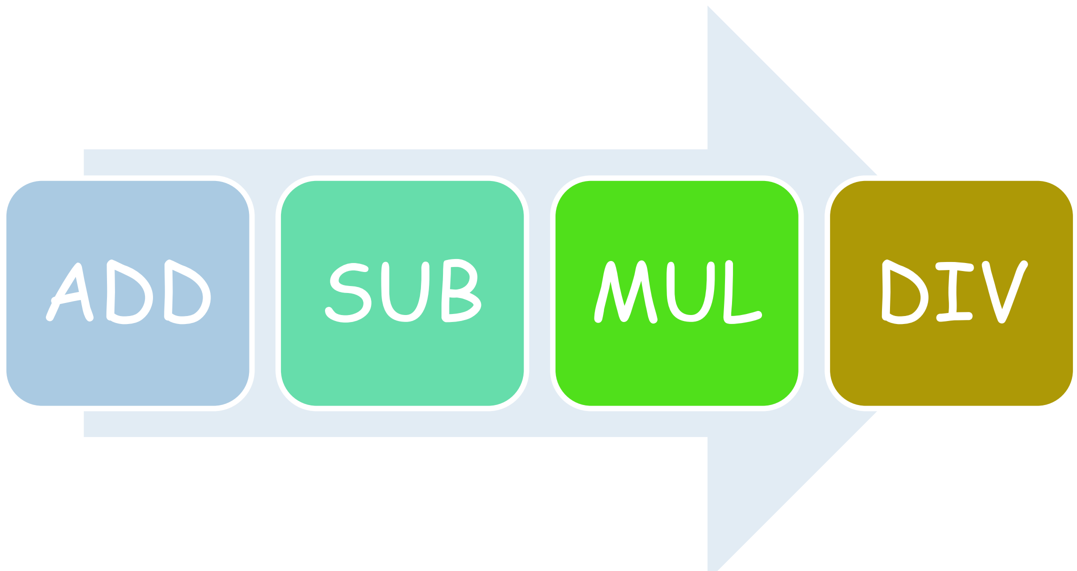
Arithmetic Operations

1000101010010101000100100101010

Decimal and Binary

Four Basic Operations



ALU and the math operations are the heart of the CPU

The computer-CPU (ALU) performs … mathematical operations ?

Main math operation of ALU ?

Main math operation of ALU: Add

- All the other mathematical operations are performed, using addition, and …
- HOW ?

Decimal addition

$$\begin{array}{r} 369 \\ +\ \ 32 \\ \hline 401 \end{array}$$

Binary addition

$$\begin{array}{r} 1 \\ +0 \\ \hline 1 \end{array} = \text{sum}$$

Binary addition

1	0
+0	+1
1 = sum	1 = sum

Binary addition

1	0	1
+0	+1	+1
1 = sum	1 = sum	0 = sum
		1 = carry

Example

```
  10110
+10011
_______

      1
```

Example

0 1 1 0 0 Carry

10110

+10011

101001 Result

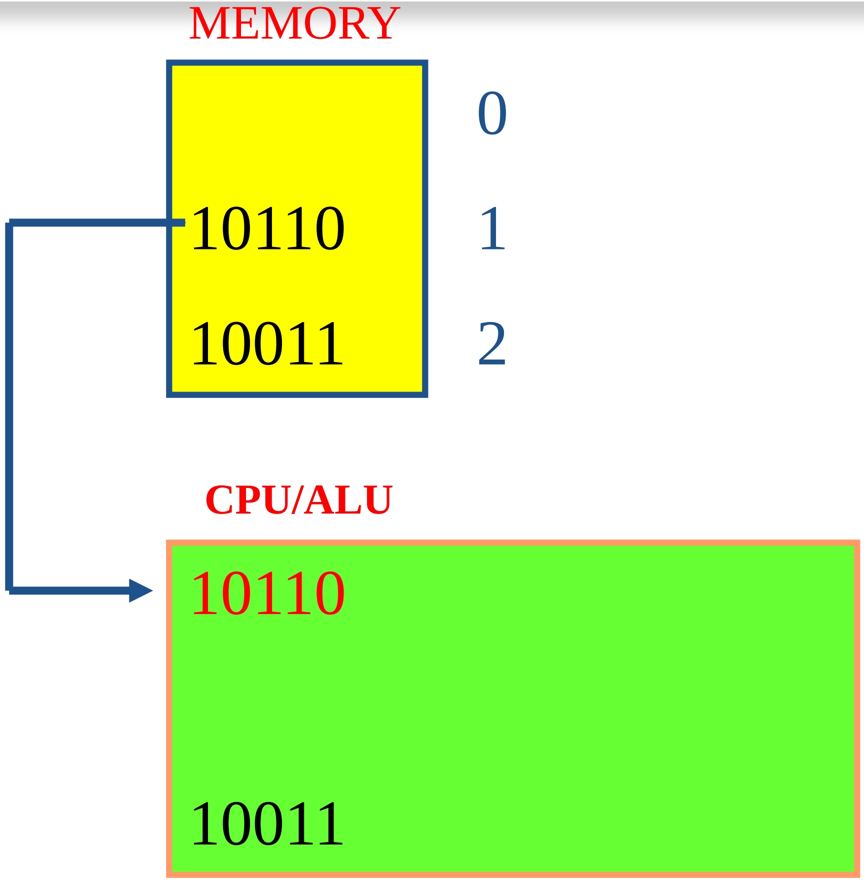
MSBit LSBit

Add two binary numbers

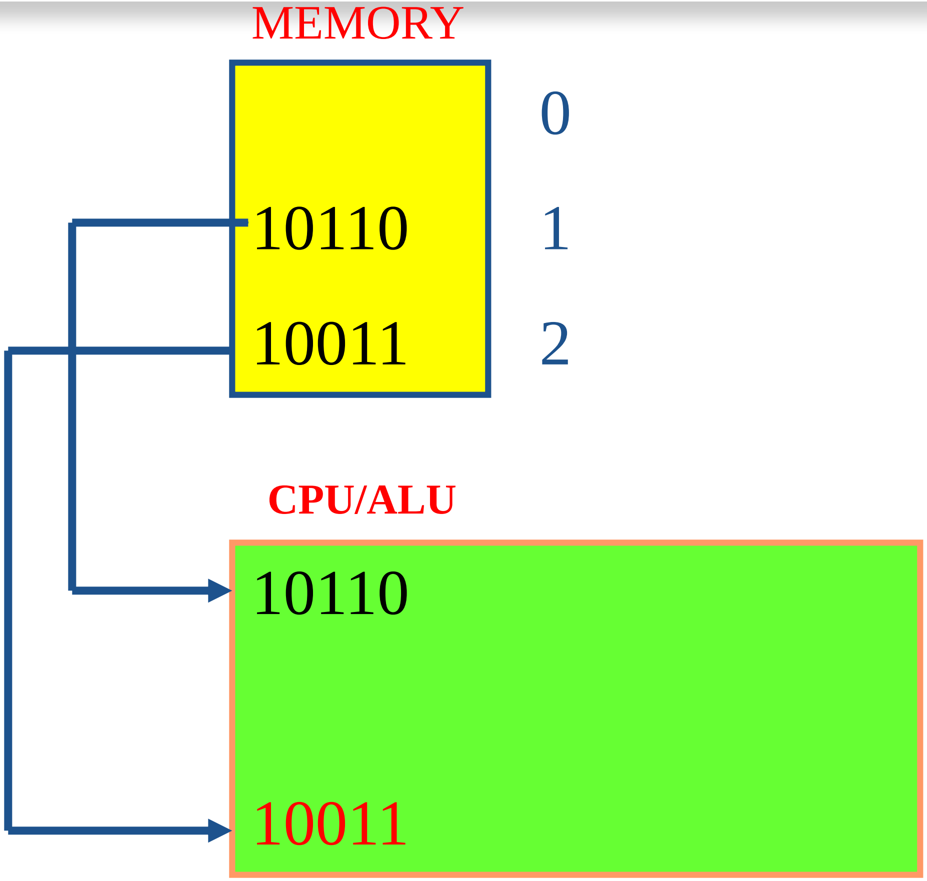
MEMORY

	0
10110	1
10011	2

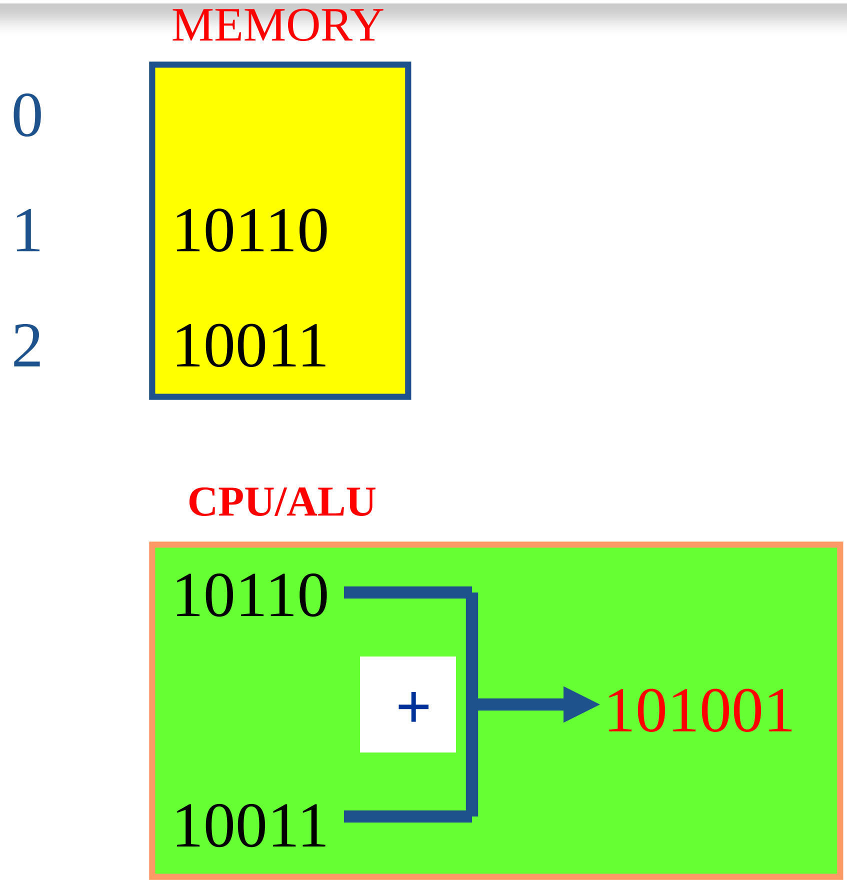
Add two binary numbers



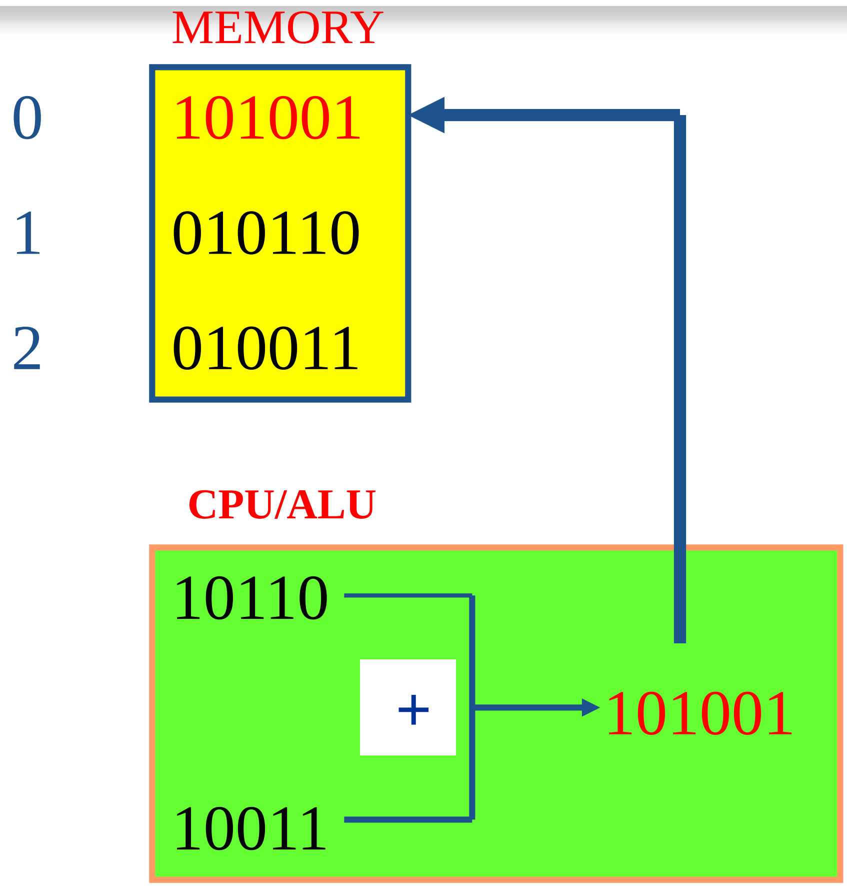
Add two binary numbers



Add two binary numbers



Add two binary numbers



10011, 10110 and 101001 are stored in the CPU/ALU in Registers

Addition using Octal (1)

$$
\begin{array}{r}
1476 \\
+3554 \\
\hline
?
\end{array}
$$

Addition using Octal (1)

- 1476
- +3554
- ―――――

?

6+4 = 10 => (1, 2)

carry sum

1

1476

3554

―――――

2

Addition using Octal (2)

- 1476
- +3554
- ———

1+7+5 = 13 => (1, 5)

carry sum

11

1476

3554

———

52

Addition using Octal (3)

- 1476
- +3554

1+4+5 = 10 => (1, 2)

carry sum

```
 111
1476
3554
----
 252
```

Addition using Octal (4)

- $\ \ 1476$
- $+3554$

$1+1+3 = 5 \Rightarrow (0, 5)$

carry sum

111
1476
3554

5252

Addition using Hex (1)

- 59F
- +E46

Addition using Hex (1)

59F
+E46

F+6 = 15 + 6 = 21 => (1, 5)

carry sum

1
59F
E46

5

Addition using Hex (2)

- 59F
- +E46

1 + 9 + 4 = 14 => (0, E)

carry sum

01
59F
E46

E5

Addition using Hex (3)

- 59F
- +E46

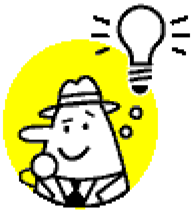
$0 + 5 + E = 19 \Rightarrow (1, 3)$

carry sum

01
59F
E46

13E5

Subtraction



Decimal subtraction

$$\begin{array}{r} 401 \\ -\ \ 32 \\ \hline 369 \end{array}$$

Binary subtraction

$$
\begin{array}{r}
0 \\
-\ 0 \\
\hline
0
\end{array}
$$

Binary subtraction

$$\begin{array}{r} 0 \\ -\ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ -\ 0 \\ \hline 1 \end{array}$$

Binary subtraction

0	1	1
- 0	- 0	- 1
0	1	0

Binary subtraction

0	1	1	0
- 0	- 0	- 1	- 1
0	1	0	1 = result
			1 = borrow

Example

$$
\begin{array}{rcccccl}
 & 1 & 0 & 1 & 1 & 0 & = 2+4+16 = 22 \\
- & 1 & 0 & 0 & 1 & 1 & = 1+2+16 = 19
\end{array}
$$

Example (1)

Since 0-1

	1	0	1	1	0
-	1	0	0	1	1

Example (1a)

Borrow 1 from next (left) column

1

$$
\begin{array}{rccccc}
 & 1 & 0 & 1 & 1 & 0 \\
- & 1 & 0 & 0 & 1 & 1 \\
\hline
\end{array}
$$

Example (1b)

Borrow 1 from next (left) column

$$
\begin{array}{rccccc}
 & & & & & 1 \\
 & 1 & 0 & 1 & 1 & 0 \\
- & 1 & 0 & 0 & 1 & 1 \\
\hline
 & & & & & 1
\end{array}
$$

Therefore 10-1 = 1

Example (1c)

Since we borrowed a 1 a 0 is left

0 1

$$\begin{array}{cccccc} & 1 & 0 & 1 & \cancel{1} & 0 \\ - & 1 & 0 & 0 & 1 & 1 \\ \hline & & & & & 1 \end{array}$$

Example (1d)

Since 0 - 1, borrow 1 from next (left) column

$$
\begin{array}{rccccc}
 & & & & 0 & 1 \\
 & 1 & 0 & 1 & \cancel{1} & 0 \\
- & 1 & 0 & 0 & 1 & 1 \\
\hline
 & & & & & 1
\end{array}
$$

Example (1e)

Borrow 1 from next (left) column

$$\begin{array}{rrrrrr} & & & 1 & 0 & 1 \\ & 1 & 0 & 1 & \cancel{1} & 0 \\ - & 1 & 0 & 0 & 1 & 1 \\ \hline & & & & & 1 \end{array}$$

Example (1f)

$$
\begin{array}{cccccc}
 & & & 1\,0 & 1 & \\
 & 1 & 0 & 1 & \cancel{1} & 0 \\
- & 1 & 0 & 0 & 1 & 1 \\
\hline
 & & & & 1 & 1
\end{array}
$$

Therefore 10-1 = 1

Example (1g)

Since we borrowed a 1 a 0 is left

$$
\begin{array}{rccccc}
 & & & 0 & & \\
 & 1 & 0 & \cancel{1} & 1 & 0 \\
- & 1 & 0 & 0 & 1 & 1 \\
\hline
 & & & & 1 & 1
\end{array}
$$

Example (1h)

Since we borrowed a 1 a 0 is left

$$
\begin{array}{rccccc}
 & & & 0 & & \\
 & 1 & 0 & \cancel{1} & 1 & 0 \\
- & 1 & 0 & 0 & 1 & 1 \\
\hline
 & & & 0 & 1 & 1
\end{array}
$$

Therefore 0-0 = 0

Example (1i)

$$
\begin{array}{rccccc}
 & 1 & 0 & 1 & 1 & 0 \\
- & 1 & 0 & 0 & 1 & 1 \\
\hline
 & & 0 & 0 & 0 & 1
\end{array}
$$

$0 - 0 = 0$

Example (1j)

$$
\begin{array}{r}
1\ 0\ 1\ 1\ 0 \\
-\ 1\ 0\ 0\ 1\ 1 \\
\hline
0\ 0\ 0\ 1\ 1
\end{array}
$$

1-1 = 0

Result: 22-19 = 3

	1	0	1	1	0
-	1	0	0	1	1
	0	0	0	1	1

Multiplication

- Shift + Add operation

Binary multiplication

101	
X 100	
000	100 add zeros
000	100 shift-add zeros
+101	100 shift-copy number
10100	Add

Binary multiplication

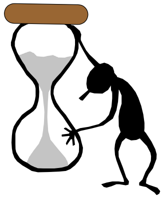
We Need:

- An Adder
- A Shifter

Binary multiplication

We Need:

- An Adder
- A Shifter

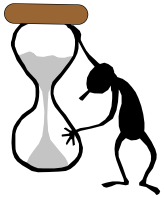


We'll … design Adders …

END-1

Please …

Read the next lecture, about complements, before you come to class.



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