Variáveis aleatórias. Distribuições de probabilidade

4.1 i) Espaço amostral = {AB, AC, AD, AE, AF, AG, BC, BD, BE, BF, BG, CD, CE, CF, CG, DE, DF, DG, EF, EG, FG}

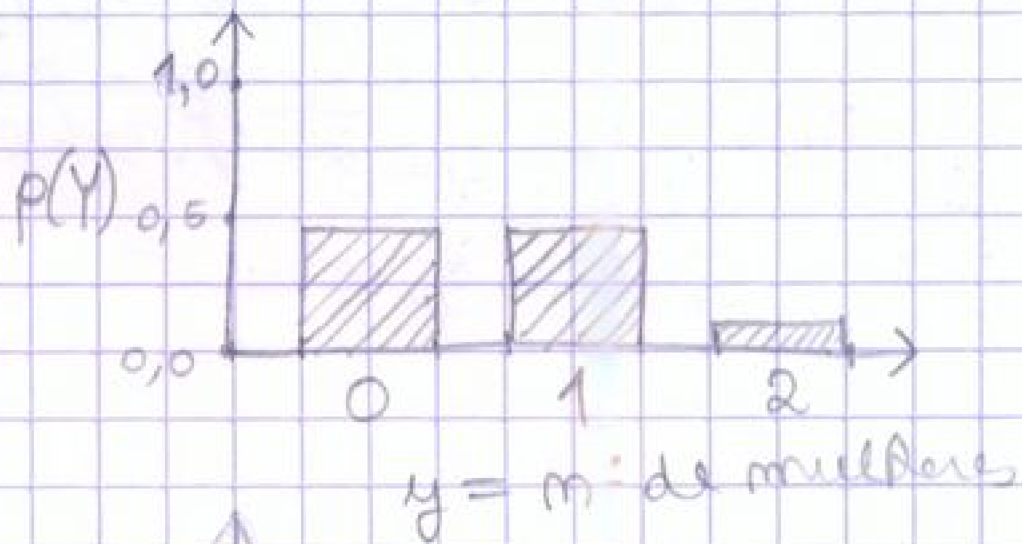
ii) $Y=0 \rightarrow \{AB, AC, AD, AE, BC, BD, BE, CD, CE, DE\}$

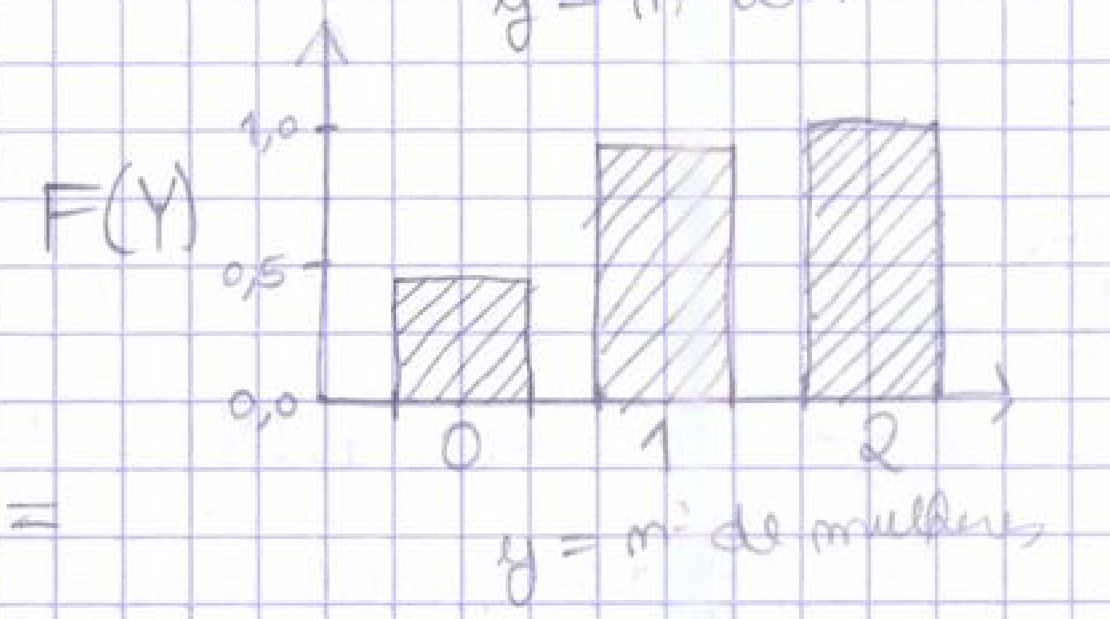
$Y=1 \rightarrow \{AF, AG, BF, BG, CF, CG, DF, DG, EF, EG\}$

$Y=2 \rightarrow \{FG\}$

iii)

Y	p(Y)	F(Y)
0	$\frac{10}{21}=0,476$	$\frac{10}{21}=0,476$
1	$\frac{10}{21}=0,476$	$\frac{20}{21}=0,952$
2	$\frac{1}{21}=0,048$	1





iv) (μ_y) $E(Y) = \sum_{y=0}^{2} y \times p(y) =$

$= 0 \times 0,476 + 1 \times 0,476 + 2 \times 0,048 =$

$= 0,572$

(σ^2) $Var(Y) = \sum_{y=0}^{2} (y-\mu_y)^2 \times p(y) = (0-0,572)^2 \times 0,476 + (1-0,572)^2 \times 0,476 + (2-0,572)^2 \times 0,048 =$

$= 0,341$ $\quad \sigma_y = \sqrt{0,341} = 0,584$ $\quad \gamma_1 = \frac{\mu_3}{\sigma_Y^3}$

$\mu_3 = \sum_{y=0}^{2} (y-\mu_y)^3 \times p(y) = (0-0,572)^3 \times 0,476 + (1-0,572)^3 \times 0,476 + (2-0,572)^3 \times 0,048 =$

$= 0,088$ $\quad \gamma_1 = \frac{0,088}{0,584^3} = 0,442$

4.2 v) $P(\Delta t > 2) = \int_2^{+\infty} e^{-\Delta t}\, d\Delta t = \left[-e^{-\Delta t}\right]_2^{+\infty} = 0 + \frac{1}{e^2} = 0{,}135$

vi) $P(\Delta t > 3) = \int_3^{+\infty} e^{-\Delta t}\, d\Delta t = \left[-e^{-\Delta t}\right]_3^{+\infty} = 0 + \frac{1}{e^3} = 0{,}0498 = 4{,}98\%$

vii) $P(\Delta t > 3 \mid \Delta t > 1) = \frac{P(\Delta t > 3 \wedge \Delta t > 1)}{P(\Delta t > 1)} = \frac{P(\Delta t > 3)}{P(\Delta t > 1)} = \frac{e^{-3}}{e^{-1}} = e^{-2} = 0{,}135$

Em geral, $P(\Delta t > t + T \mid \Delta t > t) = \frac{e^{-(t+T)}}{e^{-t}} = e^{-T} = P(\Delta t > T)$

4.3 i)

ii) $P(x \leq 1{,}50) = \int_0^{1{,}50} \frac{4}{27}(9x - 6x^2 + x^3)\,dx = \frac{4}{27}\left[\frac{9x^2}{2} - 2x^3 + \frac{x^4}{4}\right]_0^{1{,}50} =$

$= \frac{4}{27} \times 4{,}640625 = 0{,}6875 = 68{,}8\%$

$P(x \geq 2) = 1 - P(x \leq 2) = 1 - \int_0^2 \frac{4}{27}(9x - 6x^2 + x^3)\,dx = 1 - \frac{4}{27}\left[\frac{9x^2}{2} - 2x^3 + \frac{x^4}{4}\right]_0^2 =$

$= 1 - 0{,}8889 = 0{,}111 = 11{,}1\%$

$P(1 \leq x \leq 2{,}5) = \int_1^{2{,}5} \frac{4}{27}(9x - 6x^2 + x^3)\,dx = \frac{4}{27}\left[\frac{9x^2}{2} - 2x^3 + \frac{x^4}{4}\right]_1^{2{,}5} =$

$= \frac{4}{27} \times 3{,}890625 = 0{,}576 = 57{,}6\%$

iii) $F(x) = \int_0^x \frac{4}{27}(9x - 6x^2 + x^3)\,dx = \frac{4}{27}\left[\frac{9x^2}{2} - 2x^3 + \frac{x^4}{4}\right]_0^x =$

$= \frac{2}{3}x^2 - \frac{8}{27}x^3 + \frac{1}{27}x^4$

4.4 Y: "Temperatura em °C" X: "Temperatura em °F"

$Y = \frac{X - 32}{1{,}8}$ Logo, $E(Y) = \frac{E(X) - 32}{1{,}8} = \frac{153 - 32}{1{,}8} = 67{,}2\,°C$

$Var(Y) = \left(\frac{1}{1{,}8}\right)^2 \times Var(X) = \left(\frac{1}{1{,}8}\right)^2 \times 7^2 = 15{,}123\,°C$ $\sigma_Y = \sqrt{Var(Y)} = \sqrt{15{,}123} = 3{,}89\,°C$

4.5 $E(\Delta V) = 20$ $Var(\Delta V) = 225$ $V = V_0 + \Delta V \rightarrow$ transformação linear.

$E(V) = V_0 + E(\Delta V) = 400 + 20 = 420$ $Var(V) = Var(\Delta V) = 225$

$N = \phi(V) = 75{,}22 + 0{,}2511V - 0{,}000481V^2 + 0{,}386 \times 10^{-6}V^3$

$E(N) \approx \phi\, E(V) = 75{,}22 + 0{,}2511 \times 420 - 0{,}000481 \times 420^2 + 0{,}386 \times 10^{-6} \times 420^3 =$

$= 124{,}43$

$Var(N) = Var[\phi V] \approx \left[\frac{d\phi}{dV}\right]^2_{z=\mu_z} \cdot Var(V)$

$\frac{d\phi}{dV} = 0{,}2511 - 9{,}62\times10^{-4}V + 1{,}158\times10^{-6}V^2$

$\left[\frac{d\phi}{dV}\right]^2_{z} = (0{,}2511 - 9{,}62\times10^{-4}\times 400 + 1{,}158\times10^{-6}\times 400^2)^2 = 2{,}635\times10^{-3}$

$Var(N) = 2{,}635\times10^{-3}\times 225 = 0{,}593 \qquad \sigma(N) = \sqrt{0{,}593} = 0{,}77$

4.6 i) N: "número de espectadores"

$E(N) = \sum_y y\,p(y) = 0{,}20\times5000 + 0{,}20\times20000 + 0{,}10\times30000 + 0{,}50\times50000 =$
$= 33000$

transformação linear

ii) L: "lucro do concerto". $L = 4{,}5N - N - 75000 - 30000 =$
$= 3{,}5N - 105000$

$E(L) = 3{,}5\times E(N) - 105000 \Leftrightarrow$
$\Leftrightarrow E(L) = 3{,}5\times 33000 - 105000 = 10500$ €

R: Deve realizar o concerto, embora corra riscos.

iii) $E(N) = \sum_y y\,p(y) = 0{,}30\times5000 + 0{,}20\times20000 + 0{,}20\times30000 + 0{,}30\times50000 =$
$= 26500 \qquad E(L) = 3{,}5\times E(N) - 105000 = 3{,}5\times26500 - 105000 = -12250$ €

Se o concerto for realizado, o lucro é negativo, de valor −12250€.

Se o concerto for cancelado, $E(L) = -\frac{30000}{2} - 7500 = -22500$ €.

Logo, como $-12250 > -22500$, o concerto deverá ser realizado na mesma.

4.7 i) Y: "número de veículos disponíveis por dia".
N: "O veículo mais novo está disponível num dado dia".
A: "O veículo mais antigo está disponível num dado dia".
T: "O terceiro veículo está disponível num dado dia".

$P(Y=0) = P(\overline{N}\cap\overline{A}\cap\overline{T}) = P(\overline{N})\times P(\overline{A})\times P(\overline{T}) = 0{,}05\times0{,}15\times0{,}1 = 0{,}00075$

$P(Y=1) = P(N\cap\overline{A}\cap\overline{T}) + P(\overline{N}\cap A\cap\overline{T}) + P(\overline{N}\cap\overline{A}\cap T) = P(N)\times P(\overline{A})\times P(\overline{T}) + P(\overline{N})\times P(A)\times P(\overline{T}) + P(\overline{N})\times P(\overline{A})\times P(T) =$
$= 0{,}95\times0{,}15\times0{,}1 + 0{,}05\times0{,}85\times0{,}1 + 0{,}05\times0{,}15\times0{,}9 = 0{,}02525$

$P(Y=2) = P(N\cap A\cap\overline{T}) + P(N\cap\overline{A}\cap T) + P(\overline{N}\cap A\cap T) = P(N)\times P(A)\times P(\overline{T}) + P(N)\times P(\overline{A})\times P(T) + P(\overline{N})\times P(A)\times P(T) =$
$= 0{,}95\times0{,}85\times0{,}1 + 0{,}95\times0{,}15\times0{,}9 + 0{,}05\times0{,}85\times0{,}9 = 0{,}24725$

$P(Y=3) = P(N\cap A\cap T) = P(N)\times P(A)\times P(T) = 0{,}95\times0{,}85\times0{,}9 = 0{,}72675$

$F(0) = P(Y\leq0) = P(Y=0) = 0{,}00075 \quad F(1) = P(Y\leq1) = P(Y=0) + P(Y=1) = 0{,}00075 + 0{,}02525 = 0{,}026$

$F(2) = P(Y\leq2) = P(Y=0) + P(Y=1) + P(Y=2) = 0{,}026 + 0{,}24725 = 0{,}27325$

$F(3) = P(Y\leq3) = P(Y=0) + P(Y=1) + P(Y=2) + P(Y=3) = 0{,}27325 + 0{,}72675 = 1$

ii) $E(Y) = \sum_y y \times p(y) = 0 \times 0{,}00075 + 1 \times 0{,}02525 + 2 \times 0{,}24725 + 3 \times 0{,}72675 =$
$= 2{,}7$ $\quad Var(Y) = \sum_y (y - E(Y))^2 \times p(y) = (0-2{,}7)^2 \times 0{,}00075 + (1-2{,}7)^2 \times 0{,}02525 +$
$+ (2-2{,}7)^2 \times 0{,}24725 + (3-2{,}7)^2 \times 0{,}72675 = 0{,}265$ $\quad \sigma_Y = \sqrt{0{,}265} = 0{,}515$

iii) $L = 0{,}5 \times Y$ ← transformação linear. $E(L) = 0{,}5 \times E(Y) \Leftrightarrow$
$\Leftrightarrow E(L) = 50 \times 2{,}7 = 135$ € $\quad Var(L) = 50^2 \times Var(Y) = 50^2 \times 0{,}265 =$
$= 662{,}5$ € $\quad \sigma_L = \sqrt{662{,}5} = 25{,}7$ €