PH110 TEST (I) Solutions Prepared By

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Question (1)

(a) The *mass* of the *parasitic wasp* can be as small as $5 \times 10^{-6}\,\mathrm{kg}$. What is this mass in:

i. grams (g). **[2]**

ii. milligrams (mg). **[2]**

iii. micrograms $(\mu\mathrm{g})$. **[2]**

ANSWER:

i.
- We know that: $1\,\mathrm{kg} = 10^3\,\mathrm{g}$. [1 mark]
- Hence: $5 \times 10^{-6}\,\mathrm{kg} = \underline{5 \times 10^{-3}\,\mathrm{g}}$. [1 mark]

ii.
- We know that: $1\,\mathrm{kg} = 10^6\,\mathrm{mg}$. [1 mark]
- Hence: $5 \times 10^{-6}\,\mathrm{kg} = \underline{5\,\mathrm{mg}}$. [1 mark]

iii.
- We know that: $1\,\mathrm{kg} = 10^9\,\mu\mathrm{g}$. [1 mark]
- Hence: $5 \times 10^{-6}\,\mathrm{kg} = \underline{5 \times 10^3\,\mu\mathrm{g}}$. [1 mark]

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(b) State the number of significant figures in the following quantities:

i. $0.006\,\mathrm{m}^2$. **[1/2]**
ii. $0.2309\,\mathrm{m}^3$. **[1/2]**
iii. $0.006032\,\mathrm{kg}$. **[1/2]**
iv. $2.75 \times 10^3\,\mathrm{kg}$. **[1/2]**

ANSWER:

i. $0.006\,\mathrm{m}^2 = 6 \times 10^{-3}\,\mathrm{m}^2$, hence 1 s.f. [1/2 mark]
ii. $0.2309\,\mathrm{m}^3 = 2.309 \times 10^{-1}\,\mathrm{m}^3$, hence 4 s.f. [1/2 mark]
iii. $0.006032\,\mathrm{kg} = 6.032 \times 10^{-3}\,\mathrm{kg}$, hence 4 s.f. [1/2 mark]
iv. $2.75 \times 10^3\,\mathrm{kg} = 2.75 \times 10^3\,\mathrm{kg}$, hence 3 s.f. [1/2 mark]

(c) If velocity (V), time (T) and force (F) were chosen as basic quantities, find the dimensions of mass. **[3]**

ANSWER:

- We know that: $F = ma$, [1 mark]
- and that: $a = v/t$. [1 mark]
- From the above, it follows that: $m = Ft/v$, hence:

$$[\mathrm{m}] = \frac{[\mathrm{F}][\mathrm{t}]}{[\mathrm{V}]} = \mathrm{FTV}^{-1}.$$

[1 mark]

(d) The time dependence of a physical quantity, P, is found to be of the form: $P = P_0 e^{\alpha t^2}$, where t is the time and α is some constant. What are the dimensions of α?

ANSWER:

- The argument (αt^2) of the function $e^{\alpha t^2}$ is a dimensionless quantity, *i.e.*: $[\alpha t^2] = 1$. [1 mark]
- From this, it follows that: $[\alpha] = [t^{-2}] = T^{-2}$. [1 mark]

(e) i. State two applications of dimensional analysis. **[2]**

ANSWER:

A mark for each of the following:

- To check the dimensional correctness of an equation. [1 mark]
- To derive equations from a sufficient base of knowledge of known variables under a set given of assumptions. [1 mark]
- To determine the units of constants in equations. [1 mark]

ii. The period (T) of oscillation of a simple pendulum is assumed to depend on its length (l), mass of the bob (m) and the acceleration due to gravity (g). Use the *dimensional analysis approach* to derive an expression for the period. **[8]**

ANSWER:

Dimensional analysis requires that:

$$T = km^x l^y g^z,$$

where: k is a dimensionless quantity and: x, y, z, are unknown powers that make this equation dimensionally consistent. We know from dimensional analysis that:

$$[T] = [m]^x [l]^y [g]^z = M^x L^y (LT^{-2})^z = M^x L^{y+z} T^{-2z}.$$

Equating the powers, we will have:

$$\begin{array}{rcll} x & = & 0 & \ldots \quad (a) \quad [1\, mark] \\ y+z & = & 0 & \ldots \quad (b) \quad [1\, mark] \\ -2z & = & 1 & \ldots \quad (c) \quad [1\, mark] \end{array}.$$

Solving the above set of simultaneous equations, one obtains:

$$\begin{array}{rcll} x & = & 0 & \ldots \quad (a) \quad [1\, mark] \\ y & = & \dfrac{1}{2} & \ldots \quad (b) \quad [1\, mark] \\ z & = & -\dfrac{1}{2} & \ldots \quad (c) \quad [1\, mark] \end{array}.$$

hence:

$$T = k\left(\frac{l}{g}\right)^{1/2} = k\sqrt{\frac{l}{g}}.$$

[3 marks]

iii. Write down two limitations of dimensional analysis. **[2]**

ANSWER:

A mark for each of the following:

- It does not tell us the value of the dimensionless constant involved in the derived equation or expression. [1 mark]
- It does not always tell us the exact form of the relation. [1 mark]
- It does not tell whether a given physical quantity is a scalar or vector. [1 mark]

Question (2)

(a) A child obviously lost, walks $75\,\mathrm{m}$ at 25° North of East, then $100\,\mathrm{m}$ at 15° South of East, then $90\,\mathrm{m}$ South and finally $50\,\mathrm{m}$ 30° North of West. Choose the *y-axis* pointing North and the *x-axis* pointing East, and find:

(i) The total distance covered by the child. **[2]**

ANSWER:

- $d = 75\,\mathrm{m} + 100\,\mathrm{m} + 90\,\mathrm{m} + 25\,\mathrm{m}$ [1 mark]
- Hence: $\underline{d = 315\,\mathrm{m}}$ [1 mark]

(ii) The magnitude and direction of the resultant displacement of the child. **[10]**

ANSWER:

As shown in Figure (1), let the displacement vectors be $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$. Further, let the resultant displacement of the four vectors be $\boldsymbol{R}$.

- X-Component of $\boldsymbol{R}$:

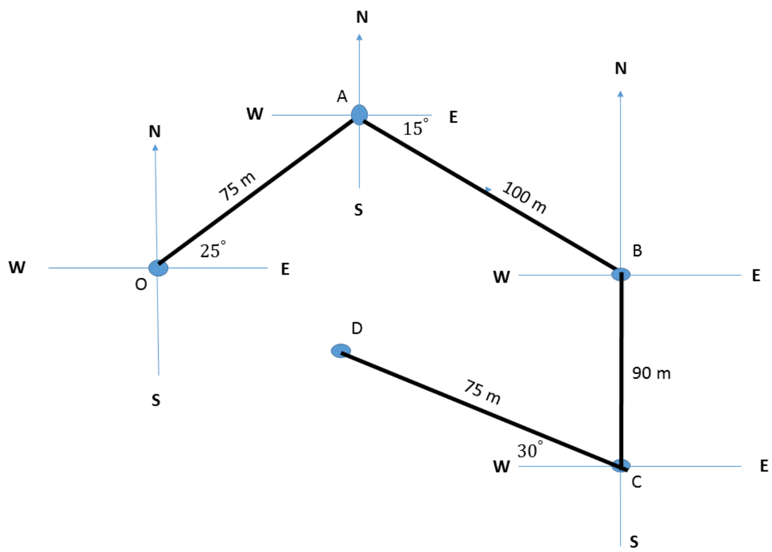

Figure (1): **Vector Diagram of the Wandering Lost Child.**

- The *x-component* of the resultant displacement vector, $\boldsymbol{R}$, is such that: $R_x = A_x + B_x + C_x + D_x$. [1 mark]
- That is to say: $R_x = (75\,\mathrm{m})\cos(75°) + (100\,\mathrm{m})\cos(345°) + (90\,\mathrm{m})\cos(270°) + (50\,\mathrm{m})\cos(150°)$ [1 mark]
- Therefore: $\underline{R_x = 121.3\,\mathrm{m}}$. [1 mark]

• Y-Component of $\boldsymbol{R}$:

- The *y-component* of the resultant displacement vector, $\boldsymbol{R}$, is such that: $R_y = A_y + B_y + C_y + D_y$. [1 mark]
- That is to say: $R_x = (75\,\mathrm{m})\sin(75°) + (100\,\mathrm{m})\sin(345°) + (90\,\mathrm{m})\sin(270°) + (50\,\mathrm{m})\sin(150°)$ [1 mark]
- Therefore: $\underline{R_x = 59.2\,\mathrm{m}}$. [1 mark]

• Magnitude:

- The magnitude of the resultant displacement is: $R = \sqrt{R_x^2 + R_y^2}$. [1 mark]
- That is to say: $R = \sqrt{(121.3\,\mathrm{m})^2 + (59.2\,\mathrm{m})^2}$. [1 mark]
- Therefore: $\underline{R = 135\,\mathrm{m}}$. [1 mark]

- Direction:
  - The direction of the resultant displacement is such that:

$$\tan\theta = \frac{R_y}{R_x}.$$

[1 mark]

  - Therefore:

$$\theta = \tan^{-1}\left(\frac{59.2\,\mathrm{m}}{121.3\,\mathrm{m}}\right) = 26^\circ.$$

Alternatively: -26°, or $334^\circ = 360^\circ - 26^\circ$. [1 mark]

(b) Given two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$, where: $\boldsymbol{A} = \hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + 3\hat{\boldsymbol{k}}$, and: $\boldsymbol{B} = 3\hat{\boldsymbol{\imath}} + 2\hat{\boldsymbol{\jmath}} + 3\hat{\boldsymbol{k}}$. Calculate the following:

(i) $\boldsymbol{A} \times \boldsymbol{B}$. **[3]**

ANSWER:

* We know that:

$$\boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & -\hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ 1 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}.$$

[1 mark]

* Expanding:

$$\boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} \hat{\boldsymbol{\imath}} - \begin{vmatrix} 1 & 3 \\ 3 & 3 \end{vmatrix} \hat{\boldsymbol{\jmath}} + \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} \hat{\boldsymbol{k}}.$$

[1 mark]

* Hence: $\boldsymbol{A} \times \boldsymbol{B} = 0\hat{\boldsymbol{\imath}} + 6\hat{\boldsymbol{\jmath}} - 4\hat{\boldsymbol{k}} = 6\hat{\boldsymbol{\jmath}} - 4\hat{\boldsymbol{k}}$. [1 mark]

(ii) The angle between $\boldsymbol{A}$ and $\boldsymbol{B}$. **[3]**

ANSWER:

There are two ways to the answer:

- FIRST METHOD:

- We know that: $\boldsymbol{A} \times \boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}|\,\hat{\boldsymbol{n}}\cos\theta$. Taking the magnitude on both-sides and re-arranging, we will have that:

$$\theta = \sin^{-1}\left[\frac{|\boldsymbol{A}\times\boldsymbol{B}|}{|\boldsymbol{A}|\,|\boldsymbol{B}|}\right].$$

[1 mark]

- Therefore:

$$\theta = \sin^{-1}\left[\frac{\left|6\hat{\boldsymbol{j}} - 4\hat{\boldsymbol{k}}\right|}{\left|\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}}\right|\left|3\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}}\right|}\right] = \sin^{-1}\left[\frac{\sqrt{6^2+4^2}}{\sqrt{(1^2+2^2+3^2)(3^2+2^2+3^2)}}\right].$$

[1 mark]

- Hence: $\theta = \sin^{-1}(\sqrt{13/77}) = 0.42\,\mathrm{rad} = 24.3^\circ$. [1 mark]

- SECOND METHOD:

  - We know that: $\boldsymbol{A}\cdot\boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}|\cos\theta$, hence:

$$\theta = \cos^{-1}\left[\frac{|\boldsymbol{A}\cdot\boldsymbol{B}|}{|\boldsymbol{A}|\,|\boldsymbol{B}|}\right].$$

[1 mark]

  - Therefore:

$$\theta = \cos^{-1}\left[\frac{\left(\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}}\right)\cdot\left(3\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}}\right)}{\left|\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}}\right|\left|3\hat{\boldsymbol{i}} + 2\hat{\boldsymbol{j}} + 3\hat{\boldsymbol{k}}\right|}\right] = \cos^{-1}\left[\frac{(1)(3)+(2)(2)+(3)(3)}{\sqrt{(1^2+2^2+3^2)(3^2+2^2+3^2)}}\right].$$

[1 mark]

  - Hence: $\theta = \cos^{-1}(8/\sqrt{77}) = 0.42\,\mathrm{rad} = 24.3^\circ$. [1 mark]

(iii) Determine the unit vector perpendicular to $\boldsymbol{A}\times\boldsymbol{B}$. **[2]**

ANSWER:

- We know that: $\hat{\boldsymbol{n}}\,|\boldsymbol{A}\times\boldsymbol{B}| = \boldsymbol{A}\times\boldsymbol{B}$, where $\hat{\boldsymbol{n}}$ is the required unit vector perpendicular to $\boldsymbol{A}\times\boldsymbol{B}$. [1 mark]
- Therefore:

$$\hat{\boldsymbol{n}} = \frac{\boldsymbol{A}\times\boldsymbol{B}}{|\boldsymbol{A}\times\boldsymbol{B}|} = \frac{6\hat{\boldsymbol{j}} - 4\hat{\boldsymbol{k}}}{\sqrt{(6)^2+(-4)^2}} = \frac{6\hat{\boldsymbol{j}} - 4\hat{\boldsymbol{k}}}{\sqrt{52}}.$$

[1 mark]

(c) A force: $\boldsymbol{F} = 2\text{N}\hat{\boldsymbol{\imath}} + 3\text{N}\hat{\boldsymbol{\jmath}} + 4\text{N}\hat{\boldsymbol{k}}$, pushes an object of mass $5\,\text{kg}$ from the origin to a position vector: $\boldsymbol{r} = 3\text{m}\hat{\boldsymbol{\imath}} - 3\text{m}\hat{\boldsymbol{\jmath}} + 5\text{m}\hat{\boldsymbol{k}}$. Using the scalar product of vectors, determine the work done by the force on the object. **[5]**

ANSWER:

- We know that: $\text{Work} = \boldsymbol{F} \cdot \boldsymbol{r}$. [1 mark]
- Therefore: $\text{Work} = \left(2\text{N}\hat{\boldsymbol{\imath}} + 3\text{N}\hat{\boldsymbol{\jmath}} + 4\text{N}\hat{\boldsymbol{k}}\right) \cdot \left(3\text{m}\hat{\boldsymbol{\imath}} - 3\text{m}\hat{\boldsymbol{\jmath}} + 5\text{m}\hat{\boldsymbol{k}}\right)$. [1 mark]
- Hence: $\text{Work} = (2) \times (3)\text{Nm} + (3) \times (-2)\,\text{Nm} + (4) \times (5)\,\text{Nm}$. [1 mark]
- Thus: $\text{Work} = 6\,\text{J} - 6\,\text{J} + 20\,\text{J}$. [1 mark]
- Work done = $20\,\text{J}$. [1 mark]

Question (3)

(a) Briefly discuss how average speed compares with average velocity. **[3]**

ANSWER:

i. A mark for any three of the following:

- Average speed is a scalar quantity while average velocity is a vector quantity both with same units $(\text{m} \cdot \text{s}^{-1})$ and dimensions (LT^{-1}). [1 mark]
- Average speed or velocity depends on time interval which it is defined. [1 mark]
- For a given time interval, average velocity is single value while average speed can have many values depending on the path followed. [1 mark]
- If after motion a particle comes back to its initial position, then the average velocity is zero (as displacement is zero) but the average speed is greater than zero. [1 mark]

(b) A ZAF officer fires a bullet that moves along the *x-axis*. Its speed as a function of time is expressed as: $v(t) = 4 + 8t$, where, $v(t)$, is in $\text{m} \cdot \text{s}^{-1}$. The position of the bullet at: $t = 1\,\text{s}$, is $25\,\text{m}$. Determine:

(i) The acceleration at: $t = 2\,\text{s}$. **[2]**

ANSWER:

- $a(t) = \dfrac{dv(t)}{dt} = \dfrac{d\,(4+8t)}{dt}$. [1 mark]
- Therefore: $a(t) = 8\,\mathrm{m}\cdot\mathrm{s}^{-2}$. [1 mark]

(ii) The position at: $t = 1.5\,\mathrm{s}$. **[3]**

ANSWER:

- We know that: $\dfrac{dx(t)}{dt} = v(t)$. [1 mark]
- Therefore: $[x(t)]_{25\,\mathrm{m}}^{x(t)} = x(t) - 25\,\mathrm{m} = \int_1^t v(t)dt = \int_0^t (4+8t)\,dt = \left[4t+4t^2\right]_1^t = 4t + 4t^2 - 8$, hence: $x(t) = 4t + 4t^2 + 17$. [1 mark]
- Therefore, at: $t = 1.5\,\mathrm{s}$, we have that: $x = 32\,\mathrm{m}$. [1 mark]

(c) A tear-gas canister thrown vertically upward is held by a student after $2.0\,\mathrm{s}$ of reaching its natural maximum height. Determine:

(i) The speed with which the tear-gas canister was thrown. **[2]**

ANSWER:

- We know that: $t_f = \dfrac{2V\sin\theta}{g}$. [1 mark]
- Hence: $V = \dfrac{gt_f}{2\sin 90^\circ} = \dfrac{(9.8\,\mathrm{m}\cdot\mathrm{s}^{-2})\times(2\,\mathrm{s})}{2\sin 90^\circ} = 9.8\,\mathrm{m}\cdot\mathrm{s}^{-1}$. [1 mark]

(ii) The maximum height the tear-gas canister reaches. **[2]**

ANSWER:

We have that: $v = 0\,\mathrm{m}\cdot\mathrm{s}^{-1}$ at: $y_{\mathrm{max}} = h_{\mathrm{max}}$ and that: $u = 9.8\,\mathrm{m}\cdot\mathrm{s}^{-1}$.

- From the given information and from our knowledge that: $v^2 = u^2 - 2gy_{max}$, it thus follows that: $0 = (9.8\,\mathrm{m}\cdot\mathrm{s}^{-1})^2 - 2(9.8\,\mathrm{m}\cdot\mathrm{s}^{-2})y_{\mathrm{max}}$. [1 mark]
- Therefore: $y_{\mathrm{max}} = 4.9\,\mathrm{m}$. [1 mark]

(d) A golfer chooses a *7-iron* club to '*chip*' the ball a short distance onto the green. His shot gives the ball a velocity of $15\,\mathrm{m}\cdot\mathrm{s}^{-1}$ at an angle of 35° to the horizontal. Ignoring air resistance, answer the following questions:

(i) What are the horizontal and vertical components of the ball's velocity just after it is hit? **[4]**

ANSWER:

- Horizontal component:
  - $v_x = v_0 \cos\theta$. [1 mark]
  - Hence: $v_x = (15\,\mathrm{m\cdot s^{-1}})\cos 35^\circ = 12.3\,\mathrm{m\cdot s^{-1}}$. [1 mark]
- Vertical component:
  - $v_y = u_0 \sin\theta$. [1 mark]
  - Hence: $u_y = (15\,\mathrm{m\cdot s^{-1}})\sin 35^\circ = 8.60\,\mathrm{m\cdot s^{-1}}$. [1 mark]

(ii) How long will the ball take to reach its maximum height? **[2]**

ANSWER:

- We know that: $v_y = v_0 \sin\theta - gt$, and that: $v_y = 0$, at maximum height. [1 mark]
- Therefore: $t = \dfrac{v_0 \sin\theta}{g} = \dfrac{8.60\,\mathrm{m\cdot s^{-1}}}{9.8\,\mathrm{m\cdot s^{-2}}} = 0.88\,\mathrm{s}$. [1 mark]

(iii) Measured along the ground, how far will the ball have travelled when it hits the ground — *i.e.*, what is its horizontal range? **[4]**

ANSWER:

There are two ways to arrive at the answer and these ways are as follows:

1. First Method:

   - We know that: $x(t) = v_0 t\cos\theta = (12.3\,\mathrm{m\cdot s^{-1}})t$. [1 mark]
   - To calculate t, we know that: $y = v_0 t\sin\theta - \dfrac{1}{2}gt^2$. At the point where the ball will hit the ground: $y = 0$, hence: $0 = (8.60\,\mathrm{m\cdot s^{-1}})t - (9.8\,\mathrm{m\cdot s^{-2}})t^2$, thus: $t = 1.76\,\mathrm{s}$. Now — substituting this into the above, we will obtain that the ball will hit the ground at the point: $x = 21.6\,\mathrm{m}$ [1 mark]

2. Second Method:

   - From symmetry, taking the time obtain in $3\,(d)(ii)$, and then multiplying this by 2 in-order to obtain the time for the full-length journey to the ground, we will have that: $t = 2\times(0.88\,\mathrm{s}) = 1.76\,\mathrm{s}$. [1 mark]
   - Hence: $x = (29.3\,\mathrm{m\cdot s^{-1}})\times(1.76\,\mathrm{s}) = 21.6\,\mathrm{m}$ [1 mark]

(iv) How would this range change if this shot had been played on the Moon? Explain why it would change and how it is going to change. **[3]**

ANSWER:

If this shot is played on the Moon, the gravitational field strength is going to be affected since the gravitational field strength on the surface of Moon, g_M, is from that on Earth, g_E: — *i.e.*, it is smaller than is on Earth: $g_M < g_E$. [1 mark]

One mark for each of the following:

- *Range* will be longer since: $R = \dfrac{V^2 \sin 2\theta}{g}$. [1 mark]
- *Maximum height* will be longer since: $h_{max} = \dfrac{V^2 \sin^2 \theta}{2g}$. [1 mark]
- *Time of flight* will be longer since: $t_f = \dfrac{2V \sin \theta}{g}$. [1 mark]