Q1. (a) Express $2.07\bar{2}$ in $\frac{a}{b}$, $b \neq 0$ form.

Sol^n Let $x = 2.07\bar{2}$

$\Rightarrow 100x = 207.\bar{2} \longrightarrow$ eq ①. +1

$\Rightarrow 1000x = 2072.\bar{2} \longrightarrow$ eq ② +1

Evaluate " eq ② − eq ①

+1

$\Rightarrow 1000x - 100x = 2072.\bar{2} - 207.\bar{2}$

$\Rightarrow 900x = 1865$ +1

$\Rightarrow x = \frac{1865}{900}$ +1

[5 Marks]

Q1 (b) Graph of $f(x) = \frac{x^2+2}{x-1}$

Sol^n Finding the Oblique Asymptote;

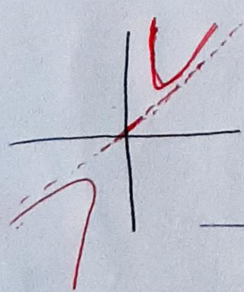
$$\begin{array}{c|ccc} x=1 & 1 & 0 & 2 \\ & & 1 & 1 \\ \hline & 1 & 1 & \underline{\underline{3}} \text{ remainder.} \end{array}$$

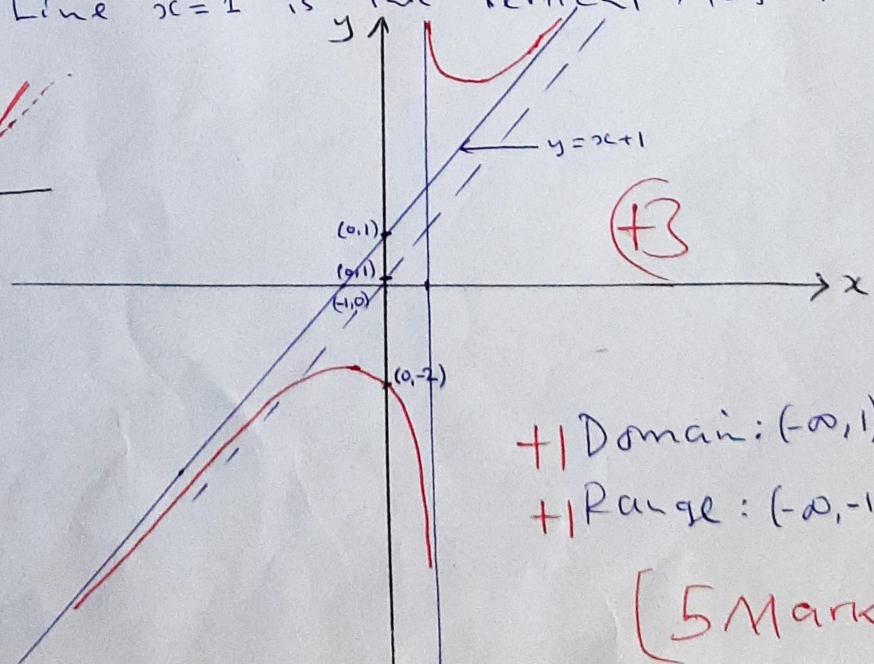
$x+1$

(+1)

* $\therefore \frac{x^2+2}{x-1} = \underline{\underline{x+1}} + \frac{3}{x-1}$, The line "$x+1=y$" is the O.A.

* The Line $x = 1$ is the Vertical Asymptote.





(+3)

+1 Domain: $(-\infty, 1) \cup (1, +\infty)$

+1 Range: $(-\infty, -1.53) \cup (5.47, +\infty)$

[5 Marks]

Q1(c) Show that "$2+\sqrt{7}$" is Irrational. If $\sqrt{7} \notin \mathbb{Q}$

Soln Suppose to the contrary that $2+\sqrt{7}$ is rational. Then it can be expressed as $\frac{a}{b}$, $b \neq 0$ and a & b do not have common factors. That is,

(+1)

$$2+\sqrt{7} = \frac{a}{b}$$ (+1)

$$\Rightarrow \sqrt{7} = \frac{a}{b} - 2$$

$$\Rightarrow \sqrt{7} = \frac{a-2b}{b}$$

$$\Rightarrow \sqrt{7} = \frac{a-2b}{b}$$ (+1), since $a, b \in \mathbb{Z}$, then $\frac{a-2b}{b} \in \mathbb{Q}$.

Let $\frac{a-2b}{b}$ be a rational, $\frac{d}{e}$.

$$\Rightarrow \sqrt{7} = \frac{d}{e}$$ (+1), $e \neq 0$.

The above is a contradiction!

(+1)

Hence, the result! [5 Marks]

Q 1 (d) Two inverse functions of each other.

Soln

If $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$

are inverses of each other, then.

i) $f \circ g(x) = x$ +1

ii) $g \circ f(x) = x$.

* i) $f \circ g(x) = f[g(x)]$

$= (\sqrt[3]{x-1})^3 + 1$ +1.5

$= x - 1 + 1$

$= x$

* ii) $g \circ f(x) = g[f(x)]$

$= \sqrt[3]{(x^3+1) - 1}$ +1.5

$= \sqrt[3]{x^3 + 1 - 1}$

$= \sqrt[3]{x^3}$

$= x$

$\because$ $f \circ g(x)$ +1 $g \circ f(x) = x$, the $g(x)$ is inverse of $f(x)$ and vice versa.

[5 Marks]

Q1 e) Binary Operation check for $a*b = a^b$

(i) Since $a, b \in \mathbb{R}$,

Raising the first number to the second number the result is another real number! +1

That is, $a * b = a^b \in \mathbb{R}$ if $a, b \in \mathbb{R}$!

ii) $a * b = a^b$ and

$b * a = b^a$ +2

clearly, $a * b = a^b \neq b^a = b * a$.

The operation is not Commutative!

iii) $(3*2) * (-2) = (3^2) * (-2)$

$= 9 * -2$

$= 9^{-2}$ +2

$= \frac{1}{(9)^2}$

$= \frac{1}{81}$

[5 Marks]

Q 2 (a) Simplify $[(A\cap B)^c \cap (A^c \cup B)]^c$

Soln $[(A\cap B)^c \cap (A^c \cup B)]^c$

$\Rightarrow [(A^c \cup B^c) \cap (A^c \cup B)]^c$ +1 De Morgan's Law

$\Rightarrow [A^c \cup (B^c \cap B)]^c$ +1 — Distributivity Property.

$\Rightarrow [A^c \cup \emptyset]^c$ +1

$\Rightarrow [A^c]^c$ +1 — Idepotent Law.

$\Rightarrow \underline{\underline{A}}$ +1 [5 Marks]

Q.2 (b) Domain for $f(x) = \sqrt{\frac{x+1}{x-1}}$.

Soln Step 1 Radicand must be positive!

+1 $\frac{x+1}{x-1} \geq 0$

Critical Points

$x=-1,\ x=1$

	$x<-1$	$-1<x<1$	$x>1$
$x+1$	$-$	$+$	$+$
$x-1$	$-$	$-$	$+$
$\frac{x+1}{x-1}$	$+$	$-$	$+$

+2

$\therefore x \leq -1$ and $x \geq 1$ is the solution set

But since $f(x)$ is undefined at +1 $x=1$, then

Domain is $\underline{\underline{(-\infty, -1] \cup (1, +\infty)}}$ +1

[+5 Marks]

Q2 (c) Quadratic Eqn with α^2 & β^2 roots.

If α, β are roots of $4x^2 + 3x - 2 = 0$, then

$\alpha + \beta = \frac{3}{4}$ and $\alpha\beta = -\frac{2}{4} = -\frac{1}{2}$ +1

Now, if Quadratic Eqn has roots α^2 and β^2, then,

* $\alpha^2 + \beta^2$ = coefficient of x

$$= (\alpha+\beta)^2 - 2\alpha\beta$$
$$= \left(\tfrac{3}{4}\right)^2 - 2\left(-\tfrac{1}{2}\right)$$
$$= \frac{9}{16} + 1$$
+1
$$= \frac{25}{16}$$

* $\alpha^2\beta^2$ = constant term

$$= (\alpha\beta)^2$$
$$= \left(-\tfrac{1}{2}\right)^2$$
+1
$$= \frac{1}{4} = \frac{4}{16}$$

Thus, the Quadratic Eqn, $ax^2 + bx + c = 0$ has

$a = 16$, $b = 25$ and $c = 4$. +1

That is, +1 $16x^2 + 25x + 4 = 0$

[5 Marks]

Q 2 (d) Solve $10 - \sqrt{2x+7} \leq 3$.

Step 1: Solve for x.

Soln

$10 - \sqrt{2x+7} \leq 3$

$\Rightarrow -\sqrt{2x+7} \leq 3 - 10$

$\Rightarrow -\sqrt{2x+7} \leq -7$

$\Rightarrow \sqrt{2x+7} \geq 7$

$\Rightarrow 2x + 7 \geq 49$

$\Rightarrow 2x + 7 \geq 49$

$\Rightarrow 2x \geq 49 - 7$

$\Rightarrow 2x \geq 42$

$\Rightarrow x \geq 21$

Step 2 consider the Radicand, it should be positive!

$2x + 7 \geq 0$

$\Rightarrow 2x \geq -7$

$\Rightarrow x \geq -\frac{7}{2}$

Therefore the solution is;

$x \geq -7/2$ and $x \geq 21$

or ~~$x = [-\frac{7}{2}, +\infty)$ $[21, +\infty)$~~

$x = [-\frac{7}{2}, +\infty) \cap [21, +\infty)$

$= [21, +\infty)$

Q2 (e) Solve $\frac{1}{x+iy} + \frac{1}{1+3i} = 1$ for x & y.

Soln $\frac{1}{x+iy} + \frac{1}{1+3i} = 1$

$\Rightarrow \frac{(1+3i) + (x+iy)}{(x+iy)(1+3i)} = 1$ +1

$\Rightarrow (1+3i) + (x+iy) = (x+iy)(1+3i)$ +1

$\Rightarrow (1+x) + (3+y)i = (x-3y) + (3x+y)i$

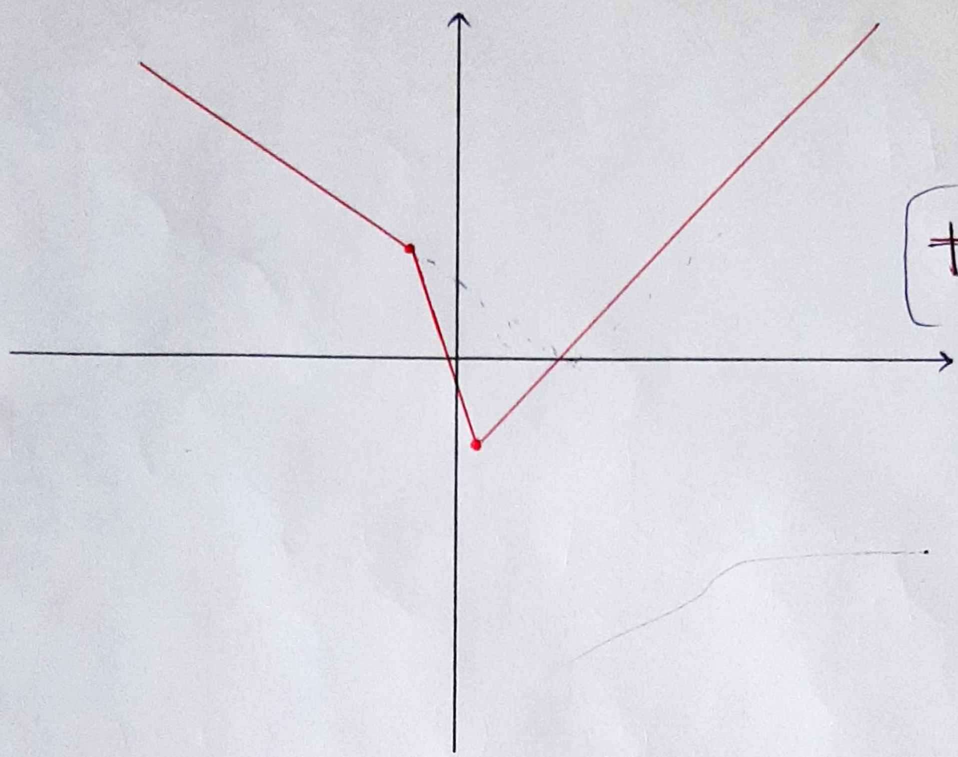
$\Rightarrow 1+x = x-3y$ and $3+y = 3x+y$

$\Rightarrow 1 = -3y$ +1 $\qquad 3 = 3x$ +1

$\Rightarrow y = -\frac{1}{3}$ $\qquad x = 1.$

$\Rightarrow (x, y) = (1, -\frac{1}{3})$ +1 [5 Marks]

Q3 (a)



+5

Q3 (b) Divide $[x-(1+i)]$ into $f(x) = x^3+2x^2+x-2$.

	1	2	1	-2
$x=1+i$		$1+i$	$1+3i$	$-1+5i$
	1	$2+i$	$2+3i$	$-3+5i$

+1 +1 +2 Remainder.

$Q(x) = x^2+(2+i)x+(2+3i)$

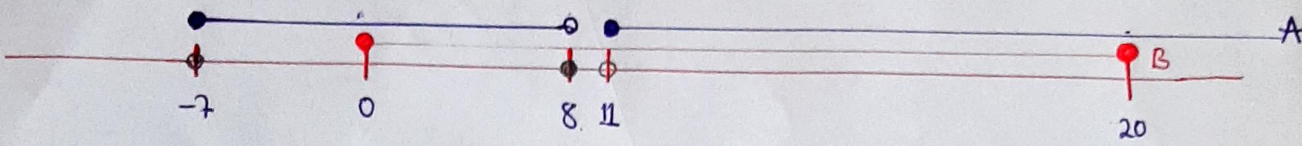
$R(x) = (-3+5i)$ +1

+5

Q3 (c) Number Line display of sets.

Soln if $A = [-7, 8) \cup [11, +\infty)$ and $B = [0, 20]$.

(i) A' is ?

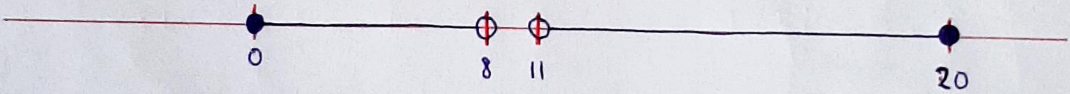


+2.5

$A' = (-\infty, -7) \cup [8, 11)$

(ii) $A \cap B$ is ?

$A \cap B = [0, 8) \cup (11, 20]$



+2.5

[5]

Q3 (d.) $\frac{\sqrt{3}+1}{\sqrt{3}-1} + \sqrt{3} - 1$ in $a + b\sqrt{3}$ form

Sol^n $\frac{\sqrt{3}+1+(\sqrt{3}-1)(\sqrt{3}-1)}{\sqrt{3}-1}$ +1

$= \frac{\sqrt{3}+1+(3+1-2\sqrt{3})}{\sqrt{3}-1}$ +1

$= \frac{(5-\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ +1

$= \frac{5-3+5\sqrt{3}-\sqrt{3}}{3-1}$ +1

$= \frac{2+4\sqrt{3}}{2}$

$= \underline{\underline{1+2\sqrt{3}}}$ +1 [+5]

Q3 (e) Checking $f(x) = x^5 + x^3 + x$ for being Odd, even.

Sol^n Take $f(-x)$. That is, +1

$f(-x) = (-x)^5 + (-x)^3 + (-x)$ +1

$= -x^5 - x^3 - x$ +1

$= -[x^5 + x^3 + x]$

$= \underline{-f(x)}$ +1

Since $f(-x) = -f(x)$, $f(x)$ is Odd ! +1

[+5]