



# The Copperbelt University

School of Mathematics And Natural Sciences

Department of Mathematics

MA 110 : (Mathematical Methods I) : Test One

February 23, 2022

## Instructions

- (1). You must write your Name, Computer number and Programme of study on your answer sheet. Time allowed is 2 hours.
- (2). Calculators and use of Cell phones are Not allowed in this paper.
- (3). There are **three (3)** questions in this paper, attempt all the questions and show detailed working for full credit.

## QUESTION ONE

- (a) Prove the De Morgan's law:  $(A \cap B)' = A' \cup B'$ .

(5 marks)

- (b) Determine whether the function given is even, odd or neither

$$f(x) = x^6 + x^4 - 10x^2.$$

(5 marks)

- (c) Given that  $\sqrt{10}$  is irrational number, prove that  $\sqrt{2} + \sqrt{5}$  is also an irrational number.

(5 marks)

- (d) Graph the rational function :  $f(x) = \frac{x^4 + 4}{(x - 3)(x + 2)}$ .

(5 marks)

- (e) Let  $z = x + iy$  be a non-zero complex number. Given that  $\frac{1}{z} = k$ , where  $k$  is a real number, prove that either  $z$  is a real number or  $|z| = 1$ .

(5 marks)

## QUESTION TWO

- (a) Express  $3.\overline{312}$  as a fraction  $\frac{a}{b}$  in the simplest form where  $a$  and  $b$  are integers and  $b \neq 0$ . (5 marks)

- (b) Show that the two functions given below are inverses of each other:

$$f(x) = 4x - 3$$

and

$$g(x) = \frac{x+3}{4}$$

(5 marks)

- (c) If  $\alpha$  and  $\beta$  are the roots of the quadratic equation  $3x^2 + 6x - 15 = 0$ , find the quadratic equation that has the roots  $\alpha^2$  and  $\beta^2$ . (5 marks)

- (d) Rationalize the denominator, expressing your answer in the form  $a + b\sqrt{xy}$  where  $a$  and  $b$  are rational numbers

$$\frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}}$$

(5 marks)

- (e) Solve the polynomial equation given below using the rational root theorem

$$x^6 - x^5 - 4x^4 + 2x^3 + 5x^2 - x - 2 = 0.$$

(5 marks)

## QUESTION THREE

- (a) Define an operation  $*$  on the set of real numbers by  $x * y = \sqrt[x]{y}$  where  $x$  is the index of the radical and  $y$  is the radicand.

- (i) Is  $*$  is a binary operation on the set of real numbers? justify your answer. (1 mark)

- (ii) Evaluate  $3 * (-64)$ . (1 mark)

- (iii) Solve the equation :  $3 * (2x - 3) = 3$ . (3 marks)

- (b) Given the quadratic function  $f(x) = -3x^2 + 6x - 5$ . Express the function  $f(x) = -3x^2 + 6x - 5$  in the vertex form  $f(x) = a(x + q)^2 + r$  and sketch its graph. (5 marks)

- (c) Determine the domain of the function, expressing the answer in interval notation

$$f(x) = 3x + \sqrt{x^2 + 4x - 12}.$$

(5 marks)

- (d) Solve the radical equation :  $\sqrt{\sqrt{2y} - \sqrt{y-1}} = 1$ . (5 marks)

- (e) Sketch the graph of  $f(x) = |2x + 1| - |x - 2|$ . (5 marks)

- (f) Sketch the graph of  $f(x) = |2x + 1|$ . (5 marks)

THE END OF TEST



# The Copperbelt University

## School of Mathematics And Natural Sciences

### Department of Mathematics

#### MA 110 : (Mathematical Methods I) : Test Two

Friday - July 22, 2022

#### Instructions

- (1). You must write your **Name, Computer number and Programme** of study on your answer sheet. Time allowed is **2 hours**.
  - (2). **Calculators** and use of **Cell phones** are **Not** allowed in this paper.
  - (3). There are **Four (4)** questions in this paper, **attempt all the questions** and show detailed working for full credit.
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#### QUESTION ONE

- (a) Find the center and radius of the circle whose equation is

$$x^2 + y^2 + 8x - 2y + 13 = 0.$$

(5 marks)

- (b) Write down the constant term in the expansion of  $\left(x - \frac{1}{2x^2}\right)^9$ .

(5 marks)

- (c) Prove that  $\log_a \left(\frac{A}{B}\right) = \log_a(A) - \log_a(B)$ .

(5 marks)

- (d) Use **Crammer's method** to solve the linear system of the equation

$$3x - 4y = -11$$

$$-5x + y = 7.$$

(5 marks)

## QUESTION TWO

- (a) Change the repeating decimal  $3.\overline{7}$  to its reduced form  $\frac{a}{b}$ , where  $a$  and  $b$  are integers and  $b \neq 0$  using sum to infinity of a geometric series. (5 marks)
- (b) Use Mathematical induction to prove that  $2^n \geq n + 1$  for all possible integer  $n$ . (5 marks)
- (c) Find the equation of the tangent at the point  $(3, 1)$  on the circle

$$x^2 + y^2 - 4x + 10y - 8 = 0.$$

(5 marks)

- (d) Graph the function  $f(x) = 2^{(x-3)} + 2$  and obtain its inverse on the same axes. (5 marks)

## QUESTION THREE

- (a) Solve  $25^x - 5^x = 12$ . (5 marks)
- (b) Express  $\frac{2x^2 - 5x + 7}{(x-2)(x-1)^2}$  into a partial fraction. (5 marks)
- (c) What is the common ratio of the G.P.  $(\sqrt{2} - 1) + (3 - 2\sqrt{2}) + \dots$ ? Find the third term of progression. (5 marks)
- (d) How long will it take K2000 to double itself at 13% interest compounded continuously? (5 marks)

## QUESTION FOUR

- (a) Find the first term and the general expansion of  $\frac{1}{(2-3x)^3}$  in ascending power of  $x$ . State the range of value of  $x$  for which this expansion is valid. (5 marks)
- (b) Show that the general term of an arithmetic sequence is given by

$$a_n = a_1 + (n-1)d.$$

(5 marks)

- (c) Solve the equation  $\log_2 x = \log_4(x+6)$ . (5 marks)

- (d) Find the inverse of the matrix

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix}.$$

(5 marks)

THE END OF TEST





# The Copperbelt University

School of Mathematics and Natural Sciences

Department of Mathematics

MA 110 : Mathematical Methods I

Sessional Exam - 2021/22

August 29, 2022

TIME ALLOWED : 3 HOURS

TOTAL MARKS : 100

## Instructions

- (1). You must write your **Group, Student Identification Number (SIN)** and **Programme** of study on your answer booklet.
- (2). Check that you have the **correct examination paper** in front of you.
- (3). There are **Seven (7)** questions in this paper. Answer any **Five (5)** questions.
- (4). All questions must be answered in the answer booklet provided **only**.
- (5). Calculators are **not allowed** in this paper.
- (6). Begin each question on a **new page**. And show all **your working** to obtain full marks.
- (7). Write down the **number of questions** that you have answered on the cover of the examination answer booklet provided.

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DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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## QUESTION ONE

- (a). Prove De Morgans law:

$$(A \cup B)' = A' \cap B'.$$

(5 marks)

- (b). If  $\alpha$  and  $\beta$  are the roots of the quadratic equation  $3x^2 + 6x - 15 = 0$ , find the quadratic equation that has the roots  $\alpha^2$  and  $\beta^2$ .

(5 marks)

- (c). Find the sum of

$$\sum_{k=0}^{\infty} 4 \left( -\frac{2}{3} \right)^k.$$

(5 marks)

- (d). Find the middle term of  $\left( \frac{1}{x} - x^2 \right)^{12}$ , making sure that you simplify your answer in simplest form.

(5 marks)

(Total Marks: 20)

## QUESTION TWO

- (a). Express  $3.3\overline{12}$  as a fraction  $\frac{a}{b}$  in simplest form where  $a$  and  $b$  are integers and  $b \neq 0$ .

(5 marks)

- (b). Graph the rational function:

$$f(x) = \frac{x+3}{(x-3)(x+2)}.$$

(5 marks)

- (c). Solve the radical equation:

$$\sqrt{\sqrt{2y} - \sqrt{y-1}} = 1.$$

(5 marks)

- (d). Let  $z = x + iy$  be a non-zero complex number. Given that  $z + \frac{1}{z} = k$ , where  $k$  is a real number, prove that either  $z$  is a real number or  $|z| = 1$ .

(5 marks)

(Total Marks: 20)

### QUESTION THREE

- (a). Solve the logarithmic equation:

$$\log_5 \sqrt{x} = \sqrt{\log_5 x}.$$

(5 marks)

- (b). Use Mathematical induction to prove that  $2^n \geq n + 1$  for all possible integer  $n$ .  
(5 marks)

- (c). Solve the equation involving absolute values:

$$|3x - 1| = |2x + 3|.$$

(5 marks)

- (d). Find the period, amplitude and phase shift of  $f(x) = 2 \sin(2x - \pi)$  and sketch its graph for the interval  $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$ .

(5 marks)

(Total Marks: 20)

### QUESTION FOUR

- (a). Prove that  $\sqrt{2}$  is an irrational number.

(5 marks)

- (b). Verify the trigonometric identity:

$$\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta.$$

(5 marks)

- (c). Solve the polynomial equation given below using the rational root theorem:

$$x^6 - x^5 - 4x^4 + 2x^3 + 5x^2 - x - 2 = 0.$$

(5 marks)

- (d). Graph the function  $f(x) = 2^{(x-3)} + 2$  and obtain its inverse on the same axis.  
(5 marks)

(Total Marks: 20)

## QUESTION FIVE

- (a). Show that the two functions given below are inverses of each other:

$$f(x) = \sqrt[3]{\frac{x-1}{3}}$$

and

$$g(x) = 3x^3 + 1.$$

(5 marks)

- (b). Find the first four terms in the expansion of  $\sqrt{4+2x}$ . State the range of the values of  $x$  for which this expansion is valid.

(5 marks)

- (c). The line  $AB$  is a diameter of a circle, where  $A$  and  $B$  are  $(-4, 9)$  and  $(10, -3)$  respectively. Find the equation of the circle.

(5 marks)

- (d). Use the crammer's rule, to solve the system of linear equations

$$-x + 2y = -5$$

$$3x + 2y - z = -4$$

$$4x + 3z = 13.$$

(5 marks)

(Total Marks: 20)

## QUESTION SIX

- (a). Prove by Mathematical induction  $3^{2n} - 1$  is divisible by 4.

(5 marks)

- (b). If  $a * b = 2a - b$  where  $a$  and  $b$  are real numbers. Solve the equation :

$$2x * (x * 3) = 5.$$

(5 marks)

- (c). How long will it take K4,000 to grow to K12,000 at 13% interest compounded continuously?

(5 marks)

- (d). Find the equations of the tangents from the origin to the circle:

$$x^2 + y^2 - 10x - 6y + 25 = 0.$$

(5 marks)

(Total Marks: 20)



## QUESTION SEVEN

- (a). The second term of an A.P is 15, and the ~~fifth~~<sup>fourth</sup> term is 21. Find the common difference, the first term and the sum of the first ten terms.

(5 marks)

- (b). Express  $\frac{2x^2 + x + 2}{(x^2 + 1)^2}$  into a partial fraction.

(5 marks)

- (c). Find the domain of the logarithmic function:

$$\log_2(x^2 + 2x - 15),$$

expressing your final answer in interval notation.

(5 marks)

- (d). Find the square roots of the complex number

$$15 + 8i.$$

(5 marks)

(Total Marks: 20)

END OF EXAM