THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 4: **MA110-Mathematical Methods** 2022

1.(a)Solve quadratic equations using factorization

(i) $4x^2 - 16x + 15 = 0$, (ii) $x^2 + 10x + 25 = 0$ (iii) $x^2 - 10x + 24 = 0$

(b) Solve quadratic equations by completing the square.

(i) $2x^2 + 2x + 5 = 0$ (ii) $5x^2 + 1 = 2x$ (iii) $10 = 3x - x^2$ (iv) $4x^2 - x = 8$

(c) Solve the following quadratic equations by using the quadratic formula, giving the solutions in simplified Surd form.

(i) $5x^2 + 2x + 1 = 0$ (ii) $7x^2 + 9x + 1 = 0$ (iii) $4x^2 - 7x = 2$ (iv) $25z^2 - 30z = -9$

2. (a)Sketch the graphs of the following equations

(i) $y = x^2 + 3x + 2$ (ii) $y = -x^2 + 6x + 7$ (iii) $f(x) = -x^2 + 2x + 5$ (iv) $f(x) = 2x^2 + 2x + 5$

(b)For what values of k will the function $f(x) = x^2 + 6x + k$

(i)cuts the x-axis twice (ii)touch the x-axis (iii)have no x intercepts

(c) For the quadratic $f(x) = 7 + 4x - 2x^2$, find

(i)The equation of the axis of symmetry (ii) Coordinates of the vertex (iii) the x and y intercepts. Hence ,sketch the graph of the function.

3. Solve the following pairs of simultaneous equations:

(i) $x + y = 6$, $x^2 + y^2 = 26$ (ii) $x + 2y = 7, x^2 - 4x + y^2 = 1$

(iii) $\frac{x}{3} - \frac{y}{2} = 1, \frac{3}{x} + \frac{2}{y} = \frac{3}{2}$ (iv) $\frac{x}{4} - \frac{y}{3} = 1$, $\frac{16}{x} + \frac{3}{y} = 3$

4.a) Given that for all values of x :

$3x^2 + 12x + 5 = p(x + q)^2 + r$. Find the values of p, q and r

b) Find, as surds, the roots of the equation:

$2(x + 1)(x - 4) - (x - 2)^2 = 0$

c) The equation $px^2 - 2(p + 3)x + p - 1 = 0$ has real roots. What is the range of values of p?

d) Find the values of k if the equation $x^2 + (k - 2)x + 10 - k = 0$ has equal roots

e) What is the largest value m can have if the roots of $3x^2 - 4x + m = 0$ are real ?

f) Show thàt the equation $a^2x^2 + ax + 1 = 0$ can never have real roots .

g) If the equation $x^2-(p-2)x+1=p(x-2)$ is satisfied by only one value of x, What are the possible values of p

h) What type of roots does the equation $5x^2-3x+1=0$ have ?

i) For what values of k will the $x-$axis be a tangent to the curve $f(x)=kx^2+(1+k)x+k$

j) With these values, find the equations of the curve

5. Show that the solution of $ax^2+bx+c=0$ are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

6. a) Determine the nature of the curves ,find the turning point and sketch the following

(i) $y=-2+2x-x^2$ (ii) $f(x)=2x^2-3x-4$ (iii) $f(x)=x^2+2x-3$ (iv) $f(x)=5-2x-4x^2$

b) If the minimum values of x^2+4x+k is -7 , find the value of k.

c)The function $f(x)=ax^2+bx+c$ has a maximum value of 4 where $x=-1$. find the value of a and b

d)The function $f(x)=1+bx+ax^2$ has a maximum value of 4 where $x=-1$.Find the value of a and b .

e) The function $f(x)=ax^2+bx+c$ has a minimum value of $-5\frac{1}{4}$ where $x=\frac{1}{4}$. and $f(0)=-5$. Find the values of , b and c.

f) Express $5-x-2x^2$ in the form $a-b(x+c)^2$ and hence or otherwise find its maximum value and the value of x where this occurs

7 a) Let α and β be the roots of the quadratic equation $4x^2+3x-2=0$

(i) Find the sum $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

(ii)Find a quadratic equation whose roots are α^2 and β^2

b) The roots of the equation $x^2-px-7=0$ are $\propto$ and β , write down in terms of p an equation whose roots are $\propto^2+p\propto^2$ and $\beta^2+p\beta^2$

c) The roots of the equation $2x^2+6x-15=0$ are $\propto$ and β. Find the value of

(i) $(\propto+\beta)(\beta+1)$ (ii) $\propto^2\beta+\propto\beta^2$ (iii) $(\propto-\beta)^2$ (iv) $\frac{1}{2\propto+\beta}+\frac{1}{\propto+2\beta}$ (v) $\frac{1}{\propto^2+1}+\frac{1}{\beta^2+1}$

(vi) $\frac{1}{\propto^2}+\frac{1}{\beta^2}$ (vii) $\frac{1}{\propto^3}-\frac{1}{\beta^3}$

d) if α and β are the roots of $ax^2+bx+c=0$, Show that $\alpha+\beta=-\frac{b}{a}$ and that $\alpha\beta=\frac{c}{a}$. Hence show that $\propto^2+\beta^2=\frac{b^2-2ac}{a^2}$ and that $\alpha-\beta=\frac{\sqrt{b^2-4ac}}{a}$

f) if α and β are the roots of $3x^2+2x+5=0$, Find new equations with the roots

(i) $3\alpha,3\beta$ (ii) $\frac{1}{\alpha^2},\frac{1}{\beta^2}$ (iii) $\frac{\alpha}{\beta},\frac{\beta}{\alpha}$ (iv) $\frac{1}{\alpha^3},\frac{1}{\beta^3}$

POLYNOMIALS

1. a) Find the remainder when
   i) $4p^3 - 5p^2 + 7p + 1$ is divided by $(p - 2)$ ii) $x^3 + 2x^2 - x - 1$ is divided by $3x + 2$
   iii) $z^3 - 20z + 3$ is divided by $(z - 4)$ iv) $-2y^4 + 2y^2 - y - 5$ is divided by $(y + 2)$
2. Express each of the following polynomials in the form $f(x) = g(x)q(x) + r(x)$ where $q(x)$ is the quotient and $r(x)$ is the remainder when the polynomial $f(x)$ is the dividend and $g(x)$ is the divisor
   i) $5 + 6x + 7x^2 - x^3$ is divided by $x + 1$ ii) $9x^3 + 4$ is divided by $3x + 2$
   iii) $x^4 - 3x^2 + x + 1$ is divided by $(x - 1)$ iv) $x^3 - 8$ is divided by $x - 2$
3. Using synthetic division find the quotient and the remainder when
   i) $x^3 - 2x^2 + 9$ is divided by $(x + 2)$ ii) $x^4 - 2x^3 - 3x^2 - 4x - 8$ is divided by $x - 2$
   iii) $8x^3 - 10x^2 + 7x + 3$ is divided by $2x - 1$ iv) $5x^6 - x^3 - 1$ is divided by $x + 1$
   v) $4x^7 + 3$ is divided by $x - 3$
4. Use the Rational root theorem to solve the following equations
   i) $x^3 - 4x^2 + 8 = 0$ ii) $x^3 - 10x - 12 = 0$ iii) $2x^5 - 5x^4 + x^3 + x^2 - x + 6 = 0$
   iv) $x^4 - 3x^3 + 2x^2 + 2x - 4 = 0$ v) $x^4 + 3x - 2 = 0$
5. Verify that the following equations have no Rational solutions.
   i) $x^4 - x^3 - 8x^2 - 3x + 1 = 0$ ii) $x^4 + 3x - 2 = 0$ iii)$2x^4 - 3x^3 + 6x^2 - 24x + 5 = 0$
6. Find $f(c)$ either by using synthetic division and the remainder theorem or by evaluating $f(c)$ directly
   i) $f(x) = 5x^6 - x^3 - 1$ and $c = -1$ ii) $f(x) = 4x^7 + 3$ and $c = 3$
   iii) $f(n) = -2n^4 + 2n^n - n - 5$ and $c = -2$ iv) $f(t) = 5t^5 - 8t^2 + 9t - 4$ and $c = -5$

7. Use the factor theorem to answer the following
   i) is $x - 2$ a factor of $3x^2 - 4x - 4$? ii) is -3 a factor of $2x^3 - 3x^2 - 10x + 3$?
   iii) is $x - 3$ a factor of $x^4 - 81$? iv) is $x - 2$ a factor of $x^3 - 8$?
8. Use synthetic division to show that $g(x)$ is a factor of $f(x)$ and complete the factorization of $f(x)$
   i) $g(x) = x + 2$; $f(x) = x^3 + 7x^2 + 4x - 12$ ii) $g(x) = x + 1$; $f(x) = x^3 - 2x^2 - 7x - 4$
   iii) $g(x) = x - 3$; $f(x) = 6x^3 - 17x^2 - 5x + 6$ iv) $g(x) = x - 5$; $f(x) = 2x^3 + x^2 - 61x + 30$

9 i) Let $g(x) = x^3 + ax^2 + 3x + 6$. Given $g(-1) = 2$, Find the remainder when $g(x)$ is divided by $(3x - 2)$.
   ii) The expression $3x^3 + 2x^2 - px + q$ is divided by $(x - 1)$ but leaves a remainder of 10 when divided by $(x + 1)$. Find the values of a and b.
   iii) Solve the equation $x^3 - 7x + 6 = 0$. Hence state the solution of the equation

$(x-2)^2 - 7(x-2) + 6 = 0$

iv) Find the remainder in terms of p *when* $x^3 + px^2 - x - 2$ is divided by $x + 3$

10. Factorize each of the following polynomials

i) $x^3 - 2x^2 - 5x + 6$ ii) $3x^3 + 2x^2 - 3x - 2$ iii) $x^4 - 1$ iv)$x^3 - 10x - 12$

11. Find the value(s) of k that makes the second polynomial a factor of the first

i) $x^3 - kx^2 + 5x + k; x - 2$ ii) $kx^3 + 19x^2 + x - 6;\ x + 3$ iii) $k^2x^4 + 3kx^2 - 4;\ x - 1$

iv) $x^3 + 4x^2 - 11x + k\ ; x + 2$

12. The remainder and factor theorem are true for any complex value of □.

Find □

a) by using synthetic division and the remainder theorem and

b) by evaluating $f(c)$ directly.

i) $f(x) = x^3 - 5x^2 + 2x + 1$ and $c = i$

ii) $f(x) = x^3 + 2x^2 + x - 2$ and $c = 2 - 3i$

iii) $f(x) = x^2 + 4x - 2$ and $c = 1 + i$

c) I) Show that $x - 2i$ is a factor of $f(x) = x^4 + 6x + 8$

ii) Show that $x + 3i$ is a factor of $f(x) = x^4 + 14x^2 + 4513$

Given that $2x^3 - 7x^2 + 7x - 5 = A(x-1)^3 + Bx(x-1) + C$ for all values of x, find the values A, B and C.

LINEAR, QUADRATIC AND RATIONAL INEQUALITIES AND EQUATIONS

1..Solve each of the following equations

i) $\frac{3x}{2x-1} - 4 = \frac{x}{2x-1}$ ii) $\frac{6}{x+3} + \frac{20}{x^2+x-6} = \frac{5}{x-2}$ iii) $\frac{4}{x-2} + \frac{x}{x+1} = \frac{x^2-2}{x^2-x-2}$ iv) $\frac{3y}{y^2+y-6} + \frac{2}{y^2+4y+3} = \frac{y}{y^2-y-2}$

v)$\frac{-1}{2x-5} + \frac{2x-4}{4x^2-25} = \frac{5}{6x+15}$

2. Solve each of the following equations

i)$\sqrt[3]{2x+3} + 3 = 0$ ii) $n^{-2} = n^{-3}$ iii) $x^{3/2} = 4x$ iv) $\sqrt{1 + 2\sqrt{x}} = \sqrt{x+1}$

v) $\sqrt{2x-1} - \sqrt{x+3} = 1$ vi) $p = \sqrt{-4p+17} + 3$ vii) $\sqrt{-2x-7} + \sqrt{x+9} = \sqrt{8-x}$

viii) $x^4 - 25x^2 + 144 = 0$ xi) $x^{2/3} + x^{1/3} - 2 = 0$ xiii) $12t^{-2} - 17t^{-1} - 5 = 0$

xiv) $x^{-2} + 4x^{-1} - 12 = 0$ xv) $2x - 11\sqrt{x} + 12 = 0$ xvi) $x + 3\sqrt{x} - 10 = 0$

3.Solve each of the following inequalities.

i) $\frac{4x-3}{6}+\frac{2x-1}{12}>\frac{2}{15}$ ii) $-3\leq\frac{4x+3}{2}\leq 1$ iii) $\frac{x}{2}-\frac{x-1}{5}\geq\frac{x+2}{10}-4$ iv) $-2\leq\frac{5-3x}{4}\leq\frac{1}{2}$

iv) $3\geq\frac{7-x}{2}\geq 1$

4.Find the set of values of x for which

i) $2x-3<5$ ii) $5x+6\leq -12-x$ iii) $x(5-x)\geq 3+x-x^2$

iv) $2(x-5)\geq 3(4-x)$ v) $1+11(2-x)<10(x-4)$

5. Find the set of values of x for which

(i) $3(x-2)>x-4$ and $4x+12>2x+17$ (ii) $15-x<2(11-x)$ and $5(3x-1)>12x+19$

(iii) $3x+8\leq 20$ and $2(3x-7)\geq x+6$

6.Find the set of values of x for which

i) $x^2-11x+24<0$ ii) $x^2+7x+12\geq 0$ iii) $11<x^2+10$ v vi) $x(x+11)<3(1-x^2)$

7.Find the set of values of x for which

i) $x^2-7x+10<0$ and $3x+5<17$ ii) $4x^2-3x-1<0$ and $4(x+2)<15-(x+7)$

iii) $x^2-x-6>0$ and $10-2x<5$ iv) $x^2-2x-3<0$ and $x^2-3x+2>0$

8.Solve each of the following inequalities, expressing the set of solution sets in interval notation.

i) $\frac{x+2}{x+4}\leq 0$ ii) $\frac{3x+2}{x-1}>0$ iii) $\frac{x}{x-1}>2$ iv) $\frac{1}{x-2}<\frac{1}{x+3}$ v) $\frac{2}{x+1}>\frac{3}{x-4}$

9. Solve each of the following equations

i) $\left|\frac{3}{k-1}\right|=4$ ii) $\left|x+\frac{1}{4}\right|=\frac{2}{5}$ iii) $|3x-1|=|2x+3|$

iv) $|-4n+5|=|-3n-5|$ v) $|-2n+1|=|-3n-1|$ vi) $\left|\frac{-2}{n+3}\right|=5$ vii) $\left|\frac{x+1}{x-2}\right|=-2$

10.Solve each of the following inequalities, expressing the set of solution sets in interval notation

i) $|x|\geq 4$ ii) $|2x-1|\leq 7$ iii) $|t-3|>5$ iv) $|x-1|+2<4$ v) $|x+4|-1>1$ vi) $\left|\frac{x+1}{x-4}\right|<3$ vii) $\left|\frac{x+4}{x-5}\right|\geq 3$ viii) $\left|\frac{n+2}{n}\right|\geq 4$ ix) $|x-1|>1-x^2$ x) $|x+1|+|x-2|\leq 5$

xi) $\left|\frac{x+1}{x^2+2x+2}\right|\leq\frac{1}{2}$ xii) $2x-x^2\geq|x-1|-1$ xiii) $|3-|x||\leq|3-{}^1/_3x^2|$

11.On the same diagram, draw the graphs of $y=|3x|$ and $y=|x-3|$ for the domain $-2\leq x\leq 3$. Hence solve the equation $|3x|=|x-3|$.

12. The range of the function $y=|x-1|$ is $0\leq y\leq 3$. Find a possible domain. What is the Widest Possible domain?

13 .Redefine each of the following modulus functions by removing the modulus, hence sketch the graph of each function:

i) $f(x) = -2|5x - 4|$ ii)$h(x) = |3x + 1| + |2x - 3|$ (iii) $k(x) = |2x - 1| - |x + 2|$

14.(a) Sketch the following modulus functions and determine their domain and range

i) $f(x) = |2x - 1| + 3$ ii) $f(x) = |x^2 + 5x + 4| - 2$ iii) $f(x) = |-3x^2 - 2x + 1| + 1$

iv)$f(x) = |x^2 + x - 6| - 3$ v) $f(x) = |-3x + 1| + 1$

(b)For each of the following functions $y = f(x)$, sketch the graph of $y = f(|x|)$ and determine their domain and range

(i) $f(x) = x^2 - 4x$ (ii) $f(x) = 1 + \sqrt{x+2}$ (iii) $f(x) = 3 + (x-2)^2$ (iv) $f(x) = \begin{cases} \dfrac{1}{x-1}, x < 1 \\ \sqrt{x} + 2, x \le 1 \end{cases}$

(v) $f(x) = \begin{cases} |x^2 - 2x|, x \ge 1 \\ |x| - 1, x < 1 \end{cases}$ (vi) $f(x) = x|x|$

APPLICATIONS OF QUADRATIC EQUATIONS

15. What are the dimensions of the largest rectangular field which can be enclosed by 1200 m of fencing?

16. A window is to be constructed in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 540 cm, find its dimensions for maximum area.

17. If the profit p in the manufacturing and sale of x units of a product is given by

$$P(x) = 200x - 0.001x^2,$$

(i) Find the number x that yields the maximum profit.

(ii) Find the maximum profit if each item is sold at K 1.50

(iii) Sketch the graph of the function P

18. Solve each of the following inequalities involving radical functions.

(a) $10 - \sqrt{2x + 7} \le 3$

(b) $3 \le \sqrt{2x + 5} < 6$

(c) $\sqrt{2x + 9} - \sqrt{9 + x} > 0$

(d) $\sqrt{2} - \sqrt{x + 6} \le -\sqrt{x}$

(e) $\sqrt{x - 3} > \sqrt{x + 4} - 1$

(f) $3 + \sqrt{2x - 7} \le 6$

(g) $\sqrt{2x+5} < \sqrt{9+x}$

(h) $\sqrt{x+3} + \sqrt{x+7} > 4$