

# **ExpEcon Methods: Multiple Hypotheses Corrections**

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ECON 8877

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## Multiple Hypothesis Corrections

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# Multiple Tests

The setup:

- You're going to run  $m$  tests (say,  $m = 2$ )
- Each test on its own has a 0.05 Type-I error
- The game is that you “win” if *at least* one is significant
  - Publish a paper, claim a result, etc.
- Problem:  $\uparrow m \Rightarrow \uparrow$  chance you win

## Definition

The **Family-Wise Error Rate (FWER)** is the probability that you “win” given that all  $m$  null hypotheses are true.

- We want FWER to be 0.05, not each test

# Multiple Tests

Suppose you run two tests and “win” if one is significant  
Each has 0.05 Type-I error. Baseline case: independent tests

**Independent** tests:

	Accept	Reject	
Accept	0.9025	0.0475	0.95
Reject	0.0475	0.0025	0.05
	0.95	0.05	<i>FWER = 0.0975</i>

**Šidák correction:** use a lower  $\alpha$ :

	Accept	Reject	
Accept	$(1 - \alpha)^2$	$(1 - \alpha)\alpha$	$1 - \alpha$
Reject	$(1 - \alpha)\alpha$	$\alpha^2$	$\alpha$
	$1 - \alpha$	$\alpha$	<i>FWER = <math>2\alpha - \alpha^2</math></i>

For  $FWER = 0.05$  use  $\alpha \approx 0.025321$ . If you have  $k$  tests:

$$1 - (1 - \alpha)^k = 0.05 \Rightarrow \alpha^* = 1 - (1 - 0.05)^{1/k} \text{ which is } > 0.05/k$$

# Multiple Tests

Worst-case situation: perfectly negative correlation  
Need a bigger correction!

## Perfect Negative Correlation:

	Accept	Reject	
Accept	0.90	0.05	0.95
Reject	0.05	0	0.05
	0.95	0.05	$Pr(R) = 0.10$

**Bonferroni correction:** what's the right  $\alpha$ ?

	Accept	Reject	
Accept	1 - 2 $\alpha$	$\alpha$	1 - $\alpha$
Reject	$\alpha$	0	$\alpha$
	1 - $\alpha$	$\alpha$	$Pr(R) = 2\alpha$

For  $Pr(R) = 0.05$  use  $\alpha \approx 0.025$

$k$  tests:  $1 - (1 - k\alpha) = 0.05 \Rightarrow k\alpha = 0.05 \Rightarrow \alpha^* = 0.05/k$

# Multiple Tests

Best case: perfect positive correlation

Extra tests don't add to the FWER!

**Perfect Positive Correlation:**

	Accept	Reject	
Accept	0.95	0	0.95
Reject	0	0.05	0.05
	0.95	0.05	$Pr(R) = 0.05$

No correction needed!

	Accept	Reject	
Accept	$1 - \alpha$	0	$1 - \alpha$
Reject	0	$\alpha$	$\alpha$
	$1 - \alpha$	$\alpha$	$Pr(R) = \alpha$

Šidák or Bonferroni would be way too conservative!

# The Bonferroni Correction

Setup:

- $k$  tests. Nulls:  $H_0^1, \dots, H_0^k$
- $\alpha_f$  is your adjusted  $p$ -value on each
- FWER (Family-Wise Error Rate) is  $Pr(R)$  on at least one test

Bonferonni Correction:  $\alpha_f = \alpha/k$

- The most popular (and conservative)
- Safe: appropriate regardless of correlation
- Too safe: likely have FWER < 0.05
- Tradeoff: high chance of Type-II error (failure to reject false  $H_0$ )

Šidák Correction:  $\alpha_f = 1 - (1 - \alpha)^{1/k}$

- Exact correction for independent tests
- In practice, Bonferroni  $\approx$  Šidák

## The Holm-Bonferroni Correction (Holm, 1979)

- More powerful while keeping FWER  $\leq 0.05$
- Order the p-values lowest to highest ( $p_1 \leq p_2 \leq \dots \leq p_k$ ). Will test nulls ( $H_0^1, H_0^2, \dots, H_0^k$ ) *sequentially*:

# The Holm-Bonferroni Correction (Holm, 1979)

- More powerful while keeping FWER  $\leq 0.05$
  - Order the p-values lowest to highest ( $p_1 \leq p_2 \leq \dots \leq p_k$ ). Will test nulls ( $H_0^1, H_0^2, \dots, H_0^k$ ) sequentially:
1. Is  $p_1 < \frac{\alpha}{k}$ ?
    - Yes: Reject  $H_0^1$  and move on to test  $H_0^2$ .
    - No: Do not reject any  $H_0^i$  (as in Bonferroni). Stop.
      - Note:  $k - 1$  tests remaining, so correction increases to  $\frac{\alpha}{k-1}$
  2. Is  $p_2 < \frac{\alpha}{k-1}$ ?
    - Yes: Reject  $H_0^2$  as well and continue.  $k - 2$  tests remain.
    - No: Do not reject  $H_0^2$  through  $H_0^k$ . Stop.
  - j. Is  $p_j < \frac{\alpha}{k+1-j}$ ?
    - Yes: Reject  $H_0^j$  as well and continue.
    - No. Do not reject  $H_0^j$  through  $H_0^k$ . Stop.

Recall: Bonferroni also allows only some nulls to be rejected. Same.  
“Win” if you reject at least one? Then Holm-Bonferroni = Bonferroni.  
Can use Šidák version assuming independence:  $1 - (1 - \alpha)^{1/(k+1-j)}$

# The Hotchberg Step-Down Procedure

- Holm-Bonferroni: Reject  $H_0^1, \dots, H_0^j$  where  $j$  is the smallest index for which  $p_{j+1} \geq \frac{\alpha}{k+1-(j+1)}$ 
  - Reject up to the “first crossing” of the threshold
- Hotchberg procedure: Reject  $H_0^1, \dots, H_0^j$  where  $j$  is the largest index for which  $p_j \leq \frac{\alpha}{k+1-j}$ 
  - Reject up to the “last crossing” of the threshold
- Alternatively, first crossing when working top-to-bottom.
- This method is more powerful than the Holm-Bonferroni correction, but it sometimes does not control the FWER (see Dmitrienko et al., 2010 for details).
  - Not valid for negative correlation (worst case)

## Balanced Resampling Using Bootstrapping

- How do we know the correlation across tests??
- Can be estimated via resampling methods! Hooray!
- Romano and Wolf (2005,2005,2016)
- This, combined with a “step-down” method like that used in Holm (1979), creates a more powerful correction.
- Furthermore, this method also creates balance, such that all tests contribute equally to error control.
- List et al. (2019) develop version of this correction for experiments where treatment is randomly assigned.

I would use these methods!

Stata: `rwolf` can be downloaded

# The Family-Wise Error Rate

- What is the “family” in the Family-Wise Error Rate? What tests should be “combined”?
  - A “family” is (frustratingly) loosely defined, but an intuitive way to think about it is a set of tests whose inference is getting at the same question.
  - An easy experimental example: suppose you have two treatments and a control group, and you want to determine if either of the treatments increased the mean, so you perform two t-tests. Both of those t-tests constitute a family.

## When to Use Corrections?

- Some people non-statisticians say we should *never* use them (O'Keefe, 2003; Perneger, 1998; Rothman, 1990)
- Other people non-statisticians say we should *always* use them (Bennett et al., 2009; Goeman & Solari, 2014; Moyé, 1998; Ottenbacher, 1998)
- Still others say we should use them only in exploratory research (Armstrong, 2014; Cramer et al., 2016; Streiner, 2015)
- Finally, some say we should use them only in confirmatory research (Bender & Lange, 2001; Schochet, 2009; Stacey et al., 2012; Tutzauer, 2003; Wason et al., 2014)

## When to Use Corrections? (Continued)

- MHT corrections in ExpEcon are fairly rare, but growing.
- List et al. (2019) argue we *should* correct when:
  1. Multiple outcomes for a given treatment (eg, GPA, SAT, ACT)
  2. Multiple subgroups for given trt (eg, Black, Hispanic, Asian)
  3. Multiple treatments (eg, different incentive schemes on effort)

PJ's view: "Can your claim be validated by *any* test being significant?"

Thus, depends on what you're claiming!

Ex: "Giving teachers incentives improves educational outcomes"

vs. "Giving teachers incentives improves GPAs"

If GPA, SAT, and ACT are 3 different hypotheses, then don't correct

Discussion: I write a paper with 20 unrelated hypotheses. MHT??

## When to Use Corrections? (Continued)

Fancy language for this idea: Rubin (2021)

- Disjunction testing: “win” if you reject at least once
  - $H_0$ : both green and red jelly beans *do not* cause acne
  - $H_1$ : either green or red jelly beans (or both) cause acne
  - Rule: reject  $H_0$  if either is significant (correction needed!)
- Conjunction testing: “win” only if you reject all
  - $H_0$ : either green or red jelly beans (or both) *do not* cause acne
  - $H_1$ : both green and red jelly beans cause acne
  - Rule: reject  $H_0$  if both are significant (no correction needed)
- Individual testing: each test has its own “win”
  - $H_0^1$ : green jelly beans do not cause acne
  - $H_1^1$ : green jelly beans cause acne
  - $H_0^2$ : red jelly beans do not cause acne
  - $H_1^2$ : red jelly beans cause acne
  - Rule: two separate tests. No ex-ante “either/both” claim

# Conclusion

- Mathematically, there are ways to correct for MHT
- Degree of correction should depend on correlation across tests
  - Resampling methods (Wolf-Romano) are best here.
- The hard question is: when to use them??
  - Be precise in what exactly you're claiming!
  - “educational outcomes” vs. “GPAs”
- Final thought: readers are decent Bayesians. They don't just blindly trust 0.05. Tie your hands with preregistration and report everything. They'll update appropriately.