Problem Set 4

Assigned: Oct. 20
Due: Nov. 3

You have a box with 6 coins.
3 of the coins are weighted so that they come up heads with probability 0.1. (Category 1)
1 is weighted so that it comes up heads with probability 0.6. (Category 2)
2 are weighted so that they come up heads with probability 0.7. (Category 3)

A. What is the probability of heads, if you pick a coin at random and flip it?

B. Suppose that you pick three coins at random from the box and you flip each of them 10 times. What is the expected total number of heads? Justify your answer. (Hint: Use random variables. This is *easy*, once you have done part (A). If you start enumerating the possible combinations of categories, you are on the wrong track.)

C. Suppose that you pick a coin at random and flip it and it comes up heads. What are the probabilities of each of the categories? What is the probability that it will come up heads again if you flip it again?

D. Suppose that you pick a coin at random, flip it twice, and it comes up heads both times. What are the probabilities of each of the categories? What is the probability that it will come up heads again if you flip it again?

E. Someone makes you the following offer: You may pick a coin at random out of the box. You will be allowed to place a $10 bet on the outcome of a flip. (That is, they will pay you $10 if you win and you will pay them $10 if you lose.) How should you bet? What is the expected payoff of the game?

F. Now you get a better offer. As in (E) you may pick a coin at random out of the box and you will be allowed to place a $10 bet on the outcome of a flip. However, before placing the bet, you are allowed to flip it once to test it. What is the proper strategy for placing the bet after you have done the test flip? What, at the start, is the expected payoff from the game?

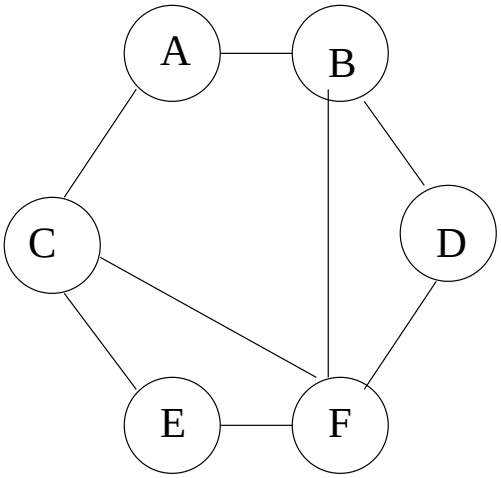

Figure 1: Network

Problem 2

Figure 1 shows a network with six nodes and eight edges. Suppose that each node fails with probability 1/10 and that failures of nodes are all independent events.

A. Given that A and E are active, what is the probability that they can communicate?

B. Given that A and E are active but cannot communicate, what is the probability that F has failed?

Problem 3

Let us continuing the example of the publisher and the reviewer discussed in the notes. Suppose that the publisher also has the option of consulting with two reviewers. Assume that the two reviewers follow the same probabilistic model, and that their reviews are conditionally independent given the actual success or failure.

A. Consider the following possible strategies:

1. Consult with one reviewer,
2. Consult with two reviewers. If both approve the manuscript, then publish, otherwise reject.
3. Consult with two reviewers. If either approves the manuscript, then publish, otherwise reject.

Suppose that a reviewer's fee is \$500. What are the expected values of these strategies? Which is the optimal strategy?