MULUNGUSHI UNIVERSITY

SCHOOL OF SCIENCE, ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS AND SCIENCES

MSM 111 - Mathematical Methods I

Tutorial Sheet 1 - 2020/2021 - Introduction to the Theory of Sets

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1. Given that $U = \{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$, $X = \{$Even numbers in $U\}$, $Y = \{$ Prime numbers in $U\}$ and $Z = \{-2, 3, 5, 6\}$, perform the following operations

   (i) XUY (ii) $(X \cap Y)^c$ (iii) $X \cap (Y \setminus Z)$ (iv) The cardinality of $X \cup Y$

2. Let $X = [0, 10]$ be the universal set and $A = [1, 4]$, $B = (2, 8)$, $C = [3, 6)$ be the subsets of X. Find each of the following and display them on the real line.

   (i) B^c,   (ii) $A \cap B$ (iii) $(A \cap B)^c$ (iv) $A \cap (B \setminus C)$ (v) $(X \setminus A) \cup (A \setminus B)$

3. Let $\mathbb{R}$ be the universal set and $D = (-7, 4]$, $E = (0, 8)$, $F = \{x : x \le 10, x \in \mathbb{R}\}$ be the subsets of X.

   (i) Find the set $E \setminus F$ and display it on the real line.

   (ii) Using the sets above, verify that $D^c \cap E^c = (D \cup E)^c$

4. (i) For each of the following sets state whether it is finite or infinite and in each case justify your answer.

   $$A = \{0, 1\} \cap \mathbb{R} \qquad B = \{x \in \mathbb{R} : x^2 < 4\} \qquad C = [0, 1] \cap \left(\tfrac{4}{3}, \tfrac{3}{2}\right) \qquad D = \{x \in \mathbb{R} : x^2 = 4, x \text{ is old }\}$$

   (ii) Which of the sets listed below are equal to other on the list?

   $$A = [1, 2] \cap (2, 3) \qquad B = [1, 2] \cap [2, 3] \qquad C = \{1, 2\} \cap \{2, 3\} \qquad D = \{1, 3\} \cap [2, 3]$$

   Justify your answer.

5. (a) Let $A = \{x \in \mathbb{Z} : x > 5\}$, $B = \{x \in \mathbb{Z} : x < 9\}$ and $C = \{x \in \mathbb{Z} : 5 \le x \le 7\}$. Find

   (i) $A \cap B$ (ii) $B \cup C$ (iii) $(A \cup C)^c$ (iv) $(A^c \cup B)^c$ (v) $n[P(C)]$

   (b) Let $X = \{x \in \mathbb{R} : 3 < x < 6\}$, $Y = \{x \in \mathbb{R} : 5 \le x \le 7\}$ and $Z = \{0, 1, 2, 3\}$. Find

   (i) $X \cup Y$ (ii) $X^c \cap Z$ (iii) $(X \cap Y) \cup Z$ (iv) $X \cap (Y \cap Z)$.

6. (a) Given that X and Y are disjoint sets, simplify $X^c \cup Y^c$.

   (b) Given that P and Q are any two sets, express the following in its simplest form:

   (i) $P^c \cap (P \cup Q)^c$ (ii) $(P \cap Q) \cup (P \cap Q^c)$ (iii) $[P^c \cap Q^c]^c$

   (v) $[(P \cap Q) \cup (P \setminus Q)]^c$ (vi) $(P \cap Q) \cup (P \cap Q^c)$ (vii) $(P \cap Q) \cup (P \cap Q^c) \cup (P^c \cap Q)$

7. Express each of the following in the form $\frac{x}{y}$, where x and y are integers and $y \ne 0$:

   (i) $0.1666...$ (ii) $0.\overline{24}$ (iii) $0.016\dot{7}$ (iv) $2.99999...$ (iv) $1.34\overline{355}$

8. Prove that the following are irrational numbers:

   (i) $\sqrt{3}$ (ii) $1 + \sqrt{2}$ (iii) $\sqrt{3} + \sqrt{2}$ (iv) $\frac{1}{1+\sqrt{2}}$

9. The operation $*$ on the set of real numbers $\mathbb{R}$ is defined by $a * b = a^2 + b^2 - 2$ where $a, b \in \mathbb{R}$.

   (i) Is $*$ a binary operation on $\mathbb{R}$?

   (ii) Determine whether $*$ is commutative

   (iii) Determine whether $*$ is associative

   (iv) Evaluate $(3 * 2) * 5$