

ExpEcon Methods:

A Theory of Testing Theories

ECON 8877

P.J. Healy

Updated 2026-01-28 at 02:19:01

Introduction

A principal wants to learn *something* about the preferences of an agent, but *not* the whole ordering
(Why not? complexity, costs, privacy, etc.)

Example: NYC school match: only list favorite 12 schools

Which properties of preferences can be elicited in an incentive compatible way?

Leading Example:

$X = \{x, y, z\}$. Let xyz denote $x \succ y \succ z$, e.g. Assume strict prefs.

All orderings:

$$\{xyz, xzy, zxy, zyx, yzx, yxz\}$$

A simple elicitation mechanism:

Pick from $\{x, y\}$

Paid what you choose

Leading Example:

$X = \{x, y, z\}$. Let xyz denote $x \succ y \succ z$, e.g. Assume strict prefs.

All orderings:

$$\underbrace{\{\textcolor{red}{xyz, xzy, zxy}\}}_{\text{pick } x}, \underbrace{\{\textcolor{blue}{zyx, yzx, yxz}\}}_{\text{pick } y}$$

A simple elicitation mechanism:

Pick from $\{x, y\}$

Paid what you choose

Leading Example:

$X = \{x, y, z\}$. Let xyz denote $x \succ y \succ z$, e.g. Assume strict prefs.

All orderings:

$$\underbrace{\{\textcolor{red}{xyz, xzy, zxy}\}}_{\text{a "type"}, \text{red}}, \underbrace{\{\textcolor{blue}{zyx, yzx, yxz}\}}_{\text{a "type", blue}}$$

A simple elicitation mechanism:

Pick from $\{x, y\}$

Paid what you choose

Leading Example:

$X = \{x, y, z\}$. Let xyz denote $x \succ y \succ z$, e.g. Assume strict prefs.

All orderings:

a “type space” or “model” or “theory”

$$\overbrace{\{\{xyz, xzy, zxy\}, \{zyx, yzx, yxz\}\}}^{\text{a ‘type’}} \quad \overbrace{\{zyx, yzx, yxz\}}^{\text{a ‘type’}}$$

A simple elicitation mechanism:

Pick from $\{x, y\}$
Paid what you choose

Leading Example:

$X = \{x, y, z\}$. Let xyz denote $x \succ y \succ z$, e.g. Assume strict prefs.

All orderings:

a “type space” or “model” or “theory”
 $\overbrace{\{\{xyz, xzy, zxy\}, \{zyx, yzx, yxz\}\}}^{\text{a ‘type’}}$
a “type”

A simple elicitation mechanism:

Pick from $\{x, y\}$
Paid what you choose

This type space is *elicitable*. Truth FOSD’s lie.

$\{xyz, xzy, zxy, zyx, yzx, yxz\}$

Mechanism:

Pick from $\{x, y\}$ and from $\{x, z\}$

We randomly pick one of your answers and pay it to you

$$\{\underbrace{\{xyz, xzy\}}_{\text{pick } x,x}, \underbrace{\{zxy\}}_{\text{pick } x,z}, \underbrace{\{zyx, yzx\}}_{\text{pick } y,z}, \underbrace{\{yxz\}}_{\text{pick } y,x}\}$$

Mechanism:

Pick from $\{x, y\}$ and from $\{x, z\}$

We randomly pick *one* of your answers and pay it to you

This type space is *elicitable*. Truth FOSD's lie.

$$\left\{ \underbrace{\{xyz, yxz\}}_{\text{dislike z}}, \underbrace{\{xzy, zxy\}}_{\text{dislike y}}, \underbrace{\{yzx, zyx\}}_{\text{dislike x}} \right\}$$

$$\left\{ \underbrace{\{xyz, yxz\}}_{\text{dislike z}}, \underbrace{\{xzy, zxy\}}_{\text{dislike y}}, \underbrace{\{yzx, zyx\}}_{\text{dislike x}} \right\}$$

There are **no** menus that generate this type space.
Generated by top two elements of X

$$\left\{ \underbrace{\{xyz, yxz\}}_{\text{dislike z}}, \underbrace{\{xzy, zxy\}}_{\text{dislike y}}, \underbrace{\{yzx, zyx\}}_{\text{dislike x}} \right\}$$

There are **no** menus that generate this type space.
Generated by top two elements of X

But it is elicitable!

$$\left\{ \underbrace{\{xyz, yxz\}}_{\text{dislike z}}, \underbrace{\{xzy, zxy\}}_{\text{dislike y}}, \underbrace{\{yzx, zyx\}}_{\text{dislike x}} \right\}$$

There are **no** menus that generate this type space.
Generated by top two elements of X

But it is elicitable!

Mechanism:
Announce least favorite,
get paid 50-50 lottery over the other two options.

Results

Preview of Main Results:

Generated by top k elements \Rightarrow elicitable \Rightarrow “convex”

Results

Preview of Main Results:

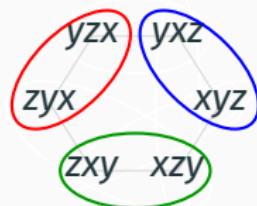
Generated by top k elements \Rightarrow elicitable \Rightarrow “convex”



Results

Preview of Main Results:

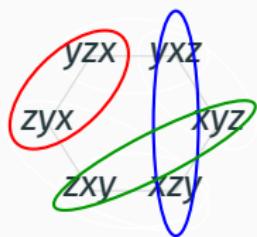
Generated by top k elements \Rightarrow elicitable \Rightarrow “convex”



Results

Preview of Main Results:

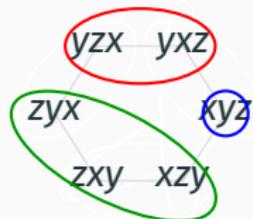
Generated by top k elements \Rightarrow elicitable \Rightarrow “convex”



Results

Preview of Main Results:

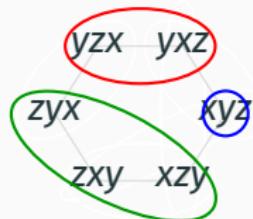
Generated by top k elements \Rightarrow elicitable \Rightarrow “convex”



Results

Preview of Main Results:

Generated by top k elements \Rightarrow elicitable \Rightarrow “convex”



We get complete characterization when:

1. Restrict to neutral type spaces, or
2. Pay in acts, not lotteries (no objective probabilities)

Some related literature

- Osband (1985), Lambert-Pennock-Shoham (2008), Lambert (2018): Scoring rules to elicit properties of beliefs
- Gibbard (1977), Bahel and Sprumont (2019): Characterizing strategy-proof random mechanisms
- Carroll (2012), Saito (2013): Sufficiency of local IC constraints
- ACH (2018, 2020): Characterizing IC mechanisms in experimental environments

The General Model

Framework

- X - a finite set of alternatives
 - Typical elements: x, y, z, w, \dots
- O - the set of strict orders over X
 - Typical elements: \succeq, \succeq', \dots

Framework

- X - a finite set of alternatives
 - Typical elements: x, y, z, w, \dots
- O - the set of strict orders over X
 - Typical elements: \succeq, \succeq', \dots

Definition

A type space $T = \{t_1, \dots, t_k\}$ is a partition of O .

- A type is any $t \in T$, so $t = \{\succeq, \succeq', \dots, \succeq''\}$
- Example: $t = \{\text{all } \succeq \text{ satisfying the Independence axiom}\}$
- Notation: $t(\succeq) \in T$ is the type containing \succeq

Examples

$$X = \{x, y, z\}$$

- Entire ranking:

$$T = \{\{xyz\}, \{yxz\}, \{yzx\}, \{zyx\}, \{zxy\}, \{xzy\}\}$$

- First-best:

$$T = \{\{xyz, xzy\}, \{yxz, yzx\}, \{zyx, zxy\}\}$$

- Top-2:

$$T = \{\{xyz, yxz\}, \{xzy, zxy\}, \{zyx, yzx\}\}$$

- Best from $\{x, y\}$:

$$T = \{\{xyz, xzy, zxy\}, \{zyx, yzx, yxz\}\}$$

- Where you rank x:

$$T = \{\{xyz, xzy\}, \{zxy, yxz\}, \{yzx, zyx\}\}$$

(This type space is not “neutral”. Labels matter.)

Mechanisms

$\Delta(X)$ is the set of lotteries on X

Definition

A T -mechanism is any $g : T \rightarrow \Delta(X)$.

- Why random payments?
 - Allows use of the RPS mechanism (and more)
 - With deterministic mechanisms very little can be elicited

Elicitable type spaces

Recall that p strictly FOSD q relative to \succeq (written $p \succ^* q$) if

$$\forall x \in X \quad p(\{y : y \succeq x\}) \geq q(\{y : y \succeq x\})$$

with strict inequality for at least one x

Elicitable type spaces

Recall that p strictly FOSD q relative to \succeq (written $p \succ^* q$) if

$$\forall x \in X \quad p(\{y : y \succeq x\}) \geq q(\{y : y \succeq x\})$$

with strict inequality for at least one x

Definition

g is IC if for every $\succeq \in O$ and every $t \neq t(\succeq)$

$$g(t(\succeq)) \succ^* g(t).$$

Elicitable type spaces

Recall that p strictly FOSD q relative to \succeq (written $p \succ^* q$) if

$$\forall x \in X \quad p(\{y : y \succeq x\}) \geq q(\{y : y \succeq x\})$$

with strict inequality for at least one x

Definition

g is IC if for every $\succeq \in O$ and every $t \neq t(\succeq)$

$$g(t(\succeq)) \succ^* g(t).$$

Definition

A type space T is *elicitable* if there exists an IC T -mechanism.

Goal: Characterize elicitable type spaces (spoiler: we can't)

Top elements of menus

“What’s your favorite thing from X' ? ”

- Every menu $X' \subseteq X$ corresponds to a type space:

$\succeq, \succeq' \in t \iff \succeq, \succeq'$ have the same favorite item in X'

Examples:

$$X' = \{x, y\} \implies T = \{\{xyz, xzy, zxy\}, \{zyx, yzx, yxz\}\}$$

$$X' = \{x, y, z\} \implies T = \{\{xyz, xzy\}, \{yxz, yzx\}, \{zyx, zxy\}\}$$

Top elements of menus

“What’s your favorite thing from X' ? ”

- Every menu $X' \subseteq X$ corresponds to a type space:

$$\succeq, \succeq' \in t \iff \succeq, \succeq' \text{ have the same favorite item in } X'$$

Examples:

$$X' = \{x, y\} \implies T = \{\{xyz, xzy, zxy\}, \{zyx, yzx, yxz\}\}$$

$$X' = \{x, y, z\} \implies T = \{\{xyz, xzy\}, \{yxz, yzx\}, \{zyx, zxy\}\}$$

- The (deterministic) mechanism that pays the revealed top element in X' is IC

RPS mechanisms

- One can elicit top elements of several menus $X_1, \dots, X_l \subseteq X$

Examples:

$$X_1 = \{x, y, z\}, X_2 = \{x, y\}$$

$$\implies T = \{\{xyz, xzy\}, \{yzx, yxz\}, \{zxy\}, \{zyx\}\}$$

$$X_1 = \{x, y\}, X_2 = \{x, z\}, X_3 = \{y, z\}$$

$$\implies T = \{\{xyz\}, \{yxz\}, \{yzx\}, \{zyx\}, \{zxy\}, \{xzy\}\}$$

- The corresponding IC mechanism randomly chooses a menu and pays the announced top element
- This is widely used in experimental economics

RPS mechanisms

- One can elicit top elements of several menus $X_1, \dots, X_l \subseteq X$

Examples:

$$X_1 = \{x, y, z\}, X_2 = \{x, y\}$$

$$\implies T = \{\{xyz, xzy\}, \{yzx, yxz\}, \{zxy\}, \{zyx\}\}$$

$$X_1 = \{x, y\}, X_2 = \{x, z\}, X_3 = \{y, z\}$$

$$\implies T = \{\{xyz\}, \{yxz\}, \{yzx\}, \{zyx\}, \{zxy\}, \{xzy\}\}$$

- The corresponding IC mechanism randomly chooses a menu and pays the announced top element
- This is widely used in experimental economics

What else is elicitable?

Top sets of menus

The top-2 type space $T = \{\{xyz, yxz\}, \{xzy, zxy\}, \{zyx, yzx\}\}$ does not reveal top elements of menus but is elicitable

Top sets of menus

The top-2 type space $T = \{\{xyz, yxz\}, \{xzy, zxy\}, \{zyx, yzx\}\}$ does not reveal top elements of menus but is elicitable

- How? If they announce “ x and y ” pay x and y with equal probability, and z with less probability.
- Every $X' \subseteq X$ and k defines a type space by

$$\succeq, \succeq' \in t \iff \succeq, \succeq' \text{ have the same top } k \text{ elements of } X'$$

- This is elicitable by paying the uniform lottery over the set of announced top- k elements
- Can elicit the top- k_i elements of $X_i \subseteq X, i = 1, \dots, l$

Top sets of menus

The top-2 type space $T = \{\{xyz, yxz\}, \{xzy, zxy\}, \{zyx, yzx\}\}$ does not reveal top elements of menus but is elicitable

- How? If they announce “ x and y ” pay x and y with equal probability, and z with less probability.
- Every $X' \subseteq X$ and k defines a type space by

$$\succeq, \succeq' \in t \iff \succeq, \succeq' \text{ have the same top } k \text{ elements of } X'$$

- This is elicitable by paying the uniform lottery over the set of announced top- k elements
- Can elicit the top- k_i elements of $X_i \subseteq X, i = 1, \dots, l$

Anything else??

Example (based on Shapley, 1971)

$$X = \{x, y, z, w\}$$

Type space:

$$\{xyzw, yxzw, xywz, yxwz\}$$

$$\{xzyw\}, \{xwyz\}, \{xzwy, xwzy\}$$

$$\{ywxz\}, \{yzxw\}, \{yzwx, ywzx\}$$

$$\{zxyw, zyxw\}, \{zywx, zwyx\}, \{zxwy, zwxy\}$$

$$\{wxyz, wyxz\}, \{wyzx, wzyx\}, \{wxzy, wzxy\}$$

Claim

\exists IC mechanism, but type space is not generated by top sets.

There is a close connection between IC mechanisms and convex TU cooperative games...

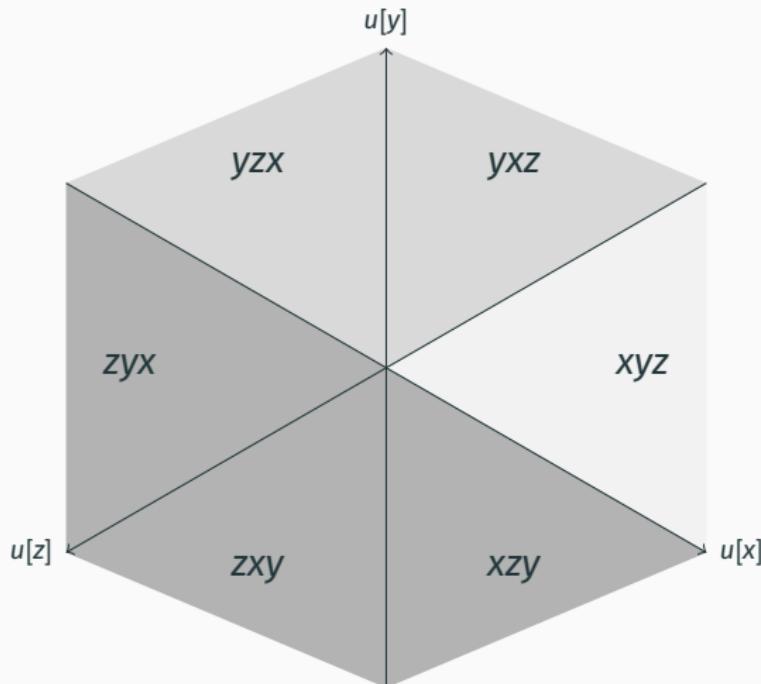
So far...

$$\begin{aligned} & \{\text{all } T\} \\ & \cup \\ & \{T : \text{elicitable}\} \\ & \cup \\ & \{T : \text{generated by top sets}\} \\ & \cup \\ & \{T : \text{generated by top elements}\} \end{aligned}$$

A convex type space - example

Necessary condition: **convex** type space

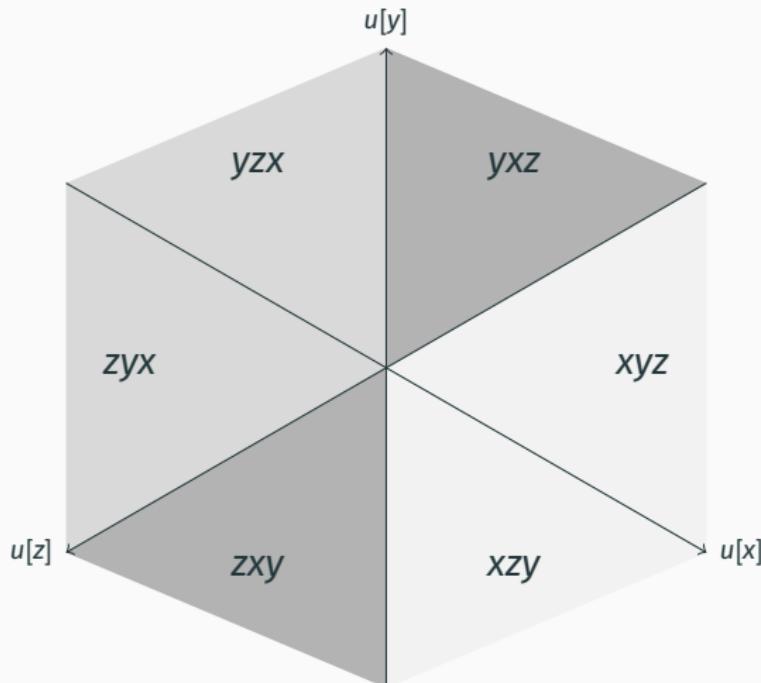
Example: $T = \{\{xyz\}, \{yxz, yzx\}, \{zyx, zxy, xzy\}\}$



A non-convex type space - example

Example of a non-convex type space:

$$T = \{\{xyz, xzy\}, \{yxz, zxy\}, \{zyx, yzx\}\}$$



Convexity is necessary

Proposition

If T is elicitable then it is convex.

Ex: Where do you rank x ?

$t = x$ is 2nd (dark gray)

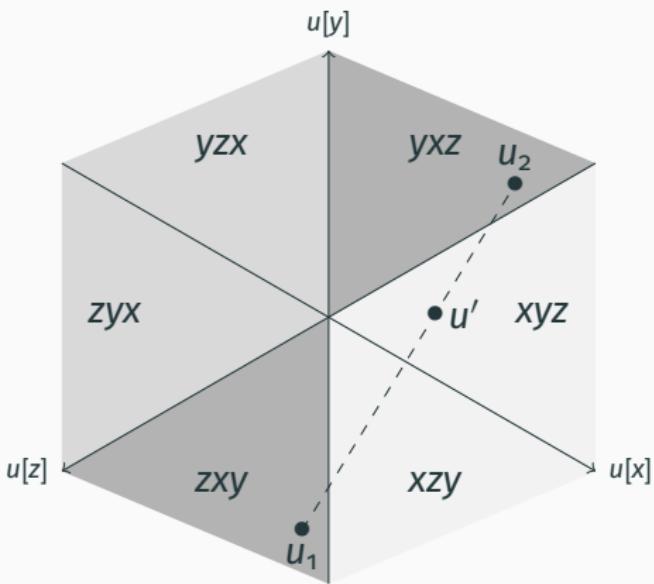
$t' = x$ is 1st (off-white)

IC requires:

$$\mathbb{E}_{g(t)}(u_1) > \mathbb{E}_{g(t')}(u_1)$$

$$\mathbb{E}_{g(t)}(u_2) > \mathbb{E}_{g(t')}(u_2)$$

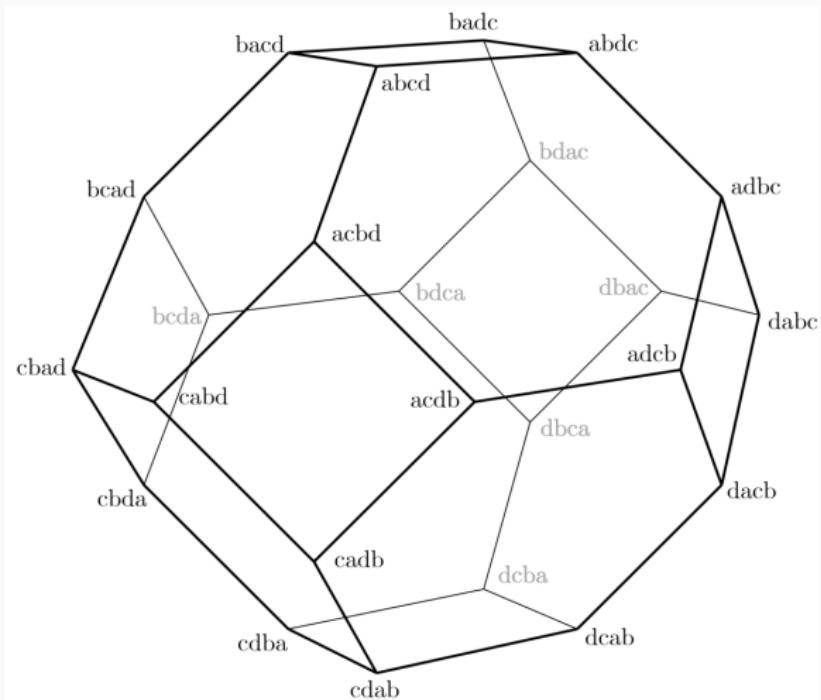
$$\implies \mathbb{E}_{g(t)}(u') > \mathbb{E}_{g(t')}(u')$$



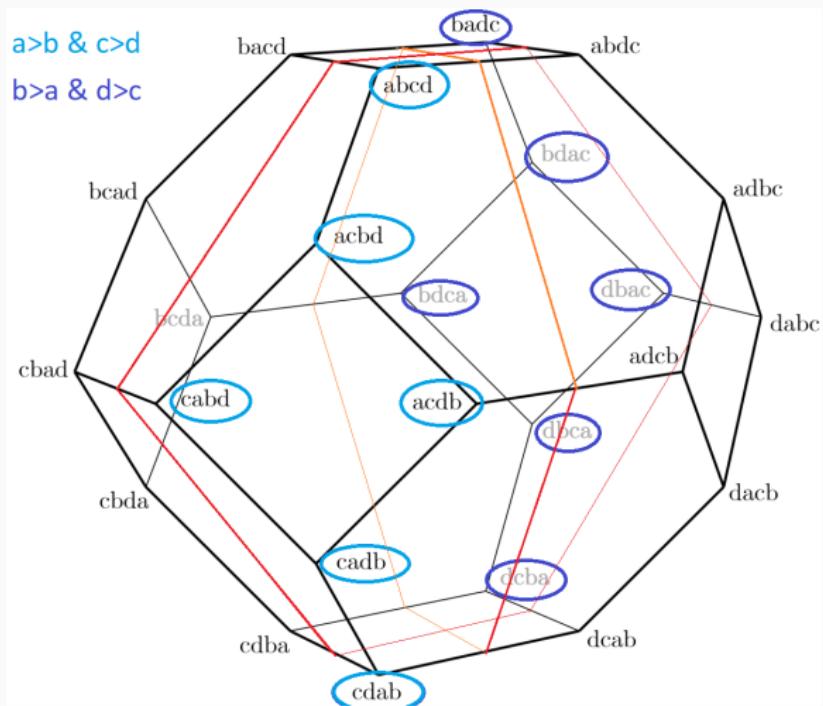
Some non-convex type spaces

- Where do you rank x ? (with $|X| \geq 3$)
- What is the k th ranked alternative for $1 < k < |X|$ (e.g. median)
- Any binary $T = \{t_1, t_2\}$, except $T = \{\{x \succeq y\}, \{y \succeq x\}\}$.
In particular, tests of most axioms of preferences!
Usually: “If $x \succeq y$ then $w \succeq z$ (and $y \succeq x \Rightarrow z \succeq w$)”

Visualizing Convexity: The Permutohedron



Visualizing Convexity: The Permutohedron



Convexity is not sufficient

$$T = \{t_1 = \{xyz\}, t_2 = \{yxz, yzx\}, t_3 = \{zyx, zxy, xzy\}\}$$

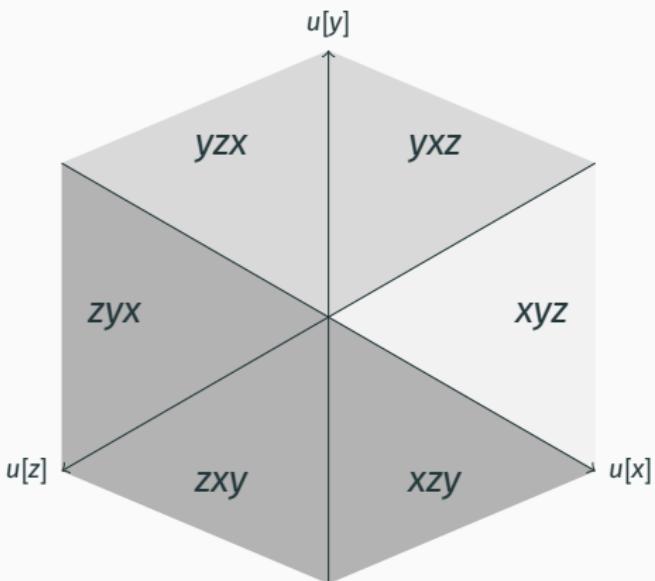
IC requires:

$$g(t_1)(x) > g(t_2)(x)$$

$$g(t_2)(x) = g(t_3)(x)$$

$$g(t_3)(x) = g(t_1)(x)$$

$$\implies g(t_1)(x) > g(t_1)(x)$$



Summary

$$\begin{aligned} & \{ \text{all } T \} \\ & \cup \text{\$} \\ & \{ T : \text{convex} \} \\ & \cup \text{\$} \\ & \{ T : \text{no bad cycles} \} \\ & \cup \text{\$} \\ & \{ T : \text{elicitable} \} \\ & \cup \text{\$} \\ & \{ T : \text{generated by top sets} \} \\ & \cup \text{\$} \\ & \{ T : \text{generated by top elements} \} \end{aligned}$$

Neutral type spaces

- Permutation: $\pi : X \rightarrow X$
- Let πT be T , but with every \succeq permuted by π

Definition

T is *neutral* if $\pi T = T$ for every π .

Neutral: “What do you rank 3rd?”

Not: “Where do you rank x?”

Neutral type spaces

- Permutation: $\pi : X \rightarrow X$
- Let πT be T , but with every \succeq permuted by π

Definition

T is *neutral* if $\pi T = T$ for every π .

Neutral: “What do you rank 3rd?”

Not: “Where do you rank x?”

Proposition

Suppose T is neutral. Then the following are equivalent:

- (1) T is elicitable
- (2) T is convex
- (3) T is generated by top sets

Neutral type spaces

$$\begin{aligned} & \{\text{all } T\} \\ & \cup \text{\texttt{\&}} \\ & \{T : \text{convex}\} \\ & \parallel \\ & \{T : \text{elicitable}\} \\ & \parallel \\ & \{T : \text{generated by top sets}\} \\ & \cup \text{\texttt{\&}} \\ & \{T : \text{generated by top elements}\} \end{aligned}$$

Robust elicitation

What if the agent has subjective beliefs about the likelihood of realizations of the randomization device (or other kinds of preferences over acts)?

Robust elicitation

What if the agent has subjective beliefs about the likelihood of realizations of the randomization device (or other kinds of preferences over acts)?

- Now payments are Savage acts, not lotteries.
- Can't pay a uniform lottery b/c can't control beliefs
- This kills our ability to elicit top sets

Robust elicitation

What if the agent has subjective beliefs about the likelihood of realizations of the randomization device (or other kinds of preferences over acts)?

- Now payments are Savage acts, not lotteries.
- Can't pay a uniform lottery b/c can't control beliefs
- This kills our ability to elicit top sets

Proposition

T is elicitable with acts iff it is generated by top elements.

Elicitation under acts

$\{\text{all } T\}$

$\cup \$$

$\{T : \text{convex}\}$

$\cup \$$

$\{T : \text{elicitable with lotteries}\}$

$\cup \$$

$\{T : \text{generated by top sets}\}$

$\cup \$$

$\{T : \text{generated by top elements}\}$

\parallel

$\{T : \text{elicitable with acts}\}$

Multiple agents

- $N = \{1, \dots, n\}$ - agents
- T_i - agent's i type space
- $T = (T_1, \dots, T_n)$ - a profile of type spaces
- $g : T \rightarrow \Delta(X)$ - a mechanism

Multiple agents

- $N = \{1, \dots, n\}$ - agents
- T_i - agent's i type space
- $T = (T_1, \dots, T_n)$ - a profile of type spaces
- $g : T \rightarrow \Delta(X)$ - a mechanism

Proposition

$T = (T_1, \dots, T_n)$ is dominant-strategy-elicitable iff each T_i is elicitable.

Conclusion

- We formulate a notion of elicability for properties of preferences
- Some necessary conditions and some sufficient conditions for elicability, but no characterization
- We do have a characterization for neutral type spaces and for robust elicitation (acts)
- Potential extensions: Weak orders, infinite sets of alternatives, domain restrictions,...

Thank You!