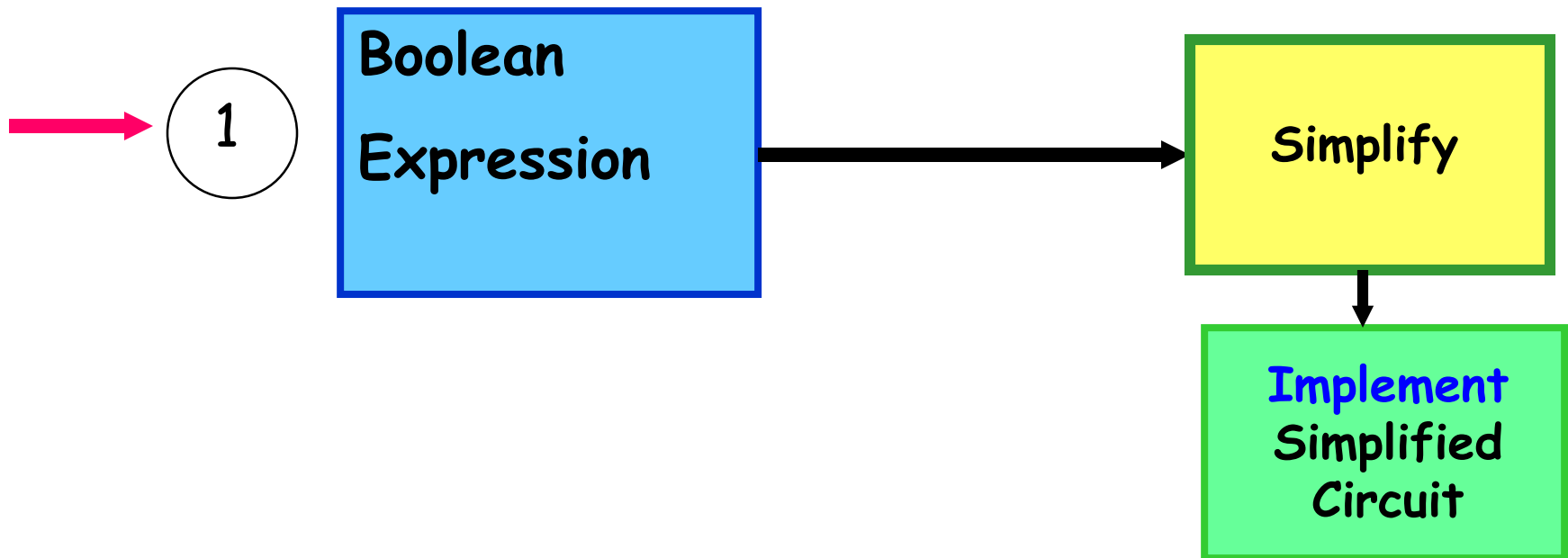


Boolean Simplification

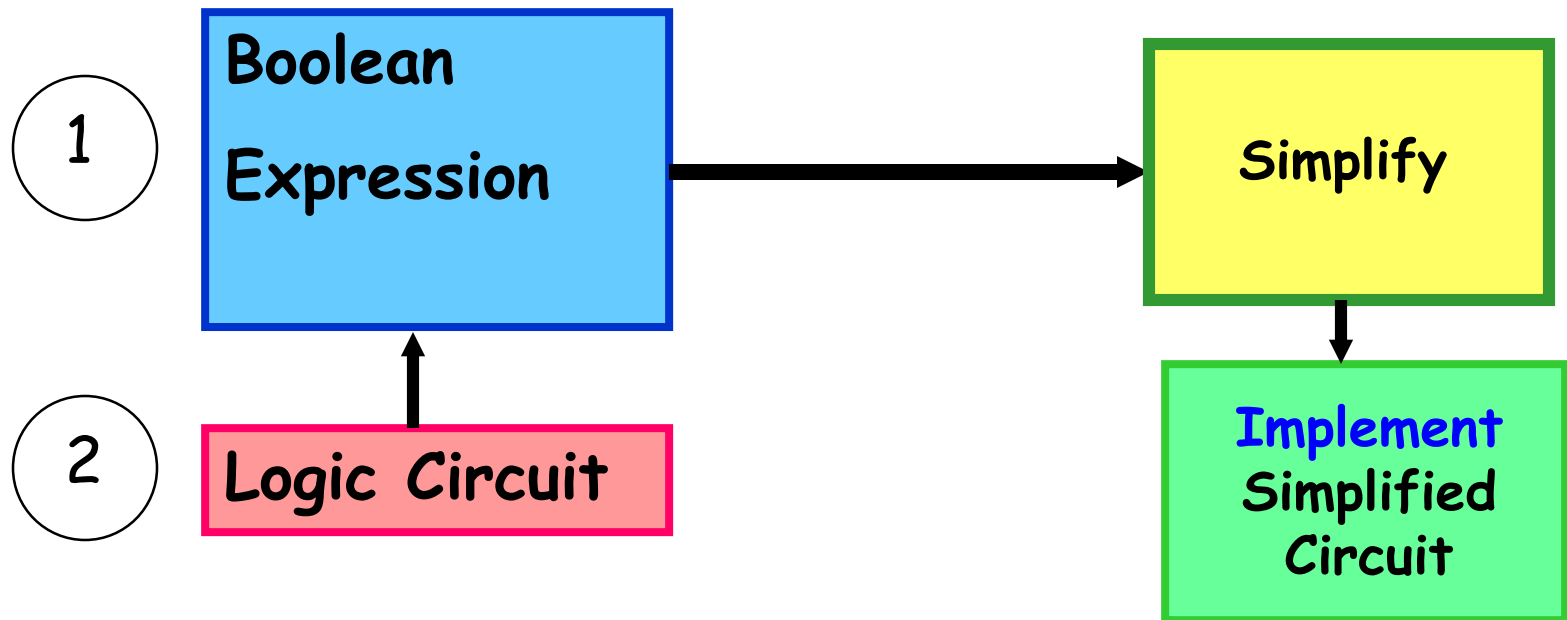
Given a Boolean expression. Simplification



1. Boolean expression simplification

- ❖ Put the Boolean expression into sum of-products (SOP) form
- ❖ Apply the Boolean simplification rules

Given a Logic Circuit ...



2. Logic circuit simplification

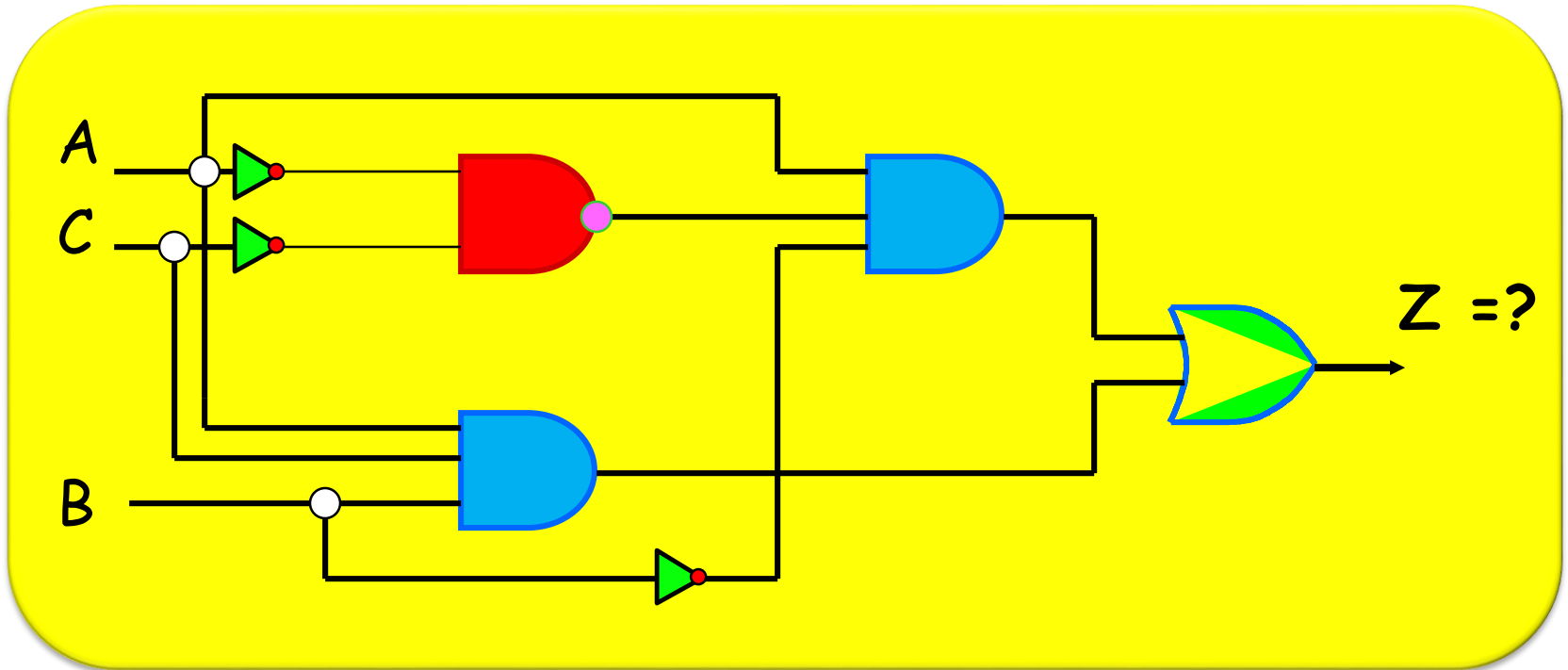
- Derive the output Boolean expression
- Apply the Boolean simplification rules



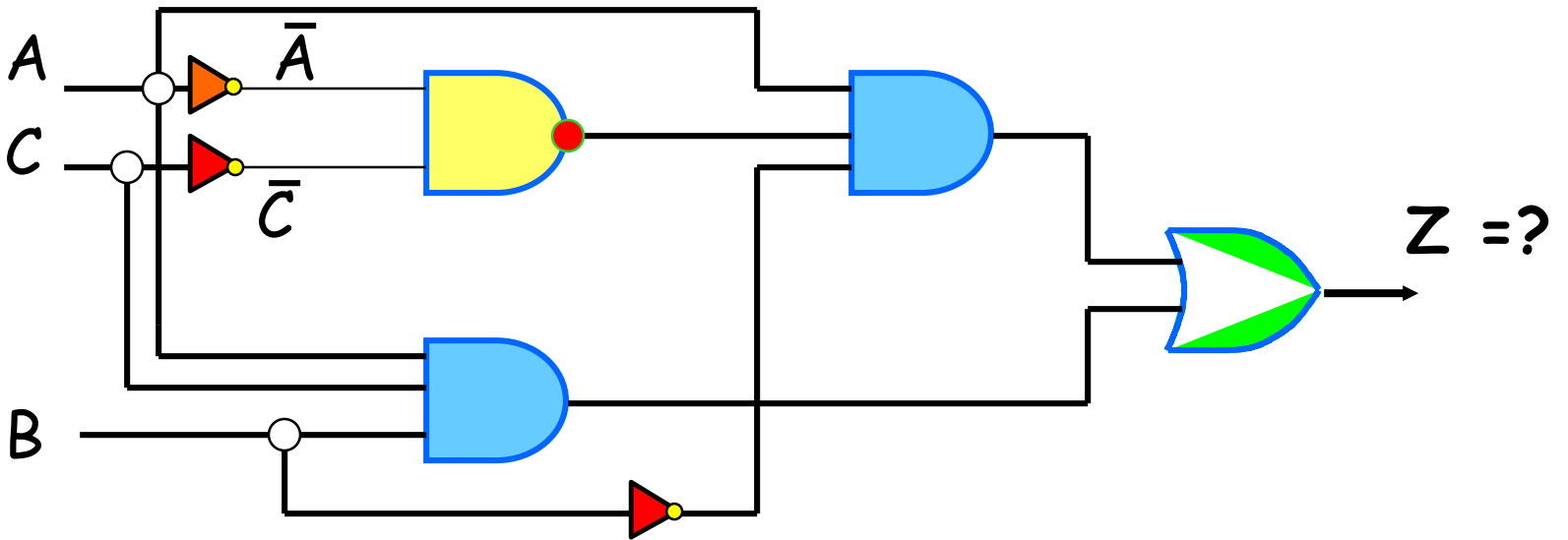
Example

Logic circuit; Derive Z

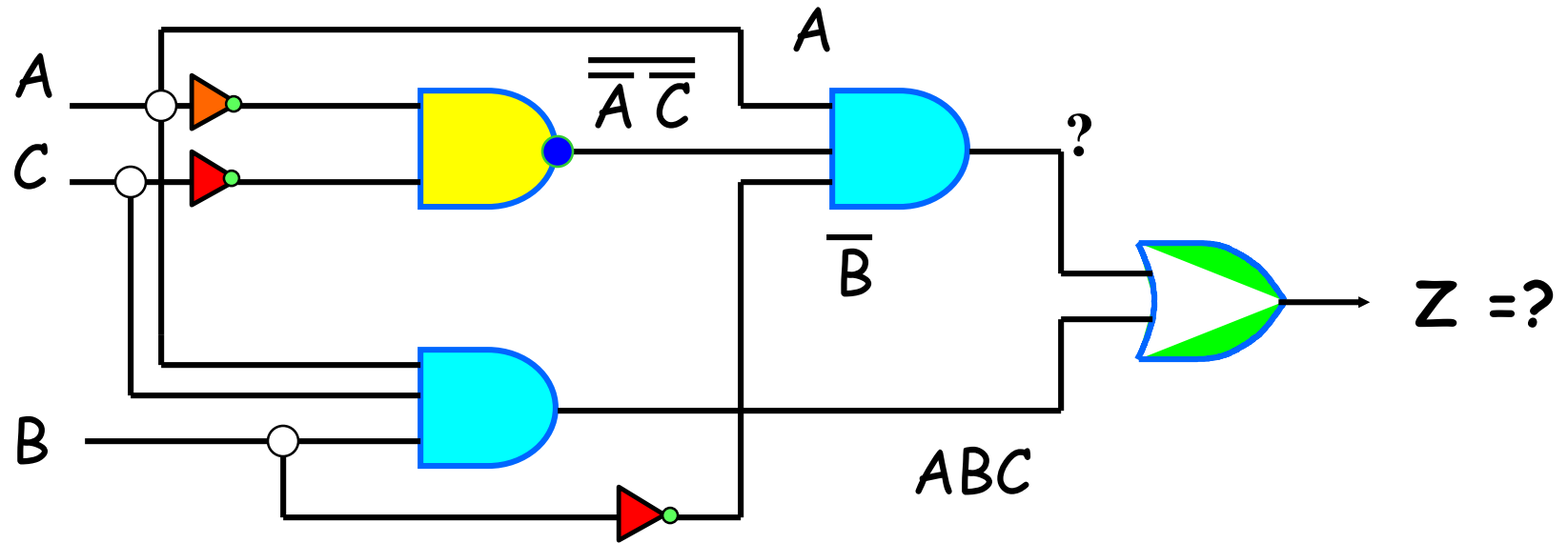
Example



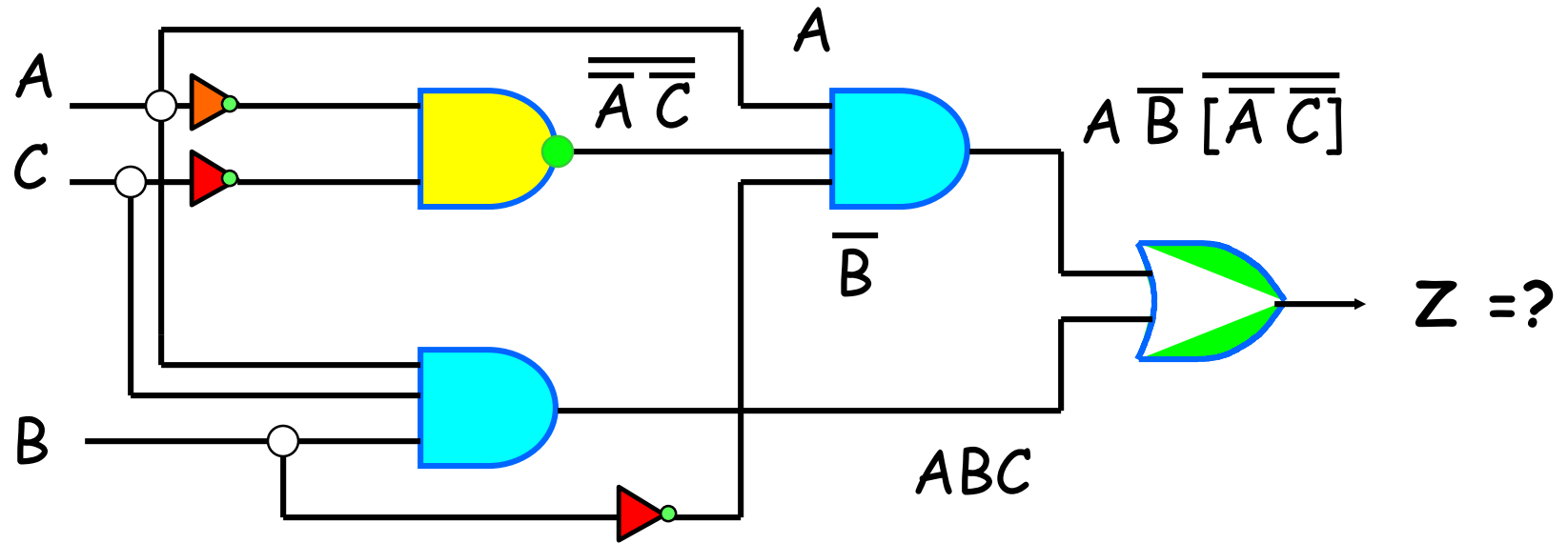
Derive Z



Derive Z

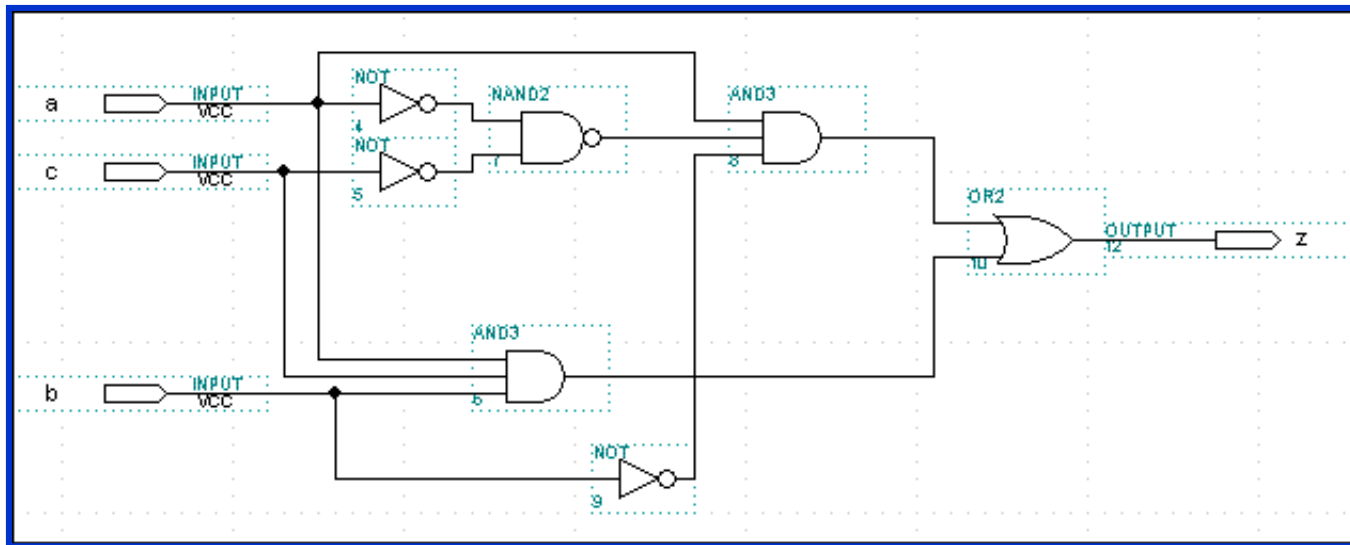


Finally the output expression is ...



Output expression

$$Z = A B C + A \bar{B} [\bar{A} \bar{C}]$$



Simplify the Boolean expression

$$Z = A B C + A \overline{B} [\overline{\overline{A} \overline{C}}]$$

- $\overline{x+y} = \overline{x} \bullet \overline{y}$

- $\overline{x \bullet y} = x+y$

DeMorgan

$$Z = A B C + A \bar{B} [\overline{\bar{A} \bar{C}}]$$

$$= A B C + A \bar{B} [\bar{\bar{A}} + \bar{\bar{C}}]$$

Sum-Of-Products (SOP) form

$$Z = A B C + A \bar{B} [\overline{\bar{A} \bar{C}}]$$

$$= A B C + A \bar{B} [\bar{\bar{A}} + \bar{\bar{C}}]$$

$$= A B C + A \bar{B} A + A \bar{B} C$$

Factor-out: AC

$$Z = A B C + A \bar{B} [\bar{A} \bar{C}]$$

$$= A B C + A \bar{B} [\bar{A} + \bar{C}]$$

$$= ABC + A\bar{B}A + A\bar{B}\bar{C}$$

$$= AC[B + \bar{B}] + A\bar{B}A$$

Simplified logic expression

$$Z = A B C + A \bar{B} [\bar{\bar{A}} \bar{\bar{C}}]$$

$$= ABC + A\bar{B}[\bar{\bar{A}} + \bar{\bar{C}}]$$

$$= ABC + A\bar{B}A + \bar{A}\bar{B}C$$

$$= AC[B + \bar{B}] + A\bar{B}A$$

$$= AC + A\bar{B}$$

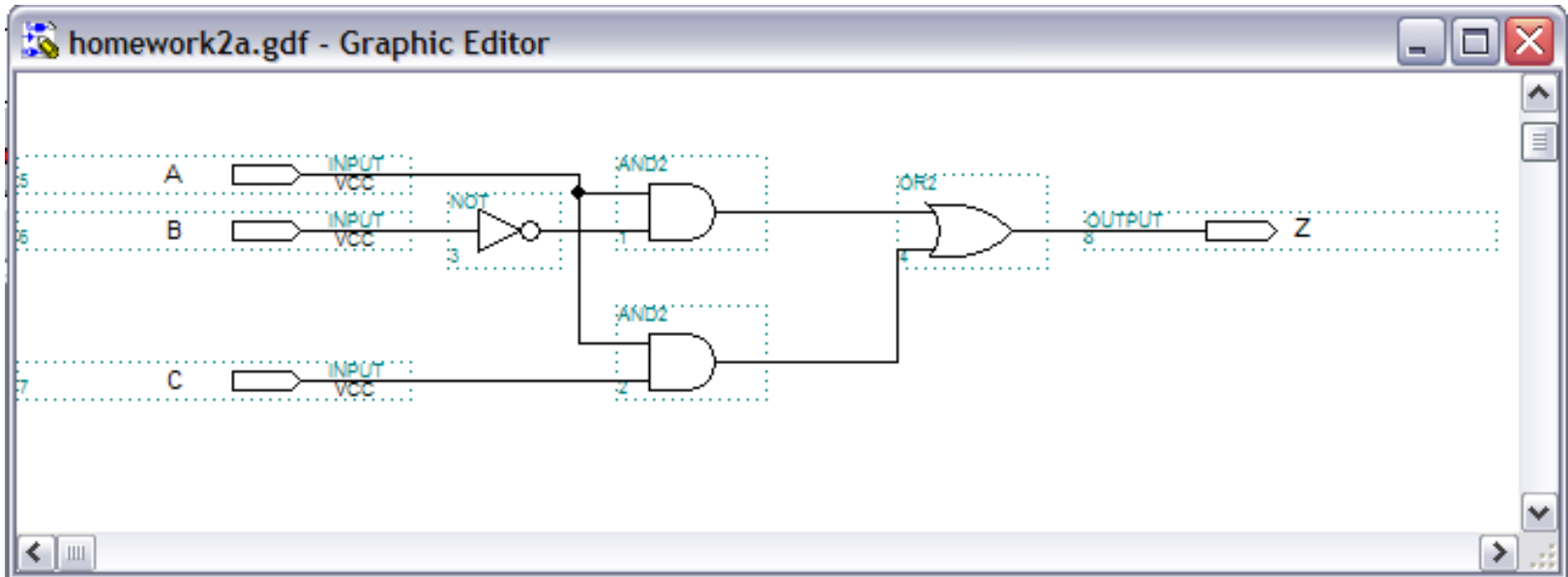
Implement (gates) the expression

$$Z = AC + A \bar{B}$$

2 Minutes...

Simplified logic circuit (gates)

$$Z = AC + A\bar{B}$$



Two-level implementation

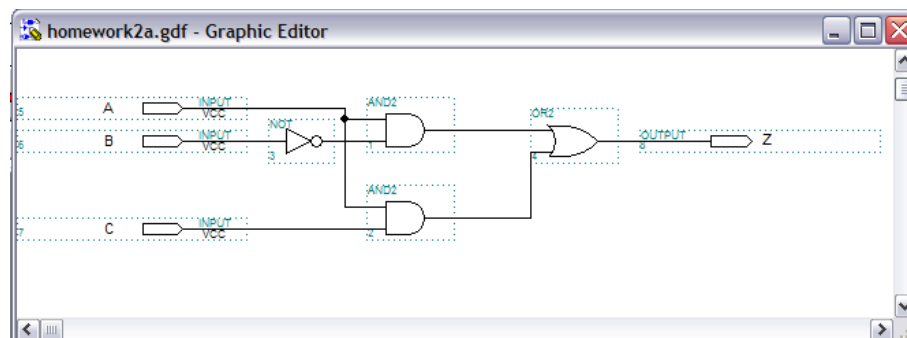
VHDL code for: $Z = AC + A\bar{B}$

```
homework2a_vhdl.vhd - Text Editor

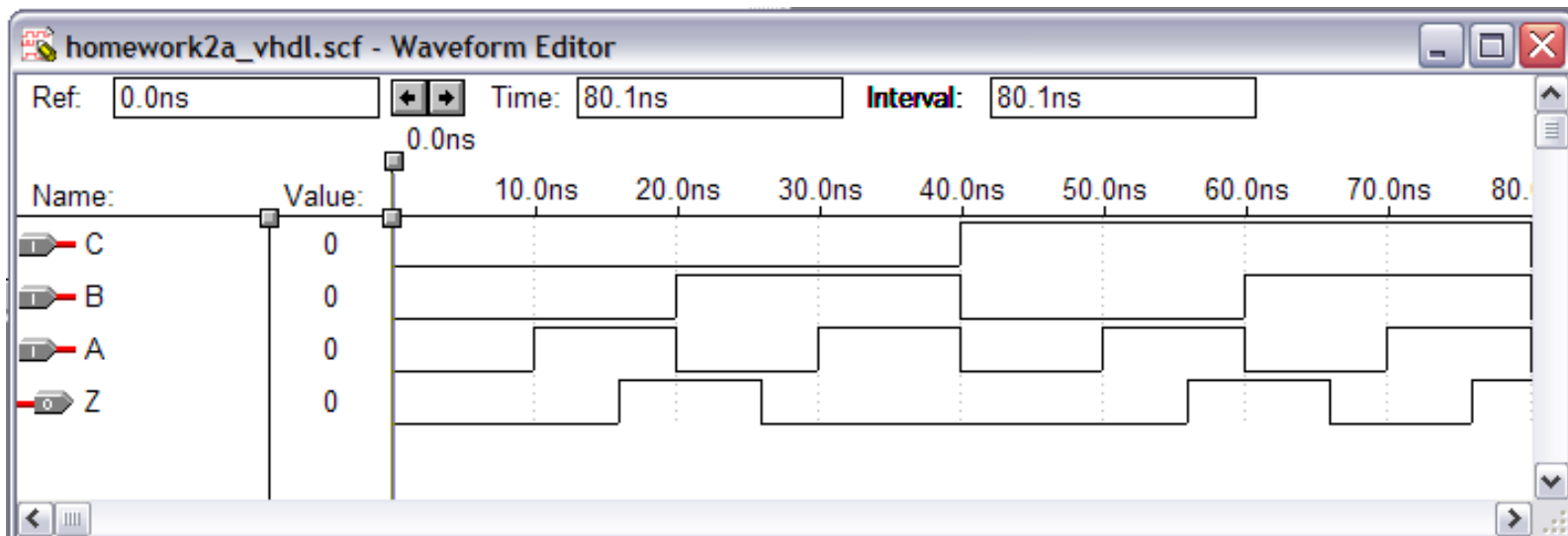
entity homework2a_vhdl is
    port (a,b,c: in bit;
          z: out bit);
end homework2a_vhdl;

architecture dataflow of homework2a_vhdl is
begin
    z<=(a and not b) or (a and c);
end dataflow;
```

Line 8 Col 33 INS



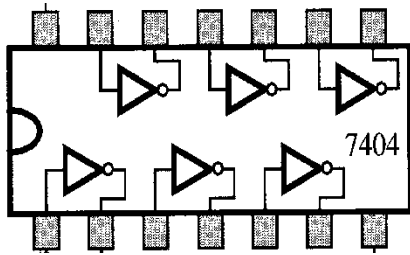
Simulation



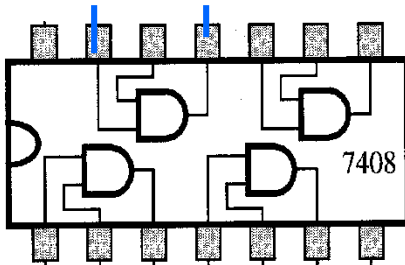
Implementation: $AC + A\bar{B} = Z$

Use the available Integrated Circuits

Chip Implementation for: $AC + A\bar{B} = Z$

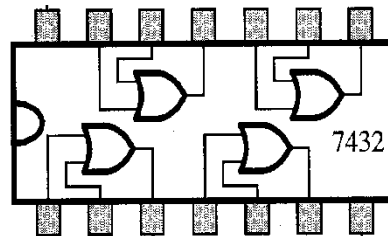


B



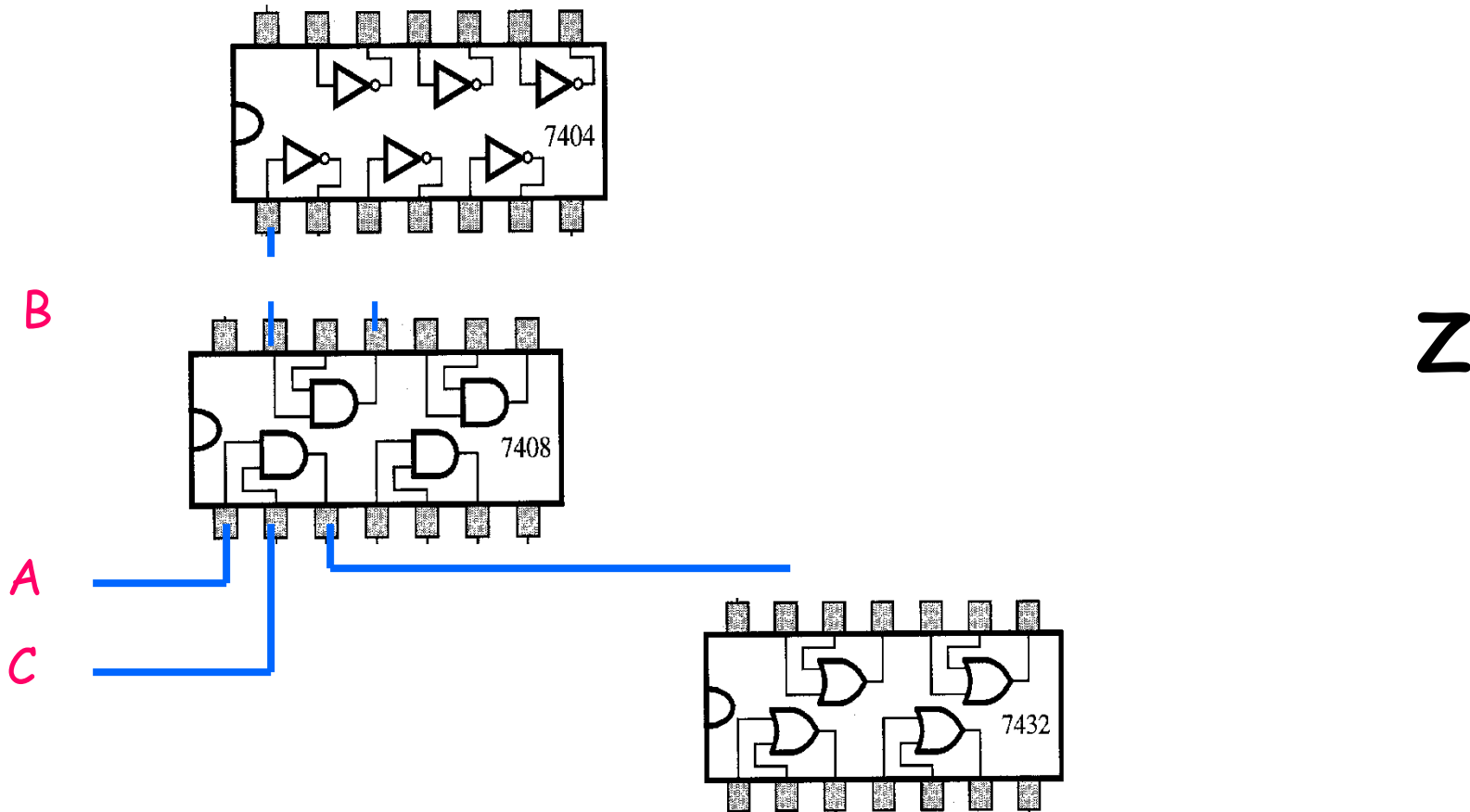
A

C

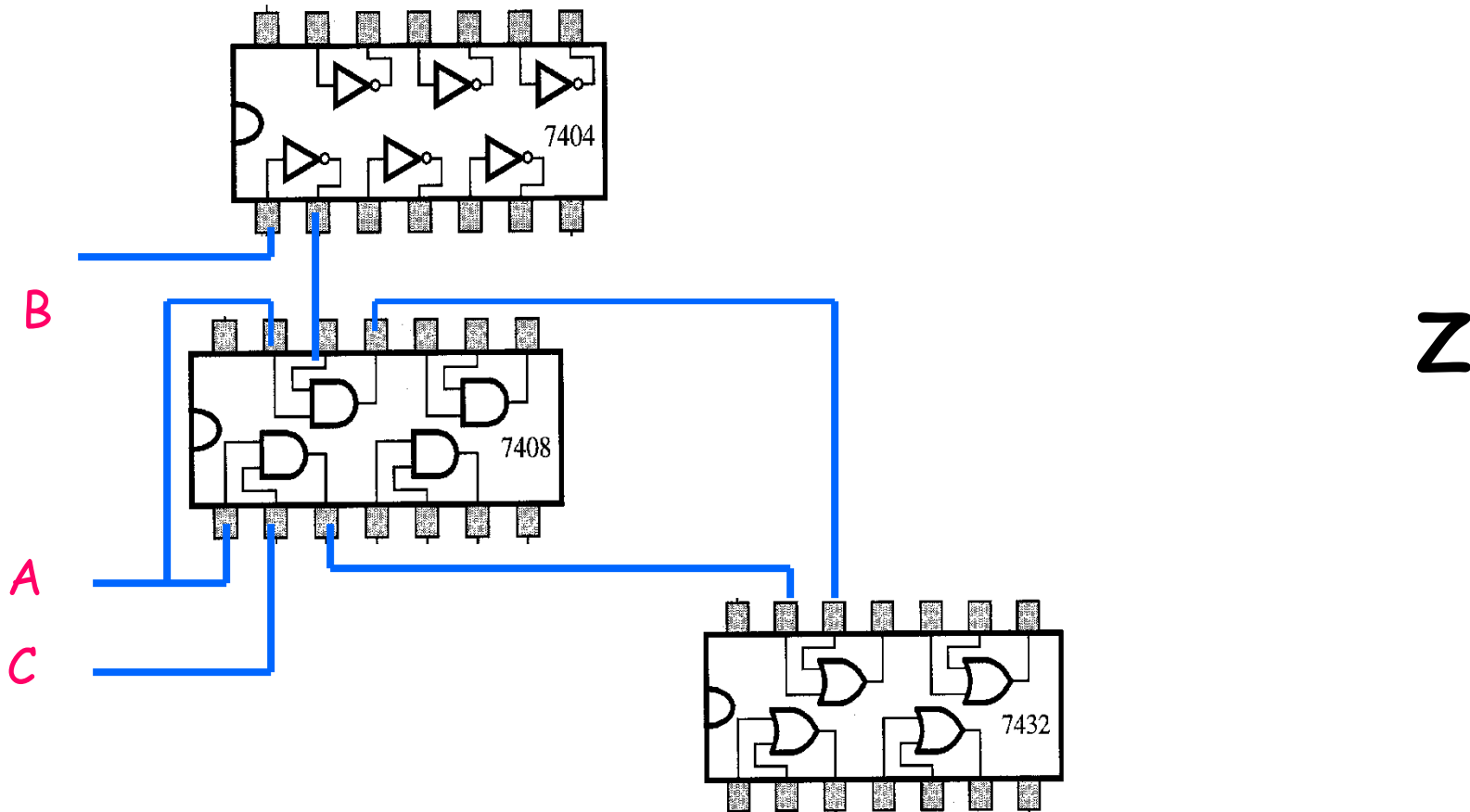


Z

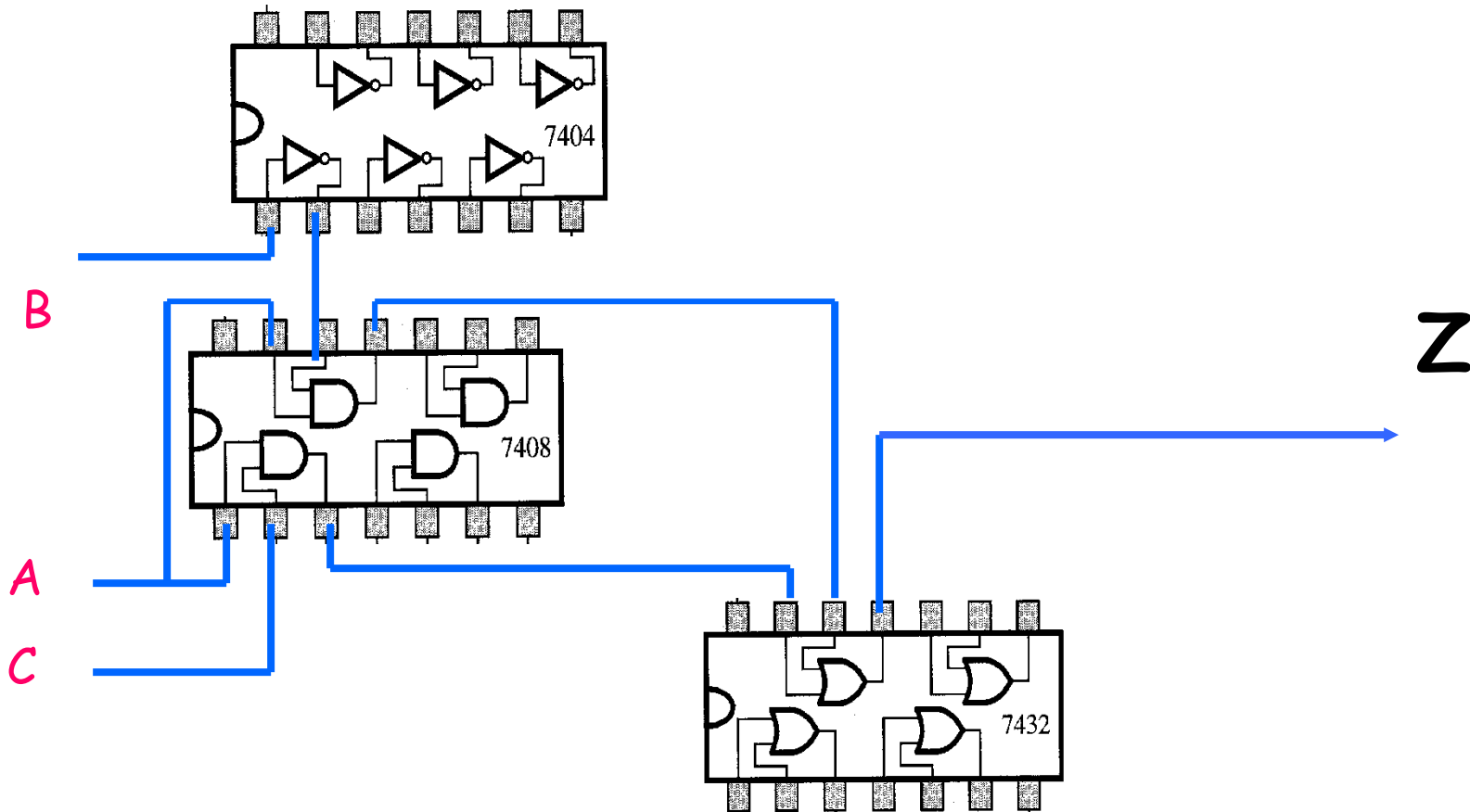
Chip Implementation for: $AC + A\bar{B} = Z$



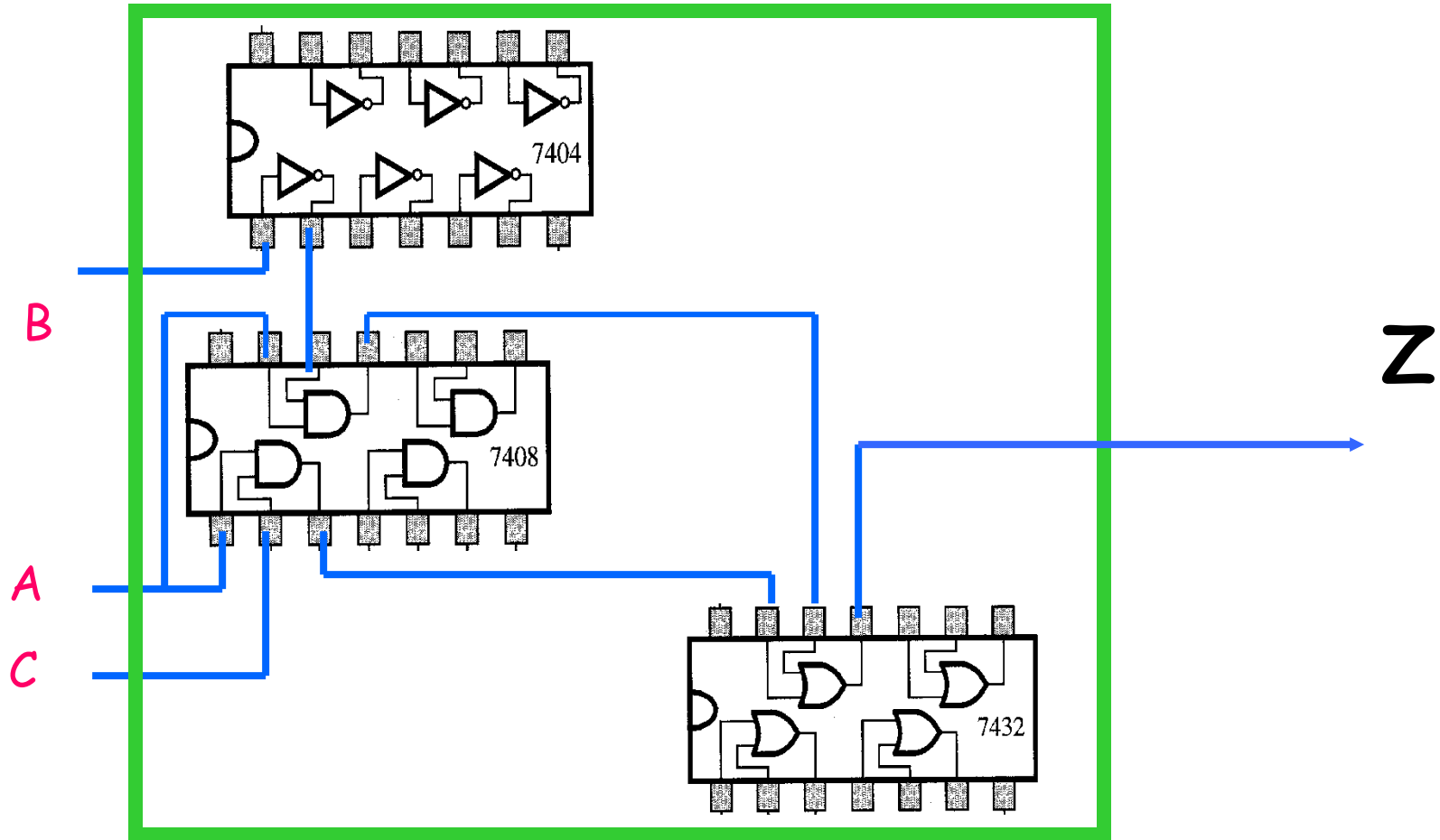
Chip Implementation for: $AC + A\bar{B} = Z$



Chip Implementation for: $AC + A\bar{B} = Z$



The board ... $AC + A\bar{B} = Z$



Simplify the expression: $ABC + AB\bar{C} + A\bar{B}C$

New Example

Simplify the expression: $ABC + AB\bar{C} + A\bar{B}C$

$$= AB(C + \bar{C}) + A\bar{B}C$$

Simplify the expression: $ABC + AB\bar{C} + A\bar{B}C$

$$= AB(C + \bar{C}) + A\bar{B}C$$

$$= AB + A\bar{B}C$$

Simplify the expression: $ABC + AB\bar{C} + A\bar{B}C$

$$= AB(C + \bar{C}) + A\bar{B}C$$

$$= AB + A\bar{B}C$$

$$= A(B + \bar{B}C)$$

Result

$$= AB(C + \overline{C}) + A\overline{B}C$$

$$= AB + A\overline{B}C$$

$$= A(B + \overline{B}C)$$

$$= A(B + C)$$

Simplify the expression: $ABC + AB\bar{C} + A\bar{B}C$

$$= AB(C + \bar{C}) + A\bar{B}C$$

$$= AB + A\bar{B}C$$

$$= A(B + \bar{B}C)$$

$$= A(B + C)$$

Another proof ?

Simplify the expression: $ABC + AB\bar{C} + A\bar{B}C$

$$= AB(C + \bar{C}) + A\bar{B}C$$

$$= AB + A\bar{B}C$$

$$= A(B + \bar{B}C)$$

$$= A(B + C)$$

Another proof:



Because: $A + A = A$

$$= ABC + ABC + ABC + ABC$$

=

Simplify the expression: $ABC + AB\bar{C} + A\bar{B}C$

$$= AB(C + \bar{C}) + A\bar{B}C$$

$$= AB + A\bar{B}C$$

$$= A(B + \bar{B}C)$$

$$= A(B + C)$$

Another proof:

Because: $A + A = A$

$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$$

$$= AB(C + \bar{C}) + AC(B + \bar{B})$$

$$= AB + AC = A(B + C)$$

Simplify the expression

$$\bar{A}C[\overline{\bar{A}BD}] + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

New Example

Simplify the expression

$$\begin{aligned} & \bar{A} C [\overline{\bar{A} B D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C \\ &= \bar{A} C [\bar{A} + \bar{B} + \bar{D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C \end{aligned}$$

Simplify the expression

$$\begin{aligned} & \bar{A} C [\overline{\bar{A} B D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C \\ &= \bar{A} C [A + \bar{B} + \bar{D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C \\ &= \bar{A} C A + \bar{A} C \bar{B} + \bar{A} C \bar{D} + \bar{A} B \bar{C} \bar{D} + A \bar{B} C \end{aligned}$$

Simplify the expression

$$\bar{A} C [\overline{\bar{A} B D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C$$

$$= \bar{A} C [A + \bar{B} + \bar{D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C$$

$$= \bar{A} C A + \bar{A} C \bar{B} + \bar{A} C \bar{D} + \bar{A} B \bar{C} \bar{D} + A \bar{B} C$$

$$\bar{A} C A = 0$$

($A \bar{A} = 0$),

Simplify the expression

$$\bar{A} C [\overline{\bar{A} B D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C$$

$$= \bar{A} C [A + \bar{B} + \bar{D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C$$

$$= \bar{A} C A + \bar{A} C \bar{B} + \bar{A} C \bar{D} + \bar{A} B \bar{C} \bar{D} + A \bar{B} C$$

$\bar{A} C A = 0$ (since $A \bar{A} = 0$), take 2nd with 5th and 3rd with 4th terms.

Simplify the expression

$$\begin{aligned}& \bar{A} C [\overline{\bar{A} B D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C \\&= \bar{A} C [A + \bar{B} + \bar{D}] + \bar{A} B \bar{C} \bar{D} + A \bar{B} C \\&= \bar{A} C A + \bar{A} C \bar{B} + \bar{A} C \bar{D} + \bar{A} B \bar{C} \bar{D} + A \bar{B} C\end{aligned}$$

($A \bar{A} = 0$), take 2nd with 5th and 3rd with 4th terms.

$$= \bar{B} C [A + \bar{A}] + \bar{A} \bar{D} [C + B \bar{C}]$$

Simplify the expression

$$\bar{A}C[\overline{\bar{A}BD}] + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

$$= \bar{A}C[A + \bar{B} + \bar{D}] + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

$$= \bar{A}CA + \bar{A}C\bar{B} + \bar{A}C\bar{D} + \bar{A}B\bar{C}\bar{D} + A\bar{B}C$$

\bar{A}
($A\bar{A} = 0$), take 2nd with 5th and 3rd with 4th terms.

$$= \bar{B}C[A + \bar{A}] + \bar{A}\bar{D}[C + B\bar{C}]$$

$$= \bar{B}C + \bar{A}\bar{D}[C + B]$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

New Example

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD]\overline{D}$$

Simplify the expression

$$(\bar{A} + B)(A + B + D)\bar{D}$$

$$= [\bar{A}A + \bar{A}B + \bar{A}D + BA + BB + BD]\bar{D}$$

$$= \bar{A}A\bar{D} + \bar{A}B\bar{D} + \bar{A}D\bar{D} + BA\bar{D} + BB\bar{D} + BD\bar{D}$$

Simplify the expression

$$(\overline{A} + B)(A + B + D)\overline{D}$$

$$= [\overline{A}A + \overline{A}B + \overline{A}D + BA + BB + BD]\overline{D}$$

$$= \overline{A}A\overline{D} + \overline{A}B\overline{D} + \overline{A}D\overline{D} + BA\overline{D} + BB\overline{D} + BD\overline{D}$$

$$= 0 + \overline{A}B\overline{D} + 0 + BA\overline{D} + B\overline{D} + 0$$

Simplify the expression

$$\begin{aligned} & (\bar{A} + B)(A + B + D)\bar{D} \\ &= [\bar{A}A + \bar{A}B + \bar{A}D + BA + BB + BD]\bar{D} \\ &= \bar{A}A\bar{D} + \bar{A}B\bar{D} + \bar{A}D\bar{D} + BA\bar{D} + BB\bar{D} + BD\bar{D} \\ &= 0 + \bar{A}B\bar{D} + 0 + BA\bar{D} + B\bar{D} + 0 \\ &= B\bar{D}[\bar{A} + A + 1] \end{aligned}$$

Simplify the expression

$$\begin{aligned}& (\bar{A} + B)(A + B + D)\bar{D} \\&= [\bar{A}A + \bar{A}B + \bar{A}D + BA + BB + BD]\bar{D} \\&= \bar{A}A\bar{D} + \bar{A}B\bar{D} + \bar{A}D\bar{D} + BA\bar{D} + BB\bar{D} + BD\bar{D} \\&= 0 + \bar{A}B\bar{D} + 0 + BA\bar{D} + B\bar{D} + 0 \\&= B\bar{D}[\bar{A} + A + 1] \\&= B\bar{D}[1 + 1]\end{aligned}$$

Simplify the expression

$$\begin{aligned}& (\bar{A} + B)(A + B + D)\bar{D} \\&= [\bar{A}A + \bar{A}B + \bar{A}D + BA + BB + BD]\bar{D} \\&= \bar{A}A\bar{D} + \bar{A}B\bar{D} + \bar{A}D\bar{D} + BA\bar{D} + BB\bar{D} + BD\bar{D} \\&= 0 + \bar{A}B\bar{D} + 0 + BA\bar{D} + B\bar{D} + 0 \\&= B\bar{D}[\bar{A} + A + 1] \\&= B\bar{D}[1 + 1] \\&= B\bar{D}(1) = B\bar{D} \quad \leftarrow\end{aligned}$$

Another example

Given:

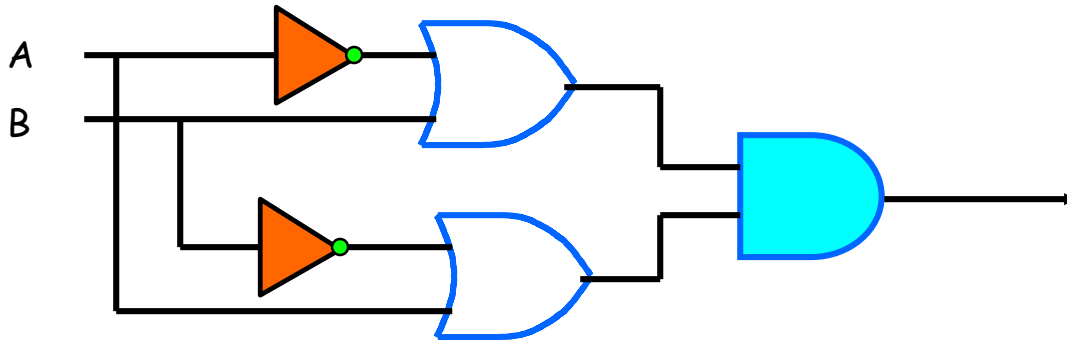
$$(A' + B)(B' + A)$$

1. Implement using basic (AND, OR, NOT) gates
2. Simplify
3. Implement the simplified expression

New Example

1. First implementation

$$(A' + B)(B' + A)$$

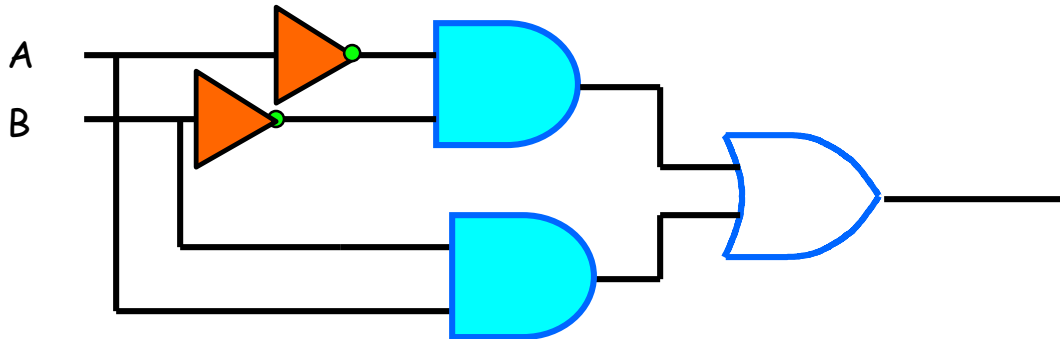


2. Simplify

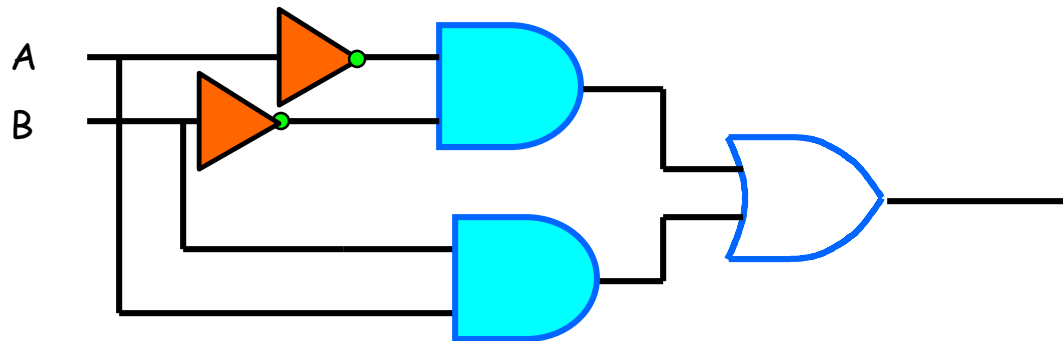
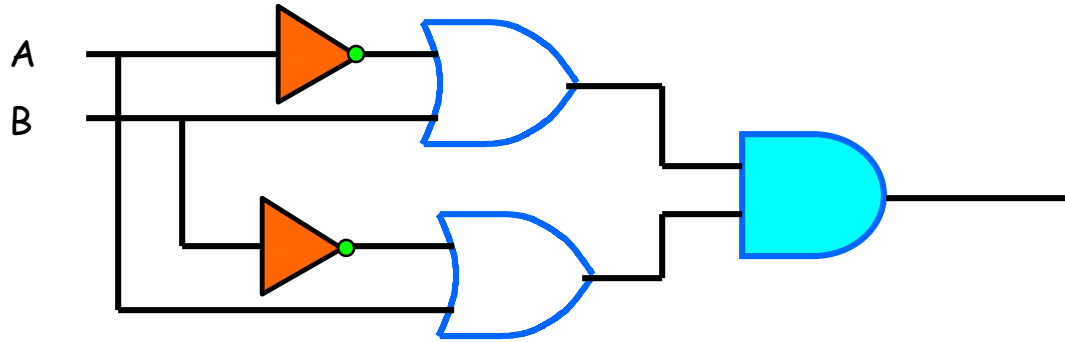
$$\begin{aligned}(A'+B)(B'+A) &= A'B'+A'A+B'B+BA \\ &= A'B' + AB\end{aligned}$$

3. Second implementation

$$\begin{aligned}(A'+B)(B'+A) &= A'B'+A'A+B'B+BA \\ &= A'B' + AB\end{aligned}$$



Both implementations are equivalent



Simplify the expression

$$Z = A\bar{C}\bar{D} + \bar{A}BD + A\bar{B}CD$$

New Example

$$Z = Z$$

$$Z = \overline{A}\overline{C}\overline{D} + \overline{A}BD + A\overline{B}CD$$

Can not be simplified

Therefore ...

- *Three cases ...*