

Aggregate properties of Platonic solids and beyond...

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Geometric and packing properties of Platonic solids are well studied, however there is **no general framework** relating the **particle geometry and packing properties** to **mechanical response** of granular aggregates.

Particle Geometry
Aspect ratio, exclusion volume, # faces, surface area...

Packing properties

Pair correlation, packing fraction, ordering...

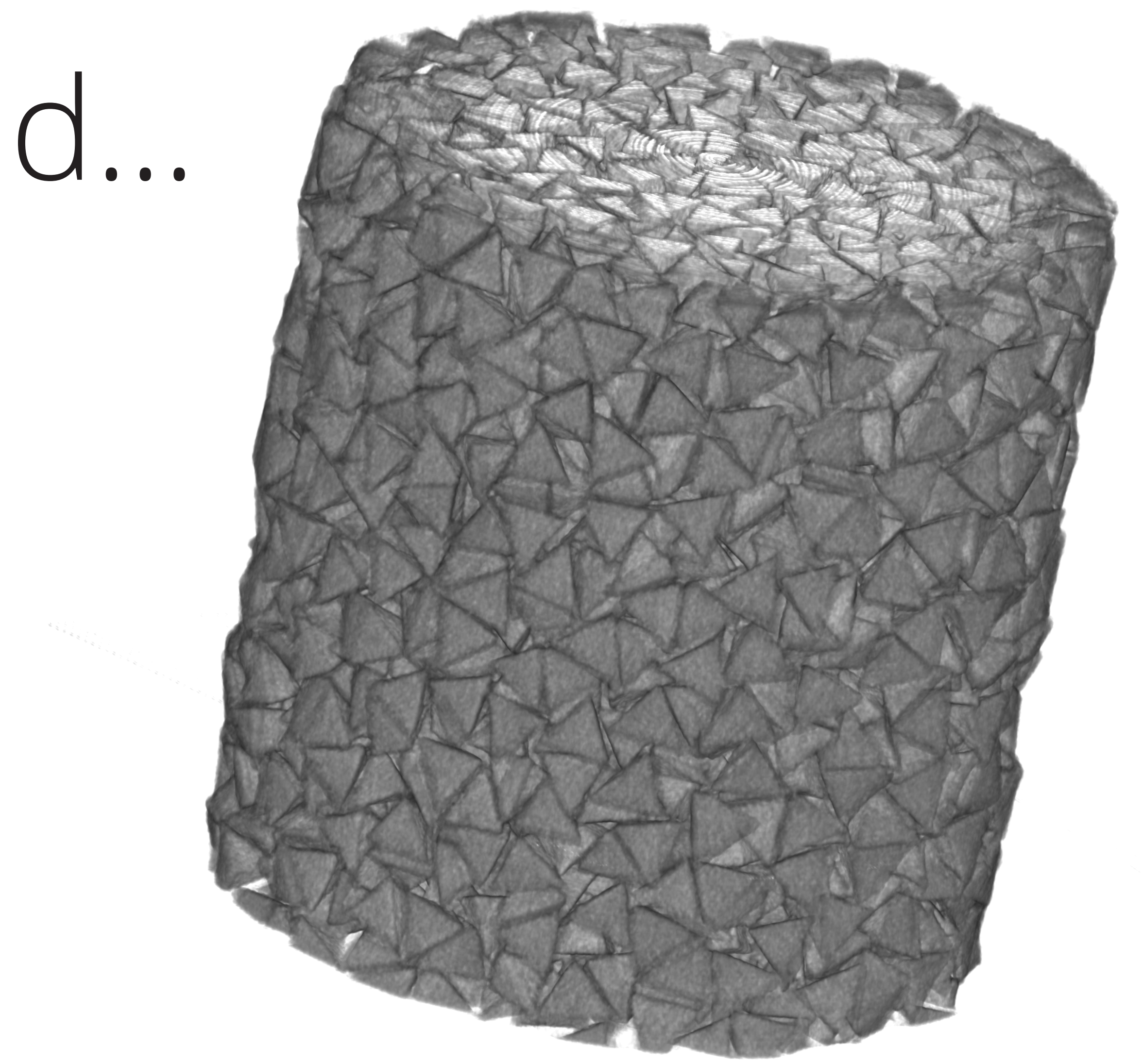
Mechanical Response

Stiffness, yield stress, toughness

What are the mechanical properties of packings of Platonic solids?

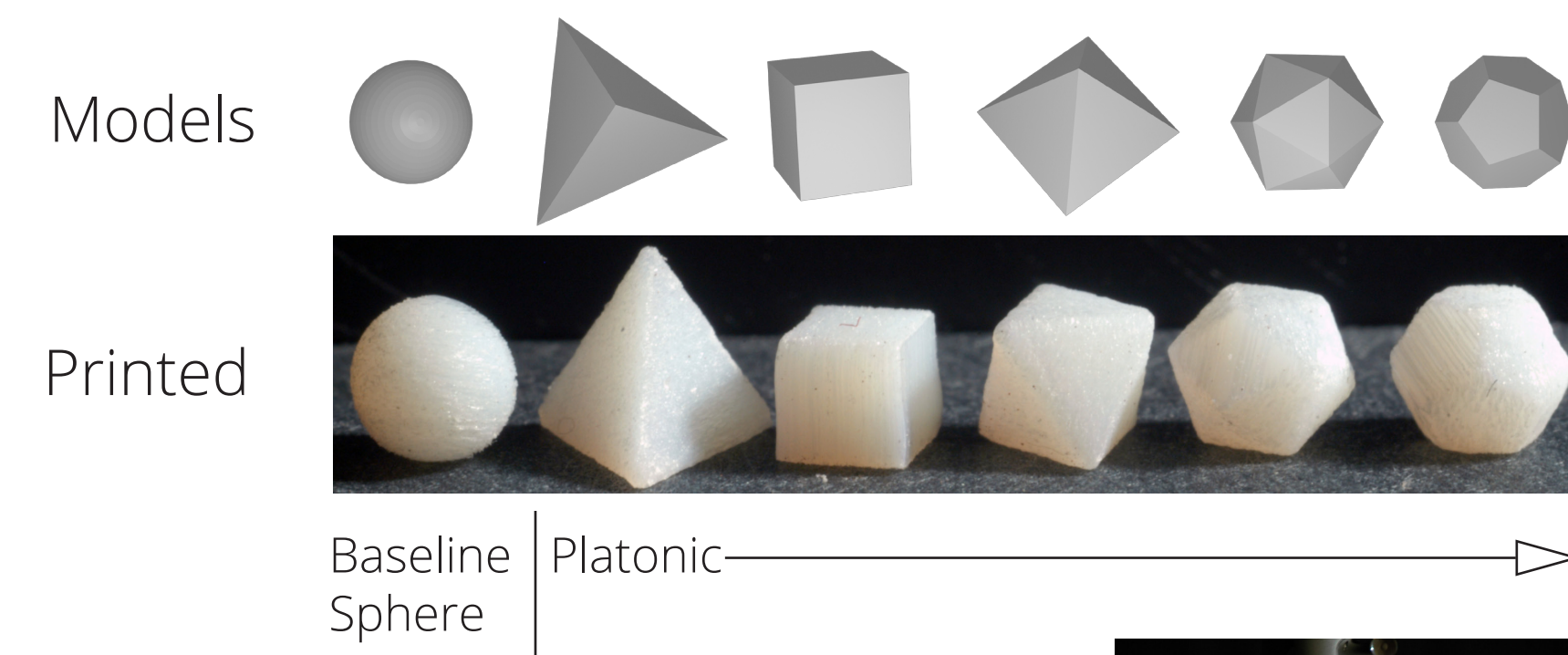
Can we directly observe the microstructure of these packings?

What is the next step in aggregate design?



The mechanical response and failure of aggregates

Fabrication and Setup



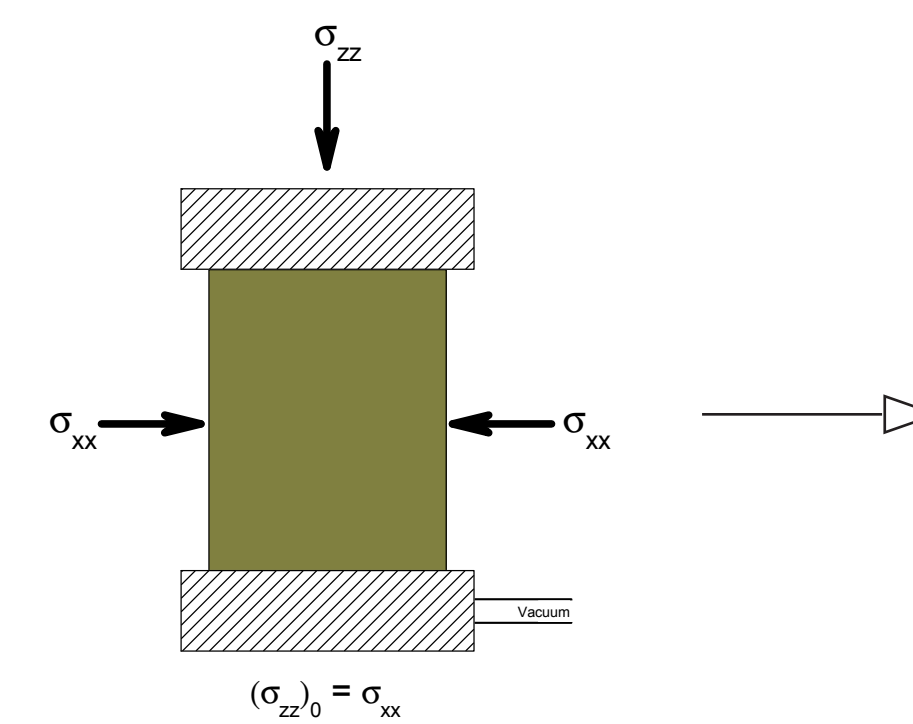
Each particle has a volume of 22.5 mm³.

Each packing contains approximately 5500 particles.

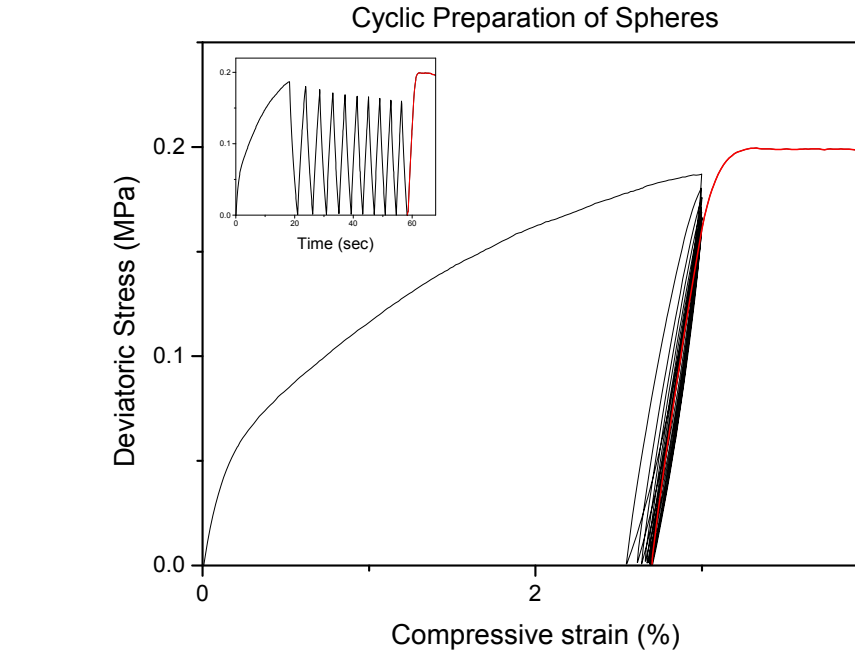


Methodology

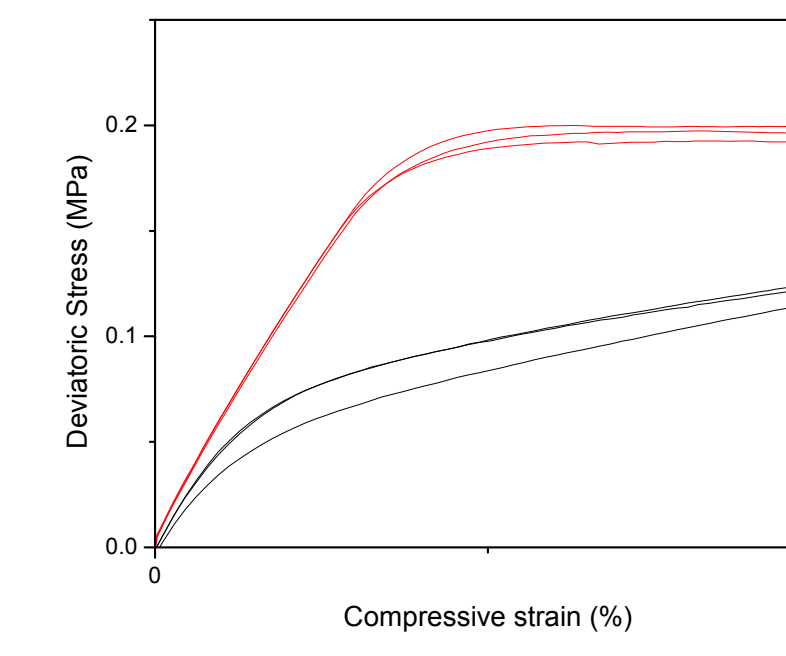
Particle packings are prepared in a cylindrical membrane, with an aspect ratio of 2:1 (h:d). Cyclic compression was used to obtain consistent initial conditions. Typically, 5 experiments are run per particle per set of boundary conditions to obtain ensemble statistics.



- Pull vacuum on the membrane, between 30kPa and 85kPa.
- Match axial stress to radial stress to attain an isotropic initial state.



- Cycle between 3% strain and initial stress condition until the asymptotic state is reached.
- Compress to desired end strain.

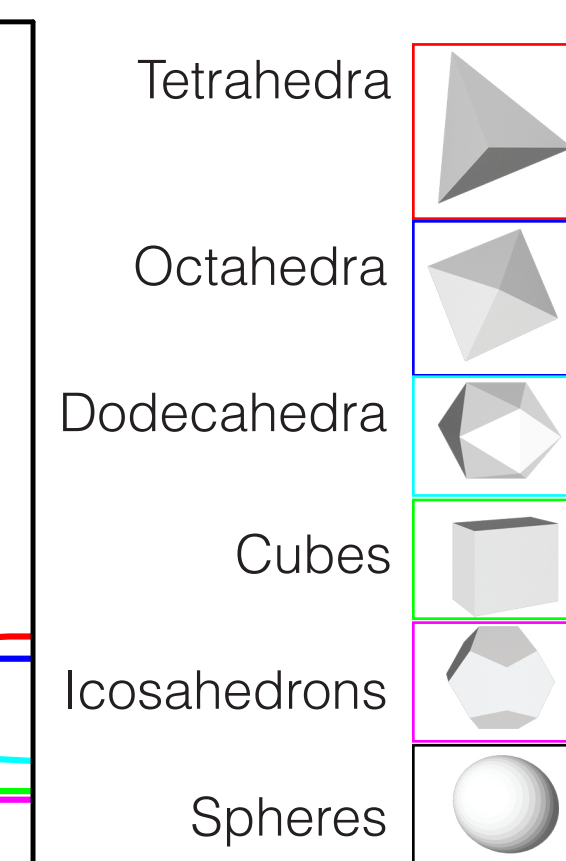
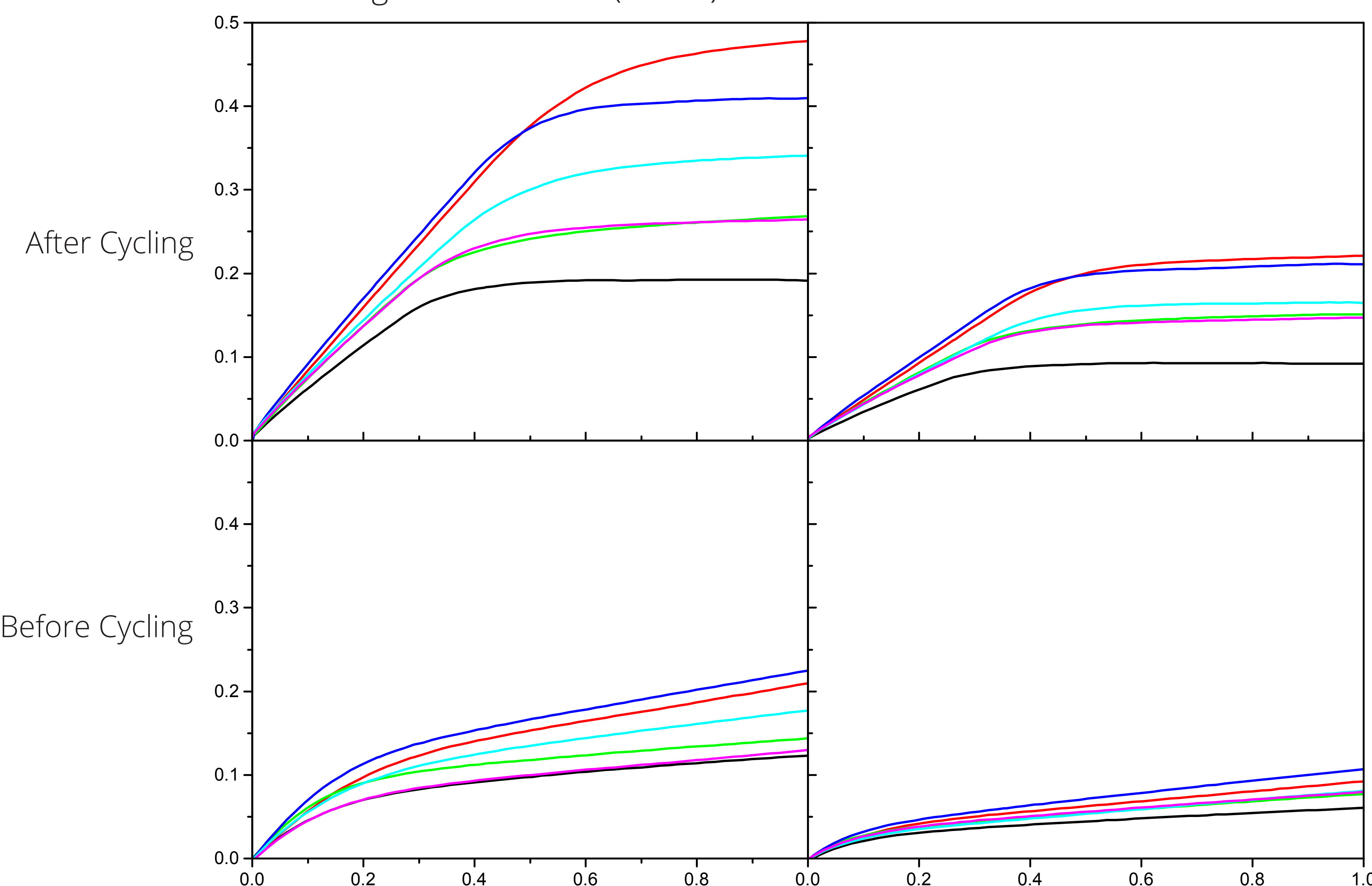


- Typical stress/strain curves before cycling (black) and after cycling (red).

Platonic aggregate response

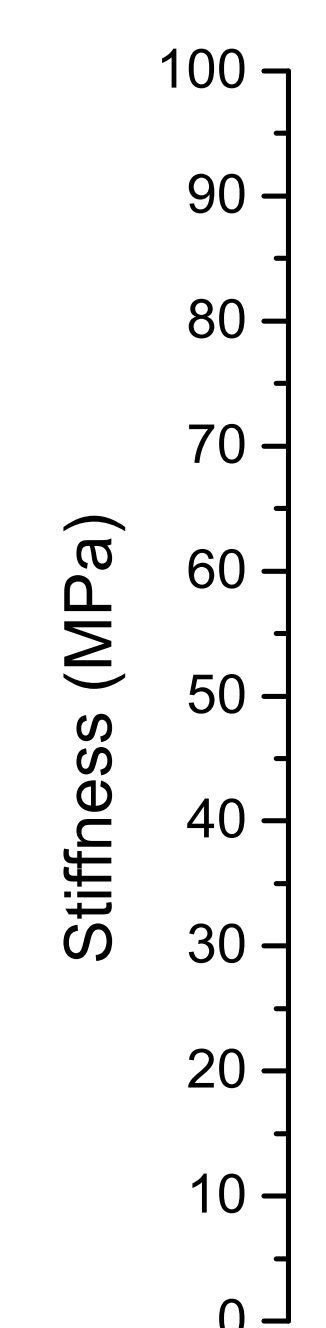
High Confinement (80kPa)

Low Confinement (35kPa)



The confining stress sets the scale of the packing's response. However the responses of the different packings maintain the same ordering at both stress scales.

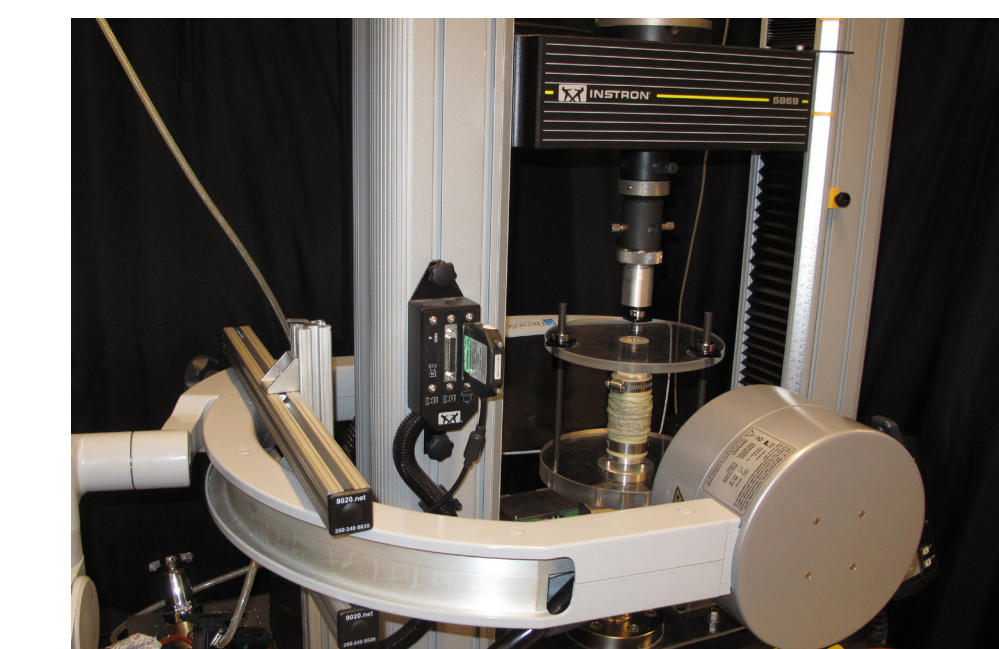
The yielding behavior of the packing is strikingly different before and after cycling, indicating a **strong dependence** of the packing's response on its **initial conditions**.



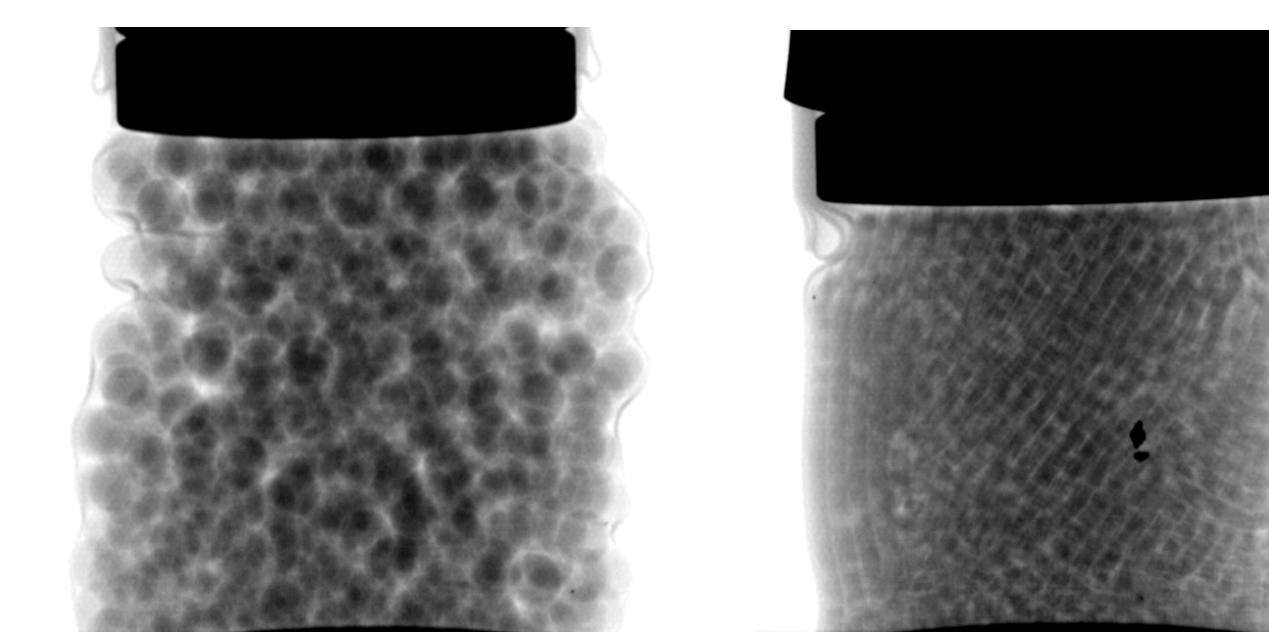
The **stiffness** of the particles, which is the derivative of the stress-strain curve near zero, varies little among the Platonic solids.

Directly probing microstructure with x-ray imaging

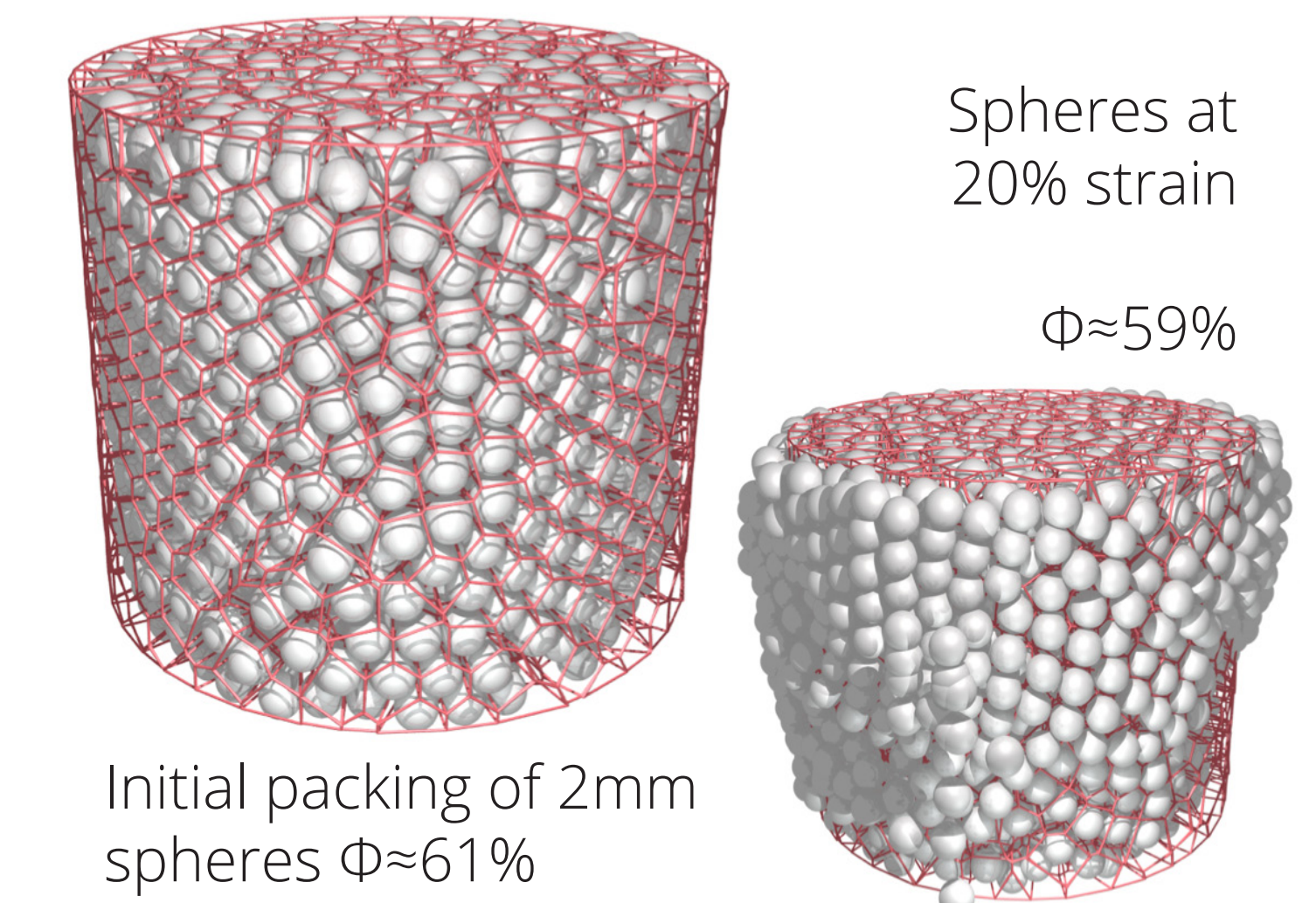
Computed Tomography (CT)



An orthopedic c-arm and a custom-designed rotation stage allows us to perform *in-situ* tomography to non-destructively probe the packing microstructure.



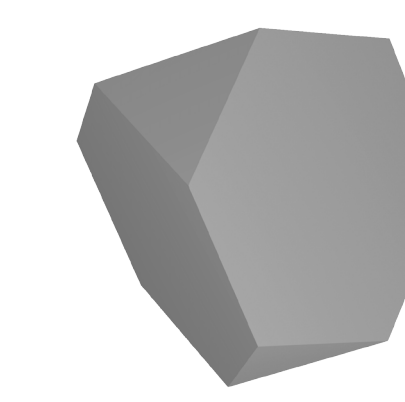
Spheres
Cubes
Projection radiographs reveal density fluctuations continuously throughout the compression.



Once reconstructed, we can analyze local relationships between particles as well as global packing properties.

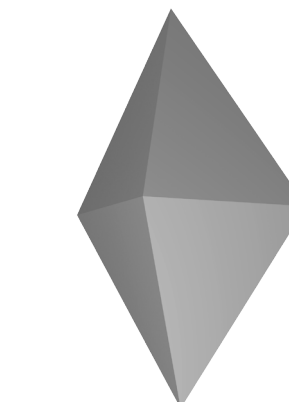
Beyond platonic solids

Archimedian Solids



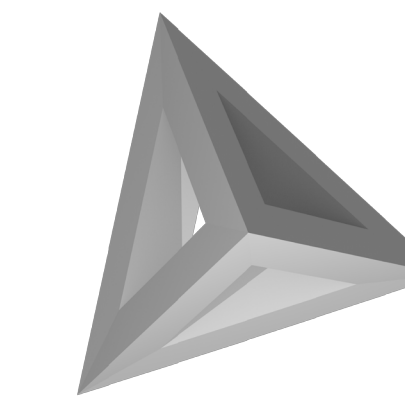
Truncated Tetrahedron

Deltahedra



Triangular Bipyramid

Open Frames



Tetrahedral Frame

- Many factors influence the response of granular materials, especially the initial and boundary conditions.
- To introduce new features into the stress strain curve, we must venture beyond simple solid shapes.
- We can probe the packing microstructure using x-rays to help develop a framework linking shape, packing, and response.

Non-convex Solids

We have seen how to drastically alter the mechanical response of granular packings using non-convex particles can interlock.

Chains



Dolos



Jacks

