CHAPTER 4
FORCE AND MOTION (DYNAMICS)

Dynamics is the branch of mechanics that is concerned with motion of bodies in relation to the physical factors that affect them; force, mass, momentum and energy. Dynamics also deals with the study of equilibrium of bodies.

4.1. Newton's laws of motion

The relationship between forces and produced acceleration is an aspect of dynamics. Sir Isaac Newton (1642-1727) studied the concept of motion in detail and formulated them in three laws, named after him.

4.1.1 First law (The law of inertia)

This law states "Every body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise"

The first law can be stated mathematically as

$$\sum F = 0, \text{ then } \begin{cases} v = 0 \\ v = cst \end{cases}$$

Newton's first law is sometimes called the law of inertia. According to this law, a body does not change its state of rest or uniform motion, unless an external force compels it to change that state. Inertia is the property of a body by which it tends to resist change in its state of rest or uniform motion in a straight line.

First law helps us to define force. According to this law, a force is required to change the state of rest or uniform motion of a body along a straight line. Hence, a force is that which changes or tends to change the state of rest or uniform motion of a body along a straight line.

4.1.2 Second law

From the first law it is clear that a force changes the state of rest of a body or changes its velocity. Thus, force produces acceleration. The second law gives us the relationship between force and acceleration. It states "The rate of change of momentum is directly proportional to the force applied and takes place in the direction of the force" or "The acceleration of a body is parallel and directly proportional to the net force acting on the body, and is inversely proportional to the mass of the body. Mathematically,

$$F = ma$$

Momentum is defined as the product of mass and its linear velocity given by:

$$p = mv$$

Thus,

$$F \propto \frac{dp}{dt}$$

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt}$$

Since $\frac{dv}{dt} = a$, then

$$\boldsymbol{F = ma}$$

The S.I unit of a force is Newton (N). It is that force which produces an acceleration of 1 m/s^2 on a mass of 1 kg.

4.1.3 Third law

From Newton's second law it is clear that when a body is in accelerated motion there is a force acting on it. This force is due to some other bodies acting on the first one. Newton's third law gives a relation between those forces. When one body exerts a force on a second body, the second body at the same time exerts an equal force on the first one. It is impossible to have a single isolated force.

Newton's third law states "To every action, there is always an equal and opposite reaction."

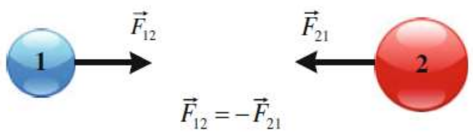

Figure 4.1 *The force exerted by body 1 on body 2 is equal in magnitude but opposite to the force exerted by body 2 on body 1*

i.e.

$$\text{Action} = -\text{Reaction}$$

$$\vec{F_1} = -\vec{F_2}$$

$$|\vec{F_1}| = |\vec{F_2}|$$

Even though action and reaction are equal and opposite, they do not cancel each other as they act on two different bodies.

4.2. Some forces in nature

4.2.1 Weight ($\vec{W}$)

The weight of a body is the force of gravity acting on the body. This force is directed toward the center of the Earth and is primarily due to an attraction between the body and the Earth. Mathematically, it is given by:

$$W = mg$$

where m is the mass of the body, and g is the acceleration due to gravity.

In vector notation, the weight of the of the body is given by:

$$\vec{W} = m\vec{g}$$

4.2.2 Normal force ($\vec{N}$)

Consider a block of weight $\vec{W}$ placed on a table as shown in the figure 4.2. When this block rests a table, the table exerts an upward force (reaction), called Normal force ($\vec{N}$). The normal force prevents the block from falling through the table, and can have any value up to the point of breaking the table.

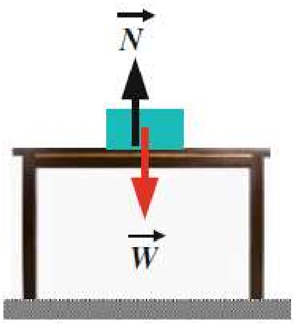

Figure 4.2 *A block resting on a table experiences a normal force $\vec{N}$ perpendicular to the table*

Therefore, we conclude that

$$\vec{W} = -\vec{N}$$

and

$$|\vec{W}| = |\vec{N}|$$

4.2.3 Tension ($\vec{T}$)

When a rope (or a cord, cable, etc) is attached to a body and pulled, the rope is said to be in tension. The rope's function is to transfer force between two bodies. The tension in the rope is defined as the force that the rope exerts on the body. A rope is considered to be massless (i.e., its mass is negligible compared to the body's mass) and non-stretchable. This force is denoted usually by the symbol $\vec{T}$, see figure 4.3.

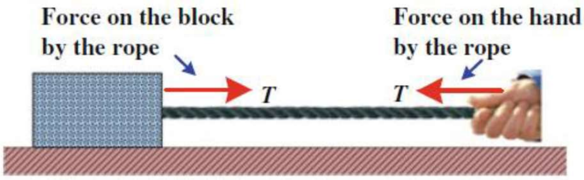

Figure 4.3 *When a rope is under the tension, it pulls the block and the hand with a force of magnitude T*

4.2.4 Friction $(\vec{f})$

If we slide or try to slide a body over a surface the motion is resisted by a bonding between the body and the surface. This resistance is represented by a single force and is called friction. Friction is the force that opposes the relative sliding motion of two surfaces in contact with one another.

The force of friction is parallel to the surface and opposite to the direction of intended motion. There are normally two types of friction: Static friction and kinetic friction.

(a) Static friction $(\vec{f_s})$

It is the force of friction between two surfaces, before relative motion actually starts. Its magnitude is always equal to the external force which tends to cause relative motion. As the external force which tries to produce relative motion increases, the force of friction also increases, till relative motion just starts.

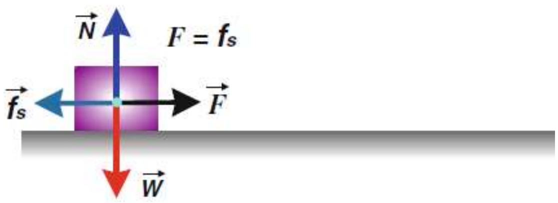

Figure 4.4 *The block resting on the horizontal table. The forces acting on the bock are weight, normal, applied force and static friction*

The laws of static friction are:

- The magnitude of static friction is independent of the area.
- The magnitude of static friction is directly proportional to the normal force (reaction). That is:

  $$f_s \propto N$$
  $$f_s = kN$$

  where k is the proportionality constant which is called the coefficient of static friction which is denoted by μ_s. Then:

  $$f_s = \mu_s N$$

(b) Kinetic friction $(\vec{f_k})$

Kinetic friction between two surfaces in contact is the force of friction which calls into play when there is relative motion between the surfaces. Kinetic friction is always less than static friction.

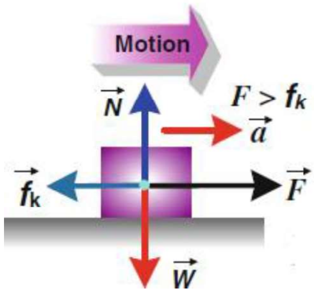

Figure 4.5 *The block moving on the horizontal table. The forces acting on the bock are weight, normal, applied force and kinetic friction*

The laws of kinetic friction are:

- The kinetic friction has a constant value which depends on the nature of the two surfaces in contact.
- The kinetic friction is directly proportional the normal force. That is

$$f_k \propto N$$
$$f_k = kN$$

  where k is the proportionality constant which is called the coefficient of kinetic friction which is denoted by μ_k. Then:

$$f_k = \mu_k N$$

  Since $f_k < f_s$, then $\mu_k < \mu_s$
  The coefficients of static and kinetic friction depend on the two surfaces in contact.

(c) Graph between applied force and force of friction

Figure 4.6 shows the graph of the variation of the force of friction with the applied force.

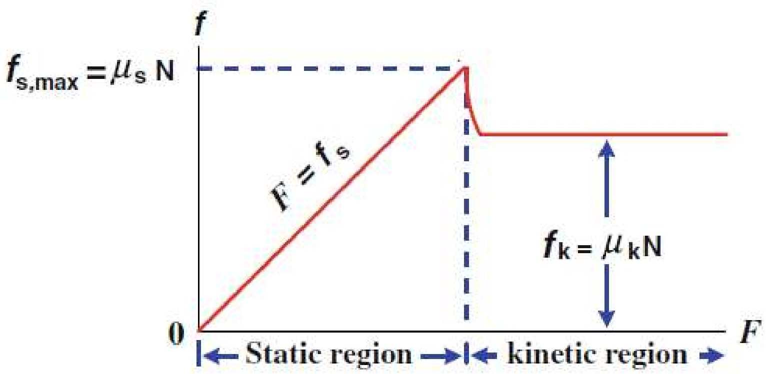

Figure 4.6 *The graph of the variation of the force of friction with the applied force*

(d) Some approximated values of coefficients μ_s and μ_k

The values of the dimensionless coefficients μ_s and μ_k depend on the nature of the surfaces, not on their areas. Typical values of the coefficients lie in the range $0.05 \leq \mu \leq 1.5$. Table 4.1 lists some reported values.

Material surfaces	μ_s	μ_k
Ice on ice	0.1	0.03
Wood on ice	0.08	0.06
Metal on metal (lubricated)	0.15	0.06
Wood on wood	0.25–0.5	0.2
Copper on steel	0.53	0.36
Glass on glass	0.94	0.4
Aluminum on steel	0.61	0.47
Steel on steel	0.74	0.57
Rubber on concrete	~ 0.9	~ 0.7

Table 4.1 *Some approximate values of coefficients of friction*

4.3 Motion of a body in a lift

Consider a body of mass *m* resting on a lift as shown in figure 4.7. The forces acting on the body are:

- The weight *mg* of the body acting vertically downwards.
- The reaction *N* acting vertically upwards.

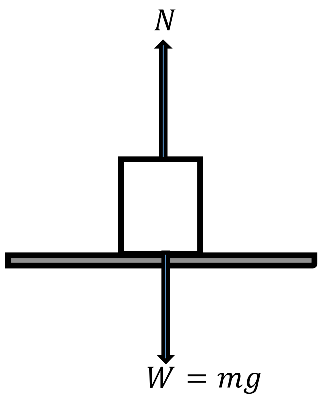

Figure 4.7 *A body mass m on a lift. The forces acting on the body are weight mg acting vertically downwards and reaction N acting vertically upwards*

We have the following conditions:

4.3.1 Lift at rest

When the lift is stationary, by Newton's second of motion, the net force is given by:

$$\sum F_y = ma$$

Since $a = 0$, then:

$$N - mg = 0$$

$$N = mg$$

In this case, the normal force (apparent weight) equals the actual weight of the body.

4.3.2 Lift moving upward or downward with constant velocity

When the lift is moving either up or down with constant velocity, by Newton's second of motion, the net force is given by:

$$\sum F_y = ma$$

Since $a = 0$, then:

$$N - mg = 0$$

$$N = mg$$

In this case, the normal force (apparent weight) equals the actual weight of the body.

4.3.3 Lift moving upward with constant acceleration *a*

When the lift moves up with a constant acceleration a**,** by Newton's second of motion, the net force is given by:

$$\sum F_y = ma$$

$$N - mg = ma$$

$$N = mg + ma = m(g + a)$$

In this case, the normal force (apparent weight) is greater than the actual weight of the body.

4.3.4 Lift moving downward with constant acceleration *a*

When the lift moves down with a constant acceleration a**,** by Newton's second of motion, the net force is given by:

$$\sum F_y = ma$$

$$mg - N = ma$$

$$N = mg - ma = m(g - a)$$

In this case, the normal force (apparent weight) is less than the actual weight of the body

4.3.5 Lift moving upward with constant acceleration $\boldsymbol{a = g}$

When the lift moves up with a constant acceleration $a = g$, by Newton's second of motion, the net force is given by:

$$\sum F_y = ma$$

$$N - mg = mg$$

$$N = mg + mg = 2mg$$

In this case, the normal force (apparent weight) is twice the actual weight of the body.

4.3.6 Lift moving downward with constant acceleration $\boldsymbol{a = g}$

When the lift moves down with a constant acceleration $a = g$, by Newton's second of motion, the net force is given by:

$$\sum F_y = ma$$

$$mg - N = mg$$

$$N = mg - mg = 0$$

In this case, the normal force (apparent weight) equals zero. In other words, there is no reaction, and as such the body is apparently weightless.

If the body is suspended from a spring balance attached to the ceiling of a lift which is moving up or down with acceleration a, there is an apparent change in the reading.

$$\text{Apparent weight } (N) = m(g \pm a)$$

4.4. Forces in equilibrium

Forces not only pull or push but also have a turning effect or moment about an axis, e.g. forces that keep bridges stationary and also leaning against the wall on a ladder.

An object is said to be in equilibrium if it is not accelerating under the action of concurrent (coplanar) forces.

4.4.1 Conditions for equilibrium

The conditions for equilibrium of a rigid body under the action of coplanar forces are as follows:

(1) The vector sum of the external forces on the rigid body must be zero.

$$\sum F = 0$$

In horizontal direction, we have:

$$\sum F_x = 0$$

And in the vertical direction, we have:

$$\sum F_y = 0$$

When this condition is satisfied we say that the body is in translational equilibrium ($a = 0$).

(2) The sum of external torques on the rigid body be equal to zero.

$$\sum \tau = 0$$

When this condition is satisfied we say that the body is in rotational equilibrium ($\alpha = 0$).

4.4.2 Moments and equilibrium

The magnitude of the moment (torque) of a force *F* about a fixed point O, is defined as the product of the force *F* and the perpendicular distance *r* from O to the line of action of a force.

Moment (or torque) = Force × perpendicular distance from axis

$$\tau = Fr$$

The S.I unit of torque is *N.m*

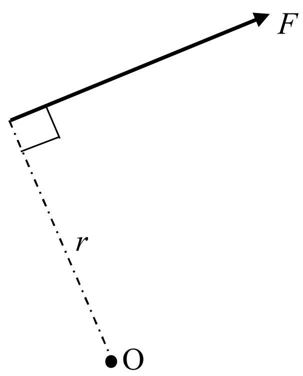

Figure 4.8 *Moment (or torque) τ of the force = Force F × perpendicular distance r from O to the line of action of a force*

The resultant of a number of forces in equilibrium is zero. So, the moment of the resultant about any point is zero. It therefore follows that the algebraic sum of the moments of all about any point is zero when those forces are in equilibrium. This means that the total clockwise moment of the forces about any point equals the total anticlockwise moment of the forces about the same point. This is known as the **principle of moments**.

$$\sum \curvearrowright \tau = \sum \curvearrowleft \tau$$

4.4.3 Couple and its torque

Two equal and opposite forces whose lines of action do not coincide are said to form a couple. The two forces always have a turning effect or moment, called a torque given by:

Moment (or torque) = One force × perpendicular distance between forces

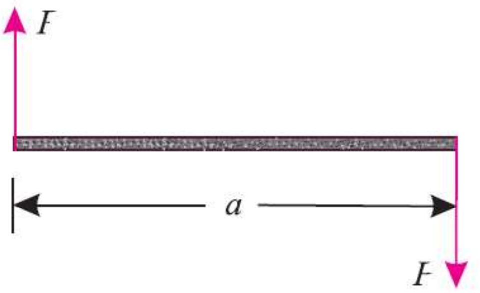

Figure 4.9 *The moment of a couple is the product of one of the forces F and the arm of the couple* a

4.4.4 Centre of gravity

It has been established, since long, that every particle of the body is attracted by the earth towards its centre. The force of attraction, which is proportional to the mass of the particle, acts vertically downwards and is known as the weight of the body. As the distance between different particles of the body and the centre of the earth is the same, therefore these forces may be taken to act along parallel lines.

A point may be found out in a body, through which the resultant of all such parallel forces act. This point, through which the whole weight of the body acts, irrespective of its position, is known as the centre of gravity.

EXERCISES

1. The value of g at the surface of the Earth is 9.8 N/kg and on the surface of Venus the magnitude of g is 8.6N/kg. A cosmonaut has a weight of 588 N on the surface of the Earth, what will be her mass and weight on the surface of Venus? **[m=60kg; W=516N]**
2. A person in a kayak starts paddling, and it accelerates from 0 to 0.8 mile/hour in a distance of 0.8km. If the combined mass of the person and the kayak is 80kg, what is the magnitude of the net force acting on the kayak? $[\mathbf{6.4 \times 10^{-3} N}]$
3. A tension of 6 kN is experienced by the elevator cable of an elevator moving upwards. If the elevator starts from rest and attains a speed of 4 m/s in 2 seconds, what is the mass of the elevator? **[508 kg]**

4. An elevator is moving with an acceleration of 2.5 m/s^2. Find the mass of a 80kg man in the elevator if the elevator is

   (a) moving up. **[100kg]**

   (b) moving down. **[60kg]**

5. A rocket of mass 5×10^4 kg is in flight. Its thrust is directed at an angle of 60 degrees above the horizontal and has a magnitude of 7×10^5 N. Find the magnitude and direction of the rocket's acceleration. Give the direction as an angle above the horizontal. **[7 m/s^2; 18°]**

6. A bullet of mass 10 g moving with velocity 100 m/s strikes a wooden plank of thickness 50 cm, emerges with a velocity 30 m/s. Find the resistance offered by the plank, assuming it uniform. **[91 N]**

7. A driver of a three-wheeler moving with a speed of 36 km/h sees a child standing in the middle of the road and brings his vehicle to rest in 4 seconds just in time to save the child. What is the average retarding force on the vehicle? The mass of the three-wheeler and the driver is 465 kg. **[1162.5 N]**

8. A 10g object is moving in a plane. The *x* and *y* coordinates of the object are given by $x(t)=2t^3-t^2$ and $y(t)=4t^3+2t^2$. Find the net force acting on the object at t=2s. **[0.56N]**

9. Two masses of 5 kg and 3 kg are suspended with help of massless inextensible string as shown in figure 4.10. Calculate the tensions T_1 and T_2 when the system is going upwards with acceleration 2 m/s^2. $[\boldsymbol{T_1 = 94.4\ N;\ T_1 = 35.4\ N}]$

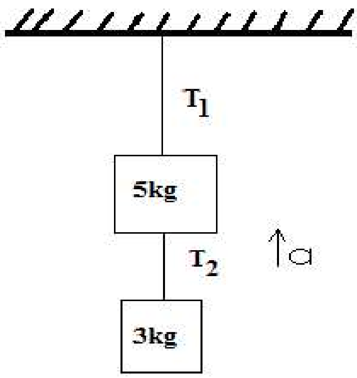

Figure 4.10 *See exercise 9*

10. A car of mass 1 tone is brought to rest from a speed of 40 m/s in a distance of 80 m. Find the braking force of the car assuming that it is constant and that there is a constant resistance to motion of 100 N. **[9900 N]**

11. A car is moving along a level road at a speed of 72 km/h. Find the shortest distance in which the car can be stopped by switching off the engine. Coefficient of kinetic friction between the road and the tyre is 0.4. Use g=10 m/s^2. **[50 m]**

12. A force of 98 N is just required to move a mass of 45 kg on a rough horizontal surface. Find the coefficient of friction and the angle of friction. **[0.22; 12.4^0]**

13. A particle of mass 5 kg slides down a smooth plane inclined at 30° to the horizontal. Find the acceleration of the particle and the reaction between the particle and the plane. $\left[\boldsymbol{a = \frac{1}{2}g\ ms^{-2};\ R = \frac{5g\sqrt{3}}{2}\ N}\right]$

14. A particle of mass 5 kg is pulled along a rough horizontal surface by a string which is inclined at 60° to the horizontal. If the acceleration of the particle is $\frac{1}{3}g\ ms^{-2}$ and the coefficient of friction between the particle and the plane is $\frac{2}{3}$, find the tension in the string. **[$\boldsymbol{T = 10g\sqrt{3}(2-\sqrt{3})}$ N]**

15. Calculate the force required to pull a train of mass 200 tons up an incline of 5° at a uniform speed of 72 km/h. Coefficient of kinetic friction = 0.02. $\left[\boldsymbol{2.1 \times 10^5\ N}\right]$

16. A hotel guest starts to pull an armchair across a horizontal floor by exerting a force of 91 N 15°above the horizontal. The normal force exerted by the floor on the chair is 221 N up. The acceleration of the chair is 0.076. Determine the mass of the chair and the coefficient of kinetic fiction between the chair and the floor. **[24.95 kg; 0.39]**

17. Calculate the force required to pull a train of mass 200 tons up an incline of 5^0 at a uniform speed of 72 km/h. Coefficient of friction = 0.02. **[209900 N]**

18. A block of mass m_1 =4 kg lies on a frictionless inclined plane of angle θ =30°. This block is connected by a cord over a massless, frictionless pulley to a second block of mass m_2 =6 kg hanging vertically, as shown in figure 4.11.

    (a) For each block, find the magnitude and direction of its acceleration.

    (b)What is the magnitude of the tension in the cord?

    (c) Repeat parts (a) and (b) after replacing each block by the other. **[(a) For m_1, a = 4m/s^2 up the plane and for m_2, a = 4m/s^2 downwards. (b) The magnitude of the tension in both cords is 36N, (c) For m_1, a = 1m/s^2 up the plane and for m_2, a = 1m/s^2 downwards. The magnitude of the tension in both cords is also 36N]**

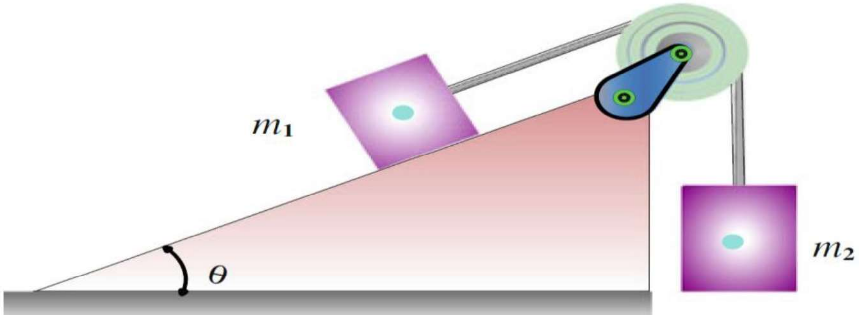

Figure 4.11 *See exercise 18*

19. A block is at rest on a rough inclined plane of angle θ

(a) When the angle is increased until the block is on the verge of slipping at angle $\theta = \theta_C$, show that the coefficient of static friction is given by

$$\mu_S = \tan \theta_C$$

(b) If the block is slightly disturbed and moves down with constant speed, find the coefficient of kinetic friction given that $\theta = 27°$. **[0.5]**

20. Three objects are connected on the table as shown in the figure 4.12. The table is rough and has a coefficient of kinetic friction of 0.35. The objects have masses of 4.00 kg, 1.00 kg, and 2.00 kg, as shown, and the pulleys are frictionless. Draw free-body diagrams of each of the objects. Determine

(a) the acceleration of each object and their directions.

(b) the tensions in the two cord. **[(a) 2.31 m/s/s (b)30.0 N; 24.2 N]**

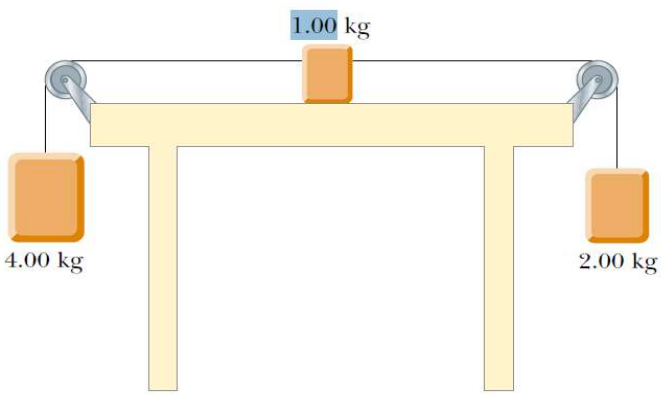

Figure 4.12 *See exercise 20*

21. Five forces acting on a particle are in equilibrium. Four of these are 2**i**+3**j**, 4**i**-7**j**, -5**i**+4**j** and **i**-**j**. What is the fifth force? **[-2i+j]**
22. A force of 15 N is applied perpendicular to the edge of a door 80 cm wide as shown. Find the moment of the force about the hinge. If this force is applied at an angle 60 ^{0} to the edge of the same door, as shown in figure 4.13, find the moment of this force. **[16 Nm; 10.4 Nm]**

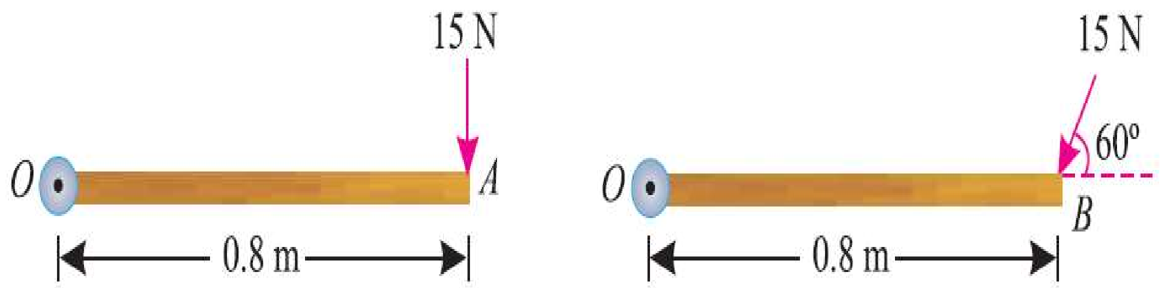

Figure 4.13 *See exercise 22*

23. A uniform plank *ABC* of weight 30 N and 2 m long is supported at one end *A* and at a point *B* 1.4 m from *A* as shown in figure 4.14. Find the maximum weight *W*, that can be placed at *C*, so that the plank does not topple. **[20 N]**

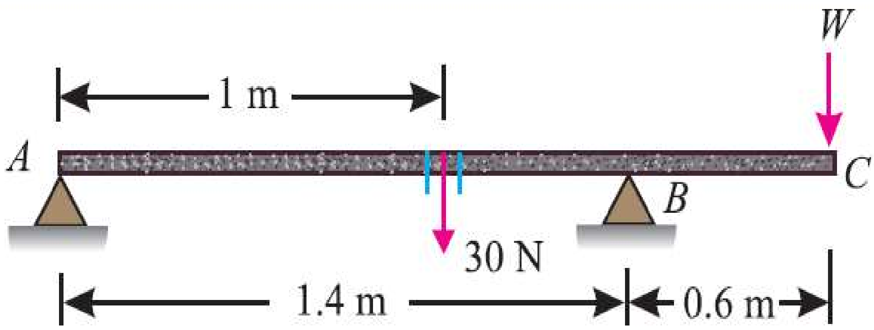

Figure 4.14 *See exercise 23*

24. A rod *AB* 2.5 m long is supported at *A* and *B*. The rod is carrying a point load of 5 kN at a distance of 1 m from *A*. What are the reactions at *A* and *B*? **[2 kN;3 kN]**
25. A painter weighing 900N stands on a massless plank 5m long that is supported at each end by a step ladder. If he stands 1m from one end of the plank, what force is exerted by each step ladder? **[F_1=180N; F_2=720N]**
26. A beam *AB* of length 1 m is supported horizontally at *A* and *B*. A weight of 500 N is attached at *C*. If the support at *A* cannot bear a pressure more than 300 N, find the distance of *C* from *A*, when the support *A* is about to fail. **[0.4 m]**

27. Two like parallel forces of 10 N and 30 N act at the ends of a rod 200 mm long. Find the magnitude of the resultant force and the point where it acts. **[40 N; 150 mm]**
28. Find the magnitude of two like parallel forces acting at a distance of 240 mm, whose resultant is 200 N and its line of action is at a distance of 60 mm from one of the forces. **[50 N; 150 N]**
29. A block of mass m =21 kg hangs from three cords as shown in figure 4.15. Taking $\sin\theta = 4/5$, $\cos\theta = 3/5$, $\sin\phi = 5/13$, and $\cos\phi = 12/13$, find the tensions in the three cords. $[\boldsymbol{T_1 = 210\ N; T_2 = 130\ N; T_3 = 200\ N}]$

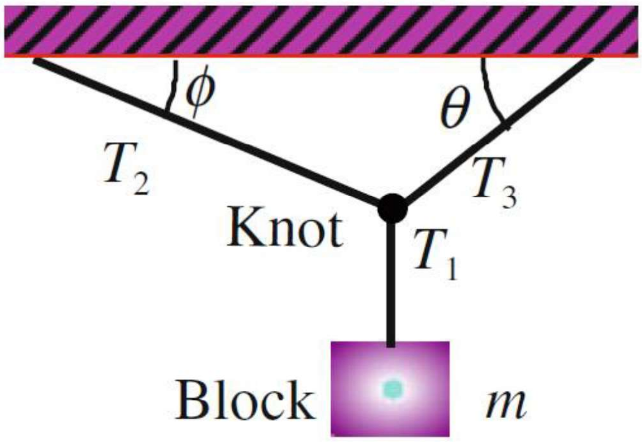

Figure 4.15 *See exercise 29*

30. An electric light fixture weighing 15 N hangs from a point C, by two strings Ac and BC. The string AC is inclined at 60 ^{0} to the horizontal and BC at 45 ^{0} to the vertical as shown in figure 4.16. Determine the forces in strings AC and BC. **[T_{AC}=10.98 N; T_{BC}=7.76 N]**

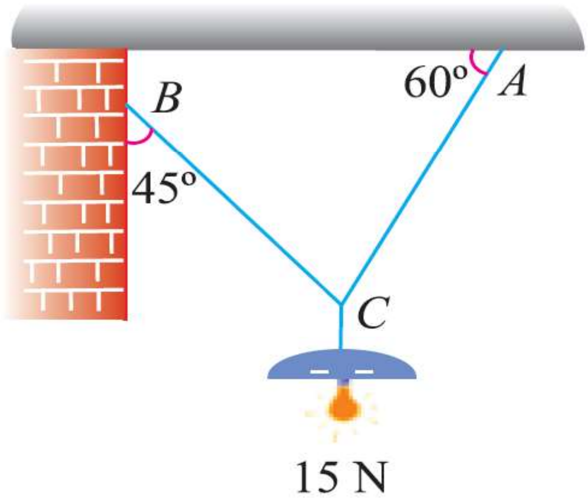

Figure 4.16 *See exercise 30*

31. A light string $ABCDE$ whose extremity A is fixed, has weights W_1 and W_2 attached to it at B and C. It passes round a small smooth peg at D carrying a weight of 300 N at the free end E as shown in figure 4.17. If the system is in equilibrium, find the
   (i) tensions in the portion AB, BC and CD. **[T_{AB}=173.2 N; T_{BC}=150 N; T_{CD}=300 N]**
   (ii) magnitudes of W_1 and W_2. **[W_1=86.6 N; W_2=259.8 N]**

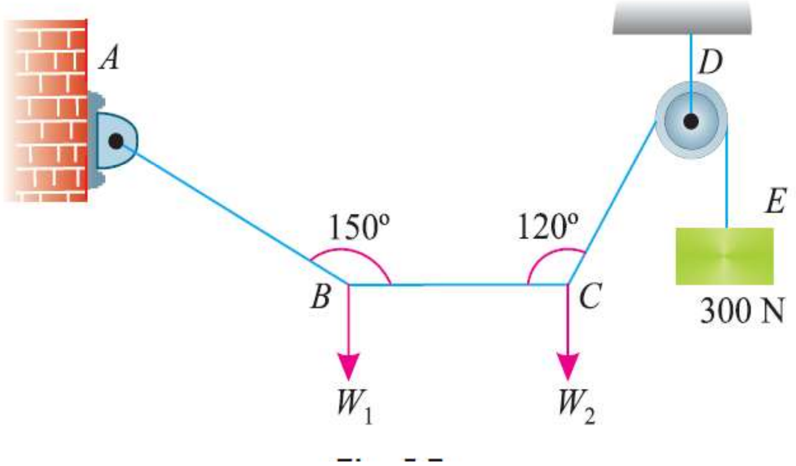

Figure 4.17 *See exercise 31*

32. A small object of weight 10 N rests in equilibrium on a rough plane inclined at 30° to the horizontal. Calculate the magnitude of the frictional force. Hence, find the coefficient of friction $\left[\mathbf{5\ N;\ \frac{\sqrt{3}}{3}}\right]$
33. A uniform ladder rests against a smooth vertical wall and on a rough horizontal ground. The weight of the ladder is 10 N and it is just about to slip when inclined 30^0 to the vertical. Calculate the coefficient of friction. **[0.289]**
34. A uniform ladder of mass m rests against a frictionless vertical wall at an angle of 60°. The lower end rests on a flat surface where the coefficient of static friction is $\mu_S = 0.4$. A student with mass $M=2m$ attempts to climb the ladder. What fraction of the length L of the ladder will the student have reached when the ladder begins to slip? **[0.789]**
35. A ladder rests in limiting equilibrium against a rough vertical wall and with its foot on rough horizontal ground, the coefficient of friction at both points of contact being $\frac{1}{2}$. The ladder is uniform and weighs 300 N. Find the angle θ which the ladder makes with the horizontal ground. Find also the normal reactions at the wall and the ground. **[36.9°; 120 N;240 N]**