

Name: _____

Assignment 4: Fourier Series Expansion

Due: Tuesday, May 12

1. _____/20

2. _____/20

3. _____/20

Total: _____/60

1. (a) Show the basis functions $\{exp(jn\omega_0 t)\}$ of Fourier series expansion are mutually orthogonal with the definition of the inner product

$$\langle f(t), g(t) \rangle = \frac{1}{T} \int_0^T f(t) g^*(t) dt$$

- (b) Extend the analysis for the basis functions $\{exp(jn\theta)\}$ of *DTFT*.
2. Use your 7-digit perm number to produce a 7-point sequence $\{F_n\}$, where $n=0,1,\dots,6$.
 - (a) Use $\{F_n\}$ as the Fourier series coefficients to formulate and plot the periodic function $f(t)$ for the interval $(0, T)$.
 - (b) Apply *DTFT* onto the sequence $\{F_n\}$. Formulate and plot the *DTFT* spectrum for the interval $(0, 2\pi)$.
 - (c) Summarize your observations based on the results from Parts (a) and (b), and explain why so.
3. (a) Consider a real periodic signal, with period T . The signal has 6 harmonics, for $n=0,1,\dots,5$.

$$f(t) = \sum_{n=0}^5 a_n \cos(n\omega_0 t + \theta_n)$$

Then we take 16 uniform samples within one period to form a short 16-point sequence $\{f(m)\}$. Subsequently, we take a 16-point *DFT* of the sequence

$$F(k) = DFT \{f(m)\}$$

Identify and list the 16-point *DFT* spectrum $F(k)$ in terms of a_n and θ_n .

- (b) Repeat the exercise with the periodic function $g(t)$

$$g(t) = \sum_{n=1}^5 b_n \sin(n\omega_0 t + \phi_n)$$