Amazing Presentation About Fibonacci Sequence

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Fibonacci Stuff

Definition

Fibonacci sequence $\{F_n\}$ is defined as:

$$F_1 = 1$$
$$F_2 = 1$$
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Goal: Finding the general formula for the Fibonacci sequence!

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$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = VDV^T$$

where $D = \text{diag}\,(\phi, \psi)$ is the diagonal matrix with entries being the two roots of $\lambda^2 - \lambda - 1 = 0$.

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where the last equality comes from V being orthogonal and D being diagonal. It is easy to now acquire the general form of F_n. (It is left as an exercise.)