CHAPTER 2
SCALARS AND VECTORS

The concept of scalars and vectors is an extremely important part of physics. Physical quantities, in general, may be divided into two main classes: **Scalar quantities and vector quantities.**

2.1 Scalar Quantities

Quantities that have only magnitude and have no direction are called scalar quantities. You can specify them by a number and a unit. Scalar quantities obey the ordinary rule of algebra. Examples of scalars include speed, distance, temperature, electric current, work, etc.

2.2 Vector quantities

Vector quantities are quantities that have both magnitude and direction. They are added and subtracted according to special laws such as the parallelogram law of addition and triangle law of addition. Examples of vectors include force, velocity, acceleration, electric field intensity, displacement, etc.

2.3 Representation of a vector

We can represent a vector in either print or graphically.

2.3.1 In print

In print, a vector quantity is represented by a bold letter such as $\mathbf{A}$ or $\boldsymbol{a}$ and in handwriting it is represented by $\vec{A}$ or $\vec{a}$. The magnitude of a vector $\vec{A}$ is written as $|\vec{A}|$ or simply A and is called modulus of the vector $\vec{A}$.

2.3.2 Graphical

A vector is represented graphically by a straight line with an arrow head denoting the direction of the vector. The length of the line denoting the magnitude.

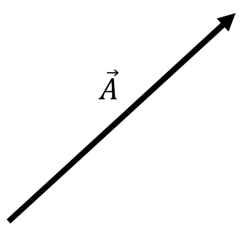

Figure 2.1 *Graphical representation of a vector*

2.4 Some important definitions about vectors

2.4.1 Equal vectors

Two vectors, say $\vec{A}$ and $\vec{B}$ are said to be equal if they have same magnitude i.e. $|\vec{A}| = |\vec{B}|$ and point in the same direction, as shown in figure 2.2

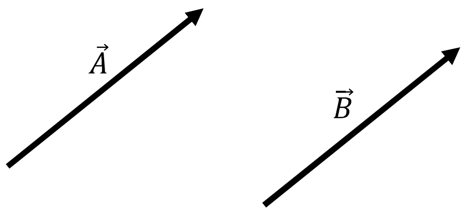

Figure 2.2 *Two equal vectors*

2.4.2 Co-planar vectors

Vectors which are confined to the same plane are called co-planar vectors, as shown in figure 2.3.

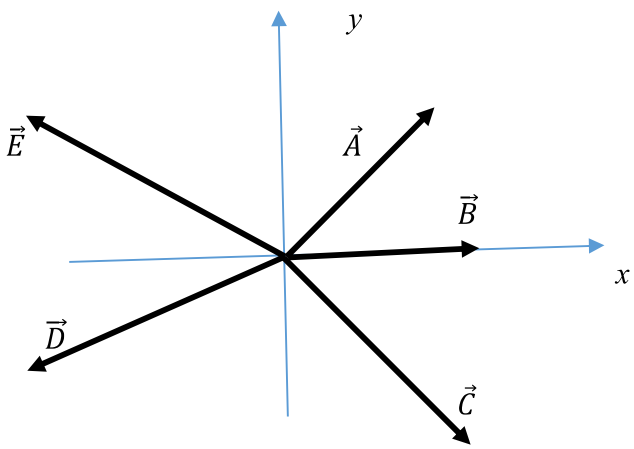

Figure 2.3 *Co-planar vectors* $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$ *and* $\vec{E}$ *lying in xy plane*

2.4.3 Negative vector

The negative of a given vector say $\vec{A}$ is a vector with the same magnitude but points in the opposite direction to that of a given vector.

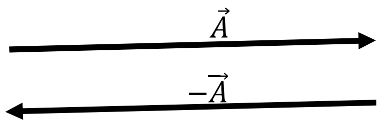

Figure 2.4 *Vector* $\vec{A}$ *and its corresponding negative vector* $\vec{A}$*, both of which have same magnitude but are in opposite direction*

2.4.4 Unit vector

A unit vector is a dimensionless vector that has a magnitude of exactly one and points in a particular direction. It is used to specify a given direction. We can define a unit vector $\vec{n}$ along any vector, say $\vec{A}$ as follows:

$$\vec{n} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$$

2.4.5 Position vector

A vector representing the position of a point with respect to an arbitrary origin is called position vector. In the diagram below, the position vector of ***P*** with respect to ***O*** is represented by $\overrightarrow{OP}$, where ***O*** is the origin and ***P*** is a point in space.

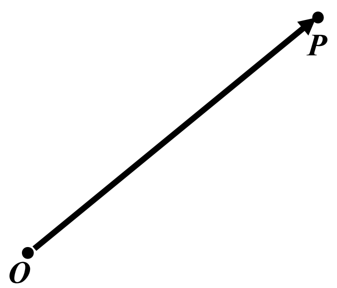

Figure 2.5 *Position vector* $\overrightarrow{OP}$

2.4.6 Zero vector

Any vector whose magnitude is zero is called a zero or simply null vector. It is represented by $\vec{0}$.

2.4.7 Parallel vectors

If $\vec{A}$ and $\vec{B}$ are parallel vectors, then

$$\vec{A} = k\vec{B}$$

and the magnitude of $\vec{A}$ being k times the magnitude of $\vec{B}$.

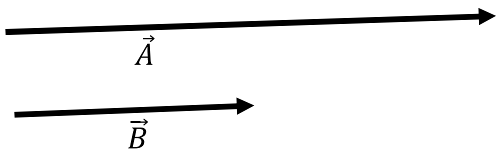

Figure 2.6 *Parallel vectors* $\vec{A}$ *and* $\vec{B}$

2.5 Addition of vectors

All vectors involved in any addition process must have same units. The rules for vector sums can be illustrated by using graphical method.

2.5.1 Triangle method

Given two vectors say $\vec{A}$ and $\vec{B}$

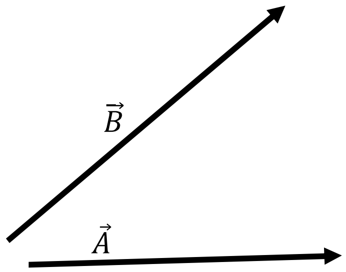

Figure 2.7 *Vectors $\vec{A}$ and $\vec{B}$*

To add vector $\vec{B}$ to vector $\vec{A}$, we first draw vector $\vec{A}$ on a graph paper with its magnitude represented by a convenient scale, and then draw vector $\vec{B}$ to the same scale with its tail coinciding with arrow head of vector $\vec{A}$. Thus, the resultant vector $\vec{R}$ is the vector drawn from tail of vector $\vec{A}$ to the head of vector $\vec{B}$.

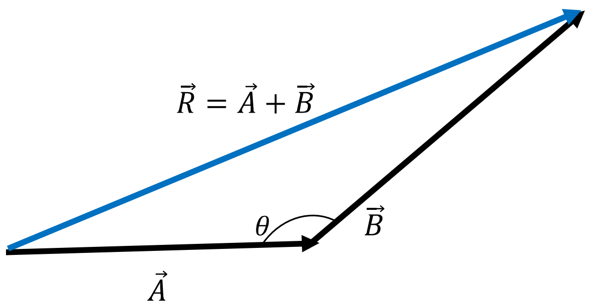

Figure 2.8 *In the triangle method of addition, the resultant vector $\vec{R}$ is the vector drawn from tail of vector $\vec{A}$ to the head of vector $\vec{B}$.*

From cosine rule, we get

$$|\vec{R}| = \sqrt{\left|\vec{A}\right|^2 + \left|\vec{B}\right|^2 - 2\left|\vec{A}\right|\left|\vec{B}\right|\cos\theta}$$

2.5.2 Parallelogram method

An alternative graphical method for adding two vectors is the parallelogram rule of addition. In this method, we superpose the tails of two vectors $\vec{A}$ and $\vec{B}$. If two adjacent sides of a parallelogram represent two given vectors, then the diagonal starting from the point of intersection of the vectors represents the vector sum, as shown in figure 2.9.

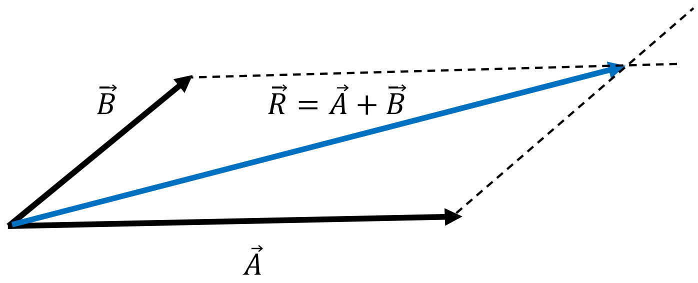

Figure 2.9 *In the parallelogram method of addition, the resultant vector $\vec{R}$ is the diagonal starting from the point of intersection of the vectors.*

2.6 Subtraction of two vectors

To subtract vector $\vec{B}$ from vector $\vec{A}$, we add the negative of vector $\vec{B}$ to vector $\vec{A}$, as follows;

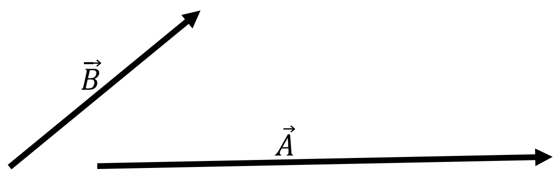

Figure 2.10 *Vectors $\vec{A}$ and $\vec{B}$*

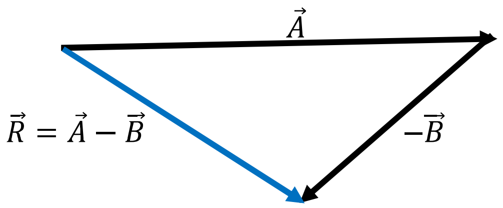

Figure 2.11 *To subtract vector $\vec{B}$ from vector $\vec{A}$, we add the negative of vector $\vec{B}$ to vector $\vec{A}$ to get $\vec{R} = \vec{A} - \vec{B}$.*

2.7 Components of a vector

Adding vectors graphically is not recommended in situations where high precision is needed or in three-dimensional problems. A better way is to make use of the projections of a vector along the axes of a rectangular coordinate system. In a Cartesian coordinate system, the unit vectors along x, y and z directions are represented by $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ and these are called orthogonal unit vectors. For example, $4\boldsymbol{i}$, $6\boldsymbol{j}$ and $2\boldsymbol{k}$ represent vectors of magnitudes 4, 6 and 2 along x, y and z directions respectively.

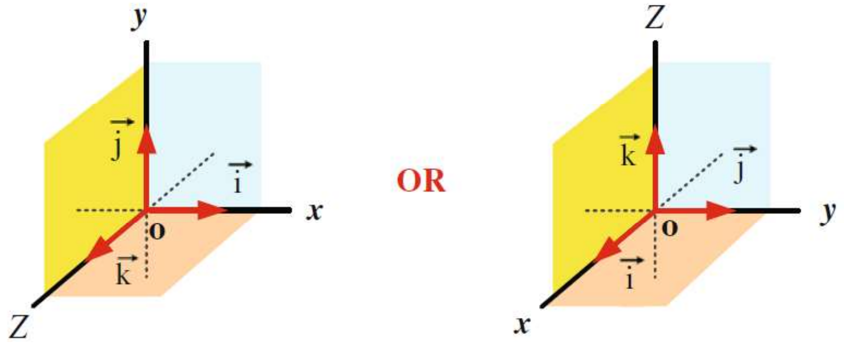

Figure 2.12 *Unit* vectors $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$ *define the direction of the commonly-used right-handed coordinate system*

Consider a vector $\vec{r}$ lying in the xy plane and making an angle θ with the positive x-axis. The vector $\vec{r}$ can be resolved into x and y components as shown in figure 2.13.

Thus $\vec{r} = x\boldsymbol{i} + y\boldsymbol{j}$ or $\vec{r} = x\vec{\imath} + y\vec{\jmath}$

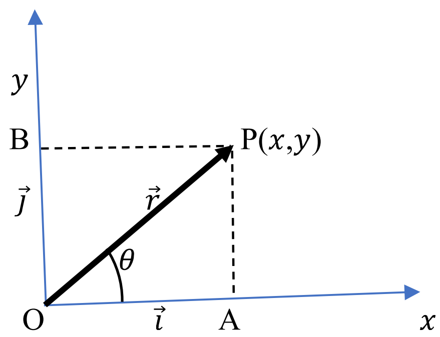

Figure 2.13 *A vector $\vec{r}$ lying in the xy plane can be represented by its rectangular components x and y and unit vectors $\vec{\imath}$ and $\vec{\jmath}$, and can be written as $\vec{r} = x\vec{\imath} + y\vec{\jmath}$*

From the triangle OAP in figure 2.12, we get the following trigonometric relations;

$$\cos\theta = \frac{x}{r} \Leftrightarrow x = r\cos\theta$$

$$\sin\theta = \frac{y}{r} \Leftrightarrow y = r\sin\theta$$

$$\tan\theta = \frac{y}{x} \Leftrightarrow \theta = \tan^{-1}({}^{y}/_{x})$$

In this case, x and y are called Cartesian coordinates; and r and θ are called polar coordinates. By applying Pythagoras theorem to triangle OAP, we obtain the relation

$$r^2 = x^2 + y^2 \iff r = \sqrt{x^2 + y^2}$$

In three dimension the components are

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

The magnitude of $\vec{r}$ is therefore given by

$$|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$$

2.8 Addition of more than two vectors (Composition of vectors)

2.8.1 Graphical method

This may be done with the help of a polygon law of vector addition. A vector polygon is drawn placing the tail end of each succeeding vector at the end of the preceding one. The resultant $\vec{R}$ is drawn with the tail end of the first vector to the head of the last one.

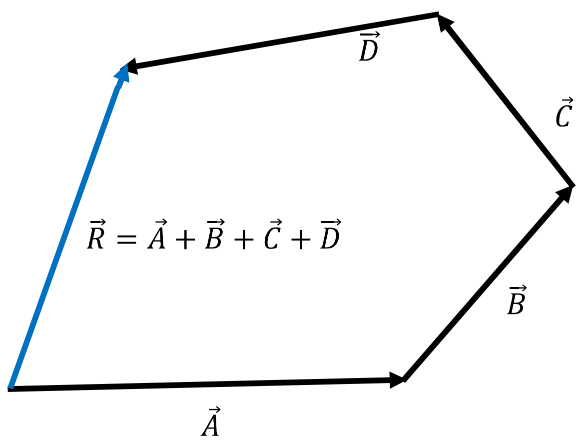

Figure 2.14 *The resultant $\vec{R}$ is drawn with the tail end of the first vector $\vec{A}$ to the head of the last vector $\vec{D}$*

2.8.2 Component method

If the vectors are in the same plane, they can be resolved into two mutually perpendicular directions. Suppose $\vec{A} = A_X\vec{i} + A_Y\vec{j}$, $\vec{B} = B_X\vec{i} + B_Y\vec{j}$, $\vec{C} = C_X\vec{i} + C_Y\vec{j}$ and $\vec{D} = D_X\vec{i} + D_Y\vec{j}$.

Then the resultant vector is given by

$$\vec{R} = R_X\vec{i} + R_Y\vec{j}$$

Where

$$R_X = A_X + B_X + C_X + D_X$$

$$R_Y = A_Y + B_Y + C_Y + D_Y$$

The magnitude of the resultant vector $\vec{R}$ is given by

$$|\vec{R}| = R = \sqrt{{R_X}^2 + {R_Y}^2}$$

The angle that $\vec{R}$ makes with the positive x-axis can be found by using the relation

$$\tan\theta = \frac{R_Y}{R_X} \Leftrightarrow \theta = \tan^{-1}\left(\frac{R_Y}{R_X}\right)$$

This can be generalized to a three-dimensional case.

2.9 Multiplying vectors

2.9.1 Multiplying a vector by a scalar

If we multiply say vector $\vec{A}$ by a scalar a, we get a new vector $\vec{B}$ i.e.

$$\vec{B} = a\vec{A}$$

The vector $\vec{B}$ has the same direction as vector $\vec{A}$ if $a > 0$ but has opposite direction if $a < 0$. In this case, the magnitude of vector $\vec{B}$ is the product of the magnitude of vector $\vec{A}$ and the absolute a.

2.9.2 The scalar (dot) product

The scalar or dot product of two vectors, say $\vec{A}$ and $\vec{B}$, denoted by $\vec{A} \cdot \vec{B}$ is defined as the product of the magnitudes of the two vectors and the cosine of the angle between them. That is

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta = AB\cos\theta$$

In the above relation, A and B are the magnitudes of the two vectors $\vec{A}$ and $\vec{B}$, and θ is the angle between them.

The following are the characteristics of a scalar product

- Scalar product is commutative i.e. $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
- Obeys associative law, i.e. $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
- If $\vec{A} \cdot \vec{B} = 0$, then the two vectors are perpendicular to each other
- If two vectors, say $\vec{A}$ and $\vec{B}$ are parallel, then $\vec{A} \cdot \vec{B} = AB$
- If $\vec{A} = A_X\vec{i} + A_Y\vec{j} + A_Z\vec{k}$ and $\vec{B} = B_X\vec{i} + B_Y\vec{j} + B_Z\vec{k}$, then

$$\vec{A} \cdot \vec{B} = A_X B_X + A_Y B_Y + A_Z B_Z$$

- By applying the definition of scalar or dot product to the unit vectors $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$, we get the following;

$$\vec{\imath}.\vec{\imath} = \vec{\jmath}.\vec{\jmath} = \vec{k}.\vec{k} = 1$$

$$\vec{\imath}.\vec{\jmath} = \vec{\jmath}.\vec{k} = \vec{\imath}.\vec{k} = 0$$

2.9.3 The vector (or cross) product

The vector product of two vectors, say vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A} \times \vec{B}$, and is vector $\vec{C}$ whose magnitude is equal to the product of the magnitude of the two given vectors and the sine of the angle between them.

$$|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta = AB \sin\theta$$

The direction of $\vec{C}$ is perpendicular to the plane that contains both $\vec{A}$ and $\vec{B}$, and can be determined using the right-hand rule. To apply this rule, we allow the tail of $\vec{A}$ to coincide with the tail of $\vec{B}$ then the four fingers of the right hand are pointed along $\vec{A}$ and then "wrapped" into $\vec{B}$ through the angle θ. The direction of the erect right thumb is the direction of $\vec{C}$, i.e. the direction of $\vec{A} \times \vec{B}$. Also, the direction of $\vec{C}$ is determined by the direction of advance of a right-handed screw as shown in figure 2.15.

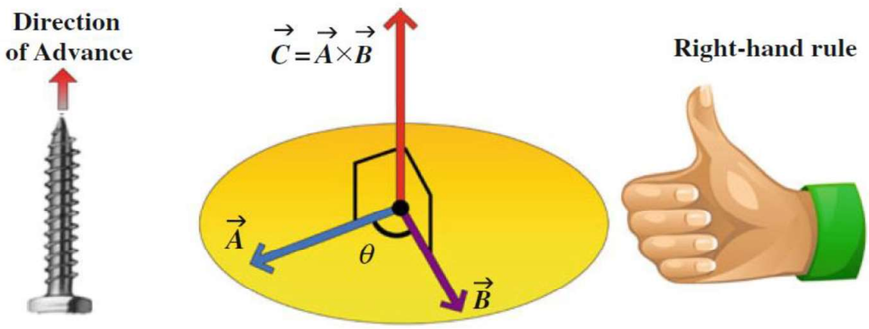

Figure 2.15 *The vector product $\vec{A} \times \vec{B}$ is a third vector $\vec{C}$ that has a magnitude of AB sin θ and a direction perpendicular to the plane containing the vectors $\vec{A}$ and $\vec{B}$. Its sense is determined by the right-hand rule or the direction of advance of a right-handed screw*

The following are the properties of the vector product.

- The vector product of two vectors obey distributive law, i.e.

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

- The vector product of two vectors does not obey the commutative law, i.e.

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \text{ but } \vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

- If $\vec{A}$ is parallel to $\vec{B}$ (i.e. $\theta = 0$) or $\vec{A}$ is antiparallel to $\vec{B}$ ($\theta = \pi$), then
$$\vec{A} \times \vec{B} = 0 \text{ (collinear vectors)}$$
- If $\vec{A}$ perpendicular to $\vec{B}$, then
$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| = AB$$
- From the definition of the vector product and unit vectors $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$, we get the following relationships;
$$\vec{\imath} \times \vec{\imath} = \vec{\jmath} \times \vec{\jmath} = \vec{k} \times \vec{k} = 0$$
$$\vec{\imath} \times \vec{\jmath} = \vec{k}, \vec{\jmath} \times \vec{k} = \vec{\imath}, \vec{k} \times \vec{\imath} = \vec{\jmath}$$
$$\vec{\jmath} \times \vec{\imath} = -\vec{k}, \vec{k} \times \vec{\jmath} = -\vec{\imath}, \vec{\imath} \times \vec{k} = -\vec{\jmath}$$
- When two vectors $\vec{A}$ and $\vec{B}$ are written in terms of unit vectors $\vec{\imath}, \vec{\jmath}$ and $\vec{k}$, then the cross product will give the result

$$\vec{A} \times \vec{B} = (A_X\vec{\imath} + A_Y\vec{\jmath} + A_Z\vec{k}) \times (B_X\vec{\imath} + B_Y\vec{\jmath} + B_Z\vec{k})$$
$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\vec{\imath} + (A_zB_x - A_xB_z)\vec{\jmath} + (A_xB_y - A_yB_x)\vec{k}$$

This result can be expressed in determinant form as follows:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{\mathrm{i}} & \vec{\mathrm{j}} & \vec{\mathrm{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{\mathrm{i}} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \vec{\mathrm{j}} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \vec{\mathrm{k}} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

EXERCISES

1. Given that the Cartesian coordinates of a point are $x = 3m$ and $y = 4m$, what are the polar coordinates of a point? **[r = 5m and $\boldsymbol{\theta} = \mathbf{53.1°}$]**
2. The polar coordinates of a point are $r = 5.50\ m$ and $\theta = 240°$. What are the Cartesian coordinates of this point? **[$\boldsymbol{x = -2.75m}$ and $\boldsymbol{y = -4.76m}$]**
3. A ship is steaming due east at a speed of 12 ms^{-1}. A passenger runs across the deck at a speed of 5 ms^{-1} toward north. What is the resultant velocity of the passenger relative to the sea? **[13m/s in the direction 22°37’ north of east]**
4. A plane is travelling as fast as it can due north at 500 miles per hour (mph). There is a strong wind blowing to the east at 50 mph. What is the maximum speed of the plane when there is no wind? **[502.5 mph]**
5. Vector **B** has x, y, and z components 4, 6, and 3, respectively. Calculate the magnitude of **B** and the angles that **B** makes with the axes of the coordinate system. **[$\mathbf{59°, 40°, 67°}$]**
6. Given the vectors: $\vec{A} = 3\boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$, $\vec{B} = 5\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$ and $\vec{C} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$. Find:
   (a) (i) $\vec{A} + \vec{B}$ **[$\boldsymbol{8i - j + 3k}$]**
   (ii) $\vec{B} - \vec{C}$ **[$\boldsymbol{3i - 4j + 3k}$]**

(b) $\vec{D}$ such that $\vec{A} + \vec{C} + \vec{D} = 0$ **[$-5\boldsymbol{i} - 5\boldsymbol{j}$]**

(c) (i) $\vec{A} \cdot \vec{B}$ **[5]**

(ii) the angle between $\vec{A}$ and $\vec{B}$ **[$81°$]**

(d) $\vec{B} \times \vec{C}$ **[$5\boldsymbol{i} + 12\boldsymbol{j} + 11\boldsymbol{k}$]**

7. Three forces are acting on a body as shown in figure in figure 2.16 where A=10 N, B=20 N, and C=15 N.
   (a) Express each force in terms of $\vec{\imath}$ and $\vec{\jmath}$.**[$\vec{A} = 8.7\,\vec{\imath} - 5\vec{\jmath}$; $\vec{B} = 17.3\vec{\imath} + 10\vec{\jmath}$;$\vec{C} = -15\,\vec{\imath}$]**
   (b) Find the magnitude and the direction of the resultant force
   (i) $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ **[12.1 N, at $24.4°$]**
   (ii) $\vec{D} = \vec{A} - \vec{B} + \vec{C}$ **[28 N, at $212.4°$]**

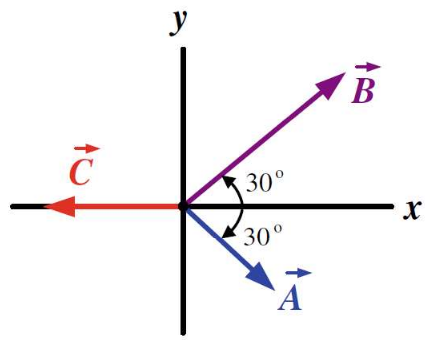

Figure 2.16 *See exercise 7*

8. When vector $\vec{B}$ is added to $\vec{A}$ we get $5\vec{\imath} - \vec{\jmath}$, and when $\vec{B}$ is subtracted from $\vec{A}$ we get $\vec{\imath} - 7\vec{\jmath}$. What is the magnitude and direction of $\vec{A}$? **[5 units, at $307°$ above the +x-axis]**
9. Vector **A** has a magnitude of 10 units and makes 60° with the positive x-axis. Vector **B** has a magnitude of 5 units and is directed along the negative x-axis. Find the vector
   (a) sum $\boldsymbol{A} + \boldsymbol{B}$ **[8.66 units at 90°]**
   (b) difference $\boldsymbol{A} - \boldsymbol{B}$ **[13.2 units at 41°]**
10. Jelita is reading a treasure map's instructions that says to walk 200 m due east. Then it says to walk 150 m 45° north of west. What direction could Jelita walk to get to the treasure quicker? How far is the treasure away? **[141.7 m, $41.5°$ north of east]**
11. You find yourself pacing, in a deep thought about a physics problem. First you walk 12 meters due east. Then, you walk 6 meters due north. Then you doze off and find yourself 50 meters from your starting place, 30° north of east. How far did you walk while you were not paying attention? **[36.6 meters]**
12. Mulenga walks 20 feet, 20° north of east. He then walks 32 feet, 40° north of west. Then 10 feet, 5° south of west. Then 65 feet, 69° south of east. What is the magnitude and direction of his resultant displacement? **[35 feet, $77°$ south of east]**
13. Fully describe the resultant of the following combination of vectors: 20 N at $\theta = 90°$, 30 N at $\theta = 210°$, 25 N at $\theta = -25°$ and 10 N at $\theta = 180°$ **[14.4 N at $\boldsymbol{\theta} = 202.7°$]**

14. Obtain a unit vector perpendicular to the vectors: $\vec{A} = 2\vec{\imath} + 3\vec{\jmath} + 4\vec{k}$; $\vec{B} = \vec{\imath} - 2\vec{\jmath} + 3\vec{k}$ $\left[\vec{c} = \frac{1}{\sqrt{342}}\left(17\vec{\imath} - 2\vec{\jmath} - 7\vec{k}\right)\right]$
15. Three vectors **A, B** and **C** are coplanar vectors acting at a single point. Vector **A** is 20 N directed at 60° west of north, vector **B** is 15 N directed south wards and vector **C** has magnitude 10 N acting at 30° north of east. Find magnitude and direction of the resultant of their combination. **[$5\sqrt{2}$ N, directed westwards]**
16. A radar device detects a rocket approaching directly from east due west. At one instant, the rocket was observed 10 km away and making an angle of 30°above the horizon. At another instant the rocket was observed at an angle of 150°in the vertical east-west plane while the rocket was 8 km away, see figure 2.17. Find the displacement of the rocket during the period of observation. **[15.62km]**

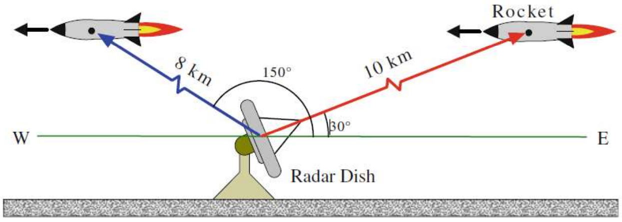

Figure 2.17 *See exercise 16*

17. Vector **A** has a magnitude of 3 units and lies along the - x-axis. Vector **B** has a magnitude of 6 units and makes an angle 30° with the + x-axis. Find the scalar product **A·B**
    (a) without using the concept of components. **[−15.59]**
    (b) by using vector components. **[−15.59]**
18. Show that for any vector $\vec{A}$:
    (a) $\vec{A} \cdot \vec{A} = A^2$
    (b) $\vec{A} \times \vec{A} = 0$
19. For vectors in question 14 above, show that
    (a) $\vec{A} \times \vec{B} = -\left(\vec{B} \times \vec{A}\right)$
    (b) $\vec{A} \cdot \left(\vec{A} \times \vec{B}\right) = 0$
20. The position vector $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ makes angles α, β, and γ with the x, y, and z axes as shown in figure 2.18. Show that the relation between what is known as the direction cosines $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ is as follows: $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

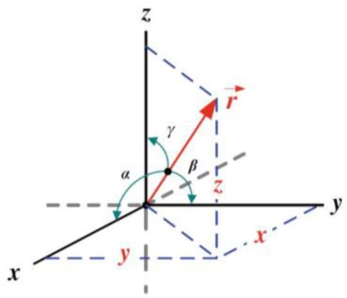

Figure 2.18 *See exercise 20*

21. Given: $\vec{A} = 5\vec{\imath} + 4\vec{\jmath} - 6\vec{k}$, $\vec{B} = -2\vec{\imath} + 2\vec{\jmath} + 3\vec{k}$, and $\vec{C} = 4\vec{\imath} + 3\vec{\jmath} + 2\vec{k}$.

(a) Determine the components and magnitude of $\vec{R}$ given that $\vec{R} = \vec{A} - \vec{B} + \vec{C}$

(b) Calculate the angle between $\vec{R}$ and the positive z axis. **[(a) $\mathbf{11\vec{\imath} + 5\vec{\jmath} - 7\vec{k}}$ (b) $\mathbf{120^0}$]**

22. Find the value of λ so that $\vec{A} = 2\vec{\imath} + \lambda\vec{\jmath} - \vec{k}$ and $\vec{B} = 4\vec{\imath} - 2\vec{\jmath} + 2\vec{k}$ are perpendicular to each other. **[$\boldsymbol{\lambda = 3}$]**

23. If the magnitudes of two vectors $\vec{A}$ and $\vec{B}$ are 3 and 4 respectively, and their scalar product is 6, find the angle between them and also find $|\vec{A} \times \vec{B}|$. **[$\mathbf{60^0; 6\sqrt{3}}$]**

24. If $\vec{A} = 3\vec{\imath} + 4\vec{\jmath}$ and $\vec{B} = 7\vec{\imath} + 24\vec{\jmath}$, find a vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$ **[$\mathbf{15\vec{\imath} + 20\vec{\jmath}}$]**

25. The $\vec{F} = q(\vec{v} \times \vec{B})$ equation gives the force on an electric point charge q moving with velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$. Find the force on a proton of $q = 1.6 \times 10^{-19}$coulomb moving with velocity $\vec{v} = (2\vec{\imath} + 3\vec{\jmath} + 4\vec{k}) \times 10^5\ m/s$ in a magnetic field of 0.5 $\vec{k}$ tesla. **[$\boldsymbol{\vec{F} = 1.6 \times 10^{-14}(1.5\vec{\imath} - \vec{\jmath})\ N}$]**