The road to success is long but never give up

~~Cqrra~~ FUNCTION
PART
6
FINAL

Content to be convered

* Rational Functions
  * Vertical ~~asm~~ asymptote
  * Horizontal asymptote
  * oblic asymptotes
* piece function
* Graphing polynomials using rational Root theorem
* Redefining Modulus Functions

[All the best]

GRAPHING Rational FUNCTIONS

A rational Function is Basically a Function that has a denominator exists in franction Form.

$$\frac{F(x)}{K(x)}, K(x) \neq 0$$

Never should you try to Break this Rule. The zero should Never be Denominator.

ASYMPTOTES

* An Asymptotes is a line that is never ~~cross~~ crossed By a ~~line~~. ~~a~~ (graph). But in more advanced Calculus it do CROSS but thats not your level.

* There are 3 types of Asymptotes depending on $F(x)$ and $g(x)$

1. Vertical asymptotes
2. Horizontal asymptotes
3. Oblique or Slant asymptotes

* Asymptotes attracts a graph towards ~~themselves~~ themselves but they don't allow a graph to Cross them

They are more Like girls who attract Data boy but they never allow the data boy to Cross them pa Room. (Lalei)

Attract but never allow to cross.

MORE ABOUT Asymptotes

① Vertical Asymptotes (V.A)

* We have vertical asymptote when the highest power of the denominator is larger than the highest power of the numerator

FOR Example: $\frac{2}{x+3}$ or $\frac{3}{x^2-1}$ or $\frac{x-1}{(x-1)(x+2)}$

* We get the vertical asymptote By equating the denominator to zero and solve for x.

Vertical Line ↕
Horizontal line ⟷
in case one don't know

e.g

$\frac{2}{x+3}$

$x+3 \neq 0$

$x \neq -3$ //

$\frac{3}{x^2-1}$

$x^2-1 \neq 0$

$x \neq \pm 1$

$x \neq 1$ or $x \neq -1$

$\frac{x-1}{(x-1)(x+2}$

$(x-1)(x+2) \neq 0$

$x \neq 1$ or $x \neq 2$

But it is possibole for a graph not have a vertical asymptote and that is when you can't get a real number after equating the denominator to zero

e.g $\frac{4}{x^2+3}$

equate the denominator to zero

$x^2+3 \neq 0$

$x^2 = -3$

$\boxed{x = \sqrt{-3}}$ This is a complex number so the graph has no vertical asymptote

② HORIZONTAL ASYMPTOTES

We get HORIZONTAL asymptotes in two ways.

① When the power of the denominator is equal to the power of the numarator

e.g $\dfrac{x+1}{x-3}$ or $\dfrac{x^3+4}{x^3-2}$

② When the highest power of the denominator is greater than the highest power of the numarator

e.g $\dfrac{x+1}{x^2-3}$ or $\dfrac{5}{x^2+8}$ or $\dfrac{x+2}{x^2+1}$

* To find the horizontal asymptote do the following

Step 1: Divide each item by the highest power of the denominator

Example

$$\frac{x+1}{\underbrace{x^2}-3}$$

highes power from the denominator

$$\frac{\dfrac{x}{x^2}+\dfrac{1}{x^2}}{\dfrac{x^2}{x^2}-\dfrac{3}{x^2}}$$ Simplify

$$\frac{\dfrac{x^1}{x^2}+\dfrac{1}{x^2}}{\dfrac{\cancel{x^2}\,1}{\cancel{x^2}}-\dfrac{3}{x^2}} = \frac{\dfrac{1}{x}+\dfrac{1}{x^2}}{1-\dfrac{3}{x^2}}$$

Step 2: Substitute $+\infty$ and $-\infty$ into the expression you got after Dividing through. By employing Infinity as follows

$$\lim_{x \to +\infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{1 + \frac{3}{x^2}} \quad \text{and} \quad \lim_{x \to -\infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{1 + \frac{3}{x^2}}$$

positive

negative

Rules For Infinity

Any number divided By Infinity the answer is zero. Read again

e.g $\frac{a}{+\infty} = 0$ | $\frac{a}{-\infty} = 0$

where "$\underline{a}$" is any number.

Now
Lets
Simplify our answer

$$\lim_{x \to +\infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{1 + \frac{3}{x^2}} \qquad \lim_{x \to -\infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{1 + \frac{3}{x^2}}$$

Substitute x with infinity

$$\lim_{x \to +\infty} \frac{\underbrace{\frac{1}{\infty}}_{\text{zero}} + \underbrace{\frac{1}{(+\infty)^2}}_{\text{zero}}}{1 + \underbrace{\frac{3}{+\infty}}_{\text{zero}}} \qquad \lim_{x \to -\infty} \frac{\frac{1}{-\infty} + \frac{1}{(-\infty)^2}}{1 + \underbrace{\frac{3}{(-\infty)^2}}_{\text{zero}}}$$

$$\therefore \frac{0 + 0}{1 + 0} \qquad \frac{0 + 0}{1 + 0}$$

$$0 \qquad 0$$

Lastly the answer you got after simplifying is the Horizontal asymptote. in this case is

$$\boxed{y = 0}$$

③ OBlique Asymptotes

* We get an oblique asymptote when we have the highest power of the Numarator larger than the highest power of the denominator

* To get the oblique asmptote Divide By long division then equate the Quotient to y after Dividing By long Division

$$\frac{F(x)}{K(x)} = Q(x) + \frac{\text{Remainder}}{K(x)}$$

Equation for oblique asymptote is $\boxed{Y = Q(x)}$

STRATEGY

The number of Asymptotes ~~corre~~ corresponds with the segments of the graph

For example.

Sketch $y = \frac{1}{x^2}$

This has two asymptotes

Vertical asymptotes

$x^2 = 0$

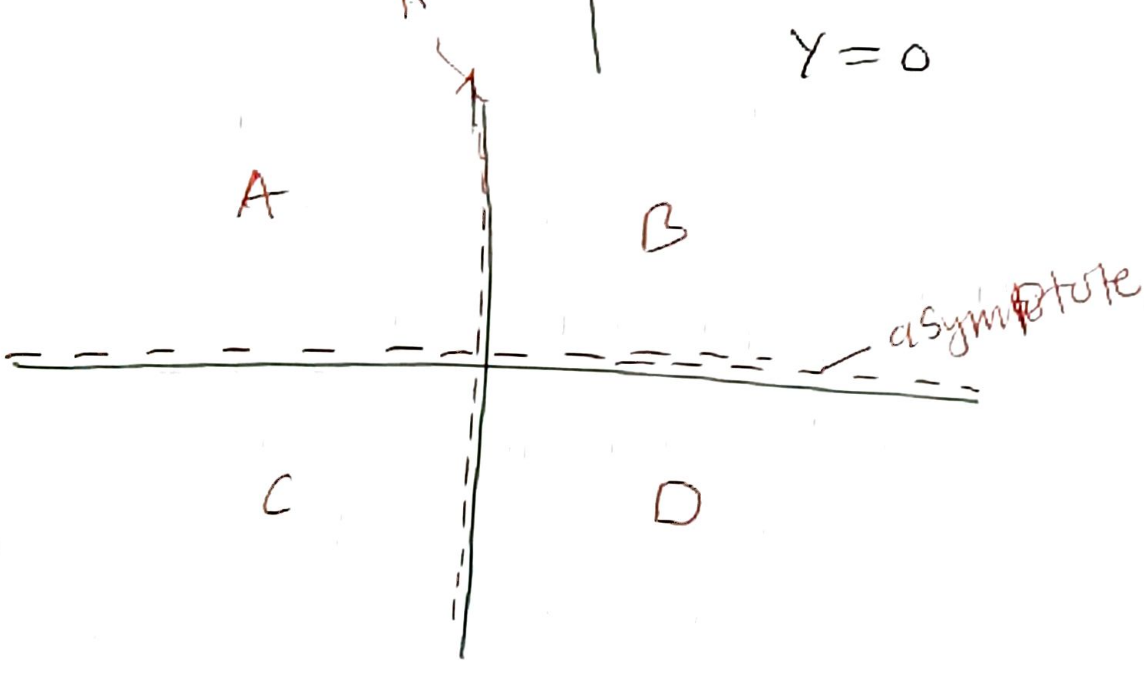
$x = 0$

Horizontal asymptotes

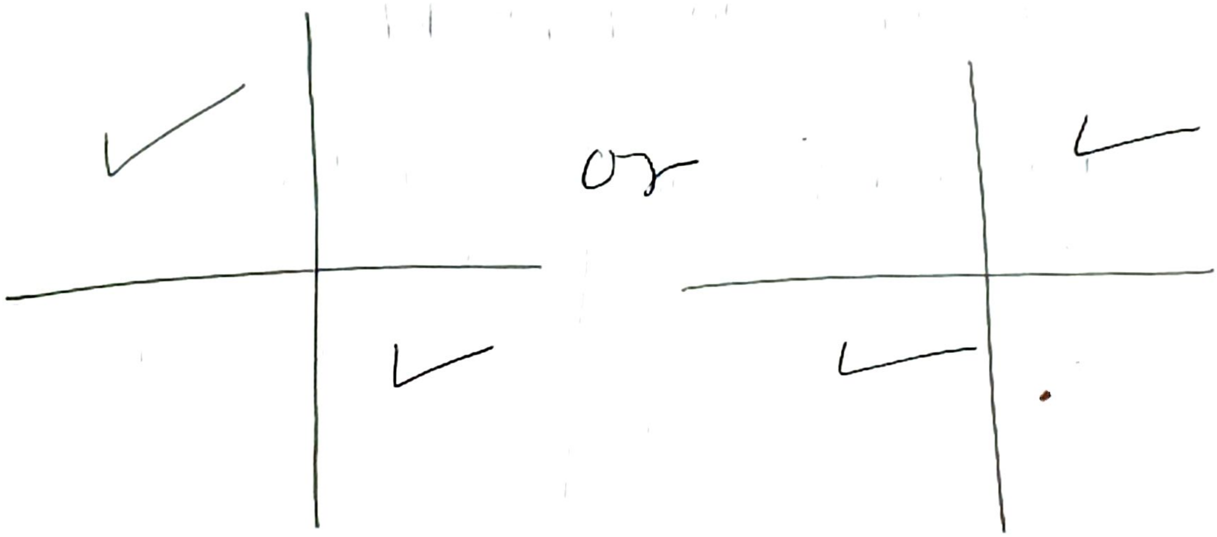
$y = \frac{1}{x^2} = \frac{1}{(\infty)^2}$

$= 0$

$y = 0$



GRAPHS Always exist in diagonals Regions, so if you can find what Region part of the ~~graph~~ segment is in. you automatically know the other segment ~~region~~ ~~diago~~ diagonal Region



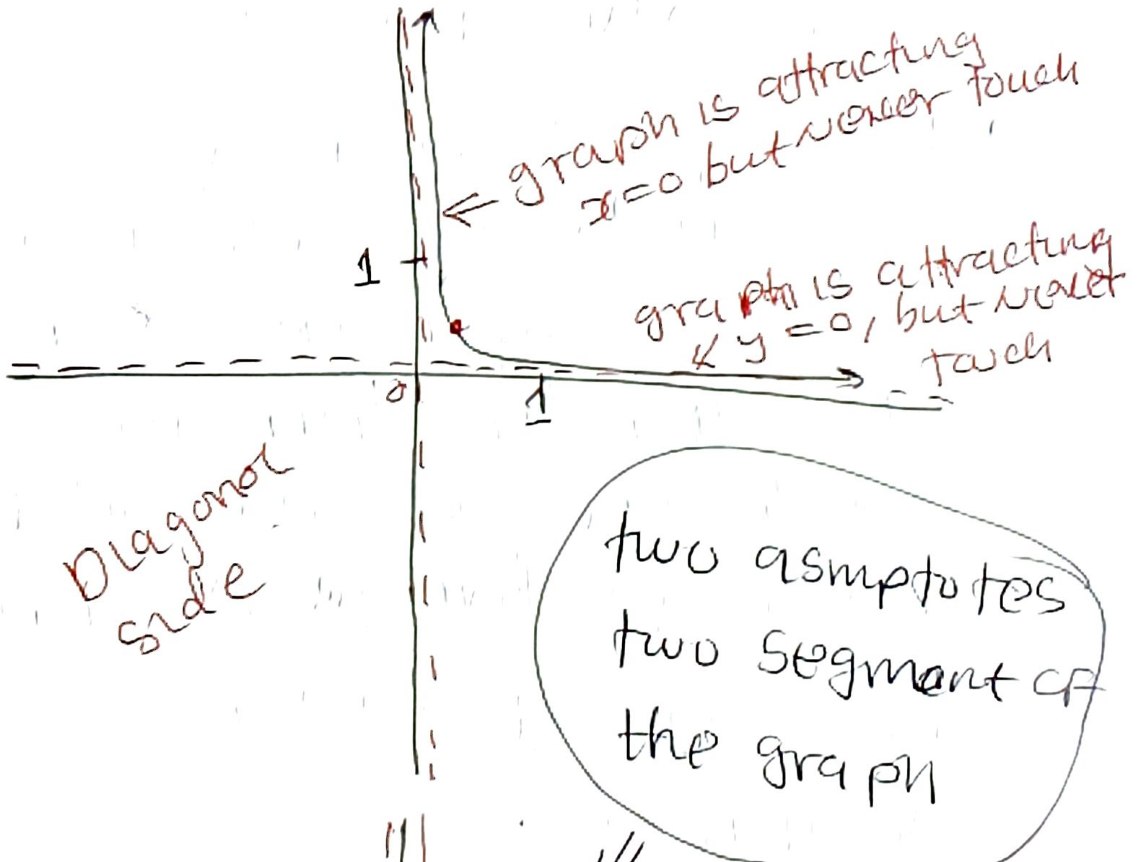
To know the Region you need to find the critical point By plugging any ~~can~~ Random value in the expression which is not among the asymptote you have. Read again

Random Value

Let $x = 1$

$$y = \frac{1}{x^2} = \frac{1}{(1)^2} = 1$$

$\therefore$ Critical point $(1, 1)$



POINTS: 23
STUDENT LOAN: 100%
ACCOMMODATION:
Bachipela

4] Exam Q: Sketch $F(x) = \frac{x+2}{x-2}$

Including all the asymptotes

Pointers

- Find the vertical asymptote By equating the denominator to zero and solve for x
- The highest power of the denominator is equal to the highest power of the Numarator so the function has Horizontal asymptote. to Find the Horizontal asymptote divided everthing By the highest power of the Numarator.

Solution

$$F(x) = \frac{x+2}{x-2}$$

vertical asypriptote: $x-2 \neq 0$

$x \neq 2$

Horizontal asymptote

- $\lim_{x \to \infty} \frac{x+2}{x-2}$ | $\lim_{x \to -\infty} \frac{x+2}{x-2}$

$\lim_{x \to +\infty} \frac{\frac{\cancel{x}}{\cancel{x}} + \frac{2}{x}}{\frac{\cancel{x}}{\cancel{x}} - \frac{2}{x}}$ | $\lim_{x \to -\infty} \frac{\frac{\cancel{x}}{\cancel{x}} + \frac{2}{x}}{\frac{\cancel{x}}{\cancel{x}} - \frac{2}{x}}$

$\lim_{x \to +\infty} \frac{1 + \frac{2}{x}}{1 - \frac{2}{x}}$ | $\lim_{x \to -\infty} \frac{1 + \frac{2}{x}}{1 - \frac{2}{x}}$

Substitute x with infinity

$\frac{1 + \frac{2}{\infty}}{1 - \frac{2}{\infty}}$ | $\frac{1 + \frac{2}{-\infty}}{1 - \frac{2}{(-\infty)}}$

$$\frac{1+\frac{2}{\infty}}{1-\frac{2}{(-\infty)}} \quad \Bigg| \quad \frac{1+\frac{2}{-\infty}}{1\cancel{+}\frac{2}{-\infty}}$$

Anything Divided By infinity the answer is zero

$$\frac{1+0}{1-0} \quad \Bigg| \quad \frac{1+0}{1-0}$$

$$\underline{1} \quad \Bigg| \quad \underline{1}$$

If you got same answer then you are doing the Right thing

$\therefore y = 1$

Shortcut way of getting Horizontal asymptote.

$$\frac{x+2}{x-2}$$

- Cancal what ever has the power less than that of the highest power of the denominator

$$\frac{x+\cancel{2}}{x-\cancel{2}}$$

$$\frac{x}{x}$$

$$= \underline{1}$$

$\therefore$ Horizontal asymptote is $\underline{1}$

Now Find the y-intercept By Plugging the random Value is is not part of asymptotes

$$\frac{x+2}{x-2}$$

Plug in zero

$x = 0$

$$y = \frac{x+2}{x-2}$$

$$y = \frac{0+2}{0-2} = \frac{+2}{-2} = -1$$

So the graph cut at $y = -1$

Note that when finding the critical point of the graph you can plug in any value which is not among asymptotes

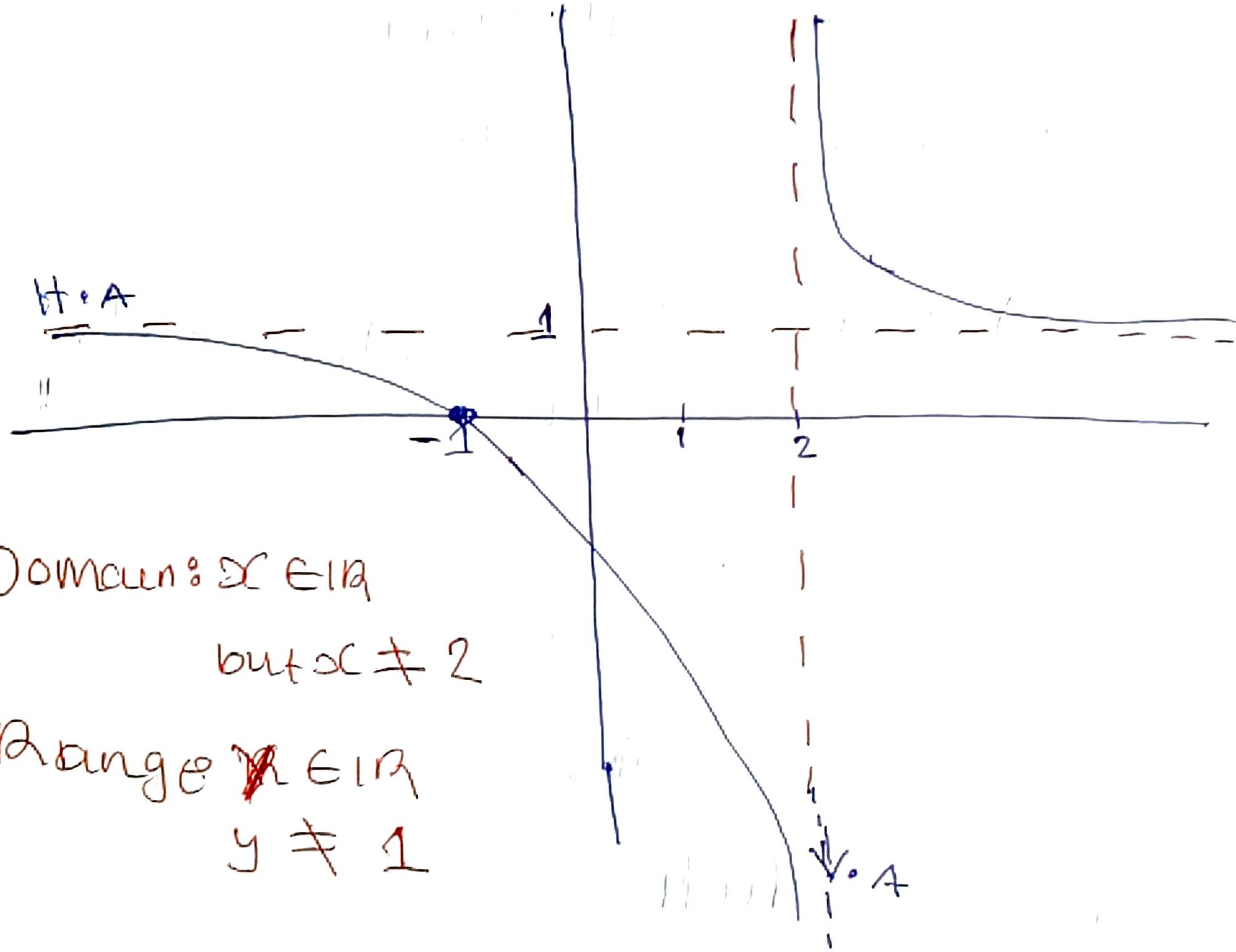
Summary

V.A $= 2$

H.A $= 1$

Critical poin $(0, -1)$

Now we can sketch



Domain: $x \in \mathbb{R}$

but $x \neq 2$

Range ~~x~~ $\in \mathbb{R}$

$y \neq 1$

It is important to Find the critical point because it tells us where the graph should be The exact Diagonal where the graphs are existing

Sketch the Following

(a) $F(x) = \dfrac{x^2 - 5x + 6}{x - 2}$

(b) $F(x) = \dfrac{2x^2}{x^2 + 4}$ (c) $F(x) = \dfrac{x^2 + 2}{x^2 - 1}$

Solutions

Pointers

(a) $F(x) = \dfrac{x^2 - 5x + 6}{x - 2}$

We will have oblique asymptotes because the highest power of the Numarator is bigger than the highest power of the alumarater

- Divide By long division, but make sure the powers are dropping nicely

$$\frac{x^2 - 5x^1 + 6}{x - 2}$$

$$\begin{array}{r|l} & x - 3 \\ \hline x - 2 & x^2 - 5x + 6 \\ & -(x^2 - 2x \downarrow) \\ \hline & 0 \quad (-3x + 6) \\ & -(-3x + 6) \\ \hline & 0 + 0 \\ & 0 \end{array}$$

Remainder = 0

The oblique asymptote is $\boxed{x - 3}$

$y = x - 3$

make a table value

x	[illegible]	0	1
y	[illegible]	-3	-2

Note that: If the rational Function has oblique asymptote it can't have Horizontal asymptote and viceversa

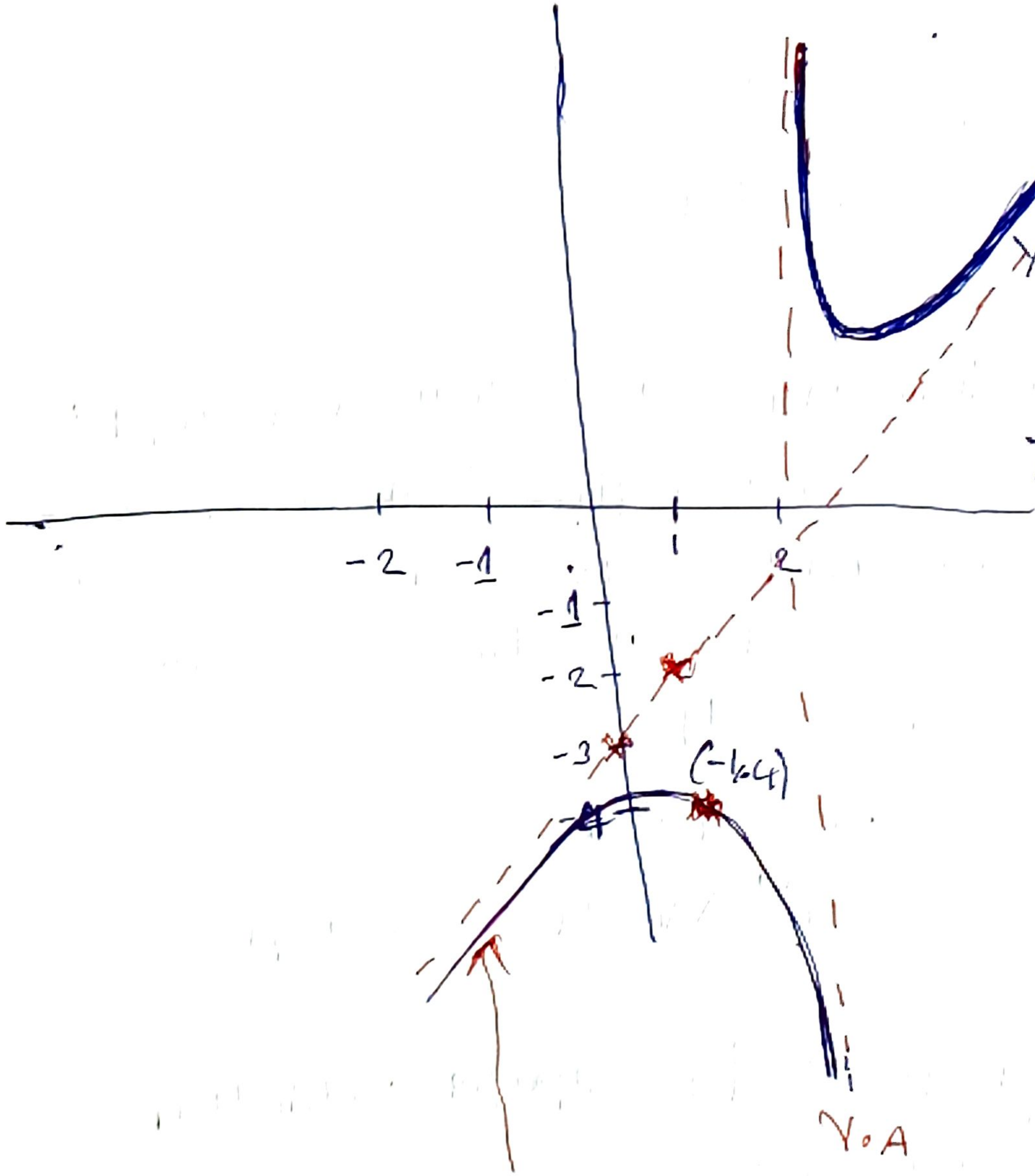
Vertical asymptote

Equate the denominator to zero

$$\frac{x^2 - 5x + 6}{x - 2}$$

V.A $x - 2 \neq 0$

$x \neq 2$



Random point

$$y = \frac{x^2 - 5x + 6}{x - 2}$$

$x = -1$

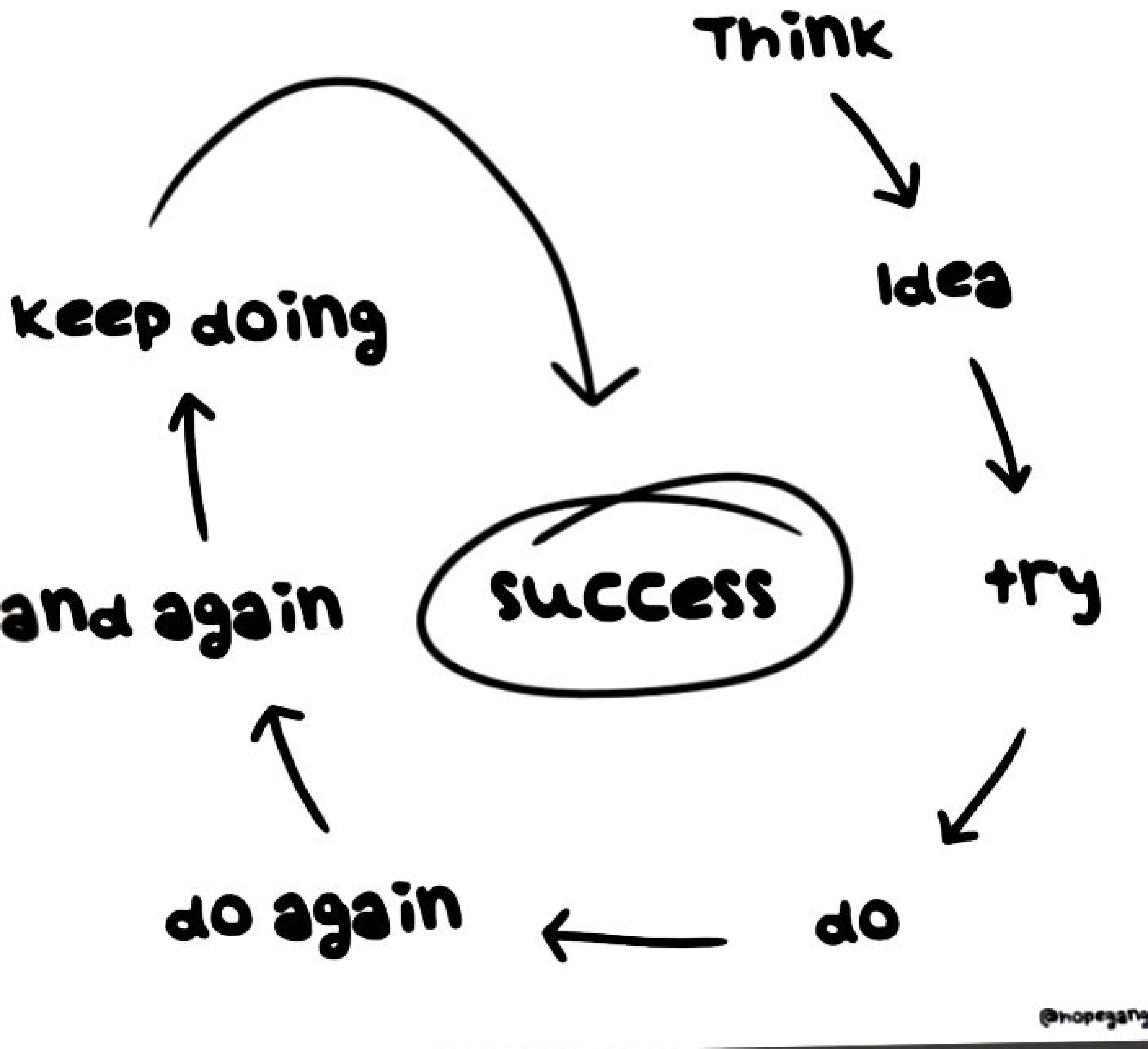
$$y = \frac{1 + 5 + 6}{-1 - 2}$$

$$y = \frac{12}{-3}$$

$y = -4$

$(-1, -4)$

Sketch this first then use the diagonal to predict where the other one will be.



Failing and trying everyday =SUCCESS

(b) $F(x) = \frac{2x^2}{x^2+4}$

Pointers

- The Function has NO Vertical asymptote because equating the denominator to zero and solve for x we will get a complex number (negative under the root)

$$x^2 - 4 \neq 0$$

$$x \neq \sqrt{-4}$$ Complex

- Nevertheless; the ~~graph~~ Function has horizonal asymptote because power to denominator and numerater are equal

∴ Horizontal Asymptote

$$\lim_{x \to +\infty} \frac{2x^2}{x^2+4}$$

$$\lim_{x \to +\infty} \frac{\frac{2x^2}{x^2}}{\frac{x^2}{x^2}+\frac{4}{x^2}}$$

$$\lim_{x \to +\infty} \frac{2}{1+\frac{4}{x^2}}$$

$$\lim_{x \to -\infty} \frac{2x^2}{x^2+4}$$

$$\lim_{x \to -\infty} \frac{\frac{2x^2}{x^2}}{\frac{x^2}{x^2}+\frac{4}{x^2}}$$

$$\lim \frac{2}{1+\frac{4}{x^2}}$$

Substitute x with $\pm\infty$

$$\frac{2}{1+\frac{4}{(\infty)^2}}$$

$$\frac{2}{1}$$

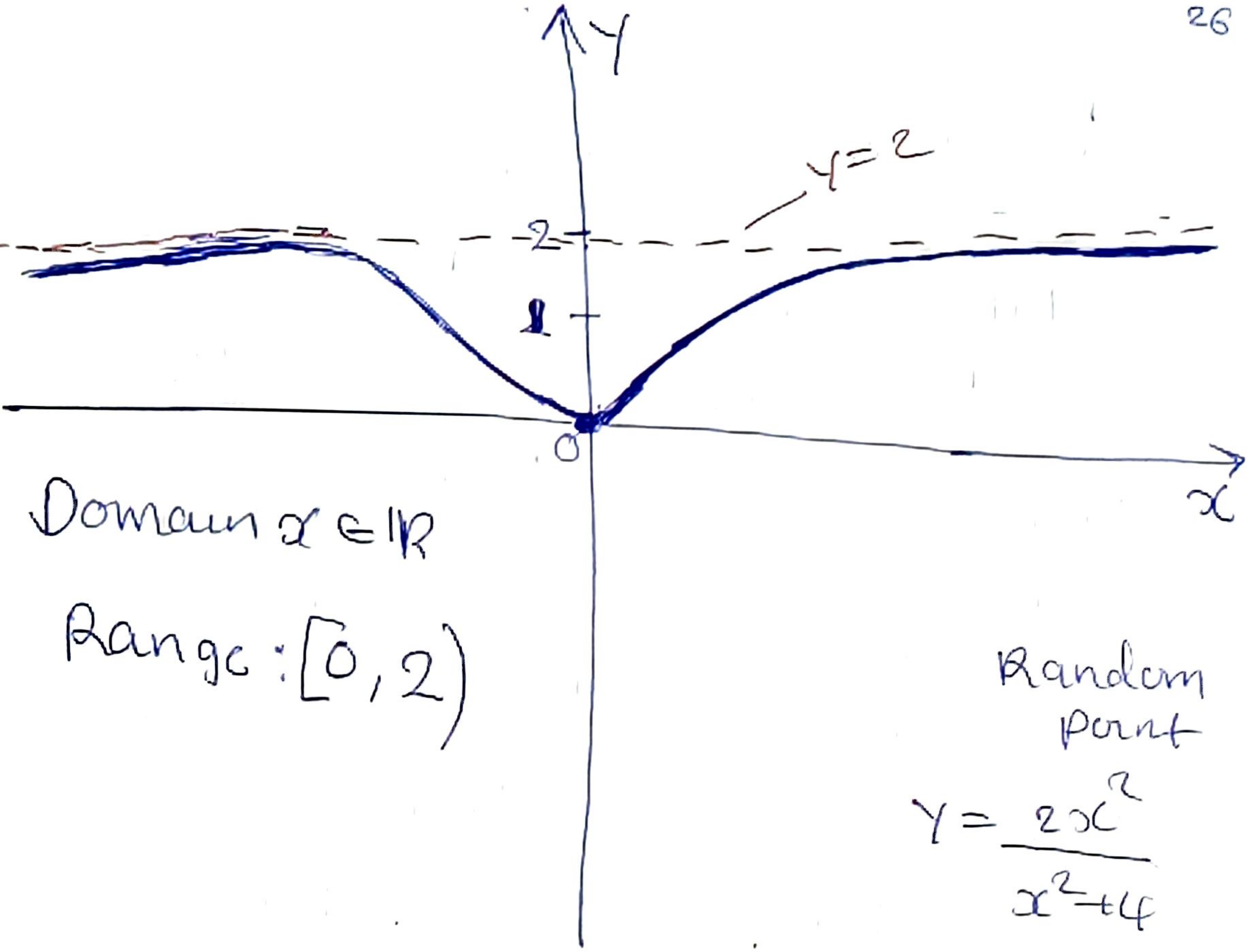
$$2$$

$$\frac{2}{1+\frac{4}{(-\infty)^2}}$$

$$\frac{2}{1}$$

$$2$$

∴ Horizontal Asymptote = 2



Domain $x \in \mathbb{R}$

Range: $[0, 2)$

Random point

$$Y = \frac{2x^2}{x^2+4}$$

$x = 0$

$$Y = \frac{2(0)^2}{(0)^2+4}$$

$$Y = \frac{0}{4} = 0$$

$(0, 0)$

(c) $F(x) = \dfrac{x^2+2}{x^2-1}$

Pointers

* The graph has vertical asymptote ~~x~~.

$x^2 - 1$

$x^2 = 1$

$x = \pm\sqrt{1}$

$x = 1$ or $x = -1$

• The function also has horizonatal asymptote. Solve for it using limits to both (−) and (+)

$$\lim_{x \to +\infty} \frac{x^2+\cancel{2}}{x^2-\cancel{1}}$$

$$\lim_{x \to +\infty} \frac{\frac{\cancel{x^2}}{\cancel{x^2}} + \frac{2}{x^2}}{\frac{\cancel{x^2}}{\cancel{x^2}} - \frac{1}{x^2}}$$

Substitute x with $+\infty$

$$\lim_{x \to -\infty} \frac{x^2+\cancel{2}}{x^2-\cancel{1}}$$

$$\lim_{x \to -\infty} \frac{\frac{\cancel{x^2}}{\cancel{x^2}} + \frac{2}{x^2}}{\frac{\cancel{x^2}}{\cancel{x^2}} - \frac{1}{x^2}}$$

Substitute x with $-\infty$

$$\frac{1+\frac{2}{(\infty)^2}}{1-\frac{1}{(\infty)^2}} \quad \Bigg| \quad \frac{1+\frac{2}{(-\infty)^2}}{1-\frac{1}{(-\infty)^2}}$$

Anything divided by infinity is equal zero

$$\frac{1+0}{1-0} \quad \Bigg| \quad \frac{1+0}{1-0}$$

$$1 \qquad\qquad 1$$

$$y = 1$$

$\therefore$ H. A $y = 1$

Randon point

$$x = 0$$

$$y = \frac{x^2+2}{x^2-1} = \frac{0^2+2}{0^2-1} = \frac{2}{-1}$$

$$= -2$$

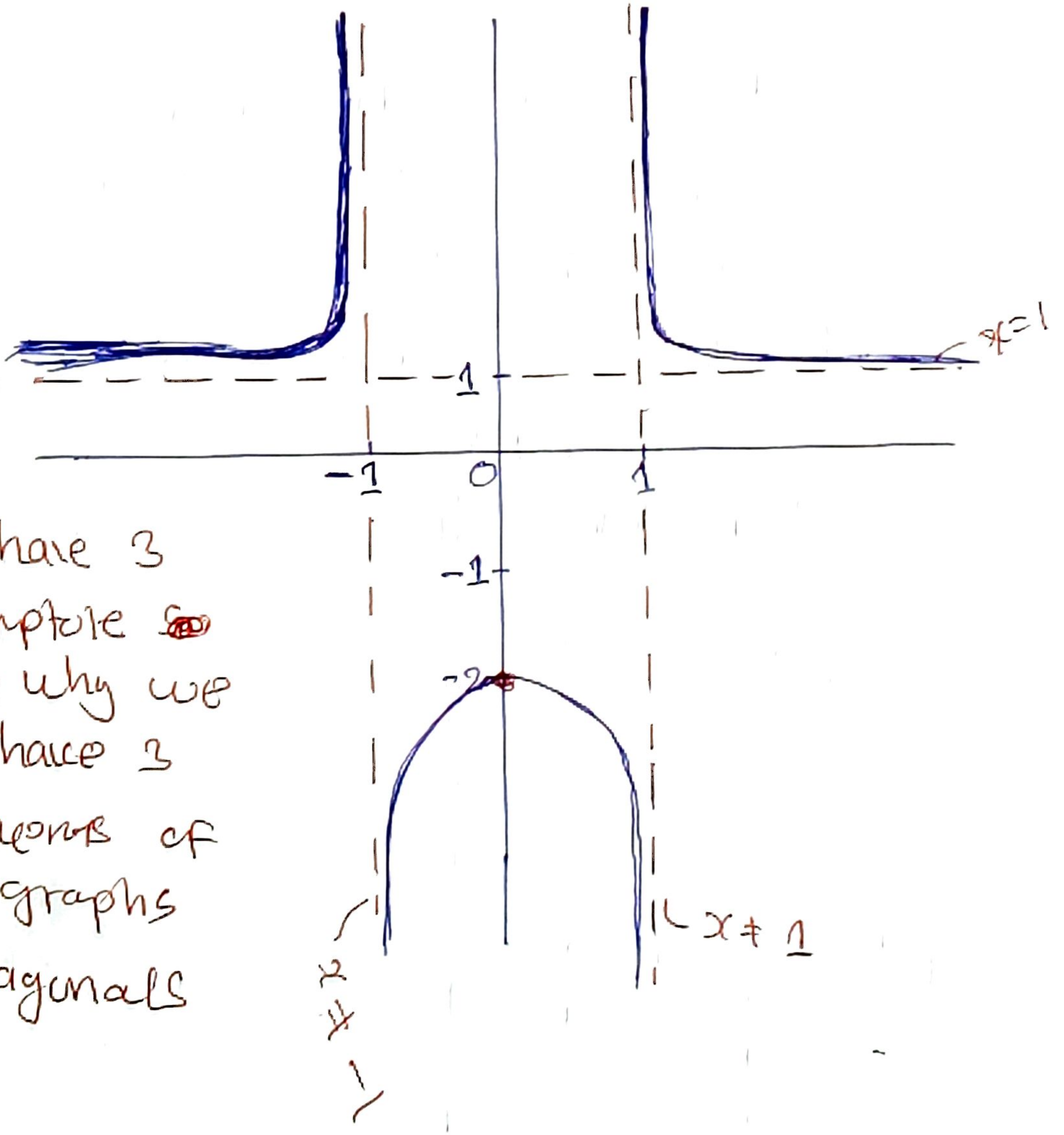
$\therefore$ Random poin $(0, -2)$

Summary

V.A : $x \neq 1$ or $x \neq -1$

H.A : $y = 1$

Random point $(0, -2)$



We have 3 Asymptote so ~~that~~ why we also have 3 segments of the graphs in diagonals

Stay organized ✊✊✊
✊

Is the motto
🎤🎤

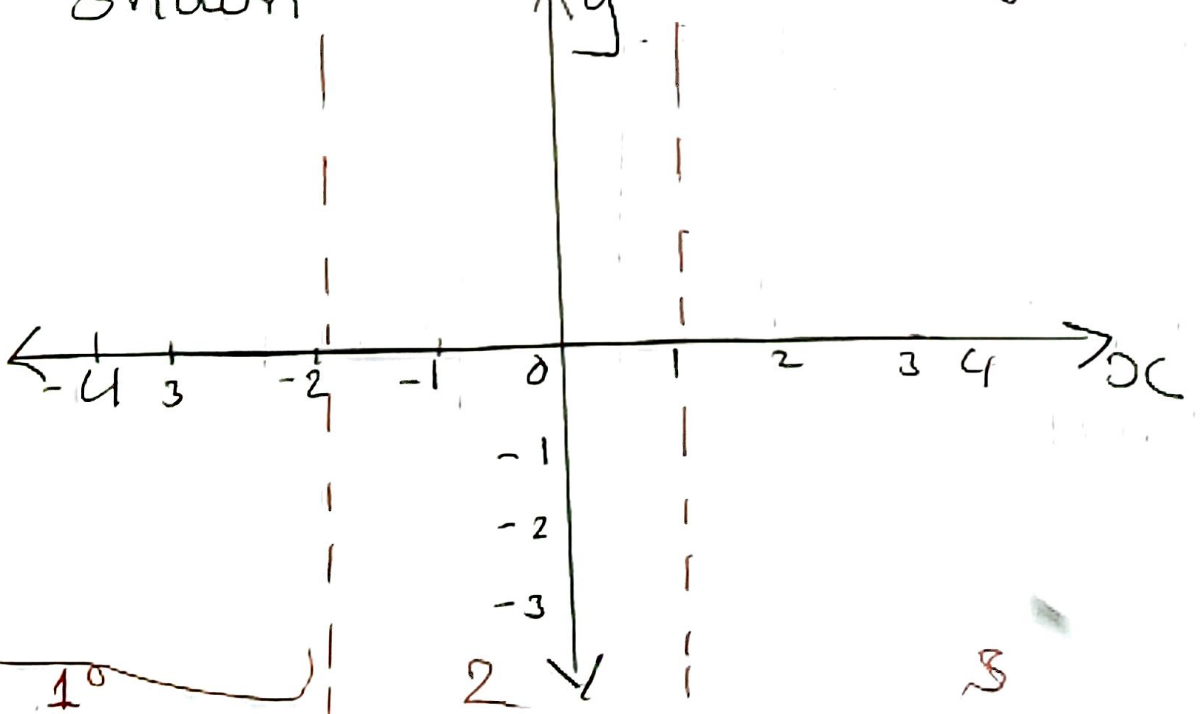
GRAPHING PIECEWISE FUNCTIONS

- These are functions drawn on pieces as the name suggested.

- A piecewise function consist of several functions whose graphs are only valid within a particular region

Example

We may have the xoy plane divided into three regions as shown



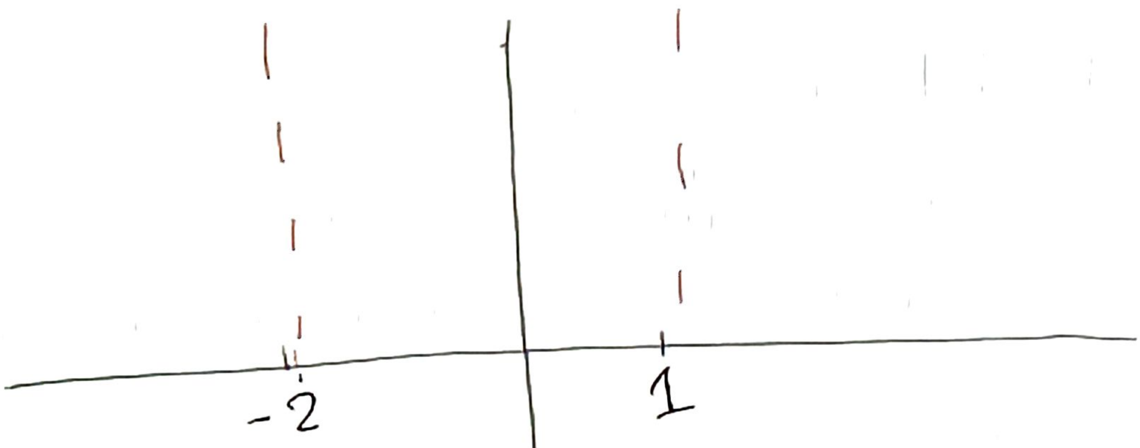
NOTICE: That there are vertical Lines that separate one Vertical Region from another

- $x = -2$ separates Region 1 From Region 2

- $x = +1$ Separate Region 2 from Region 3

- The Values of x that are common to the Regions are called <u>Break point</u>

- If each of the three Regions had a Function, then each break point should only belong to one Region



For this Region the Domain is $x \leq -2$

Domain is $-2 < x \leq 1$

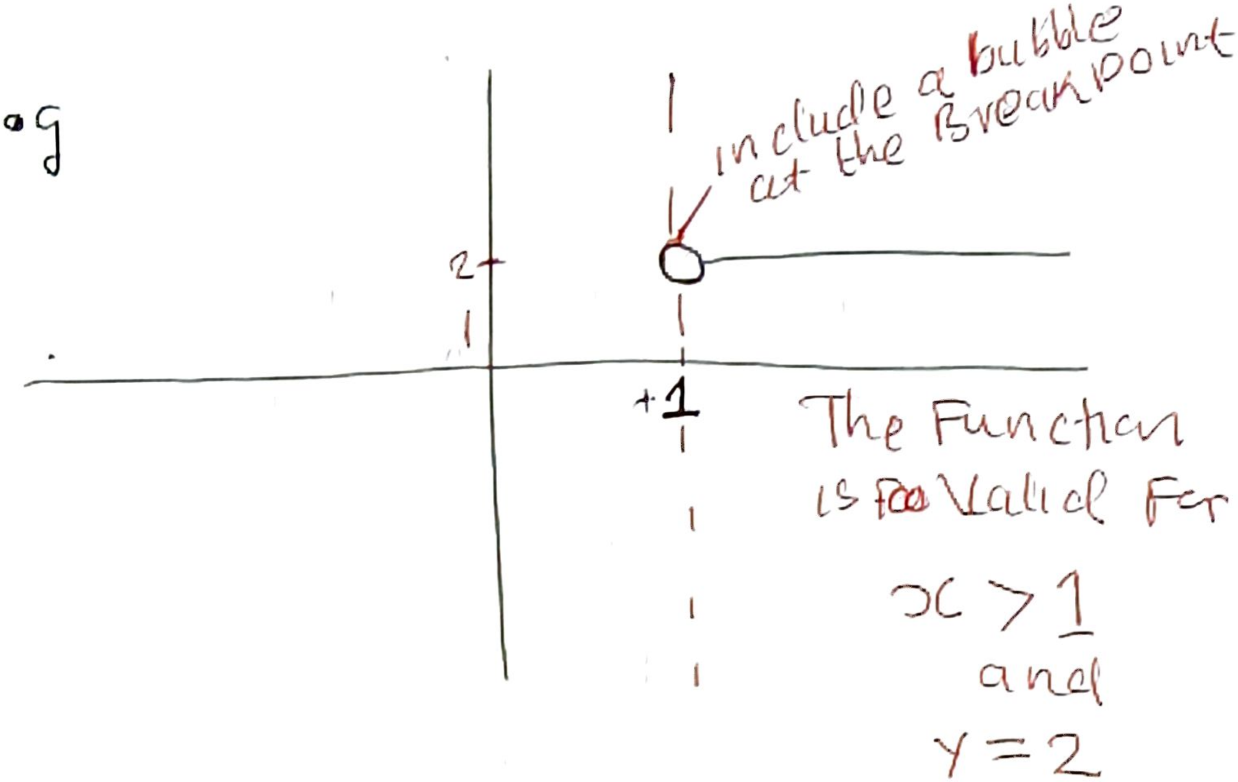
Domain is $x > 1$

The graph of each function Should stop Right ~~there~~ where the break point is and a Bubble should be drawn Right at that point

Read Again

Iwe behave ala Read

e.g

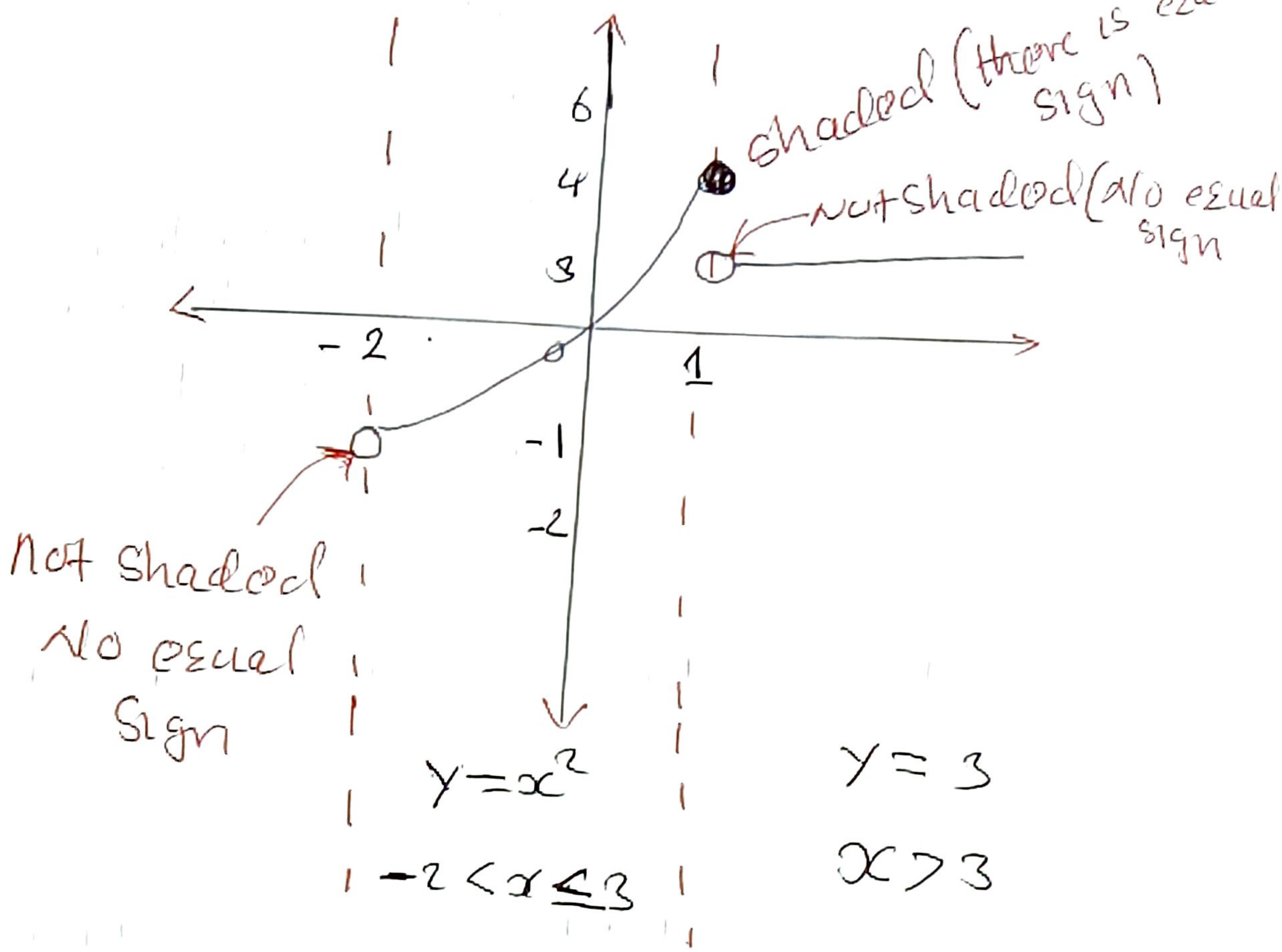


- The bubble can be shaded or un shaded

- A Bubble can only be shaded IF the breakpoint is part of the Domain (i.e IF there is equal sign)

$x \leq 3$ or $x \geq 3$

Read again



Summary

To sketch any piece wise function the Following applys

1. Find break points
2. Find the graphs of each function involved
3. Determine which one will be shaded and which on will not

Example

Q| graph the Following

ⓐ $F(x) = \begin{cases} 2x+3 & \text{IF } x<0 \\ x^2 & \text{IF } 0 \leq x < 2 \\ 1 & \text{IF } x \geq 2 \end{cases}$

ⓑ $F(x) = \begin{cases} 2x+1 & \text{IF } x \geq 0 \\ x^2 & \text{IF } x < 0 \end{cases}$

Solutions

ⓐ $F(x) = \begin{cases} 2x+3 & \text{IF } x<0 \\ x^2 & \text{IF } 0 \leq x < 2 \\ 1 & \text{IF } x \geq 2 \end{cases}$

Pointers

Step 1: Find the break point of each function, we have 3 Fuctions $2x+3$ IF $x<0$, x^2 IF $0 \leq x < 2$ and 1 IF $x \geq 2$.

- So the Break point is the ~~Restricted~~ Restricted Domain

$2x+3$ IF $x<0$

Break point is 0 (zero)

$\boxed{x<0}$

- There is no equal sign so the Buddle will not be shaded
- And we have lessthan symbol so the graph is only defined for values less than zero (0)

$(-\infty, 0)$

x^2 IF $0 \le x < 2$

Breaks points are zero and 2

$\boxed{0 \le x < 2}$

- For 0 the bubble will be shaded because we have equal sign. $\le$
- For 2 No equal sign so ~~bubble~~ bubble will not be shaded

Domain

$[0, 2)$

1 IF $x \geq 2$

Break ~~poi~~ point is 2

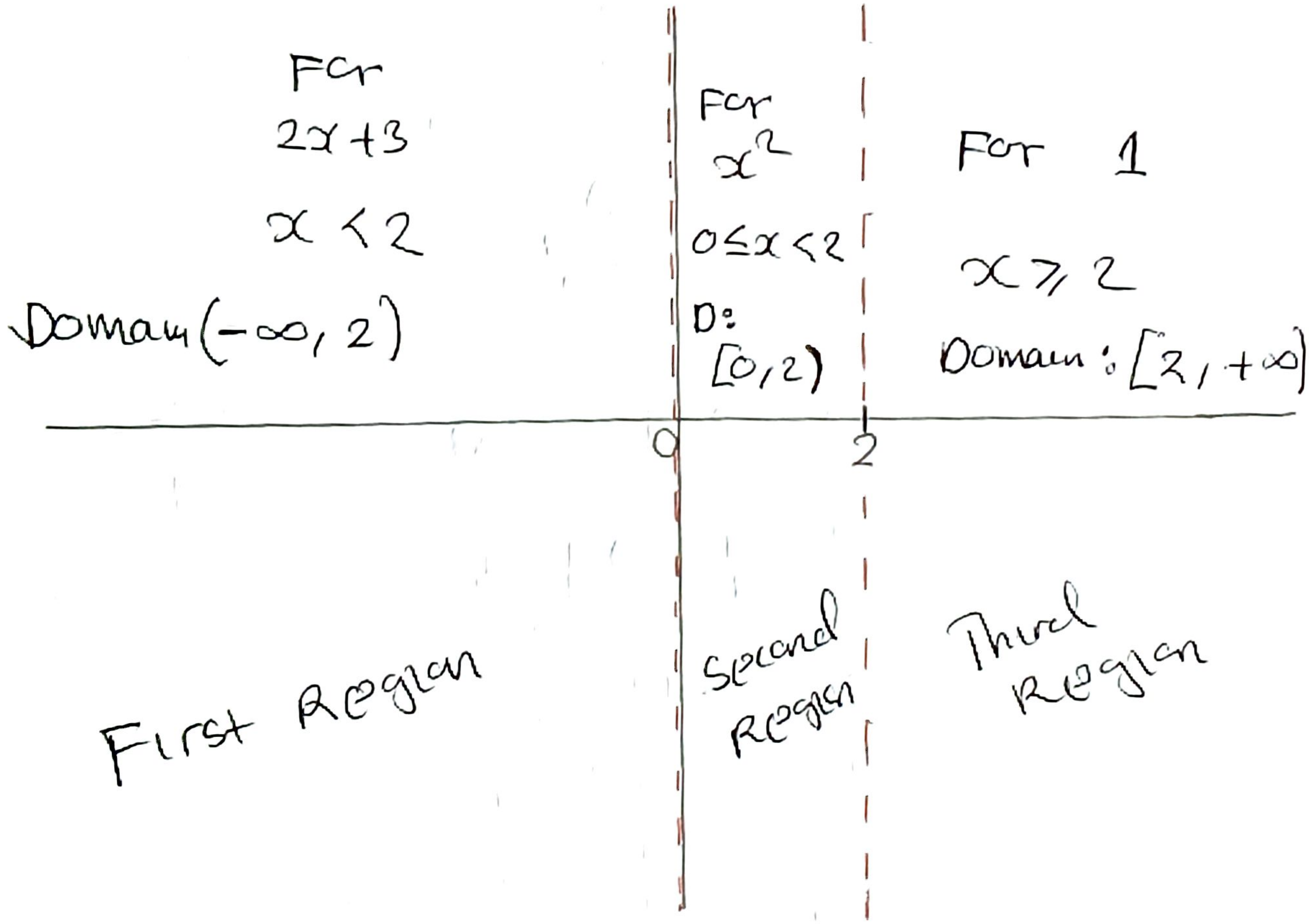
Shade the bubble Here

Domain

$x \geq 2$

$[2, +\infty)$

Now ~~Heo~~ let's Draw the Break points



Step 2: Draw the graph for each functions By Finding table values.

3b

For $2x+3$

x	-2	-1	0
y	-1	1	3

For x^2

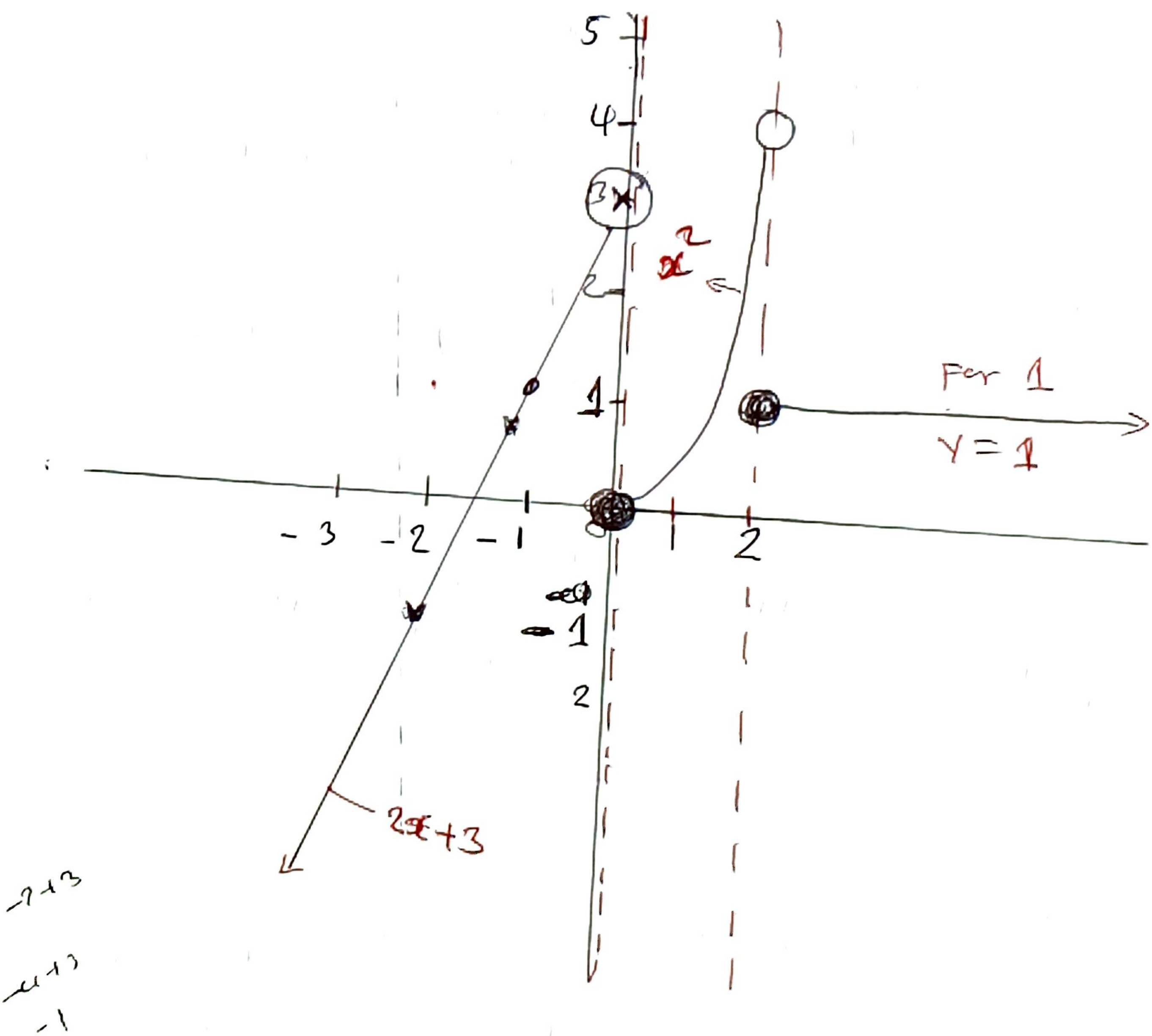
x	0	2
y	0	4

For 1

It break

$y = 1$

$x \geq 2$



- The graph for $2x+3$ can't go beyond $y=3$ because it has a break point there
- The graph of x^2 is a quadratic but it is drawn because the domain $[0, 2)$
- For $y = 1$ domain $[2, +\infty)$

(b) $F(x) = \begin{cases} 2x+1 & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}$

pointers

For $2x+1$

Break poin is at 0. Shaded Bubble

Table value

$y = 2x + 1$

x	-1	0
y	-1	1

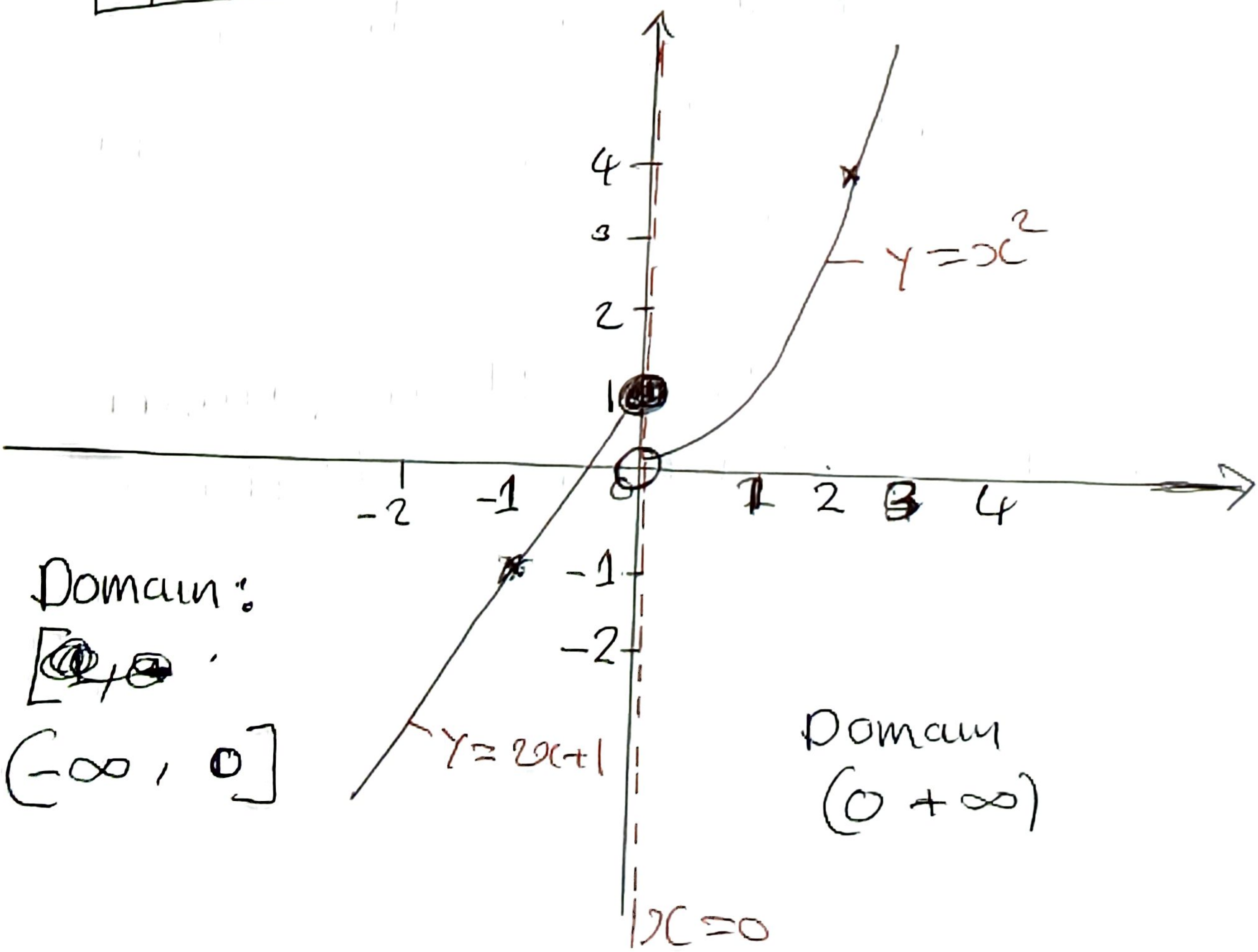
For x^2 (quadratic)

Breake point is at 0. un shaded Bubble

Table Vale

$y = x^2$

x	0	2
y	0	4



Domain:

$(-\infty, 0]$

Domain

$(0 + \infty)$

* If perhaps you haven't understood try to walk around the area and come back to try ~~it~~ again

* Try to call your friend to have a chat with you make your Brain Rest a bit abit abit and a bit the try again otherwise it's possible. to understand.

Will go deeper about piecewise in term 3 under Calculus.

GRAPHING polynomials Using Rational Root theorem

Polynomials— are Functions that has a degree power ~~greate~~ greater than 2 . e.g x^3, x^4 etc

- steps to Follow when graphing

Step 1: If the polynomial is not factorised, use the rational root ~~theo~~ theorem and synthetic Division to express it in terms of its linear factors.

For example: $F(x)\ kx^3 + px^2 + c$
can be factorised to give us

$$F(x) = (x+a)(x+b)(x-c)$$

Step 2: If you noticed any ~~repeatin~~ Repeating linear facters, Like $(x+a)$ above, express them as one with appropriate power

we got

$$F(x) = (x+a)^2 (x-c)$$

we have $F(x) = (x+a)^2 (x-c)$

so $(x+a)$ has a power of 2,

but $(x-c)$ don't have a power on top, this means the power is 1

Hence

$$F(x) = \underbrace{(x+a)^2}_{\text{even power}} \underbrace{(x-c)^1}_{\text{odd power}}$$

The powers are Important for a j

1&2 Strategy I will show you

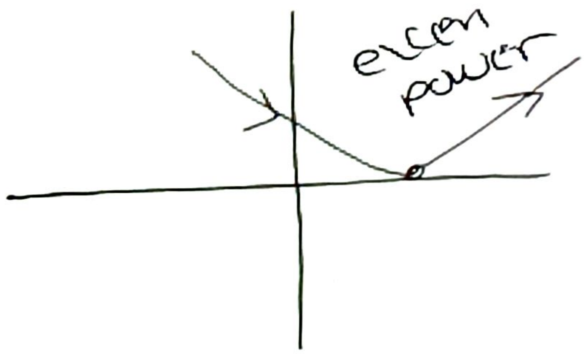
Strategy

$\Rightarrow$ If a power is even (2, 4, 6, 8, 10) then the graph will bouce off at that critical point

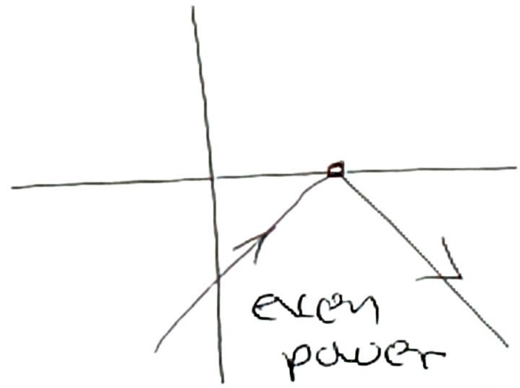
e.g $(x+2)^2$, at $x+2=0$

$x=-2$

the graph will not cross but bounce off



or

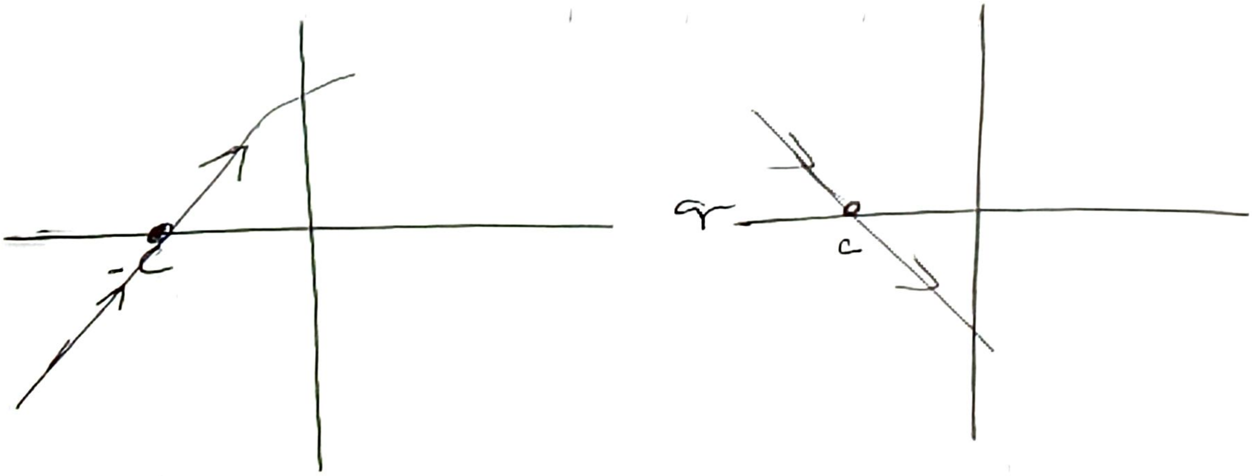


⇒ If a power is <u>odd</u> (1, 3, 5, 7) then the graph will <u>Cross</u> at that Critical Point

e.g $(x-c)^1$ odd

Critical point is

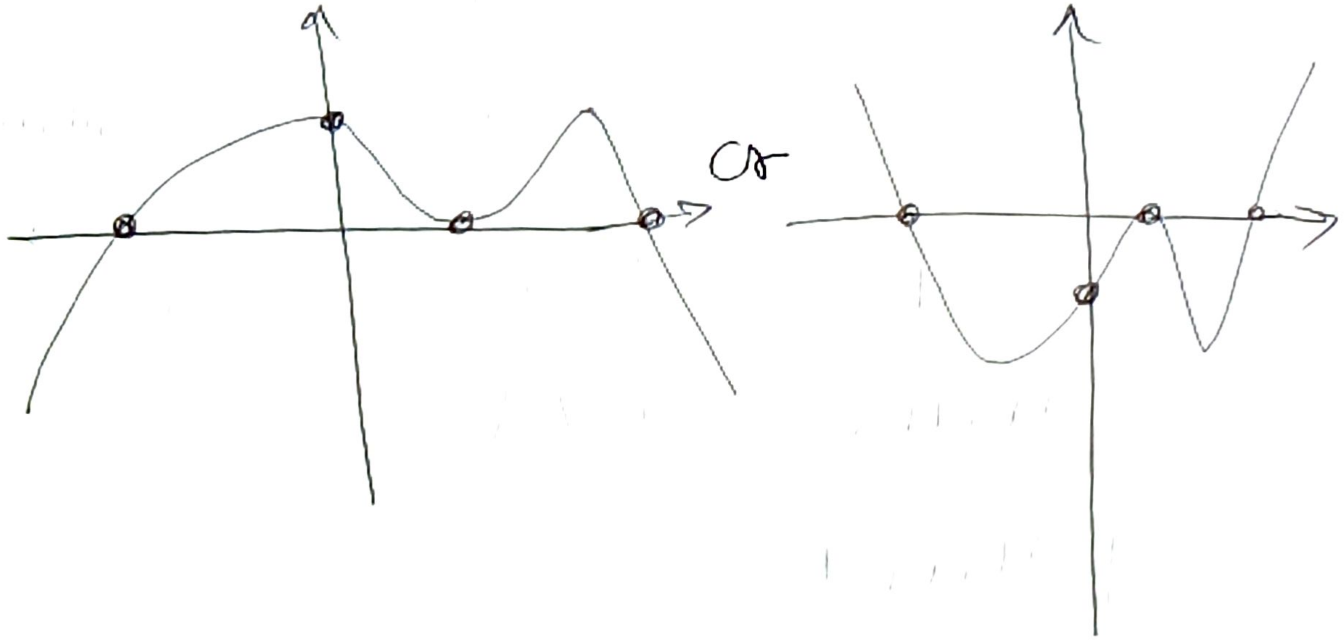
$x - c = 0$
$x = c$] the graph will Cross at $x = c$



<u>Cautions:</u> we keep drawing two possibilities for each critical points because we don't know How the graph will be moving

Redd again

To make this simple let me show you How to know How ~~the graph do to~~ the graph will always move.



To know How the graph will Move one should plug in Random value i.e $x=0$ in the Function and Solve For y IF you can positive answer it means the graph will cut positive y-axis and IF you get negative answer meaning the graph will cut negative y-axis.

Read again

Q] Sketch the Following

(a) $x^3 + x^2 - 2x$ — 5 marks

(b) $y = (x-3)(x-2)(x+1)$ — 5 marks

(c) $y = +(x-3)(x-2)^3(x+1)^2$

[5 mark each]

<u>pointers</u>

- For (a) use rational Root theorem
- Recall : ~~even root — bounce~~

  : ~~odd Root~~ Root — cross

~~[illegible]~~

~~cross~~ ~~odd~~

Root

Recall

- even power — Bounce off
- odd power — cross

C O | B E — even powe

Cross odd power | Bounce off

$\boxed{\text{COBE}}$

Solutions

(a) ~~f(x)~~ x^3+x^2-2x

Factorize x

$x(\underbrace{x^2+x-2}_{\text{quadratic}})$

quadratic solve it by ~~factor~~ Factorization

$x^2 + x - 2$

$p = -2, \; s: 1$

factors

$-1, 2$

$x^2 - x + 2x - 2$

$(x^2 - x) + (2x - 2) = 0$

$x(x-1) \; 2(x-1)$

$(x+2)(x-1)$

$\therefore$ The factoctorized version of

$x^3 + x^2 - 2x$ is $\underbrace{(x)(x+2)(x-1)}_{\text{Sketch this}}$

$$(x)^{1\,\text{odd}}\,(x+2)^{1\,\text{odd}}\,(x-1)^{1\,\text{odd}}$$

All the powers are odd so the graph will cross everywhere.

Critical Points

To Find the Critical point ignore the power and equate what's inside the Bracket to zero and solve for x

$$(x)\,(x+2)\,(x-1)$$

$$x=0,\quad x+2=0,\quad x-1=0$$

$$\boxed{x=0,\quad x=-2,\quad x=1}$$

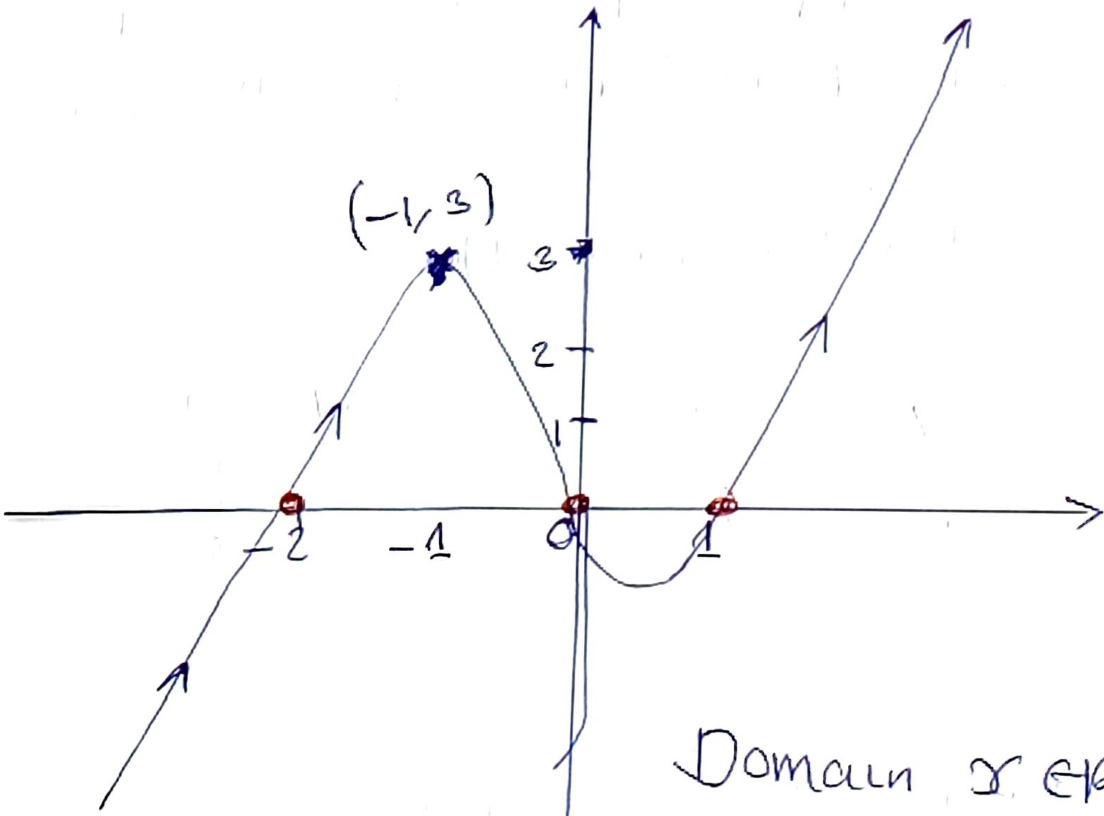
Random point of y-intercept

let $x = -1$

$$y=(x)(x+2)(x-1)$$
$$y=(-1)(-1+2)(-1-2)$$
$$y=(-1)(1)(-3 = y=+3$$

So the graph will cut positive y-axis at ~~+3~~ (-1, +3)

The Random points helps to know where the graph will starts from either bottom or top. Depends on the possibilities of the graph to pass through the critical point.



Domain $x \in \mathbb{R}$

Range $y \in \mathbb{R}$

The graph can only pass through the critical point (-1, 3) if it starts from the bottom.

(b) $(x-3)(x-2)(x+1)$

Pointers

This one is direct go ahead to find the critical points. By equating what is inside the branket to zero and solve for x

Critical points

$x-3=0$, $x-2=0$, $x+1=0$

$x=3$, $x=2$ $x=-1$

To find the Random point in the y-axis plug in any Random number into the function but make sure that number is not among the critical points

Screct Here Read again

$$y = (x-3)^{\overset{\text{odd}}{1}}(x-2)^{\overset{\text{odd}}{1}}(x+1)^{\overset{\text{odd}}{1}}$$

let $x = 0$

$y = (0-3)(0-2)(0+1)$

$y = (-3)(-2)(1)$

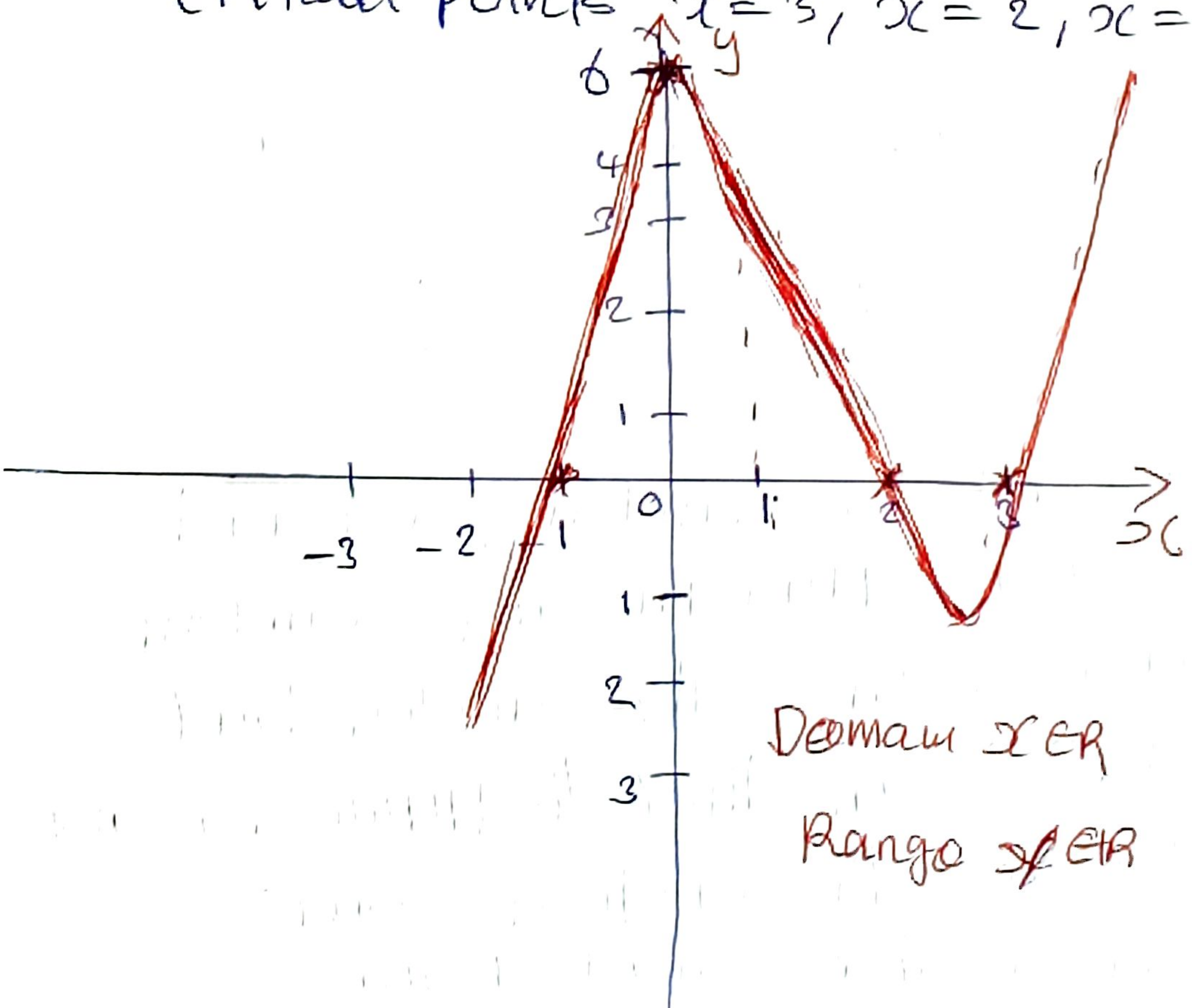
$y = (6)(1)$

$y = 6$

all the powers are odd so the graph will cross everywhere

Random point $(0, 6)$

Critical points $x = 3$, $x = 2$, $x = -1$



Domain $x \in \mathbb{R}$

Range $y \in \mathbb{R}$

(c) $+ (x-3)^{odd} (x-2)^{3\ odd} (x+1)^{2\ even}$

pointers

This is alredy Factorised so Go ahead to Find the Critical points. Ignore the powers and equate what is under (inside) Brackets to zero and Solve For x

$x-3=0$, $x-2=0$ $x+1=0$

$x=3$ $x=2$ $x=-1$

C O BE even power

Cross odd power. Bounce

Now Find the Random point

$Y=-(x-3)(x-2)^3(x+1)^2$

$x=0$

$Y=-(0-3)(0-2)^3(0+1)^2$

5

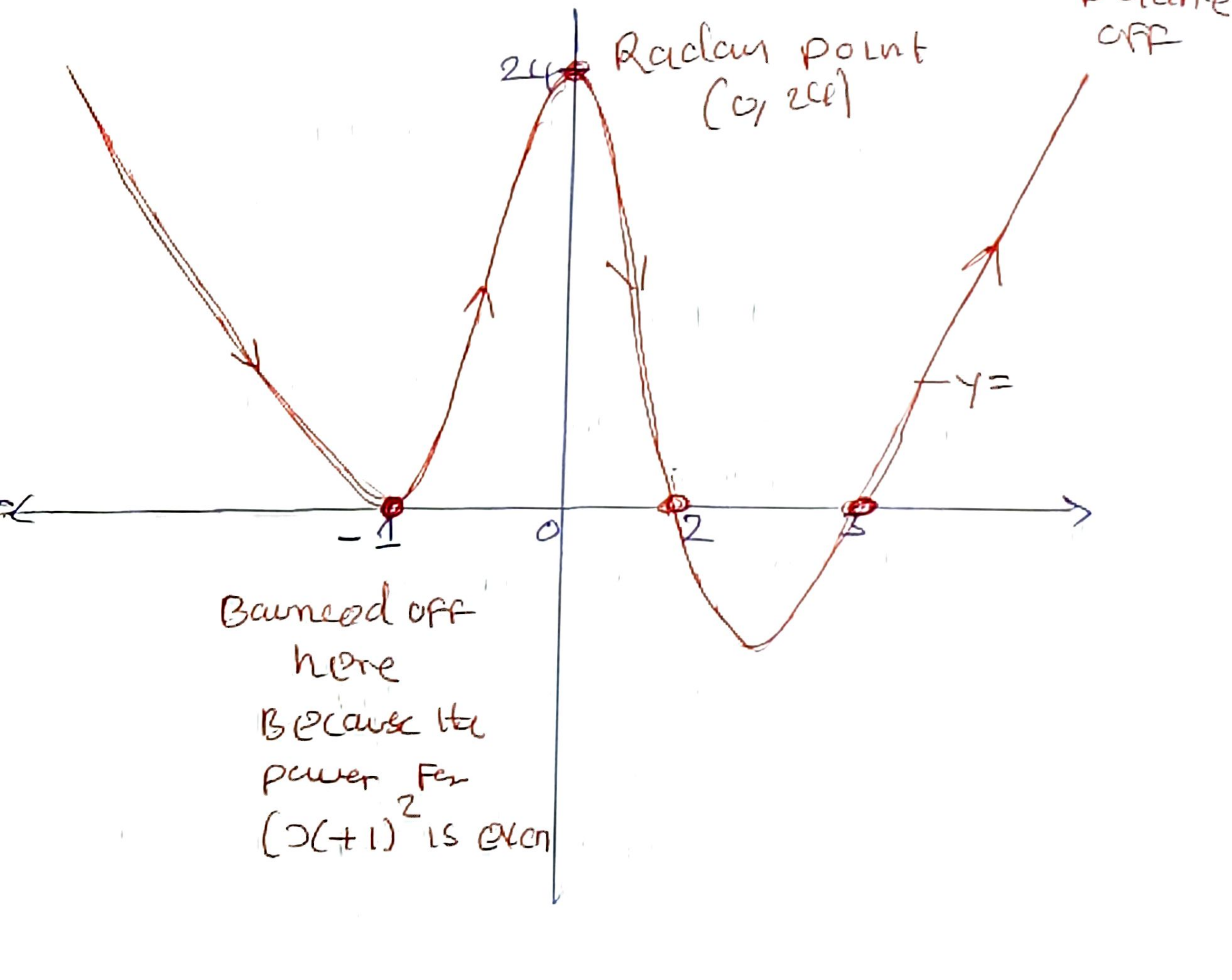
$y = (-3)(-2)^3(1)^2$

$y = -3(-8)(1)$

$y = 24$

Random $(0, 24)$

Recall $\underset{1 \text{ odd}}{(x-3)^1}$ $(x-2)^3$ odd $\underbrace{(x+1)^2}_{\text{It will Baune off}}$ even



Austin Mwelwa

Today, 10:22

You will never go wrong with Bob otherwise Biology was 😄😂😂😂😂😂😂😂😂💪💪💪🙏🙏🙏💞💞🖤🖤🖤

REDEFINING THE MODULUS FUNCTION

LIFE is about stay where people Value you

So Find where you are Valued In terms of grups, Friend, businesses because people will always Valued 20%, others 50% and others 80% but other grup will see you as total Junk So it's important to Find where you are Valued

and stay ~~there~~

Not ati' I want that beautiful lady (~~with~~ a big voice Kwati' chipuba dull). That beautiful lady see you Like Catapilar but busy Following her. Don't waste your time Iwe move on !!!

REDEFINING Modulus Functions

$|Fx| + |gx|$

Redefining a modulus ~~~~ Function is basically the process of converting a double ~~modulo~~ modulus Function to piecewise

Steps to Follow

Step 1: Find the Critical points

Step 2: Draw the sign graph

Step 3: Find the table value

Step 4: Sketch the Functions.

Steps

1. Find critical points
2. Draw the sign graph
   but not include overall over
3. Split modular functions into
   piece wise functions
4. Find the table values and
   sketch the graph

LET'S SOLVE EXAM QUESTION TO MAKE IT SIMPLE

58

Q1 Redefine the modulus function $F(x) = |3x+2| - |2x-1|$ By Romving the modulus and Hence sketch the graph of the function

Strategy

① Separate the function into $K(x)$ and $F(x)$

let $K(x) = 3x+2$ ad $g(x) = 2x-1$

② Find the critical points

To find critical point eqllate $K(x)$ and $g(x)$ to zero and solve for x

$K(x) = 0 \qquad g(x) = 0$

$3x+2=0 \qquad 2x-1=0$

$\frac{3x}{3} = \frac{-2}{3} \qquad \frac{2x}{2} = \frac{1}{2}$

$x = -\frac{2}{3} \qquad x = \frac{1}{2}$

$\therefore$ Critical points are

$x = -\frac{2}{3}$ and $x = \frac{1}{2}$

At this stage draw the sign graph but don't include the overall

$x < -\frac{2}{3}$	$-\frac{2}{3} \le x < \frac{1}{2}$	$x > \frac{1}{2}$
Region 1	Region 2	Region 3
$\lvert 3x+2\rvert - \lvert 2x-1\rvert$	$\lvert 3x+2\rvert - \lvert 2x-1\rvert$	$\lvert 3x+2\rvert - \lvert 2x-1\rvert$
Find the table of values that defines this function in this region	Find the table of values that defines this function in region 2	Find the table of values that defines this function in this region

Now From ~~[illegible]~~ piecewise Function g and K, we can get the Functions defined in each Region

Region 1: $x < -\frac{2}{3}$:

So we need negative Values

Region 2: $-\frac{2}{3} < x < \frac{1}{2}$:

So we need negative value For $-\frac{2}{3}$ and positive For $\frac{1}{2}$

Region 3: When ~~[illegible]~~ have $x > \frac{1}{2}$ postivitive Values

From Region 1: Both are negative

$\therefore x < -\frac{3}{2}$, $g(x) = -(3x+2)$

$K(x) = -(2x-1)$

$\therefore F(x) = -g(x) - (-K(x))$

$= -(3x+2) - (-(2x-1))$

$= -3x - 2 + 2x - 1$

$= -3x + 2x - 2 - 1$

$F(x) = -x - 3$ —— (i)

From Region 2: $-\frac{2}{3} < x < \frac{1}{2}$

$\therefore F(x) = g(x) - (-K(x))$

$= 3x + 2 - (-2x + 1)$

$= 3x + 2 + 2x - 1$

$F(x) = 5x + 1$ —— (ii)

From Region 3 $x > \frac{1}{2}$

$F(x) = g(x) - K(x)$

$F(x) = 3x + 2 - 2Kx + 1$

$F(x) = x + 3$ —— (iii)

we have 3 quations one from each Region. Now lets make ~~the~~ table value from each Region

R1	$-\frac{2}{3}$	R2	$\frac{1}{2}$	R3
$x < -\frac{2}{3}$		$-\frac{2}{3} < x < \frac{1}{2}$		$x > \frac{1}{2}$

R1

$y = -x - 3$

x	-2	$-\frac{2}{3}$
y	-1	-2.3

when $x = -2$

$y = -(-2) - 3$

$y = 2 - 3$

$\boxed{y = -1}$

when $x = -\frac{2}{3}$

$y = -\left(-\frac{2}{3}\right) - 3$

$y = \frac{2}{3} - \frac{3}{1}$

$y = \frac{2-9}{3}$

$y = \frac{-7}{3}$

$\boxed{y = -2.3}$

R2

$y = 5x + 1$

x	$-\frac{2}{3}$	$\frac{1}{2}$
y	$-\frac{7}{3}$	3.5

when $x = -\frac{2}{3}$

$y = 5\left(-\frac{2}{3}\right) + 1$

$y = -\frac{10}{3} + 1$

$y \frac{-10+3}{3}$

$\boxed{y = -\frac{7}{2} = -2.3}$

When $x = \frac{1}{2}$

$y = 5\left(\frac{1}{2}\right) + 1$

$y = \frac{5}{2} + \frac{1}{1}$

$y = \frac{5+2}{2}$

$\boxed{y = \frac{7}{2} = 3.5}$

R3

$y = x + 3$

x	$+\frac{1}{2}$	2
y	3.5	5

When $x = +\frac{1}{2}$

$y = \frac{1}{2} + \frac{3}{1}$

$y = \frac{1+6}{2}$ $\boxed{= \frac{7}{2} = 3.5}$

When $x = 2$

$\boxed{y = 2 + 3 = 5}$

To understand the above we use Sign graph.

$-2 \quad -\frac{3}{2} \quad 0 \quad \frac{1}{2} \quad 2$

	$x < -\frac{2}{3}$	$-\frac{2}{3} < x < \frac{1}{2}$	$x > \frac{1}{2}$
$(3x+2)$	$-$	$+$	$+$
$(2x-1)$	$-$	$-$	$+$

From R1

$3x+2$ and $2x-1$ both gives us Negative so we will Multiply each function with Negative one

From R2

- $3x+2$ gives positive so No need to Multiply
- $2x-1$ gives a negative So to make it postive Multiply with negative one

Both are ~~Bo~~ postive so No need to multiply with negative one.

We are multiplying the Function which gives us negative with Negative one because we have Monolulus in the orginal question

Modulus makes everything postive.

Recall the Function

$F(x) = |3x+2| - |2x-1|$

$g(x) = 3x+2$ and $K(x) = 2x-1$

Critical points

$3x+2=0$ / $2x-1=0$

$\frac{3x}{3} = \frac{-2}{3}$ $\quad$ $\frac{2x}{2} = \frac{1}{2}$

$x = -\frac{2}{3}$ $\quad$ $x = \frac{1}{2}$

Sign graph. Don't indicate overall

$-2 \quad -\frac{2}{3} \quad 0 \quad \frac{1}{2} \quad 2$

	$x < -\frac{2}{3}$	$-\frac{2}{3} < x < \frac{1}{2}$	$x > \frac{1}{2}$
$3x+2$	$-$	$+$	$+$
$2x-1$	$-$	$-$	$+$
	R1	R2	R3

Modulus makes everything positive so multiply every that gives a negative with (-1) negative-one to get postive.

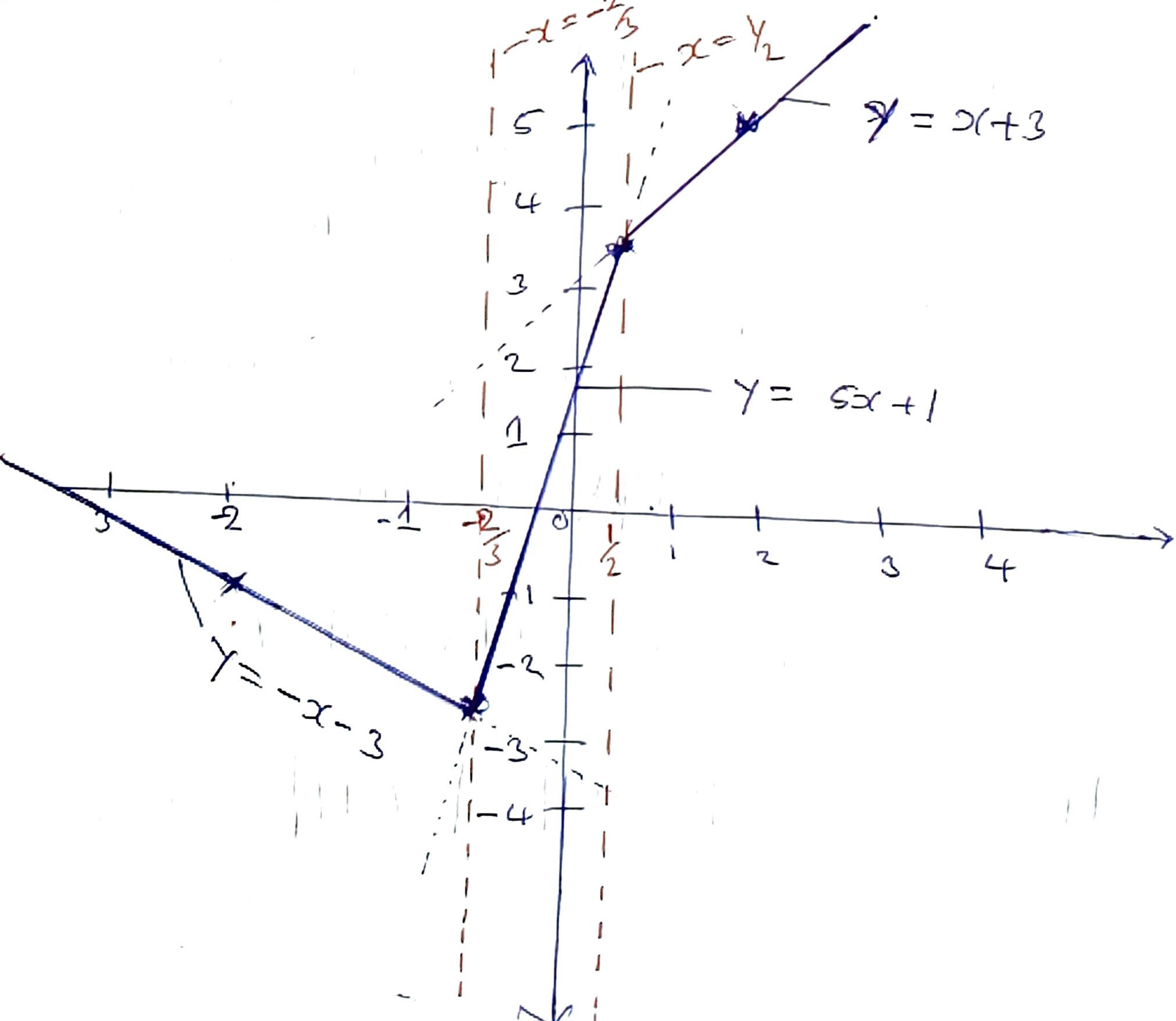
Now that we know the table value and critical points. we are Ready to sketch

$x < -\frac{2}{3}$	$-\frac{2}{3}$ critical point	$-\frac{2}{3} < x < \frac{1}{2}$	$\frac{1}{2}$ critical point	$x > \frac{1}{2}$

x	-2	$-\frac{3}{2}$
y	-1	-2.3

x	$-\frac{2}{3}$	$\frac{1}{2}$
y	-2.3	3.5

x	$\frac{1}{2}$	2
y	3.5	5




SUCCES
Step
Never give up in this life
69K
206
6K

Q] Redefine the modulus Function $F(x) = |2x-1| - |x+1|$ By Removing the modulus and Hance Sketch graph of the Function.

Pointers

Step 1. Set Functions to $g(x)$ and $h(x)$

$g(x) = 2x-1$ and $h(x) = x+1$

Step 2. Find Critical points by equating each Function to zero

Step 3. Draw Sign graph but don't Include Overall Raw.

Step 4. Focus on negative Regions of the Sign graph

Multiply them with a Negative one to make them positive because as you know modulus makes something Negative to positive.

Steps. Find linear equations on each Region and Hence Sketch.

Solutions

$F(x) = |2x-1| - |x+1|$

Critical points

Set $F(x) = g(x) - h(x)$

$g(x) = 2x-1$

$2x-1=0$

$\frac{2x}{2} = \frac{1}{2}$

$x = \frac{1}{2}$

$h(x) = x+1$

$x+1=0$

$x = -1$

Therefore Criticals are

$x = \frac{1}{2}$ and $x = -1$

At this stage Draw Sign graph

	-2	-1 0 $\frac{1}{2}$	2
$(2x-1)$	$-$	$-$	$+$
$(x+1)$	$-$	$+$	$+$
Overall	No	No	No
	Region 1	Region 2	Region 3

* Focus on the Functions that gives you a Negative on the Sign graph
* When making tuma linear equation Multiply then with a Negative (-1) to get positive
* Write the Function F(x) in each Region to see it's behavior and make the linear equation.

Region 1 | Region 2 | Region 3

-2 $\quad -1 \quad \frac{1}{2} \quad +2$

$|2x-1|-|x+1|$ $\qquad |2x-1|-|x+1|$ $\qquad |2x-1|-|x+1|$

Recall Sign graph

	-2	$-1 \quad 0 \quad \frac{1}{2}$	2
$(2x-1)$	$-$	$-$	$+$
$(x+1)$	$-$	$+$	$+$
	R1	R2	R3

So when making Linear Equation multiply all the functions ($g(x)$ and $h(x)$) that gives you a ~~ga~~ negative from the sign graph.

From Region 1

$$y = |2x-1| - |x+1|$$

both function gives us negative so multiply with a ~~good~~ (-1)

$$y = -1(2x-1) - [-(x+1)]$$

$$y = -2x+1 - [-x-1]$$

$$y = -2x+1+x+1$$

Collect the like terms

$y = -2x + x + 1 + 1$

$y = -x + 2 \longrightarrow$ First linear equation

From Region 2

$y = |2x - 1| - |x + 1|$

$y = g(x) - h(x)$

$2x - 1$ gives us a negative

$\therefore -(2x - 1)$

$x + 1$ is positive no need to multiply with a negative

$y = -(2x - 1) - (x + 1)$

$y = -2x + 1 - x - 1$

$y = -2x - x + 1 - 1$

$y = -3x$ —— second Linear Equation

From Region 3: both functions are positive

$\therefore F(x) = |2x - 1| - |x + 1|$

$y = 2x - 1 - x - 1$

$y = 2x - x - 1 - 1$

$y = x - 2$ —— Third Equation

We know have 3 equations

From Region 1 : ~~[illegible]~~

$y = -x + 2$ —— (i)

From Region 2:

$y = -3x$ —— (ii)

From Region 3

$y = x - 2$ —— (iii)

You know you are doing the right thing if equation 1 and equation (iii) differs with signs

$$\left.\begin{cases} y = -x + 2 & \text{—— (i)} \\ y = x - 2 & \text{—— (iii)} \end{cases}\right\} \text{Notice signs alternating.}$$

At this stage find table value for each equation and always use critical points as x-values

Region 1 | Region 2 | Region 3

C.P -1 | C.P $\frac{1}{2}$

$x = -2$

$x < -1$ | $-1 < x < \frac{1}{2}$ | $x > 2$ | $+2$

$Y = -x + 2$

When $x = -2$

$Y = -2(-2) + 2$

$Y = 4$

When $x = -1$

$Y = -(-1) + 2$

$Y = 1 + 2$

$Y = 3$

x	-2	-1
Y	4	3

$Y = -3x$

x	-1	0.5
Y	3	-1.5

when $x = -1$

$Y = 3(-1)$

$Y = 3$

When $x = \frac{1}{2}$

$Y = -3x$

$Y = -3(\frac{1}{2})$

$Y = -1.5$

$Y = -1.5$

$Y = x - 2$

x	$+\frac{1}{2}$	2
Y	-1.5	0

$Y = x - 2$

$Y = \frac{1}{2} - 2$

$Y = \frac{1-4}{2}$

$Y = \frac{-3}{2}$

$Y = 1.5$

When $x = 2$

$Y = x - 2$

$Y = 2 - 2$

$Y = 0$

Always plug in Critical points as x-values

$\therefore$ We get

-2 | -1 | $\frac{1}{2}$ | 2

x	-2	-1
Y	4	3

x	-1	0.5
Y	3	-1.5

x	0.5	2
Y	-1.5	0

Now Sketch the graph

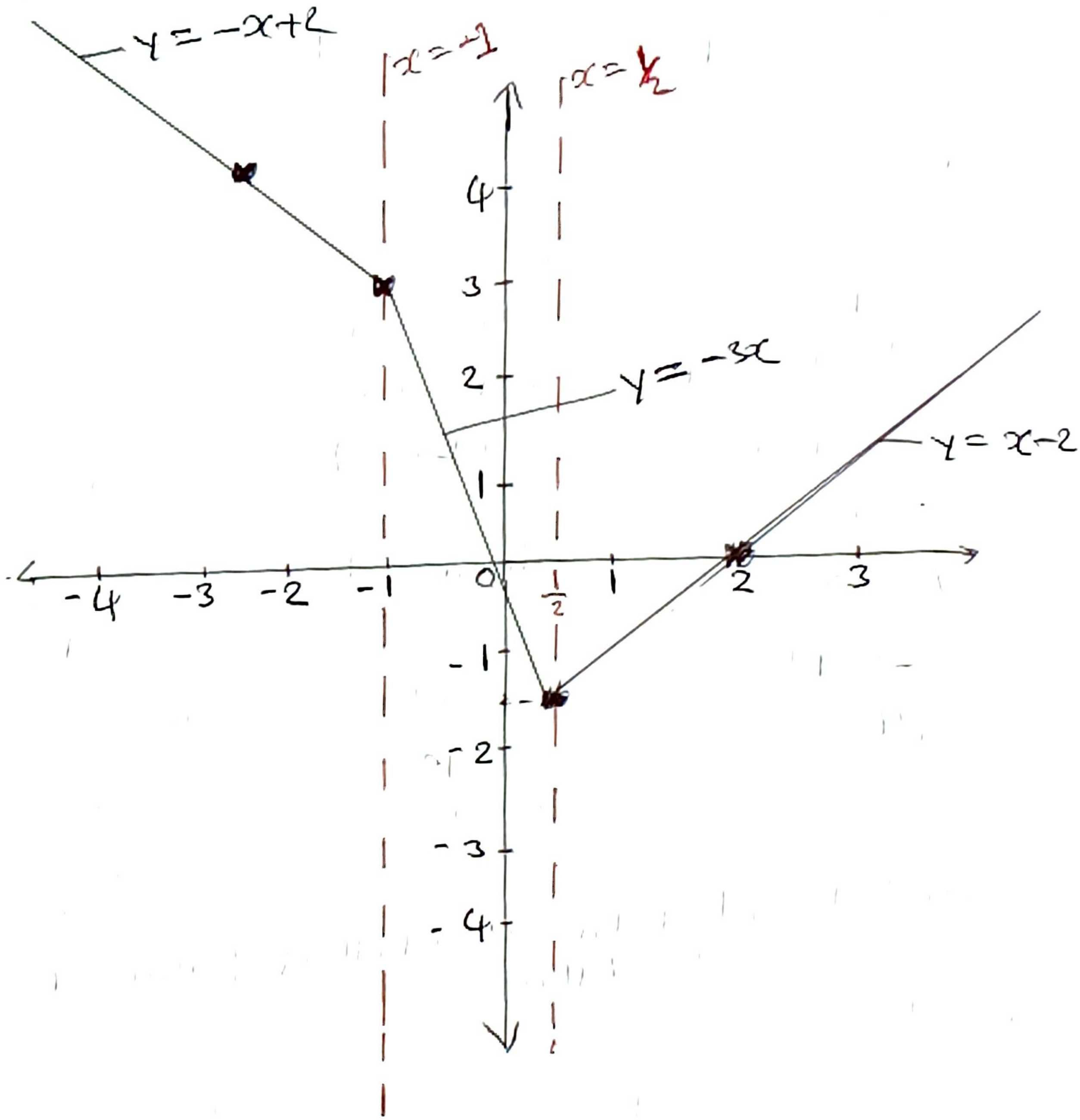
R1

x	-2	-1
y	4	3

R2

x	-1	0.5
y	3	-1.5

x	0.5	2
y	-1.5	0



This is Now End of Function
If you did'nit get it try to
Rest a bit and study again
0268790499 EMF
Docters tuitions