CMPT285 Homework 5 (due Thursday, March 20)

1. (Problem 13 on page 244 from Rosen) Suppose that a and b are integers, $a \equiv b \pmod{13}$, and $b \equiv 9 \pmod{13}$. Find the integer c with $0 \leq c \leq 12$ such that
   - $c \equiv 9a \pmod{13}$.
   - $c \equiv 11b \pmod{13}$.
   - $c \equiv a + b \pmod{13}$.
   - $c \equiv 2a + 3b \pmod{13}$.
   - $c \equiv a^2 + b^2 \pmod{13}$.
   - $c \equiv a^3 - b^3 \pmod{13}$.

2. (Problem 31 on page 245 from Rosen) Find each of these values. a and b are integers, $a \equiv b \pmod{13}$, and $b \equiv 9 \pmod{13}$. Find the integer c with $0 \leq c \leq 12$ such that
   - $(-133 \bmod 23 + 261 \bmod 23) \bmod 31$.
   - $(457 \bmod 23 \cdot 182 \bmod 23) \bmod 23$.

3. (Problem 3 on page 272 from Rosen) Find the prime factorization of each of these integers.
   - 88
   - 126
   - 729
   - 1001
   - 1111
   - 909090

4. (Problem 25 on page 273 from Rosen) What are the greatest common divisors of these pairs of integers?
   - $3^7 5^3 7^3, 2^{11} 3^5 5^9$
   - $11 \cdot 13 \cdot 17, 2^9 3^7 5^5 7^3$
   - $23^{31}, 23^{17}$
   - $41 \cdot 43 \cdot 53, 41 \cdot 43 \cdot 53$
   - $3^{13} 5^{17} 2^{12}, 2^{12} 7^{21}$
   - $1111, 0$

5. (Problem 13 on page 285 from Rosen) Find the solutions of the congruence $15x^2 + 9x \equiv 5 \pmod{11}$. [*Hint*:Show the congruence is equivalent to the congruence $15x^2 + 9x + 6 \equiv 0 \pmod{11}$. Factor the left-hand side of the congruence; show that a solution of the quadratic congruence is a solution of one of the two different linear congruences.]

6. (Problem 3 on page 292 from Rosen) A parking lot has 31 visitor spaces, numbered from 0 to 30. Visitors are assigned parking soaces using the hashing function $h(k) = k \bmod 31$, where k is the number formed from the first three digits on a visitor's license plate.

   - Which spaces are assigned by the hashing function to cars that have these first three digits on their license plates: 317, 918, 007, 100, 111, 310?
   - Describe a procedure visitors should follow to find a free parking space, when the space they are addigned is occupied.

7. (Problem 17 on page 91 from Rosen) Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

   - a proof by contraposition.
   - a proof by contradiction.

8. (Problem 31 on page 91 from Rosen) Show that these statements about the integer x are equivalent: (1) $3x + 2$ is even, (2) $x + 5$ is odd, (3) x^2 is even.