The Copperbelt University

School of Mathematics And Natural Sciences

Department of Mathematics

MA 110 : Mathematical Methods I

February 10, 2022

@**Tutorial Worksheet 3** @ **Academic year: 2021/22**

BINARY OPERATIONS

(1). Define an operation * on the set of real numbers by $x * y = x^2 + 2xy + y^2$

(a) Evaluate $(-1 * 2) * 3$.

(b) State whether or not the operation is commutative.

(c) State whether or not the operation is associative.

(2). Consider the binary operation $x * y = (x + y)^2 - 2xy$, where x and y are real number.

(a) Find $-1 * (2 * 5)$ and $(-1 * 2) * 5$.

(b) Is * commutative? Give a reason for your answer.

(c) Is * associative? Give a reason for your answer.

(3). The operation * on the set of real numbers $\mathbb{R}$ is defined by $x * y = x^2 + y^2 - 2$ where $x, y \in \mathbb{R}$

(a) Determine whether * is commutative.

(b) Detrmine whether * is associative.

(c) Evaluate $(3 * 2) * 5$.

(4). Determine whether the following operations are binary on the set of Irrational numbers.

(a) Addition (+). (b) Subtraction (-). (c) Multiplication ($\times$).

(d) Division ($\div$).

(5). Determine whether or not each of the definition of * given below gives a binary operation. In the event that * is not a binary operation, give justification for this.

(a) On $\mathbb{Z}^+$, define by $x * y = x - y$.

(b) On $\mathbb{Z}^+$, define by $x * y = xy$.

(c) On $\mathbb{R}$, define by $x * y = xy^2$.

(d) On $\mathbb{Z}^+$, define by $x * y = \mid x - y \mid$.

(6). For each operation * defined below, determine whether * is commutative or associative

(a) On $\mathbb{Z}$, define by $x * y = x - y$.

(b) On $\mathbb{Q}$, define by $x * y = xy + 1$.

(c) On $\mathbb{Q}$, define by $x * y = \dfrac{xy}{2}$.

(7). Define an operation * on the set of real numbers defined by $x * y = \dfrac{x}{y+1}$.

(a) Is * a binary operation on the set of real numbers.

(b) Is * commutative on set of real numbers.

(c) Is * associative on the set of real numbers.

(8). Is the operation * defined by $x * y = \sqrt{x - y}$ a binary operation on $\mathbb{R}$?

(9). Define an operation * on the set of real numbers defined by $x * y = \sqrt[x]{y}$ where x is the index of the radical and y is the radicand.

(a) Evaluate

(i) $3 * 216$.

(ii) $2 * (3 * 64)$.

(b) Solve $3 * (2x - 3) = 3$.

(10). If $a * b = 2a - b$ where a and b are real numbers. Solve the equation:

(a) $\mid x * 5 \mid = 1$.

(b) $2x * (x * 3) = 5$.

(c) $\mid 1 * \sqrt{x} \mid = 1$.

SKETCHING

(11). (a) Sketch the following curves and indicate clearly the points of intersection with the axes

(i) $y = (x+1)(1-x)(x+3)$. (ii) $y = x(2x-1)(x+3)$. (iii) $y = x^3+5x^2+4x$.

(iv) $y = (x+1)(x-2)(x-4)$. (v) $y = -x^3(x+2)^2(x-2)^2(x-5)$.

(12). Sketch the following functions:

(a) $f(x) = (x+2)^2 - 3$. (b) $f(x) = (x-1)^3 + 2$. (c) $f(x) = -\dfrac{1}{x}$.

(d) $f(x) = 4 - \mid x+2 \mid$. (e) $f(x) = 2 + \sqrt{-x+1}$. (f) $f(x) = 2\sqrt{1-x} - 3$.

(13). Graph the following functions:

(a) $f(x) = x^2 + 1$. (b) $f(x) = -x^2 + 2$. (c) $f(x) = -(x-2)^2 + 3$.

(d) $f(x) = 2x^2 + 12x + 17$.

(14). Let $f(x) = -2x^2 + 4x + 16$,

(a) Find the vertex of the graph of f.

(b) Find the range of f.

(c) Find the $x-$intercepts and $y-$ intercept of the graph of f.

(d) Sketch the graph of f.

(15). Suppose f is a quadratic function whose graph has a vertex at the point $(-3, 2)$ and has a $y-$intercept at the point $(0, -16)$.

(a) Find the equation for the function f.

(b) Find the $x-$ intercepts of the graph of f.

(16). Write the function $f(x) = 1 - 6x - x^2$ in the form $f(x) = a(x+h)^2 + k$ where a, h, k are constants.

(17). Find the equation of a quadratic function whose graph haa a vertical axis of symmetry $x = -2$, the range of f is given by the interval $[4, +\infty)$ and $f(2) = 8$.

(18). Graph the following rational functions:

(a) $f(x) = \dfrac{3}{x+1}$. (b) $f(x) = \dfrac{12}{(x-1)(x+2)}$. (c) $f(x) = \dfrac{x}{x-3}$.

(d) $f(x) = \dfrac{x^3+4}{(x-3)(x+2)}$. (e) $f(x) = \dfrac{x+3}{x^2-x-6}$. (f) $f(x) = \dfrac{9}{x^2-9}$.

(19). Graph the following piecewise functions:

(a) $f(x) = \begin{cases} 2 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$. (b) $f(x) = \begin{cases} 2x+1 & \text{if } x \geq 0 \\ x^2 & \text{if } x < 0 \end{cases}$.

(c) $f(x) = \begin{cases} 2x+3 & \text{if } x \geq 0 \\ x^2 & \text{if } x \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$.

(20). Find the values of the constants in each of the following:

(a) $f(x) = ax + b$, a and b are constants, $f(-2) = 7, f(1) = -1$.

(b) $f(x) = ax^2 + bx + c$, a, b and c are constants, $f(0) = 7, f(1) = 6$ and $f(-1) = 12$.

<u>FUNCTIONS</u>

(21). Determine the domain and the range of the following functions:

(a) $f(x) = x^2 - 2x + 2$. (b) $f(x) = \sqrt[3]{x^2 - 4}$. (c) $f(x) = 2x + \sqrt{x^2 + 4x - 12}$.

(d) $f(x) = \sqrt{x} + \sqrt{x-1}$.

(22). Given the function $f(x) = \begin{cases} x^2 - 6 & \text{if } x < 0 \\ 10 - x & \text{if } x \leq 0 \end{cases}$,

(a) Sketch $f(x)$.

(b) Find the values of a such that $f(a) = 43$.

(c) Find the values of the domain that get mapped to themselves in the range.

(23). Determine whether the functions f is even, odd or neither;

(a) $f(x) = x^2 + x$. (b) $f(x) = \sqrt{2 - x^2}$. (c) $f(x) = x^2$. (d) $f(x) = \dfrac{1}{x}$.

(e) $f(x) = 3x - 1$ (f) $f(x) = x^5 + x^3 + x$. (g) $f(x) = x^4 + x^2 + 1$.

(h) $f(x) = x^3 + 1$. (i) $f(x) = -x^3$.

(24). Find $(f \circ g(x))$ and$(g \circ f)(x)$and their domains for the following:

(a) $f(x) = 2x,\ g(x) = 3x - 1.$ (b) $f(x) = \dfrac{1}{x},\ g(x) = 3x - 1.$

(c) $f(x) = \sqrt{x - 2}\,,\ g(x) = 3x - 1.$ (d) $f(x) = \dfrac{4}{x + 2},\ g(x) = \dfrac{3}{2x}.$

(25). Solve each of the following:

(a) If $f(x) = x^2 - 2$ and $g(x) = x + 4$, find $(f \circ g)(2)$ and $(g \circ f)(-4)$.

(b) If $f(x) = \dfrac{1}{x}$ and $g(x) = 2x + 1$, find $(f \circ g)(1)$ and $(g \circ f)(2)$.

(c) If $f(x) = \sqrt{x + 1}$ and $g(x) = 3x - 1$, find $(f \circ g)(-4)$ and $(g \circ f)(-4)$.

(d) If $f(x) = x + 7$ and $g(x) = |\ x\ |$, find $(f \circ g)(2)$ and $(g \circ f)(2)$.

(26). Show that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.

(a) $f(x) = 2x,\ g(x) = \dfrac{1}{2x}.$ (b) $f(x) = 3x + 4,\ g(x) = \dfrac{x - 4}{3}.$

(c) $f(x) = 4x - 3,\ g(x) = \dfrac{x + 3}{4}.$

(27). If $f(x) = 3x - 4$ and $g(x) = ax + b$, find the conditions on a and b that guarantee that $(f \circ g)(x) = (g \circ f)(x)$.

(28). Let $f(x) = \dfrac{x}{x + 2}$ and $g(x) = 2x - 1$.

(a) Find $(f \circ g)(x)$. (b) Evaluate $(g \circ f)\left(\dfrac{3}{4}\right)$. (c) Verify that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

(29). Verify that the two given functions are inverses of each other:

(a) $f(x) = -\dfrac{1}{2}x + \dfrac{5}{6}$ and $g(x) = -2x + \dfrac{5}{3}$.

(b) $f(x) = \sqrt{2x - 4}$ for $x \geq 0$ and $g(x) = \sqrt[3]{x - 1}$.

(c) $f(x) = x^2 - 4$ for $x \geq 0$ and $g(x) = \sqrt{x + 4}$ for $x \geq -4$.

(30). State if the following functions is one to one or many to one:

(a) $f(x) = 3x + 2$ for the domain $\{x > 0\}$.

(b) $f(x) = x^2 + 5$ for the domain $\{x \geq 2\}$.

(c) $f(x) = \sqrt{x + 2}$ for the domain $\{x \geq -2\}$.

(31). Find $f+g, f-g, f.g$ and $\dfrac{f}{g}$ and determine their domain for the following:

(a) $f(x)=\sqrt{x-1},\ g(x)=\sqrt{x}$. (b) $f(x)=\sqrt{x+2}\,,\ g(x)=\sqrt{3x-1}$.
(c) $f(x)=x^2-2x-24,\ g(x)=\sqrt{x}$. (d) $f(x)=x^2-2x-24,\ g(x)=\sqrt{x}$.

(32). Determine the domain of the following functions:

(a) $f(x)=\dfrac{6}{\sqrt{6x-2}}$. (b) $g(y)=\sqrt{\dfrac{y}{y-8}}$. (c) $h(x)=\sqrt{x-7}+\sqrt{9-x}$.

(d) $f(x)=\sqrt{\dfrac{x+1}{x-1}}$. (e) $f(x)=\sqrt[3]{x}$.

(33). In each of the following determine f is even, odd or neither

(a) $f(x)=x^4$. (b) $f(x)=x+x^3$. (c) $f(x)=x^3-x$.

(d) $f(x)=3x-5$. (e) $f(x)=\dfrac{1}{1+x}$.

(34). If $f(x)=x^2+1$ and $g(x)=\sqrt{x-1}$. Find the domain and inverse of $(f\circ g)(x)$ and $(g\circ f)(x)$.

(35). If $f(x)=3-x$ and $g(x)=\dfrac{3x}{x-3}$, $x\neq 3$. Show that this function is its own inverse. Is $(f\circ g)(x)$ even, odd or neither.

(36). If $f(x)=1-x^2$ and $g(x)=\sqrt{x}$. Find $(g\circ f)(x)$ and $(f\circ g)(x)$ and their domains.

(37). If $f(x)=x^2$ and $g(x)=3x-4$, find $(g\circ f)(x)$ and $(f\circ g)(x)$ and determine its domain.

(38). If $f(x)=\dfrac{2}{1-x}$ and $f(x)=\dfrac{1}{x}$, find $(g\circ f)(x)$ and determine its domain.

(39). If $f(x)=1-x^2$ and $g(x)=\sqrt{x}$, find $(f\circ g)(x)$ and determine its domain.

THE END

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