THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS
2016 ACADEMIC YEAR

MA110 - Mathematical methods

INSTRUCTIONS **TIME ALLOWED: 3 hours**

(1)You must write your **NAME,POROGRAM, COMPUTER NUMBER AND GROUP on the cover of your answer sheet.**

(2)There are five questions in this paper.Attempt all Questions.

(3) Show all neccessary working and **number the pages** in your answer sheet

(4) **Calculators are NOT allowed in this paper**

1.(a) (i) Express $2.07\overline{2}$ as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$.

(ii) Express $\frac{1-\sqrt{3}}{2\sqrt{3}+1}$ in the form $a+b\sqrt{3}$ where a and b are rational numbers.

(b) (i) Sketch the graph of $f(x) = \begin{cases} 2x+3 & if\ x<0 \\ x^2 & if\ 0 \leq x < 2. \\ 1 & if\ x \geq 2 \end{cases}$

(ii)Let $f(x) = \frac{x}{x+2}$ and $g(x) = 2x-1$. Verify that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

(c) Given that $\sqrt{7}$ is an irrational number, prove that $2+\sqrt{7}$ is also an irrational number.

(d) State and prove one of de-Morgan's laws.

2. (a) Given that A, B and C are sets, simplify the following if possible
$[(A \cap B)' \cap (A' \cup B)]'$

(b) Express $f(x) = 2x^2 - 3x - 4$ in the form $f(x) = a(x+p)^2 + q$ where a, p and q are constants indicating the axis of symmetry and the coordinates of its maximum or the minimum point.

(c) Define an operation *on the set of real numbers by $a * b = a + b - 2\sqrt{ab}$

i). Is $*$ a binary operation on the set of real numbers ? Give reason for your answer.

ii). Evaluate $(1 * -1) * 2$ and $1 * (-1 * 2)$ and state whether $*$ is associative.

(d) (i) If $\sqrt{z} = \frac{3}{1+2i} + 4 - 3i$, find z in the form $x+iy$ where x, y are real numbers.

(ii) Determine whether the function $f(x) = \frac{x}{\sqrt{x^2+4}}$ is one-to-one . If it is, find the inverse graph both the function and its inverse

3.(a) Using synthetic division find the quotient and the remainder when $f(x) = x^3 + 2x^2 + x - 2$ is divided by $x - (1 + i)$.

(b) Let **R**, the set of real numbers be the universal set. If $A = [-7, 8) \cup [11, \infty)$ and $B = [0, 20]$, find the following sets and display them on the number line:

(i) A'. (ii) $A \cap B$.

(c) Let α and β be the roots of the quadratic equation $4x^2 + 3x - 2 = 0$

(i) Find the sum $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ (ii) Find a quadratic equation whose roots are α^2 and β^2

(d)Sketch the graph of the function $k(x) = |2x - 1| - |x + 2|$

4 (a)Solve each of the following inequality/equation.

(i) $|x - 1| > 1 - x^2$ (ii) $2x^{2/5} - 11x^{1/5} + 12 = 0$

(b) Given the rational function $f(x) = \frac{x^2 + 2}{x - 1}$. Sketch its graph indicating its domain and range, all the asymptotes and intercepts.

(c)Determine the domain of the following functions:

i) $f(x) = x^3 + \sqrt{x^2 - 2x - 24}$ (ii) $f(x) = \sqrt{\frac{x+1}{x-1}}$

(d) Let $z = x + iy$ be a non zero complex number. Given that $z + \frac{1}{z} = k$, where k is a real number, prove that $|z| = 1$

5. (a) Solve the inequality $\left|\frac{x+1}{x^2+2x+1}\right| \leq \frac{1}{2}$

(b) Sketch the graph of the function $f(x) = 1 + \sqrt{\frac{x}{2}}$ and find its domain and range.

(c)(i)Solve the polynomial equation below

$x^3 - 10x - 12 = 0$.

(ii) Rationalize the denominator $\frac{5\sqrt[3]{z^2}}{4\sqrt[5]{x}}$ and express the final answer in simplest radical form.

(d) (i)If the equation $x^2 - (p - 2)x + 1 = p(x - 2)$ is satisfied by only one value of x, What are the possible values of p.

(ii) Simplify $\frac{2^n - 6^n}{1 - 3^n}$