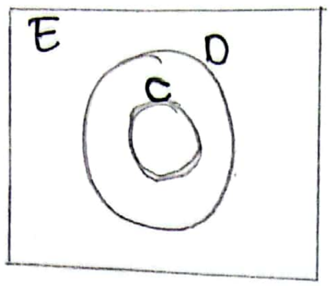
Name : Mulenga Felix
SIN : 22103841
Course : MA 110.test 1.
Group : E
Lecturer : Mr Lunjebe.
Institution : Copper belt University
Date : 23rd February, 2023.
Program : Bachelor of Science (NQ).
Venue : KQ11.

$\frac{71}{75} \times 100 = 95\%$

Q	MARKS
1	25
2	22
3	24
total	71

1a). i).

$C \subset D$



ii)

$C' \cup D'$

$(C \cap D)'$

$(C)'$

$\underline{\underline{C'}}$

5

ii)

$1.171717\ldots$

let $p = 1.171717$

$p = 1.17\overline{17}$

$p = 1.\overline{17} \quad \ldots \text{(i)}$

$\Rightarrow$ $100p = 100 \times (1.17\overline{17})$

$100p = 117.\overline{17} \quad \ldots \text{(ii)}$

Subtract equation (i) from (ii).

$100p - p = 117.\overline{17} - 1$

$$\frac{99p}{99} = \frac{116}{99}$$

$$p = \frac{116}{99}$$

$\therefore$ 1.171717 as a fraction is equal to $\frac{116}{99}$.

Q 1b). i) $a * b = a + b - 2ab$

$a * b$ is a binary operation on the set of all real numbers because for any values of a and b, the result obtained is a member of real numbers.

e.g $-1 * 2 = -1 + 2 - 2(-1)(2)$, $-\frac{1}{2} * 2 = -\frac{1}{2} + 2 - 2(-\frac{1}{2})(2)$

$= 1 + 4$

$= 5$

$= \frac{3}{2} + 2$

$= 3.5$

25/25

ii) ii) $a * b = a + b - 2ab$

$b * a = b + a - 2ba$

$b * a = a * b$ $\therefore$ the operation is commutative. ~~on the set of all r~~

For example $(1 * 0)$ and $(0 * 1)$

$1 * 0 = 1 + 0 - 2(1)(0)$, $0 * 1 = 0 + 1 - 2(0 * 1)$

$= 1 - 0$

$= 1$

$= 0 + 1 - 0$

$= 1$

$\therefore$ $a * b$ is commutative.

iii)

$a * b = a + b - 2ab$

$1 * (2 * 3)$

$2 * 3 = 2 + 3 - 2(2)(3)$
$= 5 - 12$
$= -7$

$\therefore 1 * (2 * 3) = 1 + (-7) - 2(1)(-7)$
$= 1 - 7 + 14$
$= -6 + 14$
$= 8$

$(1 * 2) * 3$

$1 * 2 = 1 + 2 - 2(1)(2)$
$= 3 - 4$
$= -1$

$\therefore (1 * 2) * 3 = -1 + 3 - 2(-1)(3)$
$= -1 + 3 + 6$
$= 2 + 6$
$= 8$

Since $1 * (2 * 3) = (1 * 2) * 3$, the operation $a * b = a + b - 2ab$ is associative.

5

Mulenga Felix 22103841.

Q1c).

$$y = \frac{x+2}{x-2}$$

x - intercepts

when $y=0$

$$0 = \frac{x+2}{x-2}$$

$$0(x-2) = x+2$$

$$0 = x+2$$

$$-2 = x$$

$$x = -2$$

$\Rightarrow$. $(-2, 0)$.

y - intercepts

when $x=0$

$$y = \frac{0+2}{0-2}$$

$$y = \frac{2}{-2}$$

$$y = -1$$

$(0, -1)$.

Vertical asymptote

Set $g(x) = x-2$

$$g(x) = 0$$

$$x-2 = 0$$

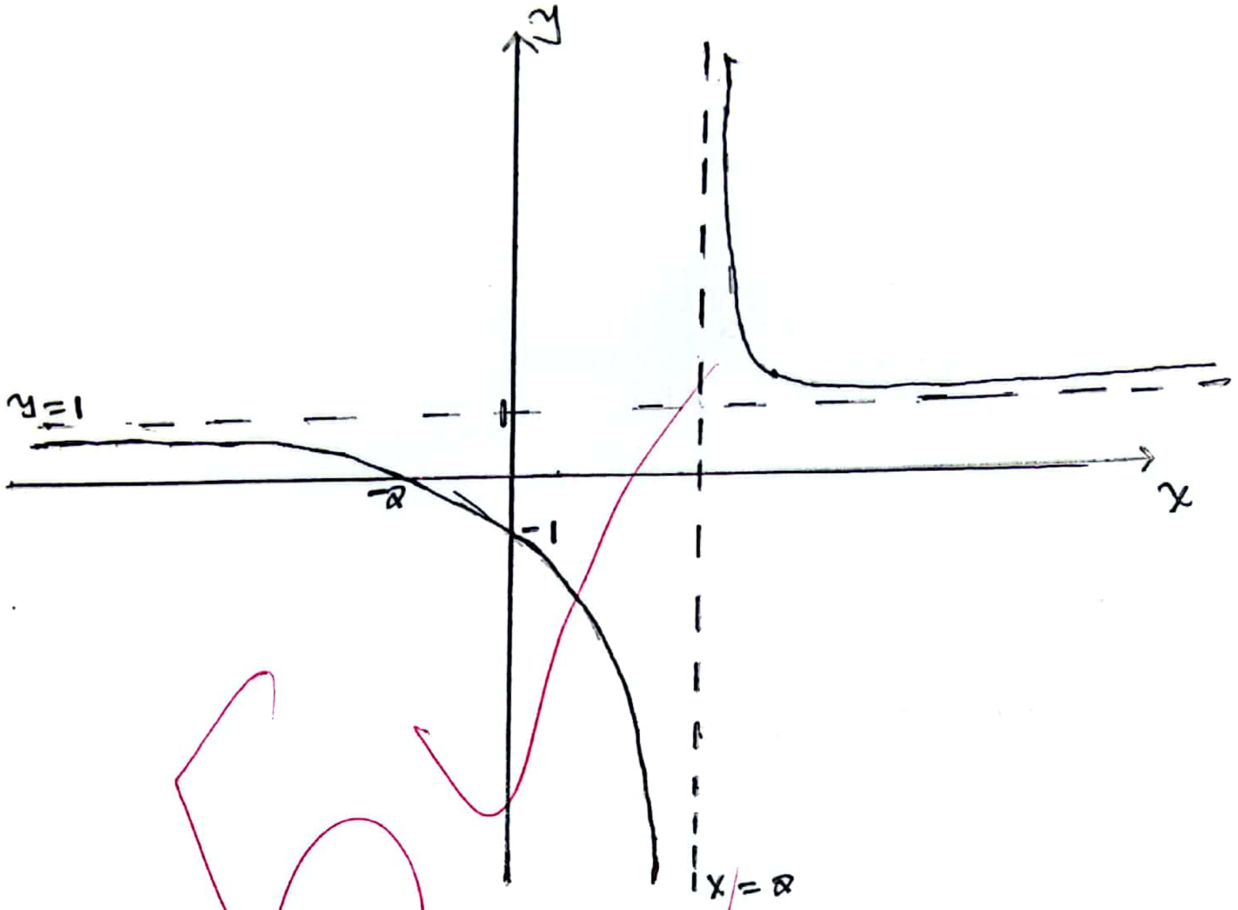
$$x = 2$$

$\therefore$ the Vertical asymptote is $x = 2$.

CamScanner

Horizontal asymptote.

$y = 1$, because the degree of the numerator is equal to the degree of the denominator.



$\Rightarrow$. Domain $(-\infty, 2) \cup (2, \infty)$

or $\{x : x \in \mathbb{R}, x \neq 2\}$.

Range $(-\infty, 1) \cup (1, \infty)$

or $\{y : y \in \mathbb{R}, y \neq 1\}$.

Proof by Contradiction.

d). Suppose $\sqrt{2}$ is rational meaning it can be expressed in the form of a/b where $a, b \in \mathbb{Z}$ and ~~both~~ they should be expressed in their lowest terms (they should have no common factors).

$$\sqrt{2} = a/b$$

$\Rightarrow$. Squaring both sides

$$(\sqrt{2})^2 = (a/b)^2$$

$$2 = \frac{a^2}{b^2}$$

$$2b^2 = a^2$$

$$a^2 = 2b^2 \quad \ldots \text{(i)}$$

a^2 is divisible by 2 meaning that a is also divisible by 2. This implies that a can be expressed in the form of $a = 2k$, where k is an integer.

$$a = 2k \quad \ldots \text{(ii)}$$

Substiting (ii) into (i)

$$(2k)^2 = 2b^2$$

$$\frac{4k^2}{2} = \frac{2b^2}{2}$$

$$2k^2 = b^2$$

b^2 is divisible by 2 meaning that b is also divisible by 2, Since a and b have a common factor which is is 2 is a contradiction because a and b should have no common factor, this means that it is not rational

Meaning it is irrational, therefore by proof of contradiction, $\sqrt{2}$ is irrational.

1)e)

$$f(x) = \frac{x+1}{x-1} \qquad g(x) = \sqrt{x}$$

$$g \circ f(x) = g[\sqrt{f(x)}].$$

$$= \sqrt{\frac{x+1}{x-1}}$$

$$\therefore gf(x) = \sqrt{\frac{x+1}{x-1}}$$

Domain;

$$\frac{x+1}{x-1} \geq 0$$

Critical points

$x+1=0$ or $x-1 \neq 0$

$x=-1$ $\qquad$ $x \neq 1.$

Sign graph

	-1		1
$x+1$	$-$	$+$	$+$
$x-1$	$-$	$-$	$+$
	$+$	$-$	$+$

$\Rightarrow$. $(-\infty, -1] \cup (1, \infty)$.

Domain $= (-\infty, -1] \cup (1, \infty)$.

Mulenga Felix.

Q 2a).

a) $A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B)$

$A \cup B = [(A \cap B) \cup (A \cap B')] \cup (A' \cap B)$

$= [(A \cup A) \cap (A \cup B') \cap (B \cup A) \cap (B \cup B')] \cup (A' \cap B)$

$= [[A \cap (A \cup B')] \cap (B \cup A) \cap E] \cup (A' \cap B)$

$= [\quad A$

a) Q2a) $A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B).$

R.H.S

$(A \cap B) \cup (A \cap B') \cup (A' \cap B)$

$[(A \cap B) \cup (A \cap B')] \cup (A' \cap B).$

$[A \cap (B \cup B')] \cup (A' \cap B)$

$[A \cap (U)] \cup (A' \cap B)$

$A \cup (A' \cap B)$

$(A \cup A') \cap (A \cup B)$

$U \cap (A \cup B)$

$A \cup B = L.H.S$

hence shown.

$\Rightarrow (A \cap B) \cup (A \cap B')$

$A \cap (B \cup B')$

$\Rightarrow$ Distributive law.

$\Rightarrow$. $A \cap U = A$

$U \cap (A \cup B) = A \cup B$

22/25

b).

$$3x^2 + 2x + 5 = 0 \quad , a=3 \quad b=2, \; c=5$$

$$\text{Sum } (\alpha+\beta) = -b/a$$

$$= \frac{-(2)}{3}$$

$$= -2/3$$

$$\text{Product } (\alpha\beta) = c/a$$

$$= \frac{5}{3}$$

$$= \frac{5}{3}$$

$$\text{Sum of new roots} = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$= \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$$

$$= \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \left[(-2/3)^2 - 2(5/3)\right] \div \left[(5/3)^2\right]$$

$$= \left[\frac{4}{9} - \frac{10}{3}\right] \div \left[\frac{25}{9}\right]$$

$$= \left[\frac{4-30}{9}\right] \div \frac{25}{9}$$

$$= \frac{-26}{9} \times \frac{9}{25}$$

Sum of new roots $= -\frac{26}{25}$

Product of new roots $= \frac{1}{\alpha^2} \times \frac{1}{\beta^2}$

$$= \frac{1}{\alpha^2\beta^2}$$

$$= \frac{1}{(\alpha\beta)^2}$$

$$= \frac{1}{(5/3)^2}$$

$$= \frac{1}{25/9}$$

$$= 1 \div \frac{25}{9}$$

$$= 1 \times \frac{9}{25}$$

$$= \frac{9}{25}$$

$\therefore$ New Equation $\Rightarrow$ $x^2 - (\text{sum of new roots})x + \text{product of new roots} = 0$

$$x^2 - \left(\frac{-26}{25}\right)x + \frac{9}{25} = 0$$

$$x^2 + \frac{26}{25}x + \frac{9}{25} = 0$$

$$25x^2 + 26x + 9 = 0$$

5

Q2 c).

$$\sqrt{2} - \sqrt{x+6} \le -\sqrt{x}$$

Domain of $\sqrt{x+6}$

$$x+6 \ge 0$$

$$x \ge -6 \quad [-6, \infty).$$

Domain of $\sqrt{x}$

$$x \ge 0 \quad [0, \infty).$$

$$\sqrt{2} - \sqrt{x+6} \le -\sqrt{x}$$

$$\sqrt{2} + \sqrt{x} \le \sqrt{x+6}$$

Squaring both sides

$$(\sqrt{2}+\sqrt{x})^2 \le (\sqrt{x+6})^2$$

$$2 + 2\sqrt{2}\cdot\sqrt{x} + x \le x+6$$

$$2 + x + 2\sqrt{2}\cdot\sqrt{x} \le x+6$$

$$2\sqrt{2}\cdot\sqrt{x} \le x - x + 6 - 2$$

$$\frac{2\sqrt{x}\cdot\sqrt{2}}{2} \le \frac{4}{2}$$

$$\sqrt{x}\cdot\sqrt{2} \le 2$$

Squaring both sides

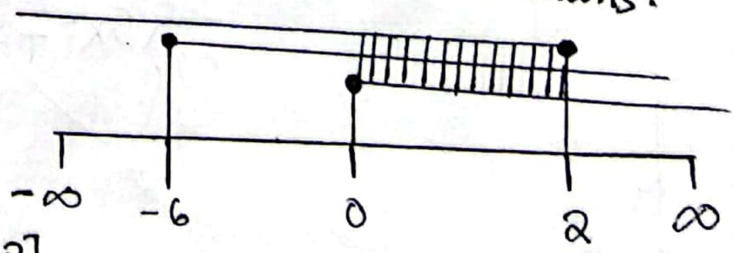
$$(\sqrt{x}\cdot\sqrt{2})^2 \le (2)^2$$

$$x \cdot 2 \le 4$$

$$\frac{2x}{2} \le \frac{4}{2}$$

$$x \le 2 \quad (-\infty, 2].$$

~~Sig.~~ Fig. Representation of the ~~splitting points~~ Domains.



$\Rightarrow$. $[0, 2]$. or $0 \le x \le 2$

$\therefore$ the solution for the inequality is $[0, 2]$.

Q 2d).

$$\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i}$$

$$\frac{x(2-i) - y(1+i)}{(1+i)(2-i)} = \frac{1-5i}{3-2i}$$

$$\frac{2x - xi - y - yi}{2 - i + 2i - i^2} = \frac{(1-5i)(3+2i)}{(3-2i)(3+2i)}$$

$$\frac{2x - y - xi - yi}{2 - i^2 + i} = \frac{3 + 2i - 15i - 10i^2}{9 + 6i - 6i - 4i^2}$$

$$\frac{2x - y - (x+y)i}{2-(-1)+i} = \frac{3 - 10(-1) - 13i}{9 - 4(-1)}$$

$$\frac{2x - y - (x+y)i}{2+1+i} = \frac{3 + 10 - 13i}{9+4}$$

$$\frac{2x - y - (x+y)i}{3+i} = \frac{13 - 13i}{13}$$

$$\frac{2x - y - (x+y)i}{3+i} = 1 - i$$

$$2x - y - (x+y)i = (1-i)(3+i)$$

$$2x - y - (x+y)i = 3 + i - 3i - i^2$$

$$2x - y - (x+y)i = 3 - (-1) - 2i$$

$$2x - y - (x+y)i = 3 + 1 - 2i$$

$$2x - y - (x+y)i = 4 - 2i$$

By compairing

$$2x - y = 4$$

$$-(x+y)i = -2i$$

$$\therefore \frac{-(x+y)}{-1} = \frac{-2}{-1}$$

$$x + y = 2$$
$$2x - y = 4$$

$$3x + y - y = 2 + 4$$

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2.$$

$$x + y = 2$$

$$2 + y = 2$$

$$y = 2 - 2$$

$$y = 0$$

$$\therefore \quad x = 2 \text{ and } y = 0.$$

Q e). $f(x) = \dfrac{2x}{x-1}$

If $f(x)$ is a bijection then it is a one to one function.

$f(a) = f(b)$ [When a function is a one to one].

$$\frac{2(a)}{a-1} = \frac{2(b)}{b-1}$$

$$\frac{2a}{a-1} = \frac{2b}{b-1}$$

$$2a(b-1) = 2b(a-1)$$

$$2ab - 2a = 2ba - 2b$$

$$2ab - 2a = 2ab - 2b$$

$$-2a + 2b = 2ab - 2ab$$

$$-2a + 2b = 0$$

$$\cancel{-2(a+b)} \; \frac{-2(a-b)}{-2} = \frac{0}{-2}$$

$$a - b = 0$$

$$a = b$$

$\therefore$ Since $f(a) = f(b)$ therefore $f(x)$ is a one to one or a bijection.

Q3 a)

$$x^3-4x^2+8=0$$

$$p=\pm 1, \pm 2, \pm 4, \pm 8$$

$$q=\pm 1$$

$$\frac{p}{q}=\pm 1, \pm 2, \pm 4, \pm 8$$

$\Rightarrow$. By trial and error, $x+1$

let $f(x)=x^3-4x^2+8$

$$x+1=0$$

$$x=-1$$

$$f(-1)=(-1)^3-4(-1)^2+8$$

$$=-1-4+8$$

$$=3$$

$\therefore$ $x+1$ is not a factor of $f(x)$.

By trial and error, $x-2$

$$x-2=0$$

$$x=2$$

$$f(2)=(2)^3-4(2)^2+8$$

$$=8-16+8$$

$$=0$$

$\therefore$ $x-2$ is a factor of $f(x)$.

Divide $f(x)$ by $x-2$ using synthetic division.

$$\begin{array}{c|cccc} 2 & 1 & -4 & 0 & 8 \\ & & 2 & -4 & -8 \\ \hline & 1 & -2 & -4 & 0 \end{array}$$

$\Rightarrow (x-2)(x^2-2x-4)=0$

$x-2=0$ or $x^2-2x-4=0$

$x=2$

$x^2-2x=4$

$x^2-2x+(-1)^2=4+(-1)^2$

$x^2-2x+1=4+1$

$(x-1)^2=5$

$x-1=\pm\sqrt{5}$

$x=1\pm\sqrt{5}$

$x=1+\sqrt{5}$, $x=1-\sqrt{5}$

$\therefore$ the values of x are $x=1+\sqrt{5}$, $x=1-\sqrt{5}$ and $x=2$.

Q b).

$$\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)} = \frac{1}{(\sqrt{2}+1)} \times \frac{1}{(\sqrt{3}-1)}$$

$$= \frac{(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$(\sqrt{2}+1)(\sqrt{2}-1)$

$\sqrt{2}(\sqrt{2}+1)+1(\sqrt{2}-1)$

$\cancel{2+\sqrt{2}}$ $2-\sqrt{2}+\sqrt{2}-1$

$\cancel{\sqrt{2}}$ $2-1$

1

$$= \frac{(\sqrt{2}-1)}{2-1} \times \frac{(\sqrt{3}+1)}{3-1}$$

$$= \frac{(\sqrt{2}-1)}{1} \times \frac{(\sqrt{3}+1)}{2}$$

$$= \frac{(\sqrt{2}-1)(\sqrt{3}+1)}{2}$$

$$= \frac{\sqrt{2}(\sqrt{3}+1)-1(\sqrt{3}+1)}{2}$$

$$= \frac{\sqrt{6}+\sqrt{2}-\sqrt{3}-1}{2}$$

24/25

$= \frac{1}{2}(\sqrt{6}) + \frac{1}{2}(\sqrt{2}) - \frac{1}{2}(\sqrt{3}) - \frac{1}{2}.$

c) i) $f(x) = x^4 + x^2 + 1$

if $f(x)$ is odd, then $-f(x) = f(-x)$

$-f(x) = -(x^4 + x^2 + 1)$

$= -x^4 - x^2 - 1$

$f(-x) = (-x)^4 + (-x)^2 + 1$

$= x^4 + x^2 + 1$

Since $-f(x) \neq f(-x)$ $\therefore$ $f(x) = x^4 + x^2 + 1$ is not odd.

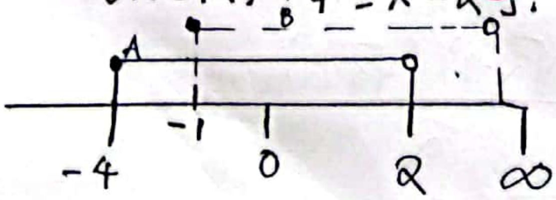
If $f(x)$ is even, then $f(x) = f(-x)$

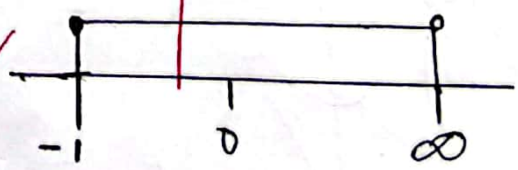
$f(x) = x^4 + x^2 + 1$

$f(-x) = x^4 + x^2 + 1$

Since $f(x) = f(-x)$, $\therefore$ $f(x) = x^4 + x^2 + 1$ is an even function.

ii) $A = \{x \in \mathbb{R} : -4 \leq x < 2\}$. $B = \{x \in \mathbb{R} : x \geq -1\}$.





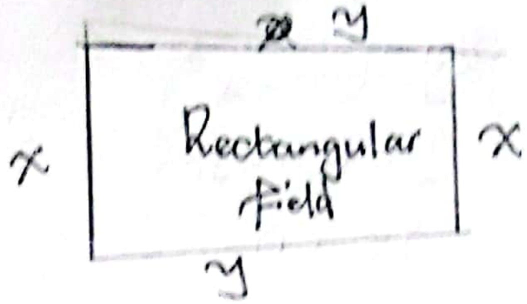
a) $\Rightarrow$. $A \cap B = [-1, 2)$. ii) $A' = [2, \infty)$.

or

$\cancel{A \cap B = \{-1, 0, 1\}}$

$\cancel{\Rightarrow [-1, 1]}$

Q 3d).



$$P = 1200$$

$$x + y + x + y = 1200$$

$$2x + 2y = 1200$$

$$\frac{2(x+y)}{2} = \frac{1200}{2}$$

$$x + y = 600$$

$$y = 600 - x.$$

$$\text{Area} = l \times b$$

$$= x(600 - x)$$

$$= 600x - \cancel{2}x^2$$

$$= 600x - x^2$$

$$\text{Area} = -x^2 + 600x$$

Maximum value ~~of~~ occurs at the turning point

$$\text{Value of } x = -b/2a$$

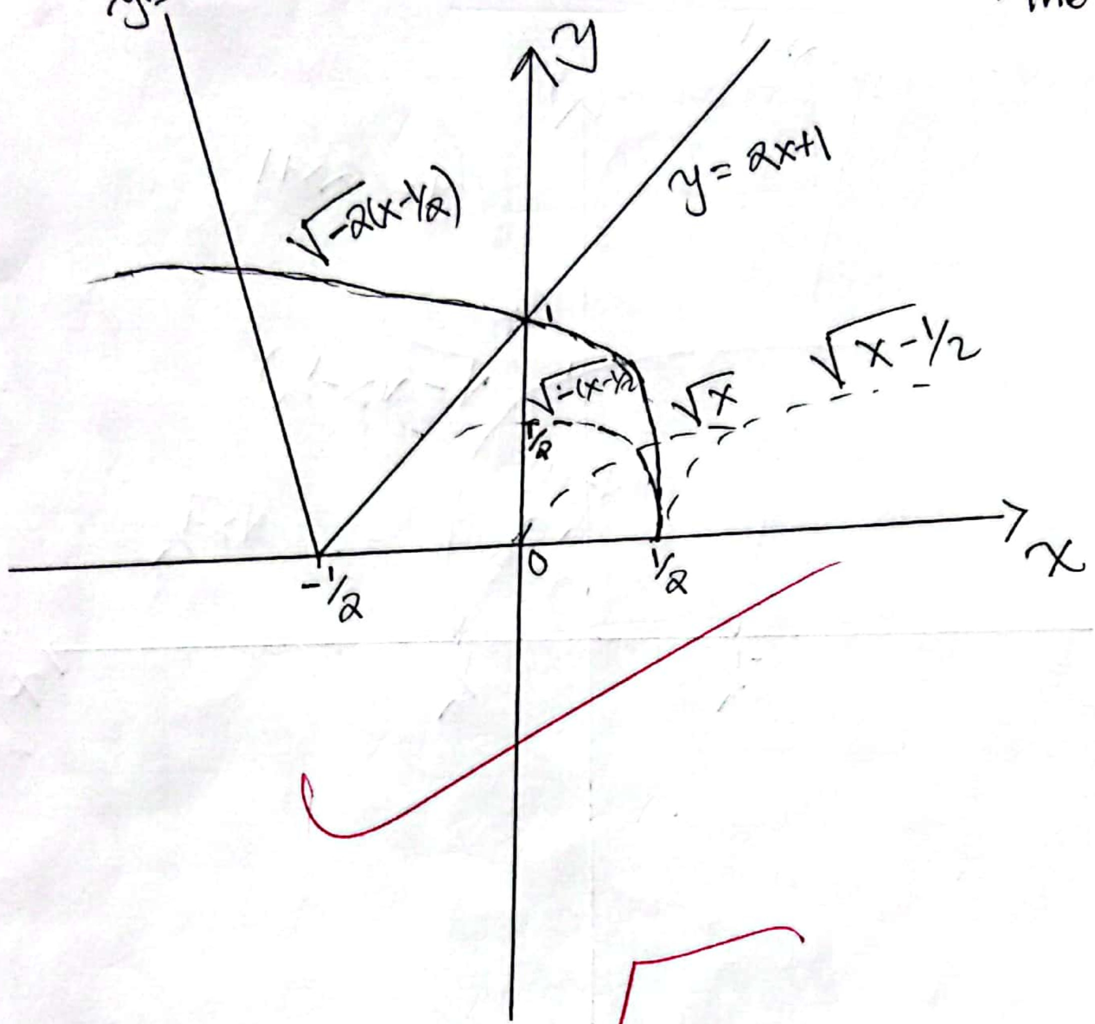
$$= \frac{-(600)}{2(-1)}$$

$$= \frac{-600}{-2}$$

$$= 300$$

5

A Clear graph is ~~on~~ at the back of this page.



5

$\Rightarrow \quad \sqrt{1-2x} > |2x+1|$

$(\sqrt{1-2x})^2 > (|2x+1|)^2$

$1-2x > 4x^2+4x+1$

$1-1 > 4x^2+6x$

$0 > 4x^2+6x$

$4x^2+6x < 0$

$x(4x+6) < 0$

Critical points

$x=0 \qquad 4x+6=0$

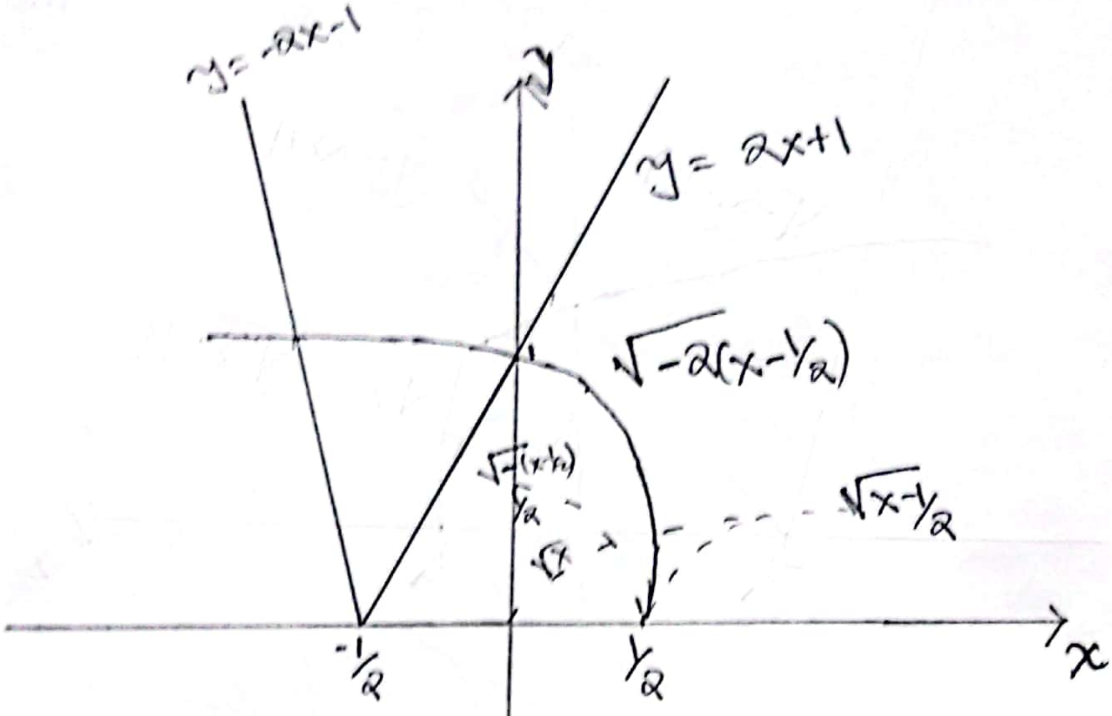
$x=-3/2$

Sign graph.

x	$-$	$-3/2$	$-$	0	$+$
4x+6	$-$		$+$		$+$
	$+$		$-$		$+$

$(-3/2, 0)$.

$\therefore$ values of x such that $\sqrt{1-2x} > |2x+1|$ are $(-3/2, 0)$.



$\Rightarrow \sqrt{1-2x} >$