Discrete Mathematics and Its Applications

Introductory Lecture

Conditional Propositions

The implication $p \rightarrow q$ is true unless p is true and q is false. This is shown in the following truth table. In the implication $p \rightarrow q$, p is called the hypothesis and q the conclusion.

p	q	$p \rightarrow q$	$\neg q \rightarrow \neg p$	$q \rightarrow p$	$\neg p \rightarrow \neg q$	$p \leftrightarrow q$
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	T	F	F	F
F	F	T	T	T	T	T

Note that the implication $p \rightarrow q$ is logically equivalent to its contrapositive $\neg q \rightarrow \neg p$, because both are true unless p is true and q is false. On the other hand, $p \rightarrow q$ is not logically equivalent to either its converse $q \rightarrow p$ or its inverse $\neg p \rightarrow \neg q$. However, the converse and inverse are equivalent to each other.

(In English, when a statement of the form "if p then q" is made, its converse is also at times implied. You need to judge whether or not this is the case from the context in which the statement is made. However, in a mathematical context, the conditional statement $p \rightarrow q$ does not imply its converse $q \rightarrow p$.)

Conditional propositions are often formed in English using various words and phrases: "p if q" ($q \rightarrow p$), "p only if q" ($p \rightarrow q$ or, equivalently, $\neg q \rightarrow \neg p$), "$p$ unless q" ($\neg q \rightarrow p$), "p whenever q" ($q \rightarrow p$), "p is sufficient for q" ($p \rightarrow q$), "p is necessary for q" ($q \rightarrow p$).

The biconditional $p \leftrightarrow q$ (p if and only if q) is equivalent to $p \rightarrow q$ and $q \rightarrow p$. It is true exactly when the two variables have the same value.

Conditional Propositions

The implication $q \to \neg p$ is true for all possible assignments of truth values to p and q except for which assignment?

A) p true, q true.

B) p true, q false.

C) p false, q true.

D) p false, q false.

Answer: A

Conditional Propositions

Which of the following positions is correct?

A) The inverse of the implication $p \to q$ is logically equivalent to $p \to q$.

B) The converse of the implication $p \to q$ is logically equivalent to the inverse of $p \to q$.

C) The contrapositive of the implication $p \to q$ is logically equivalent to the inverse of $p \to q$.

D) The converse of the implication $p \to q$ is logically equivalent to the contrapositive of $p \to q$.

Answer: B

Negation of Propositions

Suppose p represents a compound proposition. The negation of p is written $\neg p$. The truth tables for p and $\neg p$ have "opposite" truth values. That is, p is true if and only if $\neg p$ is false. The propositions p and $\neg p$ are never simultaneously true and never simultaneously false.

De Morgan's laws give rules for negating conjunctions and disjunctions:

$$\neg(p \vee q) \equiv \neg p \wedge \neg q \text{ and } \neg(p \wedge q) \equiv \neg p \vee \neg q .$$

If x, y, and z are real numbers, $x < y < z$ means that x is less than y and y is less than z. Which of the following is the negation of $x < y < z$?

A) $x \geq y$ or $y \geq z$.

B) $x \geq y \geq z$.

C) $x > y > z$.

D) $x > z$.

Answer: A

Negation of Propositions

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De Morgan's laws give rules for negating conjunctions and disjunctions:

$$\neg(p \vee q) \equiv \neg p \wedge \neg q \text{ and } \neg(p \wedge q) \equiv \neg p \vee \neg q .$$

Assume that x, y and z are real numbers. Which of the following is the negation of the statement "x is positive and y and z are negative"?

A) $x \leq 0$ or $y \geq 0$ or $z \geq 0$.

B) x is negative and y and z are positive.

C) $x \leq 0$ and $y \geq 0$ or $z \geq 0$.

D) $x \leq 0$ or $y \geq 0$ and $z \geq 0$.

Answer: A

1. Prove or disprove that $(p \to q) \to r$ and $p \to (q \to r)$ are equivalent.

Suppose that p is false, q is true, and r is false. Then $(p \to q) \to r$ is false since its premise $p \to q$ is true while its conclusion r is false. On the other hand, $p \to (q \to r)$ is true in this situation since its premise p is false. Therefore $(p \to q) \to r$ and $p \to (q \to r)$ are not equivalent.

2. Let $P(m,n)$ be "n is greater than or equal to m" where the domain (universe of discourse) is the set of nonnegative integers. What are the truth values of $\exists n \forall m\, P(m,n)$ and $\forall m \exists n\, P(m,n)$?

For every positive integer n there is an integer m such that $n < m$ (take $m = n+1$ for instance). Hence $\exists n \forall m\, P(m,n)$ is false. For every integer m there is an integer n such that $n \geq m$ (take $n = m+1$ for instance). Hence $\forall m \exists n\, P(m,n)$ is true.

Negation of Quantified Statements

Suppose p represents a quantified statement. The negation of p is written $\neg p$. The quantified statement p is true if and only if $\neg p$ is false; p and $\neg p$ are never simultaneously true and never simultaneously false.
The negation of a statement with predicates and quantifiers is carried out as follows:

$$\neg \forall x\, P(x) \equiv \exists x \neg P(x) \text{ and } \neg \exists x\, P(x) \equiv \forall x \neg P(x).$$

Note that the universal quantifier changes to an existential quantifier when a statement is negated. Similarly, the existential quantifier changes to a universal quantifier when a statement is negated.

Which of the following is the negation of $\forall x (P(x) \wedge Q(x))$?

A) $\exists x (P(x) \wedge Q(x))$
B) $\exists x (\neg P(x) \vee Q(x))$
C) $\exists x (\neg P(x) \vee \neg Q(x))$
D) $\exists x (\neg P(x) \wedge \neg Q(x))$

Answer: C

Negation of Quantified Statements

Suppose p represents a quantified statement. The negation of p is written $\neg p$. The quantified statement p is true if and only if $\neg p$ is false; p and $\neg p$ are never simultaneously true and never simultaneously false.
The negation of a statement with predicates and quantifiers is carried out as follows:

$$\neg\forall x\, P(x) \equiv \exists x \neg P(x) \text{ and } \neg\exists x\, P(x) \equiv \forall x \neg P(x).$$

Note that the universal quantifier changes to an existential quantifier when a statement is negated. Similarly, the existential quantifier changes to a universal quantifier when a statement is negated.

Which of the following is the negation of $\forall x\,(P(x) \rightarrow Q(x))$?

A) $\exists x\,(P(x) \rightarrow Q(x))$

B) $\exists x\,(P(x) \wedge \neg Q(x))$

C) $\exists x\,(\neg P(x) \rightarrow \neg Q(x))$

D) $\exists x\,(\neg P(x) \wedge Q(x))$

Answer: B

Negation of Quantified Statements

Suppose p represents a quantified statement. The negation of p is written $\neg p$. The quantified statement p is true if and only if $\neg p$ is false; p and $\neg p$ are never simultaneously true and never simultaneously false.
The negation of a statement with predicates and quantifiers is carried out as follows:

$$\neg\forall x\, P(x) \equiv \exists x \neg P(x) \text{ and } \neg\exists x\, P(x) \equiv \forall x \neg P(x).$$

Note that the universal quantifier changes to an existential quantifier when a statement is negated. Similarly, the existential quantifier changes to a universal quantifier when a statement is negated.

Which of the following is the negation of the following statement: “Everyone in the class except Lee has a laptop computer.”

A) Someone in the class other than Lee does not have a laptop computer or Lee has a laptop computer.

B) Lee and someone else in the class have a laptop computer.

C) Lee is the only student in the class with a laptop computer.

D) Someone in the class other than Lee does not have a laptop computer and Lee does not have a laptop computer.

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Negation of Quantified Statements

Which of the following is the negation of the following statement: "Everyone in the class except Lee has a laptop computer."

A) Someone in the class other than Lee does not have a laptop computer or Lee has a laptop computer.

B) Lee and someone else in the class have a laptop computer.

C) Lee is the only student in the class with a laptop computer.

D) Someone in the class other than Lee does not have a laptop computer and Lee does not have a laptop computer.

A) [CORRECT] The given statement is a conjunction: all students in the class (other than Lee) have laptop computers AND Lee does not have a laptop computer. This statement has the form $p \wedge q$. The negation has the form $\neg p \vee \neg q$. The negation $\neg q$ is "Lee has a laptop computer". However, the statement p has a universal quantifier; its negation states that "it is false that all students in the class (other than Lee) have a laptop computer," which is equivalent to "someone in the class (other than Lee) has a laptop computer." Therefore the negation of the given statement is "Someone in the class other than Lee does not have a laptop computer, or Lee has a laptop computer."

Quantified Statements

If $P(x)$ is a predicate, the truth value of $P(x)$ depends on the value x, chosen from a given set called the *universe of discourse*. The predicate could be always true, sometimes true and sometimes false, or never true. We can describe when the predicate is true or false by using *quantifiers*.

The statement $\forall x\, P(x)$ says that $P(x)$ is true for all x in the universe of discourse. The statement $\exists x\, P(x)$ says that there is at least one x in the universe for which $P(x)$ is true. You can use negation to say that a predicate is never true:

$$\neg \exists x\, P(x) \equiv \forall x \neg P(x).$$

That is, there is no x for which $P(x)$ is true, or, equivalently, no matter what x is chosen, $P(x)$ is false. You can also write

$$\neg \forall x\, P(x) \equiv \exists x \neg P(x)$$

to say that there is at least one x for which $P(x)$ is false. That is, it is false that $P(x)$ is always true.

Predicates may involve more than one variable — for example, $P(x, y)$ or $Q(x, y, z)$. Quantifiers in such statements may be "nested" — for example, $\forall x \exists y\, P(x, y)$, which means $\forall x(\exists y\, P(x, y))$, or $\forall x \exists y \forall z\, Q(x, y, z)$, which means $\forall x(\exists y(\forall z\, Q(x, y, z)))$.

Quantified Statements

Express the following statement in symbols:

"Between every two distinct real numbers there is a third real number."

In the following choices, assume that the universe for *a*, *b*, and *c* consists of all numbers.

A) $\forall a \forall b \exists c((a \neq b) \to (a < c < b))$.

B) $\forall a \forall b \exists c((a < c < b) \vee (b < c < a)$.

C) $\forall a \forall b \exists c[(a \neq b) \to ((a < c < b) \vee (b < c < a))]$.

D) $\exists a \exists b \exists c((a < c < b) \vee (b < c < a))$.

E) $\exists c \forall a \forall b (a < c < b)$.

Answer: C

Quantified Statements

Which of these statements says that

"Every number has exactly one additive inverse."

Assume that the universe for all variables consists of all numbers.

A) $\forall x \exists y \forall z[(x + y = 0) \wedge ((x + z = 0) \to (y = z))]$.

B) $\forall x \forall y \exists z (x + y = x + z = 0)$.

C) $\forall x \exists y (x + y = 0)$.

D) $\forall x \exists y \exists z[(x + y = 0) \wedge (x + z = 0)]$.

Answer: A

Quantified Statements

Suppose $P(x, y)$ is a predicate where the universe for x and y is {1, 2, 3}. Also suppose that the predicate is true in the following cases — $P(1, 2)$, $P(2, 1)$, $P(2, 2)$, $P(2, 3)$, $P(3, 1)$, $P(3, 2)$ — and false otherwise. Determine which of the following quantified statements is FALSE.

A) $\forall y \exists x \neg P(x, y)$.

B) $\exists y \forall x P(x, y)$.

C) $\forall x \exists y (x \neq y \land P(x, y))$.

D) $\exists x \forall y P(x, y)$.

Answer: A

Quantified Statements

Express the following statement in symbols:

"Every Mathematics Major is taking a Computer Science course,"

using the following: $M(x)$ is the statement "x is a Mathematics Major", $C(y)$ is the statement "y is a Computer Science course", $T(x, y)$ is the statement "x is taking y", the universe for x is the set of all students, and the universe for y is the set of all courses.

A) $\forall x \exists y [M(x) \rightarrow (C(y) \land T(x, y))]$.

B) $\forall x \exists y [M(x) \land C(y) \land T(x, y)]$.

C) $\forall x \exists y [M(x) \rightarrow T(x, C(y))]$.

D) $\exists y \forall x [M(x) \rightarrow (T(x, y) \land C(y))]$.

E) $\forall y \exists x (M(x) \land C(y) \land T(x, y))$.

Answer: A

Sets

A set S is an unordered collection of elements. If x is an element of S, we write $x \in S$. A set can be described by a list of its elements. A set can also be described by set builder notation: giving a predicate that each element must satisfy in order to belong to the set.

A set S is a subset of a set T, written $S \subseteq T$, if every element of S is an element of T. If $S \subseteq T$ and T contains an element that is not in S, then S is a proper subset of T, written $S \subset T$. Two sets S and T are equal, written $S = T$, if $S \subseteq T$ and $T \subseteq S$.

The size of a set S is denoted $|S|$. The power set of a set S is the set of all subsets of S, and is written P(S).

Which of the following is true for all sets S and T?

A) $S \cap \overline{T} = \overline{S} \cap T$

B) $\overline{S \cup T} = \overline{S} \cup \overline{T}$.

C) $\overline{\overline{S} \cap \overline{T}} = S \cup T$.

D) $(S - T) \cup (T - S) = \overline{S \cup T}$.

Answer: C

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The size of a set S is denoted $|S|$. The power set of a set S is the set of all subsets of S, and is written P(S).

If $S \subseteq T$, then

A) $\overline{T} \subseteq \overline{S}$.

B) $T \subseteq S \cap T$.

C) $T - S \neq \varnothing$.

D) $T - S \subseteq S - T$.

E) $S \cup T = S \cap T$.

Answer: A

Sets

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The size of a set S is denoted $|S|$. The power set of a set S is the set of all subsets of S, and is written P(S).

Which of the following is true for all sets A and B?

A) $A \cup \overline{B} = \overline{A \cap B}$.

B) $A \cup \overline{B} = (A \cap B) \cup B$.

C) $(A \cup B) \cap B = B \cup (A \cap B)$.

D) $A - (B - A) = A \cap \overline{B}$.

Answer: C

Sets

A set S is an unordered collection of elements. If x is an element of S, we write $x \in S$. A set can be described by a list of its elements. A set can also be described by set builder notation: giving a predicate that each element must satisfy in order to belong to the set.

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The size of a set S is denoted $|S|$. The power set of a set S is the set of all subsets of S, and is written P(S).

Let $S = \{1\}$. Which of the following is an element of P(P(S)) (the power set of the power set of S).

A) $\{\varnothing, \{1\}\}$.

B) $\{\varnothing, 1\}$.

C) $\{1\}$.

D) $\{\{\varnothing\}, 1\}$.

Answer: A

Sets

A set S is an unordered collection of elements. If x is an element of S, we write $x \in S$. A set can be described by a list of its elements. A set can also be described by set builder notation: giving a predicate that each element must satisfy in order to belong to the set.

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The size of a set S is denoted $|S|$. The power set of a set S is the set of all subsets of S, and is written $P(S)$.

Suppose $S = \{a, b, \{a\}\}$. Consider the following six statements:

i) $\{b\} \in S$ (ii) $\{a\} \subseteq P(S)$ (iii) $\{a, b\} \in P(S)$ (iv) $\{a, b\} \in S$ (v) $\{\{a\}\} \in P(S)$ (vi) $\{a, \{a\}\} \in P(S)$

A) (iii), (v), (vi).

B) (ii), (iii), (vi).

C) (i), (iii), (v).

D) (ii), (iv), (v).

E) (i), (ii), (vi).

Answer: A

Sets

Prove or disprove that if A, B, and C are sets then $A - (B \cap C) = (A - B) \cap (A - C)$.

This is false. For a counterexample take $A = \{1, 2\}$, $B = \{1\}$, and $C = \{2\}$. We have $A - (B \cap C) = \{1, 2\} - \emptyset = \{1, 2\}$, while $(A - B) \cap (A - C) = \{2\} \cap \{1\} = \emptyset$.

Let A, B, and C be sets. Prove or disprove that $A - (B \cap C) = (A - B) \cup (A - C)$.

We see that $A - (B \cap C) = A \cap \overline{B \cap C} = A \cap (\overline{B} \cup \overline{C}) = (A \cap \overline{B}) \cup (A \cap \overline{C}) = (A - B) \cup (A - C)$. These equalities follow from the definition of the difference of two sets, De Morgan's law, the distributive law for intersection over union, and the definition of the difference of two sets, respectively.

Concept of Function

A function f from a set A (called the domain) to a set B (called the codomain) is an assignment of exactly one value from B to each element of A. Write $f{:}A \to B$ when you have a function f from A to B and write $f(a) = b$ if $b \in B$ is the element assigned to $a \in A$. The set of all function values $f(a)$ for $a \in A$ is called the range of f.

In order to check whether a given rule $f{:}A \to B$ defines a function, you need to show that each element of $a \in A$ is assigned to exactly one element of B. You cannot have any elements of A left unassigned, nor can you have any element of A assigned to two or more elements of B.

Suppose $f{:}A \to B$ is a function. Which one of these statements is true?

A) If a_1 and a_2 are distinct elements of A, then $f(a_1) \neq f(a_2)$.

B) If $b \in B$, then there is at least one element $a \in A$ such that $f(a) = b$.

C) If $b \in B$, then there is exactly one $a \in A$ such that $f(a) = b$.

D) For each element $a \in A$, there is exactly one element $b \in B$ such that $f(a) = b$.

Answer: D

Concept of Function

A function f from a set A (called the domain) to a set B (called the codomain) is an assignment of exactly one value from B to each element of A. Write $f{:}A \to B$ when you have a function f from A to B and write $f(a) = b$ if $b \in B$ is the element assigned to $a \in A$. The set of all function values $f(a)$ for $a \in A$ is called the range of f.

In order to check whether a given rule $f{:}A \to B$ defines a function, you need to show that each element of $a \in A$ is assigned to exactly one element of B. You cannot have any elements of A left unassigned, nor can you have any element of A assigned to two or more elements of B.

Suppose $f{:}\mathbf{R} \to \mathbf{R}$ has the rule $f(x) = \lfloor \frac{x}{2} \rfloor$ and suppose $g{:}\mathbf{R} \to \mathbf{R}$ has the rule $g(x) = \lceil \frac{5-2x}{3} \rceil$. Find $(f \circ g)(5)$.

A) 1.

B) −1.

C) 0.

D) none of these.

Answer: B

Concept of Function

A function f from a set A (called the domain) to a set B (called the codomain) is an assignment of exactly one value from B to each element of A. Write $f:A \to B$ when you have a function f from A to B and write $f(a) = b$ if $b \in B$ is the element assigned to $a \in A$. The set of all function values $f(a)$ for $a \in A$ is called the range of f.
In order to check whether a given rule $f:A \to B$ defines a function, you need to show that each element of $a \in A$ is assigned to exactly one element of B. You cannot have any elements of A left unassigned, nor can you have any element of A assigned to two or more elements of B.

Find functions f:R → R and g:R → R such that $f \circ g$ has the rule $f \circ g(x) = \lfloor x^2 + 7 \rfloor$.

A) $g(x) = x^2$, $f(x) = \lfloor x \rfloor + 7$.

B) $g(x) = \lfloor x \rfloor + 7$, $f(x) = x^2$.

C) $g(x) = \lfloor x \rfloor$, $f(x) = x^2 + 7$.

D) $g(x) = x^2 + 7$, $f(x) = \lfloor x \rfloor$.

Answer: D

Concept of Function

A function f from a set A (called the domain) to a set B (called the codomain) is an assignment of exactly one value from B to each element of A. Write $f:A \to B$ when you have a function f from A to B and write $f(a) = b$ if $b \in B$ is the element assigned to $a \in A$. The set of all function values $f(a)$ for $a \in A$ is called the range of f.
In order to check whether a given rule $f:A \to B$ defines a function, you need to show that each element of $a \in A$ is assigned to exactly one element of B. You cannot have any elements of A left unassigned, nor can you have any element of A assigned to two or more elements of B.

Let S be the set of all bit strings of length at least 1. Determine which of the following does NOT define a function f: $S \to S$.

A) The function that takes a string as input and gives as output the reversal of the string.

B) The function that takes a string as input and gives as output the string consisting of only the last bit in the string.

C) The function that takes a string as input and gives as output the string 1.

D) The function that takes a string as input and gives as output the string consisting of the first two bits of the string.

Answer: D

Concept of Function

A function f from a set A (called the domain) to a set B (called the codomain) is an assignment of exactly one value from B to each element of A. Write $f:A \to B$ when you have a function f from A to B and write $f(a) = b$ if $b \in B$ is the element assigned to $a \in A$. The set of all function values $f(a)$ for $a \in A$ is called the range of f.

In order to check whether a given rule $f:A \to B$ defines a function, you need to show that each element of $a \in A$ is assigned to exactly one element of B. You cannot have any elements of A left unassigned, nor can you have any element of A assigned to two or more elements of B.

Let S be the set of all bit strings of length at least 1. Determine which of the following does NOT define a function f: $S \to S$.

A) The function that takes a string as input and gives as output the string obtained by removing all 0 bits from the string. (For example, $f(11010) = 111$.)

B) The function that takes a string as input and gives as output the string obtained by removing the last bit from the string and placing it at the beginning of the string. (For example, $f(1110000) = 0111000$.)

C) The function that takes a string as input and gives as output the string obtained by concatenating the string with itself. (For example, $f(0001) = 00010001$.)

D) The function that takes a string as input and gives as output the string obtained by interchanging 0's and 1's. (For example $f(1110) = 0001$.)

Answer: A

Concept of Function

A function f from a set A (called the domain) to a set B (called the codomain) is an assignment of exactly one value from B to each element of A. Write $f:A \to B$ when you have a function f from A to B and write $f(a) = b$ if $b \in B$ is the element assigned to $a \in A$. The set of all function values $f(a)$ for $a \in A$ is called the range of f.

In order to check whether a given rule $f:A \to B$ defines a function, you need to show that each element of $a \in A$ is assigned to exactly one element of B. You cannot have any elements of A left unassigned, nor can you have any element of A assigned to two or more elements of B.

Suppose $f:A \to \mathbb{R}$ has the rule $f(x) = \dfrac{1}{x^3 - x}$. Which of the following could be the domain A?

A) R.

B) R−{0, 1}.

C) R−{−1, 1}.

D) R−{−1, 0, 1}.

Answer: D

1-1 and Onto Properties of Functions

The function $f: A \to B$ is one-to-one (1–1) if the following is true for all $a_1 \in A$ and $a_2 \in A$: if $f(a_1) = f(a_2)$ then $a_1 = a_2$. In other words, for all $a_1 \in A$ and $a_2 \in A$, if $a_1 \neq a_2$ then $f(a_1) \neq f(a_2)$. To prove that a given function is not one-to-one, you need to find (or prove the existence of) two distinct elements, a_1 and a_2 in A, such that $f(a_1) = f(a_2)$.

The function $f: A \to B$ is onto B if for every $b \in B$ there is at least one element $a \in A$ such that $f(a) = b$. To prove that a given function is not onto B, you need to find (or prove the existence of) an element $b \in B$ with the following property: there is no $a \in A$ such that $f(a) = b$.

Let $f: A \to B$ where $A = \{-3, -2, -1, 0, 1, 2\}$ and f is defined by the rule $f(x) = x^2$. For which set B is the function onto B?

A) $\{-9, -4, -1, 0, 1, 4\}$.

B) $\{0, 1, 4, 9\}$.

C) $\{0, 1, 4\}$.

D) $\{-3, -2, -1, 0, 1, 2\}$.

Answer: B

1-1 and Onto Properties of Functions

The function $f: A \to B$ is one-to-one (1–1) if the following is true for all $a_1 \in A$ and $a_2 \in A$: if $f(a_1) = f(a_2)$ then $a_1 = a_2$. In other words, for all $a_1 \in A$ and $a_2 \in A$, if $a_1 \neq a_2$ then $f(a_1) \neq f(a_2)$. To prove that a given function is not one-to-one, you need to find (or prove the existence of) two distinct elements, a_1 and a_2 in A, such that $f(a_1) = f(a_2)$.

The function $f: A \to B$ is onto B if for every $b \in B$ there is at least one element $a \in A$ such that $f(a) = b$. To prove that a given function is not onto B, you need to find (or prove the existence of) an element $b \in B$ with the following property: there is no $a \in A$ such that $f(a) = b$.

Suppose the function $f: A \to \mathbf{Z}$ is defined by the rule $f(n) = \lfloor \frac{2n-1}{3} \rfloor$. On which domain A is the function 1–1?

A) $\{1, 2, 3, 4\}$.

B) $\{-1, 0, 1, 3\}$.

C) $\{-2, -1, 0, 1\}$.

D) $\{-1, 1, 3, 4\}$.

Answer: D

1-1 and Onto Properties of Functions

The function $f: A \to B$ is one-to-one (1–1) if the following is true for all $a_1 \in A$ and $a_2 \in A$: if $f(a_1) = f(a_2)$ then $a_1 = a_2$. In other words, for all $a_1 \in A$ and $a_2 \in A$, if $a_1 \neq a_2$ then $f(a_1) \neq f(a_2)$. To prove that a given function is not one-to-one, you need to find (or prove the existence of) two distinct elements, a_1 and a_2 in A, such that $f(a_1) = f(a_2)$.

The function $f: A \to B$ is onto B if for every $b \in B$ there is at least one element $a \in A$ such that $f(a) = b$. To prove that a given function is not onto B, you need to find (or prove the existence of) an element $b \in B$ with the following property: there is no $a \in A$ such that $f(a) = b$.

Let S be the set of all bit strings (strings of 0's and 1's) of length at least 2. Which of the following functions $f: S \to S$ is NOT one-to-one?

A) $f(s)$ = the string s with a 1 bit appended at the end. (For example, $f(1101) = 11011$.)

B) $f(s)$ = the reversal of s. (For example, $f(110) = 011$.)

C) $f(s)$ = the string obtained from s by interchanging 0's and 1's. (For example, $f(11000) = 00111$.)

D) $f(s)$ = the string obtained by moving all 0's (if any) in s to the end of the string. (For example, $f(101101) = 111100$.)

Answer: D

1-1 and Onto Properties of Functions

The function $f: A \to B$ is one-to-one (1–1) if the following is true for all $a_1 \in A$ and $a_2 \in A$: if $f(a_1) = f(a_2)$ then $a_1 = a_2$. In other words, for all $a_1 \in A$ and $a_2 \in A$, if $a_1 \neq a_2$ then $f(a_1) \neq f(a_2)$. To prove that a given function is not one-to-one, you need to find (or prove the existence of) two distinct elements, a_1 and a_2 in A, such that $f(a_1) = f(a_2)$.

The function $f: A \to B$ is onto B if for every $b \in B$ there is at least one element $a \in A$ such that $f(a) = b$. To prove that a given function is not onto B, you need to find (or prove the existence of) an element $b \in B$ with the following property: there is no $a \in A$ such that $f(a) = b$.

Suppose the function $f: A \to \{1, 2, 3\}$ is defined by the rule $f(n) = \lfloor \frac{2n-1}{3} \rfloor$. On which domain is the function 1–1 and onto?

A) $\{3, 4, 6\}$.

B) $\{2, 3, 4, 5\}$.

C) $\{3, 5\}$.

D) $\{2, 3, 4\}$.

Answer: A

1-1 and Onto Properties of Functions

The function $f:A \rightarrow B$ is one-to-one (1–1) if the following is true for all $a_1 \in A$ and $a_2 \in A$: if $f(a_1) = f(a_2)$ then $a_1 = a_2$. In other words, for all $a_1 \in A$ and $a_2 \in A$, if $a_1 \neq a_2$ then $f(a_1) \neq f(a_2)$. To prove that a given function is not one-to-one, you need to find (or prove the existence of) two distinct elements, a_1 and a_2 in A, such that $f(a_1) = f(a_2)$.

The function $f:A \rightarrow B$ is onto B if for every $b \in B$ there is at least one element $a \in A$ such that $f(a) = b$. To prove that a given function is not onto B, you need to find (or prove the existence of) an element $b \in B$ with the following property: there is no $a \in A$ such that $f(a) = b$.

Suppose f:R → R has the following property for all real numbers x and y: if $x < y$ then $f(x) < f(y)$. (A function with this property is called a strictly increasing function.) Which of the following is true?

A) f must be 1–1 but is not necessarily onto **R**.

B) f is onto **R**, but is not necessarily 1–1.

C) f must be both 1–1 and onto **R**.

D) f is not necessarily 1–1 and not necessarily onto **R**.

Answer: A