ECE 148 **Signal Processing Applications** **Spring 2020**

Name: ______________________________

Assignment 1: Signal Sampling and Aliasing

Due: Tuesday, April 21

1. ______________ /20

2. ______________ /20

3. ______________ /10

Total: ______________ /50

1. A simple coherent signal *f(t)* is in the form

$$f(t) = exp(j\omega_x t)$$

The Fourier spectrum of this signal consists of one single peak at $\omega = \omega_x$. This coherent waveform is first sampled, with sample spacing Δt. Subsequently, it is reconstructed through a lowpass filter with cutoff frequencies at $\pm\, \omega_o/2$, where

$$\omega_o = 2\pi/\Delta t$$

The reconstructed signal is in the form of

$$g(t) = exp(j\omega_y t)$$

And its Fourier spectrum shows a single peak at $\omega = \omega_y$. If the sampling and reconstruction are conducted adequately, the frequency remains unchanged, $\omega_y = \omega_x$. If aliasing occurs, the resultant frequency will be different than the frequency of the input waveforms.

The conversion the frequency from ω_x to ω_y is a mapping process. Formulate the mapping correspondence between ω_x and ω_y when Δt is a constant. And plot the correspondence curve.

2. Extend the function to a lowpass periodic function ,

$$g(t) \;=\; \sum_{n=-N}^{N} G_n \; exp(jn\omega_x t)$$

The frequency spectrum spans from $-N\omega_x$ *to* $+N\omega_x$ for a total bandwidth of

$$B \;=\; 2N\omega_x \;<\; \omega_0$$

Suppose we modulate each of the $2N+1$ components with an extra single-frequency term $exp(jk_n\omega_0 t)$ to form another function

$$g'(t) \;=\; \sum_{n=-N}^{N} G_n \; exp(jn\omega_x t)\; exp(jk_n\omega_0 t)$$

where k_n is an arbitrary integer.

Determine the result, if you sample $g'(t)$ with sample spacing Δt and then lowpass the sampled function with a filter with cutoff frequencies at $\pm\, \omega_0/2$.

3. Summarize your work and itemize your observations.