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Midterm Two (due Friday, May 2 @11:30am sharp)

1. A message has been encrypted using the function $f(x) = (x+5) \bmod 26$. If the message in coded form is JCFHY, decode the message.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

J = 9, C = 2, F = 5, H, 7, Y = 24

$J = 9-5 \pmod{26}$
$J = 4 \pmod{26}$
$\boxed{J = 4}$

$F = 5-5 \bmod 26$
$\boxed{F = 0}$

$Y = 24-5 \bmod 26$
$Y = 19 \bmod 26$
$\boxed{Y = 19}$

$C = 2-5 \bmod 26$
$C = 23 \bmod 26$
$\boxed{C = 23}$

$H = 7-5 \bmod 26$
$H = 2 \bmod 26$
$\boxed{H = 2}$

4 23 0 2 19
↓ ↓ ↓ ↓ ↓
E X A C T

2. Let $S = \{1, 2, 3, \cdots, 10\}$. How many subsets of S are there that contain exactly four elements, the sum of which is even?

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

✗ $C_1 = \{e, e, e, o\}$ → Any 3 even with one odd = odd
✓ $C_2 = \{o, o, o, e\}$ ⇒ Any 3 odds with one even = even
✓ $C_3 = \{o, o, o, o\}$ → all odds = even
✓ $C_4 = \{e, e, e, e\}$ → all even = even
✓ $C_5 = \{e, e, o, o\}$ → 2 even, with 2 odds = even

Using combinations formula

$C_2 = C(5,3) * C(5,1) = 50$ +
$C_3 = C(5,4) = 5$
$C_4 = C(5,4) = 5$
$C_5 = C(5,2) * C(5,2) = 100$

$\boxed{160}$

We can get 160 subsets that contain four elements which sum is even

3. In the questions below determine whether the binary relation is: (1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive.

(a)
- The relation R on N where aRb means that a has the same number of digits as b.

(b)
- The relation R on the set $\{(a, b)|a, b \in Z\}$ where $(a, b)R(c, d)$ means $a = c$ or $b = d$.

(a) • a R b

a	b
1	1
2	2
3	3
⋮	⋮

example
(1, 1), (2, 2) ....

Reflexive → all digits are the same

Symmetrict → all digits are the same
(doesn't break any rule)

transitive → we have a, b, c and aRb and bRc, but we know that the digits a and c are the same, then the digits of a and b are also the same therefore aRb and so R is transitive

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(b) (a, b) R (c, d)
⋮ ⋮

Reflexive → Since a = c or b = d then we know it is Reflexive

Symmetric → Since (a, b) exist and (b, a) also exist we know it is symmetric

4. Suppose that a and b are real numbers and the average of a and b is $\frac{a+b}{2}$. Suppose that we have the following three statements: (i) a is less than b,(ii) the average of a and b is greater than a, and (iii) the average of a and b is less b. Prove that (i) implies (ii), (ii) implies (iii), and (iii) implies (i).

I $= a < b$ II $= a < \frac{a+b}{2}$ III $= \frac{a+b}{2} < b$

Prove I $a < b$ add a to both sides

$2a < a+b$ divide by 2

$\boxed{a < \frac{a+b}{2}} =$ II

II $a < \frac{a+b}{2}$ multiply by 2

$2a < a+b$ Substract a

$a < b$ add b

$a+b < 2b$ divide by 2

$\boxed{\frac{a+b}{2} < b} =$ III

III $\frac{a+b}{2} < b$ multiply by 2

$a+b < 2b$ Substract b

$\boxed{a < b} =$ I

5. Suppose that a and b are real numbers such that $0 < b < a$. Prove, using mathematical induction, that if n is a positive integer, then

$$a^n - b^n \leq na^{n-1}(a-b)$$

basis step

$a^n - b^n \leq na^{n-1}(a-b)$ assume is true

Induction Hypothesis

$P(1) = a^1 - b^1 \leq 1a^{1-1}(a-b)$

$a - b \leq a - b$

True! it holds for Smallest case

Inductive Step

$P(k) \to P(k+1)$

$$\boxed{a^n - b^n + a^{n+1} - b^{n+1} \leq na^{n-1}(a-b) + (a^{n+1} - b^{n+1})}$$