



EXAM MADE EASY

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MA110 - MATHEMATICAL METHODS

Time allowed: Two hours (2:00 hours)

Instructions:

1. You must write your Name, your Computer Number and programme of study on your answer sheet.
2. Calculators are not allowed in this paper.
3. There are three (3) questions in this paper, Attempt All questions and show detailed working for full credit

QUESTION ONE

- a) (i) If $C \subset D$, then simplify if possible $C' \cup D'$ (2.5 marks) ✓
- (ii) Express 1.171717..... as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$. (2.5 marks) ✓
- b) Consider the binary operation $a * b = a + b - 2ab$, where a and b are real numbers.
- (i) Is $*$ a binary operation on the set of real numbers? Give reason for your answer. (1) Mark ✓
- (ii) Is the operation $*$ commutative? If not give a counter example. (1) Mark ✓
- (iii) Find the value of $1 * (2 * 3)$ and $(1 * 2) * 3$ and state whether $*$ is associative (3) Marks ✓
- c) Given the rational function $f(x) = \frac{x+2}{x-2}$. Sketch its graph indicating its domain and range, all the asymptotes and intercepts. (5 Marks) ✓
- d) Prove that $\sqrt{2}$ is an irrational number (5 Marks) ✓
- e) Let $f(x) = \frac{x+1}{x-1}$ and $g(x) = \sqrt{x}$. Find $(g \circ f)(x)$ and determine the domain (5 Marks)

QUESTION TWO

- a) Using the associative and distributive properties of union and intersection of sets. Show that

$$A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B) \quad (5 \text{ Marks})$$

- b) Let α and β be the roots of the quadratic equation $3x^2 + 2x + 5 = 0$. Find a quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ without calculating α and β (5 Marks)

- c) Solve the given radical function inequality $\sqrt{2} - \sqrt{x+6} \leq -\sqrt{x}$ 30
(5 Marks)

- d) Solve for x and y given that:

$$\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i} \quad (5 \text{ Marks})$$

- e) Show that the function f defined by $f(x) = \frac{2x}{x-1}$ $x \in \mathbb{R}$, is a bijection on \mathbb{R} on to $\{y \in \mathbb{R}; y \neq 2\}$ (5 Marks)

QUESTION THREE

- a) Use the Rational root theorem to solve $x^3 - 4x^2 + 8 = 0$ (5 Marks)

- b) Rationalize the denominator $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$ (5 Marks)

- c) (i) Determine whether the function $f(x) = x^4 + x^2 + 1$ even, odd or neither. (2.5 marks)

- (ii) Let $A = \{x \in \mathbb{R}; -4 \leq x < 2\}$ and $B = \{x \in \mathbb{R}; x \geq -1\}$.
Find a) $A \cap B$ b) A' (2.5 marks)

- d) What are the dimensions of the largest rectangular field which can be enclosed by 1200 m of fencing? (5 Marks)

- e) Sketch the graph of $f(x) = |2x+1|$. On the same diagram sketch also the graph of $g(x) = \sqrt{1-2x}$ and hence, find the values such that $\sqrt{1-2x} > |2x+1|$ (5 Marks)



MA110 - MATHEMATICAL METHODS TEST 2

Time allowed: Two hours thirty minutes (2:30)

Instructions:

1. You must write your Name, your Computer Number and programme of study on your answer sheet.
2. Calculators are not allowed in this paper.
3. There are four (4) questions in this paper, Attempt All questions and show detailed working for full credit

QUESTION ONE

- a) Express $\frac{2x+1}{x^3-1}$ in partial fractions (5marks)
- b) Find the centre and length of a radius of the given circle and graph it
 $x^2 + y^2 - 10x = 0$. (5marks)
- c) Prove the result by induction. $1 \times 3 + 2 \times 4 + \dots + n(n+2) = \frac{1}{6}n(n+1)(2n+7)$ (5marks)
- d) Find the 4th term in the binomial expansion $(2 - \frac{x}{2})^9$ (5marks)
- e) If $xy = 64$ and $\log_x y + \log_y x = \frac{5}{2}$, Find x and y . (5marks)

QUESTION TWO

- a) A is the point $(-1, 2)$, B is the point $(2, 3)$ and C is the point $(3, 5)$. P is a point which divides BC in the ratio 3 : 4 and Q lies on AB such that $AQ = \frac{2}{5}AB$.
(i) Find the coordinates of P (2.5 marks)
(ii) Find the coordinates of Q. (2.5 marks)
- b) Find λ for which the matrix $\lambda I - A$ is a singular matrix if where I is an identity Matrix given that $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{pmatrix}$ (5 marks)
- c) Show that the sum of the integers from 1 to n is $\frac{1}{2}n(n+1)$ (5marks)

- d) Solve the logarithmic equation : $\log(x - 4) + \log(x - 1) = 1$ (5marks)
- e) In the expansion of $(1 + ax)^n$, the first three terms in ascending power of x are $1 - \frac{5}{2}x + \frac{75}{8}x^2$, Find the values of n and a , and state the range of values of x for which the expansion is valid. (5marks)

QUESTION THREE

- a) Find the radius of the circle with center at $C(-2,5)$ if the line $x + 3y = 9$ is a tangent line. (5marks)
- b) Using geometrical progression, change $0.\overline{214}$ to $\frac{a}{b}$ form, where a and b are integers and $b \neq 0$. (5marks)
- c) Use mathematical induction to prove that the statement is true for all positive integers n given that $4^n - 1$ is divisible by 3 (5marks)
- d) Graph $f(x) = \log_{\frac{1}{2}}x$ by reflecting the graph of $g(x) = \left(\frac{1}{2}\right)^x$ across the line $y = x$ (5marks)

- e) (i) Find the inverse of the matrix $A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 1 & 1 \\ 2 & 2 & -1 \end{pmatrix}$ (5marks)

(ii) Use your inverse to solve the system of linear equations

$$3x - y + 2z = 4$$

$$x + y + z = 2$$

$$2x + 2y - z = 3$$

(3marks)

QUESTION FOUR

a) Write the following in sigma notation

(i) $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \dots$ (3marks)

(ii) $1^4 + 2^4 + 3^4 + \dots + n^4 + (n + 1)^4$ (2marks)

- b) The number of grams of a certain radioactive substance present after t hours is given by the equation $Q = Q_0 e^{-0.45t}$, where Q_0 represents the initial number of grams. How long will it take 2500 grams to be reduced to 1250 given $\ln\left(\frac{1}{2}\right) = -0.693$ (5marks)

- c) (i) Expand $(1 + 2x)^4$ and $(1 - 2x)^4$ in ascending powers of x . (5marks)
- (ii) Hence reduce $(1 + 2x)^4 - (1 - 2x)^4$ to its simplest form. (3marks)
- (iii) Using the results in (ii) evaluate $(1.002)^4 - (0.998)^4$ (5marks)

$$\frac{1}{10} - \frac{3}{10} = -\frac{2}{10} = -\frac{1}{5}$$

$$\begin{array}{r}
 20 \\
 + 20 \\
 + 19 \\
 + 20 \\
 + 20 \\
 \hline
 99
 \end{array}$$



THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

Department of Mathematics

2022/2023 ACADEMIC YEAR

SESSIONAL EXAMINATION

MA110-MATHEMATICAL METHODS I

TIME ALLOWED: Three (3) hours

INSTRUCTIONS:

1. You must write your **COMPUTER NUMBER, PROGRAM** and **YOUR GROUP** on each answer booklet you have used.
2. There are **seven (7)** questions in this paper, Attempt any **five (5)** questions, Each question consists of **a, b, c, d, e**. All questions carry equal marks
3. Calculators are **NOT** allowed in this paper.
4. Should you have any problem or if you need more answer booklet, put up your hand an invigilator will come to attend to you.

$$\begin{array}{r}
 90 = \\
 95 =
 \end{array}$$

QUESTION ONE

a) Given that A and B are sets, simplify the following if possible

$$[(A \cap B)' \cap (A' \cup B)]'.$$

b) Solve the logarithmic equation :

$$\log(x - 4) + \log(x - 1) = 1.$$

c) Use De Moivre's theorem to find the indicated power of

$$(1 + i)^{20} \text{ and express results in } a + bi.$$

d) Find the limit of the quotients as

$$h \rightarrow 0 \frac{\sqrt{x+h} - \sqrt{x}}{h}.$$

e) Evaluate the definite integral of

$$\int_0^{\pi^2} \frac{\cos \sqrt{x}}{\sqrt{x}} dx.$$

QUESTION TWO

a) Use the rational root theorem to solve $x^3 - 4x^2 + 8 = 0$.

b) Use mathematical induction to prove that the statement is true for all positive integers n given that $4^n - 1$ is divisible by 3.

c) Find the period, amplitude, phase shift and sketch the curve of

$$f(x) = 1 + \frac{1}{3} \sin \left(2x + \frac{\pi}{4} \right).$$

d) Let $f(x) = \frac{|x|}{x}$. Determine giving reasons whether or not the function is continuous at $x = 0$.

e) Show that $\frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2}$.

QUESTION THREE

✓ 20 (20)

- a) Define an operation $*$ on the set of real numbers by

$$a * b = a^b$$

- Is $*$ a binary operation on the set of real numbers?
- Is the operation commutative?
- Evaluate $(3 * 2) * -2$.

- b) Find λ for which the matrix $\lambda I - A$ is a singular matrix where I

is an identity Matrix given that $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{pmatrix}$. 20

- Solve for real values of x , given that $\sinh^2 x - 3 \cosh x = 3$.
- Differentiate the function $f(x) = \sin x$ using the first principle
- Evaluate the indefinite integral of

$$\int \frac{2x^2 - 5x + 2}{x(x^2 + 1)} dx.$$

QUESTION FOUR

✗

- a) Let $f(x) = 1 - x^2$ and $g(x) = \sqrt{x}$, Find $(g \circ f)(x)$ and determine the domain.

- b) Write the following in sigma notation

(i) $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \dots$ (ii) $1^4 + 2^4 + 3^4 + \dots + n^4 + (n+1)^4$

- c) Find all the critical points of the function

$f(x) = 1 + \sin^2 x + \sin x$ defined on the interval
 $0 < x < 2\pi$.

- d) Find the equation of the tangent to the graph of the

function $f(x) = 1 + \sin^2 x + \sin x$ at a point where $x = \pi$.

- e) Evaluate the indefinite integral of $\int \ln x \, dx$.

QUESTION FIVE

✓

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- a) Let α and β be the roots of the quadratic equation $3x^2 + 2x + 5 = 0$. Find a quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ without calculating α and β .

✓

- b) The number of grams of a certain radioactive substance present after t hours is given by the equation $Q = Q_0 e^{-0.45t}$, where Q_0 represents the initial number of grams. How long will it take 2500 grams to be reduced to 1250 given $\ln\left(\frac{1}{2}\right) = -0.693$.

- c) Sketch the graph of the polar equation $r^2 = \sin 2\theta$.

- d) Find the cube root of $8i$.

$$\begin{array}{r} 360 \\ 4 \overline{) 1440} \\ \underline{1600} \\ 1600 \\ \underline{1600} \\ 0 \end{array}$$

- e) Find the derivative of $xe^y - 3y \sin x = 1$.

QUESTION SIX

✓

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- a) Prove that $\sqrt{2}$ is an irrational number.

- b) Find the centre and length of a radius of the given circle and graph it $x^2 + y^2 - 10x = 0$.

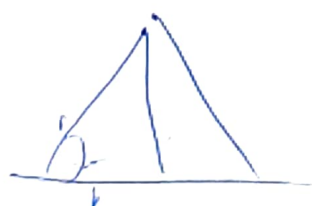
- c) Solve the trigonometric equation $\sin x + \cos x = \sqrt{2}$ if $0 \leq x \leq 2\pi$.

- d) Use the Second derivative Test to find the local extrema of $f(x) = x^4 - 8x^2 + 10$.

- e) Evaluate the indefinite integral of

$$\int e^x \sin x dx.$$

$$\begin{array}{c|c} S & A \\ \hline F & C \end{array}$$



$$\begin{array}{r} 360 \\ 72 \overline{) 2592} \\ \underline{2520} \\ 720 \\ \underline{720} \\ 0 \end{array}$$

$$\begin{array}{r} 130 \\ 210 \overline{) 27300} \\ \underline{4200} \\ 21000 \\ \underline{21000} \\ 0 \end{array}$$

QUESTION SEVEN

- a) Show that the function f defined by $f(x) = \frac{2x}{x-1}$ $x \in R$, is a bijection on R on to $\{y \in R: y \neq 2\}$.
- b) Use binomial theorem to find the value of $(1.01)^{10}$ up to the third term.
- c) Find the exact value of $\tan 67.5^\circ$
- d) Determine the interval for which $f(x)$ is continuous.

$$f(x) = \sqrt{4 - x^2}.$$
- e) Two numbers x and y are connected by the relation $x + y = 6$. Find the values of x and y which give a stationary point of the function $T = 2x^2 + 3y^2$ and determine whether they make T a maximum or minimum.



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