Monday Jun 8 :

what is signal to noise ratio?

Suppose that we have concatinated the columns of an image & created a vector $x \in \mathbb{R}^p$. Image denoising is the problem of estimating x from the observations

$$y = \beta + w, \text{ where } w_i \sim N(0, \sigma^2).$$

If σ is small, y will look like x & finding a good estimate of x is easy. However, if σ is large, it will be hard to get an estimate of x. That is why the notion of signal to noise ratio (SNR) is very important in this application. How do we define it?

$$SNR = \frac{\text{power of signal}}{\text{power of noise}} = \frac{\frac{1}{p}\sum_{i=1}^{p} x_i^2}{\sigma^2}.$$

How do we define SNR for imaging system?

In imaging systems, we have

$$y = X\beta + w, \qquad X \in \mathbb{R}^{n \times p}$$

$$w \sim N(0, 1).$$

Intuitively speaking, still SNR is important. But how do we want to define SNR. Here, usually we define SNR per measurement. Let

$$X = \begin{bmatrix} - x_1^T - \\ - x_2^T - \\ \vdots \\ - x_n^T - \end{bmatrix}$$

then, $y_i = x_i^T \beta + w_i$

$$\text{SNR per measurement} = \frac{E(x_i^T \beta)^2}{}$$

σ^2

If $x_{ij} \sim N(0, \frac{1}{n})$ $\Rightarrow$ $\frac{\mathbb{E}(x_i^T \beta)^2}{\sigma^2} = \frac{\frac{1}{n}\beta^T\beta}{\sigma^2}$.

$$= \frac{\frac{1}{n}\sum_{i=1}^{p} \beta_i^2}{\sigma^2}.$$

Please note that the statistics of the measurement matrix also affects the SNR.

PSNR(Peak Signal to noise ratio)

this is the most popular measure for evaluating the quality of the estimates.

Suppose β is the true image & based on y we get an estimate $\hat{\beta}$. Then PSNR of $\hat{\beta}$ is:

$$PSNR = 10 \log \frac{(\max |\beta_i|)^2}{\frac{1}{p} \sum_{i=1}^{p} (\beta_i - \hat{\beta}_i)^2}.$$