



The Copperbelt University

School of Mathematics And Natural Sciences

Department of Mathematics

MA 110 : (Mathematical Methods I) : Deferred Test

August 15, 2022

Instructions

- (1). You must write your Name, Student Identification Number (SIN) and Programme of study on your answer sheet. Calculators are not allowed. Time allowed is 1hr:30 minutes
- (2). There are Four (4) questions in this paper, for deferred test 1, attempt questions 1 and 2 and deferred test 2, attempt questions 3 and 4.

QUESTION ONE

- (a). Express $2.07\overline{2}$ as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$
(5 marks)
- (b). Evaluate and Simplify $\frac{2^{n-1} - 8^n}{\frac{1}{2} - 4^n}$.
(5 marks)
- (c). Rationalize the denominator of $\frac{1}{(\sqrt{2} + 1)(\sqrt{3} - 1)}$.
(5 marks)
- (d). Determine the domain of the given function $f(x) = \sqrt{\frac{x+1}{x-1}}$.
(5 marks)
- (e). Solve for x and y given that $\frac{1}{x+iy} + \frac{1}{1+3i} = 1$.
(5 marks)

QUESTION TWO

- (a). Prove the De Morgans law $(A \cap B)' = A' \cup B'$.

(5 marks)

- (b). Determine whether the given function is odd, even or neither

$$f(x) = \frac{x^3 + 2x}{2}.$$

(5 marks)

- (c). Prove that $\sqrt{2} + \sqrt{3}$ is an irrational number using the fact that $\sqrt{6}$ is an irrational number.

(5 marks)

- (d). Find the square root of a complex number $15 + 8i$.

(5 marks)

- (e). Sketch the graph of

$$f(x) = \begin{cases} 2x + 3 & x < 0 \\ x^2 & 0 \leq x < 2 \end{cases}$$

(5 marks)

(Total Marks: 25)

QUESTION THREE

- (a). Change the repeating decimal $5.\bar{7}$ to its reduced form $\frac{a}{b}$, where a and b are integers and $b \neq 0$ using sum to infinity of a geometric series.

(5 marks)

- (b). Express $\frac{3x^2 + 2x - 9}{(x^2 - 1)^2}$ into a partial fraction.

(5 marks)

- (c). Use Mathematical Induction to prove that $3^{2n} - 1$ is divisible by 8.

(5 marks)

- (d). Find the first term and the general expansion of $\frac{1}{(2 - 6x)^5}$ in ascending power of x . State the range of value of x for which this expansion is valid.

(5 marks)

- (e). Graph the function of $f(x) = 2^{(x-3)} + 2$ and obtain its inverse on the same axis.

(5 marks)

(Total Marks: 25)

QUESTION FOUR

(a) Prove that $\log_a(A^C) = C \log_a(A)$.

(5 marks)

(b) Find the center and radius of the circle whose equation is

$$x^2 + y^2 + 8x - 2y + 13 = 0.$$

(5 marks)

(c) Solve the equation $\log_3 x - 4 \log_x 3 + 3 = 0$.

(5 marks)

(d) Write down the constant term in the expansion of $\left(x - \frac{1}{2x^2}\right)^9$.

(5 marks)

(e) Use crammer's method to solve the linear system of equation

$$x + 2z = 9$$

$$2y + z = 8$$

$$4x - 3y = -2$$

(5 marks)

(Total Marks: 25)

THE END OF TEST

$$\begin{array}{r} 5.7 \\ 0.7 \\ \hline 6.4 \end{array} \quad \begin{array}{r} 5.7 \\ 0.007 \\ \hline 5.707 \end{array}$$

$$C = \sqrt{10 \times 10 - 4 \times 4}$$