Amazing Presentation About Fibonacci Sequence

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Fibonacci Stuff

Definition

Fibonacci sequence $\{F_n\}$ is defined as:

$$F_1 = 1$$
$$F_2 = 1$$
$$\forall n \in \mathbb{Z}^{\geq 0} : F_{n+2} = F_n + F_{n+1}$$

Goal: Finding the general formula for the Fibonacci sequence!

Derivation

Here is a slick way to get a general formula for Fibonacci sequence. Rewrite the recurrence equation as:

$$\begin{pmatrix} F_{n+2} \\ F_{n+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix}$$

(I know, the second equation is redundant, but bare with me...) Now, by eigendecomposition of the matrix:

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = VDV^T$$

where $D = \text{diag}\,(\phi, \psi)$ is the diagonal matrix with entries being the two roots of $\lambda^2 - \lambda - 1 = 0$.

So the recurrence equation can be written as:

$$\begin{pmatrix} F_{n+2} \\ F_{n+1} \end{pmatrix} = VDV^T \begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix}$$

By induction, one deduces:

$$\begin{aligned} \begin{pmatrix} F_{n+1} \\ F_n \end{pmatrix} &= \left(VDV^T\right)^n \begin{pmatrix} F_1 \\ F_0 \end{pmatrix} \\ &= VD^nV^T \begin{pmatrix} F_1 \\ F_0 \end{pmatrix} \end{aligned}$$

where the last equality comes from V being orthogonal and D being diagonal. It is easy to now acquire the general form of F_n. (It is left as an exercise.)