Linear Control Design II
ASSIGNMENT

Based on your state space model for the electro-mechanical system (A, B, C e D), build a simulink model with the parameters given during the class and answer the questions:

1) How the structure behaves when s0 (air gap) is varied from 5 mm to 1 mm. Simulate three different values of s0=5 mm , s0=3 mm ; s0= 1 mm.

2) What is the maximum displacement experienced by the masses when a perturbation equivalent to an initial condition of velocity of 5 mm/s acts on mass 3?

3) Sending a step function u(t) to the electromagnets how the masses will vibrate, i.e. amplitudes and frequencies.

Assignment – Mechanical & Mathematical Models

Physical System

Assumptions (simplifications)

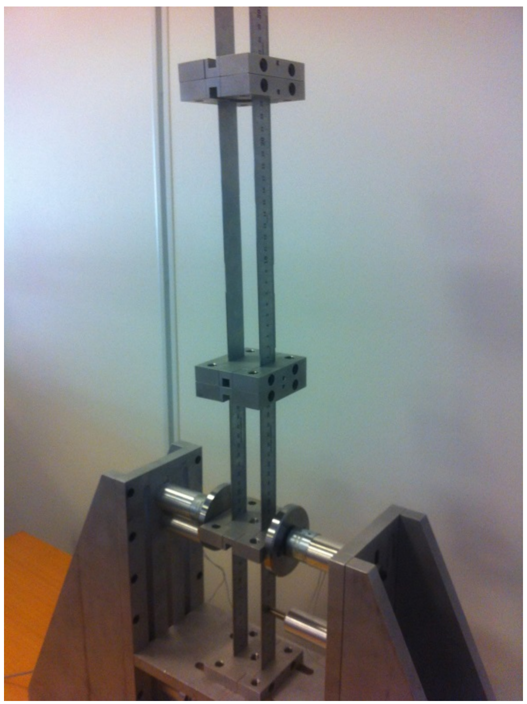
Electro-Mechanical Model

Newton, Euler, D'Alembert, Lagrange, Hamilton, Jourdain …

Coulomb, Ampère, Ohm, Kirchhoff, Faraday, Lorentz, Maxwell …

(principles & axioms)

Mathematical Model

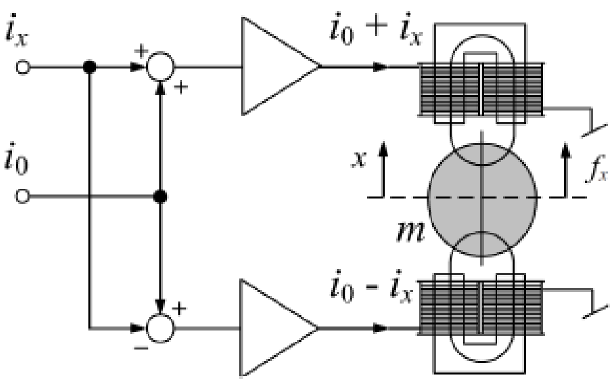


State Space Matrix **A**

$$\mathbf{A} = \begin{bmatrix} -R/L & 0 & -Ki/L & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ Ki/m1 & -(k1+k2+Ks)/m1 & 0 & k2/m1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & k2/m2 & 0 & -(k2+k3)/m2 & 0 & k3/m2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & k3/m3 & 0 & -k3/m3 & 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \end{aligned}}$$

(notes from the book)
"Magnetic Bearings – Theory, Design, and Application to Rotating Machinery" by Gerhard Schwitzer and Eric H. Maslen



Electromagnets are normally coupled in a counteracting pair such that positive and negative forces can be obtained. By applying a bias current i_0, which typically is fixed, the forces acting on the steel object in one direction can be controlled by a perturbation current i_x, as shown in Fig. 1.2. The resulting force is then given as

$$f = \frac{1}{4}\mu_0 n^2 A_a \left(\frac{(i_0 + i_x)^2}{(s_0 - x)^2} - \frac{(i_0 - i_x)^2}{(s_0 + x)^2} \right) \tag{1.13}$$

The function of force can be linearized as follows

$$f(x,y) = f(a,b) + \left.\frac{\partial f(x,y)}{\partial x}\right|_{a,b} (x-a) + \left.\frac{\partial f(x,y)}{\partial y}\right|_{a,b} (y-b) \tag{1.14}$$

where f(x,y) is the general function depending on x and y variables, which is the control current i_x and the displacement of the air gab x. f(a,b) is the steady state force evaluated at the operating point. Assuming the displacement x and the steady state current i_x is zero, this steady state force yields zero. Thus the linearized force can be expressed as

$$F = K_i i_x + K_s x \tag{1.15}$$

The coefficient K_i N/A which is used to describe the force as function of the control current i_x is given as

$$K_i = \frac{\mu_0 N^2 A_a i_0}{x_0^2} \tag{1.16}$$

The coefficient K_s N/m which is used to describe the force as function of the displacement x is given as

$$K_s = \frac{\mu_0 N^2 A_a {i_0}^2}{x_0^3} \tag{1.17}$$