CMPT285 Homework 6 (due Tuesday, April 1)

1. (Problem 7 on page 91 from Rosen) Use a direct proof to show that every odd integer is the difference between two squares.

2. (Problem 13 on page 91 from Rosen) Prove that if x is irrational, then $1/x$ is irrational.

3. (Problem 3 on page 329 from Rosen) Let $P(n)$ be the statement that $1^2+2^2+\cdots+n^2 = n(n+1)(2n+1)/6$ for the positive integer n.

   - What is the statement $P(1)$?
   - Show that $P(1)$ is true, completing the basis step of the proof.
   - What is the inductive hypothesis?
   - What do you need to prove in the inductive step?
   - Complete the inductive step, identifying where you use the inductive hypothesis.
   - Explain why these steps show that this formula is true whenever n is a positive integer.

4. (Problem 5 on page 329 from Rosen) Prove that $1^2 + 3^2 + 5^2 \cdots + (2n+1)^2 = (n+1)(2n+1)(2n+3/3$ whenever n is a positive integer

5. (Problem 19 on page 330 from Rosen) Let $P(n)$ be the statement that

$$1 + 1/4 + 1/9 + \cdots + 1/n^2 < 2 - 1/n,$$

   where n is an integer greater than 1.

   - What is the statement $P(2)$?
   - Show that $P(2)$ is true, completing the basis step of the proof.
   - What is the inductive hypothesis?
   - What do you need to prove in the inductive step?
   - Complete the inductive step, identifying where you use the inductive hypothesis.
   - Explain why these steps show that this formula is true whenever n is an integer greater than 1.

6. (Problem 3 on page 341 from Rosen) Let $P(n)$ be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 8$.

   - Show that the statements $P(8)$, $P(9)$, and $P(10)$ are true, completing the basis step of the proof.
   - What is the inductive hypothesis of the proof?

- What do you need to prove in the inductive step?
- Complete the inductive step for $k \geq 10$.
- Explain why these steps show that this formula is true whenever $n \geq 8$.

7. (Problem 5 on page 341 from Rosen) Let $P(n)$ be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 8$.

   (a) Determine which amounts of postage can be formed using just 4-cent and 11-cent stamps.

   (b) Prove your answer to (a) using the principle of mathematical induction. Be sure to state explicitly your inductive hypothesis in the inductive step.

   (c) Prove your answer to (a) using strong induction. How does the inductive hypothesis in this proof differ from that in the inductive hypothesis for a proof using mathematical induction?