Capítulo 3 - Movimento curvilíneo

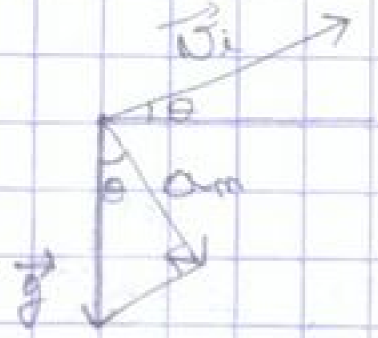
Perguntas:

1) $a_t = \frac{dv}{dt} = (5 + 3t^2 + 2t^3)' = 6t + 6t^2$ (Opção E).

2) $\alpha = \frac{d\omega}{dt} \Leftrightarrow \int_0^t \frac{3}{\pi}\, dt = \int_0^\omega d\omega \Leftrightarrow \omega = \frac{3}{\pi} t$

$\omega = \frac{d\theta}{dt} \Leftrightarrow \int_0^t \frac{3}{\pi} t\, dt = \int_0^{6\pi} \theta \Leftrightarrow 6\pi = \frac{3t^2}{2\pi} \Leftrightarrow 3t^2 = 12\pi^2 \Leftrightarrow t = 2\pi$

(Opção B).



3) (Opção A).

4) $a_m = g\cos\theta \Leftrightarrow \frac{v_i^2}{r} = g\cos\theta \Leftrightarrow$

$\Leftrightarrow r = \frac{v_i^2}{g\cos\theta}$ (Opção E).

5) $v_A = v_B \Leftrightarrow \omega_A R_A = \omega_B R_B$ (Opção A).

Problemas:

1) a) $a_t = \frac{dv}{dt} = \left(t\sqrt{4t^2+9}\right)' = \frac{8t^2+9}{\sqrt{4t^2+9}}$

b) $a^2 = a_t^2 + a_m^2 \Leftrightarrow a_m^2 = \left(\sqrt{16t^2+9}\right)^2 - \left(\frac{8t^2+9}{\sqrt{4t^2+9}}\right)^2 \Leftrightarrow a_m = \frac{6t}{\sqrt{4t^2+9}}$

c) $a_m = \frac{v^2}{r} \Leftrightarrow r = \frac{\left(t\sqrt{4t^2+9}\right)^2}{\frac{6t}{\sqrt{4t^2+9}}} \Leftrightarrow r = \frac{t}{6}(4t^2+9)^{\frac{3}{2}}$

2) $v_i = 72\,Km/h = 20\,m/s$ $\quad a_t = -\frac{4{,}5\,Km/h}{1\,s} = -1{,}25\,m/s^2$ $\quad R \simeq 16{,}7\,m$

$a_t = \frac{dv}{dt} \Leftrightarrow \int_0^4 -1{,}25\, dt = \int_{20}^{v} dv \Leftrightarrow v - 20 = -5 \Leftrightarrow v(4) = 15\,m/s$

$a_m = \frac{v(4)^2}{R} = \frac{15^2}{16{,}7} = 13{,}47\,m/s^2$ $\quad a^2 = a_t^2 + a_m^2 \Leftrightarrow a = \sqrt{a_t^2 + a_m^2} \Leftrightarrow$

$\Leftrightarrow a = \sqrt{1{,}25^2 + 13{,}47^2} \Leftrightarrow a = 13{,}53\,m/s^2$

3) a) $\vec{v} = \frac{d\vec{r}}{dt} = -32\cos(2t)\sin(2t)\,\hat{i} + 16\cos(4t)\,\hat{j}$

$v = \sqrt{(-32\cos(2t)\sin(2t))^2 + (16\cos(4t))^2} \Leftrightarrow v = \sqrt{256\cos(4t)^2 + 1024\cos(2t)^2\sin^2(2t)}$

$\Leftrightarrow v = \sqrt{256\left(\cos^2(4t) + 4\cos^2(2t)\sin^2(2t)\right)} \Leftrightarrow v = 16\sqrt{\cos^2(4t) + (2\sin(2t)\cos(2t))^2}$

$\Leftrightarrow v = 16\sqrt{\cos^2(4t) + \sin^2(4t)} \Leftrightarrow v = 16 \times 1 = 16$

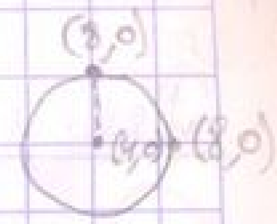
$a_t = \frac{dv}{dt} = (16)' = 0$ $\quad \vec{a} = \frac{d\vec{v}}{dt} = \left(64\sin^2(2t) - 64\cos^2(2t), -64\sin(4t)\right)$

$a^2 = \overset{=0}{a_t^2} + a_m^2 \Leftrightarrow a_m = \sqrt{a^2} = \sqrt{(64\sin^2(2t) - 64\cos^2(2t))^2 + (-64\sin(4t))^2} \Leftrightarrow$

$\Leftrightarrow a_m = 64$ Logo, o movimento é circular uniforme, pois o valor da velocidade é constante e porque o raio da curvatura é constante $\left(\frac{v^2}{a_m}\right)$.

b) Como o movimento é circular uniforme, $v = R\omega$

$R = \frac{v^2}{a_n} = \frac{16^2}{64} = 4$ $\quad \omega = \frac{v}{R} = \frac{16}{4} = 4 \text{ rad/s}$



$T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2} = 1,57 s$

c) $\vec{r}(0) = 8\cos^2(2\times 0)\vec{i} + 4\times \sin(4\times 0)\vec{j} = (8,0) \leftarrow$ posição do primeiro ponto da trajetória.

Como $R = 4$, $C = (4,0)$

4) $a_t = v\frac{dv}{ds} \Leftrightarrow a_t\, ds = v\, dv \Leftrightarrow \int_0^{1200\pi} a_t\, ds = \int_{160}^{140} v\, dv \Leftrightarrow$

$\Leftrightarrow a_t s \Big|_0^{1200\pi} = \frac{v^2}{2}\Big|_{160}^{140} \Leftrightarrow 1200\pi a_t = -3000 \Leftrightarrow a_t = -0,80 \text{ m/s}^2$

$a_t = v\frac{dv}{ds} \Leftrightarrow \int_0^{600\pi} -0,80\, ds = \int_{160}^{v} v\, dv \Leftrightarrow \frac{v^2}{2} - \frac{160^2}{2} = -0,80\times 600\pi \Leftrightarrow$

$\Leftrightarrow v_B = 150,28 \text{ m/s}$

$a_{n_B} = \frac{v_B^2}{r} = \frac{150,28^2}{1200} = 18,82 \text{ m/s}^2$

$a_B^2 = a_t^2 + a_{n_B}^2 \Leftrightarrow a_B^2 = (-0,80)^2 + 18,82^2 \Leftrightarrow a_B = 18,84 \text{ m/s}^2$

5) $v = \frac{\Delta s}{\Delta t} \Leftrightarrow \Delta t = \frac{\Delta s}{v}$ $\quad \Delta s_A = 40 + 82\pi$ m $\quad \Delta s_B = 102\pi$ m

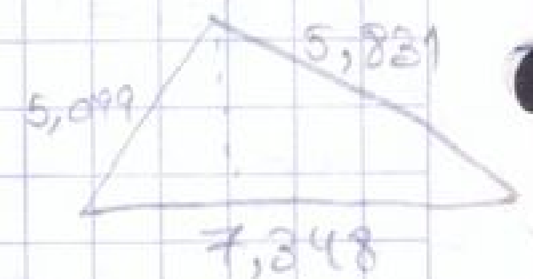
$a_n = \frac{v^2}{r} \Leftrightarrow v_A^2 = a_n \times r_A \Leftrightarrow v_A^2 = 0,8g \times 82 \Leftrightarrow v_A = 25,355 \text{ m/s}$

$a_n = \frac{v^2}{r} \Leftrightarrow v_B^2 = a_n \times r_B \Leftrightarrow v_B^2 = 0,8g \times 102 \Leftrightarrow v_B = 28,279 \text{ m/s}$

$\Delta t_A = \frac{40+82\pi}{25,355} = 11,74 s$ $\quad \Delta t_B = \frac{102\pi}{28,279} = 11,33 s$

6) a) $\overline{AB} = \sqrt{(-1-3)^2 + (2-5)^2 + (1-4)^2} = 5,831$

$\overline{AC} = \sqrt{(2-3)^2 + (-2-5)^2 + (2-4)^2} = 7,348$



$\overline{BC} = \sqrt{(2-(-1))^2 + (-2-2)^2 + (2-1)^2} = 5,099$

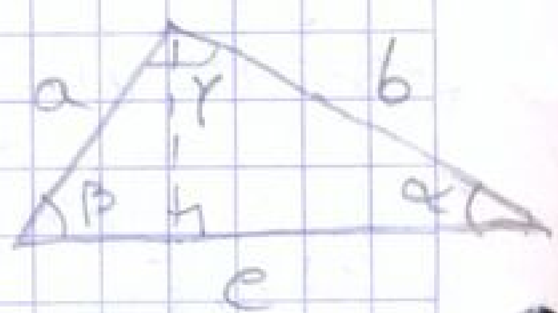
Pela fórmula de Heron: $A = \sqrt{s(s-\overline{AB})(s-\overline{AC})(s-\overline{BC})}$ $\quad s \rightarrow$ semiperímetro

$s = \frac{5,831 + 7,348 + 5,099}{2} = 9,139$

$A = \sqrt{9,139(9,139-5,831)(9,139-7,348)(9,139-5,099)} = 14,79$ u.a.

b) $A = \frac{\text{base} \times \text{altura}}{2} = \frac{c\, b \sin\alpha}{2}$

$A = \frac{\text{base} \times \text{altura}}{2} = \frac{b\, a \sin\gamma}{2}$



$A = \frac{\text{base} \times \text{altura}}{2} = \frac{a\, c \sin\beta}{2}$

$c\, b \sin\alpha = b\, a \sin\gamma = a\, c \sin\beta \Leftrightarrow$

$\Leftrightarrow b \sin\alpha = \frac{b\, a \sin\gamma}{c} = a \sin\beta \Leftrightarrow$

$\Leftrightarrow \sin\alpha = \frac{a \sin\gamma}{c} = \frac{a \sin\beta}{b} \Leftrightarrow \frac{\sin\alpha}{a} = \frac{\sin\gamma}{c} = \frac{\sin\beta}{b}$

7 a) Trajeto $\overline{AB}$: $a^2 = a_t^2 + \cancel{a_n^2}^{\,0} \Leftrightarrow a = a_t$

$a_t = v\frac{dv}{ds} \Leftrightarrow a_t\int_0^{0,60} ds = \int_0^{10} v\,dv \Leftrightarrow a_t \times 0,60 = \frac{10^2}{2} \Leftrightarrow a_B = 83,33\,m/s^2$

Trajeto $\overline{BC}$: $a^2 = \cancel{a_t^2}^{\,0} + a_n^2 \Leftrightarrow a = a_n$

$a_n = \frac{v^2}{r} = \frac{10^2}{0,60} = 166,67\,m/s^2$

Trajeto $\overline{CD}$: $a = 0\,m/s^2$ (movimento retilíneo uniforme).

Trajeto $\overline{DE}$: $a^2 = \cancel{a_t^2}^{\,0} + a_n^2 \Leftrightarrow a = a_n$

$a_n = \frac{v^2}{r} = \frac{10^2}{0,45} = 222,22\,m/s^2$

Trajeto $\overline{EF}$: $a^2 = a_t^2 + \cancel{a_n^2}^{\,0} \Leftrightarrow a = a_t$

$a_t = v\frac{dv}{ds} \Leftrightarrow a_t\int_0^{0,45} ds = \int_{10}^{0} v\,dv \Leftrightarrow 0,45\,a_t = -\frac{10^2}{2} \Leftrightarrow |a| = 111,11\,m/s^2$

b) $a_t = \frac{dv}{dt} \Leftrightarrow \int_0^{t_{AB}} 83,33\,dt = \int_0^{10} dv \Leftrightarrow 83,33\,t_{AB} = 10 \Leftrightarrow t_{AB} = 0,12\,s$

$a_t = \frac{dv}{dt} \Leftrightarrow \int_0^{t_{EF}} -111,11\,dt = \int_{10}^{0} dv \Leftrightarrow -111,11\,t_{EF} = -10 \Leftrightarrow t_{EF} = 0,09\,s$

$v = \frac{\Delta s}{\Delta t} \Leftrightarrow \Delta t_{BCDE} = \frac{\Delta s}{v} \Leftrightarrow t_{BCDE} = \frac{\frac{\pi}{2}\times 0,60 + 0,2 + \frac{\pi}{2}\times 0,45}{10} = 0,185\,s$

$\Delta t = t_{AB} + t_{BCDE} + t_{EF} = 0,12 + 0,09 + 0,185 = 0,395\,s$

$v_m = \frac{\Delta s}{\Delta t} = \frac{\frac{\pi}{2}\times 0,60 + 0,2 + \frac{\pi}{2}\times 0,45 + 0,60 + 0,45}{0,395} = 7,34\,m/s$

8 $v_A = 10\,m/s$ $\quad v_B = -35\,m/s$

$v_{A/B} = v_A - v_B = 10 - (-35) = 45\,m/s$

Logo, a roda gira no sentido horário.

$\omega = \frac{v_{A/B}}{\overline{AB}} = \frac{45}{0,09} = 500\,rad/s$

$v_{O/B} = \omega\,\overline{OB} = 500 \times 0,03 = 15\,m/s$

$v_{O/B} = v_O - v_B \Leftrightarrow v_O = 15 - 35 = -20\,m/s$

Logo, a roda desloca-se para a esquerda, com $v = 20\,m/s$ e $\omega = 500\,rad/s$.

9 a) $v_{C/P} = v_C - \cancel{v_P}^{\,0} = 2\,m/s$ $\quad \omega = \frac{v_{C/P}}{\overline{CP}} = \frac{2}{0,20} = 10\,rad/s$

$\omega = \frac{\Delta\theta}{\Delta t} \Leftrightarrow \Delta t = \frac{4\pi}{10} \Leftrightarrow \Delta t = 1,26\,s$

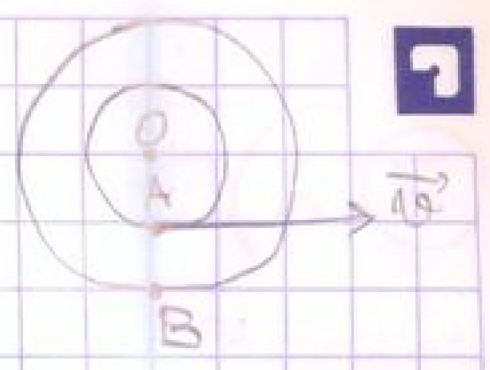
b) $\theta = \omega t = 10t$ $\quad \vec{r}_{P/C} = -0,2\left(\sin(10t)\,\hat{i} + \cos(10t)\,\hat{j}\right)$

$\vec{r}_{Q/C} = -0,1\left(\sin(10t)\,\hat{i} + \cos(10t)\,\hat{j}\right)$ $\quad \vec{r}_C = 2t\,\hat{i} + 0,2\,\hat{j}$

$\vec{r}_P = (2t - 0,2\sin(10t))\,\hat{i} + (0,2 - 0,1\cos(10t))\,\hat{j}$

$\vec{r}_Q = (2t - 0,1\sin(10t))\,\hat{i} + (0,2 - 0,1\cos(10t))\,\hat{j}$ (feito no Maxima).

10 a) $\overrightarrow{v_{A/B}} = \overrightarrow{v_A} - \cancel{\overrightarrow{v_B}}^{0} \Leftrightarrow v_{A/B} = 2{,}5\,\hat{\imath}$ (cm/s)



$\overrightarrow{v_{A/B}} = \vec{\omega} \times \overrightarrow{r_{A/B}} = \vec{\omega} \times \overrightarrow{BA} = (\omega \hat{k}) \times (2\hat{\jmath}) = -2\omega\hat{\imath}$

$-2\omega\hat{\imath} = 2{,}5\hat{\imath} \Leftrightarrow \omega = -\frac{2{,}5}{2} = -1{,}25\ s^{-1}$

R: $1{,}25\ s^{-1}$, no sentido dos ponteiros do relógio.

b) Como a velocidade angular é no sentido dos ponteiros do relógio, o ponto O desloca-se para a direita com velocidade de valor igual a:

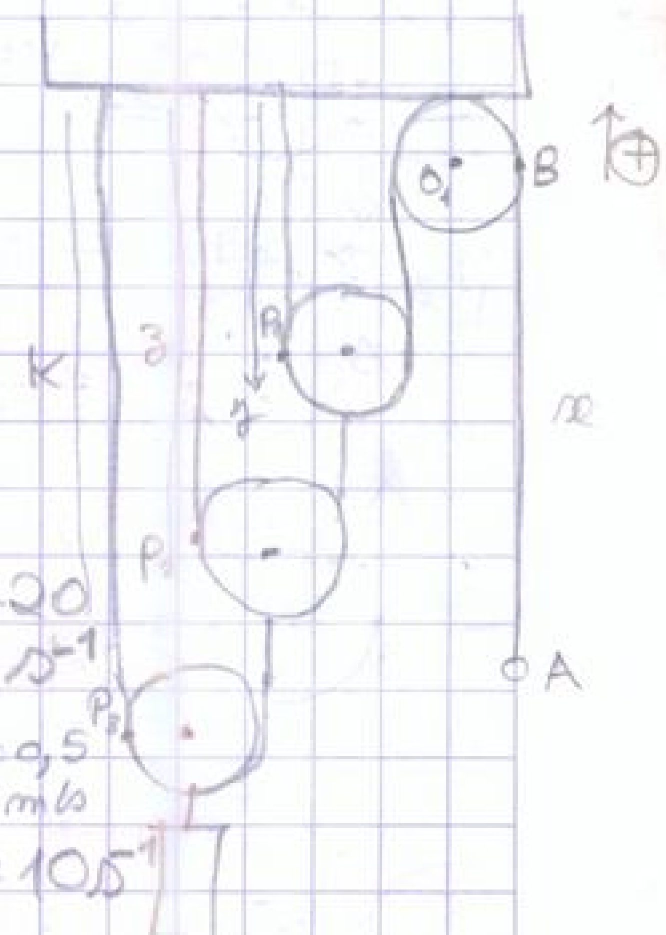
$v_{O/B} = d_{O/B} \times \omega \Leftrightarrow v_{O/B} = 6 \times 1{,}25 = 7{,}5$ cm/s

$v_{O/B} = v_O - \cancel{v_B}^{0} \Leftrightarrow v_O = v_{O/B} = 7{,}5$ cm/s

c) $\overrightarrow{v_{A/O}} = \overrightarrow{v_A} - \overrightarrow{v_O} = 2{,}5\hat{\imath} - 7{,}5\hat{\imath} = -5\hat{\imath}$ cm/s

Como este valor é negativo, os pontos A e O estão a aproximar-se, ou seja, a cada segundo enrolam-se 5 cm de corda.

11 $v_{O/B} = \cancel{v_O}^{0} - v_B = 2$ m/s $\quad v_x = -2$ m/s



$v_{O/B} = R_{O/B}\,\omega \Leftrightarrow \omega = \frac{v_{O/B}}{R_{O/B}} = \frac{2}{0{,}05} = 40\ s^{-1}$

$l_1 = x + 2y \Leftrightarrow 0 = V_x + 2V_y \Leftrightarrow V_y = -\frac{1}{2}V_x \Leftrightarrow$

$\Leftrightarrow V_y = -\frac{1}{2}\times(-2) = 1$ m/s $\quad V_O = 1$ m/s

$v_{O/P_1} = v_O - \cancel{v_{P_1}}^{0} = 1$ m/s $\quad v_{O/P_1} = R_{O/P_1}\,\omega \Leftrightarrow \omega = \frac{1}{0{,}05} = 20\ s^{-1}$

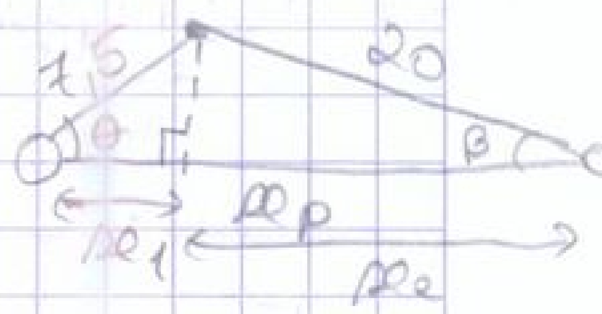
$l_2 = z + z - y \Leftrightarrow 0 = 2V_z - V_y \Leftrightarrow V_z = \frac{V_y}{2} \Leftrightarrow V_z = \frac{1}{2} = 0{,}5$ m/s

$v_{O/P_2} = v_O - \cancel{v_{P_2}}^{0} = 0{,}5$ m/s $\quad v_{O/P_2} = R_{O/P_2}\,\omega \Leftrightarrow \omega = \frac{0{,}5}{0{,}05} = 10\ s^{-1}$

$l_3 = K + K - z \Leftrightarrow 0 = 2V_K - V_z \Leftrightarrow V_K = \frac{V_z}{2} = \frac{0{,}5}{2} = 0{,}25$ m/s

$v_{O/P_3} = v_O - \cancel{v_{P_3}}^{0} = 0{,}25$ m/s $\quad v_{O/P} = R_{O/P}\,\omega \Leftrightarrow \omega = \frac{0{,}25}{0{,}05} = 5\ s^{-1}$

12 a) $x_p = x_1 + x_2 = 7{,}5\cos\theta + \sqrt{400 - 56{,}25\sin^2\theta}$



CA: $20^2 = (7{,}5\sin\theta)^2 + x_2^2 \Leftrightarrow x_2 = \sqrt{400 - 56{,}25\sin^2\theta}$

Derivando: $V_p = -7{,}5\sin\theta - \frac{56{,}25\cos\theta\sin\theta}{\sqrt{400 - 56{,}25\sin^2\theta}}$

$V_p = x_p \times \omega_m \Leftrightarrow \omega_m = -\frac{V_p}{7{,}5\sin\theta + \frac{56{,}25\sin\theta\cos\theta}{\sqrt{400-56{,}25\sin^2\theta}}}$ $\qquad \cos\beta = \frac{\sqrt{400 - 56{,}25\sin\theta}}{20}$

Pela lei dos senos:

b) $\frac{\sin\theta}{20} = \frac{\sin\beta}{7{,}5} \Leftrightarrow 7{,}5\sin\theta = 20\sin\beta \Leftrightarrow 3\sin\theta = 8\sin\beta$

Derivando: $3\cos\theta\,\dot{\theta} = 8\cos\beta\,\dot{\beta} \Leftrightarrow$ $\qquad \cos\beta = \frac{x_2}{20} = \frac{\sqrt{400 - 56{,}25\sin^2\theta}}{20}$

$\Leftrightarrow 3\cos\theta\,\omega_m = \frac{2\sqrt{400 - 56{,}25\sin^2\theta}}{5}\,\omega_b \Leftrightarrow$

$\Leftrightarrow \omega_b = \frac{15\cos\theta\,\omega_m}{2\sqrt{400 - 56{,}25\sin^2\theta}} = \frac{15\cos\theta\,\omega_m}{40\sqrt{1 - \frac{9}{64}\sin^2\theta}} = \frac{3\cos\theta\,\omega_m}{8\sqrt{1 - \frac{9}{64}\sin^2\theta}}$

c) $\omega_m = 9{,}60\ s^{-1}$ $\quad \omega_b = 2{,}843\ s^{-1}$