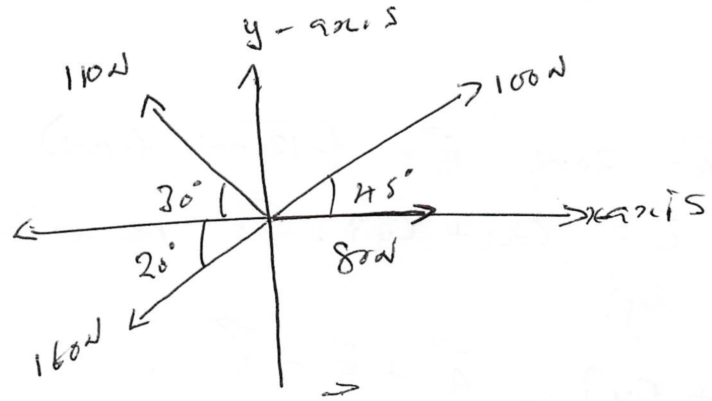
Tutorial Sheet 2

Q1

(i) Use measurements to come up the Resultant.

(ii) Define the line perpendicular to the x-axis as y-axis and Resolve the vectors into components.



$\Rightarrow$ Resultant vector $\vec{F_R} = \sum f_x + \sum f_y$

Vector components.

vector	x-comp	y-comp
80N	$80\cos 0 = 80N$	$80\sin 0 = 0N$
100N	$100\cos 45° =$	$100 \sin 45 =$
110N	$110\cos 150 =$	$110 \sin 150 =$
160N	$160\cos 200 =$	$160 \sin 200 =$
$\vec{F_R}$	$\sum f_x = -95.0N$	$\sum f_y = 71.0N$

$\therefore$ The resultant vector $\vec{F_R} = (-95.0\,\hat{i} + 71.0\,\hat{j})\,N$

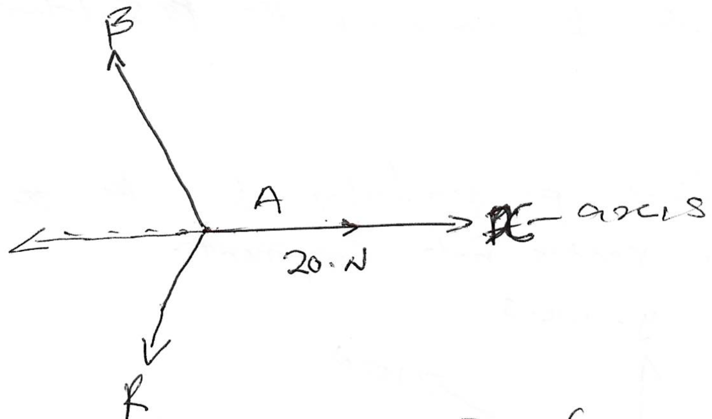
magnitude of $\vec{F_R} = |\vec{F_R}| = \sqrt{F_{Rx}^2 + F_{Ry}^2}$

$|\vec{F_R}| = \sqrt{(-95.0)^2 + (71.0)^2} = 118.6N$

Direction $(\theta) = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{71.0}{-95.0}\right) = -36.8°$

or $143.2°$

Q2



Let vector $\vec{A} = 20\,N$, $\vec{B} = (-12.0N, 6.0N)$ and the third vector be $\vec{C} = C_x\hat{i} + C_y\hat{j}$. The resultant vector $\vec{R}$ is given by:

$$\vec{R} = R_x\hat{i} + R_y\hat{j} = \vec{A} + \vec{B} + \vec{C}$$

~~$\vec{R_x} = |\vec{R}|\cos\theta$ and~~

$$R_x = R\cos\theta \quad \text{and} \quad R_y = R\sin\theta$$

$$\vec{A} = 20.0\hat{i} + 0\hat{j}$$
$$\vec{B} = -12.0\hat{i} + 6.0\hat{j}$$
$$\vec{C} = C_x\hat{i} + C_y\hat{j}$$

$\Rightarrow$

$$R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j} = 20.0\hat{i} + 0\hat{j} - 12.0\hat{i} + 6\hat{j} + C_x\hat{i} + C_y\hat{j}$$

$$-60\cos 45\hat{i} - 60\sin 45\hat{j} = 20.0\hat{i} - 12.0\hat{i} + C_x\hat{i} + 6\hat{j} + C_y\hat{j}$$

$$-42.4\hat{i} - 42.4\hat{j} = 8\hat{i} + C_x\hat{i} + 6\hat{j} + C_y\hat{j}$$

$\Rightarrow$

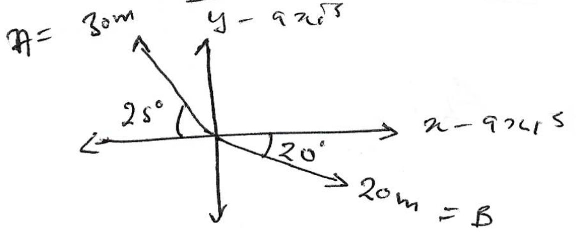
$$8\hat{i} + C_x = -42.4\hat{i} \Rightarrow C_x = -50.4\hat{i}$$
$$6\hat{j} + C_y\hat{j} = -42.4\hat{i} \Rightarrow C_y\hat{j} = -48.4\hat{j}$$

$$\therefore \vec{C} = -50.4\hat{i} - 48.4\hat{j}$$

$\vec{C}$ magnitude $= |\vec{C}| = \sqrt{C_x^2 + C_y^2} = \sqrt{(-50.4)^2 + (-48.4)^2} = 69.6\,N$

Direction of $\vec{C}$ $(\theta) = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-48.4}{-50.4}\right) = 43.8°$ below the $-x$-axis.

Q3



Resolve the vectors into components.

Vector	x-comp	y-comp
$\vec{A}$	$-30\cos 25° = -27.2$	$30 \sin 25° = 12.7$
$\vec{B}$	$20\cos 20 = 18.8$	$-20 \sin 20 = -6.84$

$\therefore \vec{A} = -27.2\hat{i} + 12.7\hat{j}$, $\vec{B} = 18.8\hat{i} - 6.84\hat{j}$

(a) $\vec{A} + \vec{B} = (-27.2\hat{i} + 12.7\hat{j}) + (18.8\hat{i} - 6.84\hat{j})$

$= (-27.2\hat{i} + 18.8\hat{i}) + (12.7\hat{j} - 6.84\hat{j})$

$= -8.4\hat{i} + 5.86\hat{j}$

(b) $\vec{A} - \vec{B} = (-27.2\hat{i} + 12.7\hat{j}) - (18.8\hat{i} - 6.84\hat{j})$

$= (-27.2\hat{i} - 18.8\hat{i}) + (12.7\hat{j} + 6.84\hat{j})$

$= -46\hat{i} + 19.5\hat{j}$

(c) $\vec{B} - \vec{A} = (18.8\hat{i} - 6.84\hat{j}) - (-27.2\hat{i} + 12.7\hat{j})$

$= (18.8\hat{i} + 27.2\hat{i}) + (-6.84\hat{j} - 12.7\hat{j})$

$= 46\hat{i} - 19.5\hat{j}$

Q4

(i) The expression of R_1 i

$\vec{R_1} = x\hat{i} + y\hat{j}$

$\vec{R_1} = a\hat{i} + b\hat{j}$

$|\vec{R_1}| = \sqrt{a\hat{i}^2 + b\hat{i}^2} = \sqrt{a^2+b^2}$.

(ii) $\vec{R_2} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{R_2} = a\hat{i} + b\hat{j} + c\hat{k}$

$|\vec{R}| = \sqrt{(a\hat{i})^2 + (b\hat{j})^2 + (c\hat{k})^2}$

$|\vec{R}| = \sqrt{a^2+b^2+c^2}$ hence shown,

Q5

Resolve the vectors into components.

Resultant-Vector	x-component	y-component
72.4	$72.4\cos(90-32)^\circ$ OR $72.4\sin 32$	$72.4\sin(90-32)^\circ$ OR $72.4\cos 32$
57.3	$-57.3\cos 36$	$-57.3\sin 36$
17.8	$17.8\cos 270$	$17.8\sin 270$
R	$R_x = -8.0$	$R_y = 9.92$

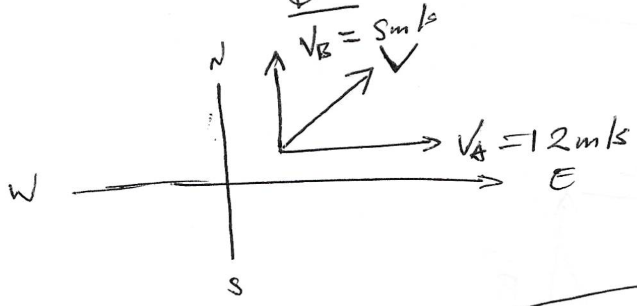
$\vec{R} = (-8.0\hat{i} + 9.92\hat{j})$ m

$|\vec{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{-8.0^2 + 9.92^2} = \sqrt{162} = 12.7$m

Direction $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{9.92}{-8}\right) = 128.9^\circ$

OR 51.1° North of west.

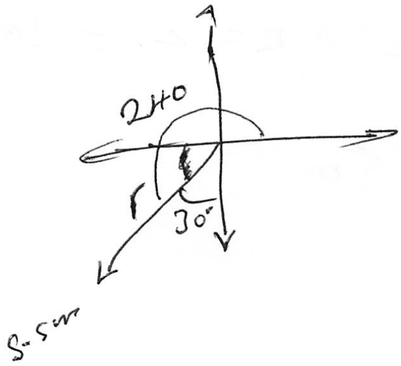
Q6



$$V_R = \sqrt{V_A^2 + V_B^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13m/s$$

$$\text{Direction } (\theta) = \tan^{-1}\left(\frac{V_B}{V_A}\right) = \tan^{-1}\left(\frac{5}{12}\right) = 22.6^\circ$$

Q7



Co-ordinate System.

~~$r = (x, y)$~~ $\quad r = (x, y) = (r_x, r_y)$

x - co-ordinate

$r_x = r \cos \theta$

$r_x = 5.5 \cos 240^\circ$

$r_x = -2.75m$

y - coordinate.

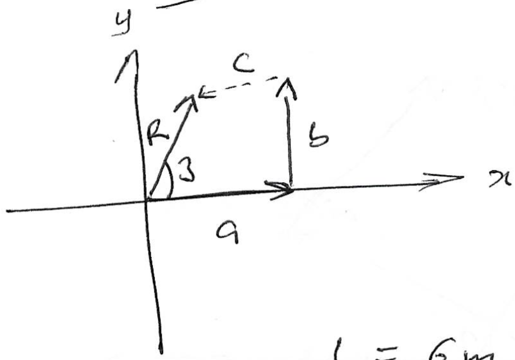
$r_y = r \sin \theta$

$r_y = 5.5 \sin 240^\circ$

$r_y = -4.76m$

$\therefore r = (-2.75m, -4.76m)$

Q8



let $a = 12\,m$, $b = 6\,m$, and c be the distance moved while dozed off. R is the resultant.

Resolve into components.

Resultant of a vector	x-axis Component	y-axis Component
$a = 12$	$12 \cos 0 = 12$	$12 \sin 0 = 0$
$b = 6$	$6 \cos 90 = 0$	$6 \sin 90 = 6$
$c = ?$	$C \cos\theta = C_x$	$C \sin\theta = C_y$
$R = 50$	$50 \cos 30 = R_x = 43.3$	$50 \sin 30 = 25$

Resultant in x-axis

$R_x = \sum x = 12 + 0 + C_x = 43.3$

$\Rightarrow C_x = 43.3 - 12 = 31.1$

Resultant in y-axis

$R_y = \sum y = 0 + 6 + C_y = 25$

$\Rightarrow C_y = 25 - 6 = 19$

$\therefore \vec{C} = C_x\hat{i} + C_y\hat{j}$

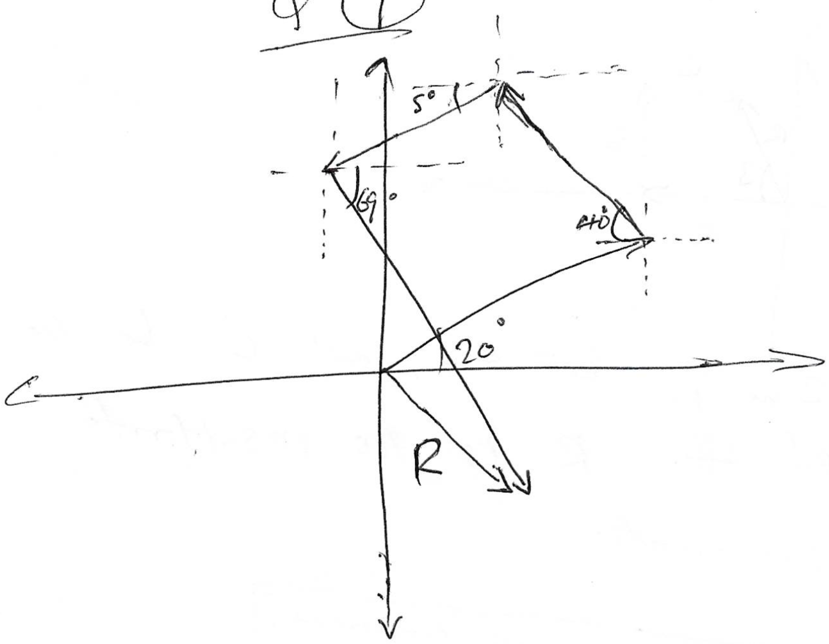
$\vec{C} = 31.1\hat{i} + 19\hat{j}$

$|\vec{C}| = \sqrt{(31.1\hat{i})^2 + (19\hat{j})^2}$

$|\vec{C}| = 36.6\,m$

Direction $(\theta) = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{19}{31.1}\right) = 31.3^\circ$

Q 9



$\vec{R} = \sum x + \sum y$

$\Rightarrow$ Resolve the vectors into Components.

x-axis	y-axis.
$20 \cos 20^\circ$	$20 \sin 20^\circ$
$-32 \cos 40^\circ$	$32 \sin 40^\circ$
$-10 \cos 5^\circ$	$-10 \sin 5^\circ$
$65 \cos 69$	$-65 \sin 69$
$\sum x = 7.63 = R_x$	$\sum y = -34.1 = R_y$

$\therefore$ The Resultant vector $\vec{R} = 7.63\hat{i} - 34.1\hat{j}$

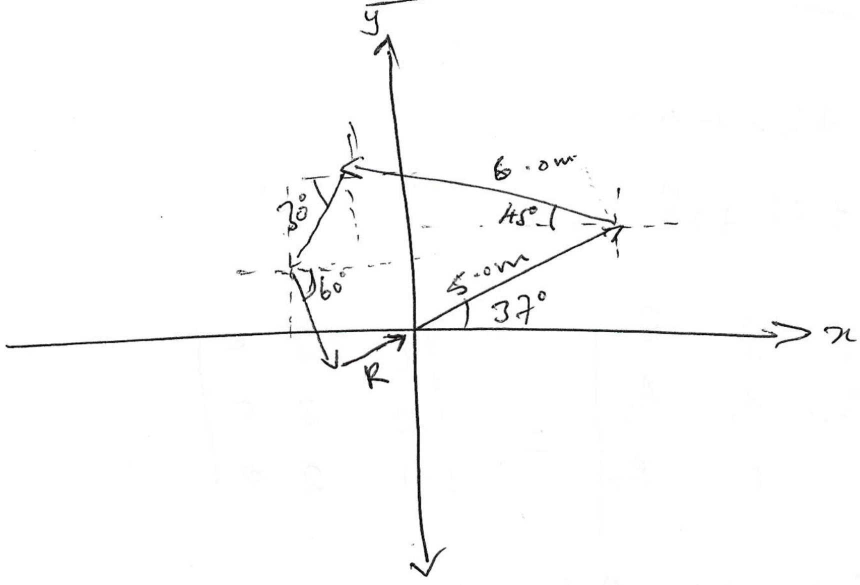
Then magnitude $|R| = \sqrt{(\sum x)^2 + (\sum y)^2} = \sqrt{(7.63)^2 + (-34.1)^2}$

$|R| = 35.0 \text{ ft}$

Direction $(\theta) = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \cancel{\tan^{-1}\left(\frac{7.63}{-34.1}\right)}$

$= \tan^{-1}\left(\frac{-34.1}{7.63}\right) = 77.7^\circ$ South of East.

Q 10



Resolve into components.

x-axis	y-axis
$5\cos 37°$	$5\sin 37°$
$-6\cos 45$	$6\sin 45$
$-4\cos 30$	$-4\sin 30$
$3\cos 60°$	$-3\sin 60°$
$R_x = \sum x = -2.21$	$R_y = \sum x = 2.65$

$$\vec{R} = R_x\hat{i} + R_y\hat{j} = -2.21\hat{i} + 2.65\hat{j}$$

$$|\vec{R}| = \sqrt{(-2.21\hat{i})^2 + (2.65\hat{j})^2} = \underline{3.45\,m}$$

$$\text{Direction } (\theta) = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{2.65}{-2.21}\right) = -50.2°$$

$$\therefore \theta = \underline{\underline{50.2°}} \quad \text{North of West}$$

Q 11

$\vec{A} = 2\hat{i} + 3\hat{j} + 0\hat{k}$

$\vec{B} = -\hat{i} + 2\hat{j} + 0\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ -1 & 2 & 0 \end{vmatrix}$$

$\vec{A} \times \vec{B} = \hat{i}(0) - \hat{j}(0)\ \hat{k}(4-(-3))$

$\vec{A} \times \vec{B} = 7\hat{k}$

Q 12

$\vec{A} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

Let the vector $\vec{B}$ be the vect ∥ to $\vec{A}$. By definition

$\vec{B} = m \dfrac{\vec{A}}{|A|}$, where $\dfrac{\vec{A}}{|\vec{A}|}$ is the unit vector of $\vec{A}$

and m is the magnitude of $\vec{B}$

$\therefore\ \vec{B} = 17 \left(\dfrac{2\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{2^2 + 6^2 + (-2)^2}} \right)$

$\vec{B} = 17 \left(\dfrac{2\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{49}} \right) = \dfrac{17}{7}(2\hat{i} + 6\hat{j} - 2\hat{k})$

Q13

$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

(a) By definition, a vector $\vec{A} \perp \vec{B}$, say $\vec{C}$ is given by,

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{C} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix}$$

$$\vec{C} = \hat{i}(10-12) - \hat{j}(5-9) + \hat{k}(4-6)$$

$$\vec{C} = -2\hat{i} + 4\hat{j} - 2\hat{k}$$

(b) it's unit vector $\hat{C} = \dfrac{C}{|C|}$

$$\hat{C} = \frac{-2\hat{i} + 4\hat{j} - 2\hat{k}}{\sqrt{(-2)^2 + 4^2 + (-2)^2}} = \frac{-2\hat{i} + 4\hat{j} - 2\hat{k}}{\sqrt{24}}$$ Answer

(ii) By definition of dot product.

$\vec{A} \cdot \vec{B} = |A||B|\cos\theta$, θ is the angle between A and B

$$\Rightarrow (\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 4\hat{j} + 5\hat{k}) = \left(\sqrt{1^2+2^2+3^2}\right) \cdot \left(\sqrt{3^2+4^2+5^2}\right) \cos\theta$$

$$3 + 8 + 15 = \sqrt{14} \cdot \sqrt{50} \cos\theta$$

$$26 = \sqrt{14 \times 50} \cos\theta$$

$$\therefore \theta = \cos^{-1} \frac{26}{\sqrt{14 \times 50}} = 10.7^\circ$$

1H

$B = 2\hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{C} = \hat{i} + \hat{j} - \hat{k}$

Let $\vec{A}$ be the vector which is $\perp$ $\vec{B}$ and $\vec{C}$. By definition

$\vec{A} = B \times C$ let vector $\vec{d}$ be parallel to $\vec{A}$ with magnitude 7. This means that it's perpendicular to $\vec{B}$ and $\vec{C}$

$\therefore \vec{d} = m\hat{A}$, where $m = |\vec{d}| = 7$

$$\Rightarrow \vec{A} = B \times C = \begin{array}{ccc} + & - & + \end{array}\begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & -3 & 5 \\ 1 & 1 & -1 \end{vmatrix} = \hat{i}(3-5) - \hat{j}(-2-5) + \hat{k}(2-(-3))$$

$$\vec{A} = -2\hat{i} + 7\hat{j} + 5\hat{k}$$

$$\therefore \hat{A} = \frac{\vec{A}}{|A|} = \frac{-2\hat{i} + 7\hat{j} + 5\hat{k}}{\sqrt{(-2)^2 + 7^2 + 5^2}} = \frac{-2\hat{i} + 7\hat{j} + 5\hat{k}}{\sqrt{78}}$$

$$\vec{d} = m\hat{A} = 7\left(\frac{-2\hat{i} + 7\hat{j} + 5\hat{k}}{\sqrt{78}}\right)$$

$$\vec{d} = \frac{-14\hat{i} + 49\hat{j} + 35\hat{k}}{\sqrt{78}}$$

Q 15

$\vec{A} = 3\hat{j}$ $A \times B = 3\hat{i}$, $\vec{A} \cdot \vec{B} = 12$

(a) $\vec{B}$?

Let $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$. using the defin of X product

$$\vec{A} \times \vec{B} = 3\hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 0 & 3 & 0 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$3\hat{i} = \hat{i}(3B_3 - 0) - \hat{j}(0-0) + \hat{k}(0 - 3B_1)$$

$$0\hat{j} + 0\hat{k} + 3\hat{i} = 3B_3\hat{i} - 3B_1\hat{k}$$

$$3B_3\hat{i} = 3\hat{i}, \quad -3B_1\hat{k} = 0$$

$$\underline{\underline{B_3 = 1}} \qquad \underline{\underline{B_1 = 0}}$$

(b) using the defin of dot Product.

$$A \cdot B = 12 = (0\hat{i} + 3\hat{j} + 0k) \cdot (B_1\hat{i} + B_2\hat{j} + B_3\hat{k})$$

$$12 = 3B_2$$

$$\therefore B_2 = 4$$

This means $\vec{B} = 0\hat{i} + 4\hat{j} + \hat{k} = \underline{\underline{4\hat{j} + \hat{k}}}$

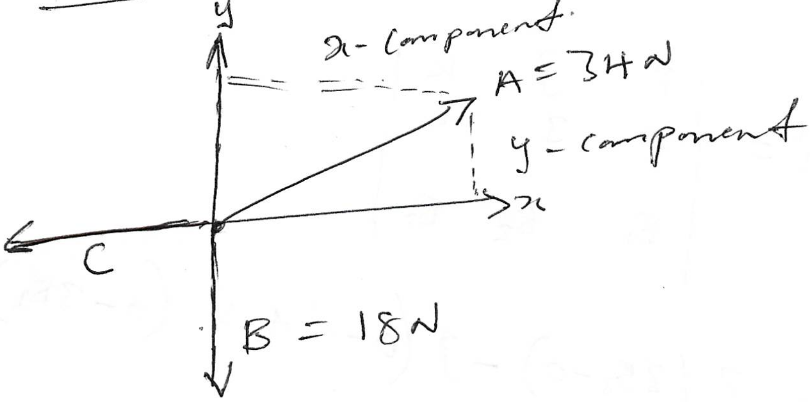
(b) $$\hat{B} = \frac{\vec{B}}{|\vec{B}|} = \frac{4\hat{j} + \hat{k}}{\sqrt{4^2 + 1^2}} = \underline{\underline{\frac{4\hat{j} + k}{\sqrt{17}}}}$$

Q (16)

(i) for the resultant $= 0$

$\Sigma x = 0$ and $\Sigma y = 0$

Taking y- components.



$\Sigma y = 0 \Rightarrow B \sin 270 + A \sin\theta = 0$

$\Rightarrow 18 \sin 270 + 34 \sin\theta = 0$

$$\frac{\cancel{34} \sin\theta}{\cancel{34}} = -\frac{18 \sin 270}{34}$$

$$\sin\theta = \frac{-18 \sin 270}{34}$$

$$\theta = \sin^{-1}\left(\frac{-18 \sin 270}{34}\right) = 32.0^\circ$$

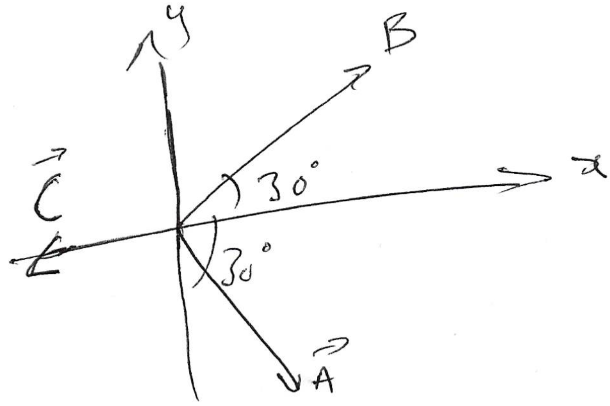
(ii) using the x- components.

$\Sigma x = 0$

$C + A \cos\theta = 0$

$C = -A\cos\theta = -34 \cos 32.0^\circ = -28.8 N$

Q 17



$$\vec{R} = \Sigma x + \Sigma y$$

$$|\vec{R}| = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

Resolving into components.

vector	x-componet	y-component.
$\vec{A}$	$10 \cos 30°$	$-10 \sin 30°$
$\vec{B}$	$20 \cos 30$	$20 \sin 30$
$\vec{C}$	$15 \cos 180$	$15 \sin 180$
$\vec{R}$	11 N.	5 N

$$\therefore \vec{R} = 11\hat{i} + 5\hat{j}$$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{11^2 + 5^2} = 12.08\,N$$

$$\text{Direction } (\theta) = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{5}{11}\right) = 24.4°$$

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF PHYSICS

PH 110 INTRODUCTORY PHYSICS
TUTORIAL SHEET 2 2023: SCALARS AND VECTORS

1. Vector **A** has a magnitude of 10 units and makes 60° with the positive x-axis. Vector **B** has a magnitude of 5 units and is directed along the negative x-axis. Find the vector
   i. sum $\boldsymbol{A} + \boldsymbol{B}$
   ii. difference $\boldsymbol{A} - \boldsymbol{B}$

2. In Fig. 2.1 Find the direction and magnitude of:
   a) the vector sum **A** + **B**
   b) the vector difference **A**-**B**
   c) the vector difference **B**-**A**

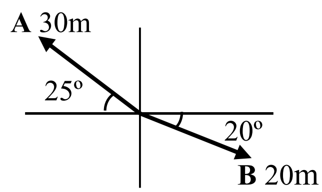


Fig. 2.1

3. The three finalists in a contest are brought to the centre of a large, flat field. Each is given a metre stick, a compass, a calculator, a shovel and the following three displacements:
   72.4 m, 32.0^0 east of north;
   57.3 m, 36.0^0 south of west;
   17.8 m straight south.
   The three displacements lead to a point where the keys to a new building are buried. Two contestants start measuring immediately; the winner first calculates where to go. What does the winner calculate in terms of magnitude and direction?

4. A ship is steaming due east at a speed of 12 ms^{-1}. A passenger runs across the deck at a speed of 5 ms^{-1} toward north. What is the resultant velocity of the passenger relative to the sea?

5. The polar coordinates of a point are $r = 5.50\ m$ and $\theta = 240°$. What are the Cartesian coordinates of this point?

6. You find yourself pacing, in a deep thought about a physics problem. First you walk 12 meters due east. Then, you walk 6 meters due north. Then you doze off and find yourself 50 meters from your starting place, 30° north of east. How far did you walk while you were not paying attention?

7. Jelita walks 20 feet, 20° north of east. He then walks 32 feet, 40° north of west. Then 10 feet, 5° south of west. Then 65 feet, 69° south of east. What is the magnitude and direction of her resultant displacement?

8. Find the magnitude and angle of the resultant of the following displacement vectors:
   A = 5.0 m at E 37^0 N
   B = 6.0 m at W 45^0 N
   C = 4.0 m at W 30^0 S
   D = 3.0 m at E 60^0 S
9. Find the cross product of $\vec{A}\times\vec{B}$ where $\vec{A}=2\hat{i}+3\hat{j}$, $\vec{B}=-\hat{i}+2\hat{j}$

10. Given a vector $\vec{A}=3\mathbf{i}+6\mathbf{j}-2\mathbf{k}$. Find another vector $\vec{B}$ which is parallel to vector $\vec{A}$ and has a magnitude of 17 units.

11. Given the $\vec{A}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\vec{B}=3\mathbf{i}+4\mathbf{j}+5\mathbf{k}$
    a) determine a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$
    b) find the angle between $\vec{A}$ and $\vec{B}$

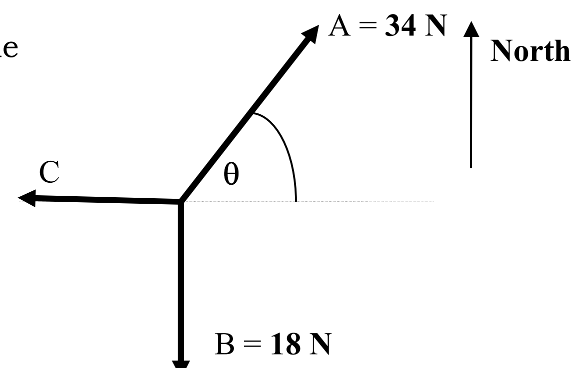
12. Find a vector whose length is 7 and which is perpendicular to each of the vectors $\vec{B}=2\mathbf{i}-3\mathbf{j}+5\mathbf{k}$ and $\vec{C}=\mathbf{i}+\mathbf{j}-\mathbf{k}$

13. If vector $\vec{A}=3\mathbf{j}$, $\vec{A}\times\vec{B}=3\mathbf{i}$ and $\vec{A}\bullet\vec{B}=12$. Find
    a) $\vec{B}$
    b) $\hat{\mathbf{B}}$

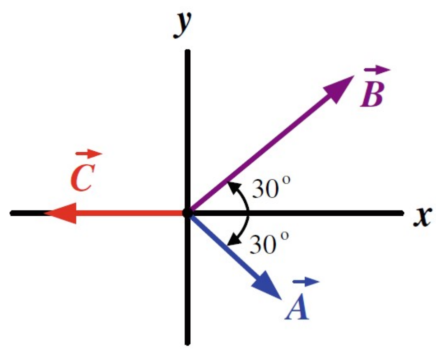
14. The vectors A, B, and C shown in the diagram alongside add together so that their resultant is zero.
    Use the method of components to find the

    (i) the bearing of vector A, and

    (ii) the magnitude of vector C.



15. Three forces are acting on a body as shown in figure in figure below where *A* = 10 N, *B* = 20 N, and *C* = 15 N. Find the magnitude and the direction of the resultant force acting on the body.



y
$\vec{B}$
$\vec{C}$
30°
x
30°
$\vec{A}$