THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 1: **MA110 - Mathematical Methods** 2022

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1. A) List the elements of each of the following sets
   - i) {x: x is natural number less than 5}
   - ii) {x: x is a negative integer greater than -3}
   - iii) {x: x is a positive number less than -5} $\cup$ {1,2,3}
   - iv) {x: x is a positive even number less than 10} $\cap$ {x: x is an integer.}
   - v) $\{x: x = 4k - 1, \text{where } k = 0,1,2,3,4,5\}$
   - vi) {x: x is an integer} $\cap \{1, \sqrt{2}, 3.14, 7\}$

   B) Given that $A = \{-2,-1,0,1,2\}$ .List the elements of the following sets

   i) $\{x^3 : x \in A\}$ ii) $\{x^2 + x : x \in A\}$ iii) $\left\{{}^{2}/_{x+1} : x \in A\right\}$ iv) $\{3x^2 + 1 : x \in A\}$

2. Describe each of the following in set builder notation
   a) $A = \{1,4,9,16,25\}$
   b) $B = \{-7,-5,-3,-1\}$
   c) C ={2,4,6,8,10,12,16}
   d) $D = \{1,2,4,8,16,32\}$

3. Let $A = \{1,2,3,4,5\}$, $B = \{2,4,6\}$,$C = \{3,4,5\}$ and let $E = \{0,1,2,3,4,5,6,7,8\}$
   Find i) B' ii) $A \cup B$ iii)$A \cap B$ iv)$(A \cup B)'$ v) $(A \cap B)'$ vi) $C - B$

   vii)$(U - A) \cap (B - C)'$ Viii) $A \cup (C - B)$

4. Verify or show the following properties:
   a) Associative Laws: $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$
   b) Distributive Laws : $A \cap (B \cup C) = (A \cap B) U (A \cap C)$
   and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

5. i) Prove the De Morgan's Laws a) $(B \cap C)' = B' \cup C'$ b) $(B \cup C)' = B' \cap C'$
   ii) Prove the $(A')' = A$
   iii) Verify or show the De Morgan's Laws a) $(B \cap C)' = B' \cup C'$ b) $(B \cup C)' = B' \cap C'$

6. If $C \subset D$, then simplify if possible
   i) $C \cap D$ ii) $C' \cup D'$ iii)$C \cup D'$ iv) $C' \cap (C \cup D)$

7. If C *and* D are disjoint, simplify if possible
   i)$C' \cap D'$ ii) $C' \cup D'$ iii) $(C \cap D)'$ iv) $(C \uplus D)'$

8. Represent each of the following on a Venn diagram

i) $A \cap B'$ ii) $(A \cap B)'$ iii) $(A \cap B') \cup (A' \cap B)$ iv) $(A \cup B) \cap (A \cup B')$
v) $[A' \cup (A \cap B)']'$ Vi) $A' \cap B' \cap C$

9. Using the associative and distributive properties of union and intersection of sets .Show that
i) $A = (A \cap B) \cup (A \cap B')$ ii) $A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B)$
iii) $A \cup (A' \cap B) = A \cup B$

10. a)Given that X, Y and Z are sets, simplify the following if possible
i) $[X' \cup (Y \cap Z)]'$ ii) $Y' \cap (X \cup Y)$ iii) $(X \cap Y) \cup (X \cap Y')$ iv) $(X \cup Y) \cap (X \cup Y')$

v) $[X' \cup (X - Z)]$

b) Given that X and Y are subsets of some universal set U, simplify the following:

(i) $X \cap (X' \cup Y)$.

(ii) $[(X \cap Y)' \cap (X' \cup Y)]'$.

11 a) Using the symbol " $\subset$ " put the set of numbers in ascending order given

$\mathbb{C}, \mathbb{N}, \mathbb{R}, \mathbb{Z}$ *and* $\mathbb{Q}$

b) Give the definition of the following sets $\mathbb{Q}^*, \mathbb{R}^*, \mathbb{C}^*$and $\mathbb{Z}^*$

12.a) Let $A = \{x \in \mathbb{R}: -4 \leq x < 2\}$ and $B = \{x \in \mathbb{R}: x \geq -1\}$. Find a) $A \cap B$ b) A'

b) Let $U = (-6,9]$ be the universal set, $= [-2,4], B = (-1,5)$ *and* $C = (-6,9]$.
Find the following sets:
i) $A \cap B$ ii) $U - C$ iii) $B'UA$ iv) $(A \cup C)'$

c) Given that $\mathbb{R}$, the set of real numbers is the universal set, $A = (-4,7]$ *and* $B = [4, \infty)$,
Find
i)A' ii)B' iii)$A - B$ iv) $B - A$

d) Let $A = (-9,9)$ be the universal set and $X = (-1,5]$, $Y = [-5,3]$ *and* $Z = [-1,7)$.
Find each of the following sets and display it on the number line:
i)X' ii) $A - X$ iii) $(X \cap Z)'$ iv) $(Y - X) \cap Z$

e) Let A = {1,2, 3, 4,5,6,7,8,9} ; B = [1, 5) and C= (3,8). The universal set

is a set of real numbers . If necessary use the real number line and find:

(i). (A ∩ B) ∪(A ∩ C) (ii). B ∩ C (iii). (B ∪ C)'

(iv). (B' ∪ C') ∩ A

13. a) Express the following in the form of $\frac{a}{b}$ where a and b are intergers, $b \neq 0$.
i) $0.\overline{33}$ ii) $0.\overline{16}$ iii) $2.\overline{143}$ iv) $3.\overline{7}$ v)1.171717..... vi) $2.5\overline{90}$

b) Prove that i) $\sqrt{3}$ is an irrational number.
ii) $\sqrt{2}$ is an irrational number

c) Given that $\sqrt{3}$ and $\sqrt{5}$ are irrational, show that the following are not rational numbers
i) $\sqrt{3} + 5$ is an irrational number.
ii) $\sqrt{5} - 1$ is an irrational number iii) $1 - \sqrt{3}$ is an irrational number