CMPT285 Homework 9 (due Thursday, April 24)

1. (Problem 7 on page 581 from Rosen) Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

   (a) $x \neq y$.

   (b) $xy \geq 1$.

   (c) $x = y + 1$ or $x = y - 1$.

   (d) $x \equiv y \pmod 7$.

   (e) x is a multiple of y.

   (f) x and y are both negative or both nonnegative.

   (g) $x = y^2$.

   (h) $x \geq y^2$.

2. (Problem 31 on page 582 from Rosen) Let A be the set of students at your school and B the set of booksin the school library. Let R_1 and R_2 be the relations consisting of all ordered pairs (a, b), where student a is required to read book b in a course, and where student a has read book b, respectively. Describe the ordered pairs in each of these relations.

   (a) $R_1 \cup R_2$.

   (b) $R_1 \cap R_2$.

   (c) $R_1 \oplus R_2$.

   (d) $R_1 - R_2$.

   (e) $R_2 - R_1$.

3. (Problem 51 on page 583 from Rosen) Show that the relation R on a set A is symmetric if and only if $R = R^{-1}$, where R^{-1} is the inverse relation.

4. (Problem 9 on page 596 from Rosen) How many nonzero entries does the matric representing the relation R on $A = \{1, 2, 3, \cdots, 100\}$ consisting of the first 100 positive integers have if R is the others lack.

   (a) $\{(a, b) | a > b\}$?

   (b) $\{(a, b) | a \neq b\}$?

   (c) $\{(a, b) | a = b + 1\}$?

   (d) $\{(a, b) | a = 1\}$?

   (e) $\{(a, b) | ab = 1\}$?

5. (Problem 19 on page 597 from Rosen) Draw the directed graphs representing each of the following relations on $\{1, 2, 3, 4\}$.

   (a) $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$

   (b) $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$

   (c) $\{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$

(d) $\{(2,4),(3,1),(3,2),(3,4)\}$

6. (Problem 3 on page 615 from Rosen) Which of these relations on the set $\mathbf{Z} \times \mathbf{Z}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

   (a) $\{(f,g) | f(1) = g(1)\}$
   (b) $\{(f,g) | f(0) = g(0) \text{ or } f(1) = g(1)\}$
   (c) $\{(f,g) | f(x) - g(x) = 1 \text{ for all } x \in \mathbf{Z}\}$
   (d) $\{(f,g) | \text{ for some } C \in \mathbf{Z}, f(x) - g(x) = C\}$
   (e) $\{(f,g) | f(0) = g(1) \text{ and } f(1) = g(0)\}$

7. (Problem 15 on page 615 from Rosen) Let R be the relation on the set of ordered pairs of positive integers such that $((a,b),(c,d)) \in R$ if and only if $a + b = c + d$. Show that R is an equivalence relation.

8. (Problem 35 on page 616 from Rosen) What is the congruence class $[n]_5$ (that is, the equivalence class of n with respect to congruence modulo 5) when n is

   (a) 2?
   (b) 3?
   (c) 6?
   (d) -3?

9. (Problem 49 on page 617 from Rosen) Show that the partition formed from congruence classes modulo 6 is a refinement of the partition formed from congruence classes modulo 3.

10. (Problem 55 on page 617 from Rosen) Find the smallest equivalence relation on the set of $\{a,b,c,d,e\}$ containing the relation $\{(a,b),(a,c),(d,e)\}$.

11. (Problem 1 on page 630 from Rosen) Which of these relations on $\{0,1,2,3\}$ are partial orderings? Determine the properties of a partial ordering that the others lack.

    (a) $\{(0,0),(1,1),(2,2),(3,3)\}$
    (b) $\{(0,0),(1,1),(2,0),(2,2),(2,3),(3,2),(3,3)\}$
    (c) $\{(0,0),(1,1),(1,2),(2,2),(3,3)\}$
    (d) $\{(0,0),(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$
    (e) $\{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,2),(3,3)\}$

12. (Problem 5 on page 630 from Rosen) Which of these are posets?

    (a) $(\mathbf{Z}, =)$
    (b) $(\mathbf{Z}, \neq)$
    (c) $(\mathbf{Z}, \geq)$
    (d) $(\mathbf{Z}, \nmid)$