Capítulo 8 - Mecânica lagrangiana

Perguntas:

1) (Opção B).

2) (Opção E).

3) (Opção B).

4) (Opção C).

5) (Opção C).

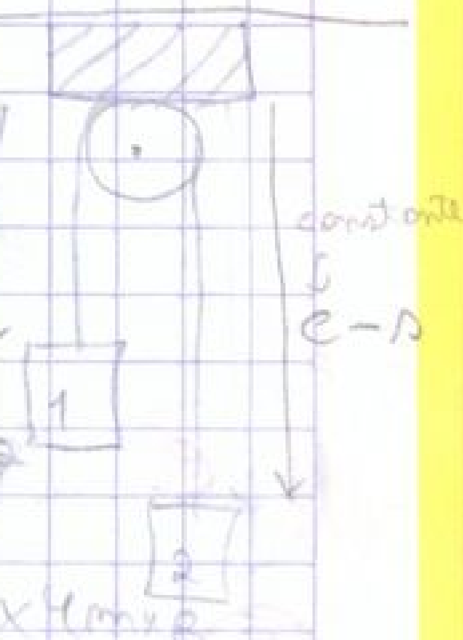
① $v_A = 0$ $\quad I_A = I_{cm} + m\left(\frac{L}{2}\right)^2 = \frac{m}{12}(3\cancel{R^2} + L^2) + \frac{L^2}{4}m =$

$= \frac{m}{12}L^2 + \frac{L^2}{4}m = \frac{4m}{12}L^2 = \frac{1}{3}mL^2$

$E_c = \frac{1}{2}m\cancel{v_A^2} + \frac{1}{2}I_A\omega^2 \quad v = l\dot{\theta} \quad \omega = \frac{l\dot{\theta}}{l} = \dot{\theta}$

$E_c = \frac{1}{2}\times\frac{1}{3}mL^2\times\dot{\theta}^2 = \frac{1}{6}mL^2\dot{\theta}^2$

② $v = \dot{s}$



$E_c = \frac{3}{2}mv_1^2 + \frac{4}{2}mv_2^2 + \frac{1}{2}\times I_x\omega^2$

$E_c = \frac{7}{2}mv_1^2 + \frac{1}{2}mr^2\times\left(\frac{v_1}{r}\right)^2$

$E_c = \frac{7}{2}mv_1^2 + \frac{1}{2}mv_1^2 = \frac{8}{2}mv_1^2$

$U = (l_1 - s)\times 3m\times g + (l_1 - (l-s))\times 4m\times g$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial v_1}\right) = \frac{d}{dt}(8mv_1) = 8ma_1 \qquad \frac{\partial E_c}{\partial s} = 0$

$8ma_1 - mg = 0 \Leftrightarrow a_1 = \frac{g}{8} \qquad \frac{\partial U}{\partial s} = -3mg + 4mg = mg$

③ $E_c = \frac{m}{2}(R^2\dot{\theta}^2 + \dot{z}^2)$

$U = \frac{az^2}{2} + \frac{b\theta^2}{2} + cz\theta$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial\dot{\theta}}\right) - \frac{\partial E_c}{\partial\theta} + \frac{\partial U}{\partial\theta} = 0$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial\dot{\theta}}\right) = \frac{d}{dt}(mR^2\dot{\theta}) = mR^2\ddot{\theta}$

$\frac{\partial E_c}{\partial\theta} = 0 \qquad \frac{\partial U}{\partial\theta} = b\theta + cz$

$mR^2\ddot{\theta} + b\theta + cz = 0 \Leftrightarrow mR^2\ddot{\theta} = -b\theta - cz \Leftrightarrow \ddot{\theta} = -\frac{b\theta + cz}{mR^2}$

④ $E_c = 5\dot{x}^2 + 11\dot{\theta}^2 \qquad U = -3x\theta \qquad \frac{d}{dt}\left(\frac{\partial E_c}{\partial\dot{\theta}}\right) - \frac{\partial E_c}{\partial\theta} + \frac{\partial U}{\partial\theta} = 0$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial\dot{\theta}}\right) = \frac{d}{dt}(22\dot{\theta}) = 22\ddot{\theta} \qquad \frac{\partial E_c}{\partial\theta} = 0 \qquad \frac{\partial U}{\partial\theta} = -3x$

$22\ddot{\theta} - 3x = 0 \Leftrightarrow 22\ddot{\theta} = 3x \Leftrightarrow \ddot{\theta} = \frac{3x}{22}$

⑤ $E_c = \frac{m}{2}(r^2\dot{\theta}^2 + \dot{r}^2) \qquad U = -\frac{4\pi^2 m}{r} \qquad \frac{d}{dt}\left(\frac{\partial E_c}{\partial\dot{r}}\right) - \frac{\partial E_c}{\partial r} + \frac{\partial U}{\partial r} = 0$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial\dot{r}}\right) = \frac{d}{dt}(m\dot{r}) = m\ddot{r} \qquad \frac{\partial E_c}{\partial r} = mr\dot{\theta}^2 \qquad \frac{\partial U}{\partial r} = \frac{4\pi^2 m}{r^2}$

$m\ddot{r} - mr\dot{\theta}^2 + \frac{4\pi^2 m}{r^2} = 0 \Leftrightarrow m\ddot{r} = mr\dot{\theta}^2 - \frac{4\pi^2 m}{r^2} \Leftrightarrow \ddot{r} = r\dot{\theta}^2 - \left(\frac{2\pi}{r}\right)^2$

Problemas:

$\vec{v}_p = \dot{s}\,\hat{e}_t \qquad \vec{v}_{C/P} = \dot{x}(\cos\theta\,\hat{e}_t + \sin\theta\,\hat{e}_n) \qquad \vec{v}_c = \vec{v}_p + \vec{v}_{C/P} = (\dot{s} + \dot{x}\cos\theta)\,\hat{e}_t + \dot{x}\sin\theta\,\hat{e}_n$

1) a) $E_c = \frac{1}{2}Mv_p^2 + \frac{1}{2}mv_c^2 \qquad v_c^2 = \dot{s}^2 + 2\dot{s}\dot{x}\cos\theta + \dot{x}^2$

$E_c = \frac{2{,}5}{2}\dot{s}^2 + 0{,}3\times(\dot{s}^2 + 2\dot{s}\dot{x}\cos\theta + \dot{x}^2) \qquad U = U_c = mgx\sin\theta$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial\dot{s}}\right) - \cancelto{0}{\frac{\partial E_c}{\partial s}} + \cancelto{0}{\frac{\partial U}{\partial s}} = 0 \Leftrightarrow \frac{d}{dt}(2{,}5\dot{s} + 0{,}6\dot{s} + 0{,}6\dot{x}\cos\theta)$

$\Rightarrow 3{,}1\ddot{s} + 0{,}6\ddot{x}\cos\theta = 0 \leftarrow$ 1ª equação de Lagrange

$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{x}}\right) - \frac{\partial Ec}{\partial x}\nearrow^{0} + \frac{\partial U}{\partial x} = 0 \Leftrightarrow \frac{d}{dt}\left(0,6\dot{s}\cos\theta + 0,6\dot{x}\right) + mg\sin\theta = 0$

$\Leftrightarrow 0,6\ddot{s}\cos\theta + 0,6\ddot{x} + mg\sin\theta = 0$ ← 2ª equação de Lagrange

$\begin{cases} 3,1\ddot{s} + 0,6\ddot{x}\cos 20^\circ = 0 \\ 0,6\ddot{s}\cos 20^\circ + 0,6\ddot{x} + 0,6\times 9,8\times \sin 20^\circ = 0 \end{cases} \Rightarrow \begin{cases} \ddot{s} = 0,735 \\ \ddot{x} = -4,043 \end{cases} (m/s^2)$

b) $\ddot{x} = \dot{x}\frac{d\dot{x}}{dx} \Leftrightarrow \int_{0,20}^{0} -4,043\,dx = \int_0^{v_c} \dot{x}\,d\dot{x} \Leftrightarrow \frac{\dot{x}^2}{2}\Big|_0^{v_c} = -4,043x\Big|_{0,20}^{0}$

$\Leftrightarrow \frac{v_c^2}{2} = 0,8086 \Leftrightarrow v_c = \pm 1,272$ Como $\ddot{x}$ é negativo, o caminho tem velocidade negativa, pelo que $v_c = -1,272$ m/s

$\ddot{x} = \frac{d\dot{x}}{dt} \Leftrightarrow \int_0^t -4,043\,dt = \int_0^{-1,272} d\dot{x} \Leftrightarrow -4,043t = -1,272 \Leftrightarrow t = 0,315\,s$

c) $\ddot{s} = \frac{d\dot{s}}{dt} \Leftrightarrow \int_0^{0,351} 0,735\,dt = \int_0^{v_p} d\dot{s} \Leftrightarrow v_p = 0,315\times 0,735 \Leftrightarrow v_p = 0,231\,m/s$

2 a) $Ec = \frac{1}{2}mv_B^2 + \frac{1}{2}I_{cm}\omega^2 \quad v_B = \dot{y} \quad I_{cm} = \frac{m}{2}R^2 \quad \omega = \frac{v_B}{R} = \frac{\dot{y}}{R}$

$U = U_B = mgh = mg\times(c-y)$ $c \rightarrow$ constante.

$Ec = \frac{1}{2}m\dot{y}^2 + \frac{1}{4}mR^2\times\left(\frac{\dot{y}}{R}\right)^2 = \frac{1}{2}m\dot{y}^2 + \frac{1}{4}m\dot{y}^2$

$\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{y}}\right) - \cancel{\frac{\partial Ec}{\partial y}}^{0} + \frac{\partial U}{\partial y} = 0 \Leftrightarrow \frac{d}{dt}\left(m\dot{y} + \frac{1}{2}m\dot{y}\right) - mg = 0 \Leftrightarrow$

$\Leftrightarrow \frac{3}{2}m\ddot{y} = mg \Leftrightarrow \ddot{y} = \frac{2g}{3}$

b) $\ddot{y} = \dot{y}\frac{d\dot{y}}{dy} \Leftrightarrow \int_0^{0,50} \frac{2g}{3}\,dy = \int_0^{v_B} \dot{y}\,d\dot{y} \Leftrightarrow \frac{2\times 9,8}{3}\times 0,50 = \frac{\dot{y}^2}{2}\Big|_0^{v_B} \Leftrightarrow$

$\Leftrightarrow v_B = 2,56\,m/s$

3 a) $y = x^2 \Rightarrow \dot{y} = 2x\dot{x} \quad \vec{v} = (\dot{x}, \dot{y}) \quad v^2 = \dot{x}^2 + \dot{y}^2$

$Ec = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{1}{2}m(\dot{x}^2 + 4x^2\dot{x}^2) = \frac{1}{2}m\dot{x}^2(4x^2+1) =$

$= \dot{x}^2(4x^2+1)$

b) $U = mgy = mgx^2 = 2\times 9,8x^2 = 19,6x^2$

c) $\frac{d}{dt}\left(\frac{\partial Ec}{\partial \dot{x}}\right) - \frac{\partial Ec}{\partial x} + \frac{\partial U}{\partial x} = 0 \Leftrightarrow \frac{d}{dt}\left(2\dot{x}(4x^2+1)\right) - 8x\dot{x}^2 + 39,2x = 0 \Leftrightarrow$

$\Leftrightarrow \ddot{x}(8x^2+2) + 16\dot{x}^2x - 8\dot{x}^2x + 39,2x = 0 \Leftrightarrow \ddot{x}(8x^2+2) = -8\dot{x}^2x - 39,2x \Leftrightarrow$

$\Leftrightarrow \ddot{x} = -\frac{2(4\dot{x}^2x + 19,6x)}{2(4x^2+1)} \Leftrightarrow \ddot{x} = -\frac{x(4\dot{x}^2+19,6)}{4x^2+1}$

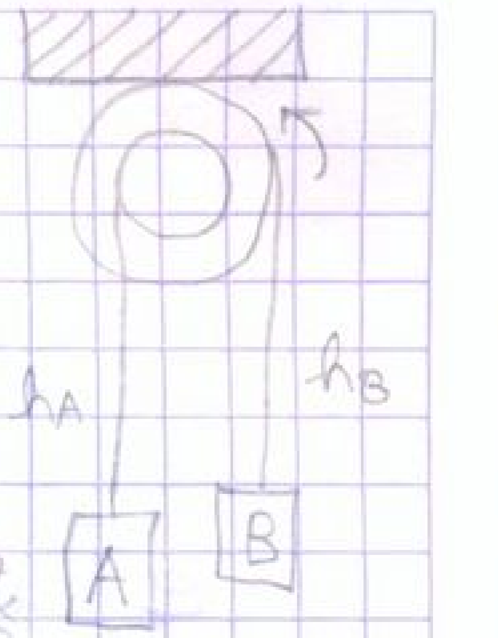
d) $\dot{x} = v \quad \dot{v} = -\frac{x(4v^2+19,6)}{4x^2+1} \quad \begin{cases} v = 0 \\ x(4v^2+19,6) = 0 \end{cases} \Leftrightarrow \begin{cases} v = 0 \\ x = 0 \end{cases}$

O único ponto de equilíbrio é o ponto (0,0) e é estável.

4) a) $y_A = h_A - 0{,}05\theta \qquad y_B = h_B + 0{,}08\theta$

$V_A = -0{,}05\,\omega \qquad V_B = 0{,}08\,\omega$

$a_A = -0{,}05\,\alpha \qquad a_B = 0{,}08\,\alpha$



$E_c = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 + \frac{1}{2} I_{cm}\,\omega^2 \Leftrightarrow$

$\Leftrightarrow E_c = \frac{1}{2}\times 36\times10^{-3}\times 0{,}05^2\omega^2 + \frac{1}{2}\times 24\times10^{-3}\times 0{,}08^2\omega^2 + \frac{1}{2}\times 4{,}43\times10^{-7}\times\omega^2$

$\Leftrightarrow E_c = 1{,}220215\times10^{-4}\,\omega^2 \qquad U = m_A g\, y_A + m_B g\, y_B = 36\times10^{-3}\times 9{,}8\times(h_A - 0{,}05\theta) + 24\times10^{-3}\times 9{,}8\times(h_B + 0{,}08\theta)$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \omega}\right) - \cancel{\frac{\partial E_c}{\partial \theta}}^{\,0} + \frac{\partial U}{\partial \theta} = 0 \Leftrightarrow \frac{d}{dt}\left(2{,}44043\times10^{-4}\,\omega\right) - 0{,}05\times 36\times10^{-3}\times 9{,}8 + 0{,}08\times 24\times10^{-3}\times 9{,}8 = 0$

$\Leftrightarrow 2{,}44043\times10^{-4}\,\alpha = -1{,}176\times10^{-3} \Leftrightarrow \alpha = -4{,}8188\ s^{-2}$

Como $\alpha < 0$, a roldana roda no sentido dos ponteiros do relógio.

$a_A = R\alpha = 0{,}05\times 4{,}8188 = 0{,}2409\ m/s^2$, para cima.

$a_B = R\alpha = 0{,}08\times 4{,}8188 = 0{,}3855\ m/s^2$, para baixo.

b) 3 graus de liberdade $\rightarrow y_A, y_B, \theta \qquad$ estado $\rightarrow (y_A, y_B, \theta, V_A, V_B, \omega)$

$E_c = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 + \frac{1}{2} I_{cm}\,\omega^2 = \frac{36\times10^{-3}}{2}\dot{y}_A^2 + \frac{24\times10^{-3}}{2}\dot{y}_B^2 + \frac{4{,}43\times10^{-7}}{2}\omega^2$

$U = U_A + U_B = m_A g\, y_A + m_B g\, y_B = 36\times10^{-3}\times 9{,}8\, y_A + 24\times10^{-3}\times 9{,}8\, y_B = 0{,}3528\, y_A + 0{,}2352\, y_B$

$f_A(y_A, \theta) = y_A + 0{,}05\theta \qquad f_B(y_B, \theta) = y_B - 0{,}08\theta \quad$ (constante).

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{y}_A}\right) - \cancel{\frac{\partial E_c}{\partial y_A}}^{\,0} + \frac{\partial U}{\partial y_A} - \lambda_A \frac{\partial f_A}{\partial y_A} - \lambda_B \cancel{\frac{\partial f_B}{\partial y_A}}^{\,0} = 0 \Leftrightarrow 36\times10^{-3}\,\ddot{y}_A + 0{,}3528 - \lambda_A = 0$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{y}_B}\right) - \cancel{\frac{\partial E_c}{\partial y_B}}^{\,0} + \frac{\partial U}{\partial y_B} - \lambda_A \cancel{\frac{\partial f_A}{\partial y_B}}^{\,0} - \lambda_B \frac{\partial f_B}{\partial y_B} = 0 \Leftrightarrow 24\times10^{-3}\,\ddot{y}_B + 0{,}2352 - \lambda_B = 0$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \omega}\right) - \cancel{\frac{\partial E_c}{\partial \theta}}^{\,0} + \cancel{\frac{\partial U}{\partial \theta}}^{\,0} - \lambda_A \frac{\partial f_A}{\partial \theta} - \lambda_B \frac{\partial f_B}{\partial \theta} = 0 \Leftrightarrow 4{,}43\times10^{-7}\alpha - 0{,}05\lambda_A + 0{,}08\lambda_B = 0$

$f_A = y_A + 0{,}05\theta \Rightarrow 0 = \ddot{y}_A + 0{,}05\alpha \qquad f_B = y_B - 0{,}08\theta \Rightarrow 0 = \ddot{y}_B - 0{,}08\alpha$

Resolvendo no Maxima: $T_A = 0{,}362\,N$ e $T_B = 0{,}226\,N$

5) a) $V_B = V_C = \dot{x} \qquad \vec{V_e} = \vec{V_C} + \vec{V_{e/c}} = \dot{x}\,\hat{i} - 0{,}2\dot{\theta}\cos\theta\,\hat{i} + 0{,}2\dot{\theta}\sin\theta\,\hat{j} =$
$= (\dot{x} - 0{,}2\dot{\theta}\cos\theta)\,\hat{i} + 0{,}2\dot{\theta}\sin\theta\,\hat{j}$

$V_e^2 = \vec{V_e}\cdot\vec{V_e} = \dot{x}^2 - 0{,}4\dot{x}\dot{\theta}\cos\theta + 0{,}04\dot{\theta}^2\cos^2\theta + 0{,}04\dot{\theta}^2\sin^2\theta = \dot{x}^2 - 0{,}4\dot{x}\omega\cos\theta + 0{,}04\omega^2$

$E_c = \frac{1}{2} m_B V_B^2 + \frac{1}{2} m_C V_C^2 + \frac{1}{2} m V_e^2 = \frac{\dot{x}^2}{2} + \frac{5}{2}\dot{x}^2 + \frac{1}{2}\times 0{,}060\times(\dot{x}^2 - 0{,}4\dot{x}\dot{\theta}\cos\theta + 0{,}04\dot{\theta}^2) =$

$= 3{,}03\,\dot{x}^2 - 0{,}012\,\dot{x}\,\omega\cos\theta + 0{,}0012\,\omega^2$

$U = U_B + U_e = -1\times 9{,}8\times x - 0{,}060\times 9{,}8\times 0{,}2\times\cos\theta =$

$= -9{,}8x - 0{,}1176\cos\theta$

b) $\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{x}}\right) - \cancelto{0}{\frac{\partial E_c}{\partial x}} + \frac{\partial U}{\partial x} = 0 \Leftrightarrow \frac{d}{dt}\left(6{,}06\dot{x} - 0{,}012\,\omega\cos\theta\right) - 9{,}8 = 0 \Leftrightarrow$

$\Leftrightarrow 6{,}06\ddot{x} - 0{,}012\,\alpha\cos\theta + 0{,}012\,\omega^2\sin\theta - 9{,}8 = 0$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \omega}\right) - \frac{\partial E_c}{\partial \theta} + \frac{\partial U}{\partial \theta} = 0 \Rightarrow 0{,}0024\alpha - 0{,}012\cos(\theta)\times\ddot{x} + 0{,}1176\sin\theta = 0$

No máximo: $\ddot{x} = \frac{3\sin(\theta)\omega^2 + 147\cos(\theta)\sin(\theta) - 2450}{15\cos^2(\theta) - 1515}$ e $\ddot{\theta} = \frac{3\cos(\theta)\sin(\theta)\omega^2 + 14897\sin(\theta) - 2450\cos(\theta)}{3\cos^2(\theta) - 303}$

e) equações de evolução: $\dot{\theta} = \omega \quad \dot{\omega} = \ddot{\theta}$

No máximo: $\begin{cases}\omega = 0\\ \ddot{\theta} = 0\end{cases} \Leftrightarrow \sin(\theta) = \frac{50\cos(\theta)}{303} \Leftrightarrow \theta = 9{,}37^\circ$, por exemplo.

$\frac{d}{d\theta}(\ddot{\theta})$ nesse ponto $= -\frac{49\,94309^{3/2}}{28300200}$ Como é negativo, é estável.

d) No máximo: $\ddot{x} = 1{,}617\ m/s^2$

(6) a) $l = x + 2y + d \Rightarrow 0 = \dot{x} + 2\dot{y} \Leftrightarrow \dot{x} = -2\dot{y} \Rightarrow \ddot{x} = -2\ddot{y}$

$E_c = \frac{1}{2}\times 20m\times 4\dot{y}^2 + \frac{1}{2}\times\frac{m}{2}R^2\times\frac{4\dot{y}^2}{R^2} + \frac{1}{2}\times mR^2\times\frac{\dot{y}^2}{R^2} + \frac{1}{2}\times 2m\times\dot{y}^2 + \frac{1}{2}\times 8m\times\dot{y}^2 \Leftrightarrow$

$\Leftrightarrow E_c = \frac{93}{2}m\dot{y}^2$

$U = U_{R2} + U_B = -2mgy - 8mgy = -10mgy$ (ignorando constantes).

b) $\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{y}}\right) - \cancelto{0}{\frac{\partial E_c}{\partial y}} + \frac{\partial U}{\partial y} = 0 \Leftrightarrow \frac{d}{dt}(93m\dot{y}) - 10mg = 0 \Leftrightarrow$

$\Leftrightarrow 93m\ddot{y} = 10mg \Leftrightarrow \ddot{y} = \frac{10\times 9{,}8}{93} \Leftrightarrow \ddot{y} = 1{,}054\ m/s^2$

$\ddot{x} = -2\ddot{y} = -2\times 1{,}054 = -2{,}108\ m/s^2$ R: $|\ddot{y}| = 1{,}054\ m/s^2$ e $|\ddot{x}| = 2{,}108\ m/s^2$

(7) Supondo que o bloco não mantém contacto com o plano inclinado, há duas coordenadas generalizadas, x e y. A equação que faz com que o bloco esteja em contacto com o plano é $y = 0$.

$E_c = \frac{1}{2}mV_B^2 \qquad V_B^2 = \dot{x}^2 + \dot{y}^2 \qquad E_c = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$

$U = mgh \qquad h = x\sin(\theta) + y\cos(\theta) \qquad U = mg(x\sin(\theta) + y\cos(\theta))$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{x}}\right) - \cancelto{0}{\frac{\partial E_c}{\partial x}} + \frac{\partial U}{\partial x} - \lambda\cancelto{0}{\frac{\partial y}{\partial x}} = Q_x \Leftrightarrow \frac{d}{dt}(m\dot{x}) + mg\sin\theta = \mu_c R_n \Leftrightarrow$

$\Leftrightarrow m(\ddot{x} + g\sin\theta) = \mu_c R_n$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{y}}\right) - \cancelto{0}{\frac{\partial E_c}{\partial y}} + \frac{\partial U}{\partial y} - \lambda\frac{\partial y}{\partial y} = Q_y \Leftrightarrow \frac{d}{dt}(m\dot{y}) + mg\cos\theta - \overset{R_n}{\lambda} = 0 \Leftrightarrow$

$\Leftrightarrow R_n = m(\ddot{y} + g\cos\theta) \qquad y = 0 \Rightarrow \dot{y} = 0 \Rightarrow \ddot{y} = 0$

$R_n = mg\cos\theta \qquad \cancel{m}(\ddot{x} + g\sin\theta) = \mu_c \cancel{m}g\cos\theta \Leftrightarrow$

$\Leftrightarrow \ddot{x} = (\mu_c\cos\theta - \sin\theta)g = -g(\sin\theta - \mu_c\cos\theta)$

8) a) $\vec{r}_{cm} = \frac{1}{2}(x, y) = \frac{1}{2}(L\cos\theta, L\sin\theta)$ $\vec{v}_{cm} = \frac{1}{2}(-L\dot{\theta}\sin\theta, L\dot{\theta}\cos\theta)$ c.q.p.

$v_{cm}^2 = \vec{v}_{cm}\cdot\vec{v}_{cm} \Leftrightarrow v_{cm}^2 = \frac{1}{4}(L^2\dot{\theta}^2\sin^2\theta + L^2\dot{\theta}^2\cos^2\theta) \Leftrightarrow v_{cm} = \frac{1}{2}\sqrt{L^2\dot{\theta}^2} \Leftrightarrow v_{cm} = \frac{L\dot{\theta}}{2}$

b) $E_c = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm}\omega^2 = \frac{1}{2} m \times \frac{L^2\dot{\theta}^2}{4} + \frac{1}{2}\times\frac{m}{12}(\cancel{3R^2} + L^2)\omega^2 =$

$= \frac{1}{8} mL^2\dot{\theta}^2 + \frac{1}{24} mL^2\dot{\theta}^2 = \frac{4}{24} mL^2\dot{\theta}^2 = \frac{1}{6} mL^2\dot{\theta}^2 = \frac{1}{6} mL^2\omega^2$

c) $U = mgh = mg\frac{y}{2} = \frac{L}{2} mg\sin\theta$

d) $\frac{d}{dt}\left(\frac{\partial E_c}{\partial \omega}\right) - \cancel{\frac{\partial E_c}{\partial \theta}}^{0} + \frac{\partial U}{\partial \theta} = 0 \Leftrightarrow \frac{d}{dt}\left(\frac{1}{3} mL^2\omega\right) + \frac{L}{2} mg\cos\theta = 0 \Leftrightarrow$

$\Leftrightarrow \frac{1}{3} mL^2\alpha = -\frac{L}{2} mg\cos\theta \Leftrightarrow \alpha = -\frac{3g}{2L}\cos\theta$

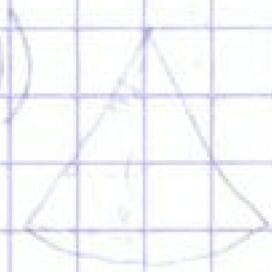
e) $\alpha = \omega\frac{d\omega}{d\theta} \Leftrightarrow -\frac{3g}{2L}\int_{30°}^{\theta}\cos\theta\, d\theta = \int \omega\, d\omega \Leftrightarrow -\frac{3g}{2L}\sin\theta\Big|_{30°}^{\theta} = \frac{\omega^2}{2} \Leftrightarrow$

$\Leftrightarrow \omega^2 = -\frac{3g}{L}\sin\theta\Big|_{30°}^{\theta} \Leftrightarrow \omega^2 = -\frac{3g}{L}\sin\theta + \frac{3g}{2L} \Leftrightarrow \omega = -\sqrt{\frac{3g}{L}\left(\frac{1}{2} - \sin\theta\right)}$

f) $\int_{\frac{\pi}{6}}^{0} -\frac{1}{\sqrt{\frac{3g}{L}\left(\frac{1}{2} - \sin\theta\right)}} = 0{,}3977\, s$ (pelo quad_gags)

9) a) $\vec{v}_B = \vec{v}_P + \vec{v}_{B/P}$ $\vec{v}_P = \frac{d}{dt}(A\cos(\omega t))\hat{j} = -A\omega\sin(\omega t)\hat{j}$

$\vec{r}_{B/P} = l\sin\theta\,\hat{i} - l\cos\theta\,\hat{j}$ $\vec{v}_B = (l\dot{\theta}\cos\theta, l\dot{\theta}\sin\theta - A\omega\sin(\omega t))$



$\vec{v}_{B/P} = l\dot{\theta}\cos\theta\,\hat{i} + l\dot{\theta}\sin\theta\,\hat{j}$

$v_B^2 = \vec{v}_B\cdot\vec{v}_B = l^2\dot{\theta}^2 + A^2\omega^2\sin^2(\omega t) - 2Al\omega\dot{\theta}\sin\theta\sin(\omega t)$

$E_c = \frac{m}{2} v_B^2 = \frac{m}{2}(l^2\dot{\theta}^2 + A^2\omega^2\sin^2(\omega t) - 2Al\omega\dot{\theta}\sin\theta\sin(\omega t))$

$U = mgh = mg(A\cos(\omega t) - l\cos\theta)$

b) $\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{\theta}}\right) - \frac{\partial E_c}{\partial \theta} + \frac{\partial U}{\partial \theta} = 0 \Leftrightarrow \frac{d}{dt}(ml^2\dot{\theta} - mAl\omega\sin\theta\sin(\omega t)) -$

$- mAl\omega\dot{\theta}\sin(\omega t)\cos\theta + mg\sin\theta) = 0 \Leftrightarrow$

$\Leftrightarrow ml^2\ddot{\theta} - mAl\omega\dot{\theta}\cos\theta\sin(\omega t) - mAl\omega^2\sin\theta\cos(\omega t) - mAl\omega\dot{\theta}\sin(\omega t)\cos\theta + mg\sin\theta = 0$

$\Leftrightarrow l^2\ddot{\theta} = 2Al\omega\dot{\theta}\cos\theta\sin(\omega t) + Al\omega^2\sin\theta\cos(\omega t) - mg\sin\theta \Leftrightarrow$

$\Leftrightarrow$

10 a) $v^2 = \dot{x}^2 + \dot{y}^2 \qquad E_c = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \Leftrightarrow$

$\Leftrightarrow E_c = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} I \dot{\theta}^2 \qquad U = m g y$

b) $\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{\theta}}\right) - \cancel{\frac{\partial E_c}{\partial \theta}}^{0} + \cancel{\frac{\partial U}{\partial \theta}}^{0} = 0 \Rightarrow L = \frac{\partial E_c}{\partial \dot{\theta}} \Leftrightarrow$

$\Leftrightarrow L = I\dot{\theta} \Leftarrow$ permanece constante.

c) Pela conservação do momento angular: $I_1 \dot{\theta}_1 = I_2 \dot{\theta}_2 \Leftrightarrow$

$\Leftrightarrow 3{,}28 \times 4 = 28{,}2 \times \dot{\theta}_2 \Leftrightarrow \dot{\theta}_2 = 0{,}465\ s^{-1}$

11 a) $E_c = \frac{1}{2} m v^2 = \frac{1}{2} m (r^2 \dot{\theta}^2 + \dot{r}^2) \qquad U = -\frac{GMm}{r}$

$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{\theta}}\right) - \cancel{\frac{\partial E_c}{\partial \theta}}^{0} + \cancel{\frac{\partial U}{\partial \theta}}^{0} = 0 \Leftrightarrow \frac{d}{dt}(m r^2 \dot{\theta}) = 0 \Rightarrow L = m r^2 \dot{\theta}$ (constante).

b) $\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{r}}\right) - \frac{\partial E_c}{\partial r} + \frac{\partial U}{\partial r} = 0 \Leftrightarrow \frac{d}{dt}(m\dot{r}) - m r \dot{\theta}^2 + \frac{GMm}{r^2} = 0 \Leftrightarrow$

$\Leftrightarrow m\ddot{r} - m r \left(\frac{L}{m r^2}\right)^2 + \frac{GMm}{r^2} = 0 \Leftrightarrow m\ddot{r} = m r \frac{L^2}{m^2 r^4} - \frac{GMm}{r^2} \Leftrightarrow$

$\Leftrightarrow \ddot{r} = \frac{L^2}{m^2 r^3} - \frac{GM}{r^2}$, onde L, m, G e M são constantes.