§Calculus $\nabla\left(\frac{1}{r}\right)=-\frac{\mathbf{r}}{r^3}$ $\quad \partial_x\left(\frac{-x}{r^3}\right)=\frac{-1}{r^3}+\frac{3x^2}{r^5}$ $\quad \frac{1}{|\mathbf{r}-\mathbf{r}'|}\overset{r\to\infty}{\sim}\frac{1}{r}+\frac{1}{r^3}\mathbf{r}\cdot\mathbf{r}'+O\left(\frac{1}{r^3}\right)$

Delta: $\int_{-\infty}^{\infty} f(x)\delta\left(x-x'\right)$ $\quad \delta\left[a\left(x-x'\right)\right]=\frac{1}{|a|}\delta\left(x-x'\right)$ $\quad \delta\left(g(x)\right)=\sum_{i=1}^{n}\frac{\delta(x-x_i)}{|g'(x_i)|}$

$\nabla\cdot\nabla\wedge=0$ (div curl) $\quad \nabla\wedge\nabla=0$ (curl grad)

Laplacian: $\nabla^2 f=\frac{1}{\rho}(\rho f_\rho)_\rho+\frac{1}{\rho^2}f_{\varphi\varphi}+f_{zz}$ (Cyl) $\nabla^2 f=\frac{1}{r}(rf)_{rr}+\frac{1}{r^2\sin\theta}(\sin\theta f_\theta)_\theta+\frac{1}{r^2\sin^2\theta}f_{\varphi\varphi}$ (Sph)

---

§Electrostatics Point Charge $\mathbf{E}=\frac{q}{4\pi\epsilon_0}\frac{\mathbf{r}-\mathbf{r}_0}{|\mathbf{r}-\mathbf{r}_0|^3}$ $\quad$ Gauss' law: $\int_\Sigma \mathbf{E}\cdot d\mathbf{S}=\frac{Q}{\epsilon_0}$ $\quad \nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_0}$

Continuum $\implies Q=\int_R \rho dV$ $\quad \mathbf{E}=\frac{1}{4\pi\epsilon_0}\int_{\mathbf{r}'\in R}\frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}(\mathbf{r}-\mathbf{r}')dV'$

"Green's function": $-4\pi\delta(\mathbf{r}-\mathbf{r}_0)=\nabla^2\left(\frac{1}{|\mathbf{r}-\mathbf{r}_0|}\right)$ $\quad (-4\pi\delta(\mathbf{r})=\nabla^2(\frac{1}{r}))$

Potential: $\phi(\mathbf{r})=\frac{q}{4\pi\epsilon_0}\frac{1}{|\mathbf{r}-\mathbf{r}_0|}$ $\quad \phi(\mathbf{r})=\frac{1}{4\pi\epsilon_0}\int_{\mathbf{r}'\in R}\frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dV'$ $\quad (\phi(\mathbf{r})\xrightarrow{r\to\infty}0$ with $O(\frac{1}{r}))$

$-\nabla\phi=\mathbf{E}\implies\nabla^2\phi=-\frac{\rho}{\epsilon_0}$ and $\nabla\wedge\mathbf{E}=\mathbf{0}$ (No magnetism case)

$W=q(\phi(\mathbf{r}_1)-\phi(\mathbf{r}_0))$ $\quad$ $\mathbf{E}$ perp. to Σ s.t. $\phi=$ const. $\quad$ Conductor in stable equili. is equipot for ϕ

$\mathbf{E}^+\cdot\mathbf{n}-\mathbf{E}^-\cdot\mathbf{n}=\frac{\sigma}{\epsilon_0}$ $\quad$ Tangential component is cts. (ie: $\mathbf{E}^+\cdot\mathbf{t}-\mathbf{E}^-\cdot\mathbf{t}=0$)

Line (analog. surface) to field: $\mathbf{E}(\mathbf{r})=\frac{1}{4\pi\epsilon_0}\int_{\mathbf{r}'\in C}\frac{\lambda(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}(\mathbf{r}-\mathbf{r}')|\frac{d\mathbf{r}'}{ds'}|ds'$

$W=\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\sum_{j<i}\frac{q_iq_j}{|\mathbf{r}_i-\mathbf{r}_j|}=\frac{1}{2}\times\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\sum_{j\neq i}\frac{q_iq_j}{|\mathbf{r}_i-\mathbf{r}_j|}=\frac{1}{2}\sum_{i=1}^{N}q_i\phi_i$ $(\phi_i=\frac{1}{4\pi\epsilon_0}\sum_{j\neq i}\frac{q_j}{|\mathbf{r}_i-\mathbf{r}_j|})$

$\xrightarrow{continuum}\frac{1}{2}\int_R\rho\phi dV=\frac{\epsilon_0}{2}\left(\int_{\partial R}\phi\mathbf{E}\cdot d\mathbf{S}+\int_R\mathbf{E}\cdot\mathbf{E}dV\right)=\frac{\epsilon_0}{2}\int_R|\mathbf{E}|^2dV$ $(\phi\xrightarrow{r\to\infty}0)\implies\mathcal{E}\coloneqq\frac{\epsilon_0}{2}|\mathbf{E}|^2$

---

§BVP in Electrostatics ρ specified in R, conductor on $\partial R\implies\sigma=\epsilon_0\frac{\partial\phi}{\partial n}|_\Sigma$

Green's Identity (i) $\int_R=(u\nabla^2v+\nabla u\cdot\nabla v)dV=\int_\Sigma u\frac{\partial v}{\partial n}dS\implies$ Uniqueness (consider homogen)

(ii) $\int_R u\nabla^2v-v\nabla udV=\int_\Sigma u\frac{\partial v}{\partial n}-v\frac{\partial u}{\partial n}dS\implies$ Green's function sol.

Green's Function $G(\mathbf{r},\mathbf{r}')$ s.t. $\nabla'^2G(\mathbf{r},\mathbf{r}')=-4\pi\delta(\mathbf{r}-\mathbf{r}')$ $\quad G(\mathbf{r},\mathbf{r}')=\frac{1}{|\mathbf{r}-\mathbf{r}'|}+F(\mathbf{r},\mathbf{r}')$ $(\nabla'^2F=0)$

GI (ii) $\implies\phi(\mathbf{r})=\frac{1}{4\pi\epsilon_0}\int_R G(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')dV'+\frac{1}{4\pi}\int_{\Sigma=\partial R}\left(G(\mathbf{r},\mathbf{r}')\frac{\partial\phi(\mathbf{r}')}{\partial n'}-\phi\left(\mathbf{r}'\right)\frac{\partial G(\mathbf{r},\mathbf{r}')}{\partial n'}\right)dS'$

For Dirichlet, $G_D(\mathbf{r},\mathbf{r}')=0$ on $\mathbf{r}'\in\Sigma=\partial R'$, $\mathbf{r}\in R$

$\implies\phi(\mathbf{r})=\frac{1}{4\pi\epsilon_0}\int_R G_D(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')dV'-\frac{1}{4\pi}\int_\Sigma\phi(\mathbf{r}')\frac{\partial G_D(\mathbf{r},\mathbf{r}')}{\partial n'}dS'$ $\quad G_D(\mathbf{r}_1,\mathbf{r}_2)=G_D(\mathbf{r}_2,\mathbf{r}_1)$ (pf: GI(ii))

Constructing GF: Test charge solution by method of image, scale for GF (remove $\frac{q}{4\pi\epsilon_0}$ factor)

Sphere: Inv pt of $\mathbf{r}_0=r\mathbf{e}_r$ is $\mathbf{r}_0^*||\mathbf{e}_r$ s.t. $\mathbf{r}_0\cdot\mathbf{r}_0^*=a^2$, $q^*=-\frac{a}{r}q$ $\quad$ GF: $G_D\left(\mathbf{r},\mathbf{r}'\right)=\frac{1}{|\mathbf{r}-\mathbf{r}'|}-\frac{a}{r'}\frac{1}{|\mathbf{r}-(a/r')^2\mathbf{r}'|}$

---

§Orthonormal Functions $\int_a^b\overline{u_m(x)}u_n(x)dx=\delta_{mn},\forall m,n\in I$

For $f(x)=\sum_{n\in I}c_nu_n(x)$, $c_m=<u_m|f>$. Substitute c_m back into expansion,

$\implies f(x)=\int_a^b f(x')\left(\sum_{n\in I}\overline{u_n(x')}u_n(x)\right)dx'\implies\sum_{n\in I}\overline{u_n(x')}u_n(x)=\delta(x'-x)$

Fourier sine series For $f:[0,a]\to\mathbb{R}$ with $f(0)=f(a)=0$, $u_n(x)\coloneqq\sqrt{\frac{2}{a}}\sin\left(\frac{\pi n}{a}x\right)$

Complex exponential For $f:[-\frac{L}{2},\frac{L}{2}]\to\mathbb{R}$, $u_n(x)\coloneqq\frac{1}{\sqrt{L}}e^{i\left(\frac{2\pi n}{L}\right)x}$

FT $u_k(x):==\frac{1}{\sqrt{2\pi}}e^{ikx}$ with $f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}C(k)e^{ikx}dk$,

$\implies C(k)=\frac{1}{\sqrt{2\pi}}\infty_{-\infty}^{\infty}e^{-ikx}f(x)dx=\int_{-\infty}^{\infty}\overline{u_k(x)}f(x)dx$

$\int_{-\infty}^{\infty}\overline{u_k(x')}u_k(x)dk=\frac{1}{2\pi}e^{ik\left(x-x'\right)}dk=\delta(x-x')$

Cartesian $\nabla^2\phi=0$ $\phi=X(x)Y(y)Z(z)\implies\frac{1}{X}\frac{d^2X}{dx^2}+\frac{1}{Y}\frac{d^2Y}{dy^2}+\frac{1}{Z}\frac{d^2Z}{dz^2}=0$ for nontrivial z BC cond.

$\implies\phi(x,y,z)=e^{\pm i\alpha x}e^{\pm i\beta y}e^{\pm\sqrt{\alpha^2+\beta^2}z}$

---

§Magnetostatics $\mathbf{J}\coloneqq\rho\mathbf{v}$ (Electric Current Density) $\quad I\coloneqq\int_\Sigma\mathbf{J}\cdot d\mathbf{S}$ (Current)

By $\int_\Sigma\mathbf{J}\cdot d\mathbf{S}=-\frac{dQ}{dt}\implies\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{J}=0$ (Continuity)

Magnetostatic: $\rho_t=0\implies\nabla\cdot\mathbf{J}=0$ (Steady Current)

Lorentz Force: $\mathbf{F}=q\mathbf{E}(\mathbf{r})+q\mathbf{u}\wedge\mathbf{B}(\mathbf{r})$

Biot-Savart: $\mathbf{B}(\mathbf{r})=\frac{\mu_0q}{4\pi}\frac{\mathbf{v}\wedge(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$ $\quad \mathbf{B}(\mathbf{r})=\frac{\mu_0I}{4\pi}\int_C\frac{d\mathbf{r}'\wedge(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$ $\quad \mathbf{B}(\mathbf{r})=\frac{\mu_0}{4\pi}\int_{\mathbf{r}'\in R}\frac{\mathbf{J}(\mathbf{r}')\wedge(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}dV'$

No magnetic monopole: $\nabla\cdot\mathbf{B}=\mathbf{0}$

$\mathbf{A}(\mathbf{r})\coloneqq\frac{\mu_0}{4\pi}\int_R\frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dV'\xRightarrow{\nabla\wedge(\varphi\mathbf{A})=\varphi(\nabla\wedge\mathbf{A})+(\nabla\varphi)\wedge\mathbf{A}}\nabla\wedge\mathbf{A}=\mathbf{B}$ (Consistent with no magnetic monopole)

$\xRightarrow{\nabla\wedge(\nabla\wedge)\mathbf{A}=\nabla(\nabla\cdot\mathbf{A})-\nabla^2\mathbf{A}}\nabla\wedge\mathbf{B}=-\nabla^2\mathbf{A}=\mu_0\mathbf{J}$ (From def of $\mathbf{A}$) (Ampère's law I)

$\iff\int_{C=\partial\Sigma}\mathbf{B}\cdot d\mathbf{r}=\mu_0\int_\Sigma\mathbf{J}\cdot d\mathbf{S}=\mu_0I$ (Ampère's law II)

For steady current, $\nabla\cdot\mathbf{A}=0$

Magnetostatic vector potential $\mathbf{B} = \nabla \wedge \mathbf{A}$ Gauge Transform: $\mathbf{A} \to \hat{\mathbf{A}} := \mathbf{A} + \nabla\psi$

If $\nabla^2\psi = -\nabla\cdot\mathbf{A}$, then $\nabla\cdot\hat{\mathbf{A}} = 0$ (Lorenz Gauge Condition)

In Lorenz gauge, $\nabla^2\mathbf{A} = -\mu_0\mathbf{J}$

---

§Macroscopic Media

Force on dipole by $\mathbf{E}_{\text{ext}}$: $\mathbf{F} = q\mathbf{E}\left(\mathbf{r}+\frac{\mathbf{d}}{2}\right) - q\left(\mathbf{E}\left(\mathbf{r}-\frac{\mathbf{d}}{2}\right)\right) = q\left(\mathbf{E}\left(\mathbf{r}\right) + \left(\frac{\mathbf{d}}{2}\cdot\nabla\right)\mathbf{E}\left(\mathbf{r}\right) + O(d^2)\right) - q(\cdot) = q\left(\left(\mathbf{d}\cdot\nabla\right)\mathbf{E}\left(\mathbf{r}\right) + O(d^2)\right) \xrightarrow{\mathbf{p}:=q\mathbf{d}} \left(\mathbf{p}\cdot\nabla\right)\mathbf{E}$ $\quad V = \mathbf{p}\cdot\nabla\phi \quad \tau \to \mathbf{p}\wedge\mathbf{E}$ (All $\mathbf{E}$ is $\mathbf{E}_{\text{ext}}$)

Dipole: $\phi(\mathbf{r})_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\mathbf{r}}{r^3} = -\frac{1}{4\pi\epsilon_0}\mathbf{p}\cdot\nabla\left(\frac{1}{r}\right)$ $\quad \mathbf{E}_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\left(\frac{3\mathbf{p}\cdot\mathbf{r}}{r^5}\mathbf{r} - \frac{\mathbf{p}}{r^3}\right)$ (pf:Cartes. expan)

Many dipoles $\to \frac{1}{4\pi\epsilon_0}\int_{\mathbf{r}'\in R}\frac{\mathbf{P}(\mathbf{r}')\cdot\left(\mathbf{r}-\mathbf{r}'\right)}{|\mathbf{r}-\mathbf{r}'|^3}dV' = \frac{1}{4\pi\epsilon_0}\int_{\mathbf{r}'\in R}\frac{\left(-\nabla'\cdot\mathbf{P}\left(\mathbf{r}'\right)\right)}{|\mathbf{r}-\mathbf{r}'|}dV'$ $P(\mathbf{r}')$: Elect polari density

Define $\rho_{\text{bound}} := -\nabla\cdot\mathbf{P}(\mathbf{r})$ (cf: $\nabla\cdot\frac{\mathbf{P}}{\epsilon_0} = -\frac{\rho_{\text{bound}}}{\epsilon_0}$)

Gauss' law: $\nabla\cdot\mathbf{E} = \frac{1}{\epsilon_0}\left(\rho_{\text{free}}+\rho_{\text{bound}}\right) = \frac{1}{\epsilon_0}\rho_{\text{free}} - \frac{1}{\epsilon_0}\nabla\cdot\mathbf{P}$ (Note $\mathbf{P}||\mathbf{E}$ in equili)

Write $\frac{\mathbf{P}}{\epsilon_0} :== \chi_e\mathbf{E} = \left(\frac{\epsilon}{\epsilon_0}-1\right)\mathbf{E} \quad \chi_e := \left(\frac{\epsilon}{\epsilon_0}-1\right)$

$\implies \nabla\cdot(\epsilon\mathbf{E}) \quad \nabla\wedge\mathbf{E} = 0 \quad \epsilon^+\mathbf{E}^+\cdot\mathbf{n} - \epsilon^-\mathbf{E}^-\cdot\mathbf{n} = \sigma$

---

§Maxwell (i) $\nabla\wedge\mathbf{B} = \mu\left(\mathbf{J}+\epsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$ (Ampère)

(ii) $\nabla\wedge\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ (Faraday) $\implies \int_{\partial\Sigma}\mathbf{E}\cdot d\mathbf{r} = -\frac{d}{dt}\int_\Sigma\mathbf{B}\cdot d\mathbf{S}$

(iii) $\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0}$ (Gauss) $\quad$ (iv) $\nabla\cdot\mathbf{B} = 0$ $\quad$ (v) $\mathbf{F} = q\left(\mathbf{E}+\mathbf{u}\wedge\mathbf{B}\right)$

Consistency: $\nabla\cdot$(i) $\implies 0 = \rho_t + \nabla\cdot\mathbf{J}$ (Continuity)

From $\mathbf{B} = \nabla\wedge\mathbf{A}$, (ii) $\implies \mathbf{0} = \nabla\wedge\left(\mathbf{E}+\frac{\partial\mathbf{A}}{\partial t}\right) \implies \exists\phi\left(\mathbf{r},t\right)\,\text{s.t.}\,\mathbf{E}+\frac{\partial\mathbf{A}}{\partial t} = -\nabla\phi$

Gauge Transform: $\mathbf{A}\to\mathbf{A}+\nabla\psi$, $\phi\to\phi-\frac{\partial\psi}{\partial t}$ $\quad$ Gauge Condition: $\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}+\nabla\cdot\mathbf{A} = 0$

In gauge: $\Box\mathbf{A} = -\mu\mathbf{J} \quad \Box\phi = -\frac{\rho}{\epsilon_0}$ where $\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\nabla^2, \quad c^2 = \frac{1}{\epsilon_0\mu_0}$

$\mathcal{E} := \frac{1}{2}\left(\epsilon_0|\mathbf{E}|^2+\frac{1}{\mu_0}|\mathbf{B}|^2\right) = \frac{\epsilon_0}{2}\left(|\mathbf{E}|^2+c^2|\mathbf{B}|^2\right)$

Poynting Theorem: $\frac{\partial\mathcal{E}}{\partial t}+\nabla\cdot\mathcal{P} = -\mathbf{E}\cdot\mathbf{J}$ where $\mathcal{P} = \frac{1}{\mu_0}\mathbf{E}\wedge\mathbf{B}$ $\quad$ Pf) $\frac{\partial\mathcal{E}}{\partial t} \overset{Elim\partial_t term}{=} \cdots$

$\mathcal{P}$ is energy flow density; $\frac{d}{dt}\int_R\mathcal{E}dV = -\int_\Sigma\mathcal{P}\cdot\mathbf{S} - \int_R\mathbf{E}\cdot\mathbf{J}dV$ (2nd term: Power by field on source)

Time Dep GF: $\Box\psi = -4\pi f(\mathbf{r},t) \quad \Box G(\mathbf{r},t;\mathbf{r}',t') = -4\pi\delta(t-t')\delta(\mathbf{r}-\mathbf{r}')$

$\implies \psi(\mathbf{r},t) = \int_{\mathbf{r}'\in\mathbb{R}^3}\int_{t'\in\mathbb{R}}G(\cdots)f(\mathbf{r}',t')dV'dt'$

Derivation: Seek $G\left(\mathbf{r},t;\mathbf{r}',t'\right) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty}\hat{G}\left(\mathbf{r},\mathbf{r}';\omega\right)e^{-i\omega\left(t-t'\right)}d\omega$, sub to equ, note:

$\frac{1}{2\pi}\int_{\omega=-\infty}^{\infty}\left(-4\pi\delta\left(\mathbf{r}-\mathbf{r}'\right)\right)e^{-i\omega\left(t-t'\right)} = -4\pi\delta\left(\mathbf{r}-\mathbf{r}'\right)\delta\left(t-t'\right)$

$\implies \left(\nabla^2+\frac{\omega^2}{c^2}\right)\hat{G}\left(\mathbf{r},\mathbf{r}',\omega\right) = -4\pi\delta\left(\mathbf{r}-\mathbf{r}'\right) \xRightarrow{\nabla^2\left(\frac{e^{ikr}}{r}\right)=-k^2\left(\frac{e^{ikr}}{r}\right)-4\pi\delta(\mathbf{r})} \hat{G}_\pm\left(\mathbf{r},\mathbf{r}',\omega\right) = \frac{e^{\pm i\omega|\mathbf{r}-\mathbf{r}'|/c}}{|\mathbf{r}-\mathbf{r}'|}$

$\xRightarrow{\delta\left(t-t'\right)=\int\frac{1}{2\pi}e^{-i\omega(t-t')}d\omega} G_\pm(\mathbf{r},t;\mathbf{r}',t') = \frac{\delta\left(t-t'\mp|\mathbf{r}-\mathbf{r}'|/c\right)}{|\mathbf{r}-\mathbf{r}'|}$ (G_+ for Retarded GF)

$\psi(\mathbf{r},t) = \int_{\mathbf{r}'\in\mathbb{R}^3}\frac{1}{|\mathbf{r}-\mathbf{r}'|}f(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c})dV'$

$\implies \phi = \frac{1}{4\pi\epsilon_0}\int_{\mathbf{r}'\in\mathbb{R}^3}\frac{\rho\left(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c}\right)}{|\mathbf{r}-\mathbf{r}'|}dV' \quad \mathbf{A} = \frac{1}{4\pi\epsilon_0}\int_{\mathbf{r}'\in\mathbb{R}^3}\frac{\mathbf{J}\left(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c}\right)}{|\mathbf{r}-\mathbf{r}'|}dV'$

Maxwell in Macroscopic: $\nabla\cdot(\epsilon\mathbf{E}) = \rho_{\text{free}} \quad \nabla\wedge\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \quad \nabla\cdot\mathbf{B} = 0 \quad \nabla\wedge\left(\frac{1}{\mu}\mathbf{B}\right) = \mathbf{J}_{\text{free}}+\frac{\partial(\epsilon\mathbf{E})}{\partial t}$

---

§Electromagnetic Waves

Source-free Maxwell (i) $\nabla\cdot\mathbf{E} = 0$ (ii) $\nabla\cdot\mathbf{E} = 0$ (iii) $\nabla\wedge\mathbf{E}+\mathbf{B}_t = \mathbf{0}$ (iv) $\nabla\wedge\mathbf{E}-\frac{1}{c^2}\mathbf{E}_t = \mathbf{0}$

$\nabla\wedge$ (iii), (iv) $\implies \Box\mathbf{E} = \mathbf{0} = \Box\mathbf{B}$ resp.

$\Box u = \mathbf{0} \implies u = f(\mathbf{e}\cdot\mathbf{r}-ct) \xrightarrow{\text{Monochromatic Sol}} u(\mathbf{r},t) = \alpha e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$

Seek $\mathbf{E}_\mathbb{C} = \mathbf{E} = \mathbf{E}_0e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \quad \mathbf{B}_\mathbb{C} = \mathbf{B} = \mathbf{B}_0e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ where $\mathbf{k} = k\mathbf{e}$, $k = \frac{\omega}{c}$

(i),(ii) $\implies \mathbf{k}\cdot\mathbf{E}_0 = 0 = \mathbf{k}\cdot\mathbf{B}_0$ (Note $\nabla\cdot\mathbf{E}_\mathbb{C} = \mathbf{E}_0\cdot\nabla e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = i\mathbf{k}\cdot\mathbf{E}_0e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$)

(iii) $\implies \mathbf{0} = \left(i\mathbf{k}\wedge\mathbf{E}_0-i\omega\mathbf{B}_0\right)e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \implies \mathbf{B}_0 = \frac{1}{\omega}\mathbf{k}\wedge\mathbf{E}_0 = \frac{1}{c}\mathbf{e}\wedge\mathbf{E}_0 \implies c|\mathbf{B}| = |\mathbf{E}|$

$\implies \mathcal{E} = \epsilon_0|\mathbf{E}_0|^2, \mathcal{P} = \frac{1}{\mu_0}\mathbf{E}\wedge\mathbf{B} = \frac{1}{\mu_0}\mathbf{E}\wedge\left(\frac{1}{c}\mathbf{e}\wedge\mathbf{E}\right) = \frac{1}{\mu_0 c}|\mathbf{E}|^2\mathbf{e} = c\mathcal{E}\mathbf{e}$

Sol: $\mathbf{E} = \mathbf{E}_0e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}, \mathbf{B} = \frac{1}{c}\mathbf{e}\wedge\mathbf{E} = \frac{1}{\omega}\mathbf{k}\wedge\mathbf{E}$

Remark: $[\mathbf{E}\cdot\mathbf{t}]_-^+ = \left[\frac{1}{\mu}\mathbf{B}\cdot\mathbf{t}\right]_-^+ = 0 \quad [\epsilon\mathbf{E}\cdot\mathbf{n}]_-^+ = [\mathbf{B}\cdot\mathbf{n}]_-^+ = 0$ (on boundary)

$n = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}$, $v^2 = \frac{1}{\epsilon\mu}$ $\quad$ Snell: $\sin\theta' = \frac{n}{n'}\sin\theta$

$\int_{z=-\infty}^{\infty}\frac{1}{\sqrt{R^2+z^2}^3}dz = \frac{2}{R^2}$