Complex Analysis Summary

Paul Joo-Hyun Kim

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0 Preface

This note is for people studying complex analysis, and got lost in the middle with bunch of technical explanations. I wil try my best to be succinct as possible, stating important results (mostly without proof, but a bit of justification).

Warning: This summary note is not a substitute for the lecture note. Make sure you study from lecture note!

1 Complex Plane and Möbius Maps

We will be working in what's known as the *extended complex plane*. Redefine the symbol $\mathbb{C} \coloneqq \mathbb{C} \cup \left\{ \underbrace{\infty}_{\text{Complex Infinity}} \right\}$; that is, whenever I mention $\mathbb{C}$, I refer to the space of complex numbers and infinity.

Note that in $\mathbb{C}$, ∞ is different from infinity in real numbers. $\infty \coloneqq \frac{1}{0}$ is a value that is not "larger" or "smaller" than any number (since we are talking about complex number...), but rather a number on a complex plane at a really far distance from origin.

It is **WRONG** to say:

- $\infty \geq a$ for any $a \in \mathbb{C}$
- $\infty \leq a$ for any $a \in \mathbb{C}$

However, it is **CORRECT**[1] to say:

- $|\infty| \geq |a|$ for any $a \in \mathbb{C}$.

[1]Subtlety here: it seems a bit dodgy to say $\infty = \infty$, but this is matter of definition; you won't really encounter this type of "philosophical" problem in your exam.