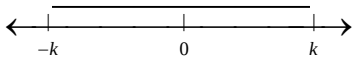
ABSOLUTE VALUE INEQUALITIES

Reminder: The absolute value of the real number x is the distance on the number line from the origin to the number x.

There are two cases:

a) Case 1: $|x| < k$

Let $k > 0$, then $|x| < k$ means the distance from the origin to the number x is less than k.



Thus $|x| < k$ is equivalent to $-k < x < k$.

Property 1-On absolute value inequalities
For any real number $k > 0$,
$\|x\| < k$ is equivalent to $-k < x < k$
and
$\|x\| \leq k$ is equivalent to $-k \leq x \leq k$

EXAMPLE 1

Solve:

i. $|2x + 1| < 5$
ii. $|6x - 1| \leq 3$

Solutions:

i. $|2x + 1| < 5$ is equivalent to
$-5 < 2x + 1 < 5$
⇨ $-5 - 1 < 2x + 1 - 1 < 5 - 1$
⇨ $-6 < 2x < 4$
⇨ $\frac{-6}{2} < \frac{2x}{2} < \frac{4}{2}$
⇨ $-3 < x < 2$

$\therefore Solution\ set = (-3, 2)$ in interval notation.

ii. $|6x - 1| \leq 3$ is equivalent to

⇨ $-3 \leq 6x - 1 \leq 3$

⇨ $-3 + 1 \leq 6x - 1 + 1 \leq 3 + 1$

⇨ $-2 \leq 6x \leq 4$

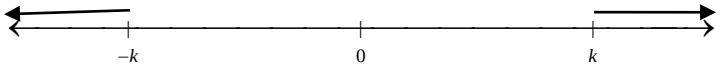
⇨ $\frac{-2}{6} \leq \frac{6x}{6} \leq \frac{4}{6}$

⇨ $-\frac{1}{3} \leq x \leq \frac{2}{3}$

$\therefore$ $Solution\ set = \left[-\frac{1}{3}, \frac{2}{3}\right]$ in interval notation.

b) Case 2: $|x| > 0$

Let $k > 0$, then $|x| > k$ means the distance from origin to the number x is greater than k.



Hence, $|x| > k$ is equivalent to double disjoint sets $x < -k$ or $x > k$.

Property 2- On absolute value inequalities

For any real number $k > 0$,

$|x| > k$ is equivalent to $x < -k \ or \ x > k$

and

$|x| \geq k$ is equivalent to $x \leq -k \ or \ x \geq k$

EXAMPLE 2

Solve $|5x - 10| > 20.$

Solution:

We write the inequality as two separate inequalities

$5x - 10 < -20$ or $5x - 10 > 20$

and solve each one for x:

either $5x - 10 < -20$

- ⇨ $5x < -20 + 10$
- ⇨ $5x < -10$
- ⇨ $\frac{5x}{5} < \frac{-10}{5}$
- ⇨ $x < -2$

Or $5x - 10 > 20$

- ⇨ $5x > 20 + 10$
- ⇨ $5x > 30$
- ⇨ $\frac{5x}{5} > \frac{30}{5}$
- ⇨ $x > 6$

Hence, the solution set of the inequality $|5x - 10| > 20$ is:

$x < -2 \ \ or \ \ x > 6$ or

$(-\infty, -2) \cup (6, \infty)$.

EXAMPLE 3

Solve the inequality:

$$\left|\frac{x-2}{x+3}\right| \geq 4.$$