1. a) i) Prove the De Morgan's laws

$$(A \cup B)' = A' \cap B'.$$

ii) Show that

$$X = (X \cap Y) \cup (X \cap Y')$$

b) i)Express the following in the form of $\frac{a}{b}$ where a and b are intergers, $b \neq 0$.

$0.5333333\ldots$

ii) Given that $\sqrt{5}$ is an irrational number, prove that $\sqrt{5} + 1$ is an irrational number .

c) Determine whether the function f is even, odd or neither.

$$f(x) = x^3 + x\,.$$

2. a) Express $\frac{\sqrt{x}}{2\sqrt{x}-1}$ in the form $a + b\sqrt{x}$, where a *and* b are real numbers.

b) i) Express $1 + 4i + \frac{5}{2-i}$ in form $a + bi$ and find its absolute value(modulus).

ii) Solve for x and y given that:

$$\frac{x+iy}{2+i} = 5 - i$$

c) Let $'*'$ be a binary operation on the set of real numbers defined by $a * b = 2^{b-a}$, where a *and* b are real numbers .

i). Is $*$ commutative on real numbers? Justify your answer.

ii). Find $-1 * (4 * 9)$.

d) Sketch the graph of the following radical function:

$$f(x) = 2 + \sqrt{-x+2}\,.$$

3. a) Given the rational function $f(x) = \frac{x+2}{x-2}$:

i). Find the domain of $f(x)$

ii). Find the vertical asymptotes

iii). Find the horizontal asymptotes

iv). Sketch the graph of $f(x)$.

b) Given the universal set $U = [1,12]$ where $A = \{1,2,3,4,5,6,7,8,9\}$ and $B = [1,5)$ .Find

i) $A \cap B$ ii) $A - B$.

c) i) Verify that the two given functions are inverses of each other

$$f(x) = x^3 + 1 \text{ and } g(x) = \sqrt[3]{x-1}.$$

ii) Determine the domain of functions:

$$f(x) = x + \sqrt{x^2 + 4x - 12}\ .$$

4. a).Use the Rational root theorem to solve the following equation:

$3x^4 + 5x^3 - 5x^2 - 5x + 2 = 0$.

b). i) Solve each of the following equation

$$\left|\frac{x+1}{x-4}\right| \leq 3$$

ii). Redefine $k(x) = |2x - 1| - |x + 2|$ by removing the modulus, hence sketch the graph of function:

c) Let $f(x) = 3x^2 + 12x + 5$ be a quadratic function.

i) By completing the square, express $f(x)$ in the form $f(x) = a(x+p)^2 + q$ where a, p and q are constants.

ii)State the maximum or the minimum point of the function f.

iii)Sketch the graph of the function $f(x) = 3x^2 + 12x + 5$

d) i)Find the values of k if the equation $x^2 + (k-2)x + 10 - k = 0$ has equal roots.

ii) The roots of the equation $2x^2 + 6x - 15 = 0$ are α and β. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.Hence find the quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.