MULUNGUSHI UNIVERSITY

SCHOOL OF SCIENCE, ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND SCIENCES

MSM 111 - Mathematical Methods I

Tutorial Sheet 4 - 2020/2021 - Polynomial Functions of Degree 3 and 4

1. Use long-division to divide

Dividend	Divisor
$x^3 + 6x^2 + 7x + 2$	$x - 1$
$2x^3 + 6x^2 - x + 3$	$2x^2 - 1$
$x^4 + 3x^2 + 1$	$x^2 - 2x + 3$

2. Use synthetic division to divide the polynomials and write the function in the form $p(x) = (x - k)q(x) + r$, where $q(x)$ is the quotient and r is the remainder :-

Dividend	Divisor
$x^3 - 10x^2 + 13x - 30$	$x - 3$
$x^3 + 15x^2 + 68x + 96$	$x + 4$
$6x^3 + x^2 - 21x - 10$	$2x + 1$

3. Factorize the following polynomials completely:

   (i) $x^3 - 2x^2 - 5x + 6$ \qquad (ii) $2x^3 + 3x^2 - 17x + 12$ \qquad $3x^3 + 10x^2 - 27x - 10$

4. Using the remainder theorem, write the function in the form $p(x) = (x - k)q(x) + r$, where $q(x)$ is the quotient and r is the remainder:

   (i) $p(x) = x^3 + x^2 - 12x + 20$, $k = 2$ \qquad (ii) $p(x) = x^3 - 2x^2 - 15x + 7$, $k = -4$

   (ii) $p(x) = x^3 + 2x^2 - 3x - 12$, $k = \sqrt{3}$ \qquad (iv) $p(x) = 3x^3 - 19x^2 + 27x - 7$, $k = 3 - \sqrt{2}$

5. Factorize the polynomial $x^3 - 3x^2 - 4x + 12$. Hence, calculate the range of values of x for which $x^3 - 3x^2 > 4x - 12$.

6. Given that $(x - 1)$ and $(x + 1)$ are the factors of $px^3 + qx^2 - 3x + 7$, find the values of p and q.

7. The expression $2x^3 - x^2 + ax + b$ gives a remainder of 14 when divided $(x - 2)$ and a remainder of -86 when divided by $(x + 3)$. Find the values of a and b.

8. The remainder obtained when $2x^3 + ax^2 - 6x + 1$ is divided by $(x + 2)$ is twice the remainder obtained when the some polynomial is divided by $(x - 1)$. Find the value of a.

9. The polynomial $3x^3 + 2x^2 - px + q$ is divisible by $(x - 1)$ but gives a remainder of 10 when divided by $(x + 1)$. Find the values of p and q.

10. Find the zeros of each of the following polynomial function. Hence, sketch its graph indicating the x and y intercepts:

    (i) $p(x) = x^3 - 2x^2 - 7x + 12$ \qquad (ii) $p(x) = 15 + 5x - 3x^2 - x^3$ \qquad (ii) $p(x) = -x^3 + x^2 + 5x - 2$

    (iv) $p(x) = x^3 + 5x^2 + 6x + 2$ \qquad (v) $p(x) = 2x^3 - 15x^2 + 24x + 16$

11. Find the coordinates of the points where the curve $y = x^3 + 6x^2 + 11x + 6$ cutes the

    (i) y-axis \qquad (ii) x-axis

    Hence, make a sketch of the curve and state the range of values for which $y < 0$.

12. Show that $x - y$ is a factor of $x^5 - y^5$, $x^6 - y^6$, $x^7 - y^7$ by using synthetic division. Find the quotient in each case.

13. Sketch the graph of each of the following rational functions, indicating the intercepts, vertical asymptotes, horizontal asymptotes and slant asymptotes if they exists:

    (i) $f(x) = \frac{-1}{x+1}$ \qquad (ii) $f(x) = \frac{x}{x^2-3x-4}$ \qquad (iii) $\frac{3x^2-12}{x^2+3x-4}$ \qquad (iii) $\frac{2x^2+1}{x}$

    Indicate the domain and range in each case.