

# **SIMULATION OF DIFFUSION IN HETEROGENEOUS MEDIA**

by

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# ABSTRACT

This is a short example of an abstract.

# ACKNOWLEDGEMENTS

Thank some people that you like here.

# AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of the University of Ontario Institute of Technology. The work is original except where indicated by special reference in the text and no part of the dissertation has been submitted for any other degree. Any views expressed in the dissertation are those of the author and in no way represent those of the University of Ontario Institute of Technology. The thesis has not been presented to any other University for examination either in Canada or elsewhere.

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March 20, 2016

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# 1. INTRODUCTION

What was/is the purpose of this thesis? Why did we develop these computational simulations? Really, we need a goal!

In this thesis, a lattice Monte Carlo approach was used to simulate diffusion. Also used a finite difference method to simulate diffusion. Both problems were boundary-value problems.

We also performed an analysis: MSD, mean position, etc.?? Analysis is kind of empty!

Overall overview to thesis? What kind of an overview? Or just a general background to some main concepts needed or used in this thesis? How does this compare with the abstract?

## 1.1 Diffusion

Introduction. Theory. Supporting Figures.

Refer to page 91 in (Patton, Thibodeau , 2013). Figure 4.1 may also be used? Or is it too ‘simple’?

## 1.2 Monte Carlo Simulations

Introduction. Theory. Supporting figures.



## 1.3 Master Equation Simulations

Introduction. Theory. Supporting figures.

## 1.4 Simple Cells and Tissues

Nearly all human cells are microscopic in size; their diameters range from  $7.5\text{ }\mu\text{m}$  to approximately  $150\text{ }\mu\text{m}$  and a cell exhibits a particular size or shape that reflects the specific task it's designated to perform. There are many different types of cells including nerve cells, muscle cells, and gland cells, but despite their anatomical and functional differences, the cells of the human body have many similarities. It is a fact that no cell contains all cellular components found in all the cell types, so often a composite cell (Figure 1.1) is used to exhibit the most important characteristics.

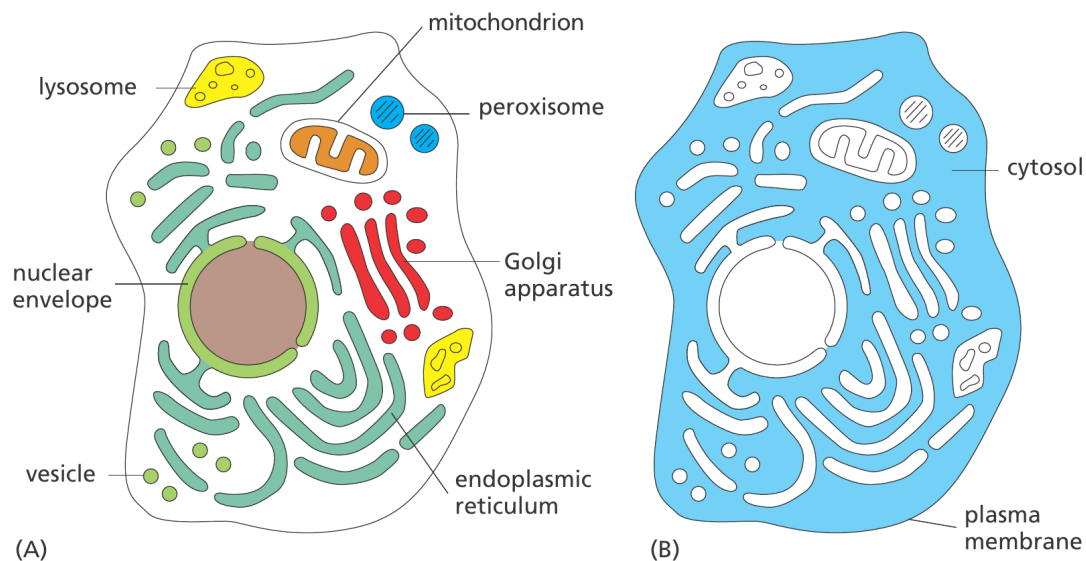


Figure 1.1: Composite cell showing various important and common internal cell structures. Most of the volume within the cell is occupied by the cytosol fluid. Figure courtesy of Essential Cell Biology, Alberts, 3rd.

Each cell is enclosed by a plasma membrane that separates the cell contents from the surrounding environment. The inside of the cell is mostly composed of a gel-like substance called cytoplasm that is a dense arrangement of proteins, organelles, and

other molecules, suspended in a watery fluid called cytosol. The dense crowding of molecules and organelles results in frequent physical interactions which promotes high metabolic efficiency (Patton, Thibodeau , 2013). All of the fluid inside the cell may be referred to as intracellular fluid or simply, cellular fluid (CF).

The cellular fluid is separated from the extracellular fluid by the cell/plasma membrane. This membrane is a phospholipid bilayer with various embedded macromolecular structures. Each phospholipid molecule is amphipathic, having both a hydrophobic and hydrophilic region. A collection of phospholipid molecules will naturally arrange themselves into a bilayer that does not allow water, polar molecules, or ions to pass through easily. However, water and other molecules or ions need to traverse this membrane and the water transporting task is accomplished by aquaporin gated channel proteins. Aquaporins facilitate the passive diffusion of water through the plasma membrane, between the intracellular and extracellular regions. (Patton, Thibodeau , 2013).

### 1.4.1 Tissues

In a multicellular organism, there are several levels of biological organization. A cell is the lowest level of organization that is considered living; tissues are the next higher level of organization and are composed of cells similar in structure and function. This ensemble of cells resides in an extracellular matrix (ECM); a medium containing water, fibrous and adhesive proteins, glycoproteins, and other molecules. The ECM varies in composition between different tissues, but providing structural support and facilitating cell-to-cell communication are common functions of the ECM. **Normally, the cytoplasm is more viscous than the extracellular matrix (is this true?).** (Campbell, Reece , 2008).

## 2. MODELS AND SIMULATIONS

A simulation is intended to imitate in many cases, a real-world process or system, that may be too difficult or costly to analyze directly. Before any such simulation can begin, a model of the system studied must be constructed. Models capture the characteristics and behaviours of the system they represent and in general, a model should be as simple as possible (since resources are limited) while still explaining experimental observations and making predictions with a given degree of accuracy. The simulation is the implementation of the model and can be executed on a computer to produce data for testing, analysis, and visual presentation.

In our simplified model of a biological tissue, the relatively ordered and periodic nature of cells in most simple tissues is captured as a series of repeating *unit cells*. These unit cells are the building blocks of the heterogeneous 1D and 2D models. Each unit cell is characterized by a cellular domain, separated from an extracellular domain by a semi-permeable membrane. The domains are isotropic except at the boundaries where a change in viscosity and semi-permeable barrier exist. Within each domain, the only characteristic modelled is viscosity, and is implemented as a directional stepping probability (Section 1.1). Regarding the boundaries, there exists two kinds in our models. The first kind is a totally-reflecting boundary; it forms the absolute boundary of the model system and represents an insurmountable physical barrier. The second kind is a semi-permeable non-active/passive boundary and represents the selectively permeable nature of the plasma membrane. In a real biological plasma membrane, the integral membrane proteins can facilitate either active or passive transport. In

the simple cell model developed, the semi-permeable membranes behave in a passive transport manner and this is implemented as a boundary transition probability, a concept explained in Section 1.1.

The diffusion of idealized particles, which are non-interacting, experience no net force, and boundary constraints. Particle motion is therefore undirected but occurs in only 1 direction (along a line) in the 1D system, and in 2 orthogonal directions in the 2D system. It was decided from the start that a fixed particle jump/step-size, compared to a Gaussian step-size, would yield data sufficiently accurate for our analytical purposes.

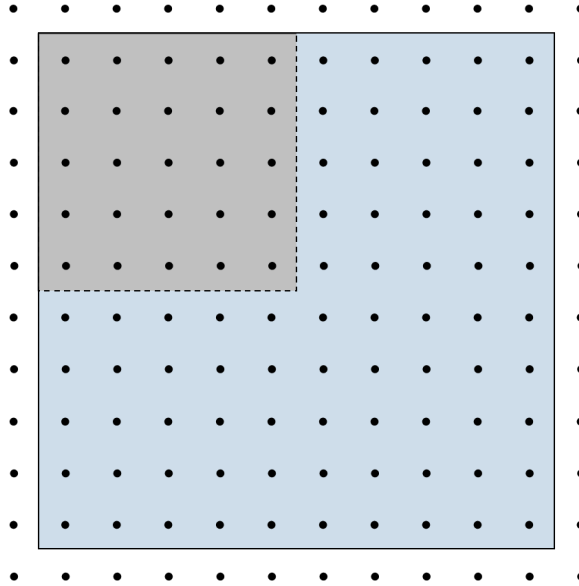


Figure 2.1: A single 2D unit cell forms the building block of the 2D model. An example lattice arrangement is overlayed on the model to show possible particle positions. The dimensions of each domain can be adjusted individually by increasing or decreasing the number of lattice points used in the simulation. Dashed lines represent semi-permeable boundaries and lattice points outside the cell belong to adjacent cells.

Particle diffusion in both the 1D and 2D systems were simulated using Monte Carlo and master equation approaches.

Using Monte Carlo, information on the individual state (i.e. current position and path history) can be maintained. However, due to the finite number of particles used in the simulation and the stochastic nature of individual particle motion, statistical

fluctuations lead discontinuous (?) distributions.

Using the master equation methods and evolving the particle density distribution in time, discontinuities (?) in the distributions are no longer an issue, however at the expense of no information on individual particle state.

For both simulation types, the computed density distribution at every time step was written to a file. From the data generated, information such as mean particle (ensemble?) position was extracted for computation of the mean-square-displacement at every time step.

The density distribution data was also turned into visual graphic that could show the evolution of the particle diffusion within the system, in time.

## 2.1 1-Dimensional Systems

In 1D, homogenous and heterogeneous simple cell systems were simulated using Monte Carlo (MC) and master equation approaches.

### 2.1.1 Homogenous System

\*include figure of homogenous model\*

### 2.1.2 Heterogeneous System



Figure 2.2: 2D lattice unit cell with cellular and extracellular regions. Dashed lines indicate semi-permeable barrier.

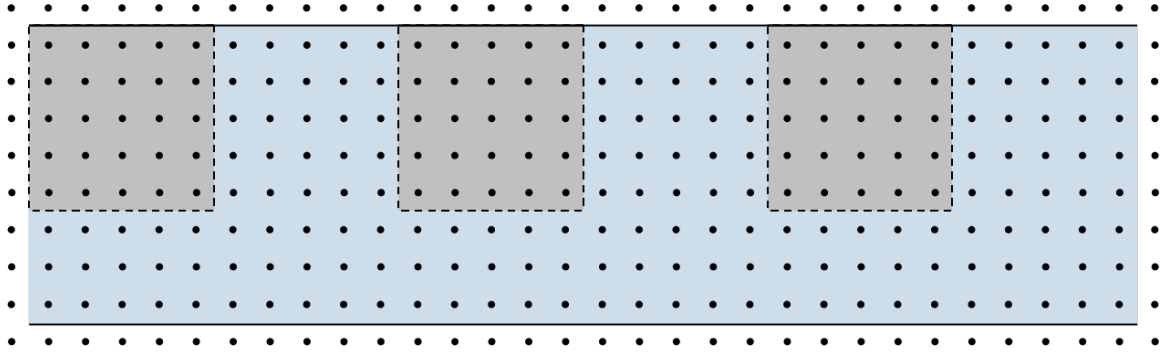


Figure 2.3: 2D lattice unit cell with cellular and extracellular regions. Dashed lines indicate semi-permeable barrier.

## 2.2 2-Dimensional System

\*include figure of heterogeneous model\*

### **3. RESULTS AND ANALYSIS**

## 4. FUTURE WORK AND CONCLUSION

May just put this into the results section near the end as conclusions and future work.



# REFERENCES

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## A. APPENDIX

This is just an example of an appendix.