CMPT285 Homework 10 (due Friday, May 2)

1. (Problem 17 on page 451 from Rosen) What is the probability that a five-card poker hand contains a straight, that is, five cards that have consecutive kinds? (Note that an ace can be considered either the lowest card of an A-2-3-4-5 straight or the highest card of a 10-J-Q-K-A straight.)

2. (Problem 37 on page 452 from Rosen) Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of 9 when three dice are rolled?

3. (Problem 23 on page 467 from Rosen) What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up tails?

4. (Problem 31 on page 468 from Rosen) Find the probability that a family with five children does not have a boy, if the sexes of children are independent and if
   - a boy and a girl are equally likely.
   - the probability of a boy is 0.51.
   - the probability that the ith child is a boy is $0.51 - (i/100)$.

5. (Problem 1 on page 475 from Rosen) Suppose that E and F are evemts in a sample space and $p(E) = 1/3$, $p(F) = 1/2$,and $p(E|F) = 2/5$.

6. (Problem 9 on page 476 from Rosen) Suppose that 8% of the patients tested in a clinic are infected with HIV. Furthermore, suppose that when a blood test for HIV is given, 98% of the patients infected with HIV test positive and that 3% of the patients not infected with HIV test positive. What is the probability that
   (a) a patient testing positive for HIV with this test is infected with it?
   (b) a patient testing positive for HIV with this test is not infected with it?
   (c) a patient testing negative for HIV with this test is infected with it?
   (d) a patient testing negative for HIV with this test is not infected with it?

7. (Problem 11 on page 492 from Rosen) Suppose that we roll a fair die until a 6 comes up or we have rolled it 10 times. What is the expected number of times we roll the die?

8. (Problem 19 on page 492 from Rosen) Let X be the number appearing on the first die when two fair dice are rolled and let Y be the sum of the numbers appearing on the two dice. Show that $E(X)E(Y) \neq E(XY)$.

9. (Problem 29 on page 493 from Rosen) Let X_n be the random variable that equals the number of tails minus the number of heads when n fair coins are flipped.
   (a) What is the expected value of X_n?
   (b) What is the variance of X_n?

10. (Problem 33 on page 493 from Rosen) Suppose that X_1 and X_2 are independent Bernoulli trials, each with probability $1/2$, and let $X_3 = (X_1 + X_2) \bmod 2$.

(a) Show that X_1, X_2 and X_3 are pairwise independent, but X_3 and $X_1 + X_2$ are not independent.

(b) Show that $V(X_1 + X_2 + X_3) = V(X_1) + V(X_2) + V(X_3)$.

(c) Explain why a proof by mathematical induction of Theorem 7 does not work by considering the random variables X_1, X_2 and X_3.