln(x) + x e^{\frac{1}{x}}}{x^2} =$$
$$= -\frac{e^{\frac{1}{x}}(\ln(x)+x)}{x^2}$$

15) $$f'(x) = \left(\frac{\ln(x^2+1)}{\ln(x^4+x+1)}\right)' = \frac{\frac{2x}{x^2+1}\left(\ln(x^4+x+1)\right) - \frac{4x^3+1}{x^4+x+1}\left(\ln(x^2+1)\right)}{\left(\ln(x^4+x+1)\right)^2}$$

16) $$f'(x) = \left(\frac{e^x\log(x+1)}{x^2}\right)' = \frac{x^2\left(e^x\log(x+1) + \frac{1}{(x+1)\ln(10)}\times e^x\right) - 2x\left(e^x\log(x+1)\right)}{(x^2)^2} =$$
$$= \frac{x e^x\log(x+1) + \frac{x e^x}{(x+1)\ln(10)} - 2e^x\log(x+1)}{x^3} = \frac{x e^x\frac{\ln(x+1)}{\ln(10)} + x e^x\frac{1}{(x+1)\ln(10)} - 2e^x\frac{\ln(x+1)}{\ln(10)}}{x^3} =$$
$$= \frac{x e^x\ln(x+1)\times(x+1) + x e^x - 2e^x\ln(x+1)\times(x+1)}{x^3\times\ln(10)\times(x+1)}$$

17) $$f'(x) = \left(\frac{\log(x)}{2^x}\right)' = \frac{\frac{1}{x\ln(10)}\times 2^x - \log(x)\times 2^x\times\ln(2)}{(2^x)^2} =$$
$$= \frac{1 - \log(x)\times\ln(2)\times x\times\ln(10)}{x\times 2^x\times\ln(10)} = \frac{1 - \ln(10)\ln(2)x\times\log(x)}{\ln(10)\times 2^x\times x}$$

18) $$f'(x) = \left(x^2 + e^x + 5^{x-1}\right)' = 2x^{2-1} + e^x + 5^{x-1}\times\ln(5)$$

19) $$f'(x) = \left(5\cos\left(\frac{1}{x}\right)\right)' = 5\times\left[-\left(-\frac{1}{x^2}\sin\left(\frac{1}{x}\right)\right)\right] = \frac{5}{x^2}\sin\left(\frac{1}{x}\right)$$

20) $$f'(x) = \left(x^4\cos x\right)' = 4x^3\cos x - \sin x\times x^4 = 4x^3\cos(x) - x^4\sin(x).$$

21) $$f'(x) = \left(x\sin\left(\frac{\pi}{2}x\right)\sin(x^{-1})\right)' = \left(\sin\left(\frac{\pi}{2}x\right) + \frac{\pi}{2}x\cos\left(\frac{\pi}{2}x\right)\right)\sin(x^{-1}) -$$
$$- \frac{1}{x^2}\cos(x^{-1})\left(x\sin\left(\frac{\pi}{2}x\right)\right) = \frac{2x\times\sin\left(\frac{\pi}{2}x\right)\sin\left(\frac{1}{x}\right) + \pi x^2\cos\left(\frac{\pi}{2}x\right)\sin\left(\frac{1}{x}\right) - 2\sin\left(\frac{\pi x}{2}\right)\times\cos\left(\frac{1}{x}\right)}{2x}$$

$(\tan(u))' = u'\sec^2(u)$ $\quad (\operatorname{cosec}(u))' = -u'\operatorname{cosec}(u)\operatorname{cotan}(u)$ $(\sec(u))' = u'\sec u \tan u$	$\sec = \frac{1}{\cos x}$

22) $f'(x) = (\sin(xe^x))' = (e^x + xe^x)\cos(xe^x) = e^x(x+1)\cos(xe^x)$

23) $f'(x) = (\tan(x^2+1))' = 2x\sec^2(x^2+1)$

24) $f'(x) = (\ln(x)\tan(x^2))' = \frac{\tan(x^2)}{x} + 2x\sec^2(x^2)\ln(x)$

25) $f'(x) = (\sec(x)+\log(x)) = \sec(x)\tan(x) + \frac{1}{x\ln(10)}$ ← ?

26) $f'(x) = (\sec(x)+\operatorname{cosec}(x)) = \sec(x)\tan(x) - \operatorname{cosec}(u)\operatorname{cotan}(u)$

27) $f'(x) = (x^2\tan(x^3))' = 2x\tan(x^3) + 3x^2\sec^2(x^3)\times x^2 = 2x\tan(x^3) + 3x^4\sec^2(x^3)$

28) $f'(x) = (\log(\cos(x)) + \tan(\log(x)))' = \frac{-\sin x}{\cos(x)\times\ln(10)} + \frac{1}{x\ln(10)}\times\sec^2(\log(x)) =$

$= \frac{-x\sin(x) + \cos(x)\sec^2(\log(x))}{x\ln(10)\cos(x)}$

29) $f'(x) = (\sin^2(x+1) + \tan(x^3))' = 2\sin(x+1)\times(\sin(x+1))' + 3x^2\sec^2(x^3) =$

$= 2\sin(x+1)\times\cos(x+1) + 3x^2\sec^2(x^3) = \sin(2x+2) + 3x^2\sec^2(x^3)$

30) $f'(x) = (\cos(\sqrt{x}) + \sin^2(x))' = -\frac{1}{2\sqrt{x}}\sin(\sqrt{x}) + 2\sin(x)\cos(x)$

31) $f'(x) = \left(2\tan(e^x) - 5\sec(x) + \frac{\pi}{2}\right)' = 2e^x\sec^2(e^x) - 5\sec(x)\tan(x)$

32) $f'(x) = \left(\frac{\tan(x)+\sin(x^2+1)}{\cos^2 x}\right)' = \frac{(\sec^2(x)+2x\cos(x^2+1))\cos^2 x + 2\sin(x)(\tan(x)+\sin(x^2+1))}{\cos^4 x}$

33) $f'(x) = ((\sin x)^{\tan x})' = \tan x \times \sin x^{\tan x - 1}\times\cos x + \sin x^{\tan x}\times\sec^2(x)\times\ln(\sin(x)) =$

$= \sin x^{\tan x}\left(\tan x\times\cos x\times\frac{1}{\sin x} + \sec^2(x)\times\ln(\sin(x))\right) =$

$= (\sin x)^{\tan x}\left(\tan x\times\frac{1}{\tan x} + \sec^2(x)\ln(\sin(x))\right) = \sin x^{\tan x}\left(1+\sec^2(x)\ln(\sin(x))\right)$

34) $f'(x) = (e^{\tan x\ln(\sin(x))})' = \left(\sec^2(x)\ln(\sin(x)) + \tan(x)\times\frac{\cos x}{\sin x}\right)\times e^{\tan x\ln(\sin(x))} =$

$= (\sec^2(x)\ln(\sin(x)) + 1)\times e^{\tan x\ln(\sin(x))}$

35) $f'(x) = \left((\ln(x))^{\frac{1}{x}}\right)' = \frac{1}{x}\times\ln(x)^{\frac{1}{x}-1}\times\frac{1}{x} + \ln(x)^{\frac{1}{x}}\times\left(-\frac{1}{x^2}\right)\times\ln(\ln(x)) =$

$= \ln(x)^{\frac{1}{x}}\left(\frac{1}{x^2}\times\frac{1}{\ln(x)} - \frac{1}{x^2}\times\ln(\ln(x))\right) = (\ln(x))^{\frac{1}{x}}\left[\frac{1}{x^2\ln(x)} - \frac{\ln(\ln(x))}{x^2}\right]$

	$\tan^2(y)+1=\sec^2(y)$	

36) $$f'(x)=\left[(\sec(x)+3)^{\ln(x)}\right]'=\ln(x)\times(\sec(x)+3)^{\ln(x)-1}\times\sec(x)\tan(x)+(\sec(x)+3)^{\ln(x)}\times\frac{1}{x}\times\ln(\sec(x)+3)=$$

$$=(\sec(x)+3)^{\ln(x)}\left[\ln(x)\sec(x)\tan(x)\times\frac{1}{\sec(x)+3}+\frac{\ln(\sec(x)+3)}{x}\right]=$$

$$=(\sec(x)+3)^{\ln(x)}\left[\frac{\ln(x)\sec(x)\tan(x)}{\sec(x)+3}+\frac{\ln(\sec(x)+3)}{x}\right]$$