Complements

2's = 1's + 1

TO SIMPLIFY THE SUBTRACTION OPERATION AND FOR LOGICAL MANIPULATIONS USE

Complements

Complements

- Are used in digital computers (ALU) to simplify the **subtraction** operation and for logical manipulations

Complements; r's & (r-1)'s

- Are used in digital computers (ALU) to simplify the **subtraction** operation and for logical manipulations

  - r 's or N(r) complement
  - (r-1)' s or N(r-1) complement

Complements; 2's

- Decimal: r's = 10's
- **Binary: r's = 2's**

Complements; 2's & 1's

- Decimal: r's = 10's
- **Binary: r's = 2's**

- Decimal: (r-1)'s = 9's
- **Binary: (r-1)'s = 1's**

N(r) Complement-formula

$$\boxed{N(r) = r^n - N}$$

r = our base

n = integer digits of the number

N = our number

N(r) = r's complement of N

DECIMAL NUMBER EXAMPLE

Find r's (10's) complement: 52520.0

Example

- 52520.0

Find r's (10's) complement: 52520.0

- 52520.0

  n = 5

  r = 10

  N = 52520

$\boxed{N(r) = r^n - N}$

Find r's (10's) complement: 52520.0

- 52520.0

  n = 5

  r = 10

  N = 52520

$N(r) = r^n - N$

Answer = 10^5 - 52520 = 47480

Find r's (10's) complement: 52520.0

- 52520.0

  n = 5

  r = 10

  N = 52520

N(10) = 10^5 - 52520 = 47480

Note: 52520+47480 = 100000 = 10^5

BINARY NUMBER EXAMPLE

Find r's (2's) complement: 101100

- 101100

NewExample

$\boxed{N(r) = r^n - N}$

Find r’s (2’s) complement: 101100

- <u>101100</u>

n = 6

r = 2

N = 101100

$\boxed{N(r) = r^n - N}$

Find r's (2's) complement: 101100

- **101100**

n = 6

r = 2

N = 101100

$\boxed{N(r) = r^n - N}$

$$N(2) = (2^6)_{10} - (101100)_2$$
$$= (64)_{10} - (101100)_2$$
$$= (1000000)_2 - (101100)_2$$
$$= (\mathbf{0010100})_2$$

N(r-1) complement-formula

$$N(r-1) = r^{n} - r^{-m} - N$$

where,

- m = number of fraction digits of N
- N = our number
- r = base
- n = number of integer digits of N
- N(r-1) = (r-1)'s complement of N

DECIMAL NUMBER EXAMPLE

Find r-1 (9's) complement of 52520

- r = 10
- n = 5
- m = 0

Example

$$\boxed{N(r-1) = r^{n} - r^{-m} - N}$$

(r-1) 9's complement of 52520

- $r = 10$
- $n = 5$
- $m = 0$
- $N = 52520$

$$\boxed{N(r-1) = r^{n} - r^{-m} - N}$$

$$N(9) = 10^5 - 10^0 - 52520$$
$$= 47479$$

(r-1) 9's complement of 52520

- r = 10
- n = 5
- m = 0
- N = 52520

N(9) = 10^5 - 10^0 - 52520

= 47479

Note: 52520+47479 = 99999 = 10^5 - 1

BINARY NUMBER EXAMPLE

Find (r-1) 1's complement of $(101100)_2$

- $r = 2$
- $n = 6$
- $m = 0$
- $N = 101100$

NewExample

$$\boxed{N(r-1) = r^{n} - r^{-m} - N}$$

(r-1) 1's complement of $(101100)_2$

- $r = 2$
- $n = 6$
- $m = 0$
- $N = 101100$

$$\boxed{N(r-1) = r^{n} - r^{-m} - N}$$

$$N(1) = 2^6 - 2^0 - (101100)_2$$
$$= (1000000)_2 - (1)_2 - (101100)_2$$
$$= (\mathbf{010011})_2$$

$N(1) = 1's$

1's complement; Quick method

- Easy memorization rule for finding the 1's complement (binary).
  - **Change 1's to 0's and 0's to 1's**

$$a = 1\ 1\ 0\ 1\ 0\ 0\ 1$$

$$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow$$

$$\text{1's of } a = 0\ 0\ 1\ 0\ 1\ 1\ 0$$

2's complement; Quick method ?

Derive : 2's complement formula

- 2's = 2^{n} - N
- 1's = 2^{n} - 2^{- m} - N

N(2)	=	2's
N(1)	=	1's

Derive : 2's complement formula

- 2's = $2^{n} - N$
- 1's = $2^{n} - 2^{-m} - N$

or

- 1's = $\boxed{2^{n} - N} - 2^{-m}$

Derive : 2's complement formula

- 2's = $2^{n} - N$
- 1's = $2^{n} - 2^{-m} - N$

  or

- 1's = $\boxed{2^{n} - N} - 2^{-m}$

  or

  1's = 2's $- 2^{-m}$

Derive : 2’s complement formula

- $2's = 2^{n} - N$
- $1's = 2^{n} - 2^{-m} - N$

  or
- $1's = 2^{n} - N - 2^{-m}$

  or

  $1's = 2's - 2^{-m}$

  or
- $2's = 1's + 2^{-m}$

Derive : 2's complement formula

- $2's = 2^{n} - N$
- $1's = 2^{n} - 2^{-m} - N$

  or
- $1's = 2^n - N - 2^{-m}$

  or

  $1's = 2's - 2^{-m}$

  or
- $2's = 1's + 2^{-m}$, for $m = 0$

2's = 1's + 1

- 2's = $2^{n} - N$
- 1's = $2^{n} - 2^{-m} - N$

  or
- 1's = $2^{n} - N - 2^{-m}$

  or

  1's = 2's $- 2^{-m}$

  or
- 2's = 1's $+ 2^{-m}$, for $m = 0$
- **2's = 1's + 1**

2’s complement-example

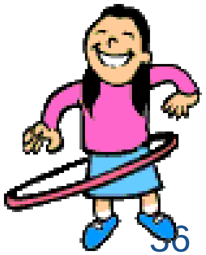
1	0	1	0	1	0	0

1's complement

1	0	1	0	1	0	0	
0	1	0	1	0	1	0	← 1's

2’s complement

1	0	1	0	1	0	0	
0	1	0	1	0	1	0	1’s
					+	1	
0	1	0	1	1	0	0	2’s



Why do we learn about complements ?

- To simplify subtraction …
- Subtraction performed by digital computers is much more efficient using 2’s complements

Binary Subtraction using Complements

(M-N) algorithm using (2's)

1. Find 2's complement of N
2. Add M to 2's complement of N
   a) If an end carry occurs, discard it and whatever is left is your answer
   b) If an end carry does not occur, take the 2's complement of the number obtained in step 1 and place a minus (-) sign in front of it

Subtract 84-68, using 2’s complement

- M = 1010100 = 84
- -N = 1000100 = -68

Subtraction using complements (M-N)

- M = 1010100 = 84
- -N = 1000100 = -68

.

2's of N ?

1's of N =	0111011
	+1
2's of N =	0111100

Subtraction using complements (M-N)

- M = 1010100 = 84
- -N = 1000100 = -68
-

.

Therefore,

1010100

+ 0111100

10010000,

out

2's of N ?

1's of N = 0111011

+1

2's of N = 0111100

Answer: (10000)$_2$ = (16)$_{10}$

Overflow

- Note that the answer (**10010000**) is *8*-bits long, while the inputs were only 7 bits. This is … overflow
- The answer 10010000 is correct, but the result cannot be used in further computations

Subtract 68-84, using 2’s complement

Subtract 68-84, using 2's complement

- M = 100 0100 = 68
- -N = -101 0100 = -84

-16

Subtraction using complements

- M = 100 0100 = 68
- -N = -101 0100 = -84

-16

2's of 1010100 ?

1's = 0101011

+ 1

0101100

Subtraction using complements

- M = 100 0100 = 68
- -N = -101 0100 = -84

-16

2's of 1010100 ?

1's = 0101011

+ 1

0101100

Therefore,

100 0100

+ 010 1100

NC 111 0000

The answer is: - 2's of (111 0000) = -10000 = $(-16)_{10}$

Therefore...

$$
\begin{aligned}
M - N &= M + \text{2's comp. of } N \\
&= M + \{\, (\text{1's comp. of } N) + 1 \,\}
\end{aligned}
$$

Therefore…

$$M - N = M + \text{2's comp. of } N$$
$$= M + \{\, (\text{1's comp. of } N) + 1 \,\}$$

(Will be realized with gates…)

To avoid using the minus sign … in front of …

To avoid using the minus sign … in front of …

- Use **signed binary numbers** ⟹

Signed binary numbers

- Positive numbers (binary) are represented by placing a bit (0) in the leftmost position

Signed binary numbers

- Positive numbers (binary) are represented by placing a bit (0) in the leftmost position
- Negative numbers use a bit (1) in the leftmost position

  - bit 0 = + (Positive number)
  - bit 1 = - (Negative number)

Examples: Signed/unsigned binary numbers

- 01001 = 9; if unsigned
- 01001 = +9; if signed
- 11001 = 25; if unsigned
- 11001 = -9; if signed

8-bit negative signed number; 2’s comp

- +9 signed 0000 1001

8-bit negative signed number; 2's comp

- +9 signed 0000 1001
- -9 singed - mag. 1000 1001

8-bit negative signed number; 2's comp

- +9 signed 0000 1001
- -9 singed - mag. 1000 1001
- -9 singed - mag.1's 1111 0110

8-bit negative signed number; 2's comp

• +9 signed	0000 1001
• -9 singed - mag.	1000 1001
• -9 singed - mag.1's	1111 0110
	+ 1
• -9 singed - mag. 2's	1111 0111

Addition of signed numbers: 2’s complement

Example-1

+ 6	0000 0110
+ 13	0000 1101
+ 19	

Example-1

$+\ 6$	0000 0110
$+\ 13$	0000 1101
$+\ 19$	0001 0011 $= (19)_{10}$

Example-2

$$(-6) + 13 = 7$$

Example-2

- $-\ 6$
- $+\ 13$

$+\ 7$

Example-2

- \- 6
- \+ 13

\+ 7

2's of (0000 0110) = 6 ?

1111 1001

Example-2

- \- 6
- \+ 13

\+ 7

2's of (0000 0110) = 6

1111 1001

+1

1111 1010 = (-6)

Example-2

- \- 6
- \+ 13

\+ 7

2's of (0000 0110) = 6

1111 1001

+1

1111 1010 = (-6)

1111 1010

+0000 1101 = $(+13)_{10}$

1 0000 0111

Example-2

- \- 6
- \+ 13

\+ 7

2's of (0000 0110) = 6

1111 1001

+1

1111 1010 = (-6)

1111 1010

+0000 1101 = $(+13)_{10}$

1 0000 0111

out

Answer: $(0000\ 0111)_2 = (7)_{10}$

Example-3

$$6 + (-13) = -7$$

Example-3

- + 6
- - 13

- 7

(0000 1101) = 13

Example-3

- \+ 6
- \- 13

- 7

2's of (0000 1101) = 13?

1111 0010

+1

1111 0011 = -13

Example-3

- \+ 6
- \- 13

\- 7

2's of (0000 1101) = 13

1111 0010

+1

1111 0011 = -13

0000 0110 = + 6

+1111 0011 = 2's of $(-13)_{10}$

Example-3

- \+ 6
- \- 13

\- 7

2's of (0000 1101) = 13

1111 0010

+1

1111 0011 = -13

0000 0110 = + 6

+1111 0011 = 2's of $(-13)_{10}$

1111 1001 = $(-7)_{10}$, **in 2's complement form**

0000 0110
+ 1

00000 111

Example-4

$$(-6) + (-13) = -19$$

Example-4

- \- 6
- \- 13

\+

\- 19

2's of 6 = 1111 1010

2's of 13 = 1111 0011

Example-4

- \- 6
- \- 13

\+

- 19

2's of 6 = 1111 1010

2's of 13 = 1111 0011

```
  1111 1010
 +1111 0011
 ----------
1 1110 1101
```

Example-4

- - 6
- - 13

+

- 19

2's of 6 = 1111 1010

2's of 13 = 1111 0011

```
  1111 1010
 +1111 0011
 ----------
1 1110 1101
```

out

Example-4

- $-\ \ 6$
- -13

$+$

-19

2's of 6 = 1111 1010

2's of 13 = 1111 0011

```
  1111 1010
 +1111 0011
 ----------
1 1110 1101
```

out

Answer: 1110 1101 ⇦ $(-19)_{10}$, in 2's comp.form

Procedure:

1. If both numbers are positive the addition is as known (discard the carry if any)
2. If one of the numbers is negative, then:

   a) Find the 2's complement of the negative number
   b) Add the result of (a) to the positive number (discard the carry if any)

Signed 2's complement

- Is the most preferable form in computing (CMPT280)

END