THE COPPERBELT UNIVERSITY

PHYSICS DEPARTMENT

TEST 1 – AUGUST 2020

PH 110 – *INTRODUCTORY PHYSICS*

TIME: **2** HOURS

MAX MARKS: **100**

ATTEMPT **ALL** QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

CLEARLY INDICATED YOUR STUDENT IDENTIFICATION NUMBER AND LECTURE GROUP ON THE FRONT COVER OF THE ANSWER BOOKLET

You may use the following information:

Acceleration due to gravity, $g = 9.8\ m/s^2$

Q1. (a) A car travels 1 km between two stops. It starts from rest and accelerates at 2.5 m/s^2 until it attains a velocity of 12.5 m/s. The car continues at this velocity for some time and decelerates at 3 m/s^2 until it stops. Calculate the total time for the journey. **[10 marks]**

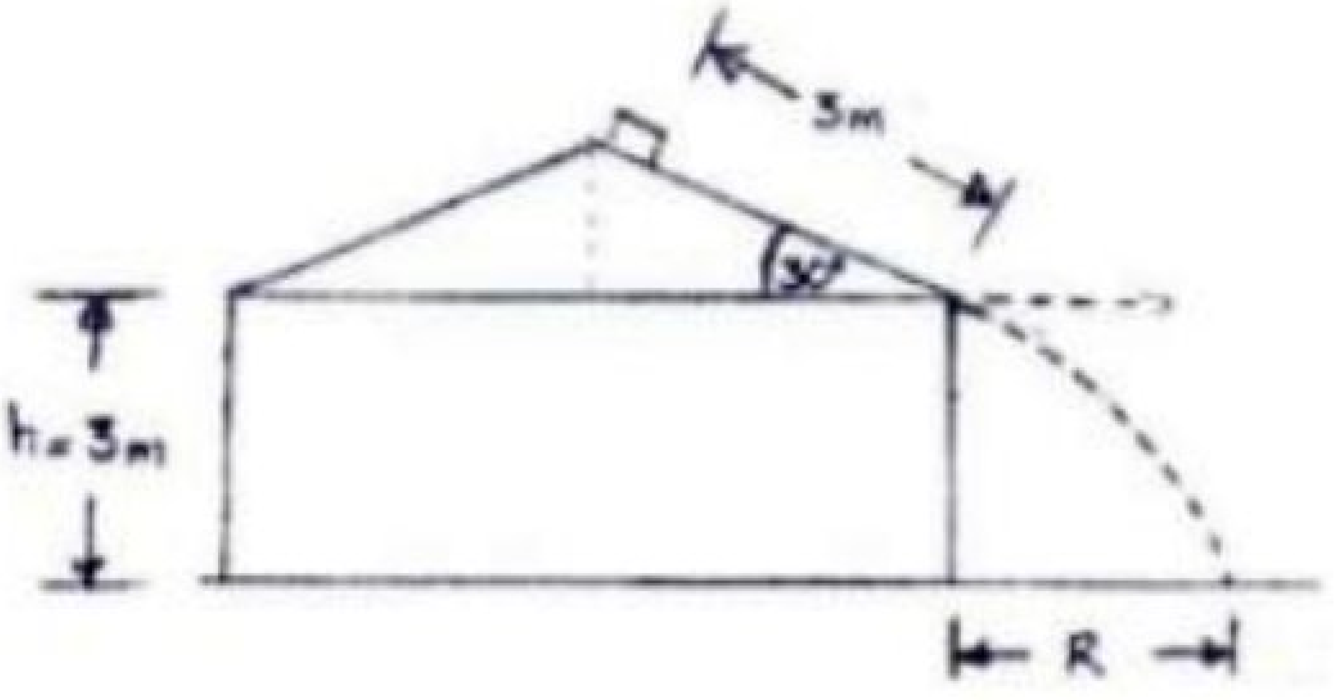
(b) A crate slides from rest and accelerates uniformly at 4.9 m/s^2 along a frictionless roof 3 m long which is inclined at an angle of 30° to the horizontal as indicated in the Figure below. Determine:

(i) the velocity of the crate just after losing contact with the roof,

(ii) the velocity (magnitude and direction) of the crate just before it hits the ground,

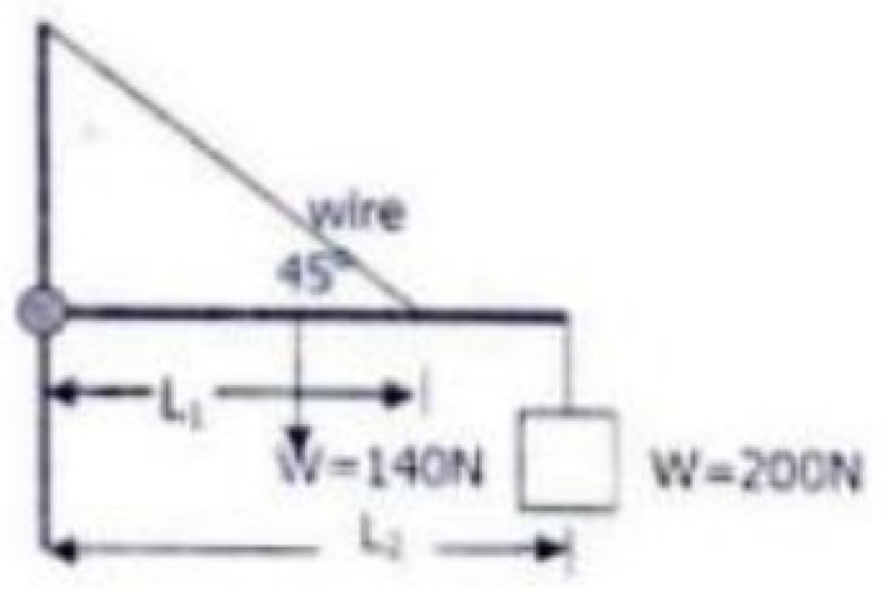
(iii) the time the crate takes to hit the ground after losing contact with the roof, and

(iv) the horizontal distance between the point directly below the roof and the landing Point (i.e. the range). **[15 marks]**



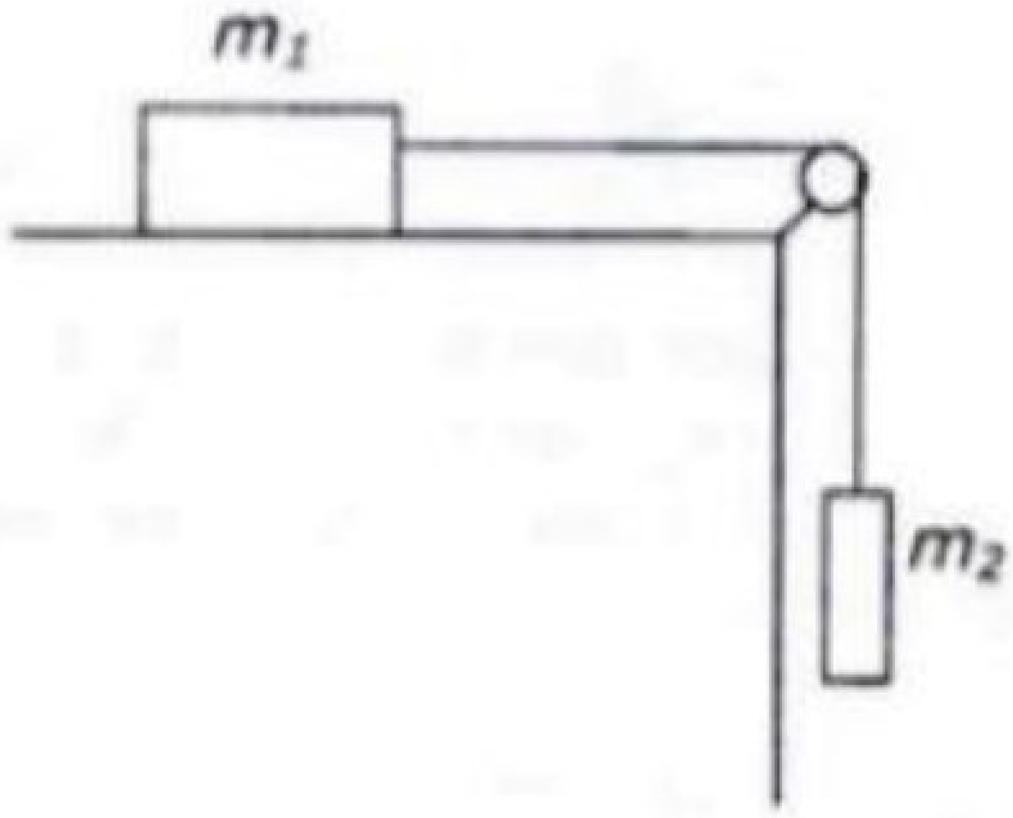
Q2. (a) A block of weight $W = 200$ N is supported by a uniform beam of weight 140 N as shown in the Figure below. If $L_1 = 1.1$ m and $L_2 = 1.4$ m, find the tension in the wire and the vertical and horizontal components of the force exerted by the hinge on the beam.

[10 marks]

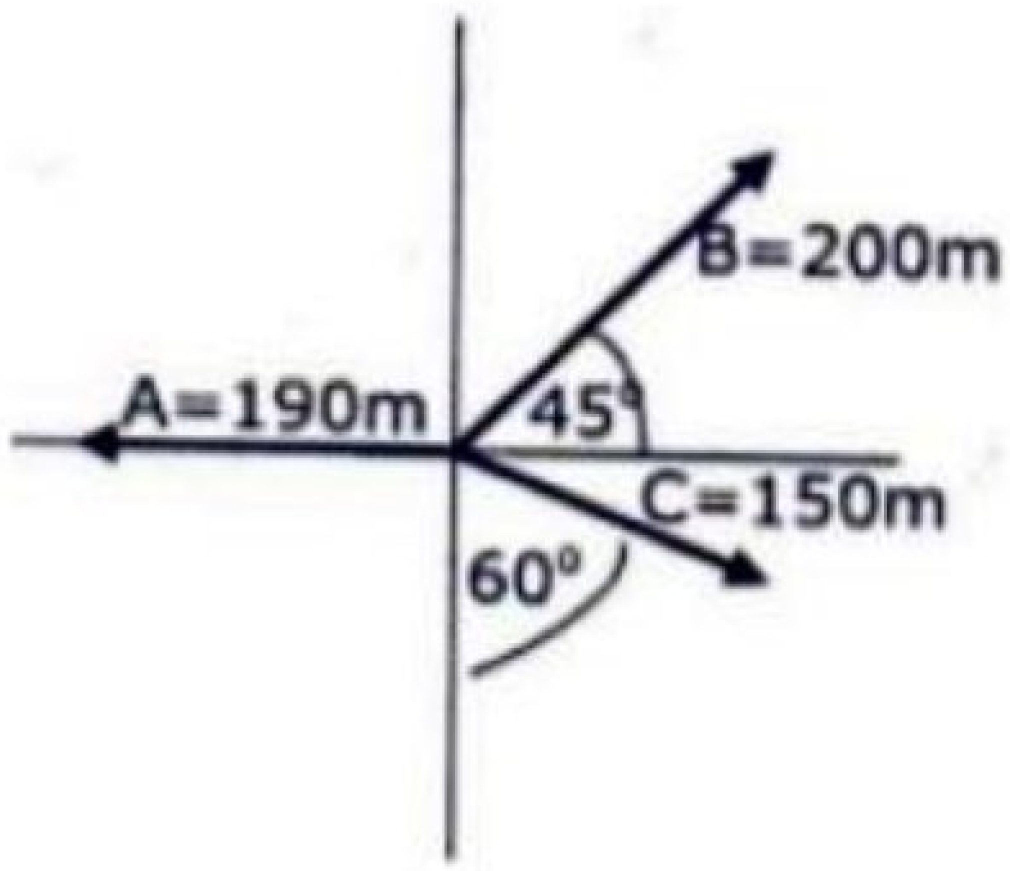


(b) (i) Give two conditions required for an object to be static equilibrium. **[4 marks]**

(ii) Two objects with masses $m_1 = 10$ kg and $m_2 = 5$ kg are connected by a light string that passes over a frictionless pulley as shown in the Figure below. If, when the system starts from rest, m_2 falls 1 m in 1.2 seconds, determine the coefficient of kinetic friction between m_1 and the table. **[11 marks]**



Q3. (a) The magnitude and directions of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are as shown in the Figure below. Find the magnitude and direction of a fourth vector $\vec{D}$ which when added to these three vectors will give a resultant of zero. **[12 marks]**



(b) Two people pull as hard as they can on ropes attached to a 200 kg object. If they pull in the same direction the object accelerates at 1.52 m/s^2 to the right. If they pull in opposite directions the object accelerates at 0.518 m/s^2 to the left. Ignoring any other forces, what is the force exerted by each person on the object? **[9 marks]**

(c) If $\vec{A}$ and $\vec{B}$ are nonzero vectors, is it possible for $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$ both to be zero? Explain.

[4 marks]

Q4. (a) An acre-foot is the volume of water that would cover 1 acre of flat land to a depth of 1 foot. How many gallons are in 1 acre-foot? **[5 marks]**

(b) You are using water to dilute small amounts of chemicals in the laboratory, drop by drop. How many drops of water are in a 1.0-L bottle? **[9 marks]**

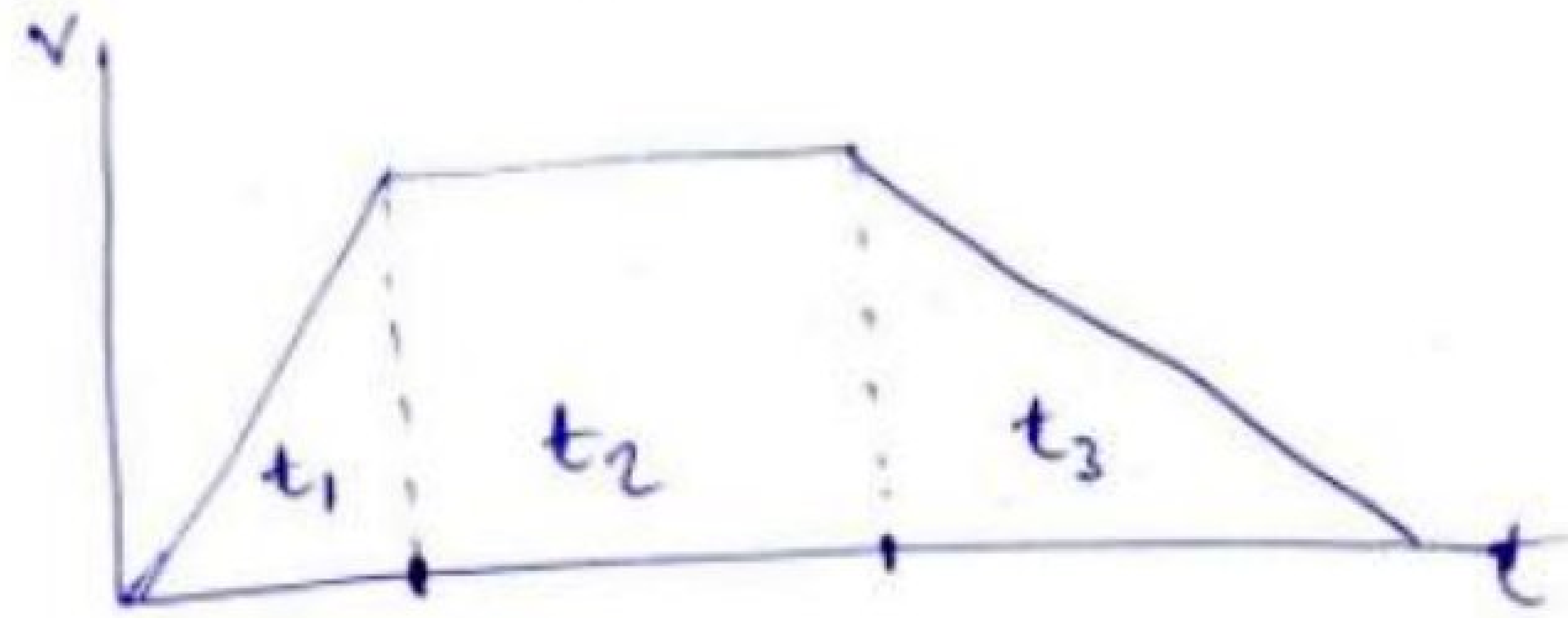
(c) (i) State the principle of homogeneity. **[2 marks]**

(ii) The wavelength λ associated with a moving particle depends on its mass m, velocity v and Planck's constant h which is measured in kgm^2s^{-1}. Show dimensionally, that

$$\lambda \propto \frac{h}{mv}$$ **[9 marks]**

QUESTION 1.

(a)



$t_1 \rightarrow$ time to reach $12.5\ m/s$.

Assuming motion to be in x direction

$$V_f = V_0 + at_1$$
$$= 0 + at_1$$
$$\Rightarrow t_1 = \frac{V_f}{a} = \frac{12.5\ m/s}{2.5\ m/s^2} = 5s$$

3

$x_1 \rightarrow$ distance travelled in t_1.

$$x_1 = V_0 t_1 + \tfrac{1}{2} a t_1^2 = 0 + \tfrac{1}{2} \times 2.5 \times 5 \times 5$$
$$= 31.25\ m$$

$t_3 \rightarrow$ time to decelerate from $12.5\ m/s$ to 0 (stop).

$$V_f = V_0 + at_3$$
$$0 = 12.5\ m/s - 3t_3$$
$$\Rightarrow t_3 = \frac{12.5\ m/s}{3\ m/s^2} = 4.2\ s.$$

3

$x_3 \rightarrow$ Distance traveled in t_3

$$x_3 = V_0 t_3 + \tfrac{1}{2} a t_3^2$$

$x_3 = 12.5 \times 4.2 - 0.5 \times 3 \times (4.2)^2$

$= 26.04$ m//

$x_2 \rightarrow$ distance traveled in t_2

$x_2 = 1000 - (31.25 + 26.04)$

$= 942.71$

since speed was constant during this interval,

3

$x_2 = \bar{v}\, t_2$

$\Rightarrow t_2 = \dfrac{x_2}{\bar{v}} = \dfrac{942.71\,m}{12.5\,m/s}$

$= 75.42$ s

$\therefore$ total time for the journey is

$t = t_1 + t_2 + t_3$

$= 5 + 75.42 + 4.2$

$= 84.62$ s//

1

Q1. contd

(b) (i) From

$$x = V_0 t + \tfrac{1}{2}at^2$$
$$= 0 + \tfrac{1}{2}at^2$$

$$\Rightarrow t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2\times 3}{4.9}} = 1.11\,s$$

Substitute in

4

$$\cancel{V_f = V_0 + a}$$

$$V_f = V_0 + at = 0 + 4.9\,m/s^2 \times 1.11\,s$$
$$= 5.42\,m/s$$

(ii) Take downward +ve

$$V_{0x} = V_0 \cos\theta = 5.42 \times \cos 30^\circ$$
$$= 4.7\,m/s$$

2

$$V_{0y} = V_0 \sin\theta = 5.42 \times \sin 30^\circ$$
$$= 2.7\,m/s$$

Time of flight

$$y = V_{0y}t + \tfrac{1}{2}gt^2$$
$$3 = 2.7t + 4.9t^2$$
$$4.9t^2 + 2.7t - 3 = 0$$

4

$$t = \frac{-2.7 \pm \sqrt{(2.7)^2 - 4(4.9)(-3)}}{9.8}$$
$$= \frac{-2.7 \pm 8.13}{9.8} = 0.6\,s \text{ or } -1.11\,s$$

$\therefore \; t_f = 0.6\,s.$

From

$$V_y = V_{oy} + g t_f$$
$$= 2.7 + 9.8 \times 1.11$$
$$= 13.53 \; m/s$$

Hence

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(4.7)^2 + (13.53)^2}$$

2

$$= 14.32 \; m/s$$

$$\theta = \tan^{-1}\left(13.53/4.7\right) = 71^\circ \text{ below +ve } x\text{-axis}$$

(iii) from (ii)

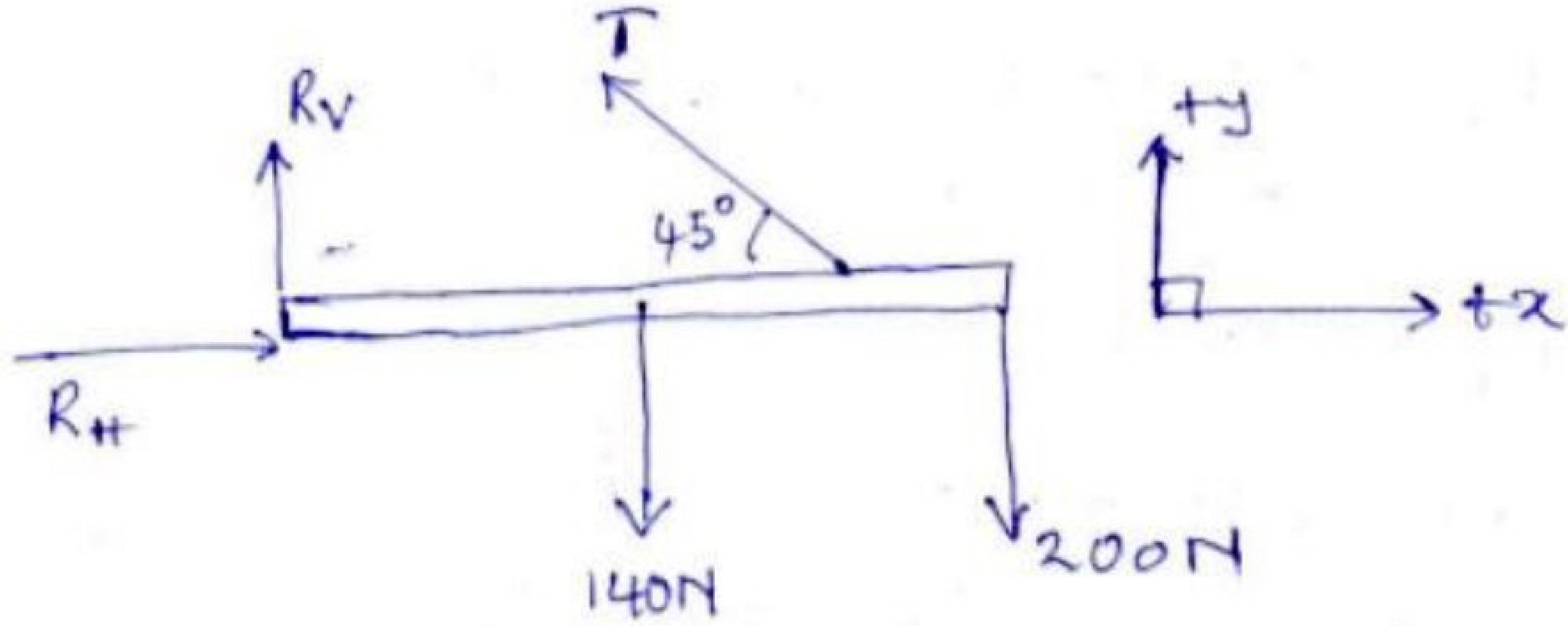
$$t_f = 0.6\,s.$$ 1

(iv) $R = V_{ox} t_f = 4.7\,m/s \times 0.6\,s$

$$= 2.82\,m$$ 2

QUESTION 2

(a) FBD for beam



$R_v \rightarrow$ vertical rxn of the hinge

$R_H \rightarrow$ horizontal " " " "

From 2nd condition for equilibrium

~~$\sum F_x$~~ $\sum \tau = 0$

Take pivot at the hinge.

$$T\sin 45^\circ L_1 - \left(140\frac{L_2}{2} + 200L_2\right) = 0$$

$$\Rightarrow T = \frac{\left(\frac{140}{2} + 200\right)L_2}{L_1 \sin 45^\circ} = \frac{270 \times 1.4}{1.1 \sin 45^\circ}$$

$$= 486\,N$$

From 1st condition for equilibrium

$$\sum F_x = 0$$

$$R_H - T\cos 45^\circ = 0$$

$$R_H = T\cos 45 = 486 \times \cos 45^\circ = 343.64\,N$$

Q2 (a) contd

$$\sum F_y = 0$$

$$R_v + T\sin 45^\circ - 140 - 200 = 0$$

$$\therefore R_v = 340 - T\sin 45^\circ = 340 - 486\sin 45^\circ$$
$$= -3.65\text{ N}$$

$$\Rightarrow R_v = 3.65\text{ N downwards.}$$

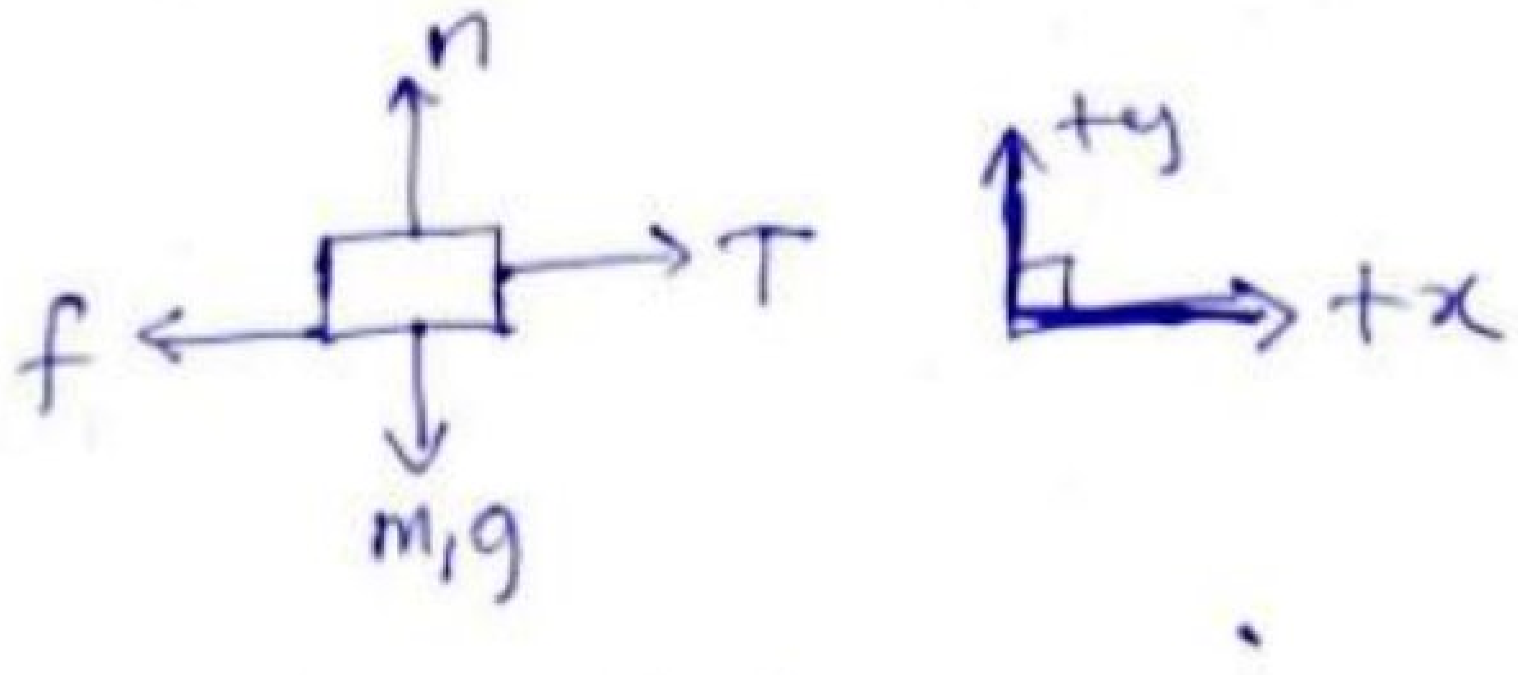
(b) (i) The net force acting on the object must be equal zero.

$$\sum \vec{F} = 0$$

The net torque acting on the object must equal zero.

$$\sum \vec{\tau} = 0$$

(ii) FBD for m_1



Newton's 2nd law gives

$$\sum F_x = m_1 a$$

$$T - f = m_1 a \quad \cdots (1)$$

$$\sum F_y = 0$$

$$n - m_1 g = 0$$

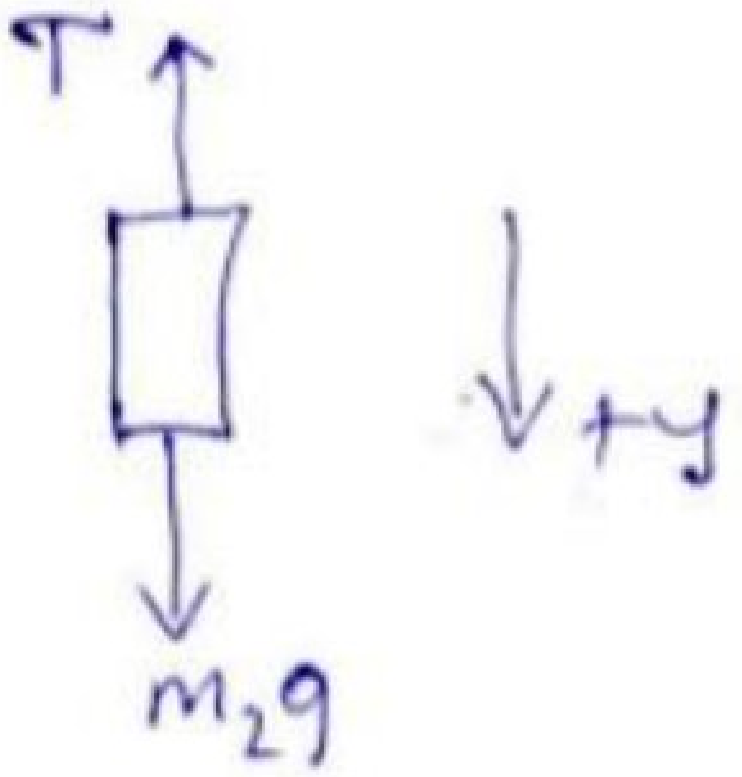
$$n = m_1 g \quad \cdots (2)$$

Now

$$f = \mu_k N = \mu_k m_1 g$$

$$\therefore \quad T - \mu_k m_1 g = m_1 a \quad \cdots \quad (3)$$

FBD for m_2



Newton's 2nd law gives

$$m_2 g - T = m_2 a \quad \cdots \quad (4)$$

Adding eqns (3) & (4) gives

$$m_2 g - \mu_k m_1 g = (m_1 + m_2) a$$

$$\Rightarrow \mu_k = \frac{m_2 g - (m_1 + m_2) a}{m_1 g} \quad \cdots \quad (5)$$

From Kinematics

$$y = v_0 t + \frac{1}{2} a t^2 = 0 + \frac{a}{2} t^2$$

$$\Rightarrow a = \frac{2y}{t^2} = \frac{2 \times 1}{1.2 \times 1.2} = 1.39\, m/s^2$$

Substituting in Eqn (5) gives

$$\mu_k = \frac{50 \times 9.8 - (15 \times 1.39)}{98} = \frac{49 - 20.85}{98} = 0.29$$

QUESTION 3

(a) Let $\vec{R}$ be the resultant of the three vectors.

$$R_x = 190\cos 180^\circ + 200\cos 45^\circ + 150\cos 330^\circ$$

$$= 81.33\,m$$

$$R_y = 190\sin 180^\circ + 200\sin 45^\circ + 150\sin 330^\circ$$

$$= 66.42\,m$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(81.33)^2 + (66.42)^2}$$

$$= 105.01\,m$$

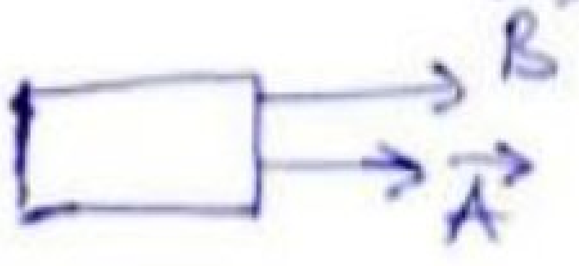
$$\theta = \tan^{-1}\left(\frac{66.42}{81.33}\right) = 39.24^\circ$$

$$\therefore \vec{D} = 81.33\hat{i} + 66.42\hat{j}$$

or

$$D = 105.01\,m,\ 219.24^\circ$$

(b) Assume the force to be applied as indicated below.



$$\Rightarrow \vec{B} > \vec{A}$$

Now

$B - A = ma_L$ (Object accelerates left)

$B + A = ma_R$ (Object accelerates right) substituting numerical values yields

$$B - A = 200 \times 0.518 = 103.6 \quad \cdots (1)$$

$$B + A = 200 \times 1.52 = 304 \quad \cdots (2)$$

solving Eqns (1) & (2) simultaneously we obtain

$$2B = 407.6$$

$$B = \frac{407.6}{2} = 203.8\ N$$

$$\therefore \quad A = 304 - 203.8 = 100.2\ N$$

(c) From

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

and

$$\vec{A} \times \vec{B} = AB\sin\theta\ \hat{n}$$

$\hat{n} \rightarrow$ unit vector $\perp$ to both $\vec{A}$ and $\vec{B}$

If $\vec{A} \cdot \vec{B} = 0$, then $\theta = 90^\circ$ and $\sin 90^\circ = 1$, so $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$ can't both be equal to zero.

QUESTION 4

(a) $1\,ft = 0.3048\,m$

$1\ acre = 4046.86\,m^2$

$1\,m^3 = 264.172\ gal$

$1\ acre\text{-}foot = 4046.86\,m^2 \times 0.3048\,m$

$= 1,233.482928\,m^3$

$= 1233.482928\,m^3 \times 264.172\,\frac{gal}{m^3}$

$= 325,851.65\ gal$ //

(b) $1\,L = 10^{-3}\,m^3$.

Assume a typical drop of water has a radius of $1\,mm = 10^{-3}\,m$.

Volume of drop

$$V_d = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{21}{7} \times (10^{-3}\,m)^3$$

$$= 4 \times 10^{-9}\,m^3$$

NB: since we are only approximating we can take $\pi = \frac{21}{7}$ instead of $\frac{22}{7}$!

$\therefore$ Number of drops is

$$N = \frac{V_w}{V_d} = \frac{10^{-3}\,m^3}{4 \times 10^{-9}\,m^3} = 0.25 \times 10^{6}\ drops$$

$$= 2.5 \times 10^{5}\ drops$$ //

(c) (i) The principle of homogeneity states that the powers of fundamental units on one side of an equation must be equal to their respective powers on the other side of the equation.

(ii)

$$\lambda \propto m^a v^b h^c$$

$$\lambda = k m^a v^b h^c \quad \dots \quad ①$$

where k is a dimensionless constant of proportionality.

In dimensional form Eqn ① becomes

$$[L] = [M^a][LT^{-1}]^b [ML^2T^{-1}]^c$$

or

$$[M^0 L^1 T^0] = [M^a L^b T^{-b} M^c L^{2c} T^{-c}]$$

$$= [M^{a+c} L^{b+2c} T^{-b-c}]$$

From the principle of homogeneity

$a + c = 0 \quad \dots \quad (i)$

$b + 2c = 1 \quad \dots \quad (ii)$

$-b - c = 0 \quad \dots \quad (iii)$

$\Rightarrow c = -b$

$b + 2(-b) = 1$

$-b = 1$

$\therefore \quad b = -1$

$c = -b = -(-1) = 1$

$a = -c = -1$

Hence

$\lambda = k m^{-1} v^{-1} h^{1}$

$= k \frac{h}{mV}$

$\therefore \quad \lambda \propto \frac{h}{mV}$