Chapter 1

Units and Measurements

1.0 Introduction

- Measurements in everyday life are necessary. For example, measurement of mass, volume or length of an iPhone.
- There is need for measurements in physics to understand any phenomenon. For example, carrying out experiments we may need to measure mass, time, temperature, pressure, etc.

2.0 Physical quantity

- A physical quantity or property is that which can be **measured** and **described** by a number. For example, mass of a laptop is 3.5kg. Thus, mass is a physical quantity described by 3.5 (**measurement)** kg (unit/**description**).

2.1 Types of physical quantities

- There are two types namely: **fundamental (basic)** physical quantities and **derived** physical quantities.
- **Fundamental (basic)** physical quantities do not depend on any other physical quantities for their **measurement** and **description.** For example, mass (Kg), time(s), length (m), temperature (K), electric current (A), amount of substance (mole, mol.), luminous intensity (candela, cd) are **all** fundamental (basic) physical quantities.
- **Derived** physical quantities do depend on one or more fundamental quantities for their **measurement** and **description.** For example, area depends on two dimensions of length or speed depends on length and time.

  Mental exercise: What physical quantities the following derived physical quantities depend:

  a) Acceleration

  b) Force

3.0 Units for measurements

- The standard used for the measurement of a physical quantity is called a **unit.**
- Units in physics can be broadly categorized into two types: **base (fundamental)** units and **derived** units.

3.1 Base units

- **Base** units are the fundamental units that are not derived from any other units.
- The **base** units include:
  a) Meter (m) for length.
  b) Kilogram (kg) for mass.
  c) Second (s) for time.
  d) Ampere (A) for electric current.
  e) Kelvin (K) for temperature.
  f) Mole (mol.) for amount of substance.
  g) Candela (cd) for luminous intensity.

3.2 Derived units

- **Derived** units are derived from combinations of base units and represent quantities that are defined in terms of one or more base units.
- Examples of **derived** units include:
  a) Square meter (m^2) for area.
  b) Cubic meter (m^3) for volume.
  c) Newton (N) for force ($1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2$).
  d) Joule (J) for energy and work ($1 \text{ J} = 1 \text{ N}\cdot\text{m}$).
  e) Watt (W) for power (1 W = 1 J/s).
  f) Hertz (Hz) for frequency (1 Hz = 1/s).

4.0 Standard units for fundamental physical quantities

- Standard unit is an internationally accepted measurement for a physical quantity.
- An international committee established a set of standards for measurements of physical quantities. It is called the SI (Systeme International). The

committee established units of length, mass and time as the meter, kilogram and second respectively.

5.0 Standard prefixes

- **Standard prefixes** are added to units to represent multiples or fractions of those units.
- Here are some prefixes arranged in order from small negative exponents to large positive exponents:
  1. Pico- (p): (10^{-12}) e.g. $20pm = 20 \times 10^{-12}m$
  2. Nano- (n): (10^{-9}) e.g. $20nm = 20 \times 10^{-9}m$
  3. Micro- (μ): (10^{-6}) e.g. $20\mu m = 20 \times 10^{-6}m$
  4. Milli- (m): (10^{-3}) e.g. $20mm = 20 \times 10^{-3}m$
  5. Centi- (c): (10^{-2}) e.g. $20cm = 20 \times 10^{-2}m$
  6. Deci- (d):): (10^{-1}) e.g. $20dm = 20 \times 10^{-1}m$
  7. Base Unit (no prefix): (10^{0}) e.g. $20m = 20 \times 10^{0}m$
  8. Deca- (da): (10^{1}) e.g. $20dam = 20 \times 10^{1}m$
  9. Hecto- (h): (10^{2}) e.g. $20hm = 20 \times 10^{2}m$
  10. Kilo- (k): (10^{3}) e.g. $20km = 20 \times 10^{3}m$
  11. Mega- (M): (10^{6}) e.g. $20Mm = 20 \times 10^{6}Mm$
  12. Giga- (G): (10^{9}) e.g. $20Gm = 20 \times 10^{9}m$
  13. Tera- (T): (10^{12}) e.g. $20Tm = 20 \times 10^{12}m$
  14. Peta- (P): (10^{15}) e.g. $20Pm = 20 \times 10^{15}m$

6.0 Conversion of units

- The conversion of units is crucial for expressing quantities in different **measurement systems** or **units**.
- A **measurement system** is a set of units and standards used to quantify and express various physical quantities. **Examples** are: **metric system**, **Imperial measurement system**, **centimeter-gram-second (CGS)**, **International system of units (SI units)**, etc.
- The International System of Units (SI) is the **most widely** used measurement system in physics and science in general.

- **Prefixes** and **conversion factors** play a significant role in this process. **Prefixes** make it easier to express measurements of different magnitudes without using **excessively large** or **small numbers**. For example, 2 Megawatts (MW) = 2,000, 000W or 2 Micrometers (μm) = 0.000002meters.
- To perform unit conversions, you can use the **conversion factors** associated with these prefixes. For example, to convert 5 kilometers to meters, you would use the conversion factor 1km = 1000m:

$$5km \times \frac{1000m}{1km} = 5000m$$

  Similarly, to convert 500millimeters to meters, you would use the conversion factor 1m = 1000mm:

$$500mm \times \frac{1m}{1000m} = 0.5m$$

  Homework:

  a) Imagine you are setting up a secure server room with square walls, and you need to determine the surface area of the walls for proper security measures. The height of the walls is 8.00 ft. and each side of the square room measures 12 ft. Calculate the total surface area of the walls in square meters.

  b) Express the following in terms of prefixes

  (i) $0.00085\ l$ (ii) $5.44 \times\ 10^{-11}$ g (iii) $73{,}000{,}000\ m$ (iv) $9.450\ s$

7.0 Dimension

- The **dimension** of physical quantity may be defined as the **number of times** the fundamental units of mass, length and time appear in the physical quantity.
- For example, the dimensions of density are 1 in mass and –3 in length. The powers are determined as follows:

$$[Density] = \frac{[Mass]}{[Volume]}$$

$$[\,] - symbol\ means\ "dimension\ of"$$

$$[\rho] = \frac{[M]}{[L] \times [L] \times [L]}$$

$$[\rho] = \frac{[M]}{[L^3]}$$

$$[\rho] = [M^1 L^{-3}]$$

Homework:

- Compute the dimensions of the following (show your work clearly):
  a) Velocity
  b) Acceleration
  c) Energy
  d) Force
  e) Power
  f) Pressure
  g) Frequency

8.0 Dimensional Analysis

8.1 What dimensional analysis is

- **Dimensional analysis** is the analysis of a **relationship** by considering its **units** of measure.
- For example, we can analyse dimensionally the following equations:
  a) $s = vt$ or distance = speed x time

$$[s] = [v] \times [t]$$

$$L = \frac{L}{T} \times T$$

$$L = L$$

  b) $v = u + at$

$$[v] = [u] + [a] \times [t]$$

$$LT^{-1} = LT^{-1} + LT^{-2} \times T$$

$$LT^{-1} = LT^{-1} + LT^{-1}$$

$$LT^{-1} = 2LT^{-1}$$

- Equations (a) and (b) are said to be **dimensionally homogenous** or **dimensionally consistent**. This means that the equations have consistent dimensions. In other words, each physical quantity in an equation must have the **same** dimensions on **both sides** of the equation.

- In addition, equations can also be **dimensionally non-homogenous** or **dimensionally inconsistent.** This means that the equations have no consistent dimensions. That is, each physical quantity in an equation must have **different** dimensions on **both sides** of the equation. For example, the following equation is dimensionally inconsistent.

$$s = vt^2$$
$$[s] = [v] \times [t]^2$$
$$L = LT^{-1} \times T^2$$
$$L = LT^1$$

8.2 Crucial terms

- **Dimensional variables** are the quantities which actually **vary** during a given case and can be plotted against each other. They even have dimensions.
- **Dimensional constants** are normally held constant during a case. But they may vary from case to case. They also have dimensions.
- **Pure constants** have no dimensions but while performing the mathematical manipulation they can arise.
- These terms (i.e. dimensional variables, dimensional constants, pure constants) can be explained using the following example:

  The displacement of a free falling body is given as,

$$\Delta s = ut + \frac{1}{2}gt^2$$

  Dimensional variables in this equation are: Δs, u, and t

  Dimension constant in this equation is g:

  Pure constant is $\frac{1}{2}$

- **Dimensionless quantities** have no dimensions. The following are the examples of dimensionless quantities:

  a) Angle measures

$$C = 2\pi r, \qquad 2\pi = 360° = \theta$$

$$C = \theta r$$

$$[\theta] = \frac{C}{r} = \frac{[C]}{[r]} = \frac{L}{L} = 1$$

b) All trigonometric functions

$$[\sin\theta] = \frac{O}{H} = \frac{[O]}{[H]} = \frac{L}{L}$$

$$[\cos\theta] = \frac{A}{H} = \frac{[A]}{[H]} = \frac{L}{L}$$

$$[\tan\theta] = \frac{O}{A} = \frac{[O]}{[A]} = \frac{L}{L}$$

c) Exponential functions

Consider the general form of an exponential function

$$f(x) = ae^{bx}$$

Here:

a is constant multiplier

e is the base of the natural logarithm (approximately 2.71828)

b is a constant exponent

x is the variable

A specific example is the expression for the radioactive decay of a at time, t.

$$N(t) = N_o e^{-\lambda t}$$

Here:

$N(t)$ is the quantity (number) of the radioactive substance at t

N_o is the initial quantity (number) of the substance

λ is the decay constant

e is the base of the natural logarithm

We can analyse the dimensions as:

$N(t)$ and N_O have no dimensions since are numbers

$[\lambda] = T^{-1}$

$[t] = T$

$[e^{-\lambda t}] = e^{T^{-1}T} = e^1 = e$ (no dimensions)

d) Quantities which are simply counted, such as the number of people in the room

e) Plain numbers such as (1, 2, 3… e, π, etc.)

8.3 Applications of dimensional analysis

8.3.1 To check the correctness of equations

- Consider the equation of displacement,

$$\Delta x = ut + \frac{1}{2}at^2$$

$$[\Delta x] = [u] \times [t] + \left[\frac{1}{2}\right] \times [a] \times [t]^2$$

$$L = LT^{-1} \times T + \left[\frac{1}{2}\right] \times LT^{-2} \times T^2$$

$$L = L + 1 \times L$$

$$L = L + L$$

$$L = [2]L$$

$$L = L$$

8.3.2 To convert units

- Using dimensional analysis, we can convert **one system of units to another** or **within a system**. For example, we can convert Newton (SI Units of force) into dyne (CGS) or within the SI system (e.g. Kilo Newton to Newton). The latter is within the scope of this course.

8.3.3 To derive a formula

- Using dimensional analysis, we can derive a formula showing the relationship of physical quantities.
- **Example:**

The centripetal force, F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v) and radius (r) of the circle. Derive the formula for F using dimension analysis.

Solution:

$$\boldsymbol{F \propto (m)^a (v)^b (r)^c}$$

$$\boldsymbol{F = k(m)^a (v)^b (r)^c \ldots (1)}$$

Here, k is a dimensionless constant of proportionality. Writing the dimensions of RHS and LHS in eqn (1), we have:

$$[F] = [k][m]^a[v]^b[r]^c$$

$$MLT^{-2} = M^a[LT^{-1}]^b[L]^c$$

$$MLT^{-2} = M^aL^bT^{-b}L^c$$

$$MLT^{-2} = M^aL^{b+c}T^{-b}$$

$$a = 1, b + c = 1, -b = -2$$

$$a = 1, b = 2, c = -1$$

Thus eqn (1) becomes:

$$F = km^1v^2r^{-1} \ldots (2)$$

Applying indices and where k=1 eqn (2) is simplified as follows:

$$F = \frac{mv^2}{r}$$

8.4 Limitations of dimensional analysis

- **Homework:**

  List at least three limitations of dimensional analysis.

9.0 Measuring instruments

- In this course, it is important to revise on how to use basic measuring instruments such as: **vernier calipers, micrometer screw gauge, the beam balance, stop watch,** etc.

9.1 Key concepts related to measurements

- **Least count:** the smallest value that can be measured by the measuring instrument. This is also called the **resolution.** For example, the least count of the following measuring instruments:

  i. Ruler scale = Least count = 1mm

  ii. Vernier caliper = Least count = 0.1mm

  iii. Micrometer screw gauge = Least count = 0.01mm

- **Accuracy of measurement:** this refers to the closeness of a measurement to the true value of the physical quantity. For example:

  True value of mass = 35.67kg

  Mass measured by student A = 35.61kg

Mass measured by student B= 35.65kg

The measurent made by student is more **accurate.**

- **Precision of measurement:** this refers to the limit to which a physical quantity is measured. For example:

  Time measured by student A = 3.6s

  Time measured by student B = 3.69s

  Time measured by student C = 3.695s

  The measurement made by student C is most precise.

- **Uncertainty in measurements:** it is important to recognize that all measurements are wrong in that the measured value (the result) and the right answer (the 'true' value) are different. The difference between these two is the measurement error, which is meant in the sense of a discrepancy, rather than an avoidable mistake. Unfortunately, the true value is never precisely known and by the same token, neither is the measurement error. Rather, one makes a statement about measurement **uncertainty** whose purpose may be summarised as follows:

- **A statement of uncertainty indicates how large the measurement error might be:**

  For instance:

  $T = T_m \pm U_r$

  where

  $T \text{ is the true value}$

  $T_m \text{ is the measured value, and}$

  The range $\pm U_r$ is the **measurement uncertainty.**

  This statement can be taken to mean that the true value, T, probably lies somewhere between $T_m - U_r$ and $T_m + U_r$. On its own this statement is incomplete, since there is a need to quantify what the word probably means.

- **Example:** The ambient air pressure is measured with a barometer and found to be $105.40kPa$. This result might be reported either as:

  $(105.40 \pm 0.10)kPa$ or

$(105.40 \pm 0.07)kPa$

Note that these different ways of reporting the **same** measurement. The is greater confidence in a measured value when it is quoted with a large uncertainty and less confidence when the same value is with a smaller uncertainty.

10.0 Significant figures

- The number of **significant figures** in a result is simply the number of figures that are known with some degree of reliability.
- The number 13.2 is said to have 3 significant figures. The number 13.20 is said to have 4 significant figures.
- **Rules for deciding the number of significant figures in a measured quantity:**

1) *All nonzero digits are significant:*

   1.235g has 4 significant figures,

   1.2g has 2 significant figures.

2) *Zeroes between nonzero digits are significant:*

   1003kg has 4 significant figures,

   3.09mL has 3 significant figures

3) *Zeroes to the left of the first nonzero digits are not significant; such zeroes merely indicate the position of the decimal point:*

   0.001m has only 1 significant figure,

   0.015cm has 2 significant figures

4) *Zeroes to the right of a decimal point in a number are significant:*

   0.034mL has 2 significant figures,

   0.200g has 3 significant figures

5) *When a number ends in zeroes that are not to the right of a decimal point, zeroes are not necessarily significant:*

   180 miles may be 2 or 3 significant figures,

   30,500 joules may be 3, 4 or 5 significant figures

   The potential ambiguity in the last rule can be avoided by the use of standard exponential or "scientific" notation. For example, depending on whether 3, 4 or 5 significant figures is correct, we could write 30, 500 joules as:

$3.05 \times 10^4\ joules$ (3 significant figures),

$3.050 \times 10^4\ joules$ (4 significant figures) or

$3.0500 \times 10^4\ joules$ (5 significant figures)

11.0 Errors in measurements

- **Homework:**

1) Define an **error** in measurements.
2) State and explain three types of **errors** i.e. **systematic, gross and random errors.**