

ExpEcon Methods: A Theory of Testing Theories

ECON 8877

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Introduction

A principal wants to learn *something* about the preferences of an agent, but *not* the whole ordering
(Why not? complexity, costs, privacy, etc.)

Example: NYC school match: only list favorite 12 schools

Which properties of preferences can be elicited in an incentive compatible way?

Leading Example:

$X = \{x, y, z\}$. Let xyz denote $x \succ y \succ z$, e.g. Assume strict prefs.

All orderings:

$\{xyz, xzy, zxy, zyx, yzx, yxz\}$

A simple elicitation mechanism:

Pick from $\{x, y\}$

Paid what you choose

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$$\underbrace{\{xyz, xzy, zxy\}}_{\text{pick } x}, \underbrace{\{zyx, yzx, yxz\}}_{\text{pick } y}$$

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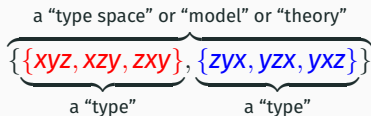
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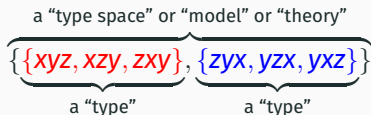
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This type space is *elicitable*. Truth FOSD's lie.

$\{xyz, xzy, zxy, zyx, yzx, yxz\}$

Mechanism:

Pick from $\{x, y\}$ *and* from $\{x, z\}$

We randomly pick *one* of your answers and pay it to you

$$\underbrace{\{\{xyz, xzy\}\}}_{\text{pick } x,x}, \underbrace{\{zxy\}}_{\text{pick } x,z}, \underbrace{\{zyx, yzx\}}_{\text{pick } y,z}, \underbrace{\{yxz\}}_{\text{pick } y,x}$$

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$$\underbrace{\{\{xyz, yxz\}\}}_{\text{dislike } z}, \underbrace{\{\{xzy, zxy\}\}}_{\text{dislike } y}, \underbrace{\{\{yzx, zyx\}\}}_{\text{dislike } x}$$

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There are **no** menus that generate this type space.
Generated by top *two* elements of X

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Mechanism:
Announce least favorite,
get paid 50-50 lottery over the other two options.

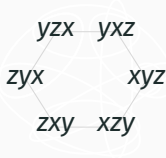
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Generated by top k elements \Rightarrow elicitable \Rightarrow “convex”

Results

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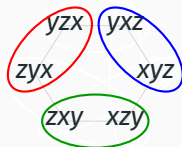
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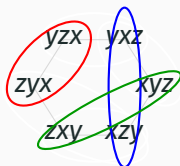
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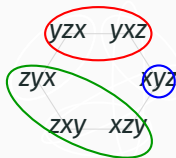
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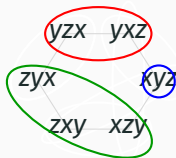
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Preview of Main Results:

Generated by top k elements \Rightarrow elicitable \Rightarrow “convex”



We get complete characterization when:

1. Restrict to neutral type spaces, or
2. Pay in acts, not lotteries (no objective probabilities)

Some related literature

- Osband (1985), Lambert-Pennock-Shoham (2008), Lambert (2018): Scoring rules to elicit properties of beliefs
- Gibbard (1977), Bahel and Sprumont (2019): Characterizing strategy-proof random mechanisms
- Carroll (2012), Saito (2013): Sufficiency of local IC constraints
- ACH (2018, 2020): Characterizing IC mechanisms in experimental environments

The General Model

Framework

- X - a finite set of alternatives
 - Typical elements: x, y, z, w, \dots
- O - the set of strict orders over X
 - Typical elements: \succ, \succ', \dots

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Definition

A *type space* $T = \{t_1, \dots, t_k\}$ is a partition of O .

- A *type* is any $t \in T$, so $t = \{\succeq, \succeq', \dots, \succeq''\}$
- Example: $t = \{\text{all } \succeq \text{ satisfying the Independence axiom}\}$
- Notation: $t(\succeq) \in T$ is the type containing \succeq

Examples

$$X = \{x, y, z\}$$

- Entire ranking:

$$T = \{\{xyz\}, \{yxz\}, \{yzx\}, \{zyx\}, \{zxy\}, \{xzy\}\}$$

- First-best:

$$T = \{\{xyz, xzy\}, \{yxz, yzx\}, \{zyx, zxy\}\}$$

- Top-2:

$$T = \{\{xyz, yxz\}, \{xzy, zxy\}, \{zyx, yzx\}\}$$

- Best from $\{x, y\}$:

$$T = \{\{xyz, xzy, zxy\}, \{zyx, yzx, yxz\}\}$$

- Where you rank x:

$$T = \{\{xyz, xzy\}, \{zxy, yxz\}, \{yzx, zyx\}\}$$

(This type space is not “neutral”. Labels matter.)

$\Delta(X)$ is the set of lotteries on X

Definition

A T -mechanism is any $g : T \rightarrow \Delta(X)$.

- Why random payments?
 - Allows use of the RPS mechanism (and more)
 - With deterministic mechanisms very little can be elicited

Elicitable type spaces

Recall that p strictly FOSD q relative to \succeq (written $p \succ^* q$) if

$$\forall x \in X \quad p(\{y : y \succeq x\}) \geq q(\{y : y \succeq x\})$$

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A type space T is *elicitable* if there exists an IC T -mechanism.

Goal: Characterize elicitable type spaces (spoiler: we can't)

Top elements of menus

“What’s your favorite thing from X' ?”

- Every menu $X' \subseteq X$ corresponds to a type space:

$$\succeq, \succeq' \in t \iff \succeq, \succeq' \text{ have the same favorite item in } X'$$

Examples:

$$X' = \{x, y\} \implies T = \{\{xyz, xzy, zxy\}, \{zyx, yzx, yxz\}\}$$

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- The (deterministic) mechanism that pays the revealed top element in X' is IC

- One can elicit top elements of several menus $X_1, \dots, X_l \subseteq X$

Examples:

$$X_1 = \{x, y, z\}, X_2 = \{x, y\}$$

$$\implies T = \{\{xyz, xzy\}, \{yzx, yxz\}, \{zxy\}, \{zyx\}\}$$

$$X_1 = \{x, y\}, X_2 = \{x, z\}, X_3 = \{y, z\}$$

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- The corresponding IC mechanism randomly chooses a menu and pays the announced top element
- This is widely used in experimental economics

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What else is elicitable?

Top sets of menus

The top-2 type space $T = \{\{xyz, yxz\}, \{xzy, zxy\}, \{zyx, yzx\}\}$ does not reveal top elements of menus but is elicitable

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- How? If they announce “x and y” pay x and y with equal probability, and z with less probability.
- Every $X' \subseteq X$ and k defines a type space by

$$\succeq, \succeq' \in \mathbf{t} \iff \succeq, \succeq' \text{ have the same top } k \text{ elements of } X'$$

- This is elicitable by paying the uniform lottery over the set of announced top- k elements
- Can elicit the top- k_i elements of $X_i \subseteq X, i = 1, \dots, l$

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Anything else??

Example (based on Shapley, 1971)

$$X = \{x, y, z, w\}$$

Type space:

$\{xyzw, yxzw, xywz, yxwz\}$
 $\{xzyw\}, \{xwyz\}, \{xzw y, xwzy\}$
 $\{ywxz\}, \{yzxw\}, \{yzwx, ywzx\}$
 $\{zxyw, zywx\}, \{zywx, zwyx\}, \{zxwy, zwxy\}$
 $\{wxyz, wyxz\}, \{wyzx, wzyx\}, \{wxzy, wzxy\}$

Claim

\exists IC mechanism, but type space is not generated by top sets.

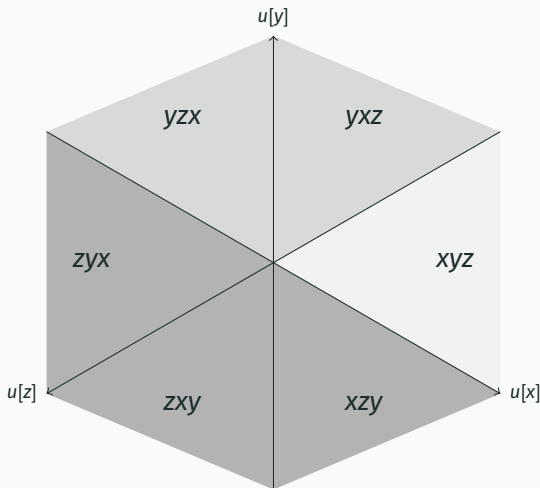
There is a close connection between IC mechanisms and convex TU cooperative games...

$$\begin{aligned} & \{\text{all } T\} \\ & \cup \\ & \{\textcolor{red}{T} : \text{elicitable}\} \\ & \cup \\ & \{T : \text{generated by top sets}\} \\ & \cup \\ & \{T : \text{generated by top elements}\} \end{aligned}$$

A convex type space - example

Necessary condition: **convex** type space

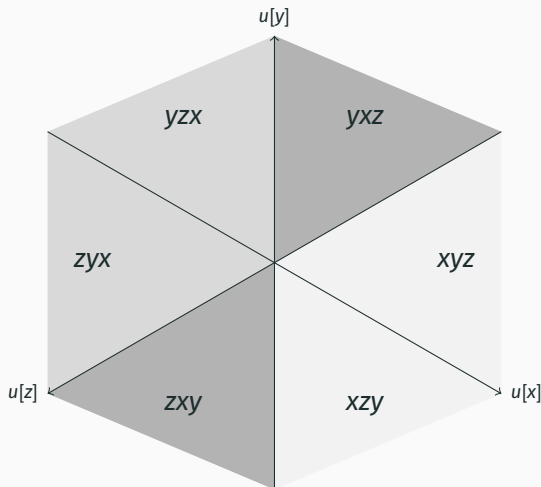
Example: $T = \{\{xyz\}, \{yxz, yzx\}, \{zyx, zxy, xzy\}\}$



A non-convex type space - example

Example of a non-convex type space:

$$T = \{\{xyz, xzy\}, \{yxz, zxy\}, \{zyx, yzx\}\}$$



Convexity is necessary

Proposition

If T is elicitable then it is convex.

Ex: Where do you rank x ?

$t = x$ is 2nd (dark gray)

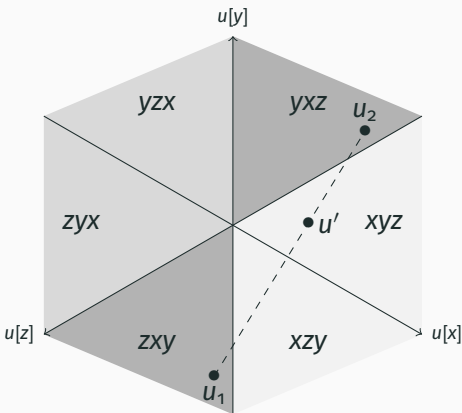
$t' = x$ is 1st (off-white)

IC requires:

$$\mathbb{E}_{g(t)}(u_1) > \mathbb{E}_{g(t')}(u_1)$$

$$\mathbb{E}_{g(t)}(u_2) > \mathbb{E}_{g(t')}(u_2)$$

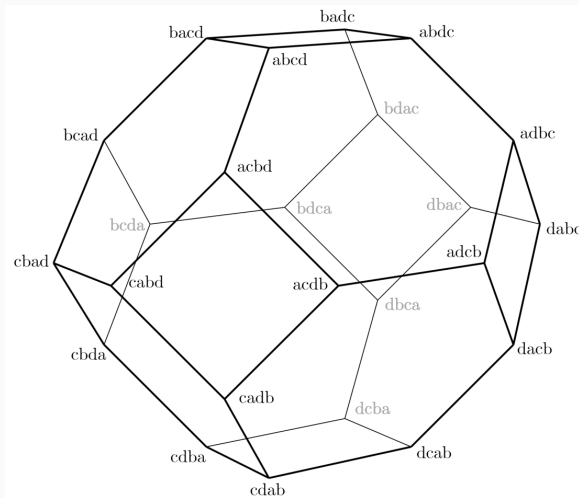
$$\implies \mathbb{E}_{g(t)}(u') > \mathbb{E}_{g(t')}(u')$$



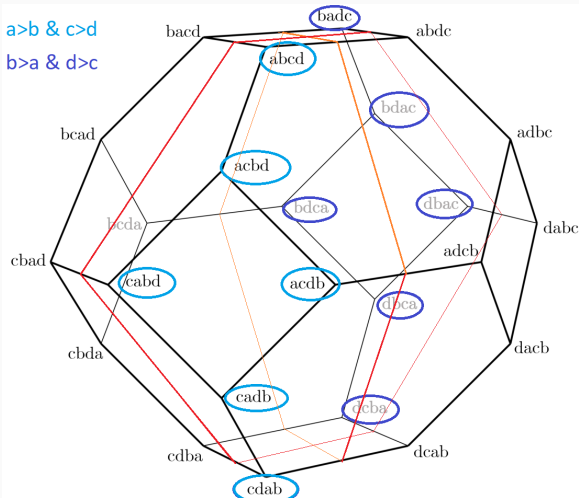
Some non-convex type spaces

- Where do you rank x ? (with $|X| \geq 3$)
- What is the k th ranked alternative for $1 < k < |X|$ (e.g. median)
- Any binary $T = \{t_1, t_2\}$, *except* $T = \{\{x \succeq y\}, \{y \succeq x\}\}$.
In particular, tests of most axioms of preferences!
Usually: “If $x \succeq y$ then $w \succeq z$ (and $y \succeq x \Rightarrow z \succeq w$)”

Visualizing Convexity: The Permutohedron



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Convexity is not sufficient

$$T = \{t_1 = \{xyz\}, t_2 = \{yxz, yzx\}, t_3 = \{zyx, zxy, xzy\}\}$$

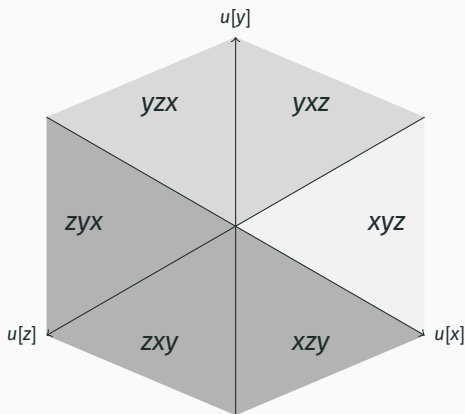
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$$g(t_1)(x) > g(t_2)(x)$$

$$g(t_2)(x) = g(t_3)(x)$$

$$g(t_3)(x) = g(t_1)(x)$$

$$\implies g(t_1)(x) > g(t_1)(x)$$



Summary

$$\begin{aligned} & \{\text{all } T\} \\ & \cup \\ & \{T : \text{convex}\} \\ & \cup \\ & \{T : \text{no bad cycles}\} \\ & \cup \\ & \{T : \text{elicitable}\} \\ & \cup \\ & \{T : \text{generated by top sets}\} \\ & \cup \\ & \{T : \text{generated by top elements}\} \end{aligned}$$

Neutral type spaces

- Permutation: $\pi : X \rightarrow X$
- Let πT be T , but with every \succeq permuted by π

Definition

T is *neutral* if $\pi T = T$ for every π .

Neutral: “What do you rank 3rd?”

Not: “Where do you rank x ?”

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Not: “Where do you rank x ?”

Proposition

Suppose T is neutral. Then the following are equivalent:

- (1) T is elicitable
- (2) T is convex
- (3) T is generated by top sets

Neutral type spaces

$$\begin{aligned} & \{\text{all } T\} \\ & \cup \\ & \{T : \text{convex}\} \\ & \parallel \\ & \{T : \text{elicitable}\} \\ & \parallel \\ & \{T : \text{generated by top sets}\} \\ & \cup \\ & \{T : \text{generated by top elements}\} \end{aligned}$$

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- This kills our ability to elicit top sets

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Proposition

T is elicitable with acts iff it is generated by top elements.

$$\begin{aligned} & \{ \text{all } T \} \\ & \cup \\ & \{ T : \text{convex} \} \\ & \cup \\ & \{ T : \text{elicitable with lotteries} \} \\ & \cup \\ & \{ T : \text{generated by top sets} \} \\ & \cup \\ & \{ T : \text{generated by top elements} \} \\ & \parallel \\ & \{ T : \text{elicitable with acts} \} \end{aligned}$$

Multiple agents

- $N = \{1, \dots, n\}$ - agents
- T_i - agent's i type space
- $T = (T_1, \dots, T_n)$ - a profile of type spaces
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Proposition

$T = (T_1, \dots, T_n)$ is dominant-strategy-elicitable iff each T_i is elicitable.

Conclusion

- We formulate a notion of elicibility for properties of preferences
- Some necessary conditions and some sufficient conditions for elicibility, but no characterization
- We do have a characterization for neutral type spaces and for robust elicitation (acts)
- Potential extensions: Weak orders, infinite sets of alternatives, domain restrictions,...

Thank You!