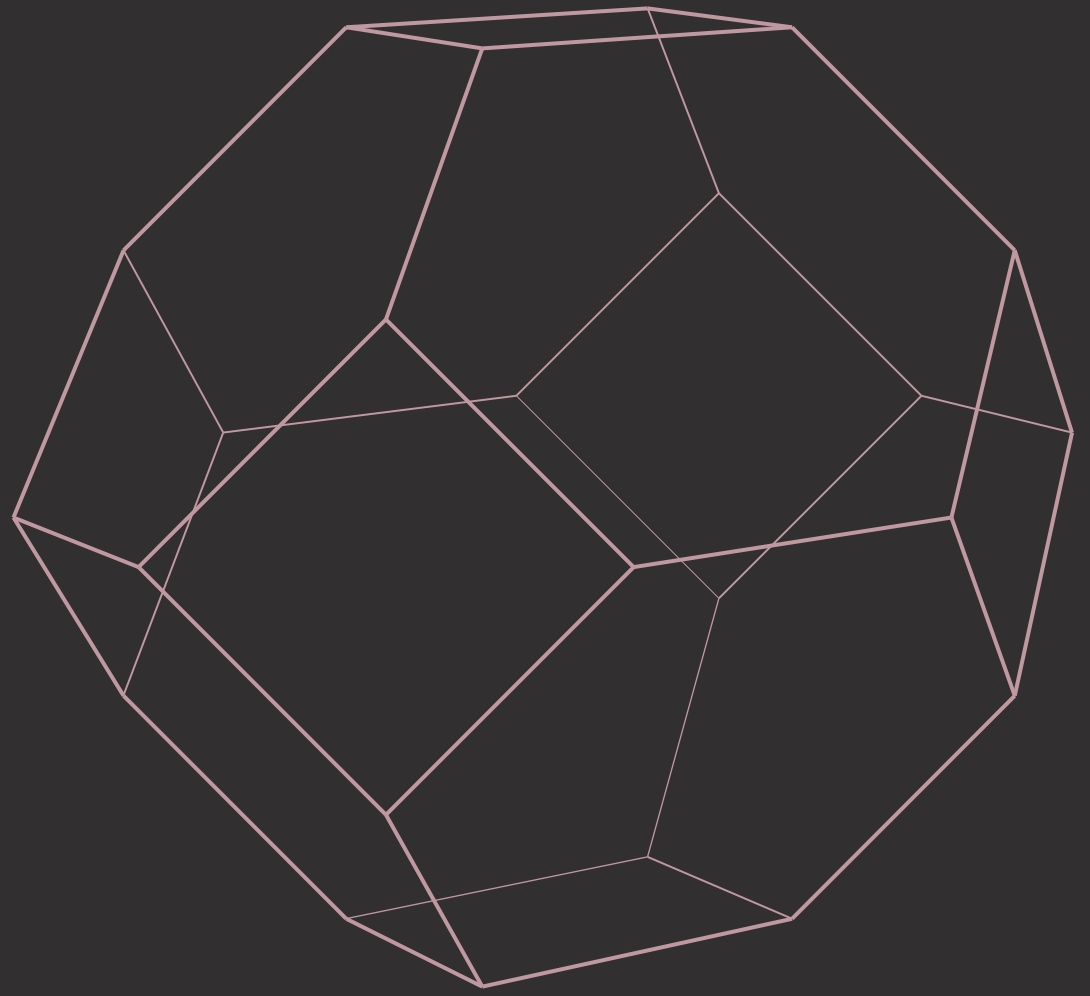
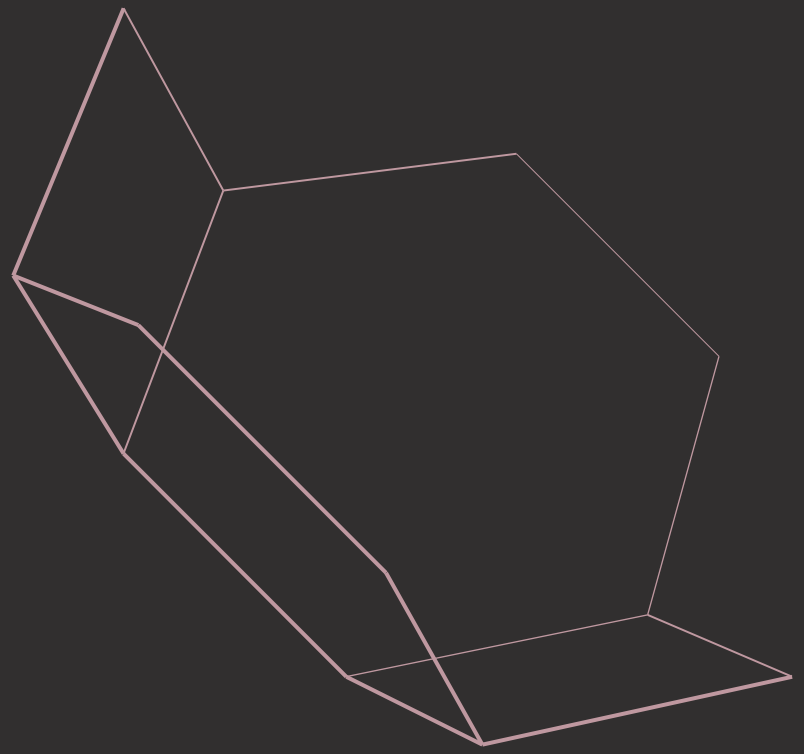
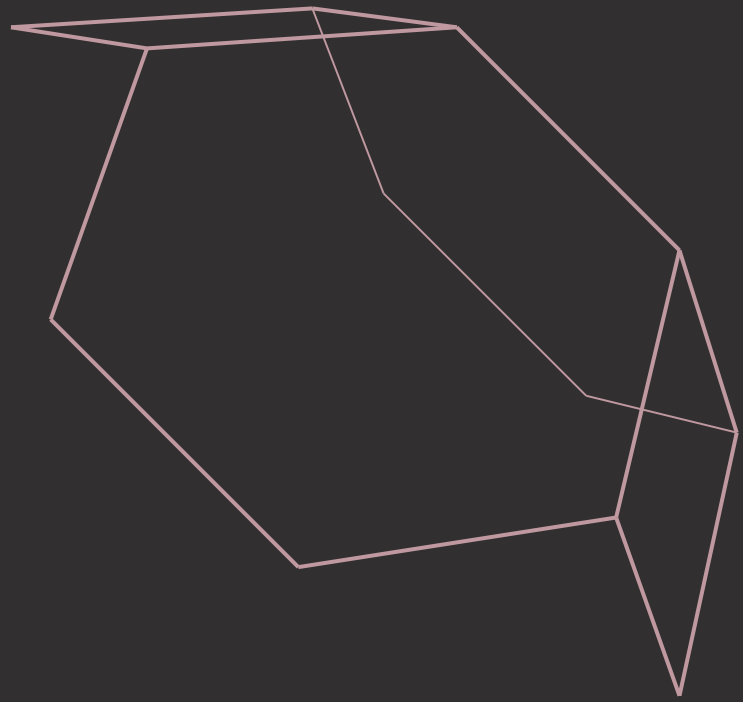




Minimal Experiments

Healy

Leo



ROUSSEAS HART 1951

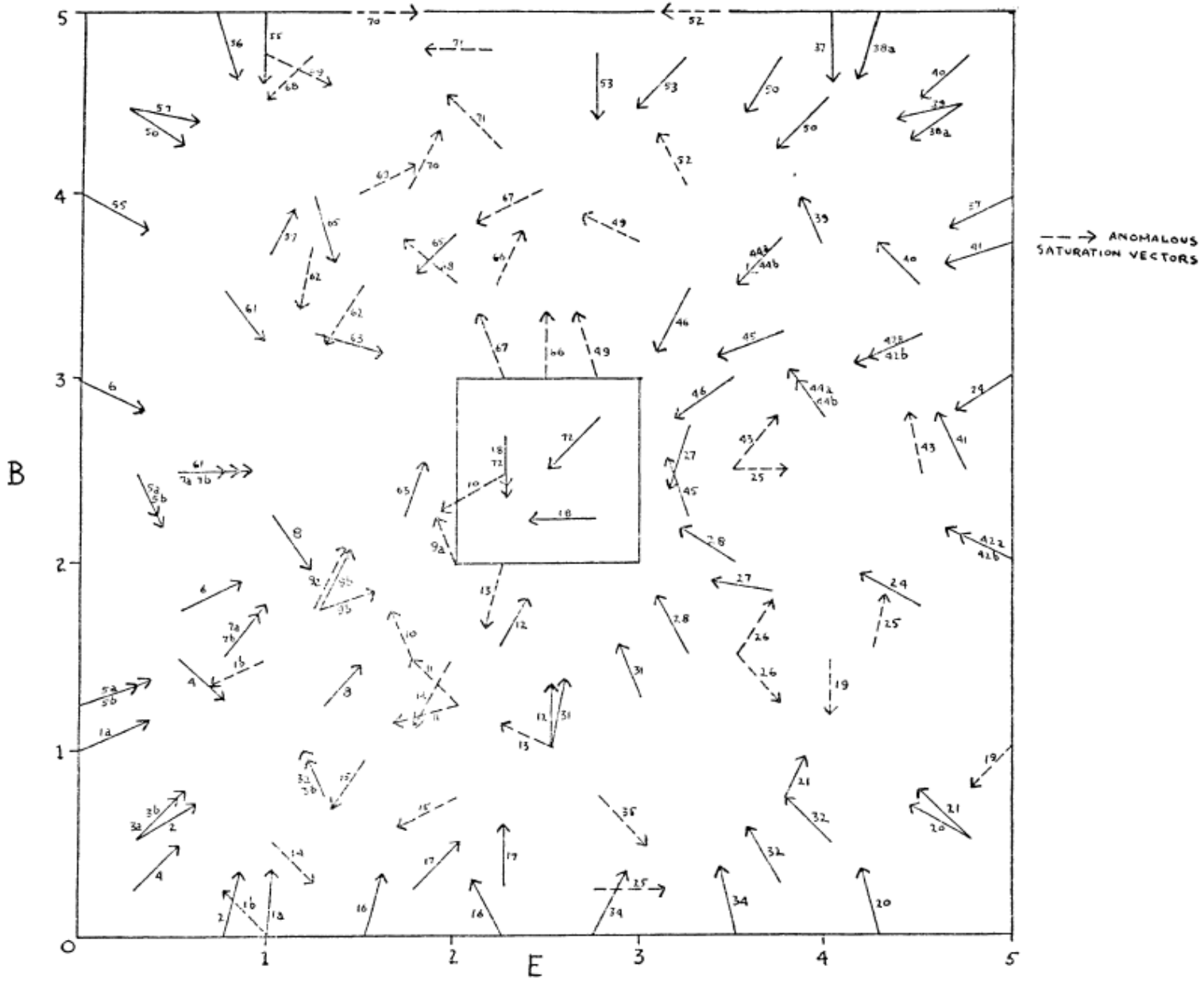


DIAGRAM 14

9a

NAME JONES, JOHN J.
Last First Middle
(Please Print)

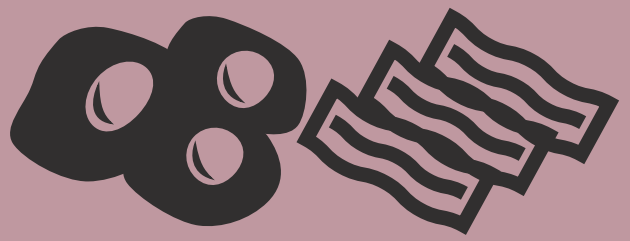
Choice	Bacon	Eggs	Order of Pref.
A	1 3/4	1 1/4	
B	2 3/4	1 3/4	
C	2	2	

Instructions:

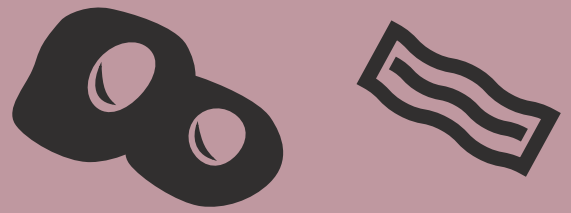
1. List your order of preference using the numbers 1, 2 and 3.
2. If you are indifferent to two or all three of the choices offered write "N" in the appropriate two or all three boxes.

ROUSSEAS HART
1951

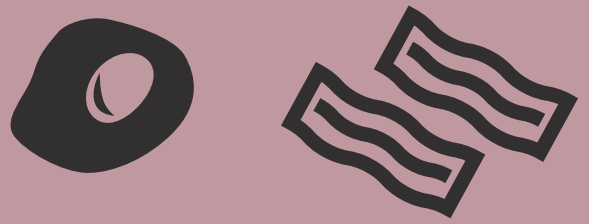
MONOTONICITY



a

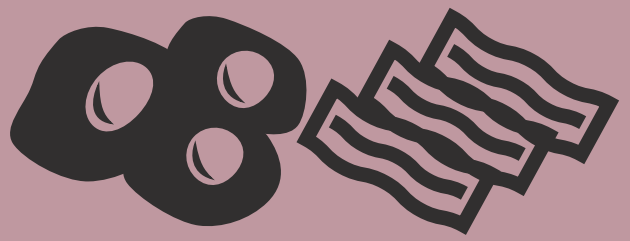


b

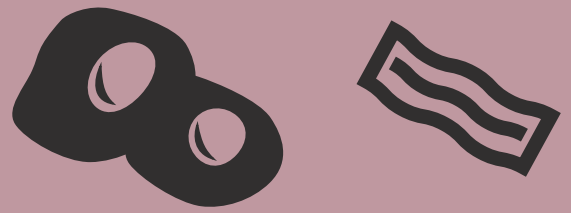


c

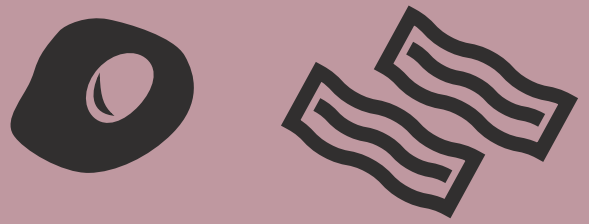
MONOTONICITY



a



b



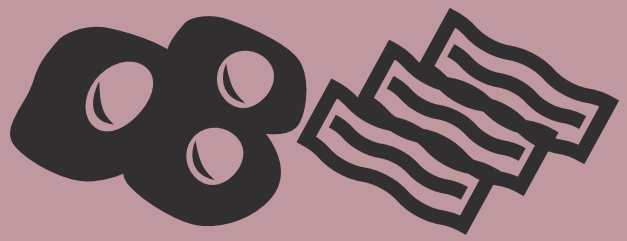
c

$$a \succ b$$

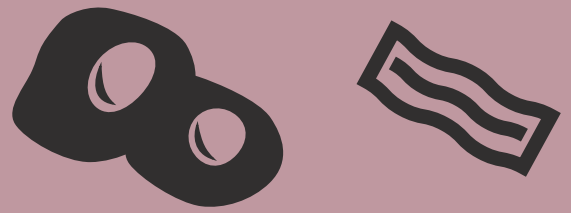
and

$$a \succ c$$

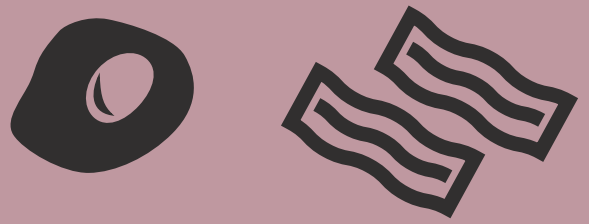
MONOTONICITY



a



b



c

$$a \succ b$$

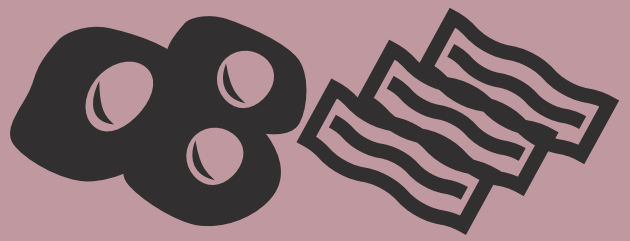
and

$$a \succ c$$

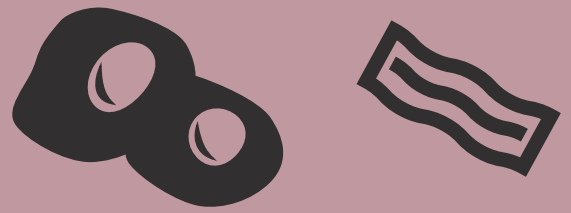
{abc,acb}

{bac,bca,cab,cba}

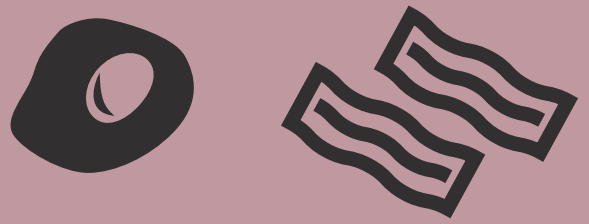
MONOTONICITY



a



b



c

Model.

{abc,acb}
t_1

{bac,bca,cab,cba}
t_2

{abc,acb} {bac,bca,cab,cba}

t_1 t_2

$10 Now.

$20 Next Week.

$30 Next Month.

$10 for you, $0 for other

$8 for you, $2 for other

$5 for you, $5 for other

$5 with a 100% chance

$10 with a 50% chance

$8 with a 75% chance

$10 if the Braves win the 2022 world series

$10 if the Astros with the 2022 world series

$10 with a 50% chance

{abc,acb} {bac,bca,cab,cba}

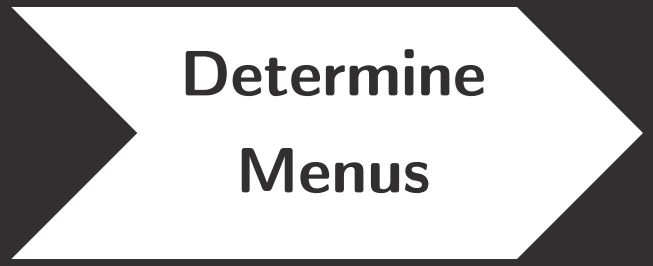
t_1 t_2

Differentiated Pairs.

{abc,acb} t_1

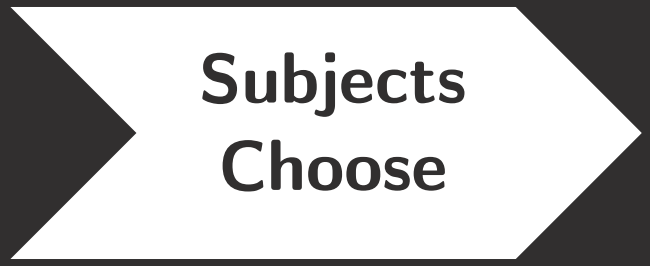
{bac,bca,cab,cba} t_2

Choice-from-Sets Experiments.



{a,b,c}
{b,c}

Choice-from-Sets Experiments.

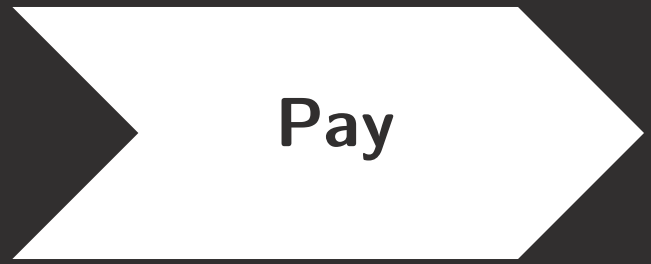


{a,b,c}
{b,c}

Choice-from-Sets Experiments.

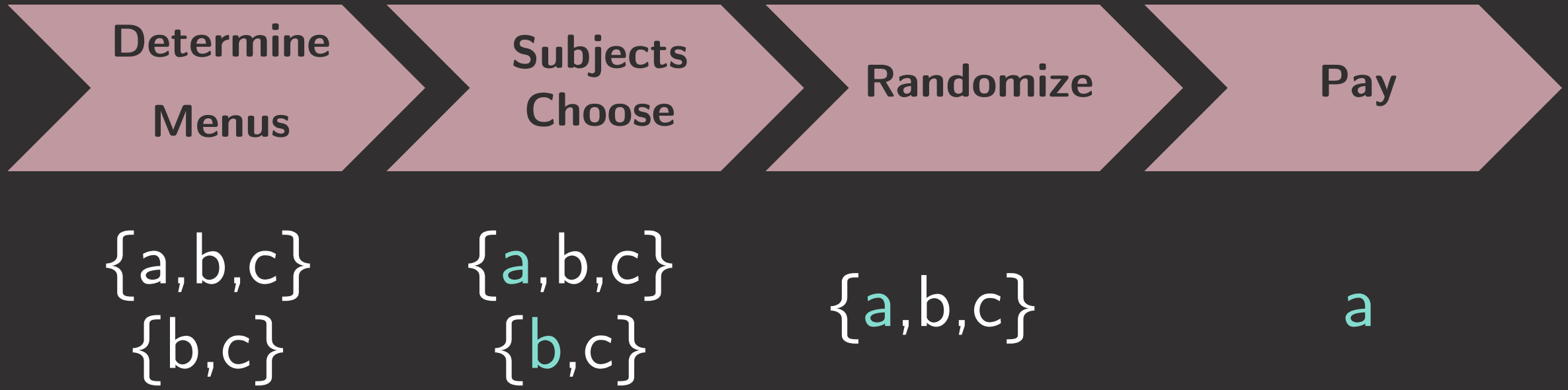
{a,b,c}

Choice-from-Sets Experiments.



a

Choice-from-Sets Experiments.



Experiments.

{abc,acb} {bac,bca,cab,cba}

t_1 t_2

{a,b,c}

Separates.

{abc,acb} {bac,bca,cab,cba}

t_1 t_2

{a,b,c}

Classifies Model.

$\{abc, acb\}$ $\{bac, bca, cab, cba\}$

t_1 t_2

$\{a, b, c\}$

Minimal Experiment.

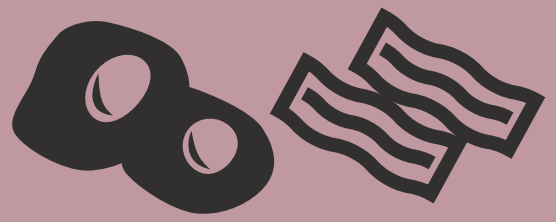
$\{abc, acb\}$ $\{bac, bca, cab, cba\}$

t_1 t_2

$\{a, b, c\}$

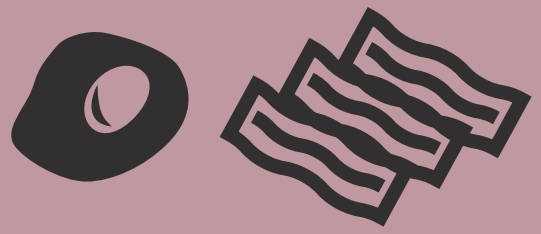
Smallest Experiment that Classifies Model

CONVEXITY



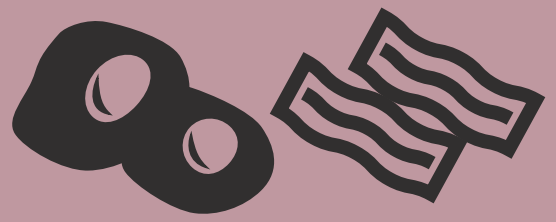
a

b



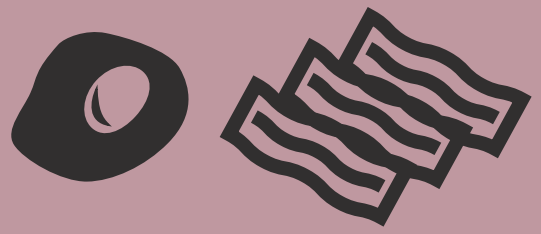
c

CONVEXITY



a

b



c

$$a \succ b$$

or

$$a \succ c$$

{abc,acb,bac,cab}

{bca,cba}

$\{abc, acb, bac, cab\}$ $\{bca, cba\}$

t_1 t_2

{abc,acb,bac,cab} t_1

{bca,cba} t_2

{a,b,c}

{abc,acb,bac,cab} t_1

{bca,cba} t_2

{a,b,c}

Fails to Separate

{abc,acb,bac,cab} {bca,cba}

t_1 t_2

{a,b} {a,c}

Minimal

{abc,acb}	{bac}	{cab}	{bca,cba}
t_1	t_2	t_3	t_4

{abc,acb} {bac} {cab} {bca,cba}

t_1 t_2 t_3 t_4

{a,b} {a,c}

Minimal

An *Incomplete* Model.

{abc,acb}	{bac}	{cab}	{bca,cba}
t_1	t_2	t_3	M_0

{abc,acb}	{bac}	{cab}
t_1	t_2	t_3

{abc,acb}	{bac}	{cab}
t_1	t_2	t_3

{abc,acb} {bac} {cab}

t_1 t_2 t_3

{a,b,c}

Is this hard?

{ecihfgbda,efdhcigab,hiacefdgb}

t_1

{bcfeagihd,dgbaeifhc,ibcafgehd,dgbceahif}

t_2

{afchedbgi,fcdahgieb,gdfhibcae}

t_3

$\{\ldots\}$

t_4

13,093,562,431,584,567,480,052,
758,787,310,396,608,866,568,184,
172,259,157,933,165,472,384,535,
185,618,698,219,533,080,369,303,
616,628,603,546,736,510,240,284,
036,869,026,183,541,572,213,314,
110,357,504

{ecihfgbda,efdhcigab,hiacefdgb}

t_1

{bcfeagihd,dgbaeifhc,ibcafgehd,dgbceahif}

t_2

{afchedbgi,fcdahgieb,gdfhibcae}

t_3

$\{\ldots\}$

t_4

{b,c},{a,b,c,i},{c,f,h},{d,e,f},{a,e},{a,g},{g,i},{h,i},
{d,h},{d,g},{d,f,g},{b,g},{a,b},{a,c,e},{e,i},{f,i},
{c,h},{a,f,h},{e,h},{e,g,h},{d,e},{b,d},{c,d,e},{a,d},{a,h},{b,e}

{ecihfgbda,efdhcigab,hiacefdgb}

t_1

{bcfeagihd,dgbaeifhc,ibcafgehd,dgbceahif}

t_2

{afchedbgi,fcdahgieb,gdfhibcae}

t_3

{ecihfgbda,efdhcigab,hiacefdgb}

t_1

{bcfeagihd,dgbaeifhc,ibcafgehd,dgbceahif}

t_2

{afchedbgi,fcdahgieb,gdfhibcae}

t_3

{b,e,f}

{ecihfgbda,efdhcigab,hiacefdgb}

t_1

{bcfeagihd,dgbaeifhc,ibcafgehd,dgbceahif}

t_2

{afchedbgi,fcdahgieb,gdfhibcae}

t_3

{b,e,f}

{b,c},{a,b,c,i},{c,f,h},{d,e,f},{a,e},{a,g},{g,i},{h,i},
{d,h},{d,g},{d,f,g},{b,g},{a,b},{a,c,e},{e,i},{f,i},
{c,h},{a,f,h},{e,h},{e,g,h},{d,e},{b,d},{c,d,e},{a,d},{a,h},{b,e}

Geometry of Experiments

abc

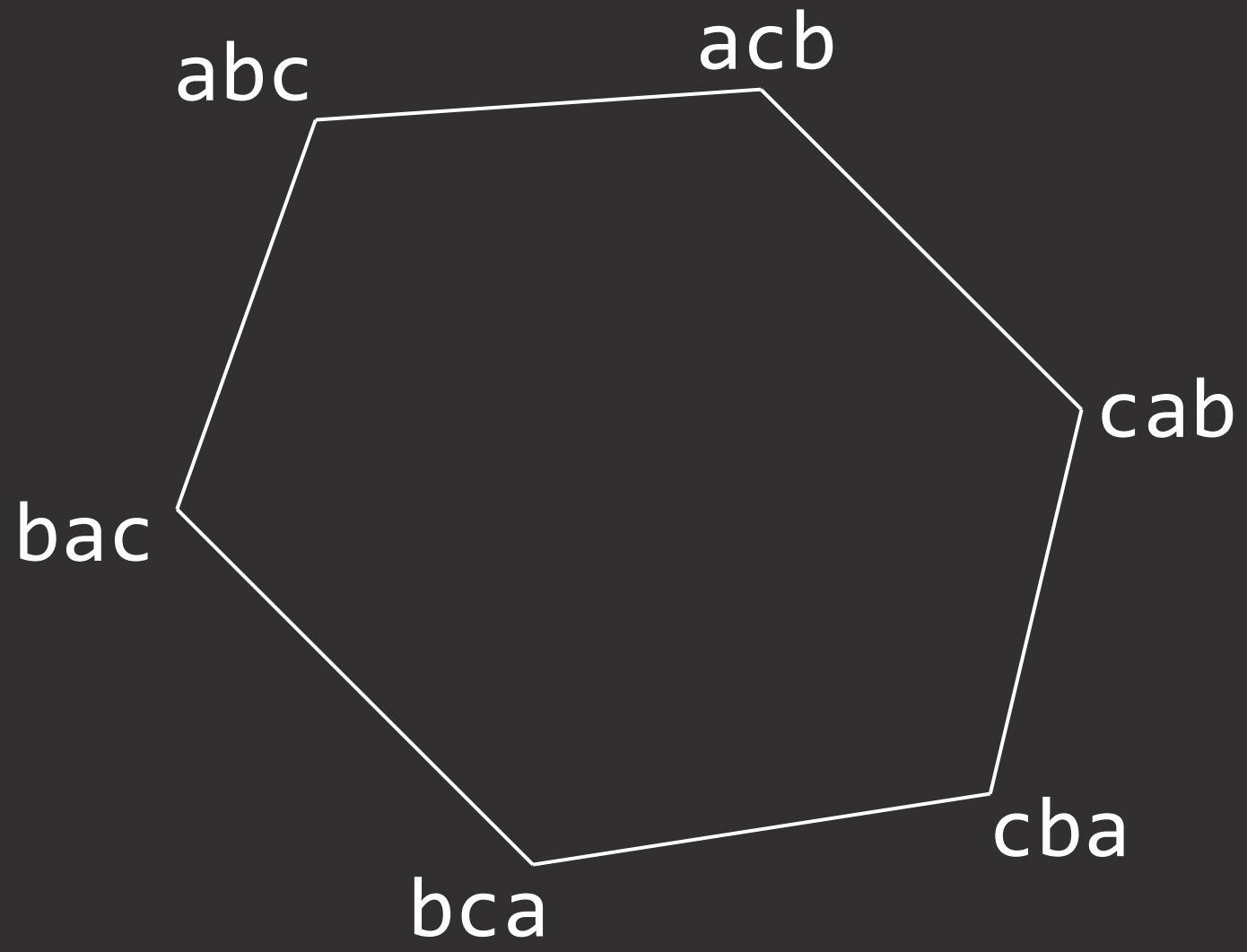
acb

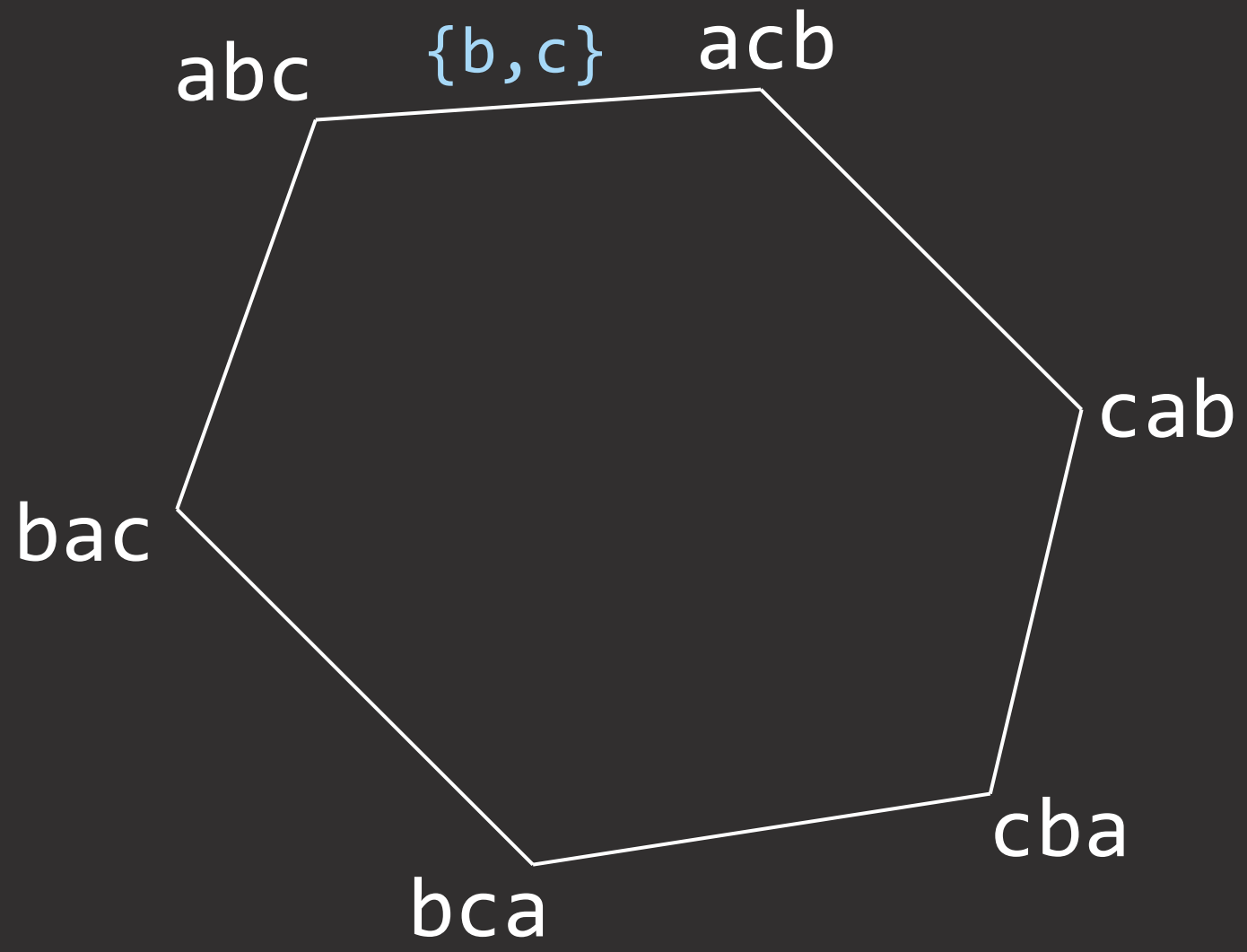
cab

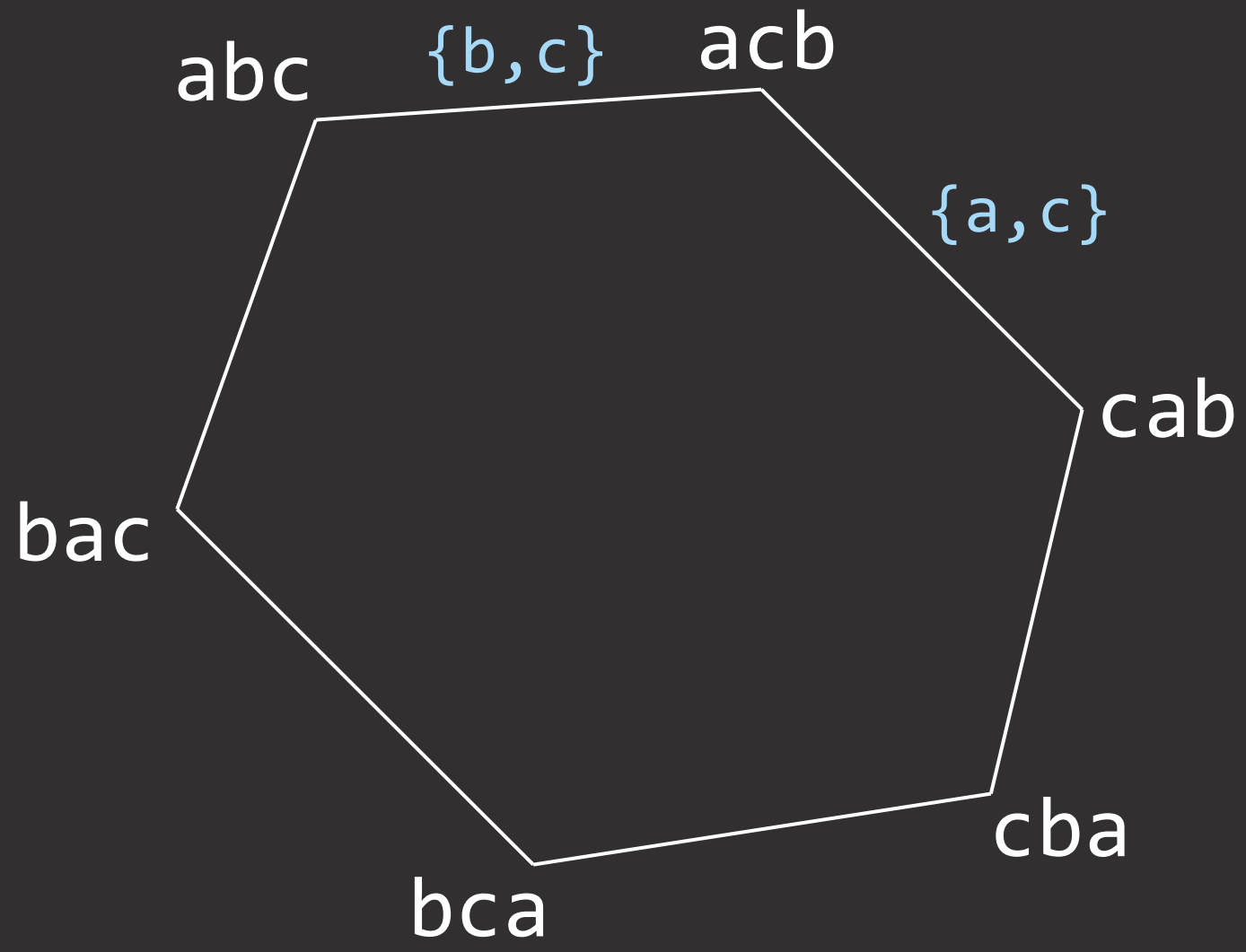
bac

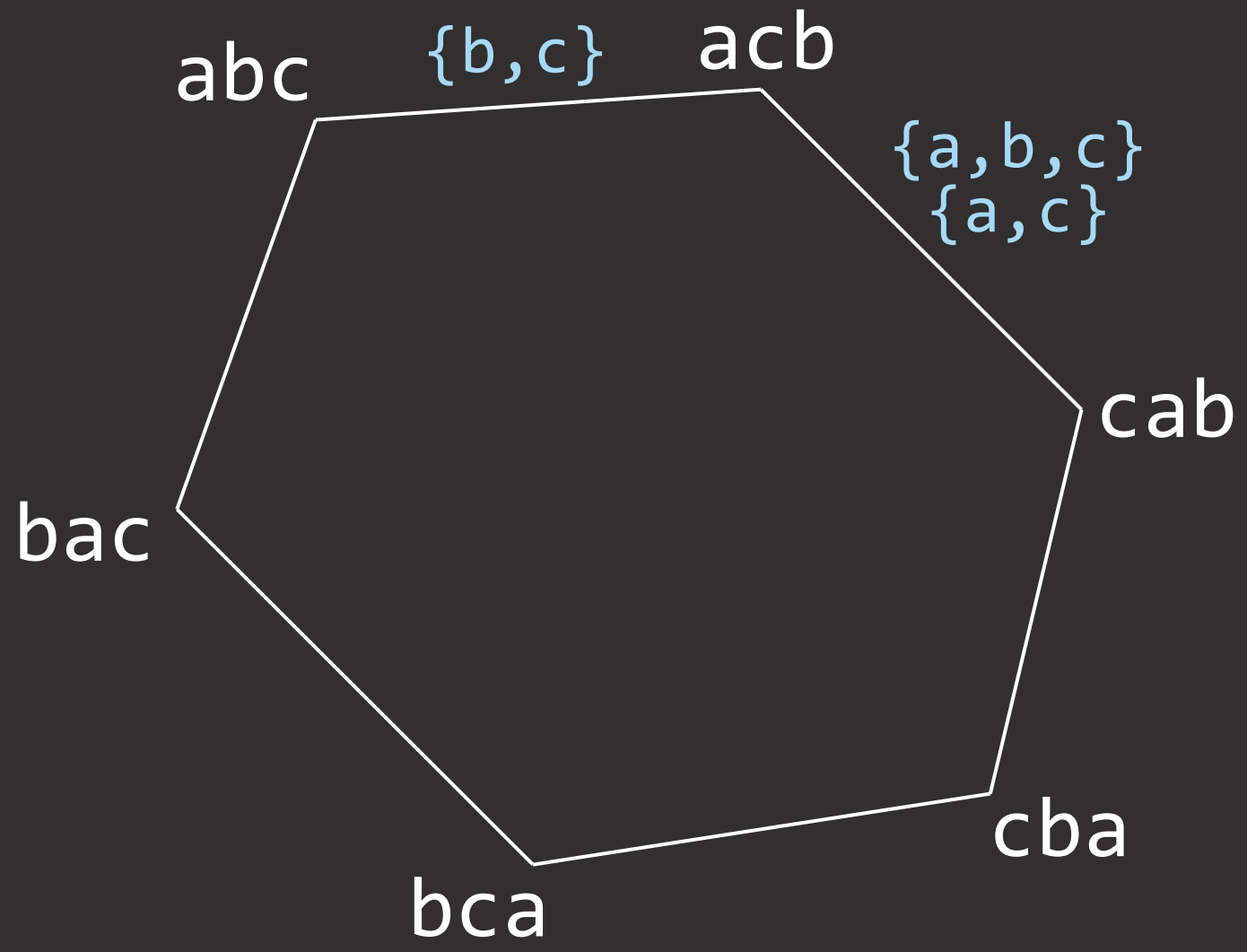
cba

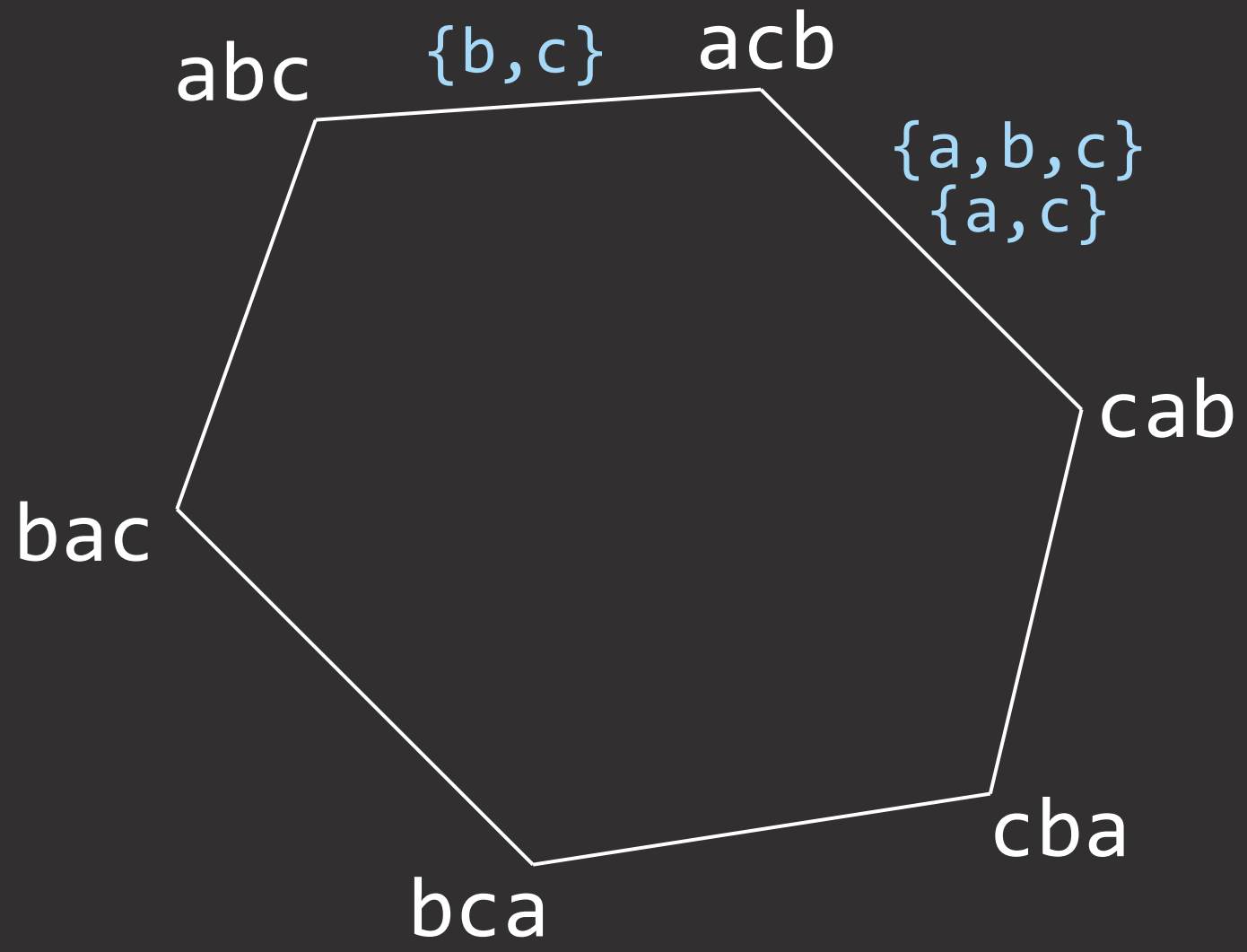
bca

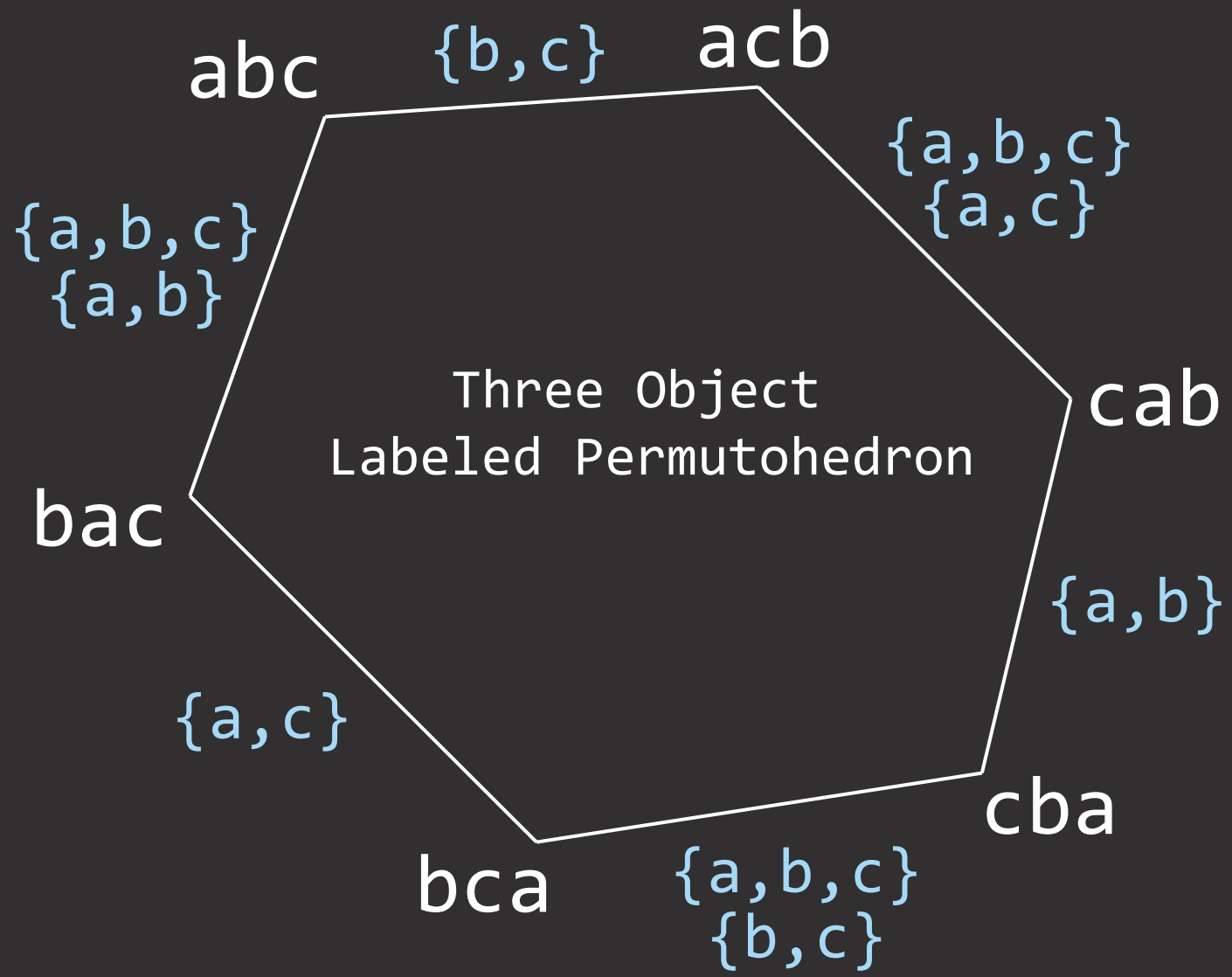

abc
acb
cab
bac
cba
bca


abc
{b,c}
acb
cab
bac
cba
bca

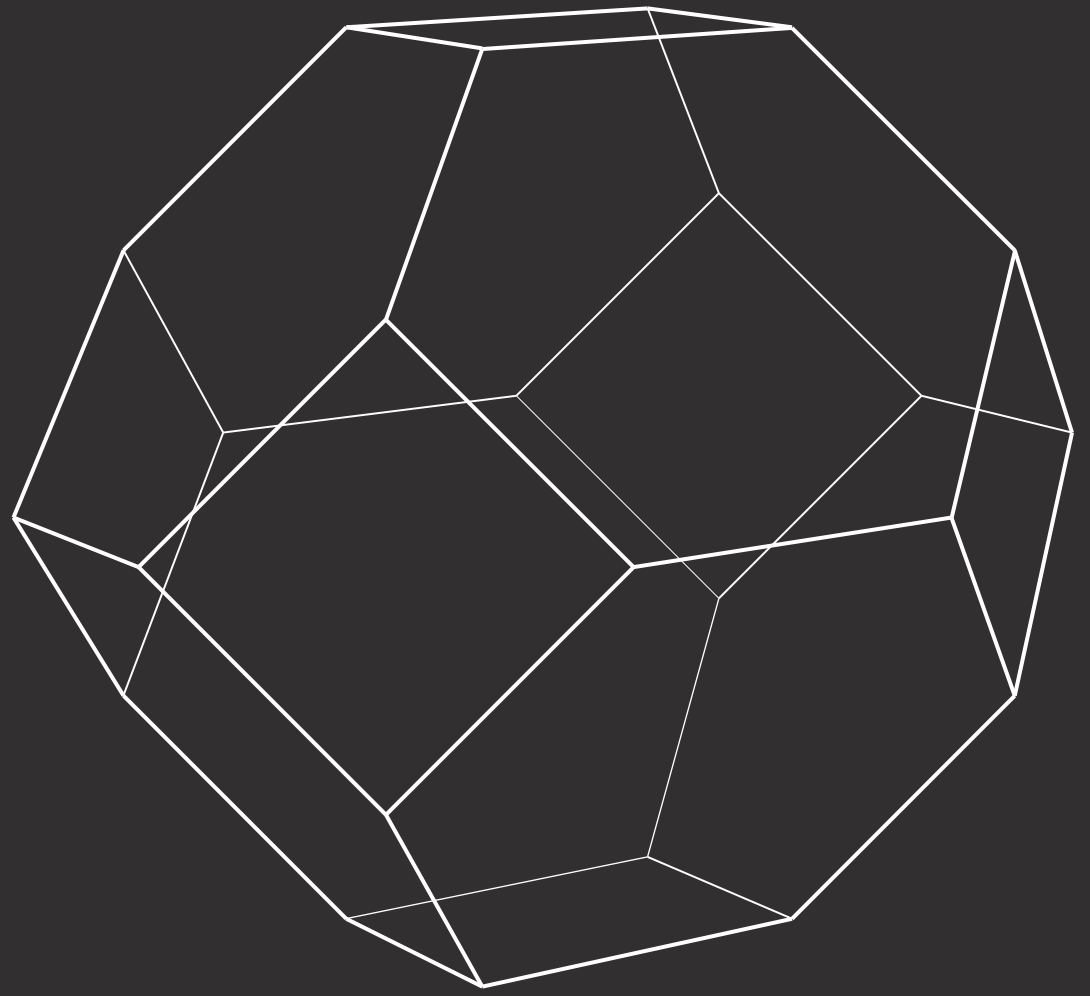

abc
{b,c}
acb
{a,c}
cab
bac
cba
bca

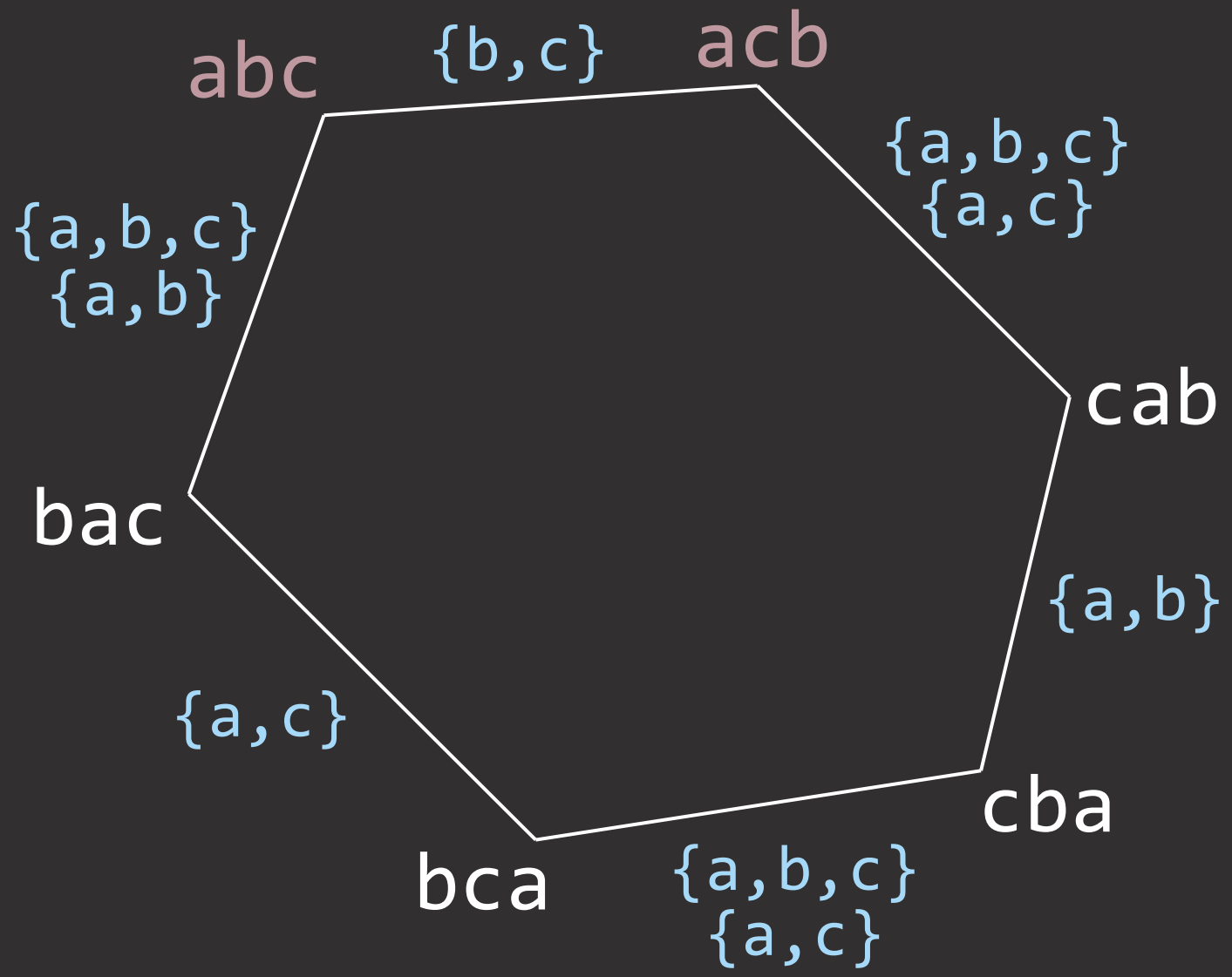

abc
{b,c}
acb
{a,b,c}
{a,c}
cab
bac
cba
bca

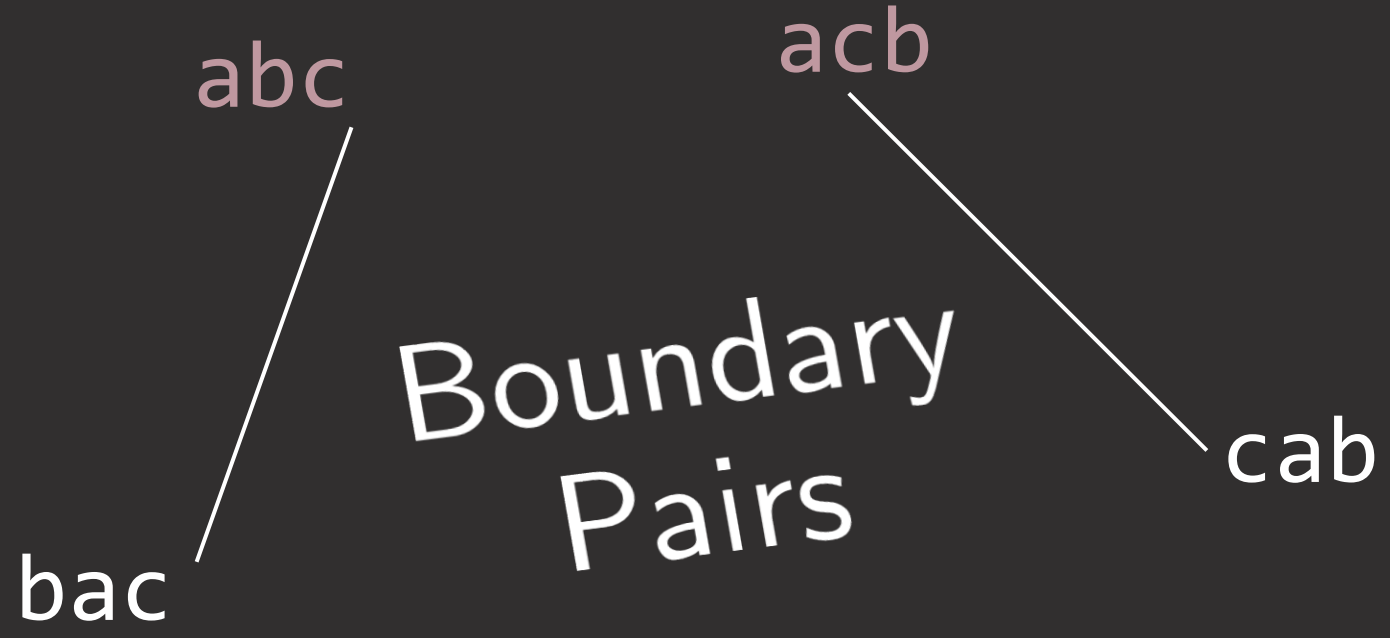

abc
{b,c}
acb
{a,b,c}
{a,c}
cab
bac
cba
bca


abc
{b,c}
acb
{a,b,c}
{a,c}
{a,b,c}
{a,b}
Three Object
Labeled Permutohedron
cab
bac
{a,b}
{a,c}
cba
bca
{a,b,c}
{b,c}

Four Object
Permutohedron




abc
{b,c}
acb
{a,b,c}
{a,c}
{a,b,c}
{a,b}
cab
bac
{a,b}
{a,c}
cba
bca
{a,b,c}
{a,c}

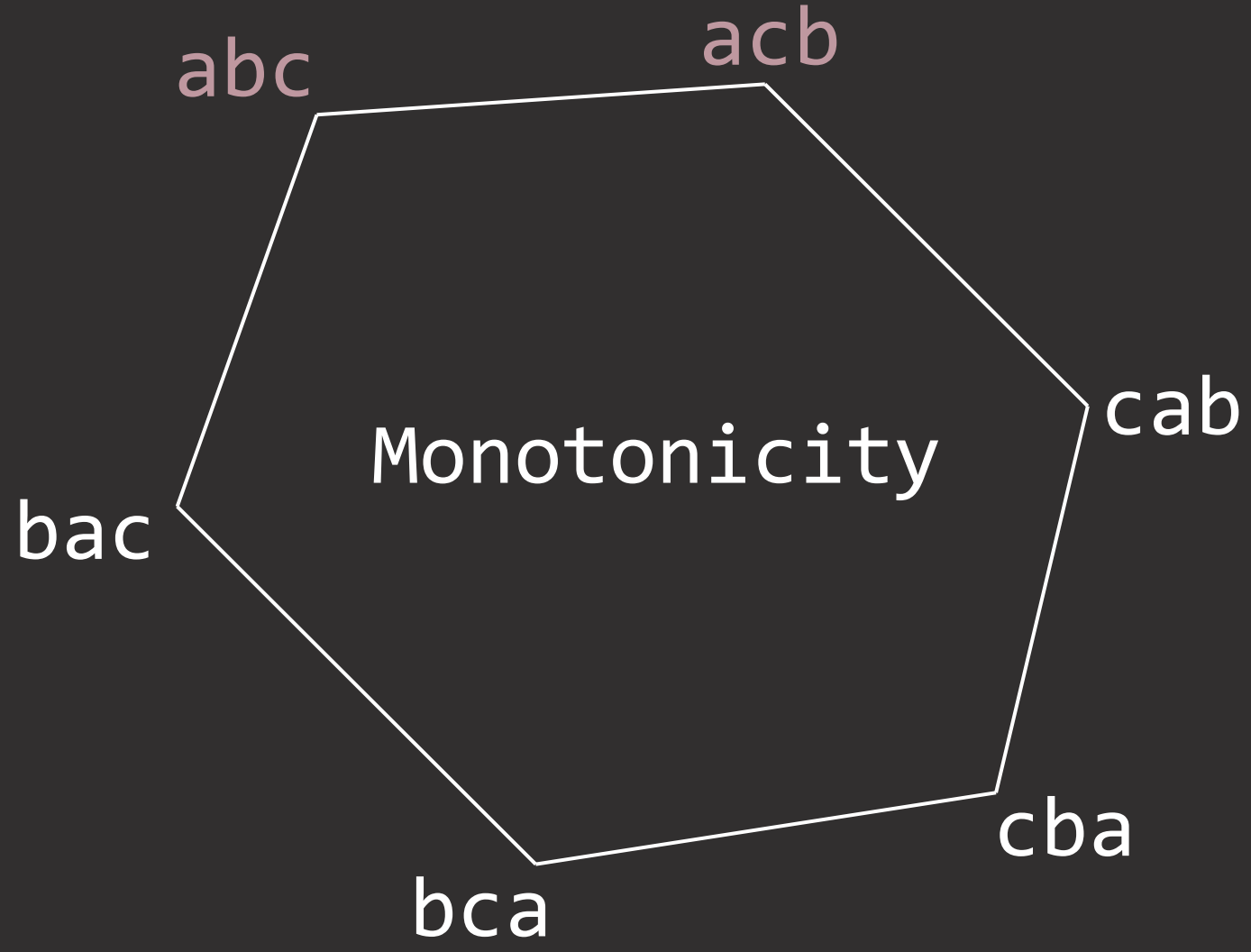

abc
acb
Boundary
Pairs
bac
cab

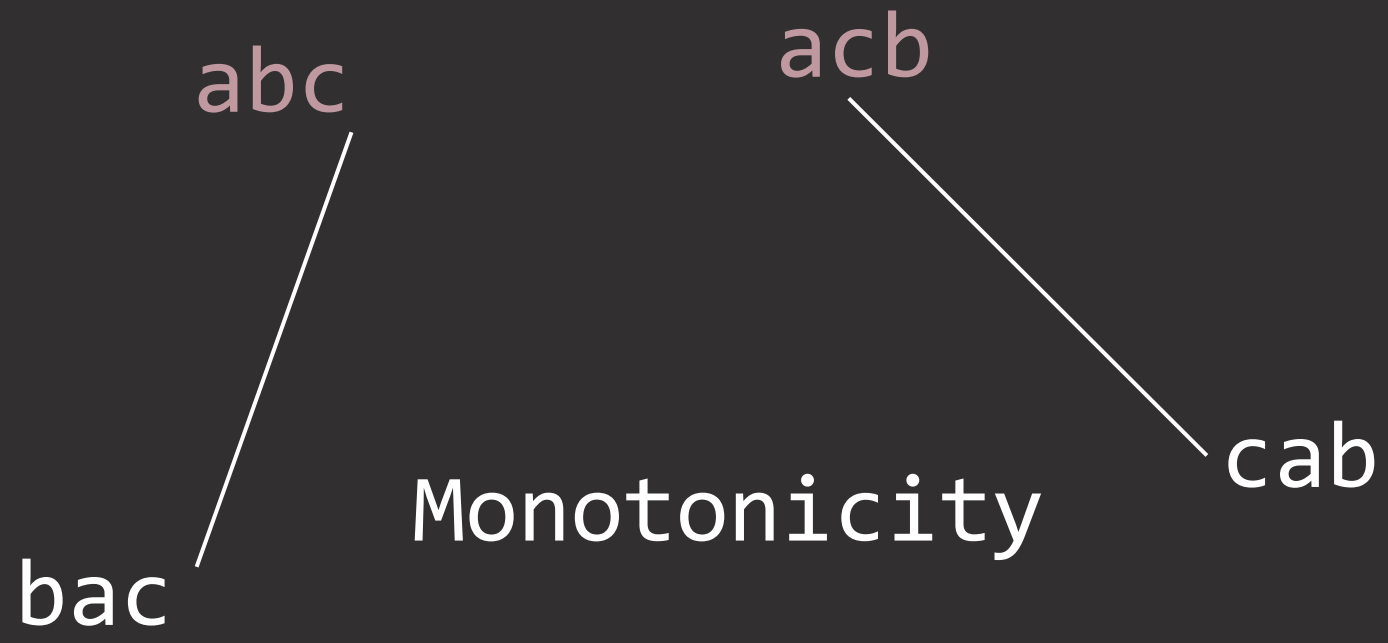
Theorem.

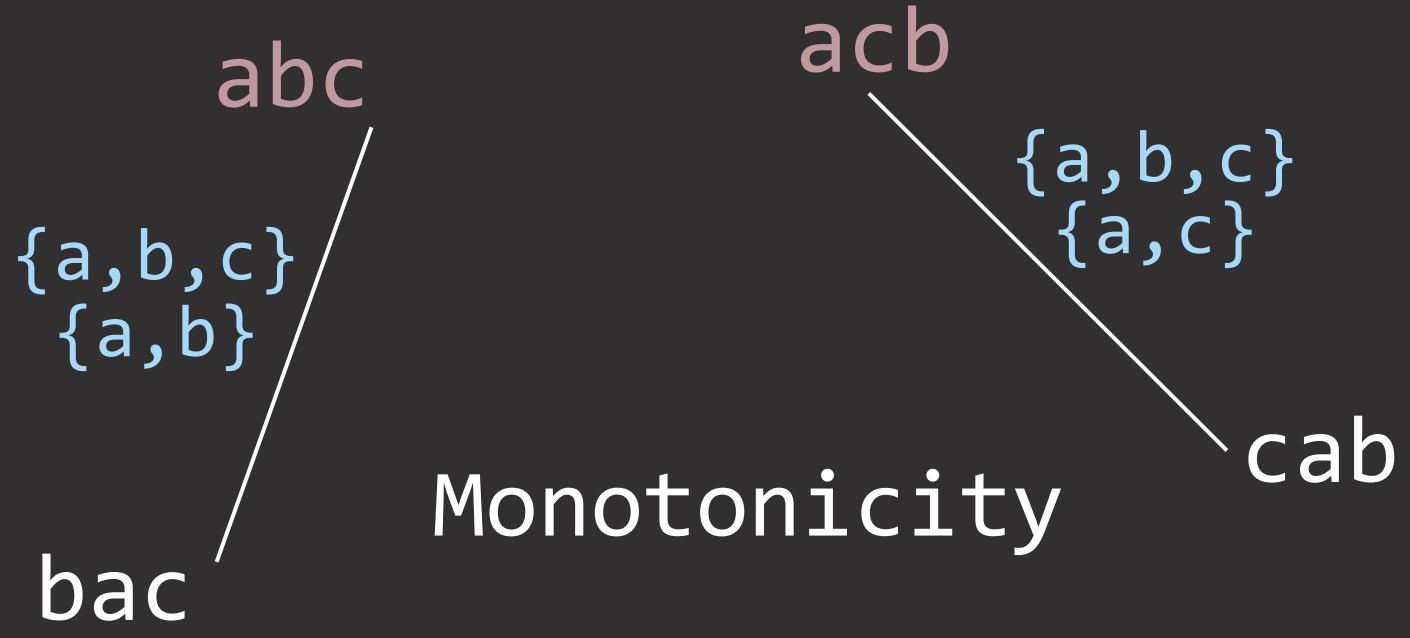
Experiment $\mathcal{D}$ classifies complete model $\mathcal{M}$

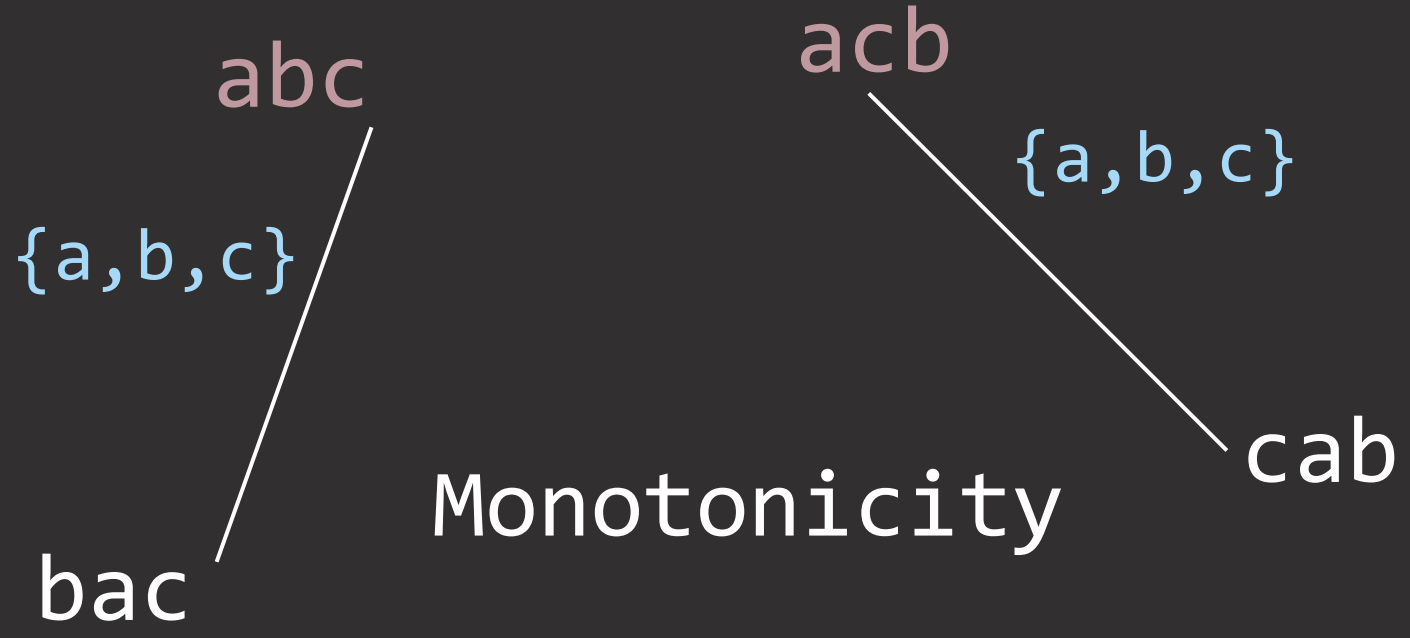
if and only if:

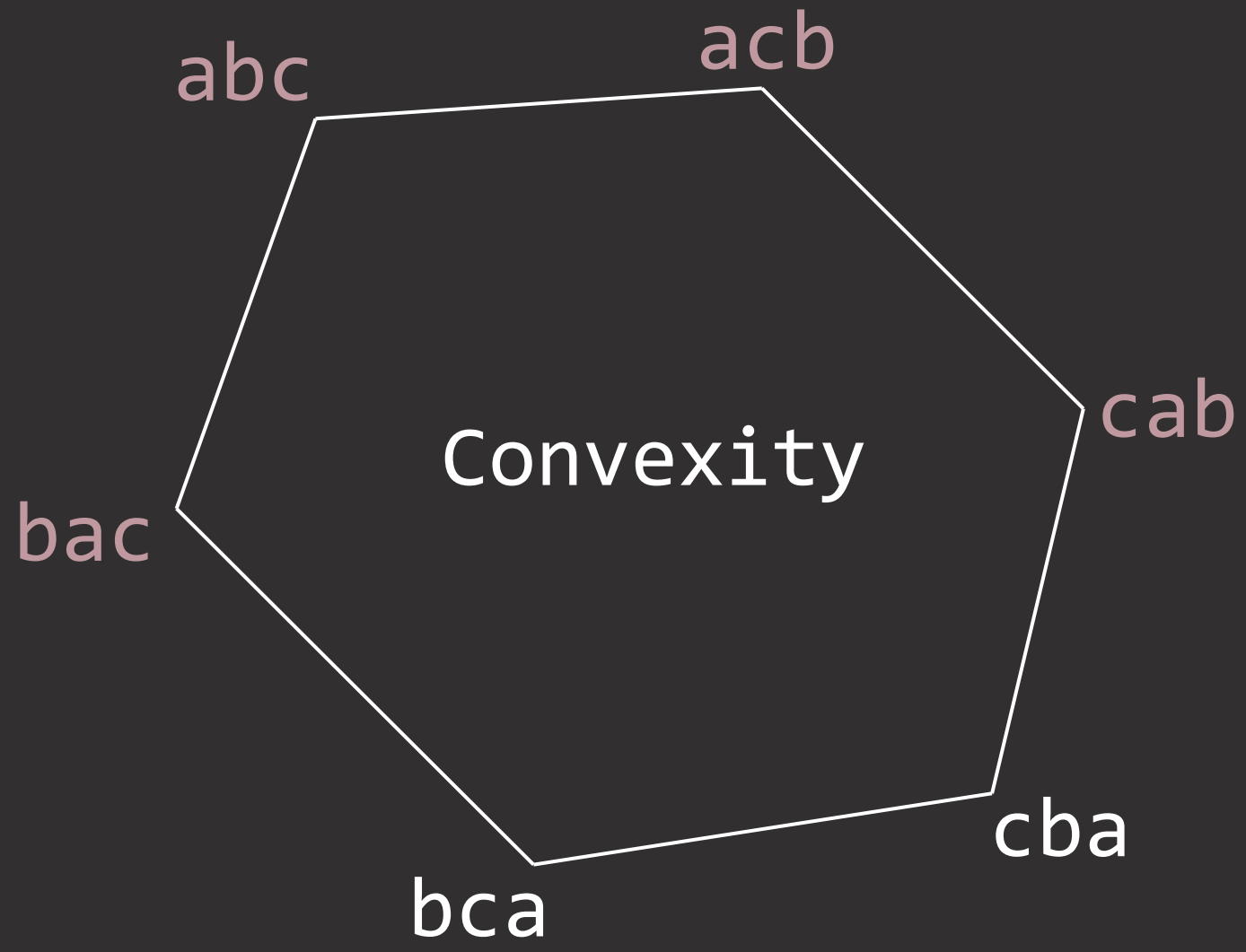
$\mathcal{D}$ separates every boundary pair of $\mathcal{M}$.

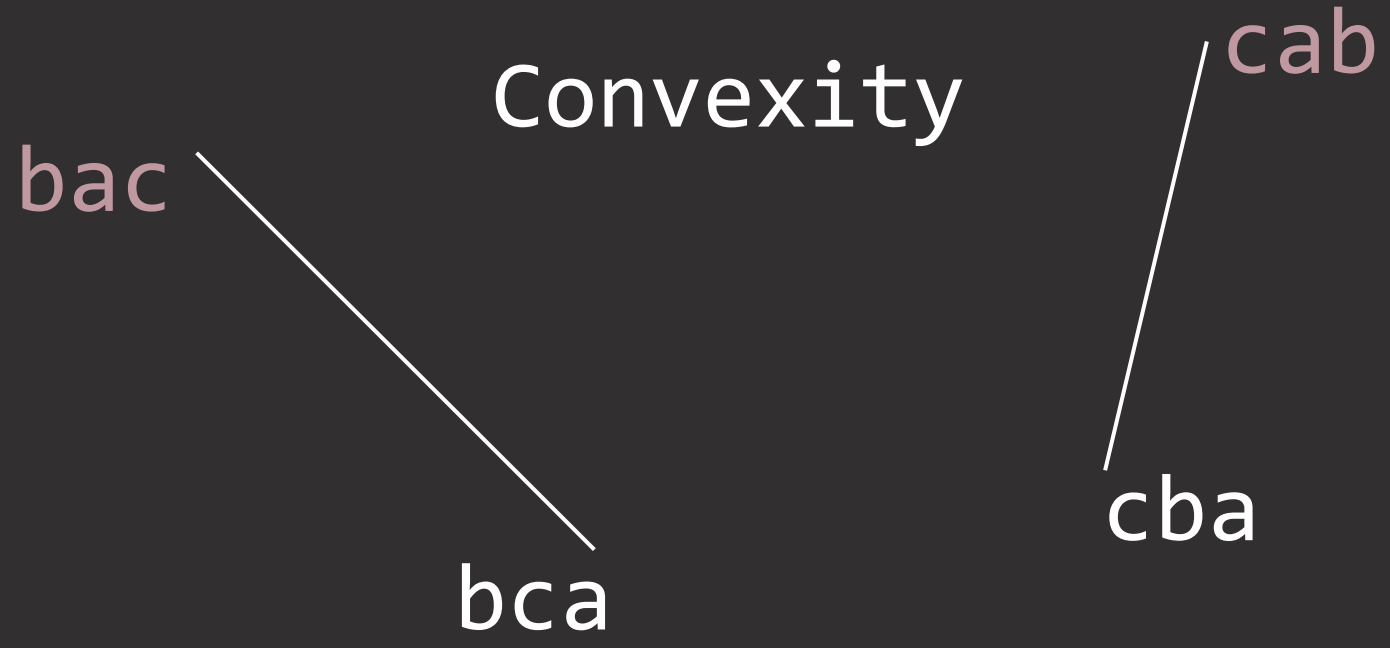

abc
acb
cab
Monotonicity
bac
cba
bca

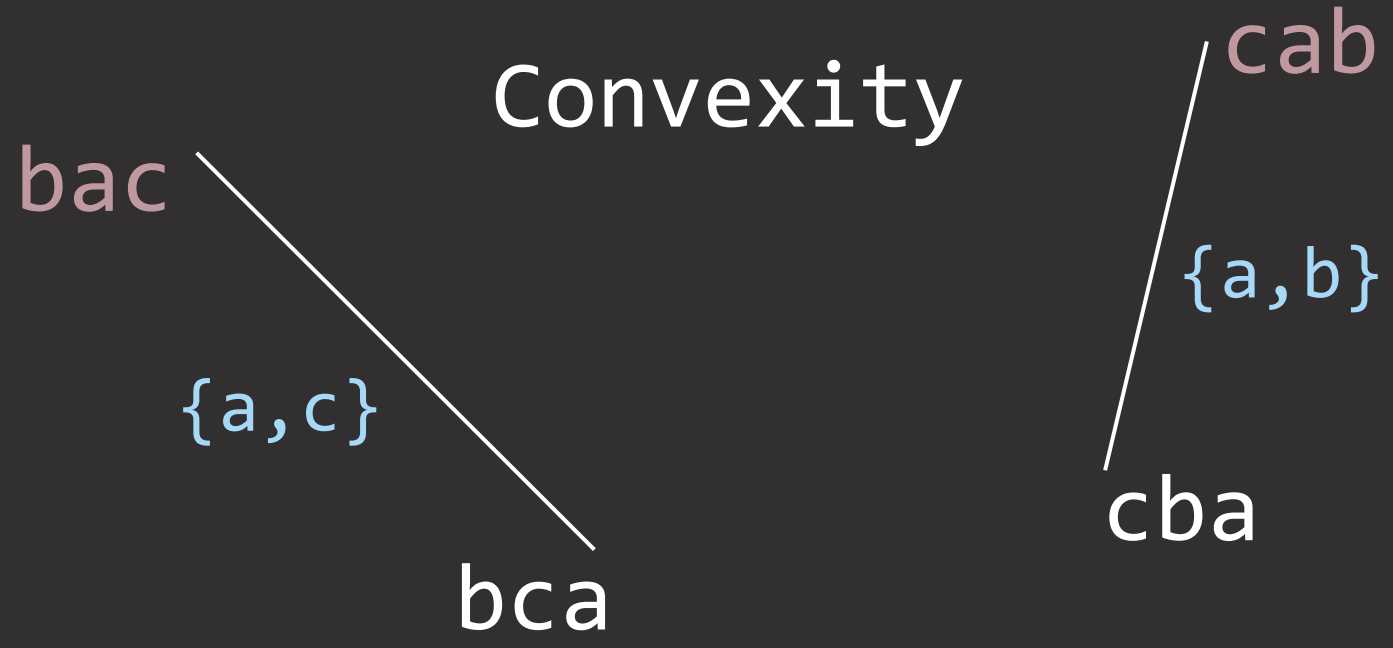

abc
acb
Monotonicity
cab
bac


abc
{a,b,c}
{a,b}
bac
acb
{a,b,c}
{a,c}
cab
Monotonicity


abc
{a,b,c}
bac
acb
{a,b,c}
cab
Monotonicity


abc
acb
cab
Convexity
bac
cba
bca

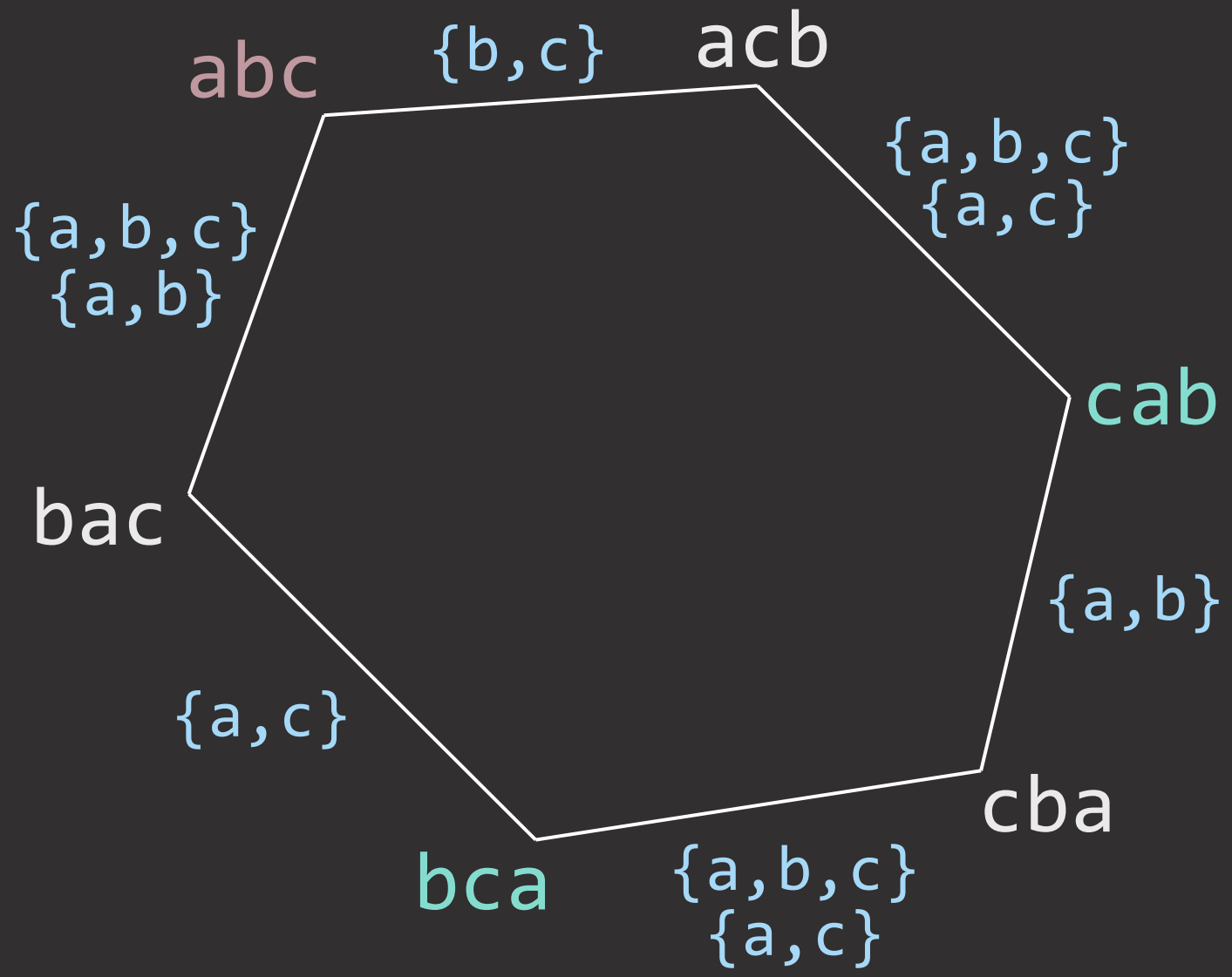

Convexity
bac
bca
cab
cba

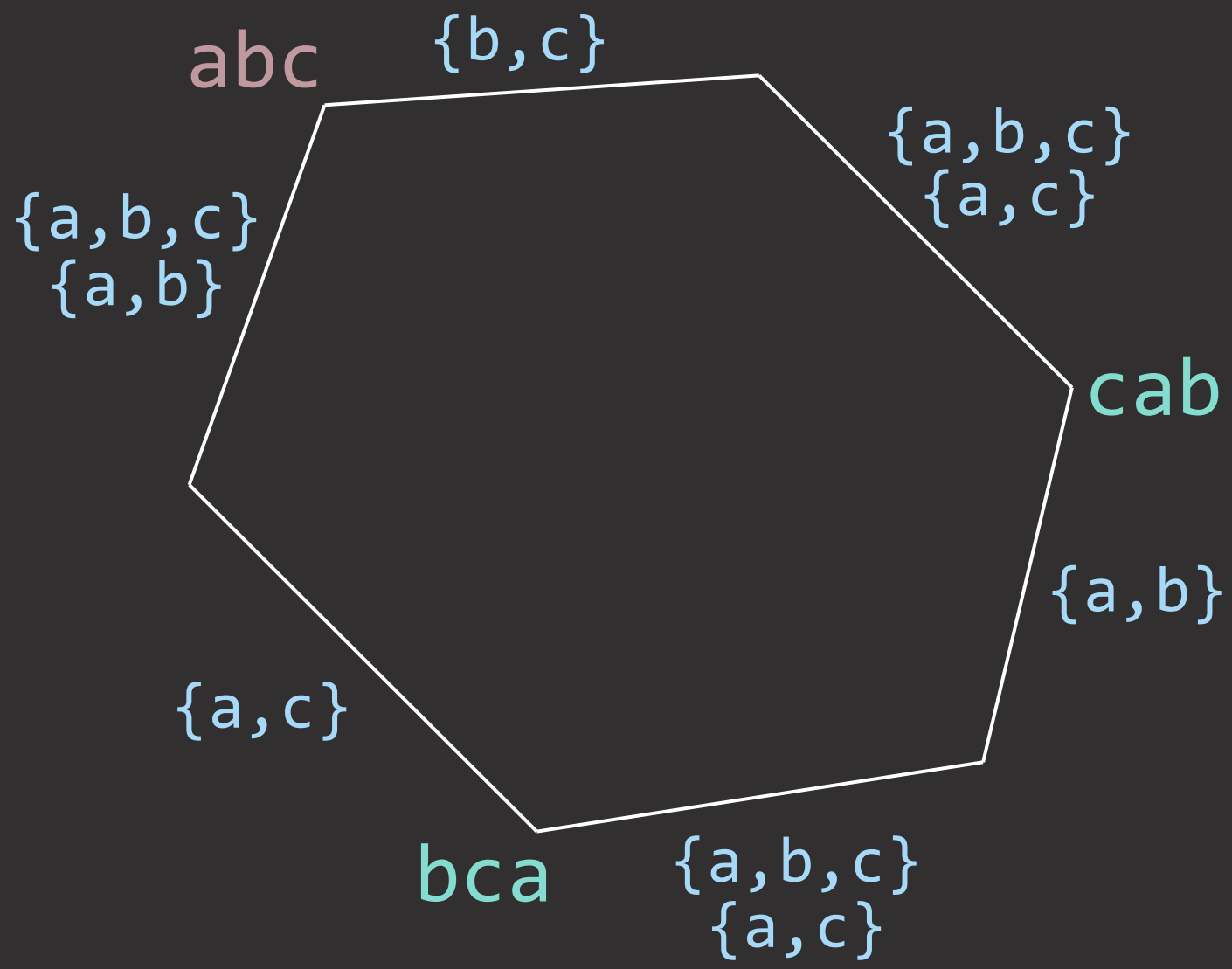

Convexity
bac
{a,c}
bca
cab
{a,b}
cba

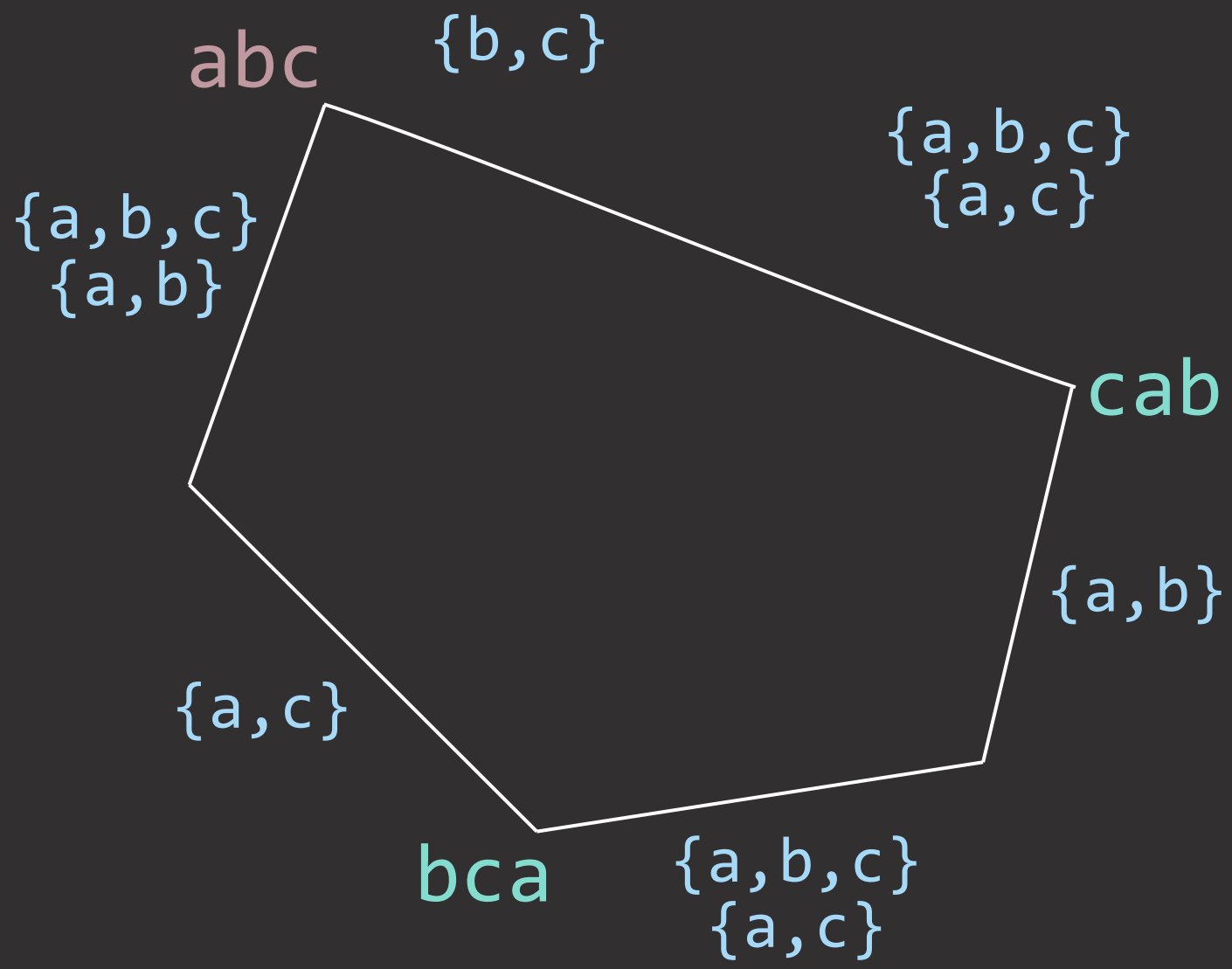
Classifying *Incomplete* Models?

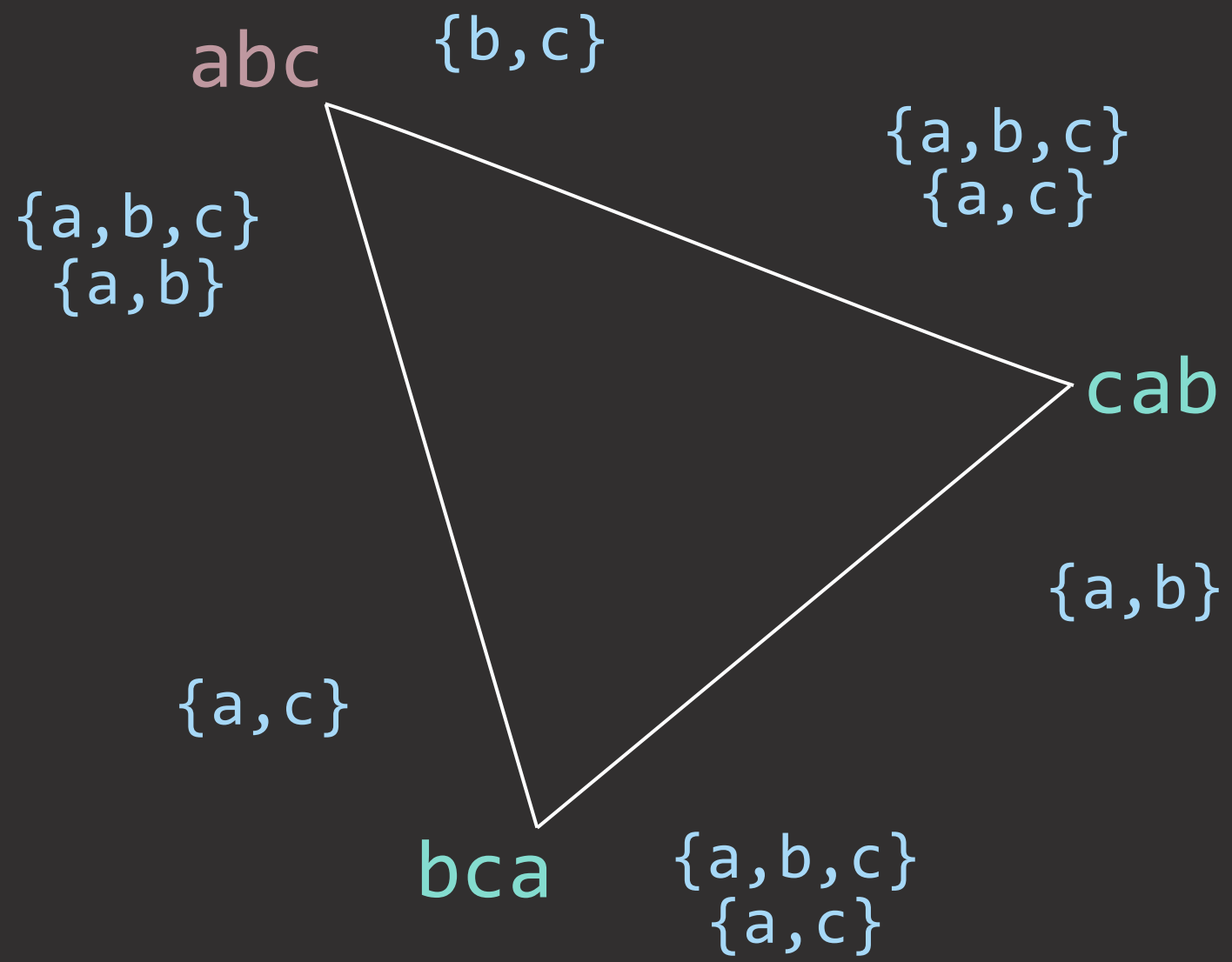
$\{\text{abc}\}$ $\{\text{bca,cab}\}$

t_1 t_2


abc
{b,c}
acb
{a,b,c}
{a,c}
cab
{a,b}
cba
{a,b,c}
{a,c}
bca
{a,c}
bac
{a,b,c}
{a,b}


abc
{b,c}
{a,b,c}
{a,c}
cab
{a,b}
{a,b,c}
{a,c}
bca
{a,b,c}
{a,c}
{a,c}
{a,b,c}
{a,b}


abc
{b,c}
{a,b,c}
{a,c}
{a,b,c}
{a,b}
cab
{a,b}
{a,c}
bca
{a,b,c}
{a,c}


abc
{b,c}
{a,b,c}
{a,c}
{a,b,c}
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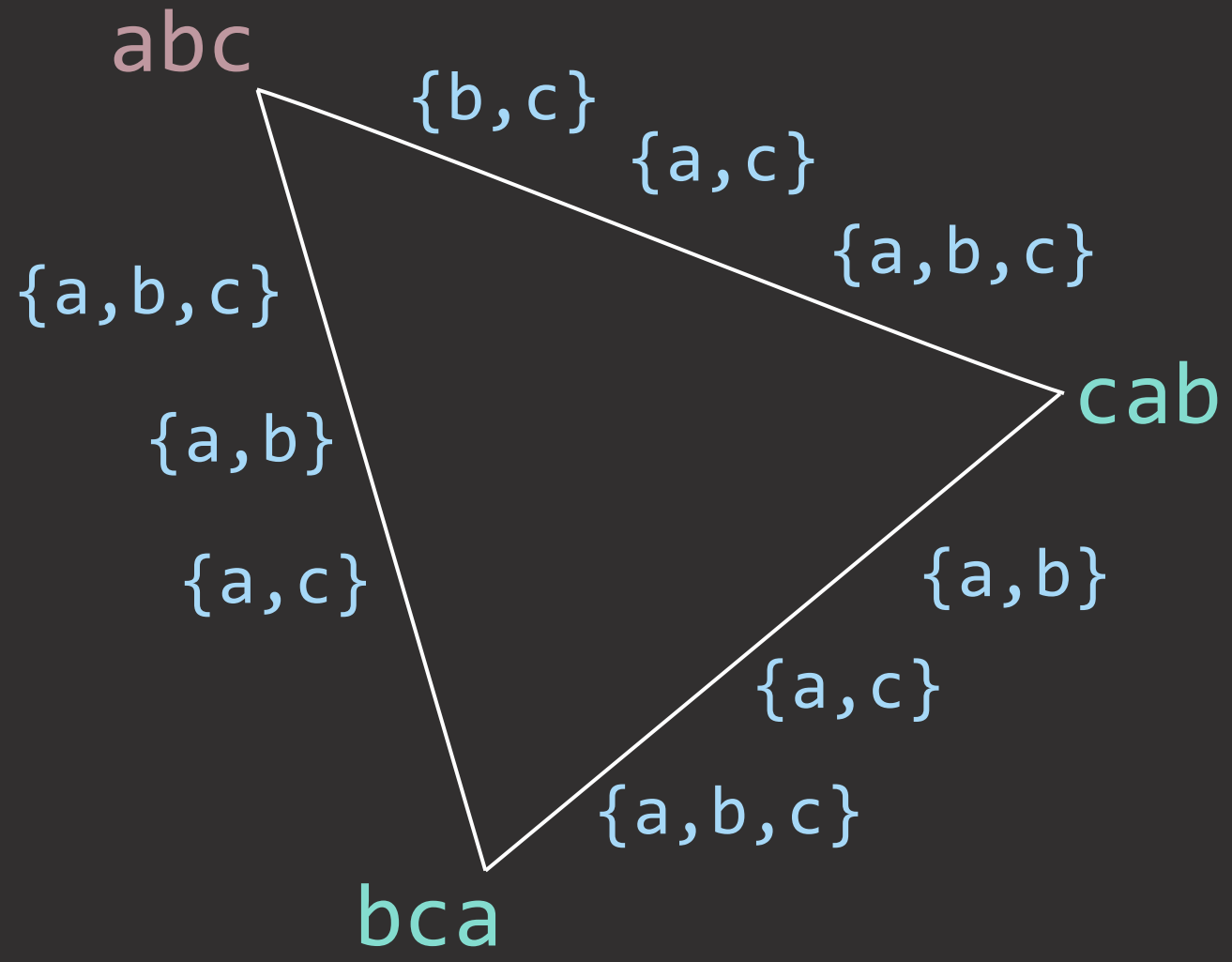
cab
{a,b}
{a,c}
bca
{a,b,c}
{a,c}

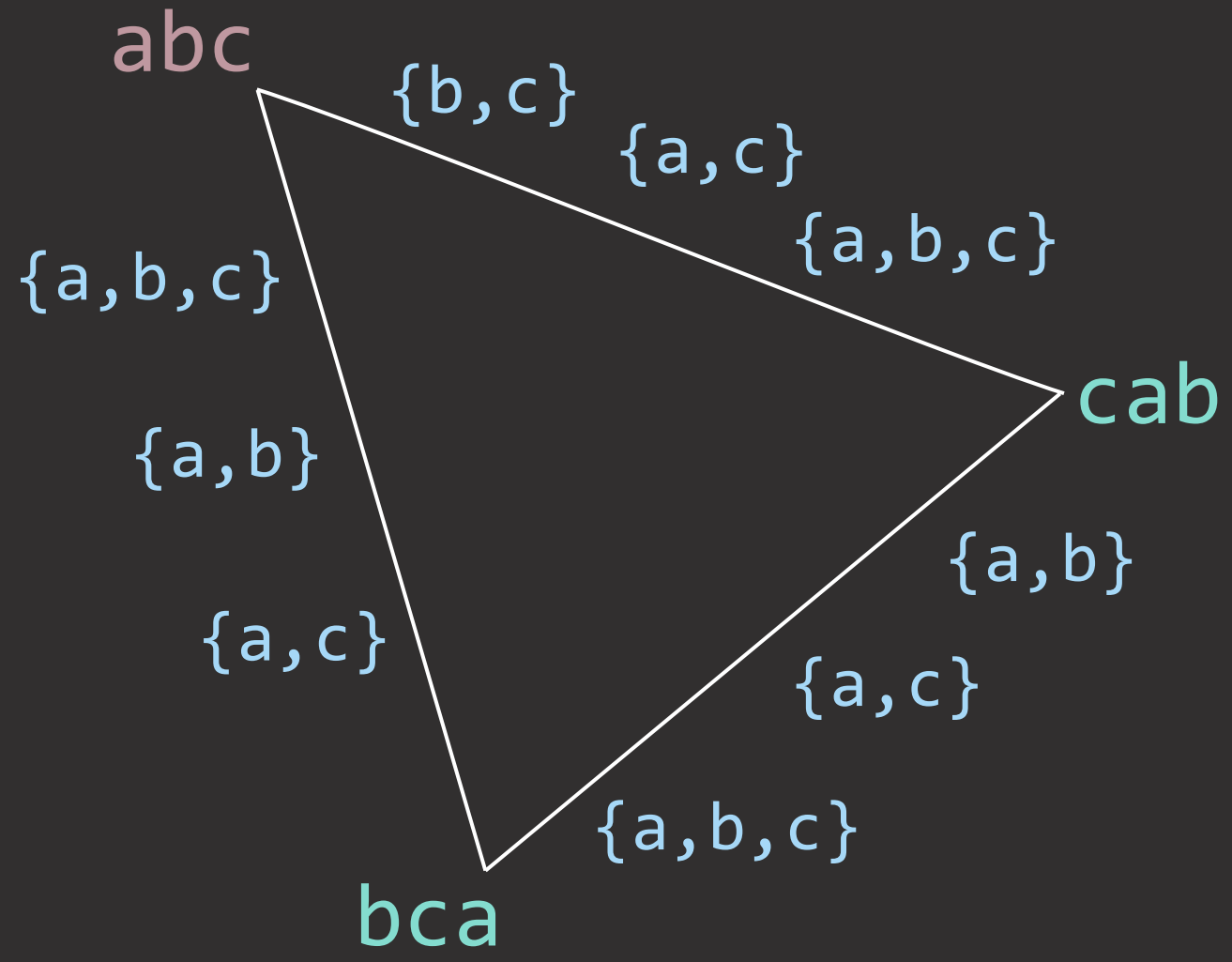

abc
{b,c}
{a,c}
{a,b,c}
{a,b,c}
{a,b}
{a,c}
cab
{a,b}
{a,c}
{a,b,c}
bca

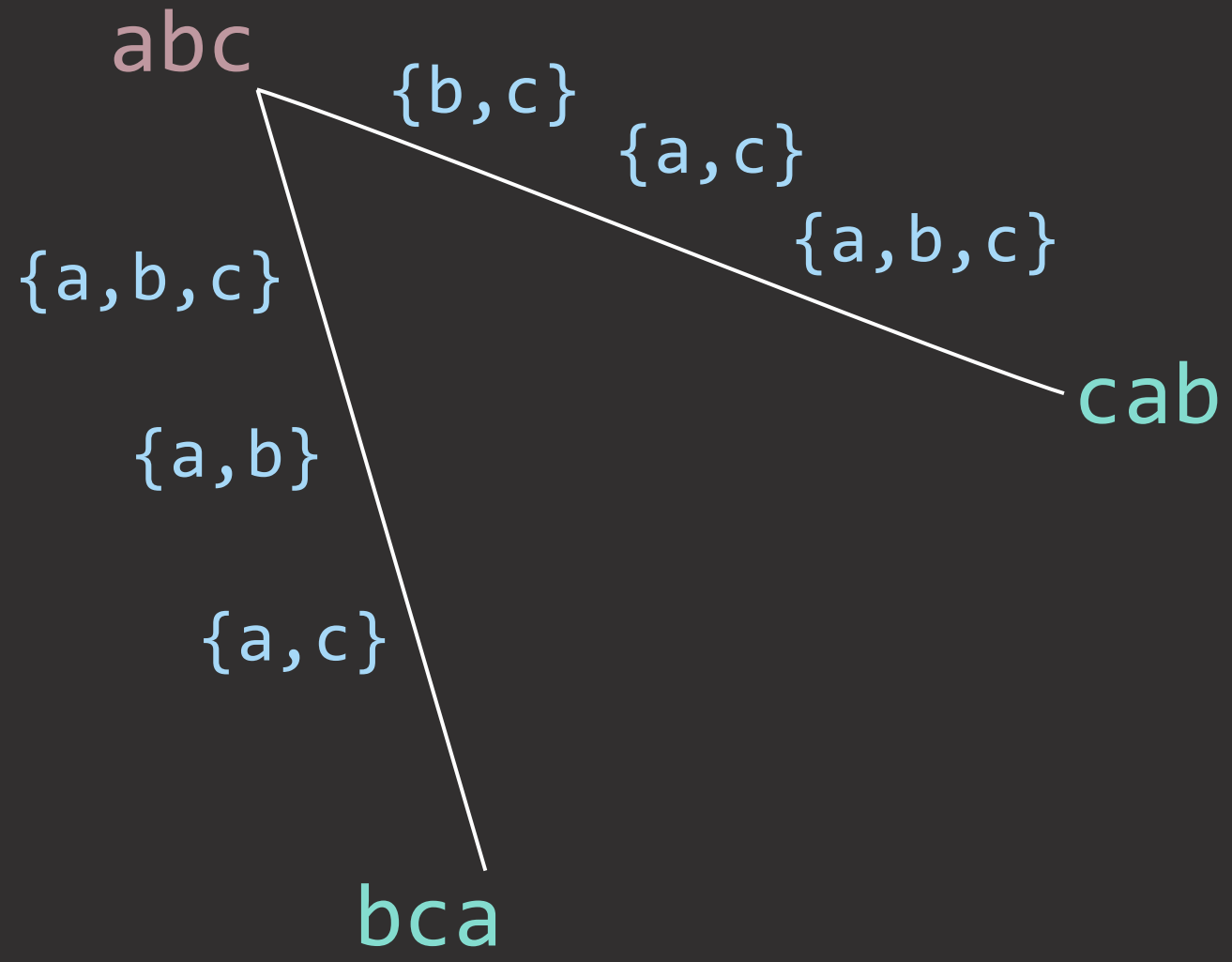
Theorem.

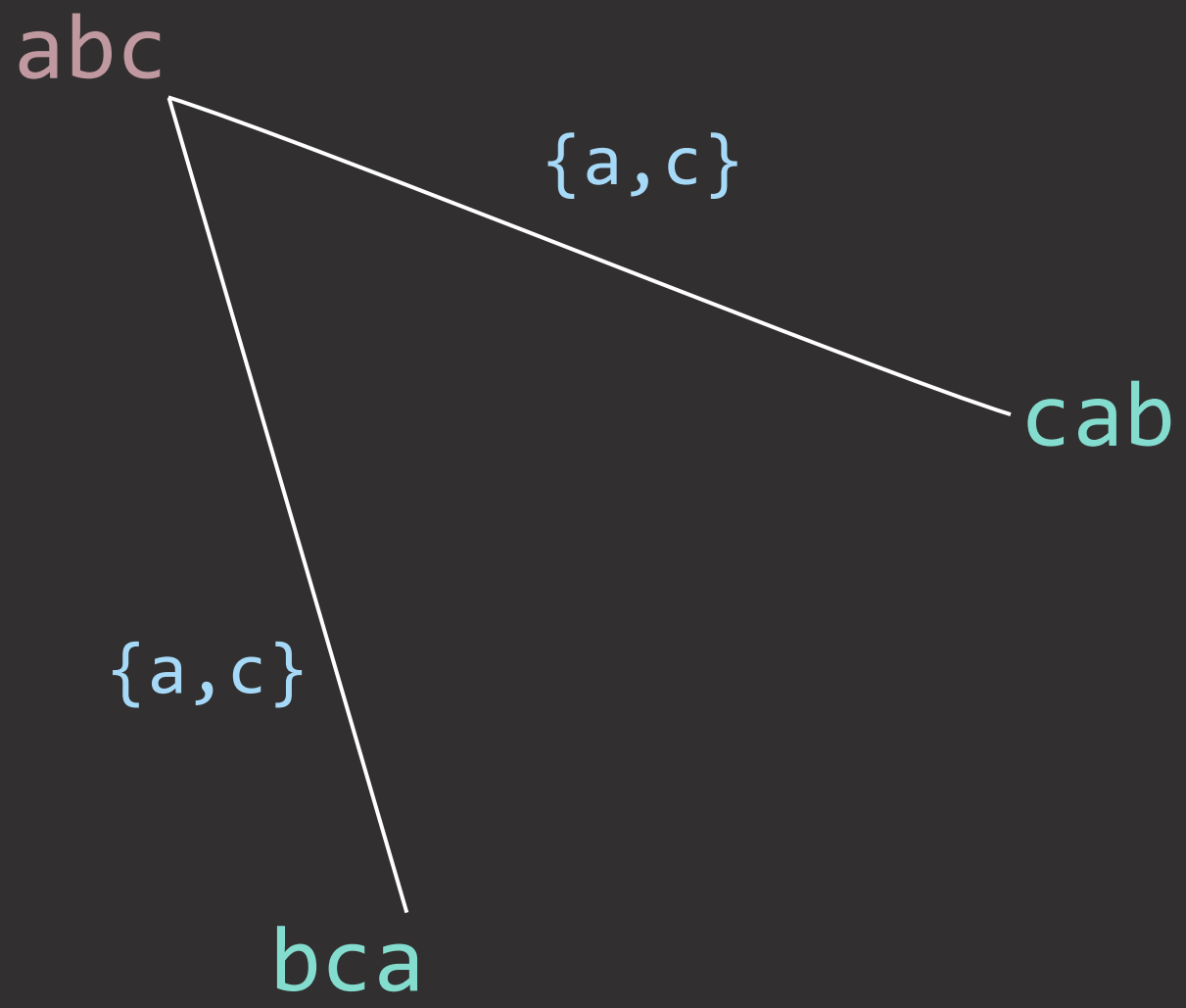
Experiment $\mathcal{D}$ classifies incomplete model $\mathcal{M}$

if and only if:

$\mathcal{D}$ separates every restricted boundary pair of $\mathcal{M}$.


abc
{b,c}
{a,c}
{a,b,c}
{a,b,c}
cab
{a,b}
{a,b}
{a,c}
{a,c}
{a,b,c}
bca


abc
{b,c}
{a,c}
{a,b,c}
cab
{a,b,c}
{a,b}
{a,c}
bca


abc
{a,c}
cab
{a,c}
bca

Will the Braves Win the World Series?

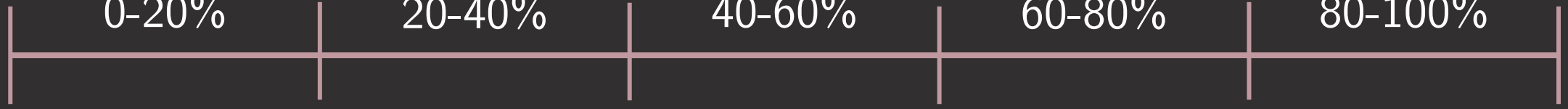
0-33% | 33-66% | 66-100%

Will the Braves Win the World Series?

0-20% | 20-40% | 40-60% | 60-80% | 80-100%

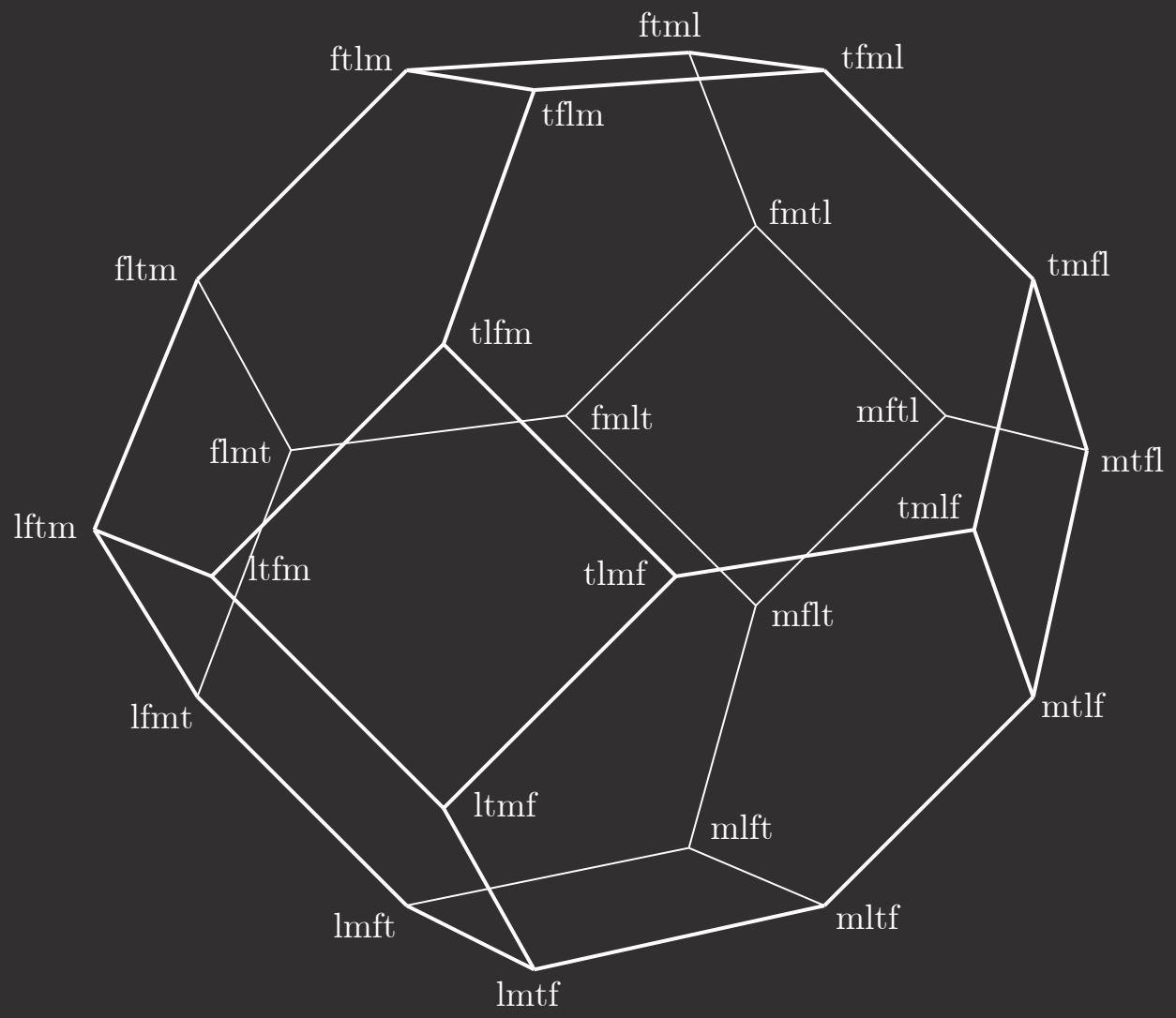
t f l m

\$10 if ***Braves*** *Win*, \$10 if ***Astros*** *Win*, \$10 with **60%,** \$10 with **80%**



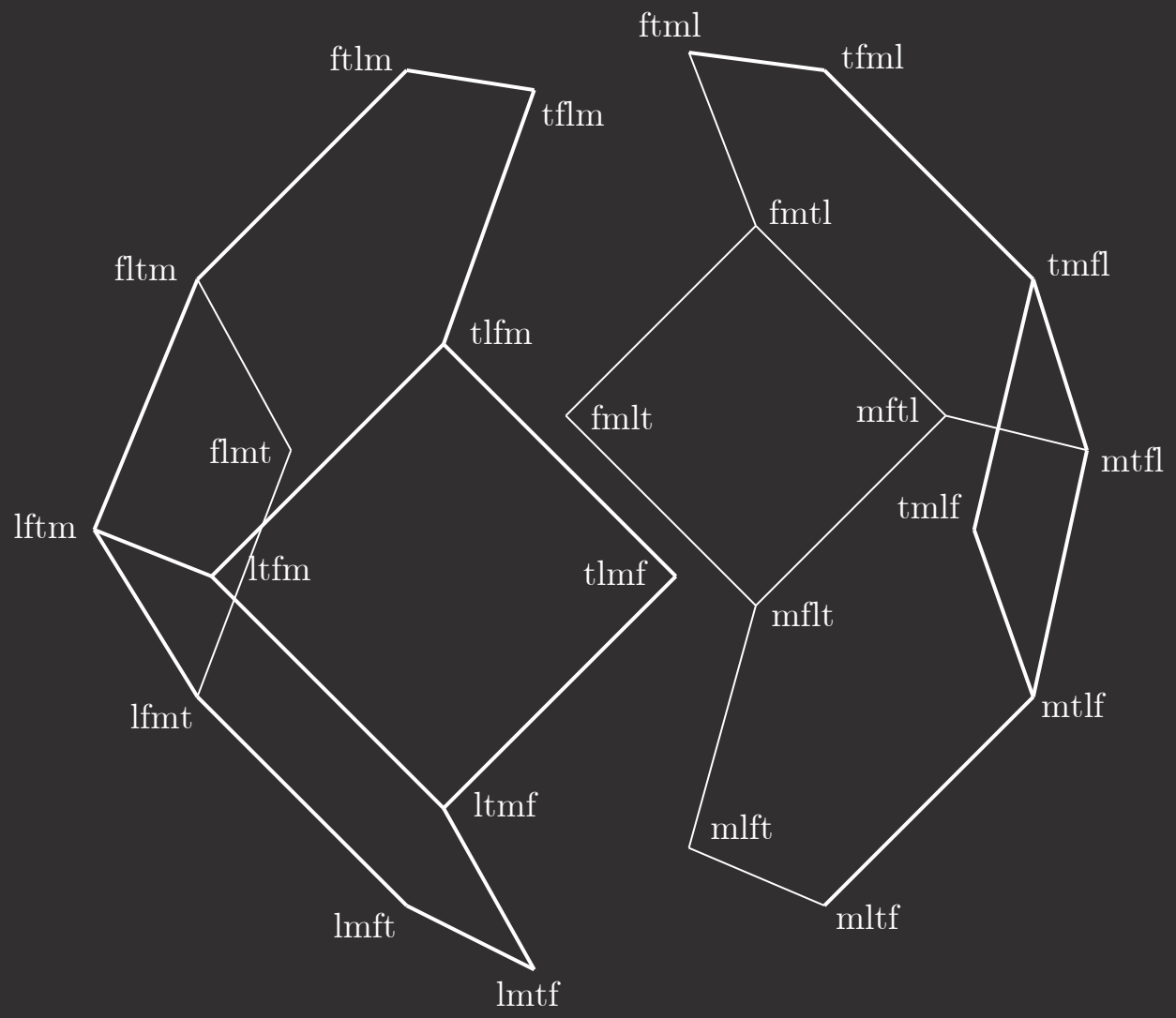
t f l m

\$10 if ***Braves*** *Win*, \$10 if ***Astros*** *Win*, \$10 with **60%**, \$10 with **80%**



$fmlt$ $mflt$ $\{mlft, mltf\}$ mtlf $tmlf$

0-20% 20-40% 40-60% 60-80% 80-100%


ftml
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tflm
fmtl
tmfl
fltm
tlfm
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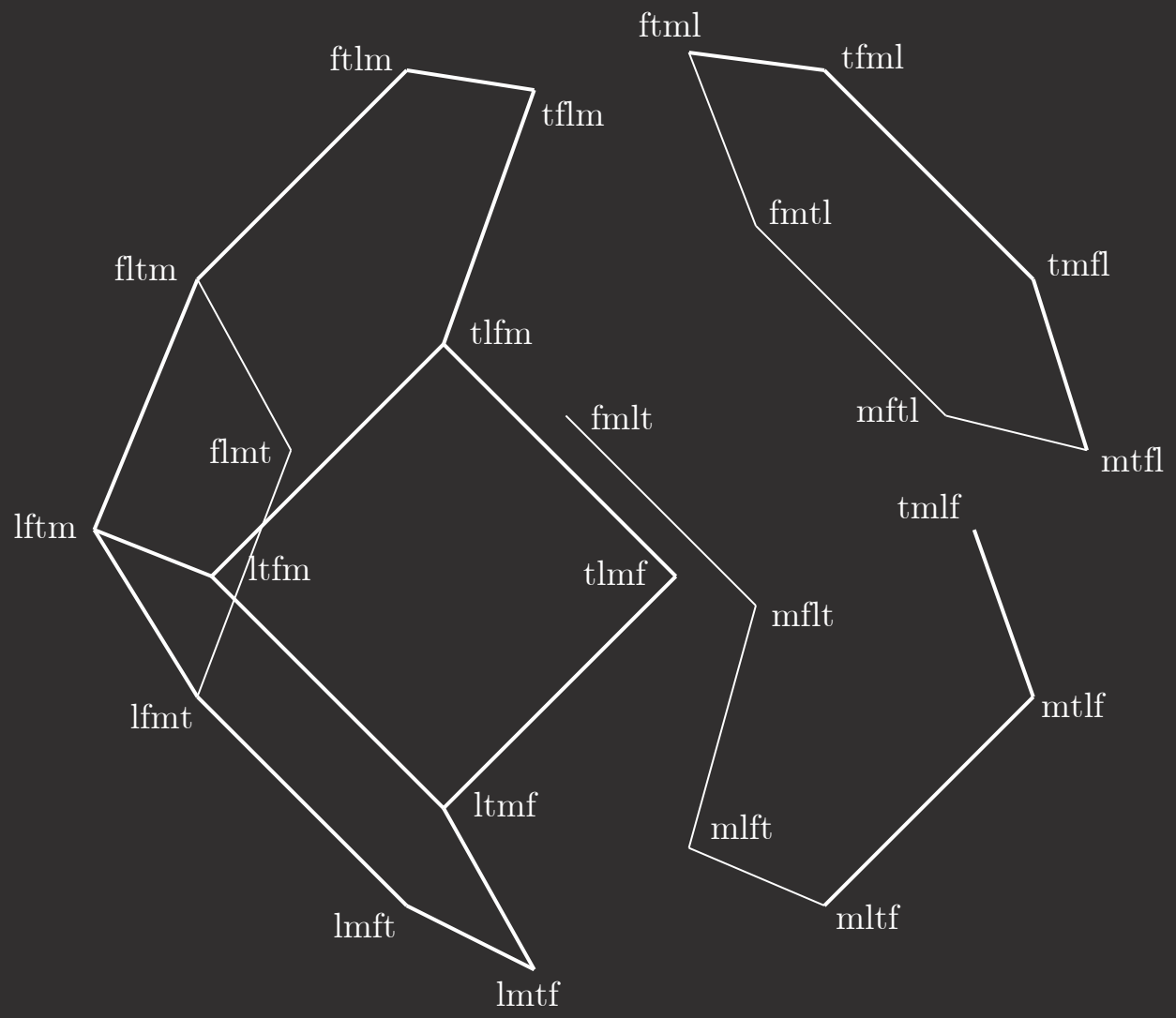
mftl
mtfl
flmt
tmlf
lftm
ltfm
tlmf
mflt
lfmt
mtlf
ltmf
mlft
lmft
mltf
lmtf


$fmlt$
$mflt$
$\{mlft, mltf\}$
mtlf
$tmlf$
0-20%
20-40%
40-60%
60-80%
80-100%


ftml
tfml
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tflm
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fltm
tlfm
mftl
mtfl
fmlt
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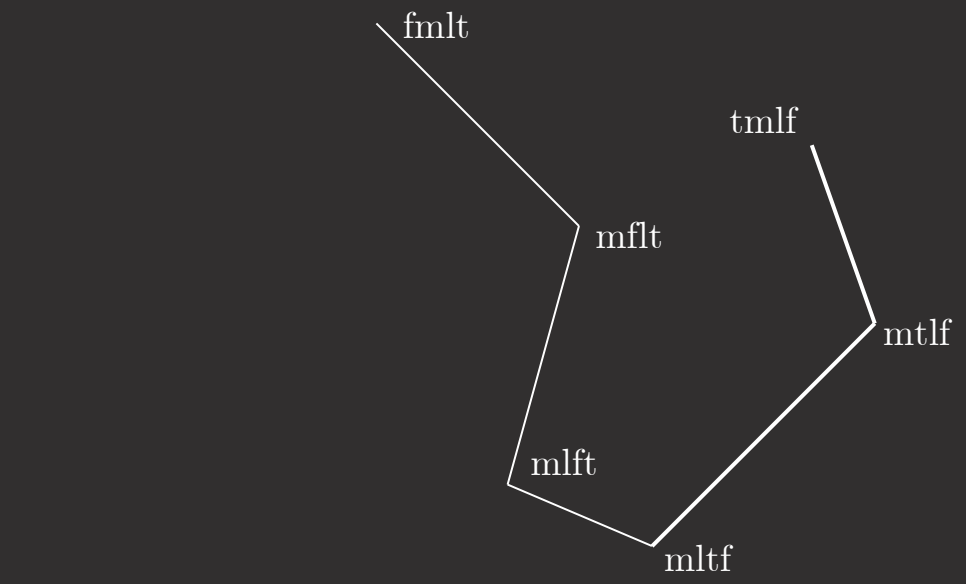
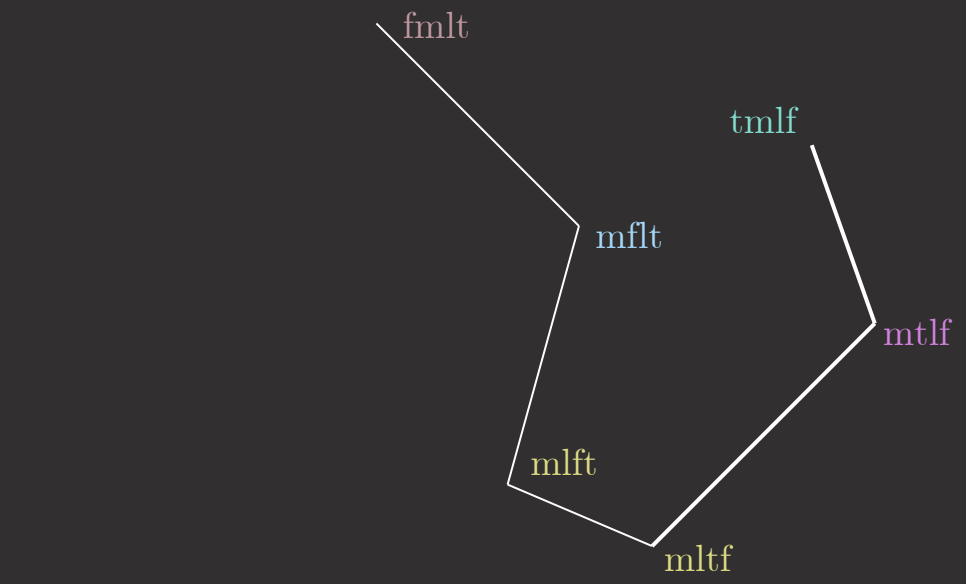
lftm
ltfm
tlmf
mflt
mtlf
lfmt
ltmf
mlft
lmft
mltf
lmtf

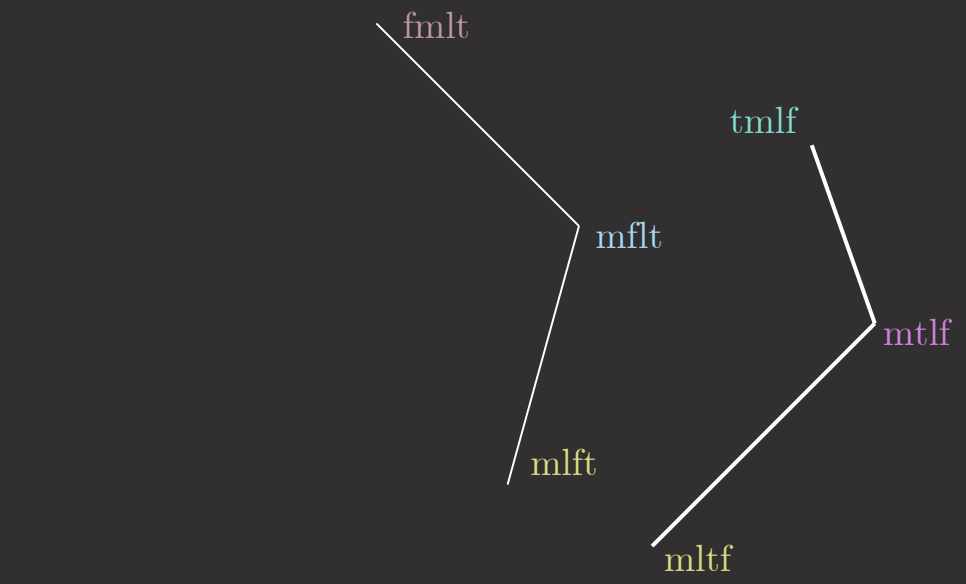

fmlt
mflt
{mlft, mltf}
mtlf
tmlf
0-20%
20-40%
40-60%
60-80%
80-100%



$fmlt$	$mflt$	$\{mlft, mltf\}$	mtlf	$tmlf$
0-20%	20-40%	40-60%	60-80%	80-100%

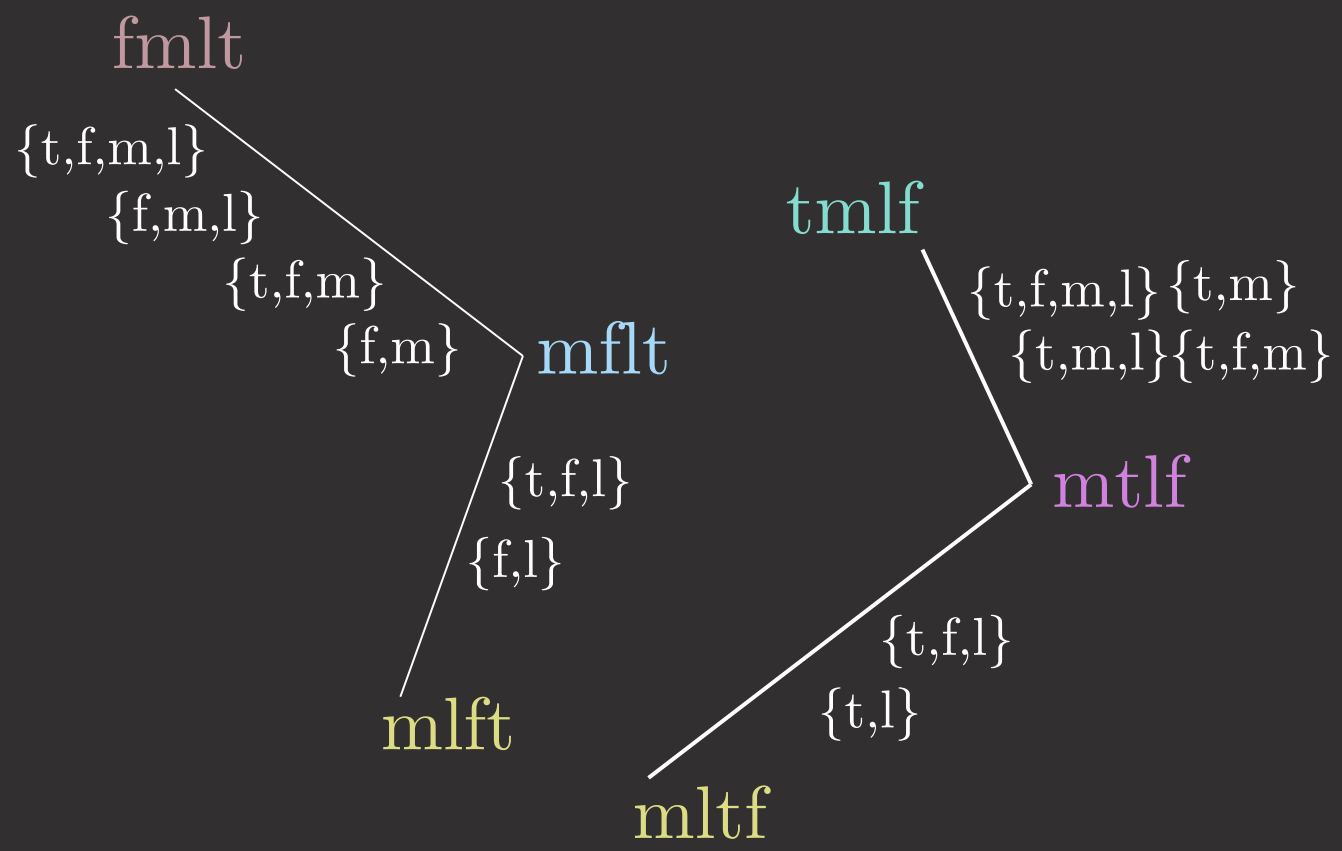


$fmlt$	$mflt$	$\{mlft, mltf\}$	$mtlf$	$tmlf$
0-20%	20-40%	40-60%	60-80%	80-100%



$fmlt$ $mflt$ $\{mlft, mltf\}$ $mtlf$ $tmlf$

0-20% 20-40% 40-60% 60-80% 80-100%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{t,f,m,l}
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mflt
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mlft
tmlf
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mtlf
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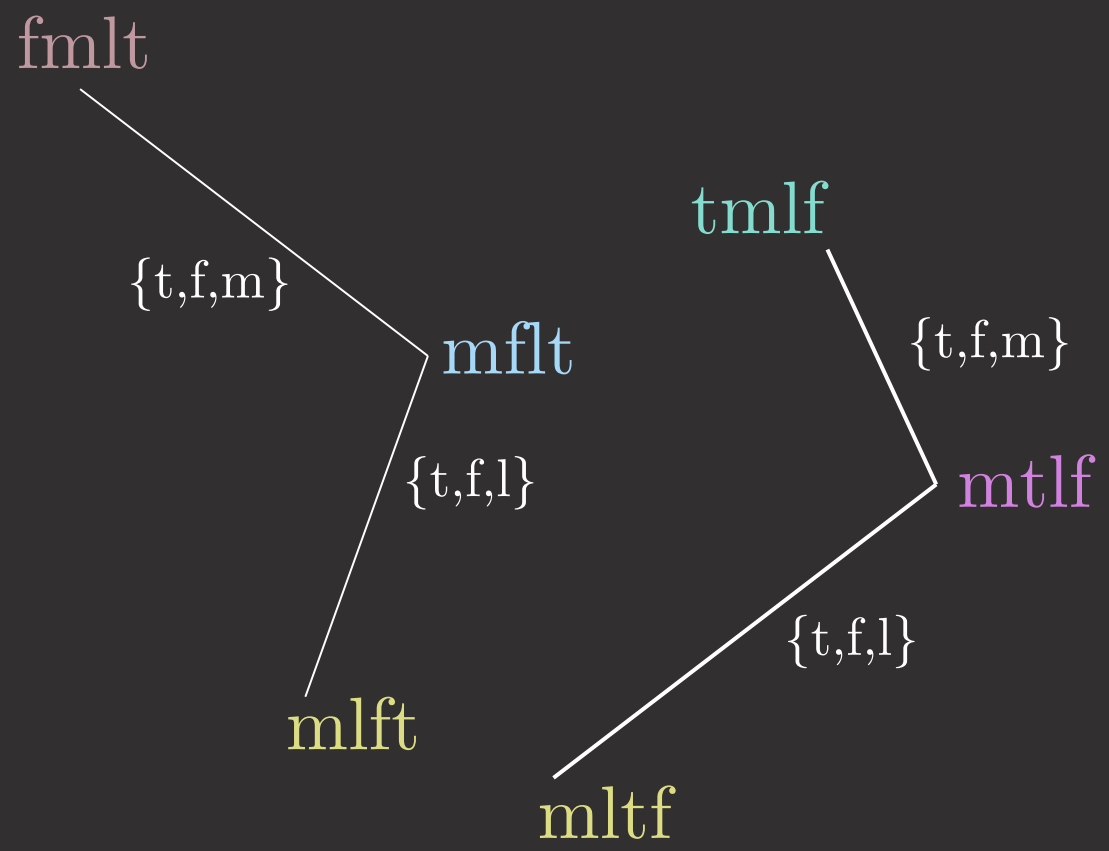

fmlt
{t,f,m}
mflt
{t,f,l}
mlft
tmlf
{t,f,m}
mtlf
{t,f,l}
mltf

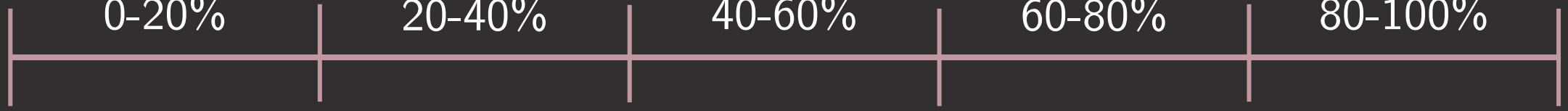
{t,f,m}

{t,f,l}

Pick one from each menu...

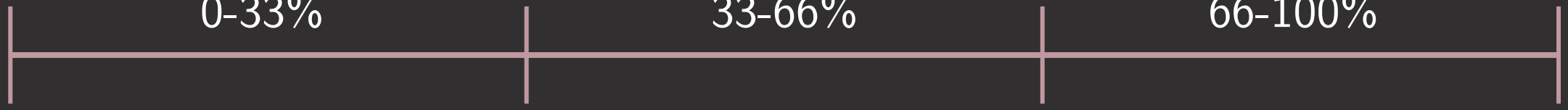
$10 if ***Braves*** *Win*, $10 if ***Astros*** *Win*, $10 with **80%**

$10 if ***Braves*** *Win*, $10 if ***Astros*** *Win*, $10 with **60%**



Pick one...

$10 if ***Braves*** *Win*, $10 if ***Astros*** *Win*, $10 with **66%**



Paper

App

Counting Experiments

Counting Experiments

{a,b},{b,c},{a,c},{a,b,c}

Counting Experiments

{a,b},{b,c},{a,c},{a,b,c}

$$2^N - (N + 1)$$

Counting Experiments

{a,b},{b,c},{a,c},{a,b,c}

$$2^N - (N+1)$$

{a,b}	{a,b},{a,c}	{a,b},{a,b,c}	{a,b},{a,c},{a,b,c}	{a,b},{a,c},{b,c},{a,b,c}
{a,c}	{a,c},{b,c}	{a,c},{a,b,c}	{a,c},{b,c},{a,b,c}	{a,b},{a,c},{b,c}
{b,c}	{b,c},{a,b}	{b,c},{a,b,c}	{b,c},{a,b},{a,b,c}	{a,b,c}

$$2^{\left(2^N-(N+1)\right)} - 1$$

N=3

15

N=3	15
N=4	2,047

N=3	15
N=4	2,047
N=5	67,108,863

N=9

13,093,562,431,584,567,480,052,
758,787,310,396,608,866,568,184,
172,259,157,933,165,472,384,535,
185,618,698,219,533,080,369,303,
616,628,603,546,736,510,240,284,
036,869,026,183,541,572,213,314,
110,357,504

Minimal Experiments

P.J. Healy[1] Greg Leo[2]

1 The Ohio State University, Columbus Ohio
2 Vanderbilt University, Nashville Tennessee

June 12, 2022