CMPT285 Homework 7 (due Tuesday, April 8)

1. (Problem 11 on page 330 from Rosen)

   (a) Find a formula for
   $$1/2 + 1/4 + 1/8 + \cdots + 1/2^n$$
   by examining the values of this expression for small values of n.

   (b) Prove the formula you conjectured in part (a).

2. (Problem 13 on page 330 from Rosen) Prove that $1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1}n^2 = (-1)^{n-1}n(n+1)/2$ whenever n is a positive integer.

3. (Problem 5 on page 396 from Rosen) Six different airlines fly from New York to Denver and seven fly from Denver to Dan Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and and airline for the continuation flight to San Francisco?

4. (Problem 15 on page 405 from Rosen) How many numbers must be selected from the set $\{1, 2, 3, 4, 5, 6\}$ to guarantee that at least one pair of these numbers add up to 7?

5. (Problem 27 on page 406 from Rosen) Show that in a group of 10 people (where any two people are either friends or enemies), there are either three mutual friends or four mutual enemies, and there are either three mutual enemies or four mutual friends.

6. (Problem 11 on page 413 from Rosen) How many bit strings of length 10 contain

   - exactly four 1s?
   - at most four 1s?
   - at least four 1s?
   - an equal number of 0s and 1s?

7. (Problem 19 on page 413 from Rosen) A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes

   - are there in total?
   - contain exactly two heads?
   - contain at most three tails?
   - contain the same number of heads and tails?

8. (Problem 31 on page 413 from Rosen) The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain

   - exactly one vowel?

- exactly two vowels?
- at least one vowel?
- at least two vowels?