THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PURE AND APPLIED MATHEMATICS

MA 110 - Mathematical Methods I

Tutorial Worksheet 1

1. Let $A = \{0,2,4,6,8,10\}$ $B = \{1,3,5,7,9\}$ $C = \{1,2,4,5,7,8\}$ and $D = \{1,2,3,5,7,8,9\}$ find
   (a) $(A - B) - C$ (b) $(A \cap B) \cup (A \cap C)$ (c) $D \cap (A \cap C)$,
   (d) $(A \cup B) - (C \cap D)$
2. Using the laws of set theory, simplify the following
   (a) $A \cap (B - C)$ (b) $(A - B) \cup (A \cap B)$, (c) $(A - B)'$
3. Let $U = (3,4,5,6)$ be the universal set for the sets $A = (3,4)$, and $B = (4,5)$. Find
   (a) $P(A) - P(B)$, (b) $P(A) \cap P(B)$, (c) $P(A') - P(B')$, (d) $P(A) \cup P(B)$,
4. Let A, B and C be subsets of U. Prove that
   (a) $A - (B \cap C) = (A - B) \cup (A - C)$ (b) $A \cup (U - A) = U$ (c) $A \cap (U - B) = A - B$
   (d) $(A')' = A$ (e) $(A \cup B)' = A' \cap B'$
5. If $A \subset B$, simplify the following
   (a) $A' \cap B'$ (b) $A' \cup B'$
6. Sow that $[(A \cap B) \cup (A \cap B')]' = A'$
7. Determine whether the following subsets of $\mathbb{Z}$ are closed in $\mathbb{Z}$ under addition, division and multiplication.
   (a) $\mathbb{Z}^+$ (b) Even numbers (c) Odd numbers
8. Let the universal set be a set of real numbers. Let $A = [3,8)$, $B = [2,7]$, $C = (1,5)$ and $D = (6,\infty)$, Find
   (a) $A \cup C$ (b) $A \cap B$ (c) A' (d) $B - D$ (e) $B - (A \cup C)$ (f) $(A \cup C) - (B \cap D)$
   (g). $A - B$ (h) $B \cap D$ (i) D' (j) $D - A$ (k) $B \cap C$
9. Write the following in the form $\frac{a}{b}$ where a and b are integers that are relatively prime:
   (a) $3.5\overline{57}$ (b) $-0.25\overline{25}$ (c) $3.312\overline{12}$ (d) $0.123\overline{123}$ (e) $0.203\overline{03}$
   (f) 2.25
10. Prove that the following numbers are irrational
    (a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $2 + \sqrt{3}$ (d) $\sqrt{2} + \sqrt{3}$ (e) $\frac{1}{3}\sqrt{2} + \sqrt{5}$

11. Simplify the following as much as possible.

(a) $(-2x^2y^3)^3$ (b) $\left[\frac{2x^3y^4}{xy^7}\right]^2$ (c) $\sqrt{\frac{-27r^6}{125s^9}}$ (d) $\left[\frac{16x^3y^{12}z^2}{2x^{-6}z^8}\right]^{\frac{2}{3}}$

(e) $\left(\frac{81}{100}\right)^{-\frac{3}{2}}$ (f) $\left(16^{\frac{2}{3}}\right)^{-\frac{3}{4}}$ (g) $3|1-2x|+x-1$ (h) $(5x^2z^5)^3(5x^3z)^{-3}$

(i) $\frac{3(1+x)^{\frac{1}{3}}-x(1+x)^{-\frac{2}{3}}}{(x+1)^{\frac{2}{3}}}$ (j) $\frac{2}{x+1}+\frac{1}{x-1}+\frac{1}{x^2-1}$ (k) $\frac{\frac{1-(x-h)}{2+h}-\frac{1-x}{2+x}}{h}$

12. Simplify the expression to the form $a+bi$ where;

(a) $\frac{3i}{2+i}$ (b) $(2+3i)(-3+7i)$ (c) $3\sqrt{-50}+\sqrt{-72}$ (d) $\frac{2+4i}{3-i}$

13. Determine algebraically whether the function f is even, odd or neither

(a) $f(x)=x+\frac{1}{x}$ (b) $f(x)=3+|x-3|$ (c) $f(x)=x^4+x^2$

(d) $f(x)=\frac{1}{x^3}$ (e) $f(x)=2x^4-3x^2+7$ (f) $f(x)=(x-2)^2-1$

(g) $f(x)=x^5-x$

14. Find the functions $f\circ g$, $g\circ f$, $f\circ f$, $g\circ g$ and their doamins.

(a) $f(x)=2x+3$, $g(x)=4x-1$ (b) $f(x)=|x|$ (d) $f(x)=x^3$ (e) $f(x)=\sqrt{x}$

15. Find the domain and range of the following functions;

(a) $f(x)=\frac{2}{x^2-x}$ (b) $g(x)=\frac{x}{\sqrt{x+1}}$ (c) $h(x)=\sqrt{4-x^2}$ (d) $k(x)=\frac{1}{x^2+1}$

16. Find the inverse of f and its domain;

(a) $f(x)=\frac{x-2}{x+2}$ (b) $f(x)=4x+7$ (c) $f(x)=\sqrt{2x-1}$ (d)$f(x)=1+\sqrt{1+x}$

17. Find $f\circ g\circ h$

(a) $f(x)=x-1$, $g(x)=\sqrt{x}$, $h(x)=x-1$ (b) $f(x)=\sqrt{x}$, $g(x)=\frac{x}{x-1}$, $h(x)=\sqrt{x}$

18. Use $f(x)=3x-7$ and $g(x)=2-x^2$ to evaluate

(a) $f\big(g(0)\big)$ (b) $g(g(3))$ (c) $g(f(0))$ (d) $f(g(-2)$, (e) $f(f(-1))$

19. Sketch the graph of the function below by starting with the graph of a standard function and applying the transformations

(a) $f(x)=(x+7)^2$ (b) $g(x)=1+\sqrt{x}$ (c) $h(x)=\frac{1}{2}\sqrt{x+4}-3$

(d) $f(x)=|x+3|+3$ (e) $g(x)=5+(x-2)^2$, (f) $f(x)=2-|x|$

20. Show that the functions f and g are inverse functions of each other

(a) $f(x) = \frac{3-x}{4}$ and $g(x) = 3 - 4x$ (b) $f(x) = \frac{1}{x-1}$ and $g(x) = 1 + \frac{1}{x}$

21. Determine whether the given functions are one-to-one, onto or objective

(a) $f: \mathbb{R}^+ \to \mathbb{R}$ defined by $f(x) = x^2 + 2$ (b) $f(x) = \frac{3}{2x-1}, \quad x \neq \frac{1}{2}$

(c) g: $[0, \infty) \to 0, \infty)$ be a function given by $g(x) = x^2$ (d) $h(x) = \sqrt{2-x}, x \leq 2$

22. Sketch the following piecewise defined function

(a) $f(x) = \begin{cases} |x+2| & if -4 \leq x < 0 \\ x^2 & if\ 0 \leq x \leq 2 \\ 3 & if\ 2 < x < 4 \end{cases}$ (b) $g(x) = \begin{cases} \frac{1}{x} & if -3 \leq x < 0 \\ \sqrt{x} & if\ 0 \leq x \leq 4 \\ |x+2| & if\ 4 < x < 8 \end{cases}$

(c) $h(x) = \begin{cases} 3x - 1 & if\ x < -1 \\ 4 & if -1 \leq x \leq 1 \\ x & if\ x > 1 \end{cases}$

23. A function f is given and the indicated are applied to its graph (in the given order). Write the equation of the final transformed graph.
    (a) (a) $f(x) = x^2$ shift upward by 3 units and shift 2 units to the right.
    (b) $f(x) = \frac{1}{x}$, shrink by a factor of 0.1 units, shift 6 units to the right and shift 3 units downwards.
    (c) $f(x) = |x|$ shift to the left by 1 unit, stretch vertically by a factor of 3 and shift upwards by 10 units.
    (d) $f(x) = \sqrt{x}$, shift to the left by 2 units, and stretch horizontally by a factor of 3.

24.
    (a) Is the binary operation $*$ defined on Z by $x * y = 1 + x + y$ both commutative and associative.
    (b) Is the binary operation $*$ defined by $a * b = a + b^2$ both commutative and associative.
    (c) Suppose the binary operation $*$ is defined on integers by $x * y = 1 - 2xy$. Show that $*$ is both associative and commutative
    (d) Is the binary operation $*$ defined by $a * b = a + b - ab$ both commutative and associative

25. List the ordered pairs $A \times B$ Aand $B \times A$ in each case;
    (a) $A = \{1,3,5\}, \ B = \{a, e, k, n, r\}$ (b) $A = \{1,2,\{1,2\}\}, B = \{q, \{t\}, s\}$

26. Simplify each of the following expressions in the form $a + bi$ where a and b are real numbers.

(a). $(3 - 2i)^3$ (b) $\sqrt{-25}\sqrt{-4}$ (c) $3i^7 - 2i^6 + 5i^5 - 8i^{103} + 4i^2 - 6i - 10$

(d) $i\sqrt{2}\left(2\sqrt{2} - 3i\sqrt{2} + i - 2\right) + i\sqrt{2}$ (e) $(2 + i)^2 + (4 - i)i$ (f) $\frac{3-5i}{4i}$

27. Find the complex square root in the form $a + bi$ where a and b are real numbers

(a). $-5 + 12i$ (b) $21 - 20i$ (c) $3 + 4i$

28. Solve for x and y where x and y are real numbers

(a). $2y + ix = 4 + x - i$ (b) $(4 - 3i)^2 + 5(x - iy) = x + iy$ (c) $\frac{x}{1+i} + \frac{y}{3+i} = 4 - 2i$

29. Rationalize the denominator

(a) $\frac{x+\sqrt{y}}{2x-\sqrt{y}}$ (b) $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$ (c) $\frac{2+\sqrt{3}}{3+\sqrt{3}}$ (d) $\frac{\sqrt{x+h}-\sqrt{x}}{h}$ e) $\frac{\sqrt{3}+\sqrt{4}}{\sqrt{3}-\sqrt{4}}$ (f) $\frac{\sqrt{x}-\sqrt{y}}{y+\sqrt{x}}$

30. Express each of the following in simplest form:

(a) $\frac{\sqrt[3]{45x^2}}{\sqrt[3]{9x}}$ (b) $\sqrt[3]{54} + \sqrt[3]{16} - \sqrt[3]{128}$ (c) $\sqrt[3]{-16x^4y^{-3}}$ (d) $\sqrt{72x^3y^2}$

(e) $\frac{\sqrt{3}}{\sqrt{2}}$ (f) $\sqrt[3]{16x^2} + \sqrt[3]{250x^2} - \sqrt[3]{128x^2}$