The Copperbelt University

School of Mathematics And Natural Sciences

Department of Mathematics

MA 110 : Mathematical Methods I

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@Tutorial Worksheet 5 **@ Academic year : 2021/22**

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(1). Express each of the following as a single fraction :

(a) $\dfrac{3}{x+3} - \dfrac{2}{x-2}$. (b) $\dfrac{1}{(x+2)^2} - \dfrac{2}{x+2} + \dfrac{1}{3x-1}$. (c) $\dfrac{4}{2+3x^2} - \dfrac{1}{1-x}$.

(d) $\dfrac{3}{x^2+1} - \dfrac{1}{1-x} + \dfrac{2}{(x-1)^2}$.

(2). Express the following fractions into partial fractions

(a) $\dfrac{2x}{(3+x)(3-x)}$. (b) $\dfrac{a}{a^2-b^2}$. (c) $\dfrac{1}{p(1-p)}$.

(d) $\dfrac{s}{s^3+1}$. (e) $\dfrac{2}{s(s+1)}$.

(3). Find the values of the constants A, B, C in the following identities

(a) $31x - 8 \equiv A(x-5) + B(4x+1)$.

(b) $8 - x \equiv A(x-2)^2 + B(x-2)(x+1) + C(x+1)$.

(c) $71 + 9x - 2x^2 \equiv A(x+5)(x+2) + B(x+2)(x-3) + C(x-3)(x+5)$.

(d) $4x^2 + 4x - 26 \equiv A(x+2)(x-4) + B(x-4)(x-1) + C(2x^2+1)(x+2)$.

(4). Can values of A, B, C, D be found which makes the following pairs of expression is identical?

(a) $2x^2 - 22x + 53$ and $A(x-5)(x-3) + B(x-3)(x+2) + C(x+2)(x-5)$.

(b) $x + 7$ and $A(x-2) + B(x+1)^2$.

(c) $x + 1$ and $A(x-2) + B(x^2+1)$.

(d) $x^3 + 2x^2 - 4x - 2$ and $(Ax + B)(x - 2)(x + 1) + C(x + 1) + D(x - 2)$.

(5). (a) Find the values of A, B, C if $x^3 - 1$ is expressed in the form $(x-1)(Ax^2+Bx+C)$.
(b) Express $x^3 + 1$ in the form $x(x - 1)(x - 2) + Ax(x - 1) + Bx + C$.

(6). Find the partial fraction representation of

(a) $\dfrac{6}{(x+3)(x-3)}$. (b) $\dfrac{x-1}{3x^2-11x+10}$. (c) $\dfrac{6x^2+27x-3}{(x+2)(x+1)(x-3)}$.
(d) $\dfrac{3-4x}{2+3x-2x^2.}$

(7). Express the following into partial fractions

(a) $\dfrac{6-x}{(1-x)(4+x^2)^2}$. (b) $\dfrac{4}{(x+1)(2x^2+x+3)}$. (c) $\dfrac{3+2x}{(2-x)(3+x^2)}$
(d) $\dfrac{3x+7}{(x+1)(x^2-4)}$.

(8). Express the following fractions into partial fractions

(a) $\dfrac{x+1}{(x+3)^2}$. (b) $\dfrac{2x^2-5x+7}{(x-2)(x-1)^2}$. (c) $\dfrac{1}{x^4+5x^2+6}$. (d) $\dfrac{2x+1}{x^3-1}$.

(9). Express the following fractions into partial fractions

(a) $\dfrac{x^3+2x^2-2x+2}{(x-1)(x+3)}$. (b) $\dfrac{x^3-x^2-4x+1}{x^2-4}$. (c) $\dfrac{x^4+3x-1}{(x+2)(x-1)^2}$.
(d) $\dfrac{2x^4-17x-1}{x^2+5}$. (e) $\dfrac{3x^2+2x-9}{(x^2-1)^2}$.

(10). Find the partial fraction decomposition for each of the following rational expressions

(a) $\dfrac{11x-10}{x^2-x-2}$. (b) $\dfrac{-2x-8}{x^2-1}$. (c) $\dfrac{20x-3}{6x^2+7x-3}$. (d) $\dfrac{-6x^2+7x+1}{x(2x-1)(4x+1)}$.
(e) $\dfrac{-6x^2+19x+21}{x^2(x+3)}$. (f) $\dfrac{-9x^2+7x-4}{x^3-3x^2-4x}$. (g) $\dfrac{10x^2-73x+144}{x(x-4)^2}$.
(h) $\dfrac{4x^2+3x+14}{x^3-8}$. (i) $\dfrac{3x^2+10x+9}{(x+2)^3}$. (j) $\dfrac{2x^2+x+2}{(x^2+1)^2}$. (k) $\dfrac{2x^4-17x-1}{x^3-2x^2+5x-10}$.
(l) $\dfrac{3x^2+10x+9}{x^3+6x^2+12x+8}$. (m) $\dfrac{3x+7}{x^3+x^2-2x}$. (n) $\dfrac{2x^2-5x+7}{x^3-4x^2+5x-2}$.

ARITHMETIC AND GEOMETRIC SERIES AND SEQUENCES

(1). Which of the following sequences are arithmetic sequence? write down the common difference of those that are.

(a) $1, 6, 11, 16, \ldots$. (b) $\frac{1}{3}, 1, \frac{5}{3}, \frac{7}{3}, \ldots$. (c) $4, -1, -6, -11, \ldots$. (d) $9, 12, 16, \ldots$.
(e) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$. (f) $7, 9 + 3p, 11 + 6p, 13 + 9p, \ldots$.
(g) $7, 8\frac{1}{2}, 10, 11\frac{1}{2}$. (h) $n, 2n, 3n, 4n$. (i) $1^2 + 2^2 + 3^2 + 4^2$.

(2). write down the terms indicated in each of the following $A.Ps$

(a) $3, 11, \cdots, 10^{th}, 19^{th}.$ (b) $\frac{1}{4}, \frac{7}{8}, \cdots, 12^{th}, n^{th}.$

(c) $3 + 7 + \cdots + 200^{th} + (n+1)^{th}.$

(3). Find the number of terms in the following $A.Ps$

(a) $x, 2x, \cdots, nx.$ (b) $a + (a+d) + \cdots + (a + (n-1)d).$

(c) $2, 4, \cdots, 4n.$ (d) $2 - 9 - \cdots - 130.$

(4). Find the sum of the following $A.Ps$:

(a) $x + 3x + 5x + \cdots + 12x.$ (b) $a + (a+1) + \cdots + (a+n-1).$

(c) $2.01 + 2.02 + 2.03 + \cdots + 3.00.$ (d) $1 + 3 + 5 + \cdots + 101.$

(5). Find the sums of the following arithmetical progressions as far as the terms indicated:

(a) $4 + 10 + \cdots$ $(12^{th}\ term).$ (b) $1\frac{1}{4} + 1 \cdots$ $(n^{th}\ term).$

(c) $15 + 13 + \cdots$ $(20^{th}\ term).$ (d) $1 + 2 + \cdots$ $(200^{th}\ term).$

(6). (a) Show that the sum of the first n terms of the $A.P$ with first term a and common difference d is $\frac{1}{2}n(2a + (n-1)d)$ or $S_n = \frac{n}{2}(a_1 + a_n)$.

(b) Show that sum of the integers from 1 to n is $\frac{1}{2}n(n+1)$.

(c) Show that the general term of an arithmetic sequence is given by $a_n = a_1 + (n-1)d$ where a_1 is the first term and d is the common difference.

(7). (a) The second term of an $A.P$ is 15, and the firth term is 21. Find the common difference, the first term and the sum of the first ten terms.

(b) The fourth term of an $A.P$ is 18, and the common difference is -5. Find the first term and the sum of the sixteen terms.

(c) Find the sum of the odd numbers between 100 and 200.

(d) Find the sum of even numbers, divisible by three, lying between 400 and 500.

(8). Write the first five terms of the sequence that has the indicated general term

(a) $a_n = 3n^2 - 1.$ (b) $a_n = 2^{n+1}.$ (c) $a_n = n(n-1).$

(d) $a_1 = 3, \quad a_{n+1} = a_n - 1, n \geq 1.$ (e) $S_0 = 1, S_{n+1} = x^{n+1} + S_n, n \geq 0.$

(f) $a_n = \frac{(-1)^{n+1}}{n(n+1)}.$ (g) $a_n = \frac{2}{n+1}.$

(9). Find the sum of each of the following arithmetic series.

(a) $-193 - 189 - 185 - \cdots - 21 - 17.$ (b) $5 + 8 + 11 + 14 + \cdots + 59 + 62.$

(c) $2\frac{1}{4} + 2\frac{17}{20} + 3\frac{9}{20} + \cdots + 20\frac{1}{4} + 20\frac{17}{20}.$

(10). Find the 15^{th} and 30^{th} terms of the sequence for which $a_n = -5n - 4.$

(11). Find the general term (*the n^{th} term*) for each of the arithmetic sequences.

(a) $11, 13, 15, 17, 19, \cdots$. (b) $7, 10, 13, 16, 19, \cdots$. (c) $-3, -6, -9, -12, -15, \cdots$.

(12). Find the required term for each of the arithmetic sequences

(a) The 15^{th} term of $3, 8, 13, 18, \cdots$. (b) The 52^{th}term of $1, \frac{5}{3}, \frac{7}{3}, 3, \cdots$.

(13). (a) If the 6^{th} term of arithmetic sequence is 12 and the 10^{th} term is 16, find the first term.

(b) If the 3^{th}term of the arithmetic sequence is 20 and the 7^{th} term is 32, find the 25^{th} term.

(c) Find the sum of the first 40 terms of the arithmetic sequences $2, 6, 10, 14, 18, \cdots$.

(d) Find the sum of the first 50 terms of the arithmetic sequences $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \cdots$.

(e) Find the sum of the first 200 odd whole numbers.

(g) Find the sum of all even numbers between 18 and 482, inclusive.

(14). Write each series in expanded form

(a) $\sum_{i=1}^{5}(i^2-2)$. (b) $\sum_{i=1}^{5}(16-4i)$. (c) $\sum_{k=2}^{8}\frac{k+1}{k-1}$. (d) $\sum_{j=1}^{n}(-1)^j\left(\frac{1}{3}\right)^{j-1}$.

(e) $\sum_{j=1}^{\infty}(-1)^{j+1}\left(\frac{1}{2}\right)^{j}$.

(15). Find the sum of the following arithmetic series.

(a) $\sum_{j=1}^{7}2^j$. (b) $\sum_{i=1}^{5}3^{i=1}$. (c) $\sum_{k=2}^{5}\left(\frac{3}{4}\right)^{k-2}$. (d) $\sum_{i=4}^{8}\left(\frac{1}{2}\right)^{i}$

(16). Find each of the following sums

(a) $\sum_{i=1}^{45}(5i+2)$. (b) $\sum_{i=1}^{5}i^2$. (c) $\sum_{i=3}^{3}(2i^2+i)$. (d) $\sum_{n=10}^{20}4n$

(17). Write in the $\sum$ notation

(a) $1+2+3+\cdots+n$. (b) $1^4+2^4+\cdots+n^4+(n+1)^4$. (c) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$.

(d) $3^2+3^3+3^4+3^4$. (e) $2\times 7+3\times 8+4\times 9+5\times 10+6\times 11$.

(18). Write in full:

(a) $\sum_{1}^{4}m^3$. (b) $\sum_{1}^{3}\frac{m}{m(m+1)}$. (c) $\sum_{3}^{6}\frac{(-1)^m}{m}$. (d) $\sum_{1}^{4}(-1)^m m^2$.

(19). Write the first five terms of each sequence

(a) $a_n=\begin{cases} 2n+1 \text{ for } n \text{ odd} \\ 2n-1 \text{ for } n \text{ even} \end{cases}$. (b) $a_n=\begin{cases} 2n+1 \text{ for } n \leqslant 3 \\ 2n-1 \text{ for } n>3 \end{cases}$.

(20). Write the first five terms of each sequence

(a) $\begin{cases} a_1=4 \\ a_n=3a_{n-1} \text{ for } n\geq 3 \end{cases}$. (b) $\begin{cases} a_1=2 \\ a_2=3 \\ a_n=2a_{n-2}+3a_{n-1} \text{ for } n\geq 3 \end{cases}$.

(20). A sequence is generated according to the formula $a_n=bn+c$ where b and c are constants. Given that $a_3=14$ and $a_5=38$. Find the values of b and c.

(22). A freely falling body, starting from rest falss $16ft$ during the first second, $48ft$ during the second, $80ft$ during the third second, etc. Calculate the distance it falls during the fifteenth second and the total distance it falls in 15 seconds from rest.

(23). Given that p is an integer, find the value of p that makes the sequence arithmetic $2p, 3p^2, 11p - 2$?

(24). The first 3 terms of a sequence are $(x - 1), (2x - 4)$ and $(x + 3)$. Which is the first term in the sequence that will exceed 98?