THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 2 : **MA110 - Mathematical Methods** 2022

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1. Evaluate each of the following using the definition of Absolute value.

   a) $|x-2|=6$ b) $|2n+1|=11$ c) $\left|\frac{3}{k-1}\right|=4$ d) $\left|x-\frac{2}{3}\right|=\frac{3}{4}$ e) $|-4|$ f) $|4|$

   g) $|2x-3|\leq 5$ h) $|5x-4|\leq 8$

2. State the property that justifies each of the statements

   a) $x(2)=2(x)$ b) (7+4)+6=7+(4+6) c) 1(x)=x d) $\left(\frac{2}{3}\right)\left(\frac{3}{2}\right)=1$

3. Evaluate each of the following if x is a nonzero real number.

   a) $\frac{|x|}{x}$ b) $\frac{x}{|x|}$ c) $\frac{|-x|}{-x}$ d) $|x|-|-x|$

4. Evaluate each of the algebraic expressions for the given values of the variables

   a) $|x-y|-|x+y|; if\ x=-4\ and\ y=-7$

   b) $|3x+y|+|2x-4y|;\ if\ x=5\ and\ y=-3$

   c) $\left|\frac{x-y}{y-x}\right|\ if\ x=-6\ and\ y=13$

   d) $\left|\frac{2a-3b}{3b-2a}\right|\ if\ a=-4\ and\ b=-8$

5. Evaluate each of the following numerical expressions

   a) $(3^{-4}+4^{-1})^{-1}$ b) $2^{-3}+3^{-1}$ c) $\left(\frac{2^{-1}}{3^{-3}}\right)^{-2}$ d) $\left(\frac{3}{4}\right)^{-1}-\left(\frac{2}{3}\right)^{-1}$

6. Simplify each of the following ; express final results without using zero or negative integers as exponents

   a) $(a^2b^{-1}c^{-2})^{-4}$ b) $\left(\frac{x^{-2}}{y^{-3}}\right)^{-2}$ c) $\left(\frac{3x^2y}{4a^{-1}b^{-3}}\right)^{-1}$ d) $\left(\frac{24x^5y^{-5}}{-8x^6y^{-1}}\right)^{-3}$

7. Evaluate each of the following in simplest radical form. All variables represent positive real number

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a) $\sqrt{64x^4y^7}$ b) $\sqrt[3]{81x^5y^6}$ c) $\frac{\sqrt[3]{12xy}}{\sqrt[3]{3x^2y^5}}$ d) $\sqrt[4]{162x^6y^7}$ e) $\frac{\sqrt[3]{2y}}{\sqrt[3]{3x}}$ f) $\sqrt[3]{\frac{5}{2x}}$

8. Simplify the following

a) $2\sqrt{28}-3\sqrt{63}+8\sqrt{7}$ b) $4\sqrt[3]{2}+2\sqrt[3]{16}-\sqrt[3]{54}$ c) $\frac{2\sqrt{8}}{3}-\frac{3\sqrt{18}}{5}-\frac{\sqrt{50}}{2}$ d) $\frac{3\sqrt[3]{54}}{2}+\frac{5\sqrt[3]{16}}{3}$

e) $4\sqrt{50}-9\sqrt{32}$ f) $5\sqrt{12}+2\sqrt{3}$

9 . Multiply and express the results in simplest radical form. All variables represent non-Negative real numbers

a) $2\sqrt{3}(5\sqrt{2}+4\sqrt{10})$ b) $(2\sqrt{x}-3\sqrt{y})^2$ c) $(3\sqrt{x}+5\sqrt{y})(3\sqrt{x}-5\sqrt{y})$

d) $\sqrt{6y}(\sqrt{8x}+\sqrt{10y^2})$ e) $(\sqrt{x}+\sqrt{y})^2$

10. For each of the following, rationalize the denominator and simplify. All variables Represent positive real numbers.

a) $\frac{3}{\sqrt{5}+2}$ b) $\frac{\sqrt{x}}{\sqrt{x}-1}$ c) $\frac{5}{5\sqrt{2}-3\sqrt{5}}$ d) $\frac{3\sqrt{x}-2\sqrt{y}}{2\sqrt{x}+5\sqrt{y}}$ e) $\frac{5}{3-2\sqrt{3}}$

f) $\frac{7}{\sqrt{10}-3}$ g) $\frac{\sqrt{x}}{\sqrt{x}+2}$ h) $\frac{2\sqrt{x}}{\sqrt{x}-\sqrt{y}}$

11. Evaluate each of the following

a) $-8^{2/3}$ b) $-16^{5/4}$ c) $(0.01)^{3/2}$ d) $\left(\frac{1}{27}\right)^{-2/3}$

12. Perform the indicated operations and express the answers in simplest radical form.

a) $\frac{\sqrt[3]{16}}{\sqrt[6]{4}}$ b) $\frac{\sqrt[4]{x^9}}{\sqrt[3]{x^2}}$ c) $\sqrt{ab}\sqrt[3]{a^4b^5}$ d) $\sqrt[3]{x}\sqrt[5]{x^3}$

13. Rationalize the denominators and express the final answers in simplest radical form.

a) $\frac{5}{\sqrt[3]{x}}$ b) $\frac{2\sqrt{x}}{3\sqrt[3]{y}}$ c) $\frac{5\sqrt[3]{y^2}}{4\sqrt[4]{x}}$ d) $\frac{\sqrt{xy}}{\sqrt[3]{a^2b}}$

e) $\frac{\sqrt[3]{x}}{\sqrt{y}}$ f) $\frac{\sqrt[4]{x}}{\sqrt{y}}$ g) $\frac{3}{\sqrt[3]{x^2}}$

14. Simplify each of the following, expressing the final result as one radical.

a) $\sqrt{\sqrt[3]{2}}$ b) $\sqrt[3]{\sqrt[4]{3}}$ c) $\sqrt[3]{\sqrt{x^3}}$ d) $\sqrt{\sqrt[3]{x^4}}$

15. Add or subtract as indicated

a) $(5+3i)+(7-2i)(-8-i)$ b) $\left(4+i\sqrt{3}\right)+\left(-6-2i\sqrt{3}\right)$

c) $(5-7i)-(6-2i)-(1-2i)$ d) $\left(\frac{5}{8}+\frac{1}{2}i\right)-\left(\frac{7}{8}+\frac{1}{5}i\right)$

16. Write each of the following in terms of i, perform the indicated operations , and Simplify if possible.

a) $\sqrt{-4}\sqrt{-16}$ b) $\sqrt{-25}\sqrt{-9}$ c) $\frac{\sqrt{-36}}{\sqrt{-4}}$ d) $\frac{\sqrt{-64}}{\sqrt{-16}}$ f) $\frac{\sqrt{-18}}{\sqrt{-3}}$

17. Find each of the following products and express the answers in standard form

a) $(-2+5i)^2$ b) $(5+3i)(5-3i)$ c) $(1+i)(2-i)$ d) $(5i)(2+6i)$ e)$(-5i)(8i)$

f) $(5i)(2+6i)$

18. Find each of the following quotients and express the answers in standard form

a) $\frac{2+3i}{3i}$ b) $\frac{3-5i}{4i}$ c) $\frac{4+7i}{2-3i}$ d) $\frac{3-7i}{4i+2}$

e) $\frac{1+\sqrt{2}i}{\sqrt{3}-2i}$ f) $\frac{1+2i}{1-i}+\frac{1-2i}{1+3i}$

19. Plot each complex number and find its absolute value

a) $3+4i$ b)-4 c) $\frac{3}{5}-\frac{4}{5}i$ d) $-5i$ e) $1-2i$ f) $3-2i$

e) $\frac{1}{(2+i)(\sqrt{3}-2i)}$ f) $5-4i+\frac{5}{3-4i}$

20. Let $z_1=2+i$, $z_2=1-i\sqrt{3}$ and $z_3=3+4i$. Verify the following identities

(i) $\overline{z_1z_2}=\overline{z_1}\cdot\overline{z_2}$ (ii) $z_3\cdot\overline{z_3}=\overline{z_3}\cdot z_3=|z_3|^2$ (iii) $\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\overline{z_1}}{\overline{z_2}}$

21. Solve for x and y given that:

a) $(x+iy)(4i)=8$ b) $\frac{1}{x+iy}+\frac{1}{1+3i}=1$

c) $\frac{x}{1+i}-\frac{y}{2-i}=\frac{1-5i}{3-2i}$

22. Let $z=x+iy$ be a non zero complex number

a) Express $\frac{1}{z}$ in the form $a+ib$

b) Given that $z+\frac{1}{z}=k$, where k is a real number, prove that either

z is real or $|z|=1$

23 Express each of the following in the form $a+ib$ where a and b are real numbers:

a) $\frac{1}{i^3}$ b) i^{15} c) i^{1002}

24 . a) Express $\frac{\sqrt{3}+1}{\sqrt{3}-1}+\sqrt{3}-1$ in the form $a+b\sqrt{3}$ where a and b are rational numbers.

b) Rationalize the denominator of each of the following:

(i) $\frac{2\sqrt{3}-\sqrt{2}}{4\sqrt{3}}$ (ii) $\frac{x}{x+\sqrt{y}}$ (iii) $\frac{2\sqrt{7}+\sqrt{3}}{3\sqrt{7}-\sqrt{3}}$

(iv) $\frac{x-\sqrt{x^2-9}}{x+\sqrt{x^2-9}}$ (v) $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$

(c) Rationalize the numerator in each of the following:

(i) $\frac{\sqrt{5+h}-3}{h}$ (ii) $\frac{\sqrt{3}+\sqrt{5}}{7}$ (iii) $\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}+\sqrt{x+h}}$

25a) Give a reason why $Z-a\ set\ of\ integers$, $N-a\ set\ of natural\ numbers$ are not Fields while $Q-a\ set\ of\ rational\ number, R-a\ set\ of\ real\ numbers\ and\ C-a\ set\ of\ complex\ numbers\ are\ Fields.$

b) Prove that if $a+c=b+c\ then\ a=b$ when $a,b,c\in R$

c) Prove that if $ac=bc\ then\ a=b$ when $a,b,c\in R$ and $c\neq 0$