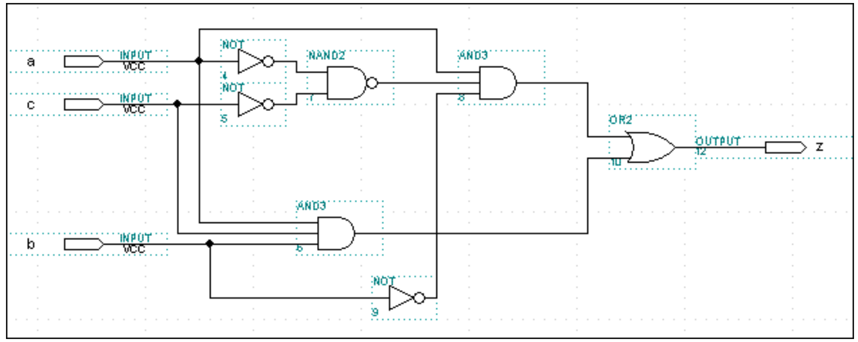
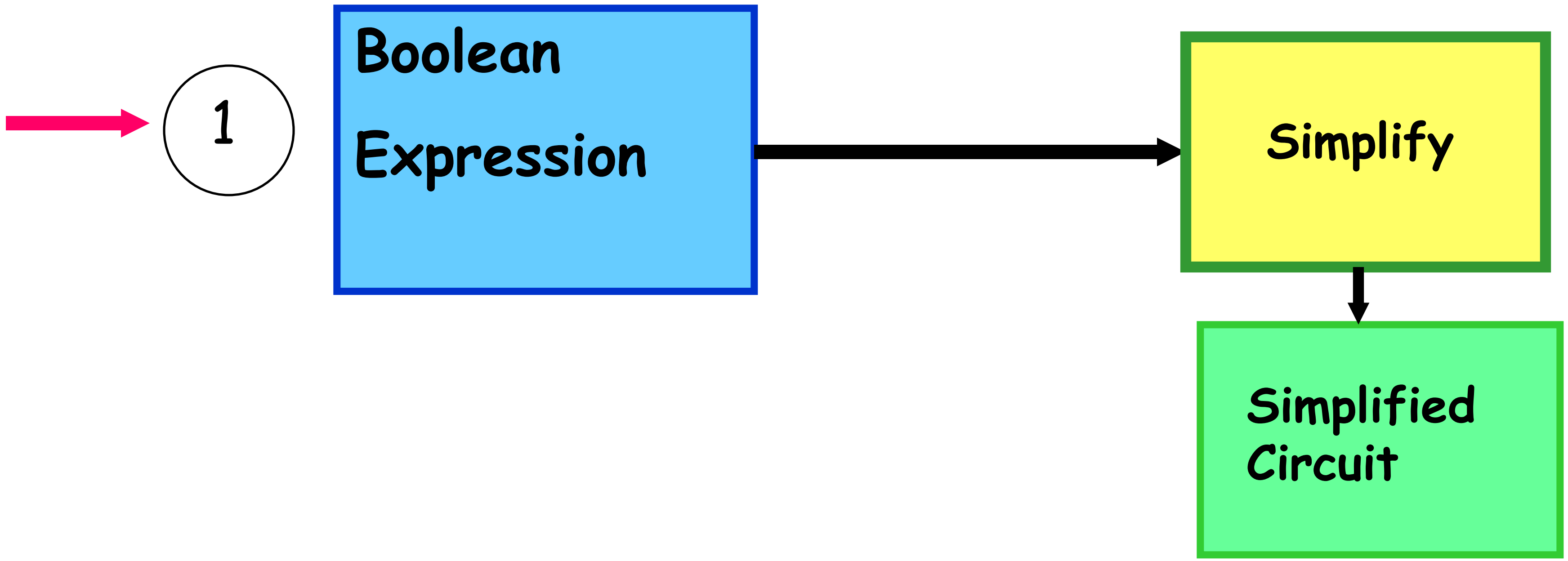


Boolean Simplification

CMPT2?0

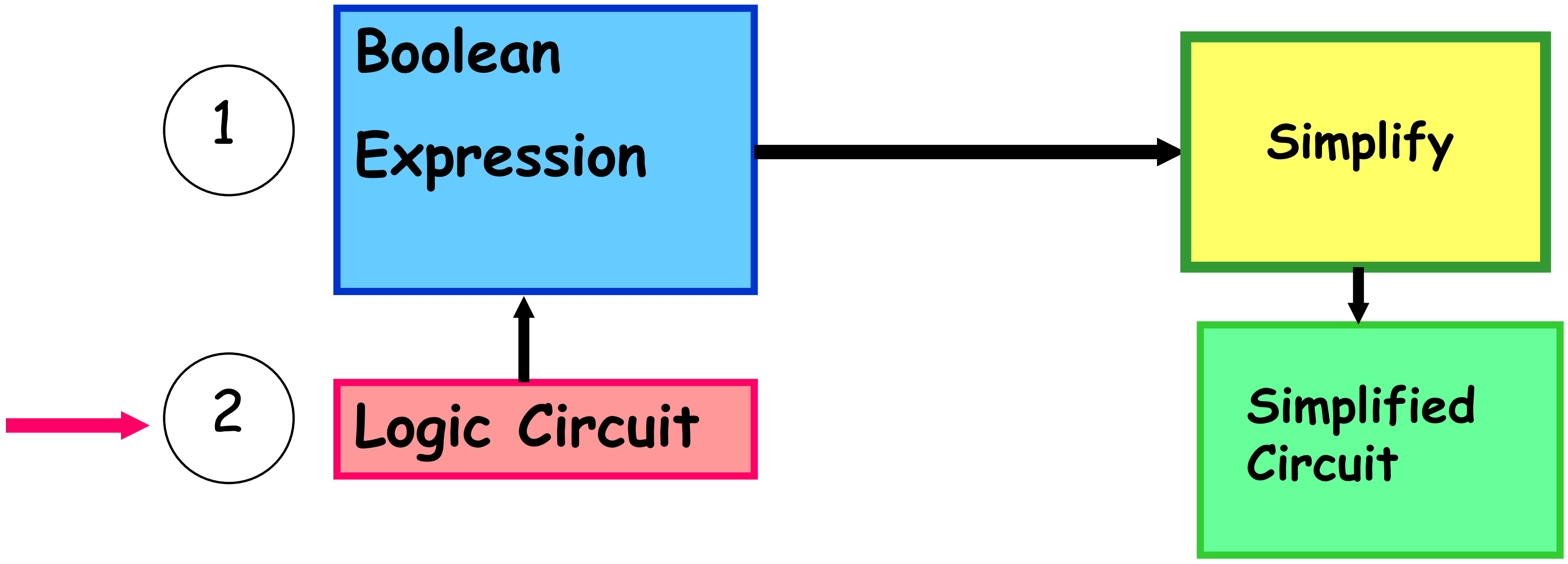
Simplification



1. Boolean expression simplification

- Put the Boolean expression into sum of-products (SOP) form
- Apply the Boolean simplification rules

Derive the Boolean expression



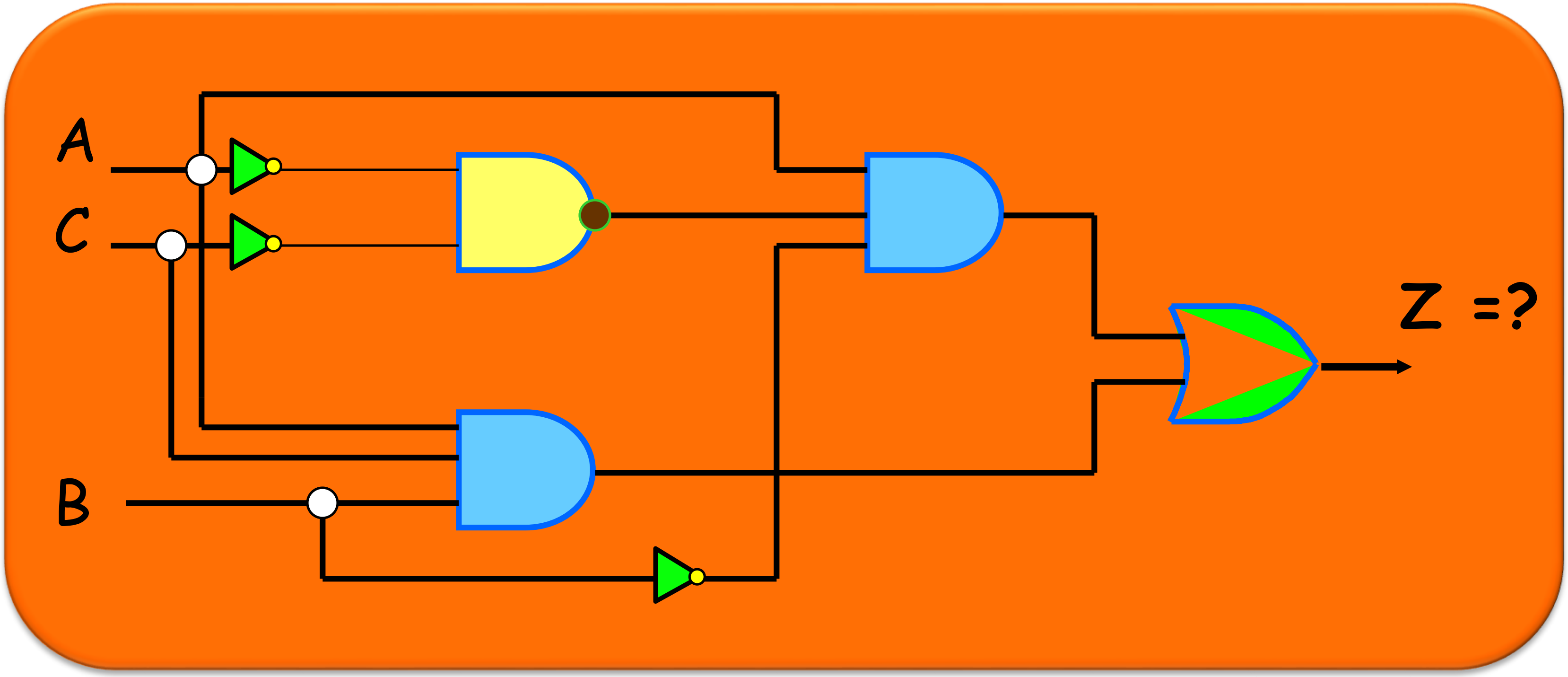
2. Logic circuit simplification

- Derive the output Boolean expression
- Apply the Boolean simplification rules

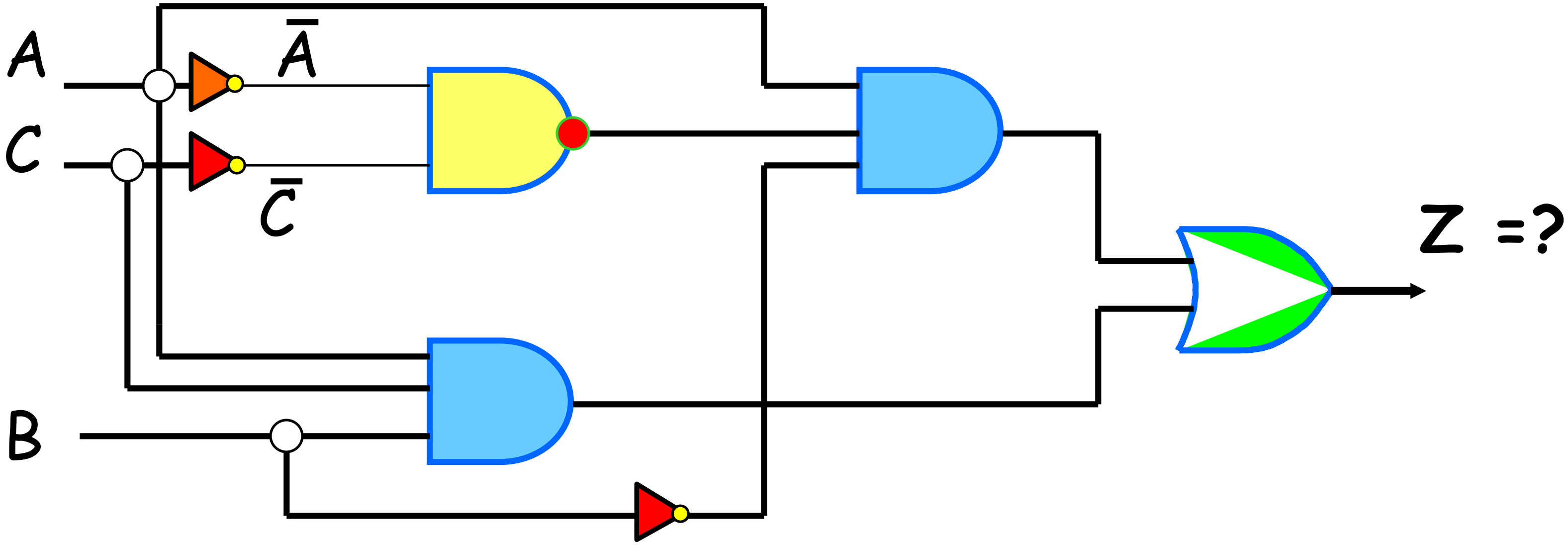
Example

Logic circuit; Derive Z

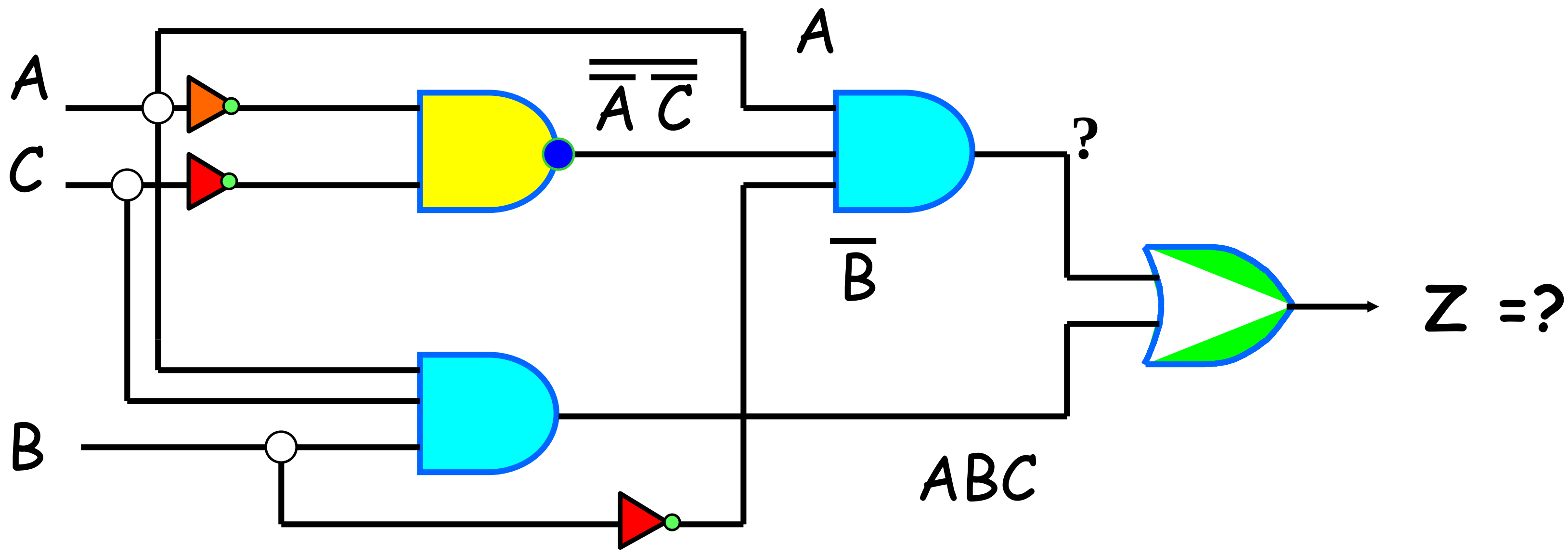
Example



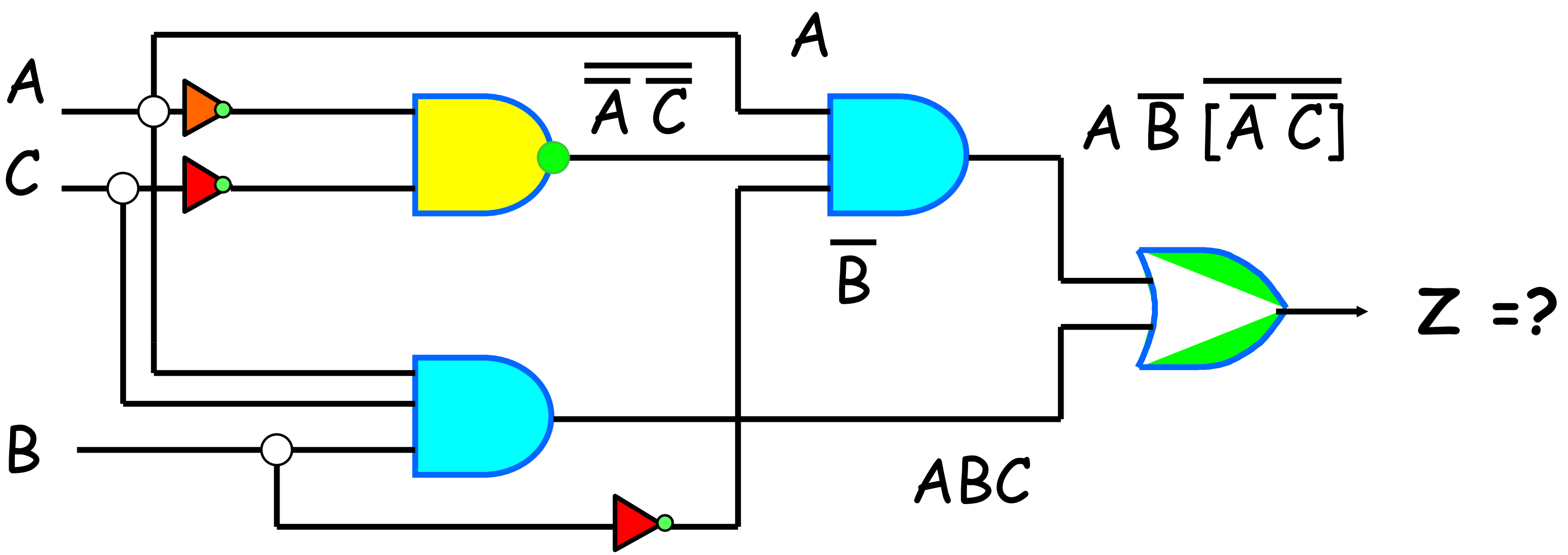
Derive Z



Derive Z

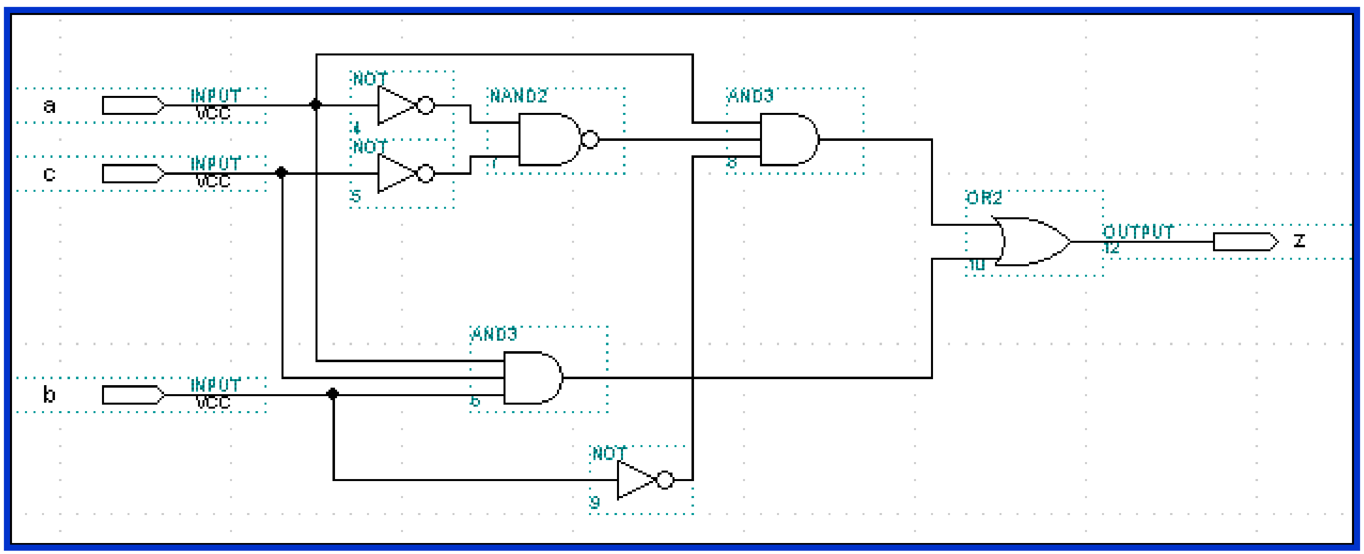


Finally the output expression is …



Output expression

$$Z = A\,B\,C + A\,\bar{B}\,[\overline{\bar{A}\;\bar{C}}]$$



Simplify the Boolean expression

$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\overline{C}}\,]$$

DeMorgan

$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\;\overline{C}}\,]$$

$$= ABC + A\overline{B}[\overline{\overline{A}} + \overline{\overline{C}}]$$

Sum-Of-Products (SOP) form

$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\;\overline{C}}\,]$$

$$= ABC + A\overline{B}[\overline{\overline{A}} + \overline{\overline{C}}]$$

$$= ABC + A\overline{B}A + A\overline{B}C$$

Factor-out: AC

$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\;\overline{C}}\,]$$

$$= ABC + A\overline{B}[\overline{\overline{A}} + \overline{\overline{C}}]$$

$$= ABC + A\overline{B}A + A\overline{B}C$$

$$= AC[B+\overline{B}] + A\overline{B}A$$

Simplified logic expression

$$Z = A\,B\,C + A\,\overline{B}\,[\,\overline{\overline{A}\;\overline{C}}\,]$$

$$= ABC + A\overline{B}[\overline{\overline{A}} + \overline{\overline{C}}]$$

$$= ABC + A\overline{B}A + A\overline{B}C$$

$$= AC[B+\overline{B}] + A\overline{B}A$$

$$= AC + A\overline{B}$$

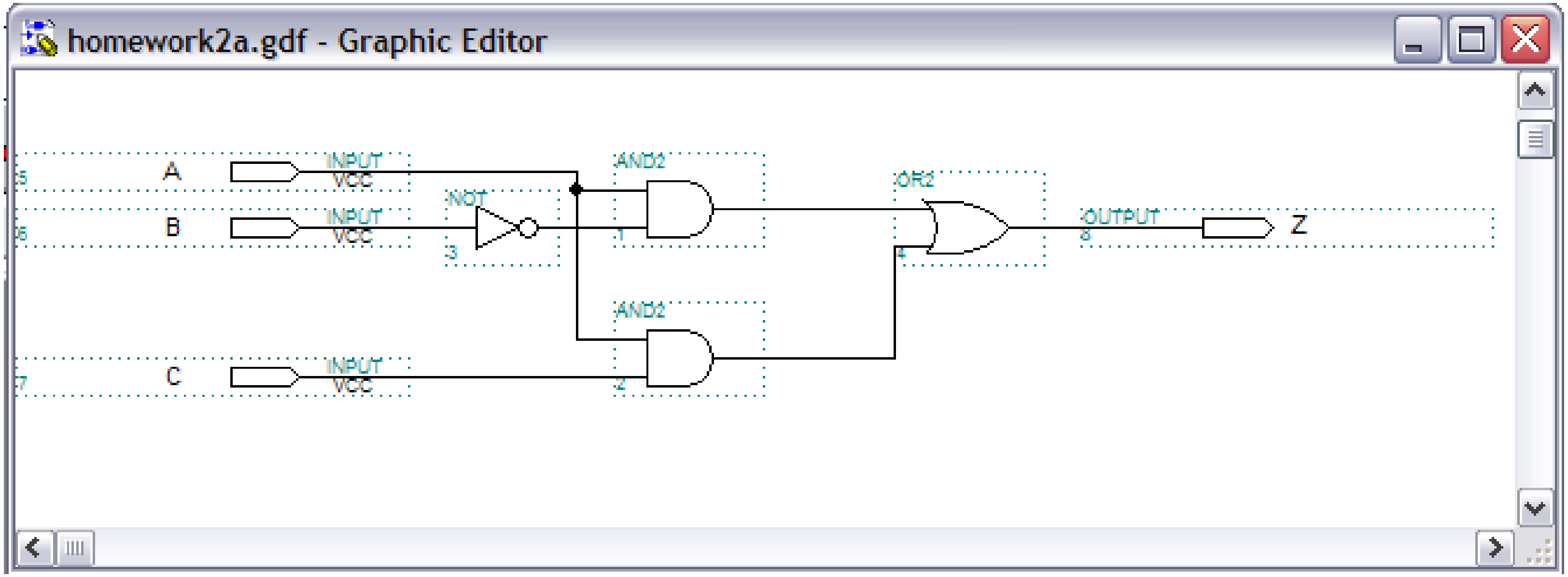
Implement (gates) the expression

$Z = AC + A\overline{B}$

2 Minutes …

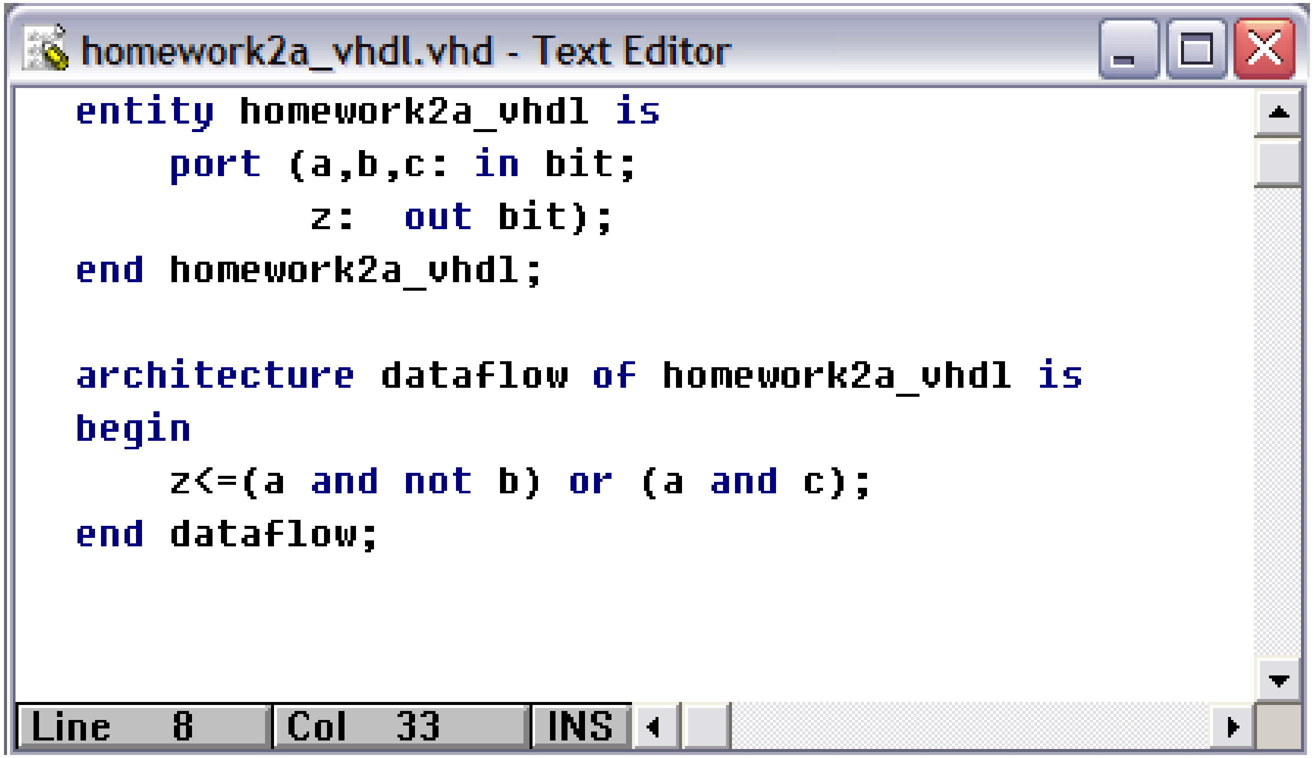
Simplified logic circuit (gates)

$$Z = AC + A\overline{B}$$



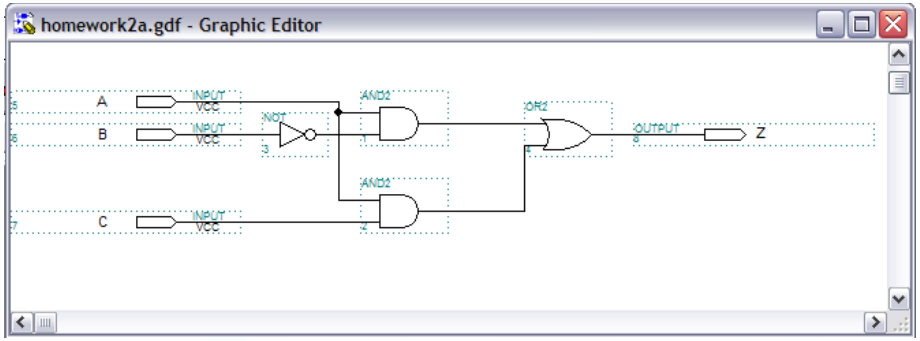
Two-level implementation

VHDL code for: $z = AC + A\overline{B}$

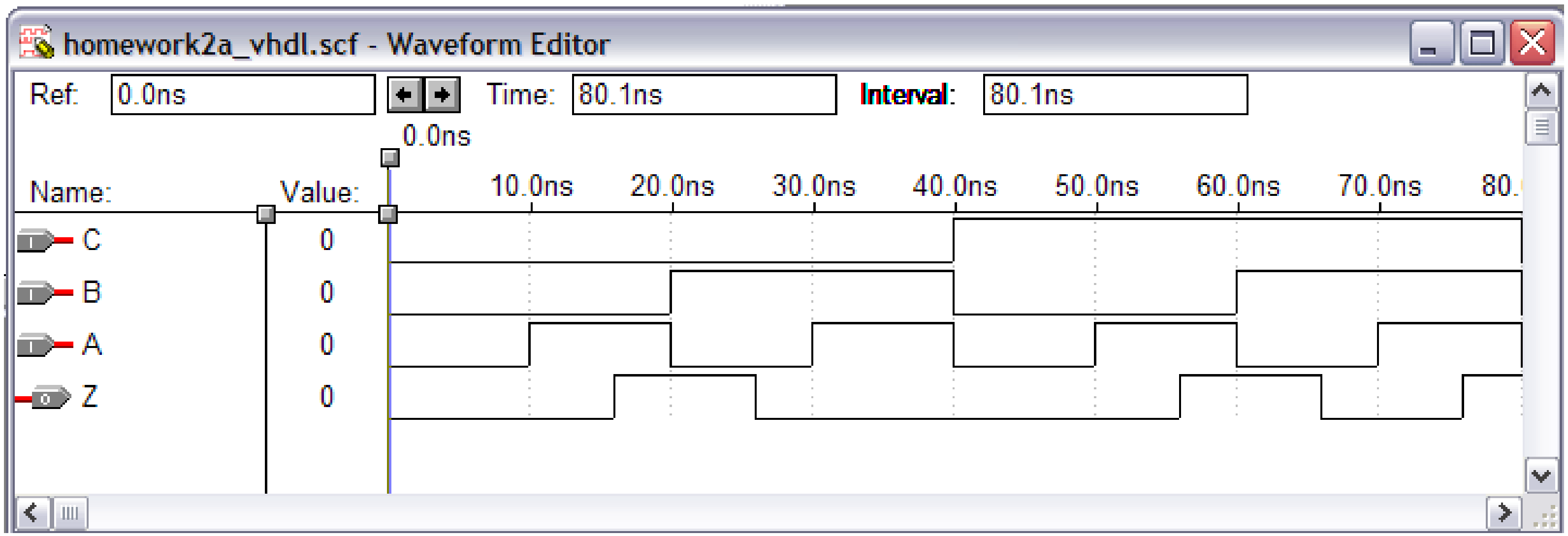



```vhdl
entity homework2a_vhdl is
    port (a,b,c: in bit;
          z:  out bit);
end homework2a_vhdl;

architecture dataflow of homework2a_vhdl is
begin
    z<=(a and not b) or (a and c);
end dataflow;
```



Simulation

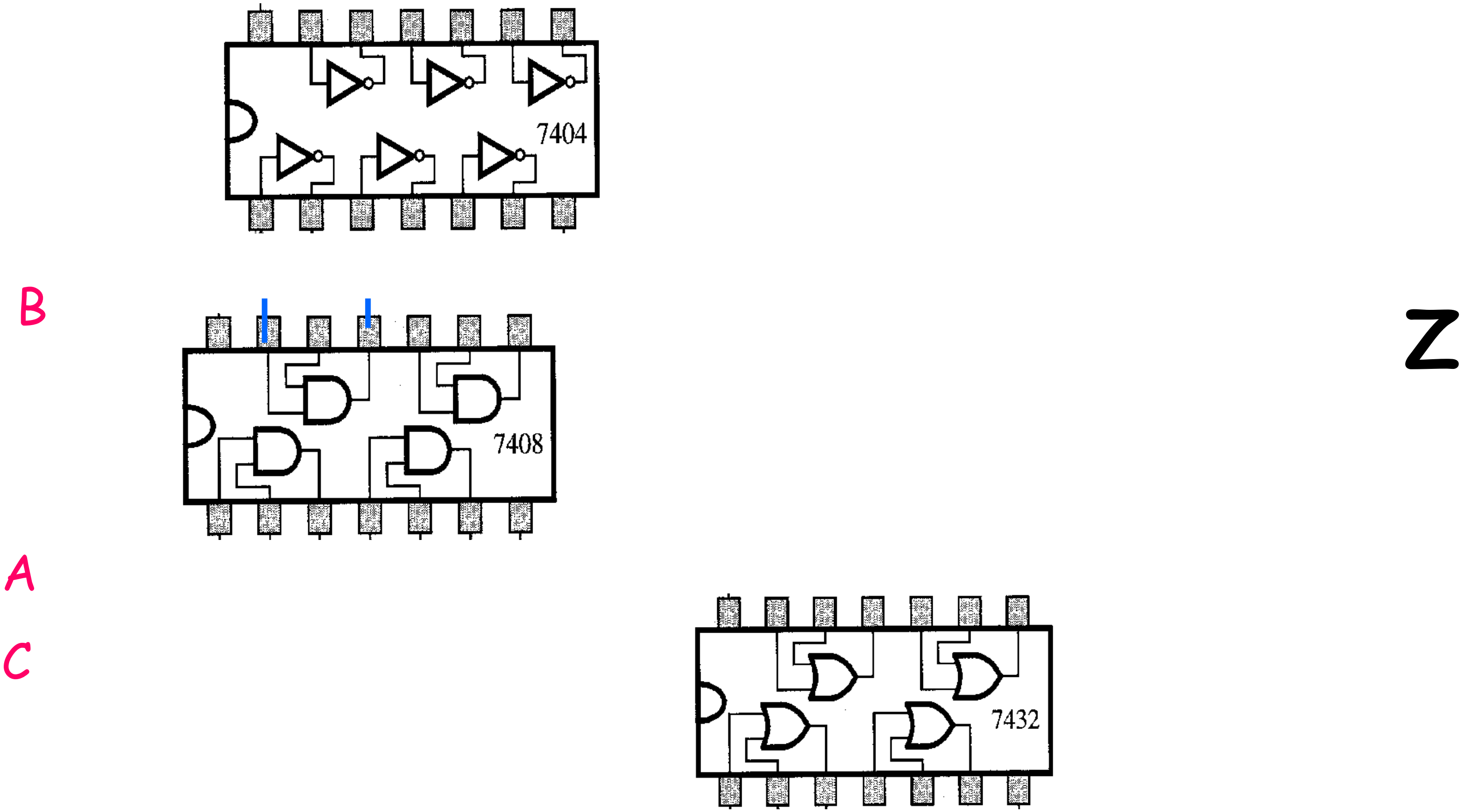


homework2a_vhdl.scf - Waveform Editor
Ref: 0.0ns
Time: 80.1ns
Interval: 80.1ns
0.0ns
Name:
Value:
10.0ns
20.0ns
30.0ns
40.0ns
50.0ns
60.0ns
70.0ns
80.
C
B
A
Z
0
0
0
0

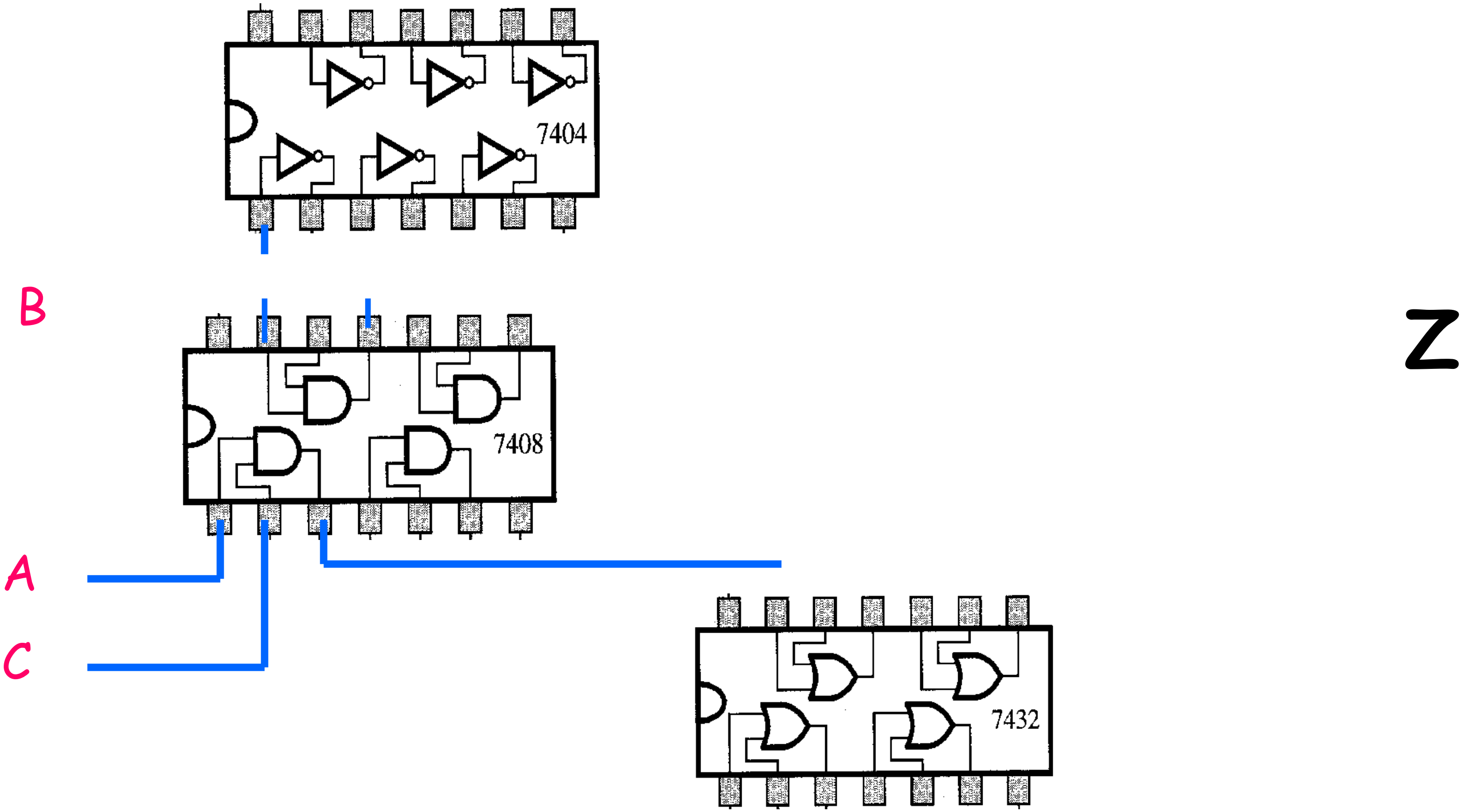
Implementation: $AC+A\overline{B} = Z$

Use the available Integrated Circuits

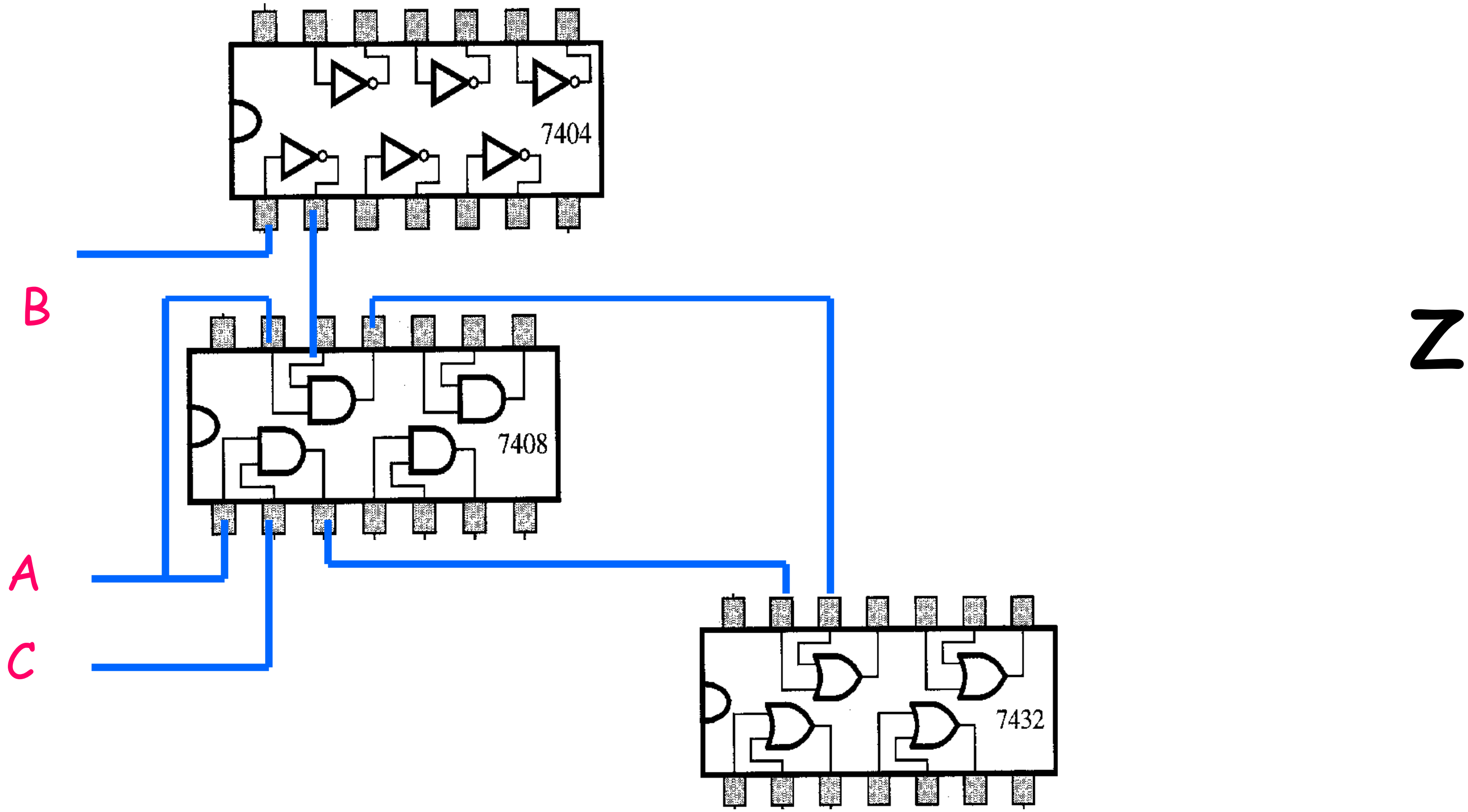
Chip Implementation for: $AC+A\bar{B} = Z$



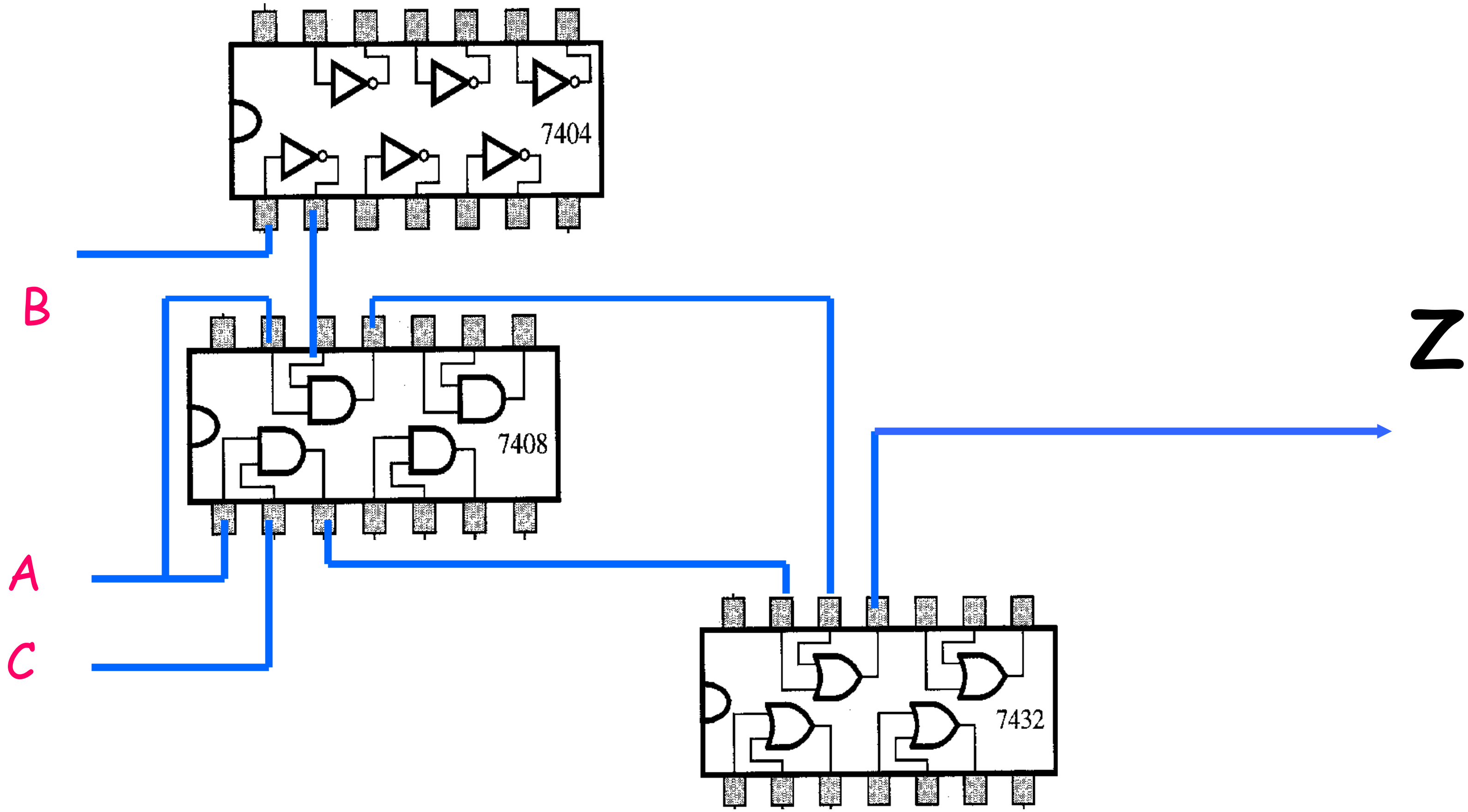
Chip Implementation for: AC+A$\overline{B}$ = Z



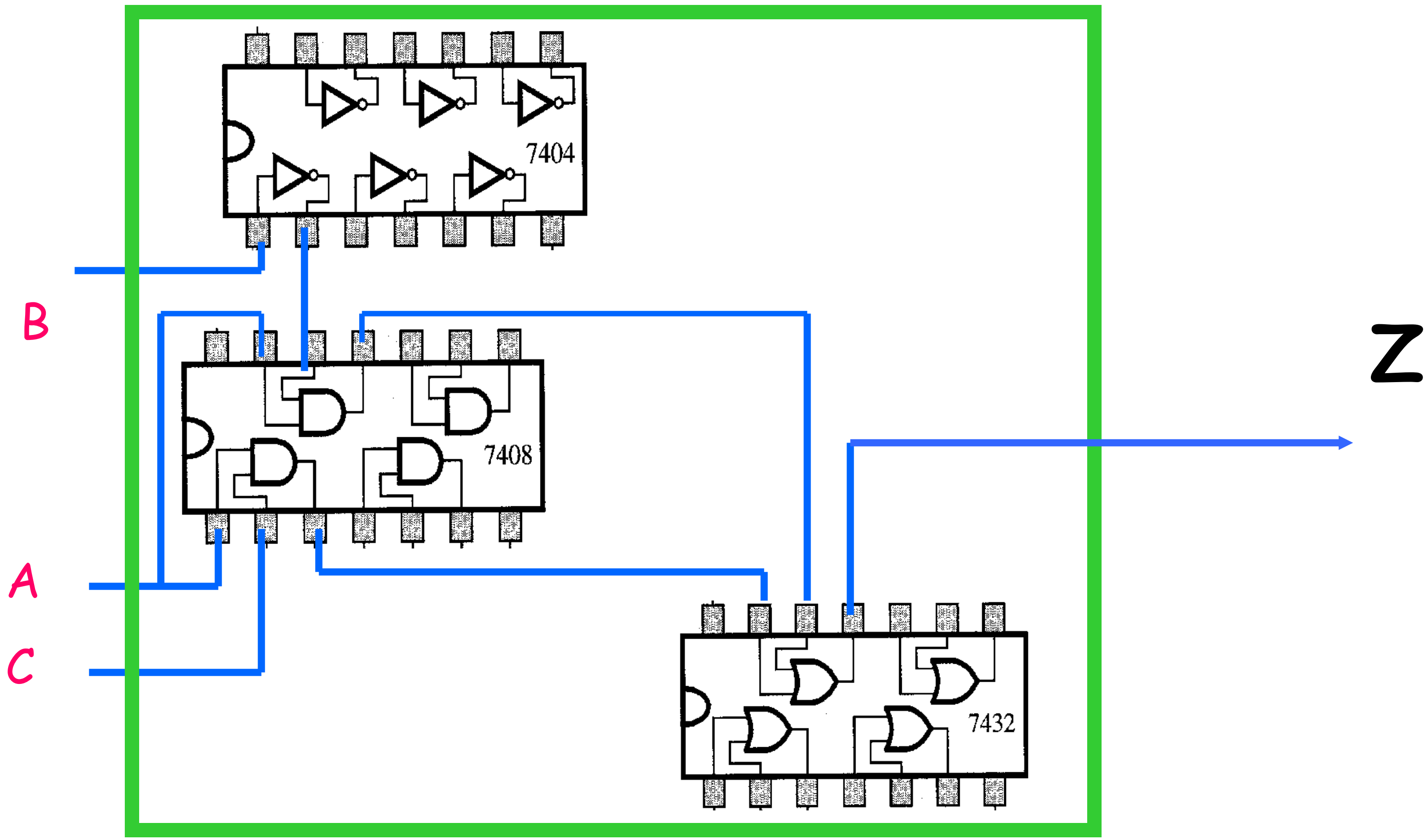
Chip Implementation for: $AC+A\overline{B} = Z$



Chip Implementation for: $AC+A\overline{B} = Z$



The board … $A C + A\overline{B} = Z$



Simplify the expression: $ABC+AB\overline{C}+A\overline{B}C$

New Example

Simplify the expression: $ABC+AB\overline{C}+A\overline{B}C$

$$= AB(C + \overline{C}) + A\overline{B}C$$

Simplify the expression: $ABC+AB\overline{C}+A\overline{B}C$

$$= AB(C+\overline{C}) + A\overline{B}C$$

$$= AB + A\overline{B}C$$

Simplify the expression: $ABC+AB\overline{C}+A\overline{B}C$

$$= AB(C + \overline{C}) + A\overline{B}C$$

$$= AB + A\overline{B}C$$

$$= A(B + \overline{B}C)$$

Result

$$= AB(C + \overline{C}) + A\overline{B}C$$

$$= AB + A\overline{B}C$$

$$= A(B + \overline{B}C)$$

$$= A(B + C)$$

Simplify the expression: $ABC+AB\overline{C}+A\overline{B}C$

$= AB(C + \overline{C}) + A\overline{B}C$

$= AB + A\overline{B}C$

$= A(B + \overline{B}C)$

$= A(B + C)$

Another proof ?

Simplify the expression: $ABC+AB\overline{C}+A\overline{B}C$

$= AB(C + \overline{C}) + A\overline{B}C$

$= AB + A\overline{B}C$

$= A(B + \overline{B}C)$

$= A(B + C)$

Another proof:

Because: $A + A = A$

$= ABC + ABC + AB\overline{C} + A\overline{B}C$

$=$

Simplify the expression: $ABC+AB\overline{C}+A\overline{B}C$

$= AB(C + \overline{C}) + A\overline{B}C$

$= AB + A\overline{B}C$

$= A(B + \overline{B}C)$

$= A(B + C)$

Another proof:

Because: $A + A = A$

$= ABC + ABC + AB\overline{C} + A\overline{B}C$

$= AB(C + \overline{C}) + AC(B + \overline{B})$

$= AB + AC \quad = \quad A(B + C)$

Simplify the expression

$$\bar{A}\,C\,[\,\overline{\bar{A}\,B\,D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

New Example

Simplify the expression

$$\bar{A}\,C\,[\,\overline{\bar{A}\,B\,D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,[\,\bar{\bar{A}} + \bar{B} + \bar{D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

Simplify the expression

$$\bar{A}\,C\,[\,\overline{\bar{A}\,B\,D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,[\,A + \bar{B} + \bar{D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,A + \bar{A}\,C\,\bar{B} + \bar{A}\,C\,\bar{D} + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

Simplify the expression

$$\bar{A}\,C\,[\,\overline{\bar{A}\,B\,D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,[\,A + \bar{B} + \bar{D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,A + \bar{A}\,C\,\bar{B} + \bar{A}\,C\,\bar{D} + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$(A\,\bar{A} = 0)$,

Simplify the expression

$$\bar{A}\, C\, [\,\overline{\bar{A}\, B\, D}\,] + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

$$= \bar{A}\, C\, [\,A + \bar{B} + \bar{D}\,] + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

$$= \bar{A}\, C\, A + \bar{A}\, C\, \bar{B} + \bar{A}\, C\, \bar{D} + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

($A\,\bar{A} = 0$), take 2nd with 5th and 3rd with 4th terms.

Simplify the expression

$$\bar{A}\,C\,[\,\overline{\bar{A}\,B\,D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,[\,A + \bar{B} + \bar{D}\,] + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$$= \bar{A}\,C\,A + \bar{A}\,C\,\bar{B} + \bar{A}\,C\,\bar{D} + \bar{A}\,B\,\bar{C}\,\bar{D} + A\,\bar{B}\,C$$

$(A\,\bar{A} = 0)$, take 2nd with 5th and 3rd with 4th terms.

$$= \bar{B}\,C\,[\,A + \bar{A}\,] + \bar{A}\,\bar{D}\,[\,C + B\,\bar{C}\,]$$

Simplify the expression

$$\bar{A}\, C\, [\, \overline{\bar{A}\, B\, D}\,] + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

$$= \bar{A}\, C\, [\, A + \bar{B} + \bar{D}\,] + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

$$= \bar{A}\, C\, A + \bar{A}\, C\, \bar{B} + \bar{A}\, C\, \bar{D} + \bar{A}\, B\, \bar{C}\, \bar{D} + A\, \bar{B}\, C$$

$(A\, \bar{A} = 0)$, take 2nd with 5th and 3rd with 4th terms.

$$= \bar{B}\, C\, [\, A + \bar{A}\,] + \bar{A}\, \bar{D}\, [\, C + B\, \bar{C}\,]$$

$$= \bar{B}\, C + \bar{A}\, \bar{D}\, [\, C + B\,]$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

New Example

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\ \overline{A}\,A + \overline{A}\,B + \overline{A}\,D + B\,A + B\,B + B\,D\ ]\,\overline{D}$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD]\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + BB\overline{D} + BD\overline{D}$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\,\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD\,]\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + BB\overline{D} + BD\overline{D}$$

$$= 0 + \overline{A}B\overline{D} + 0 + BA\overline{D} + B\overline{D} + 0$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\,\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD\,]\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + BB\overline{D} + BD\overline{D}$$

$$= 0 + \overline{A}B\overline{D} + 0 + BA\overline{D} + B\overline{D} + 0$$

$$= B\overline{D}\,[\,\overline{A} + A + 1\,]$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD]\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + BB\overline{D} + BD\overline{D}$$

$$= 0 + \overline{A}B\overline{D} + 0 + BA\overline{D} + B\overline{D} + 0$$

$$= B\overline{D}[\overline{A} + A + 1]$$

$$= B\overline{D}[1 + 1]$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD]\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + BB\overline{D} + BD\overline{D}$$

$$= 0 + \overline{A}B\overline{D} + 0 + BA\overline{D} + B\overline{D} + 0$$

$$= B\overline{D}[\overline{A} + A + 1]$$

$$= B\overline{D}[1 + 1]$$

$$= B\overline{D}(1) = B\overline{D}$$

Another example

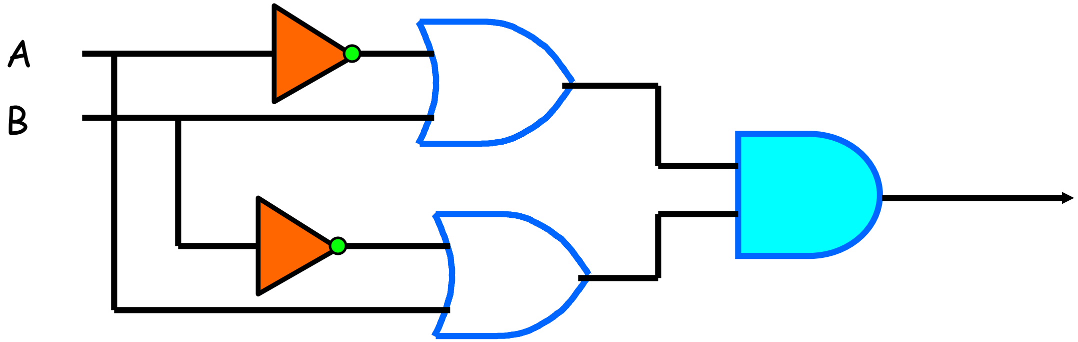
Given: $(\bar{A} + B)(\bar{B} + A)$

1. Implement using basic (AND, OR, NOT) gates
2. Simplify
3. Implement the simplified expression

New Example

1. First implementation

$$(\bar{A} + B)(\bar{B} + A)$$



2. Simplify

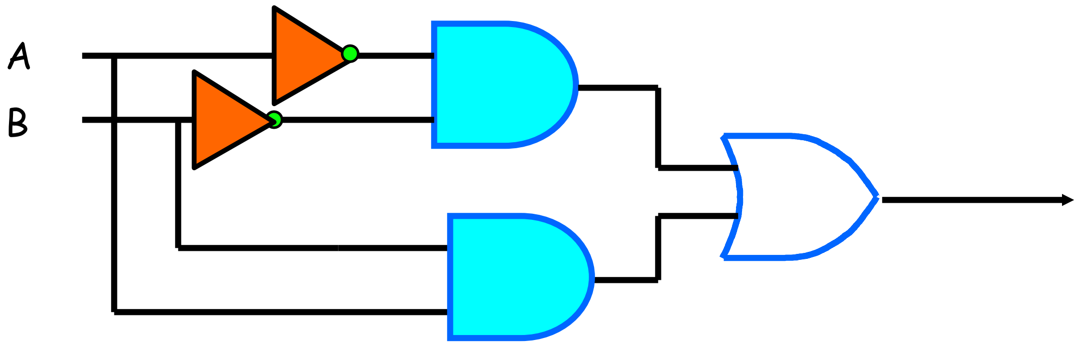
$$(\bar{A} + B)(\bar{B} + A) = \bar{A}\,\bar{B} + \bar{A}\,A + \bar{B}\,B + B\,A$$

$$= \bar{A}\,\bar{B} + A\,B$$

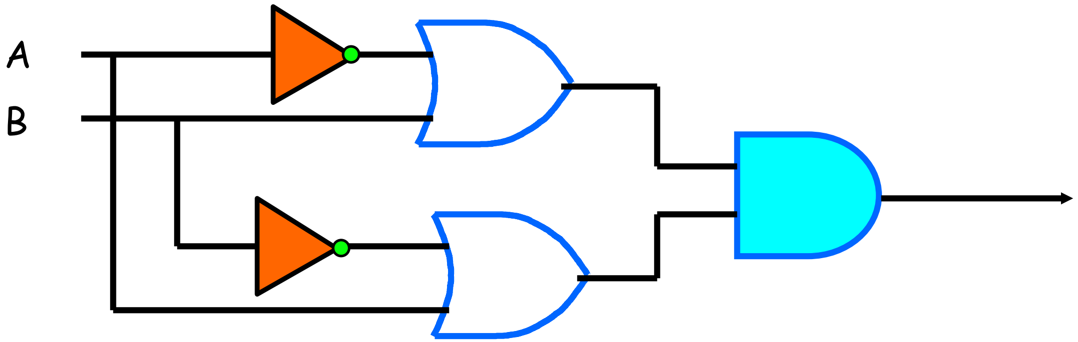
3. Second implementation

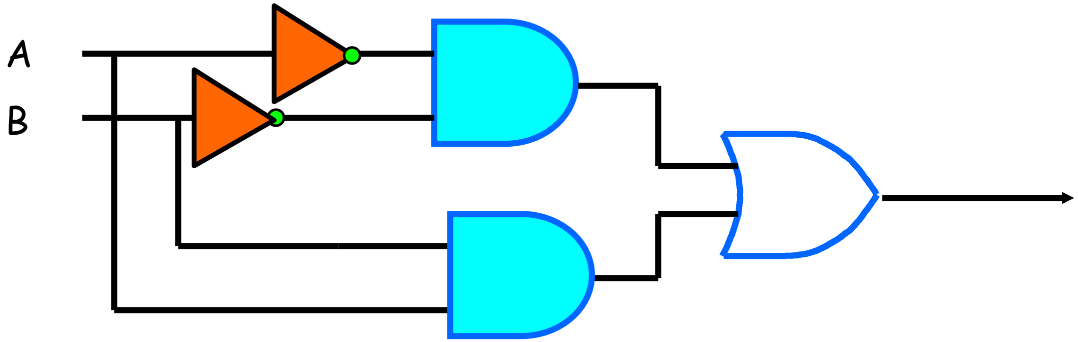
$$(\bar{A} + B)(\bar{B} + A) = \bar{A}\,\bar{B} + \bar{A}A + \bar{B}B + BA$$

$$= \bar{A}\,\bar{B} + AB$$



Both implementations are equivalent





Simplify the expression

$$Z = A\overline{C}\,\overline{D} + \overline{A}BD + A\overline{B}CD$$

New Example

$Z = Z$

$$Z = A\overline{C}\,\overline{D} + \overline{A}BD + A\overline{B}CD$$

Can not be simplified

Therefore …

- *Three cases* …