CMPT285 Homework 2 (due Tuesday, Feb. 11)

1. Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is tautology.

2. Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

   - $\exists x P(x)$.
   - $\forall x P(x)$.
   - $\exists \neg x N(x)$.
   - $\forall \neg x P(x)$.

3. Determine the truth value of each of the following if the domain consists of all integers.

   - $\forall n(n + 1 > n)$.
   - $\exists n(2n = 3n)$.
   - $\exists n(n = -n)$.
   - $\forall n(3n \leq 4n)$.

4. Suppose that the domain of the propositional function $P(x)$ consists of the integers 1, 2, 3, 4, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

   - $\exists x P(x)$.
   - $\forall x P(x)$.
   - $\neg \exists x N(x)$.
   - $\neg \forall x P(x)$.
   - $\forall x((x \neq 3) \to P(x)) \wedge \exists \neg x P(x)$.

5. Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

   - $\exists x \forall y(xy = y)$.
   - $\forall x \forall y(((x < 0) \wedge (y < 0) \to (xy > 0))$.
   - $\exists x \exists y((x^2 > y) \wedge (x < y))$.
   - $\forall x \forall y \exists z(x + y = z)$.

6. Suppose the domain of the propositional function $P(x, y)$ consists of all pairs x and y, where x is 1, 2, or 3 and y pis 1, 2, or 3. Write our these propositions ysing disjuctions and conjunctions.

   - $\forall x \forall y P(x, y)$.
   - $\exists x \exists y P(x, y)$.
   - $\exists x \forall y P(x, y)$.
   - $\forall x \exists y P(x, y)$.

7. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

   - $\forall x \exists y \forall z T(x, y, z)$.
   - $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$.
   - $\forall x \exists y (P(x, y) \wedge \exists z R(x, y, z))$.
   - $\forall x \exists y (P(x, y) \rightarrow Q(x, y)$.