



MA110 - MATHEMATICAL METHODS

Time allowed: Two hours (2:00 hours)

Instructions:

1. You must write your Name, your Computer Number and programme of study on your answer sheet.
2. Calculators are not allowed in this paper.
3. There are three (3) questions in this paper, Attempt All questions and show detailed working for full credit

QUESTION ONE

- a) (i) If $C \subset D$, then simplify if possible $C' \cup D'$ (2.5 marks)
- (ii) Express $1.171717\ldots$ as a fraction $\frac{a}{b}$ in its simplest form where a and b are integers and $b \neq 0$. (2.5 marks)

* b) Consider the binary operation $a * b = a + b - 2ab$, where a and b are real numbers.

(i) Is $*$ a binary operation on the set of real numbers? Give reason for your answer. (1) Mark

(ii) Is the operation $*$ commutative? If not give a counter example. (1) Mark

(iii) Find the value of $1 * (2 * 3)$ and $(1 * 2) * 3$ and state whether $*$ is associative (3) Marks

c) Given the rational function $f(x) = \frac{x+2}{x-2}$. Sketch its graph indicating its domain and range, all the asymptotes and intercepts. (5 Marks)

d) Prove that $\sqrt{2}$ is an irrational number (5 Marks)

e) Let $f(x) = \frac{x+1}{x-1}$ and $g(x) = \sqrt{x}$. Find $(g \circ f)(x)$ and determine the domain (5 Marks)

QUESTION TWO

- a) Using the associative and distributive properties of union and intersection of sets. Show that

$$A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B) \quad (5 \text{ Marks})$$
- b) Let α and β be the roots of the quadratic equation $3x^2 + 2x + 5 = 0$. Find a quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ without calculating α and β (5 Marks)
- c) solve the given radical function inequality $\sqrt{2} - \sqrt{x+6} \leq -\sqrt{x}$ (5 Marks)
- d) Solve for x and y given that:

$$\frac{x}{1+i} - \frac{y}{2-i} = \frac{1-5i}{3-2i} \quad (5 \text{ Marks})$$
- e) Show that the function f defined by $f(x) = \frac{2x}{x-1}$ $x \in \mathbb{R}$, is a bijection on \mathbb{R} on to $\{y \in \mathbb{R} : y \neq 2\}$ (5 Marks)

QUESTION THREE

- a) Use the Rational root theorem to solve $x^3 - 4x^2 + 8 = 0$ (5 Marks)
- b) Rationalize the denominator $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$ (5 Marks)
- c) (i) Determine whether the function $f(x) = x^4 + x^2 + 1$ even, odd or neither. (2.5 marks)
- (ii) Let $A = \{x \in \mathbb{R} : -4 \leq x < 2\}$ and $B = \{x \in \mathbb{R} : x \geq -1\}$.
 Find a) $A \cap B$ b) A' (2.5 marks)
- d) What are the dimensions of the largest rectangular field which can be enclosed by 1200 m of fencing? (5 Marks)
- e) Sketch the graph of $f(x) = |2x+1|$. On the same diagram sketch also the graph of $g(x) = \sqrt{1-2x}$ and hence, find the values such that $\sqrt{1-2x} > |2x+1|$ (5 Marks)