
It's been a journey... not easy but tuff. but one thing is for sure we all stood strong despite everything . that just defines what group E means, EXCELLENCE. Therefore let's not give up on our dreams cause it's never too late, we can and I believe we will do this. we will go home knowing that we shall return for that great dream to be fulfilled and therefore don't quick your day dream cause it's your life that you are making it aren't big enough if it doesn't scare the hell out of you.
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SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 1: **MA110 - Mathematical Methods** 2022

1. A) List the elements of each of the following sets
   i) {x: x is natural number less than 5}
   ii) {x: x is a negative integer greater than -3}
   iii) {x: x is a positive number less than -5} $\cup \{1,2,3\}$
   iv) {x: x is a positive even number less than 10} $\cap$ {x: x is an integer.}
   v) $\{x: x = 4k - 1$, where $k = 0,1,2,3,4,5\}$
   vi) {x: x is an integer} $\cap \{1, \sqrt{2}, \ 3.14, 7\}$

   B) Given that $A = \{-2, -1, 0, 1, 2\}$ .List the elements of the following sets
   i) $\{x^3: x \in A\}$ ii) $\{x^2 + x: x \in A\}$ iii) $\left\{{}^{2}/_{x+1} : x \in A\right\}$ iv) $\{3x^2 + 1: x \in A\}$

2. Describe each of the following in set builder notation
   a) $A = \{1,4,9,16,25\}$
   b) $B = \{-7, -5, -3, -1\}$
   c) C ={2,4,6,8,10,12,16}
   d) $D = \{1,2,4,8,16,32\}$

3. Let $A = \{1,2,3,4,5\}$, $B = \{2,4,6\}$,$C = \{3,4,5\}$ and let $E = \{0,1,2,3,4,5,6,7,8\}$
   Find i) B' ii) $A \cup B$ iii)$A \cap B$ iv)$(A \cup B)'$ v) $(A \cap B)'$ vi) $C - B$

   vii)$(U - A) \cap (B - C)'$ Viii) $A \cup (C - B)$

4. Verify or show the following properties:
   a) Associative Laws: $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$
   b) Distributive Laws : $A \cap (B \cup C) = (A \cap B) U (A \cap C)$
   and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

5. i) Prove the De Morgan's Laws a) $(B \cap C)' = B' \cup C'$ b) $(B \cup C)' = B' \cap C'$
   ii) Prove the $(A')' = A$
   iii) Verify or show the De Morgan's Laws a) $(B \cap C)' = B' \cup C'$ b) $(B \cup C)' = B' \cap C'$

6. If $C \subset D$, then simplify if possible
   i) $C \cap D$ ii) $C' \cup D'$ iii)$C \cup D'$ iv) $C' \cap (C \cup D)$

7. If C *and* D are disjoint, simplify if possible
   i)$C' \cap D'$ ii) $C' \cup D'$ iii) $(C \cap D)'$ iv) $(C \uplus D)'$

8. Represent each of the following on a Venn diagram

i) $A \cap B'$ ii) $(A \cap B)'$ iii) $(A \cap B') \cup (A' \cap B)$ iv) $(A \cup B) \cap (A \cup B')$
v) $[A' \cup (A \cap B)']'$ Vi) $A' \cap B' \cap C$

9. Using the associative and distributive properties of union and intersection of sets .Show that
i) $A = (A \cap B) \cup (A \cap B')$ ii) $A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B)$
iii) $A \cup (A' \cap B) = A \cup B$

10. a)Given that X, Y and Z are sets, simplify the following if possible
i) $[X' \cup (Y \cap Z)]'$ ii) $Y' \cap (X \cup Y)$ iii) $(X \cap Y) \cup (X \cap Y')$ iv) $(X \cup Y) \cap (X \cup Y')$

v) $[X' \cup (X - Z)]$

b) Given that X and Y are subsets of some universal set U, simplify the following:
(i) $X \cap (X' \cup Y)$.
(ii) $[(X \cap Y)' \cap (X' \cup Y)]'$.

11 a) Using the symbol " $\subset$ " put the set of numbers in ascending order given

$\mathbb{C}, \mathbb{N}, \mathbb{R}, \mathbb{Z}$ *and* $\mathbb{Q}$

b) Give the definition of the following sets $\mathbb{Q}^*, \mathbb{R}^*, \mathbb{C}^*$and $\mathbb{Z}^*$

12.a) Let $A = \{x \in \mathbb{R}: -4 \leq x < 2\}$ and $B = \{x \in \mathbb{R}: x \geq -1\}$. Find a) $A \cap B$ b) A'
b) Let $U = (-6,9]$ be the universal set, $= [-2,4], B = (-1,5)$ *and* $C = (-6,9]$.
Find the following sets:
i) $A \cap B$ ii) $U - C$ iii) $B'UA$ iv) $(A \cup C)'$
c) Given that $\mathbb{R}$, the set of real numbers is the universal set, $A = (-4,7]$ *and* $B = [4, \infty)$,
Find
i)A' ii)B' iii)$A - B$ iv) $B - A$
d) Let $A = (-9,9)$ be the universal set and $X = (-1,5]$, $Y = [-5,3]$ *and* $Z = [-1,7)$.
Find each of the following sets and display it on the number line:
i)X' ii) $A - X$ iii) $(X \cap Z)'$ iv) $(Y - X) \cap Z$
e) Let A = {1,2, 3, 4,5,6,7,8,9} ; B = [1, 5) and C= (3,8). The universal set

is a set of real numbers . If necessary use the real number line and find:

(i). (A ∩ B) ∪(A ∩ C) (ii). B ∩ C (iii). (B ∪ C)'

(iv). (B' ∪ C') ∩ A

13. a) Express the following in the form of $\frac{a}{b}$ where a and b are intergers, $b \neq 0$.
i) $0.\overline{33}$ ii) $0.\overline{16}$ iii) $2.\overline{143}$ iv) $3.\overline{7}$ v)1.171717……. vi) $2.5\overline{90}$

b) Prove that i) $\sqrt{3}$ is an irrational number.
ii) $\sqrt{2}$ is an irrational number

c) Given that $\sqrt{3}$ and $\sqrt{5}$ are irrational, show that the following are not rational numbers
i) $\sqrt{3}+5$ is an irrational number.
ii) $\sqrt{5}-1$ is an irrational number iii) $1-\sqrt{3}$ is an irrational number

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TUTORIAL SHEET 2 : **MA110 - Mathematical Methods** 2022

1. Evaluate each of the following using the definition of Absolute value.

   a) $|x-2|=6$ b) $|2n+1|=11$ c) $\left|\frac{3}{k-1}\right|=4$ d) $\left|x-\frac{2}{3}\right|=\frac{3}{4}$ e) $|-4|$ f) $|4|$

   g) $|2x-3|\leq 5$ h) $|5x-4|\leq 8$

2. State the property that justifies each of the statements

   a) $x(2)=2(x)$ b) (7+4)+6=7+(4+6) c) 1(x)=x d) $\left(\frac{2}{3}\right)\left(\frac{3}{2}\right)=1$

3. Evaluate each of the following if x is a nonzero real number.

   a) $\frac{|x|}{x}$ b) $\frac{x}{|x|}$ c) $\frac{|-x|}{-x}$ d) $|x|-|-x|$

4. Evaluate each of the algebraic expressions for the given values of the variables

   a) $|x-y|-|x+y|; if\ x=-4\ and\ y=-7$

   b) $|3x+y|+|2x-4y|;\ if\ x=5\ and\ y=-3$

   c) $\left|\frac{x-y}{y-x}\right|\ if\ x=-6\ and\ y=13$

   d) $\left|\frac{2a-3b}{3b-2a}\right|\ if\ a=-4\ and\ b=-8$

5. Evaluate each of the following numerical expressions

   a) $(3^{-4}+4^{-1})^{-1}$ b) $2^{-3}+3^{-1}$ c) $\left(\frac{2^{-1}}{3^{-3}}\right)^{-2}$ d) $\left(\frac{3}{4}\right)^{-1}-\left(\frac{2}{3}\right)^{-1}$

6. Simplify each of the following ; express final results without using zero or negative integers as exponents

   a) $(a^2b^{-1}c^{-2})^{-4}$ b) $\left(\frac{x^{-2}}{y^{-3}}\right)^{-2}$ c) $\left(\frac{3x^2y}{4a^{-1}b^{-3}}\right)^{-1}$ d) $\left(\frac{24x^5y^{-5}}{-8x^6y^{-1}}\right)^{-3}$

7. Evaluate each of the following in simplest radical form. All variables represent positive real number

a) $\sqrt{64x^4y^7}$ b) $\sqrt[3]{81x^5y^6}$ c) $\frac{\sqrt[3]{12xy}}{\sqrt[3]{3x^2y^5}}$ d) $\sqrt[4]{162x^6y^7}$ e) $\frac{\sqrt[3]{2y}}{\sqrt[3]{3x}}$ f) $\sqrt[3]{\frac{5}{2x}}$

8. Simplify the following

a) $2\sqrt{28}-3\sqrt{63}+8\sqrt{7}$ b) $4\sqrt[3]{2}+2\sqrt[3]{16}-\sqrt[3]{54}$ c) $\frac{2\sqrt{8}}{3}-\frac{3\sqrt{18}}{5}-\frac{\sqrt{50}}{2}$ d) $\frac{3\sqrt[3]{54}}{2}+\frac{5\sqrt[3]{16}}{3}$

e) $4\sqrt{50}-9\sqrt{32}$ f) $5\sqrt{12}+2\sqrt{3}$

9 . Multiply and express the results in simplest radical form. All variables represent non-Negative real numbers

a) $2\sqrt{3}(5\sqrt{2}+4\sqrt{10})$ b) $(2\sqrt{x}-3\sqrt{y})^2$ c) $(3\sqrt{x}+5\sqrt{y})(3\sqrt{x}-5\sqrt{y})$

d) $\sqrt{6y}(\sqrt{8x}+\sqrt{10y^2})$ e) $(\sqrt{x}+\sqrt{y})^2$

10. For each of the following, rationalize the denominator and simplify. All variables Represent positive real numbers.

a) $\frac{3}{\sqrt{5}+2}$ b) $\frac{\sqrt{x}}{\sqrt{x}-1}$ c) $\frac{5}{5\sqrt{2}-3\sqrt{5}}$ d) $\frac{3\sqrt{x}-2\sqrt{y}}{2\sqrt{x}+5\sqrt{y}}$ e) $\frac{5}{3-2\sqrt{3}}$

f) $\frac{7}{\sqrt{10}-3}$ g) $\frac{\sqrt{x}}{\sqrt{x}+2}$ h) $\frac{2\sqrt{x}}{\sqrt{x}-\sqrt{y}}$

11. Evaluate each of the following

a) $-8^{2/3}$ b) $-16^{5/4}$ c) $(0.01)^{3/2}$ d) $\left(\frac{1}{27}\right)^{-2/3}$

12. Perform the indicated operations and express the answers in simplest radical form.

a) $\frac{\sqrt[3]{16}}{\sqrt[6]{4}}$ b) $\frac{\sqrt[4]{x^9}}{\sqrt[3]{x^2}}$ c) $\sqrt{ab}\sqrt[3]{a^4b^5}$ d) $\sqrt[3]{x}\sqrt[5]{x^3}$

13. Rationalize the denominators and express the final answers in simplest radical form.

a) $\frac{5}{\sqrt[3]{x}}$ b) $\frac{2\sqrt{x}}{3\sqrt[3]{y}}$ c) $\frac{5\sqrt[3]{y^2}}{4\sqrt[4]{x}}$ d) $\frac{\sqrt{xy}}{\sqrt[3]{a^2b}}$

e) $\frac{\sqrt[3]{x}}{\sqrt{y}}$ f) $\frac{\sqrt[4]{x}}{\sqrt{y}}$ g) $\frac{3}{\sqrt[3]{x^2}}$

14. Simplify each of the following, expressing the final result as one radical.

a) $\sqrt{\sqrt[3]{2}}$ b) $\sqrt[3]{\sqrt[4]{3}}$ c) $\sqrt[3]{\sqrt{x^3}}$ d) $\sqrt{\sqrt[3]{x^4}}$

15. Add or subtract as indicated

a) $(5+3i)+(7-2i)(-8-i)$ b) $(4+i\sqrt{3})+(-6-2i\sqrt{3})$

c) $(5-7i)-(6-2i)-(1-2i)$ d) $\left(\frac{5}{8}+\frac{1}{2}i\right)-\left(\frac{7}{8}+\frac{1}{5}i\right)$

16. Write each of the following in terms of i, perform the indicated operations , and Simplify if possible.

a) $\sqrt{-4}\sqrt{-16}$ b) $\sqrt{-25}\sqrt{-9}$ c) $\frac{\sqrt{-36}}{\sqrt{-4}}$ d) $\frac{\sqrt{-64}}{\sqrt{-16}}$ f) $\frac{\sqrt{-18}}{\sqrt{-3}}$

17. Find each of the following products and express the answers in standard form

a) $(-2+5i)^2$ b) $(5+3i)(5-3i)$ c) $(1+i)(2-i)$ d) $(5i)(2+6i)$ e) $(-5i)(8i)$

f) $(5i)(2+6i)$

18. Find each of the following quotients and express the answers in standard form

a) $\frac{2+3i}{3i}$ b) $\frac{3-5i}{4i}$ c) $\frac{4+7i}{2-3i}$ d) $\frac{3-7i}{4i+2}$

e) $\frac{1+\sqrt{2}i}{\sqrt{3}-2i}$ f) $\frac{1+2i}{1-i}+\frac{1-2i}{1+3i}$

19. Plot each complex number and find its absolute value

a) $3+4i$ b) -4 c) $\frac{3}{5}-\frac{4}{5}i$ d) $-5i$ e) $1-2i$ f) $3-2i$

e) $\frac{1}{(2+i)(\sqrt{3}-2i)}$ f) $5-4i+\frac{5}{3-4i}$

20. Let $z_1=2+i$, $z_2=1-i\sqrt{3}$ and $z_3=3+4i$. Verify the following identities

(i) $\overline{z_1 z_2}=\overline{z_1}\cdot\overline{z_2}$ (ii) $z_3\cdot\overline{z_3}=\overline{z_3}\cdot z_3=|z_3|^2$ (iii) $\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\overline{z_1}}{\overline{z_2}}$

21. Solve for x and y given that:

a) $(x+iy)(4i)=8$ b) $\frac{1}{x+iy}+\frac{1}{1+3i}=1$

c) $\frac{x}{1+i}-\frac{y}{2-i}=\frac{1-5i}{3-2i}$

22. Let $z=x+iy$ be a non zero complex number

a) Express $\frac{1}{z}$ in the form $a+ib$

b) Given that $z+\frac{1}{z}=k$, where k is a real number, prove that either

z is real or $|z|=1$

23 Express each of the following in the form $a+ib$ where a and b are real numbers:

a) $\frac{1}{i^3}$ b) i^{15} c) i^{1002}

24 . a) Express $\frac{\sqrt{3}+1}{\sqrt{3}-1}+\sqrt{3}-1$ in the form $a+b\sqrt{3}$ where a and b are rational numbers.

b) Rationalize the denominator of each of the following:

(i) $\frac{2\sqrt{3}-\sqrt{2}}{4\sqrt{3}}$ (ii) $\frac{x}{x+\sqrt{y}}$ (iii) $\frac{2\sqrt{7}+\sqrt{3}}{3\sqrt{7}-\sqrt{3}}$

(iv) $\frac{x-\sqrt{x^2-9}}{x+\sqrt{x^2-9}}$ (v) $\frac{1}{(\sqrt{2}+1)(\sqrt{3}-1)}$

(c) Rationalize the numerator in each of the following:

(i) $\frac{\sqrt{5+h}-3}{h}$ (ii) $\frac{\sqrt{3}+\sqrt{5}}{7}$ (iii) $\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}+\sqrt{x+h}}$

25a) Give a reason why $Z - a\ set\ of\ integers$, $N - a\ set\ of natural\ numbers$ are not Fields while $Q - a\ set\ of\ rational\ number, R - a\ set\ of\ real\ numbers\ and\ C - a\ set\ of\ complex\ numbers\ are\ Fields.$

b) Prove that if $a+c=b+c$ then $a=b$ when $a,b,c \in R$

c) Prove that if $ac=bc$ then $a=b$ when $a,b,c \in R$ and $c \neq 0$

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TUTORIAL SHEET 3: **MA110-Mathematical Methods** 2022

BINARY OPERATIONS

1. Define an operation *on the set of real numbers by $a * b = b^a$
   i). Is $*$ a binary operation on the set of real numbers ? Give reason for your Answer.
   ii). Is the operation commutative?
   iii). Evaluate $(3 * 2) * -2$
2. Consider the binary operation $a * b = a + b - 2ab$, where a *and* b are real numbers .
   i). Compute 1*(2*3) and (1*2)*3
   ii). Is * commutative? Justify your answer

3. Let '$*$' be a binary operation on the set of real numbers defined by $a * b = -2^{b-a}$,
   Where a *and* b are real numbers .

   i). Is $*$ commutative on real number? Justify your answer.
   ii). Find $-1 * (4 * 9)$

4. Determine whether the following operations are binary on the set of Irrational

   numbers .
   a). Addition [+] (b). Multiplication [×] (c). Subtraction [−]

   Justify your answers.

5 a) Determine whether the binary operation $*$ defined is commutative and whether $*$ is associative
   i) $*$ defined on Z by letting $a * b = a - b$
   ii) $*$ defined on Q by letting $a * b = ab + 1$
   iii) $*$ defined on $Z^+$by letting $a * b = 2^{ab}$
   iv) . $*$ defined on $Z^+$by letting $a * b = a^b$

b) Determine whether the definition of $*$ does give a binary operation on the set and give a reason why.

i) On Z^+ define $*$ by letting $a * b = a - b$

ii) On Z^+ define $*$ by letting $a * b = a^b$

iii) On R define $*$ by letting $a * b = a - b$

iv) On Q, define $*$ by letting $a * b = a/b$

v) On Z^+ define $*$ by letting $a * b = a/b$

6. a) A binary operation $*$ is defined on the set of real numbers as follows:

$$a * b = 2^{-a} + b, \qquad a, b \in \mathbf{R}$$

(i) Is the operation $*$ commutative? If not give a counter example.

(ii) Find the value of $-1*(0*1)$ and $(-1*0)*1$, and state whether $*$ is associative.

b) State whether each of the following operation is a binary operation on $\mathbf{Z}$, the set of integers, where a and b are integers:

(i) $a*b = 3a - b$ (ii) $a*b = (ab)^2$ (iii) $a*b = \sqrt{a-b}$

c) Each of the following operations in I, II, III, IV, V is a binary operation on $\mathbf{R}$

I: $a*b = (a+b)(a-b)$ II: $a*b = ab$

III: $a*b = 2^{a-b}$ IV: $a*b = a+2b$ V: $a*b = a+b-ab$

a) Determine which ones are commutative and / or associative.

b) For each of the operations I, II, III, IV and V, evaluate $3*(7*4)$

d) Given the sets X = {0 , 1} and Y = {0 , 1 , 2},

(i) Determine whether each of the following operations $+, -, \times, \div$ is a binary operation on X and on Y.

(ii) Also check whether each of the operations is commutative or associative.

7. Let A={1,2} and B={3,4,5} be two set . List the element of $A \times B$ and $\times A$. State if

$A \times B \neq B \times A$?

SKETCHING

1 a) Sketch the following curves and indicate clearly the points of intersection with the axes:

i) $y = (x-3)(x-2)(x+1)$ ii) $y = (x+1)(1-x)(x+3)$

iii) $y = x(x+1)(x-1)$ iv) $y = x(2x-1)(x+3)$

b) Sketch the curves with the following equations:

i) $y = (x+1)^2(x-1)$ ii) $y = (x+2)(x-1)^2$

iii) $y = (x-1)^2 x$ iv) $y = x^2(x-2)$

c) Factories the following equations and then sketch the curves

i) $y = x^3 + x^2 - 2x$ ii) $y = x^3 + 5x^2 + 4x$

iii) $y = x - x^3$ iv) $y = 12x^3 - 3x$

d). Sketch the following curves and show their positions relative to the curve $y = x^3$

i) $y = (x-2)^3$ ii) $y = (2-x)^3$ (iii) $y = -(x+2)^3$

iv) $y = (x+2)^3$

e). Sketch the following and indicate the coordinates of the points where the curves cross the axes:

i) $y = (x+3)^3$ ii) $y = (1-x)^3$ iii) $y = -\left(x - \frac{1}{2}\right)^3$ iv) $y = (-x+2)^3$

f). Apply the following transformation to the curves with equations $y = f(x)$ where:

(i) $f(x) = x^2$ (ii) $f(x) = x^3$ (iii) $f(x) = \frac{1}{x}$

In each case state the coordinates of points where the curves cross the axes and in **(iii)** state the equations of any asymptotes.

a) $f(x+2)$ b) $f(x)+2$ c) $f(x-1)$ d) $f(x)-1$

g). Apply the following transformation to the curves with equations $y = f(x)$ where:

i) $f(x) = x^2$ ii) $f(x) = x^3$ iii) $f(x) = \frac{1}{x}$

In each case show both $f(x)$ and the transformation on the same diagram.

(a) $f(2x)$ (b) $f(-x)$ (c) $2f(x)$ (d) $4f(x)$ (e) $\frac{1}{4}f(x)$

2. Sketch the following rational functions:

i) $y = \frac{3}{x+1}$ ii) $f(x) = \frac{x^2+2}{x-1}$ iii) $f(x) = \frac{x^2}{x^2+x-3}$ iv) $f(x) = \frac{x+2}{x-2}$ v) $f(x) = \frac{x^2-5x+6}{x-2}$

vi) $f(x) = \frac{2x^4}{x^4+1}$ vii) $f(x) = \frac{2x^2}{x^2+4}$

3. A) Sketch the following and find the domains :

i) $f(x) = 1 - \sqrt{2-3x}$ ii) $f(x) = \sqrt{x+3}$ iii) $f(x) = 1 + \sqrt{\frac{x}{2}}$ iv) $f(x) = -\sqrt{-x+3}$

v) $f(x) = 2 + \sqrt{-3x+2}$ vi) $f(x) = -\sqrt{x}$ vii) $f(x) = 2 + 3\sqrt{-x+1}$ ix) $f(x) = 2\sqrt{-x+1}$

B) Graph each of the following:

i) $f(x) = \begin{cases} -x^2 & for\ x \geq 0 \\ 2x^2 & for\ x < 0 \end{cases}$ ii) $f(x) = \begin{cases} 2x+3 & if\ x < 0 \\ x^2 & if\ 0 \leq x < 2 \\ 1 & if\ x \geq 2 \end{cases}$

iii) $f(x)=\begin{cases} 2 & if\ x>2 \\ 1 & if\ 0<x\leq 2 \\ -1 & if\ x\leq 0 \end{cases}$ iv) $f(x)=\begin{cases} 2x+1 & if\ x\geq 0 \\ x^2 & if\ x<0 \end{cases}$

Functions

1. Determine whether each of the given relation below is a function or not .

(i) $y=x^2$ (ii) $y=\begin{cases} 2x+3\ , & x\leq 1 \\ 6-x^2\ , & x\geq 1 \end{cases}$ (iii) $y=\begin{cases} x^3, & x<2 \\ \frac{1}{x}, & x\geq 2 \end{cases}$ (iv) $y=3x-1$

2. For each of the following functions, find the image of -2, $-\frac{1}{2}$, 0 , 4 ,7

(i) $f(x)=1-6x$ (ii) $g(x)=(x-1)^2$ (iii) $h(x)=\frac{x-1}{x+1}$

3. Find the values of the constants in each of the following:

(i) $f(x)=ax+b, a$ and b are constants, $f(-2)=7, f(1)=-1$

(ii) $f(x)=ax^2+bx+c,\ \ a, b$ and c are constants, $f(0)=7, f(1)=6$and $f(-1)=12$

4. Determine the domain of the following functions:

i) $f(x)=2x+\sqrt{x^2+4x-12}$ ii) $h(x)=x+\sqrt{3x-4}$

iii) $k(x)=\sqrt{x}+\sqrt{x-1}$ iv) $L(x)=-x^2+2x-7$ v) $f(x)=x^2-2x+2$

vi) $f(x)=\sqrt{x^2-2x-24}+2$ vii)$f(x)=\sqrt[3]{x^2-4}$ viii) $f(x)=\sqrt[5]{x+3}$

ix) $k(x)=\sqrt[4]{2x+1}$ x) $h(x)=\sqrt[8]{x^3-2}$ xi) $f(x)=x^2+4x-1$

xii) $f(x)=x^2+\sqrt{x^2+4x-12}$ xiii) $h(x)=x^2+\sqrt{x-1}$

NOT that for Questions 4 and 5 , determine the domain and the range of the function

5. The function f is defined by

$$f(x)=\begin{cases} x^2-6, & x<0 \\ 10-x & x\geq 0 \end{cases}$$

a) Sketch $f(x)$.
b) Find the value(s) of a such that $f(a)=43$.
c) Find the values of the domain that get mapped to themselves in the range.

6. a) The domain of the function $g(x) = \frac{2x-1}{x^2-1}$ is { 1,2,3,4}. Find the range of the function

b) The range of the function $g(x) = 1 - \frac{3}{x}$ is { -2,4, 5} , Find the domain of the function.

7. For each of the given functions find $\frac{f(a+h)-f(a)}{h}$.

(i) $f(x) = 4x + 5$ (ii) $f(x) = x^2 - 3x$ (iii) $f(x) = -x^2 + 4x - 2$

8. Determine whether the function f is even, odd or neither.

(i) $f(x) = x^2 + x$ (ii) $f(x) = \sqrt{2 - x^2}$ (iii) $f(x) = x^2$ (iv) $f(x) = \frac{1}{x}$

(v) $f(x) = 3x - 1$ (vi) $f(x) = x^5 + x^3 + x$ vii) $f(x) = x^4 + x^2 + 1$

(viii) $f(x) = x^3 + 1$ ix) $f(x) = x^2 + 1$ $f(x) = -x^3$

9. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ for the following and their domains

(i) $f(x) = 2x$, $g(x) = 3x - 1$. (ii) $f(x) = \frac{1}{x}$, $g(x) = 3x - 1$

(iii) $f(x) = \sqrt{x-2}$, $g(x) = 3x - 1$ (iv) $f(x) = \frac{4}{x+2}$, $g(x) = \frac{3}{2x}$

10. Solve each of the following problems

(i) If $f(x) = x^2 - 2$ and $g(x) = x + 4$, find $(f \circ g)(2)$ and $(g \circ f)(-4)$

(ii) If $f(x) = \frac{1}{x}$ and $g(x) = 2x + 1$, find $(f \circ g)(1)$ and $(g \circ f)(2)$

(iii) If $f(x) = \sqrt{x+1}$ and $g(x) = 3x - 1$, find $(f \circ g)(4)$ and $(g \circ f)(4)$

(iv) If $f(x) = x + 5$ and $g(x) = |x|$, find $(f \circ g)(-4)$ and $(g \circ f)(-4)$

11. Show that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$

(i) $f(x) = 2x$, $g(x) = \frac{1}{2}x$ (ii) $f(x) = 3x + 4$, $g(x) = \frac{x-4}{3}$

(iii) $f(x) = 4x - 3$, $g(x) = \frac{x+3}{4}$

12. If $f(x) = 3x - 4$ and $g(x) = ax + b$, find the conditions on a and b that guarantee that

$$(f \circ g)(x) = (g \circ f)(x)$$

13. Let $f(x) = \frac{x}{x+2}$ and $g(x) = 2x - 1$.

(i) Find $(f \circ g)(x)$

(ii) Evaluate $(g \circ f)\left(\frac{3}{4}\right)$

(iii) Verify that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

14. Verify that the two given functions are inverses of each other

(i) $f(x) = -\frac{1}{2}x + \frac{5}{6}$ and $g(x) = -2x + \frac{5}{3}$

(ii) $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$

(iii) $f(x) = \sqrt{2x-4}$ for $x \geq 0$ and $g(x) = \frac{x^2+4}{2}$

(iv) $f(x) = x^2 - 4$ for $x \geq 0$ and $g(x) = \sqrt{x+4}$ for $x \geq -4$

15. Find f^{-1} and verify that $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = (x)$

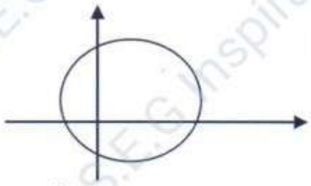
(i) $f(x) = \sqrt{x}$ for $x \geq 0$ (ii) $f(x) = \frac{1}{x}$ for $x \neq 0$ (iii) $f(x) = \frac{3}{4}x - \frac{5}{6}$

16. The function $f(x)$ is defined by $f(x) = x^2 - 3$ $\{x \in R,\ x > 0\}$

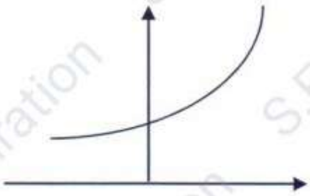
i) Find $f^{-1}(x)$ ii) Sketch $f^{-1}(x)$ iii) Find values of x such that $f(x) = f^{-1}(x)$

17. State whether or not the following graphs represent functions. Give reasons for your answers and describe the type of the function.

i)



ii)



18. The following mappings f and g are defined on all the real numbers by

$$f(x) = \begin{cases} 4-x, & x<4 \\ x^2+9, & x \geq 4 \end{cases} \qquad g(x) = \begin{cases} 4-x, & x<4 \\ x^2+9, & x>4 \end{cases}$$

Explain why $f(x)$ is a function and $g(x)$ is not .

19. State if the following functions is one to one or many to one

i) $f(x) = 3x + 2$ for the domain $\{x > 0\}$

ii) $f(x) = x^2 + 5$ for the domain $\{x \geq 2\}$

iii) $f(x) = +\sqrt{x+2}$ for the domain $\{x \geq -2\}$

21. Find $f+g,\ f-g,\ f\cdot g,\ and\ \frac{f}{g}$ and determine their domain

i) $f(x) = \sqrt{x-1}$, $g(x) = \sqrt{x}$ ii) $f(x) = \sqrt{x+2}$, $g(x) = \sqrt{3x-1}$

iii) $f(x) = x^2 - 2x - 24$, $g(x) = \sqrt{x}$ iv) $f(x) = -6x - 1$, $g(x) = -x - 1$

22) Find $(f^\circ g)(x)$ *and* $(g^\circ f)(x)$ .Also specify the domain for each.

i) $f(x) = \frac{1}{x}$, $g(x) = 2x + 7$ ii)$f(x) = \sqrt{x-2}$, $g(x) = 3x - 1$

iii) $f(x) = \frac{1}{x-1}$, $g(x) = \frac{2}{x}$ iv) $f(x) = \frac{4}{x+2}$, $g(x) = \frac{3}{2x}$

23). Determine whether the function is one-to-one . If it is,find the inverse and graph both the function and its inverse.

(i) $f(x) = x^3 - 2$ ii)$f(x) = \frac{x}{\sqrt{x^2+4}}$ iii)$f(x) = \sqrt{x^2+1}$ iv) $f(x) = \frac{x}{x+4}$

v) $f(x) = x^2 + 3$ vi) $f(x) = x^3$

24) . Determine the domain of the following fuctions

a). $f(x) = \frac{6}{\sqrt{6x-2}}$ (b). $g(y) = \sqrt{\frac{y}{y-8}}$ (c). $h(x) = \sqrt{x-7} + \sqrt{9-x}$

d). $f(x) = \sqrt{\frac{x+1}{x-1}}$ (e). $f(x) = \sqrt[3]{x}$

25) . Graph the function

$\Bigg\{$

$$f(x) = \begin{cases} 4 - x^2 \ , & \text{if} \quad x \leq 1 \\ x - 3 \ , & \text{if} \quad x > 1 \end{cases}$$

26). Given $f(x) = \frac{x}{|1-x|}$

(i). Find the domain of $f(x)$.
(ii). Sketch the graph of $f(x)$.
(iii). Find the range of $f(x)$

27). Suppose $f(x) = \frac{1+x}{x^2-2x+1}$

(i). Find the domain of $f(x)$
(ii). Find the vertical asymptotes if any
(iii). Find the horizontal asymptotes if any
(iv). Sketch the graph of $f(x)$.

28.) Sketch the graph of $f(x) = |2x+1|$. On the same diagram sketch also the graph of $g(x) = \sqrt{1-2x}$ and, hence, find the values such that $\sqrt{1-2x} > |2x+1|$

29). In each of the following determine whether f is even, odd or neither

i). $f(x) = x^4$ (ii). $f(x) = x + x^3$ (iii). $f(x) = x^3 - x$ (iv). $f(x) = 2 - x$

30. Given the functions $f(x) = 3x + 1$, $h(x) = \frac{1}{x+1}$, $g(x) = x^3$ and $k(x) = \sqrt{x}$;
Find (i). $(fog)(x)$ (ii). $(gof)(x)$ (iii). $(kok)(x)$ (iv). $f[g(k(x))]$

31) . (i) If $f(x) = 3x - 4$ and $g(x) = ax + b$, find the conditions of a and b such that $(fog)(x) = (gof)(x)$.

(ii) If $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-1}$. Find the domain and inverse of $(fog)(x)$ and $(gof)(x)$.

32). If $f(x) = 3 - x$ and $g(x) = \frac{3x}{x-3}$, $x \neq 3$. Show that this function is its own inverse. Is $(fog)(x)$ even, odd or neither.

33) . If $f(x) = 1 - x^2$ and $(x) = \sqrt{x}$. Find $(gof)(x)$ and $(fog)(x)$ and their domains

34). Show that the function f defined by $f(x) = {}^{x}/_{\sqrt{x^2+1}}$, $x \in R$, is a bijection on R on to $\{y: -1 < y < 1\}$

35) .Let $A = B = \{x \in R: -1 \leq x \leq 1\}$ and consider the subset $C = \{(x,y): x^2 + y^2 = 1\}$ of $A \times B$.Is this set a function ? Explain

36). a) If $f(x) = x^2$ and $g(x) = 3x - 4$,find $(gof)(x)$ and$(fog)(x)$and determine its domain

b) If $f(x) = \sqrt{x}$ and $g(x) = 2x - 1$,find $(fog)(x)$ and determine its domain.

c) If $f(x) = \frac{2}{x-1}$ and $(x) = \frac{1}{x}$, find $(fog)(x)$ and determine its domain

d) If $f(x) = 1 - x^2$ and $g(x) = \sqrt{x}$, find $(gof)(x)$ and$(fog)(x)$ determine their domains

THE COPPERBELT UNIVERSITY
SCHOOL OF MATHEMATICS AND NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 4: **MA110-Mathematical Methods** 2022

1.(a)Solve quadratic equations using factorization

(i) $4x^2 - 16x + 15 = 0$, (ii) $x^2 + 10x + 25 = 0$ (iii) $x^2 - 10x + 24 = 0$

(b) Solve quadratic equations by completing the square.

(i) $2x^2 + 2x + 5 = 0$ (ii) $5x^2 + 1 = 2x$ (iii) $10 = 3x - x^2$ (iv) $4x^2 - x = 8$

(c) Solve the following quadratic equations by using the quadratic formula, giving the solutions in simplified Surd form.

(i) $5x^2 + 2x + 1 = 0$ (ii) $7x^2 + 9x + 1 = 0$ (iii) $4x^2 - 7x = 2$ (iv) $25z^2 - 30z = -9$

2. (a)Sketch the graphs of the following equations

(i) $y = x^2 + 3x + 2$ (ii) $y = -x^2 + 6x + 7$ (iii) $f(x) = -x^2 + 2x + 5$ (iv) $f(x) = 2x^2 + 2x + 5$

(b)For what values of k will the function $f(x) = x^2 + 6x + k$

(i)cuts the x-axis twice (ii)touch the x-axis (iii)have no x intercepts

(c) For the quadratic $f(x) = 7 + 4x - 2x^2$, find

(i)The equation of the axis of symmetry (ii) Coordinates of the vertex (iii) the x and y intercepts. Hence ,sketch the graph of the function.

3. Solve the following pairs of simultaneous equations:

(i) $x + y = 6, \quad x^2 + y^2 = 26$ (ii) $x + 2y = 7, x^2 - 4x + y^2 = 1$

(iii) $\frac{x}{3} - \frac{y}{2} = 1, \frac{3}{x} + \frac{2}{y} = \frac{3}{2}$ (iv) $\frac{x}{4} - \frac{y}{3} = 1, \ \frac{16}{x} + \frac{3}{y} = 3$

4.a) Given that for all values of x :

$3x^2 + 12x + 5 = p(x + q)^2 + r$. Find the values of p, q and r

b) Find, as surds, the roots of the equation:

$2(x + 1)(x - 4) - (x - 2)^2 = 0$

c) The equation $px^2 - 2(p + 3)x + p - 1 = 0$ has real roots. What is the range of values of p?

d) Find the values of k if the equation $x^2 + (k - 2)x + 10 - k = 0$ has equal roots

e) What is the largest value m can have if the roots of $3x^2 - 4x + m = 0$ are real ?

f) Show that the equation $a^2x^2 + ax + 1 = 0$ can never have real roots .

g) If the equation $x^2 - (p-2)x + 1 = p(x-2)$ is satisfied by only one value of x, What are the possible values of p

h) What type of roots does the equation $5x^2 - 3x + 1 = 0$ have ?

i) For what values of k will the x –axis be a tangent to the curve $f(x) = kx^2 + (1+k)x + k$

j) With these values, find the equations of the curve

5. Show that the solution of $ax^2 + bx + c = 0$ are $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

6. a) Determine the nature of the curves ,find the turning point and sketch the following
(i) $y = -2 + 2x - x^2$ (ii) $f(x) = 2x^2 - 3x - 4$ (iii) $f(x) = x^2 + 2x - 3$ (iv) $f(x) = 5 - 2x - 4x^2$

b) If the minimum values of $x^2 + 4x + k$ is -7 , find the value of k.

c)The function $f(x) = ax^2 + bx + c$ has a maximum value of 4 where $x = -1$. find the value of a and b

d)The function $f(x) = 1 + bx + ax^2$ has a maximum value of 4 where $x = -1$.Find the value of a and b .

e) The function $f(x) = ax^2 + bx + c$ has a minimum value of $-5\frac{1}{4}$ where $x = \frac{1}{4}$. and $f(0) = -5$. Find the values of , b and c.

f) Express $5 - x - 2x^2$ in the form $a - b(x+c)^2$ and hence or otherwise find its maximum value and the value of x where this occurs

7 a) Let α and β be the roots of the quadratic equation $4x^2 + 3x - 2 = 0$

(i) Find the sum $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(ii)Find a quadratic equation whose roots are α^2 and β^2

b) The roots of the equation $x^2 - px - 7 = 0$ are $\propto$ and β , write down in terms of p an equation whose roots are $\propto^2 + p\propto^2$ and $\beta^2 + p\beta^2$

c) The roots of the equation $2x^2 + 6x - 15 = 0$ are $\propto$ and β. Find the value of
(i) $(\propto + \beta)(\beta + 1)$ (ii) $\propto^2\beta + \propto\beta^2$ (iii) $(\propto - \beta)^2$ (iv) $\frac{1}{2\propto+\beta} + \frac{1}{\propto+2\beta}$ (v) $\frac{1}{\propto^2+1} + \frac{1}{\beta^2+1}$
(vi) $\frac{1}{\propto^2} + \frac{1}{\beta^2}$ (vii) $\frac{1}{\propto^3} - \frac{1}{\beta^3}$

d) if α and β are the roots of $ax^2 + bx + c = 0$, Show that $\alpha + \beta = -\frac{b}{a}$ and that $\alpha\beta = \frac{c}{a}$. Hence show that $\propto^2 + \beta^2 = \frac{b^2-2ac}{a^2}$ and that $\alpha - \beta = \frac{\sqrt{b^2-4ac}}{a}$

f) if α and β are the roots of $3x^2 + 2x + 5 = 0$, Find new equations with the roots
(i) $3\alpha, 3\beta$ (ii) $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$ (iii) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ (iv) $\frac{1}{\alpha^3}, \frac{1}{\beta^3}$

POLYNOMIALS

1. a) Find the remainder when

   i) $4p^3 - 5p^2 + 7p + 1$ is divided by $(p-2)$ ii) $x^3 + 2x^2 - x - 1$ is divided by $3x + 2$

   iii) $z^3 - 20z + 3$ is divided by $(z-4)$ iv) $-2y^4 + 2y^2 - y - 5$ is divided by $(y+2)$

2. Express each of the following polynomials in the form $f(x) = g(x)q(x) + r(x)$ where $q(x)$ is the quotient and $r(x)$ is the remainder when the polynomial $f(x)$ is the dividend and $g(x)$ is the divisor

   i) $5 + 6x + 7x^2 - x^3$ is divided by $x + 1$ ii) $9x^3 + 4$ is divided by $3x + 2$

   iii) $x^4 - 3x^2 + x + 1$ is divided by $(x-1)$ iv) $x^3 - 8$ is divided by $x - 2$

3. Using synthetic division find the quotient and the remainder when

   i) $x^3 - 2x^2 + 9$ is divided by $(x+2)$ ii) $x^4 - 2x^3 - 3x^2 - 4x - 8$ is divided by $x - 2$

   iii) $8x^3 - 10x^2 + 7x + 3$ is divided by $2x - 1$ iv) $5x^6 - x^3 - 1$ is divided by $x + 1$

   v) $4x^7 + 3$ is divided by $x - 3$

4. Use the Rational root theorem to solve the following equations

   i) $x^3 - 4x^2 + 8 = 0$ ii) $x^3 - 10x - 12 = 0$ iii) $2x^5 - 5x^4 + x^3 + x^2 - x + 6 = 0$

   iv) $x^4 - 3x^3 + 2x^2 + 2x - 4 = 0$ v) $x^4 + 3x - 2 = 0$

5. Verify that the following equations have no Rational solutions.

   i) $x^4 - x^3 - 8x^2 - 3x + 1 = 0$ ii) $x^4 + 3x - 2 = 0$ iii) $2x^4 - 3x^3 + 6x^2 - 24x + 5 = 0$

6. Find $f(c)$ either by using synthetic division and the remainder theorem or by evaluating $f(c)$ directly

   i) $f(x) = 5x^6 - x^3 - 1$ and $c = -1$ ii) $f(x) = 4x^7 + 3$ and $c = 3$

   iii) $f(n) = -2n^4 + 2n^n - n - 5$ and $c = -2$ iv) $f(t) = 5t^5 - 8t^2 + 9t - 4$ and $c = -5$

7. Use the factor theorem to answer the following

   i) is $x - 2$ a factor of $3x^2 - 4x - 4$? ii) is -3 a factor of $2x^3 - 3x^2 - 10x + 3$?

   iii) is $x - 3$ a factor of $x^4 - 81$? iv) is $x - 2$ a factor of $x^3 - 8$?

8. Use synthetic division to show that $g(x)$ is a factor of $f(x)$ and complete the factorization of $f(x)$

   i) $g(x) = x + 2$; $f(x) = x^3 + 7x^2 + 4x - 12$ ii) $g(x) = x + 1$; $f(x) = x^3 - 2x^2 - 7x - 4$

   iii) $g(x) = x - 3$; $f(x) = 6x^3 - 17x^2 - 5x + 6$ iv) $g(x) = x - 5$; $f(x) = 2x^3 + x^2 - 61x + 30$

9 i) Let $g(x) = x^3 + ax^2 + 3x + 6$. Given $g(-1) = 2$, Find the remainder when $g(x)$ is divided by $(3x - 2)$.

   ii) The expression $3x^3 + 2x^2 - px + q$ is divided by $(x-1)$ but leaves a remainder of 10 when divided by $(x+1)$. Find the values of a and b.

   iii) Solve the equation $x^3 - 7x + 6 = 0$. Hence state the solution of the equation

$$(x-2)^2 - 7(x-2) + 6 = 0$$

iv) Find the remainder in terms of p *when* $x^3 + px^2 - x - 2$ is divided by $x + 3$

10. Factorize each of the following polynomials

i) $x^3 - 2x^2 - 5x + 6$ ii) $3x^3 + 2x^2 - 3x - 2$ iii) $x^4 - 1$ iv) $x^3 - 10x - 12$

11. Find the value(s) of k that makes the second polynomial a factor of the first

i) $x^3 - kx^2 + 5x + k; x - 2$ ii) $kx^3 + 19x^2 + x - 6;\ x + 3$ iii) $k^2x^4 + 3kx^2 - 4;\ x - 1$

iv) $x^3 + 4x^2 - 11x + k\ ; x + 2$

12. The remainder and factor theorem are true for any complex value of □.

Find □

a) by using synthetic division and the remainder theorem and

b) by evaluating $f(c)$ directly.

i) $f(x) = x^3 - 5x^2 + 2x + 1$ and $c = i$

ii) $f(x) = x^3 + 2x^2 + x - 2$ and $c = 2 - 3i$

iii) $f(x) = x^2 + 4x - 2$ and $c = 1 + i$

c) I) Show that $x - 2i$ is a factor of $f(x) = x^4 + 6x + 8$

ii) Show that $x + 3i$ is a factor of $f(x) = x^4 + 14x^2 + 4513$

Given that $2x^3 - 7x^2 + 7x - 5 = A(x-1)^3 + Bx(x-1) + C$ for all values of x, find the values A, B and C.

LINEAR, QUADRATIC AND RATIONAL INEQUALITIES AND EQUATIONS

1..Solve each of the following equations

i) $\frac{3x}{2x-1} - 4 = \frac{x}{2x-1}$ ii) $\frac{6}{x+3} + \frac{20}{x^2+x-6} = \frac{5}{x-2}$ iii) $\frac{4}{x-2} + \frac{x}{x+1} = \frac{x^2-2}{x^2-x-2}$ iv) $\frac{3y}{y^2+y-6} + \frac{2}{y^2+4y+3} = \frac{y}{y^2-y-2}$

v) $\frac{-1}{2x-5} + \frac{2x-4}{4x^2-25} = \frac{5}{6x+15}$

2. Solve each of the following equations

i) $\sqrt[3]{2x+3} + 3 = 0$ ii) $n^{-2} = n^{-3}$ iii) $x^{3/2} = 4x$ iv) $\sqrt{1 + 2\sqrt{x}} = \sqrt{x+1}$

v) $\sqrt{2x-1} - \sqrt{x+3} = 1$ vi) $p = \sqrt{-4p + 17} + 3$ vii) $\sqrt{-2x-7} + \sqrt{x+9} = \sqrt{8-x}$

viii) $x^4 - 25x^2 + 144 = 0$ xi) $x^{2/3} + x^{1/3} - 2 = 0$ xiii) $12t^{-2} - 17t^{-1} - 5 = 0$

xiv) $x^{-2} + 4x^{-1} - 12 = 0$ xv) $2x - 11\sqrt{x} + 12 = 0$ xvi) $x + 3\sqrt{x} - 10 = 0$

3.Solve each of the following inequalities.

i) $\frac{4x-3}{6}+\frac{2x-1}{12}>\frac{2}{15}$ ii) $-3\leq\frac{4x+3}{2}\leq 1$ iii) $\frac{x}{2}-\frac{x-1}{5}\geq\frac{x+2}{10}-4$ iv) $-2\leq\frac{5-3x}{4}\leq\frac{1}{2}$

iv) $3\geq\frac{7-x}{2}\geq 1$

4.Find the set of values of x for which

i) $2x-3<5$ ii) $5x+6\leq -12-x$ iii) $x(5-x)\geq 3+x-x^2$

iv) $2(x-5)\geq 3(4-x)$ v) $1+11(2-x)<10(x-4)$

5. Find the set of values of x for which

(i) $3(x-2)>x-4$ and $4x+12>2x+17$ (ii) $15-x<2(11-x)$ and $5(3x-1)>12x+19$

(iii) $3x+8\leq 20$ and $2(3x-7)\geq x+6$

6.Find the set of values of x for which

i) $x^2-11x+24<0$ ii) $x^2+7x+12\geq 0$ iii) $11<x^2+10$ v vi) $x(x+11)<3(1-x^2)$

7.Find the set of values of x for which

i) $x^2-7x+10<0$ and $3x+5<17$ ii) $4x^2-3x-1<0$ and $4(x+2)<15-(x+7)$

iii) $x^2-x-6>0$ and $10-2x<5$ iv) $x^2-2x-3<0$ and $x^2-3x+2>0$

8.Solve each of the following inequalities, expressing the set of solution sets in interval notation.

i) $\frac{x+2}{x+4}\leq 0$ ii) $\frac{3x+2}{x-1}>0$ iii) $\frac{x}{x-1}>2$ iv) $\frac{1}{x-2}<\frac{1}{x+3}$ v) $\frac{2}{x+1}>\frac{3}{x-4}$

9. Solve each of the following equations

i) $\left|\frac{3}{k-1}\right|=4$ ii) $\left|x+\frac{1}{4}\right|=\frac{2}{5}$ iii) $|3x-1|=|2x+3|$

iv) $|-4n+5|=|-3n-5|$ v) $|-2n+1|=|-3n-1|$ vi) $\left|\frac{-2}{n+3}\right|=5$ vii) $\left|\frac{x+1}{x-2}\right|=-2$

10.Solve each of the following inequalities, expressing the set of solution sets in interval notation

i) $|x|\geq 4$ ii) $|2x-1|\leq 7$ iii) $|t-3|>5$ iv) $|x-1|+2<4$ v) $|x+4|-1>1$ vi) $\left|\frac{x+1}{x-4}\right|<3$ vii) $\left|\frac{x+4}{x-5}\right|\geq 3$ viii) $\left|\frac{n+2}{n}\right|\geq 4$ ix) $|x-1|>1-x^2$ x) $|x+1|+|x-2|\leq 5$

xi) $\left|\frac{x+1}{x^2+2x+2}\right|\leq\frac{1}{2}$ xii) $2x-x^2\geq|x-1|-1$ xiii) $|3-|x||\leq|3-{}^1/_3\,x^2|$

11.On the same diagram, draw the graphs of $y=|3x|$ and $y=|x-3|$ for the domain $-2\leq x\leq 3$. Hence solve the equation $|3x|=|x-3|$.

12. The range of the function $y=|x-1|$ is $0\leq y\leq 3$. Find a possible domain. What is the Widest Possible domain?

13 .Redefine each of the following modulus functions by removing the modulus, hence sketch the graph of each function:

i) $f(x) = -2|5x-4|$ ii) $h(x) = |3x+1| + |2x-3|$ (iii) $k(x) = |2x-1| - |x+2|$

14.(a) Sketch the following modulus functions and determine their domain and range

i) $f(x) = |2x-1| + 3$ ii) $f(x) = |x^2+5x+4| - 2$ iii) $f(x) = |-3x^2-2x+1| + 1$

iv) $f(x) = |x^2+x-6| - 3$ v) $f(x) = |-3x+1| + 1$

(b)For each of the following functions $y = f(x)$, sketch the graph of $y = f(|x|)$ and determine their domain and range

(i) $f(x) = x^2 - 4x$ (ii) $f(x) = 1+\sqrt{x+2}$ (iii) $f(x) = 3+(x-2)^2$ (iv) $f(x) = \begin{cases} \frac{1}{x-1}, x<1 \\ \sqrt{x}+2, x \le 1 \end{cases}$

(v) $f(x) = \begin{cases} |x^2-2x|, x \ge 1 \\ |x|-1, x<1 \end{cases}$ (vi) $f(x) = x|x|$

APPLICATIONS OF QUADRATIC EQUATIONS

15. What are the dimensions of the largest rectangular field which can be enclosed by 1200 m of fencing?

16. A window is to be constructed in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 540 cm, find its dimensions for maximum area.

17. If the profit p in the manufacturing and sale of x units of a product is given by

$$P(x) = 200x - 0.001x^2,$$

(i) Find the number x that yields the maximum profit.

(ii) Find the maximum profit if each item is sold at K 1.50

(iii) Sketch the graph of the function P

18. Solve each of the following inequalities involving radical functions.

(a) $10 - \sqrt{2x+7} \le 3$

(b) $3 \le \sqrt{2x+5} < 6$

(c) $\sqrt{2x+9} - \sqrt{9+x} > 0$

(d) $\sqrt{2} - \sqrt{x+6} \le -\sqrt{x}$

(e) $\sqrt{x-3} > \sqrt{x+4} - 1$

(f) $3 + \sqrt{2x-7} \le 6$

(g) $\sqrt{2x+5} < \sqrt{9+x}$

(h) $\sqrt{x+3} + \sqrt{x+7} > 4$

TERM 2 2023 TEST 2 ON 18/05/23 WEEK 9. 1

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

[redacted]

TUTORIAL SHEET 5: MA110-Mathematical Methods

1. Express each of the following as a single fraction
   a) $\frac{3}{x+3} - \frac{2}{x-2}$ b) $\frac{1}{(x+2)^2} - \frac{2}{x+2} + \frac{1}{3x-1}$ c) $\frac{4}{2+3x^2} - \frac{1}{1-x}$ d) $\frac{3}{x^2+1} - \frac{1}{x-1} + \frac{2}{(x-1)^2}$
2. Express in partial fractions
   a) $\frac{2x}{(3+x)(3-x)}$ b) $\frac{a}{a^2-b^2}$ c) $\frac{1}{p(1-p)}$
3. Find the values of the constants A, B, C in the following identities
   a) $31x - 8 \equiv A(x-5) + B(4x+1)$
   b) $8 - x \equiv A(x-2)^2 + B(x-2)(x+1) + C(x+1)$
   c) $71 + 9x - 2x^2 \equiv A(x+5)(x+2) + B(x+2)(x-3) + C(x-3)(x+5)$
   d) $4x^2 + 4x - 26 \equiv A(x+2)(x-4) + B(x-4)(x-1) + C(2x^2+1)(x+2)$
4. Can values of A, B, C, D be found which make the following pairs of expression identical?
   a) $2x^2 - 22x + 53$ and $A(x-5)(x-3) + B(x-3)(x+2) + C(x+2)(x-5)$
   b) $x + 7$ and $A(x-2) + B(x+1)^2$
   c) $x + 1$ and $A(x-2) + B(x^2+1)$
   d) $x^3 + 2x^2 - 4x - 2$ and $(Ax+B)(x-2)(x+1) + C(x+1) + D(x-2)$
5. A) Find the values of A, B, C if $x^3 - 1$ is expressed in the form $(x-1)(Ax^2 + Bx + C)$
   B) Express $x^3 + 1$ in the form $x(x-1)(x-2) + Ax(x-1) + Bx + C$
6. Express in partial fractions
   a) $\frac{6}{(x+3)(x-3)}$ b) $\frac{x-1}{3x^2-11x+10}$ c) $\frac{3x+1}{(x+2)(x+1)(x-3)}$ d) $\frac{3-4x}{2+3x-2x^2}$
7. Express in partial fractions
   a) $\frac{6-x}{(1-x)(4+x^2)^2}$ b) $\frac{4}{(x+1)(2x^2+x+3)}$ c) $\frac{3+2x}{(2-x)(3+x^2)}$ d) $\frac{5x+2}{(x+1)(x^2-4)}$
8. Express in partial fractions
   a) $\frac{x+1}{(x+3)^2}$ b) $\frac{2x^2-5x+7}{(x-2)(x-1)^2}$ c) $\frac{1}{x^4+5x^2+6}$ d) $\frac{2x+1}{x^3-1}$ e) $\frac{3x+7}{x(x+2)(x-1)}$
9. Express in partial fractions
   a) $\frac{x^3+2x^2-2x+2}{(x-1)(x+3)}$ b) $\frac{x^3-x^2-4x+1}{x^2-4}$ c) $\frac{x^4+3x-1}{(x+2)(x-1)^2}$ d) $\frac{2x^4-17x-1}{(x-2)(x^2+5)}$ e) $\frac{3x^2+2x-9}{(x^2-1)^2}$
10. Find the partial fraction decomposition for each of the following rational expressions.
    a) $\frac{11x-10}{x^2-x-2}$ b) $\frac{-2x-8}{x^2-1}$ c) $\frac{20x-3}{6x^2+7x-3}$ d) $\frac{-6x^2+7x+1}{x(2x-1)(4x+1)}$
    e) $\frac{-6x^2+19x+21}{x^2(x+3)}$ f) $\frac{-9x^2+7x-4}{x^3-3x^2-4x}$ g) $\frac{10x^2-73x+144}{x(x-4)^2}$ h) $\frac{4x^2+3x+14}{x^3-8}$
    i) $\frac{3x^2+10x+9}{(x+2)^3}$ j) $\frac{2x^2+x+2}{(x^2+1)^2}$ k) $\frac{2x^4-17x-1}{x^3-2x^2+5x-10}$ l) $\frac{3x^2+10x+9}{x^3+6x^2+12x+8}$
    m) $\frac{3x+7}{x^3+x^2-2x}$ n) $\frac{2x^2-5x+7}{x^3-4x^2+5x-2}$

ARITHMETIC AND GEOMETRIC SERIES

1. Which of the following series are arithmetical progressions? Write down the common difference of those that are.
   a) $7+8\frac{1}{2}+10+11\frac{1}{2}$, b) $n+2n+3n+4n$, c) $1^2+2^2+3^2+4^2$,
   d) $1-2+3-4+5$ e) $1+0.8+0.6+0.4$.
2. Write down the terms indicated in each of the following A.Ps
   a) $3+11+\ldots\ldots, 10th,\ 19th$, b) $\frac{1}{4}+\frac{7}{8}+\ldots\ldots, 12th, nth$
   c) $3+7+\ldots\ldots, 200th, (n+1)th$
3. Find the number of terms in the following A.Ps:
   a) $x+2x+\ldots\ldots+nx$, b) $a+(a+d)+\ldots\ldots+(a+(n-1)d)$
   c) $2+4+\ldots\ldots+4n$, c) $2-9-\ldots\ldots-130$
4. Find the sum of the following A.Ps:
   a) $x+3x+5x+\ldots+21x$ b) $a+(a+1)+\ldots+(a+n-1)$
   c) $2.01+2.02+2.03+\cdots+3.00$ d) $1+3+5+\cdots+101$
5. Find the sums of the following arithmetical progressions as far as the terms indicated:
   a) $4+10+\ldots\ 12th\ term$ b) $1\frac{1}{4}+1+\ldots\ nth\ term$
   c) $15+13+\cdots 20th\ term$ d) $1+2+\cdots 200th\ term$
6. a) Show that the sum of the first n terms of the A.P. with first term a and common difference d is $\frac{1}{2}n(2a+(n-1)d)$ or $S_n=\frac{n}{2}(a_1+a_n)$
   b) Show that the sum of the integers from 1 to n is $\frac{1}{2}n(n+1)$
   c) Show that the general term of an arithmetic sequence is given by $a_n=a_1+(n-1)d$ where a_1 is the first term and d is the common difference.
7. a) The second term of an A.P is 15, and the fifth term is 21. Find the common difference, the first term and the sum of the first ten terms.
   b) The fourth term of an A. P is 18, and the common difference is -5 . Find the first term and the sum of the sixteen terms
   c) Find the sum of the odd numbers between 100 and 200.
   d) Find the sum of even numbers, divisible by three, lying between 400 and 500.
8. . Write the first five terms of the sequence that has the indicated general term
   a) $a_n=3n^2-1$ b) $a_n=2^{n+1}$ c) $a_n=n(n-1)$ d) $a_1=3,\quad a_{n+1}=a_n-1, n\geq 1$
   e) $s_o=1,\ s_{n+1}=x^{n+1}+s_n,\ n\geq 0$ f) $a_n=\dfrac{(-1)^{n+1}}{n(n+1)}$ g) $a_n=\dfrac{2}{n+1}$
9. Find the sum of each the following arithmetic series.
   a) $-193-189-185-\ldots-21-17$
   b) $5+8+11+14+\ldots+59+62$
   c) $2\frac{1}{4}+2\frac{17}{20}+3\frac{9}{20}+\ldots+20\frac{1}{4}+20\frac{17}{20}$
10. Find the 15[th] and 30[th] terms of the sequence for which $a_n=-5n-4$

11. Find the general term (the $nth\ term$) for each of the arithmetic sequences .

a) 11,13,15,17,19,...... b) 7,10,13,16,19,....... c) -3,-6,-9,-12,-15,........

12. Find the required term for each of the arithmetic sequences

a) The 15th term of 3,8,13,18,........ b) The 52th term of $1, \frac{5}{3}, \frac{7}{3}, 3, ...$

13. a) If the 6th term of an arithmetic sequence is 12 and the 10th term is 16, find the first term.

b) If the 3th term of an arithmetic sequence is 20 and the 7th term is 32, find the 25th term.

c) Find the sum of the first 40 terms of the arithmetic sequences 2, 6, 10, 14, 18, ...

d) Find the sum of the first 50 terms of the arithmetic sequences $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, ...$

e) Find the sum of the first 200 odd whole numbers.

b) Find the sum of all even numbers between 18 and 482, inclusive.

14. Write each series in expanded form

a) $\sum_{i=1}^{5}(i^2-2)$ b) $\sum_{i=1}^{5}(16-4i)$ c) $\sum_{k=2}^{8}\frac{k+1}{k-1}$ d) $\sum_{j=1}^{n}(-1)^j\left(\frac{1}{3}\right)^{j-1}$ e) $\sum_{j=1}^{\infty}(-1)^{j+1}\left(\frac{1}{2}\right)^j$

15. Find the sum of the following arithmetic series.

a) $\sum_{j=1}^{7}2^j$ b) $\sum_{i=1}^{5}3^{i-1}$ c) $\sum_{k=2}^{5}\left(\frac{3}{4}\right)^{k-2}$ d) $\sum_{i=4}^{8}\left(\frac{1}{2}\right)^i$

16 Find each of the following sums

a) $\sum_{i=1}^{45}(5i+2)$ b) $\sum_{i=1}^{5}i^2$ c) $\sum_{i=3}^{8}(2i^2+i)$ d) $\sum_{n=10}^{20}4n$

17. Write in the Σ notation:

a)$1+2+3+ \ .... +n$ b) $1^4+2^4+\ ...+n^4+(n+1)^4$ c)$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$,

d)$3^2+3^3+3^4+3^4$ e) $2\times7+3\times8+4\times9+5\times10+6\times11$

18. Writ in full:

a) $\sum_1^4 m^3$ b) $\sum_1^3\frac{m}{m(m+1)}$ c) $\sum_3^6\frac{(-1)^m}{m}$ d) $\sum_1^4(-1)^m m^2$

18. Write the first five terms of each sequence

a) $a_n = \begin{cases}2n+1 & for\ n\ odd\\ 2n-1 & for\ n\ even\end{cases}$ b) $a_n = \begin{cases}3n+1\ for\ n\le 3\\ 4n-3\ for\ n>3\end{cases}$

19. Write the first five terms of each sequence

a) $\begin{cases}a_1 = 4\\ a_n = 3a_{n-1}\ for\ n\ge 2\end{cases}$ b) $\begin{cases}a_1 = 2\\ a_2 = 3\\ a_n = 2a_{n-2}+3a_{n-1}\ \ for\ n\ge 3\end{cases}$

2023

TERM 2

2

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 6 **MA110 - Mathematical Methods** [redacted]

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1. Which of the following series are geometrical progressions? Write down the common ratio of those that are .

   a) $-1+2-4+8$ b) $a+a^2+a^3+a^4$ b)$1+1.1+1.21+1.331$ c)$\frac{3}{4}+\frac{9}{2}+27+162$

2. Write down the terms indicated in each of the following geometrical progressions. Do not simplify your answers.

   a) $\frac{2}{7}-\frac{3}{7}+\cdots, 9th,\ \ nth$ b) $3-2+\cdots, 8th, nth$ c) $3+1\frac{1}{2}+\cdots, 19th, 2nth$

3. Find the number of terms in the following geometrical progressions:

   a)$0.03+0.06+0.12+\cdots+1.92$, b)$5+10+20+\cdots+5\times 2^n$,

   c) $a+ar+ar^2+\cdots+ar^{n-1}$ d) $\frac{8}{81}-\frac{4}{27}+\frac{2}{9}-\cdots-1\frac{11}{16}$

4. Find the sums of the geometrical progression in number 3. Simplify, but do not

   evaluate, your answers

5. Find the sums of the following geometrical progressions as far as the terms indicated. Simplify, but do not evaluate, your answers

   a) $4+12+36+\cdots, 12th\ term;$ b) $3+6+12+\cdots, nth$

   c) $1-\frac{1}{3}+\frac{1}{9}-\cdots, nth$ d) $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots, 13th\ term$

6. The three numbers ,$n-2, n, n+3$, are consecutive terms of a geometrical progression. Find n, and the term after $n+3$

7. Show that the sum of the series $4+12+36+108+\cdots\ to\ 20\ term$ is greater than 3×10^9

8. The numbers $n-4, n+2, 3n+1$ are in geometrical progression. Find the two possible values of the common ratio.

9. What is the common ratio of the $G.P.$ $(\sqrt{2}-1)+(3-2\sqrt{2})+\cdots$? Find the third term of the progression.

10. Find the ratio of the sum of the first 10 terms of the series

$$\log x + \log x^2 + \log x^4 + \log x^8 + \cdots$$

11 Find the first term of the geometric sequence with 5th term $\frac{32}{3}$ and common ratio 2

12 Find the common ratio of the geometric sequence with 3rd term 12 and 6th term 96

a) $4+2+1+\cdots+\frac{1}{512}$ b) $1+(-2)+4\cdots+256$

13 Find each of the indicated sums

a) $\sum_{i=3}^{9} 2^{i-3}$ b) $\sum_{i=1}^{6} 3\left(\frac{1}{2}\right)^{i}$ c) $\sum_{i=2}^{5}(-3)^{i+1}$

14 Find the sum of each infinite geometric sequence. If the sequence has no sum then state

a) $2,1,\frac{1}{2},\frac{1}{4},\ldots$ b) $9, -3, 1, -\frac{1}{3},\ldots$ c) $4,8,16,32,\ldots$ d) $\frac{3}{4}+3+12+\ldots$

e) $3+\frac{9}{4}+\frac{27}{16}+\cdots$ f) $4+\frac{8}{3}+\frac{16}{9}+\cdots$

15 Change each repeating decimal to a/b form, where a and b are integers and $b \neq 0$. Express a/b in reduced form

a) $0.\overline{3}$ b) $0.\overline{26}$ c) $0.2\overline{14}$ d) $3.\overline{7}$

16. Determine if the sequence is geometrical or arithmetical or nether

a) $\sum_{i=10}^{20} 4i$ b) $\sum_{i=1}^{5} i^2$ c) $\sum_{i=1}^{6} 3^i$ d) $\sum_{x=3}^{8}(2x^2+x)$

17. Find a_5 and S_5 of (a) the G.P. $\frac{12}{25}+\frac{6}{5}+3+\ldots$

(b)the G.P. in which $a=27$ and

18. Write the following geometric series in sigma notation.

a) $3+9+27+81$

b) $16+8+4+2+1$

c) $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+\ldots$

19. For each of the following geometric series, find the range of values of x for which the sum to infinity of the series exists. a) $x+x^2+x^3+x^4+x^5+\ldots$

(b) $1+\frac{x}{3}+\frac{x^2}{9}+\frac{x^3}{27}+\ldots$ (c) $\left(x+\frac{1}{2}\right)+\left(x+\frac{1}{2}\right)^2+\left(x+\frac{1}{2}\right)^3+\left(x+\frac{1}{2}\right)^4+\ldots$

20. a) Show that the general term of an arithmetic sequence is given by $a_n = a_1 + (n-1)d$
    b) Show that the sum of an arithmetic sequence is given by $S_n = \frac{n}{2}(a_1 + a_n)$
    c) Show that the general term of a geometric sequence is given by $a_n = a_1 r^{n-1}$
    d) Show that the sum of a geometric sequence is given by $S_n = \frac{a_1 r^n - a_1}{r-1}$ $r \neq 1$
    e) Show that the sum of an infinite geometric sequence is given by $S_\infty = \frac{a_1}{1-r}$, $|r| < 1$

MATHEMATICAL INDUCTION

1. Use mathematical induction to prove that each statement is true for all positive integers n.
   a) $3^n \geq 2n+1$ b) $4^n \geq 4n$ c) $n^2 \geq n$ d) $2^n \geq n+1$
   e) $4^n - 1$ is divisible by 3 f) $6^n - 1$ is divisible by 5 g) $9^n - 1$ is divisible by 4
   h) $n^2 + n$ is divisible by 2 I) $n^2 - n$ is divisible by 2

2. Use mathematical induction to prove each of the sum formulas for the indicated sequence. They are to hold for all positive integers n
   a) $S_n = \frac{n(n+1)}{2}$ for $a_n = n$ b) $S_n = 2(2^n - 1)$ for $a_n = 2^n$ c) $S_n = \frac{n(5n+9)}{2}$ for $S_n = 5n+2$
   d) $S_n = \frac{n(n+1)(2n+1)}{6}$ for $a_n = n^2$ e) $S_n = \frac{n(n+1)(n+2)}{3}$ for $a_n = n(n+1)$
   f) $S_n = \frac{n}{n+1}$ for $a_n = \frac{1}{n(n+1)}$ g) $S_n = \frac{3(3^n-1)}{2}$ for $a_n = 3^n$

3. Prove the following results by induction:
   a) $1 + 2 + \cdots n = \frac{1}{2}n(n+1)$ b) $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$
   c) $1 \times 3 + 2 \times 4 + .. + n(n+2) = \frac{1}{6}n(n+1)(2n+7)$
   d) $\frac{1}{1\times3} + \frac{1}{2\times4} + \cdots + \frac{1}{n(n+2)} = \frac{3}{4} - \frac{2n+3}{2(n+1)(n+2)}$

4. Show that $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$, and prove by induction that

   $(1+x)^n = 1 + nx + \cdots + \binom{n}{r}x^r + \cdots + x^n$ where $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

   and r is a positive integer, less than or equal to n

2023

TERM 2

3

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 4 MA110 - Mathematical Methods [redacted]

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1. Evaluate

   a) $\frac{11!}{7!4!}$ b) $\frac{6!2!}{8!}$ c) $(2!)^2$ d) $\frac{6!}{(3!)^2}$

2. Express in factorial notation:

   a) $12 \times 11 \times 10 \times 9$, b) $n(n-1)(n-2)$, c) $(n+2)(n+1)n$ d) $\frac{n(n-1)}{2\times1}$

   e) $\frac{(n+1)n(n-1)}{3\times2\times1}$, f) $n(n-1)\ldots.(n-r+1)$.

3. Express in factors:

   a) $20! + 21!$ b) $14! - 2(13!)$ c) $n! + 2(n-1)!$ d) $(n+2)! + (n+1)! + n!$

   f) $(n-1)! - (n-2)!$

4. Simplify:

   a) $\frac{15!}{11!4!} + \frac{15!}{12!3!}$, b) $\frac{16!}{9!7!} + \frac{2\times16!}{10!6!} + \frac{16!}{11!5!}$, c) $\frac{n!}{(n-r)!r!} + \frac{n!}{(n-r+1)!(r-1)!}$,

   d) $\frac{n!}{(n-r)!r!} + \frac{2\times n}{(n-r+1)!(r-1)!} + \frac{n!}{(n-r+2)!(r-2)!}$

5. Evaluate

   a) 10_{C_2} b) 6_{C_4} c) 7_{C_3} d) 9_{C_5}

6 Express in factors

   a) n_{C_2} b) $n_{C_{n-2}}$ c) $n+1_{C_2}$ d) $n+1_{C_{n-1}}$

7. a) In how many ways can 13 cards be selected from a pack of 52 playing cards?

   b) In how many ways can r objects be chosen from n unlike objects?

   c) A mixed hockey team containing 5 men and 6 women is to be chosen from 7 men and 9 women. In how many ways can this be done?

d) There are ten possible players for the VI to represent a tennis club, and of these the captain and the secretary must be in the term. In how many ways can the team be selected?

8. Use the Pascal's Triangle to find the expansions of :

a) $(1-z)^4$ b) $\left(x-\frac{1}{x}\right)^5$ c) $(3x-y)^4$ d) $\left(\frac{x}{2}+\frac{2}{x}\right)^4$

9. Use the Pascal's Triangle to Simplify, leaving surds in the answers, where appropriate:

a) $\left(1+\sqrt{2}\right)^3+\left(1-\sqrt{2}\right)^3$ b) $\left(2+\sqrt{3}\right)^4+\left(2-\sqrt{3}\right)^4$ c) $\left(1+\sqrt{2}\right)^3-\left(1-\sqrt{2}\right)^3$

d) $\left(\sqrt{6}+\sqrt{3}\right)^3-\left(\sqrt{6}-\sqrt{2}\right)^3$

10. Use the Pascal's Triangle

a) Write down the expansion of $(2+x)^5$ in ascending powers of x. Taking the first three terms of the expansion, put $x = 0.001$, and find the value of $(2.001)^5$ correct to five places of decimals.

b) Write down the expansion of $\left(1+\frac{1}{4}x\right)^4$. Taking the first three terms of the expansion, put $x = 0.1$, and find the value of $(1.025)^4$ correct to three places of decimals

11. Write down the terms indicated, in the expansion of the following , and simplify your answers:

a) $(x+2)^8$ term in x^5 ; b$(3u-2)^5$ term in u^3 ; c) $\left(2t-\frac{1}{2}\right)^{12}$ term in t^7 ;

d) $(2x+y)^{11}$ term in x^3 ;

12.Write down, and simplify, the terms indicated, in the expansions of the following in ascending powers of x:

a) $(1+x)^9$, 4th term ; b) $\left(2-\frac{x}{2}\right)^{12}$, 4th term ; c) $(3+x)^7$, 5th term ; d) $(x+1)^{20}$,3rd term ;

13. Write down, and simplify, the coefficients of the terms indicated, in the expansions of the following:

a) $\left(\frac{1}{2}t+\frac{1}{2}\right)^{10}$,term in t^4; b) $\left(4+\frac{3}{4}x\right)^6$,term in x^3; c) $(2x-3)^7$,term in x^5;

d) $\left(3+\frac{1}{3}y\right)^{11}$,term in y^5;

14. Write down the terms involving:

a) $x^4\left(\frac{1}{x}\right)^2$ b) $x^3\left(\frac{1}{x}\right)^3$, in the expansion of $\left(x+\frac{1}{x}\right)^6$

15.(i) Write down the constant terms in the expansions of

a) $\left(x-\frac{1}{x}\right)^8$, b) $\left(2x^2+\frac{1}{2x}\right)^6$

(ii)Find the term independent of x in $\left(x-\frac{1}{2x^2}\right)^9$

16. Find the coefficients of the terms indicated in the expansions of the following:

a) $\left(x+\frac{1}{x}\right)^6$, term in x^4: b) $\left(2x+\frac{1}{x}\right)^7$, term in $\frac{1}{x^5}$; c) $\left(x-\frac{2}{x}\right)^8$, term in x^6.

17. Use the binomial theorem to find the values of

a)$(1.01)^{10}$, correct to three places of decimals:

b) $(2.001)^{10}$, correct to six significant figures:

c) $(0.997)^{12}$, correct to three places of decimals:

d) $(1.998)^8$, correct to two places of decimals:

e) $\frac{1}{(1.02)^2}$, correct to four places of decimals:

f) $\frac{1}{\sqrt{(0.98)}}$,correct to four places of decimals:

g) $\sqrt{(0.998)}$ correct to six places of decimals:

18. Expand the following as far as the terms in x^3:

a) $(1+x+x^2)^3$ b) $(3-2x+x^2)^4$ c) $(2+x+x^2)^5$ d) $(3+x+x^3)^4$

19. Evaluate the following binomial coefficients:

a) $\binom{5}{3}$ b) $\binom{-2}{4}$ c) $\binom{\frac{1}{2}}{2}$ d) $\binom{-\frac{1}{4}}{3}$

20. Expand the following in ascending powers of x, as far as the terms in x^3, and state the values of x for which the expansions are valid.

a) $\left(1+\frac{x}{2}\right)^{-3}$ b) $\frac{1}{(3-x)^2}$ c) $\frac{1}{\sqrt{(2+x^2)}}$ d) $\frac{1}{2+x}$

21. Find the first four terms of the expansions of the following in ascending powers of x

a) $\frac{1+x}{1-x}$ b) $\frac{x+2}{(1+x)^2}$ c) $\sqrt{\frac{(1-x)^3}{1+x}}$ d) $\frac{x+3}{\sqrt[3]{(1-3x)}}$

22. Find the first terms and the general terms in the expansions of the following functions in ascending powers of . State the ranges of values of x for which the expansions are valid.

a) $(2+x)^{-1}$ b) $\frac{1}{(3-x)^2}$ c) $\frac{1}{(2-3x)^3}$ d) $\sqrt{(1+x)}$

23. Express the following in partial fractions and find the first three terms and the general terms in their expansions in ascending powers of . For what values of x are the expansions valid.

a) $\frac{x-1}{x^2+2x+1}$ b) $\frac{5}{1-x-6x^2}$ c) $\frac{x+3}{(x-2)^2}$ d) $\frac{x+2}{x^2-1}$

24. Expand the following functions in ascending powers of x, giving the first three terms and the general term, and state the necessary restrictions on the values of x:

a) $\frac{1-x}{1+x}$ b) $(1+x)(1-x)^{10}$ c) $\frac{x+7}{(x+1)^2(x-2)}$ d) $\frac{x+5}{(3-2x)(x-1)}$

25. Expand the following functions in ascending powers of $\frac{1}{x}$, giving the first three terms and the general term, and state the necessary restrictions on the values of x:

a) $\frac{1}{x^2-5x+6}$ b) $\frac{x+2}{x+1}$ c) $\frac{x-1}{(x+2)^2}$ d) $\frac{1}{1-x+x^2-x^3}$

26. a) Obtain the first three terms in the expansion of $\left(a+\frac{x}{b}\right)^6$ in ascending powers of x. If the first and the third terms are 64 and $\frac{80x^2}{3}$ respectively, find the values of a and b and the second term.

b) (i) Expand $(1+2x)^5$ and $(1-2x)^5$ in ascending powers of x.

(ii) Hence reduce $(1+2x)^5-(1-2x)^5$ to its simplest form.

(iii) Using the result in (ii) evaluate $(1.002)^5-(0.998)^5$

c) Expand $(1+x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^4 and hence find an approximation for $\sqrt{1.08}$.

(ii) In the expansion of $(1+ax)^n$, the first three terms in ascending powers of x are $1-\frac{5}{2}x+\frac{75x^2}{8}$, find the values of n and a, and state the range of values of x for which the expansion is valid.

d) i) Find the coefficient of the term in x^3 in the expansion of $(2+3x)^3(5-x)^3$

ii) The coefficient of x^2 in the expansion of $(2+ax)^2$ is 54. Find the possible values of the constant a

iii) The coefficient of x^2 in the expansion of $(2-x)(2+bx)^3$ is 54. Find the possible values of the constant b

e) . If x is so small that terms of x^3 and higher can be ignored, show that

(i) $(2+x)(1-3x)^5 \approx 2-29x+165x^2$

(ii) $(2-x)(3+x)^4 \approx a+bx+cx^2$, find the values of the constants a, b and c.

f) Given that

$$(2+x)^5+(2-x)^5 \equiv A+Bx^2+Cx^3$$

(i) find the values of the constants A, B and C

(ii) Using the substitution $y=x^2$ and your answers to part (i), solve

$$(2+x)^5+(2-x)^5=349$$

27. a) The coefficient of x^2 in the binomial expansion of $\left(1+\frac{x}{2}\right)^n$, where n is a positive interger is 7. (i) Find the value of n

(ii) Using the value of n found in part (i), find the coefficient of x^4

b) I n the binomial expansion of $(2k+x)^n$, where k is a constant and n is positive integer, the coefficient of x^2 is equal to the coefficient of x^3

(i) Prove that $n=6k+2$.

28. (i) a) Express $\frac{8x+4}{(1-x)(2+x)}$ as partial fractions

b) Hence or otherwise expand $\frac{8x+4}{(1-x)(2+x)}$ in ascending powers of x as far as the term in x^2

c) State the set of value of x for which the expansion is valid.

(ii) a) Express $\frac{-2x}{(2+x)^2}$ as partial fraction

b) Hence prove that $\frac{-2x}{(2+x)^2}$ can be expressed in the form $0-\frac{1}{2}x+Bx^2+Cx^3$ where constants B and C are to be determined.

c) State the set of value of x for which the expansion is valid

(iii) a) Express $\frac{6+7x+5x^2}{(1+x)(1-x)(2+x)}$ as partial fraction

b) Hence or otherwise expand $\frac{6+7x+5x^2}{(1+x)(1-x)(2+x)}$ in ascending powers of x as far as the term in x^3.

c) State the set of value of x for which the expansion is valid

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SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 8 MA110 - Mathematical Methods [redacted]

MATRICES AND DETERMINANTS

1. Given that $A = \begin{pmatrix} 3 & 1 & 2 \\ 5 & 1 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -1 & 2 \\ 3 & 1 & 3 \end{pmatrix}$ evaluate

a) $3A$ b) $2B$ c) $3A + 2B$ d) $3A - 2B$.

2. Find, where possible, the following products. When it is not possible to form the product, state this clearly and give the reason for your conclusion.

a) $\begin{pmatrix} 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 4 & 7 \\ 1 & 6 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 6 \end{pmatrix}$. d) $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 7 \\ 8 \end{pmatrix}$

3. Evaluate the matrix products:

a) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 1 & 0 & 7 \end{pmatrix}$, b) $\begin{pmatrix} 2 & \frac{1}{2} & \frac{3}{4} \\ 1 & 0 & \frac{1}{4} \\ 3 & 1 & 0 \end{pmatrix}\begin{pmatrix} 8 & -4 \\ 0 & 12 \\ 4 & 0 \end{pmatrix}$

4. Verify that if $M = \begin{pmatrix} -5 & 10 & 8 \\ 4 & -7 & -6 \\ -3 & 6 & 5 \end{pmatrix}$ and $N = \begin{pmatrix} -1 & 2 & 4 \\ 2 & 1 & -2 \\ -3 & 0 & 5 \end{pmatrix}$, then $MN = NM = I$, where I is the 3×3 unit matrix. Use this to solve the matrix equation $\begin{pmatrix} -5 & 10 & 8 \\ 4 & -7 & -6 \\ -3 & 6 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$

5. The matrix A is $\begin{pmatrix} 3 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 3 & -2 \end{pmatrix}$. Show that A satisfies the matrix equation $A^3 = 13A - 12I$.

Assuming that A^{-1} exists, show that this equation can be written $A^{-1} = \frac{1}{12}(13I - A^2)$, and hence find A^{-1}

6. a) Solve the equation $\begin{vmatrix} x-3 & 1 & -1 \\ -7 & x+5 & -1 \\ -6 & 6 & x-2 \end{vmatrix} = 0.$

b) Solve the equation $\begin{vmatrix} x+3 & 5 & 6 \\ -1 & x-3 & -1 \\ 1 & 1 & x+4 \end{vmatrix} = 0$

7. Evaluate a) $\begin{vmatrix} 2 & -1 & 0 \\ 3 & 2 & 0 \\ 4 & 7 & 3 \end{vmatrix}$ b) $\begin{vmatrix} 3 & 4 & 2 \\ 1 & 2 & 0 \\ -2 & 3 & 5 \end{vmatrix}$ c) $\begin{vmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{vmatrix}$ d) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{vmatrix}$

8. Find, where possible, the inverses of the following matrices.

a) $\begin{pmatrix} 1 & -1 & 2 \\ 1 & 2 & 3 \\ 3 & 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 1 & 7 \\ 3 & 4 & 5 \\ 1 & -2 & 9 \end{pmatrix}$ c) $\begin{pmatrix} 1 & p & q \\ 0 & 1 & r \\ 0 & 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos x & \sin x \\ 0 & \sin x & -\cos x \end{pmatrix}$

9.The matrix A is $\begin{pmatrix} 3 & 1 & 1 \\ 1 & 0 & -2 \\ 10 & 3 & 1 \end{pmatrix}$. Show that A is singular. Find a non-zero matrix B, such that

$$AB = BA = 0$$

10.Find the inverse A^{-1} of the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$.Find also B^{-1} and $(AB)^{-1}$ where $B = \begin{pmatrix} 1 & 4 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$. Given that $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, find x, y and z.

11. M is the matrix $\begin{pmatrix} 3 & 1 & -3 \\ 1 & 2a & 1 \\ 0 & 2 & a \end{pmatrix}$.

a) Find two values of a for which M is singular.

b) Solve the equation $M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3\frac{1}{2} \\ 5\frac{1}{2} \\ 5 \end{pmatrix}$ in the case $a = 2$, and determine whether or not solutions exist for each of the two values of a found in a)

12. Use Cramer's method to solve the following systems of linear equation

a) $x - y + 2z = 1$
$2x + y + z = 2$
$x - 3y + z = 1$

b) $2x + 3y - 5z = 4$
$x + 7y - 2z = 1$
$5x - 11y + 2z = -2$

13. Let $A=\begin{pmatrix}3 & 0\\ -1 & 5\end{pmatrix}, B=\begin{pmatrix}4 & -2 & 1\\ 0 & 2 & 3\end{pmatrix}, C=\begin{pmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{pmatrix}, D=\begin{pmatrix}0 & -3\\ -2 & 1\end{pmatrix}$.

Find

(a) A + 2D (b) 3D − 2A (c) AB (d) A^3

(e) D + BC (f) B^TB (g) $B-C^T$

(h) $B^TC^T-(CB)^T$ (i) $(I_2-D)^2$

14. (a) Find the matrix of cofactors of the matrix $A=\begin{pmatrix}1 & 2 & -1\\ 2 & 2 & 4\\ 1 & 3 & -3\end{pmatrix}$

15. (a) Given that
$$\begin{aligned} x+y+3z&=5\\ 2x-y+4z&=11\\ -y+z&=3\end{aligned}$$

Express the system in the form $AX=B$ where A is the matrix of coefficients hence solve the system of equations.

b) Find the inverse of the matrix $A=\begin{pmatrix}1 & -1 & 2\\ -1 & 0 & 3\\ 2 & 1 & 0\end{pmatrix}$. Hence solve the equation $A\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}-\frac{5}{2}\\ 0\\ \frac{1}{2}\end{pmatrix}$.

c) (i) Find the inverse of the matrix $A=\begin{pmatrix}3 & -1 & 2\\ 1 & 1 & 1\\ 2 & 2 & -1\end{pmatrix}$

(ii) Use your inverse to solve the system of linear equations

$$\begin{aligned} 3x-y+2z&=4\\ x+y+z&=2\\ 2x+2y-z&=3\end{aligned}$$

d) Use inverse method to solve the following Equations

i) $x+2y+3z=-2$
$x+3y+4z=-3$
$x+4y+3z=-6$

ii) $x+3y-2z=5$
$x+4y-z=3$
$-2x-7y+5z=-12$

iii) $x-2y+z=-3$
$-2x+5y+3z=34$
$3x-5y+7z=14$

iv) $x+4y-2z=2$
$-3x-11y+z=-2$
$2x+7y+3z=-2$

16. a) Find λ for which the matrix $\lambda I - A$ is a singular matrix if where I is an identity Matrix

(a) $A=\begin{pmatrix}0 & 3\\ 2 & -1\end{pmatrix}$ (b) $A=\begin{pmatrix}1 & 0 & 2\\ 0 & -1 & -2\\ 2 & -2 & 0\end{pmatrix}$

(c) $\begin{pmatrix}11 & -2 & 8\\ 19 & -3 & 14\\ -8 & 2 & -5\end{pmatrix}$

(b) (i) Find values of x such that the matrix $xI - A$ is a singular matrix if $A=\begin{pmatrix}3 & 2 & 4\\ 2 & 0 & 2\\ 4 & 2 & 3\end{pmatrix}$

and $I=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$.

17. Use Crammer's Rule to solve the following

a) $x-2y+z=3$
$3x+2y+z=-3$
$2x-3y-3z=-5$

b) $x-2y+z=1$
$3x+y-z=2$
$2x-4y+2z=-1$

c) $-2x+y-3z=-4$
$x+5y-4z=13$
$7x-2y-z=37$

d) $-x-y+3z=-2$
$-2x+y+7z=14$
$3x+4y-5z=12$

18 . Let $A=\begin{bmatrix}1 & 2 & -3\\ 3 & 1 & 4\\ 4 & 3 & -2\end{bmatrix}$. Find the following minors:

a) $\det(M_{13})$ b) $\det(M_{23})$ c) $\det(M_{31})$ d) $\det(M_{32})$

19. Let $A = \begin{bmatrix} -1 & 2 & 3 \\ -2 & 5 & 4 \\ 0 & 1 & -3 \end{bmatrix}$. Find the following cofactors :

a) A_{13} b) A_{21} c) A_{32} d) A_{33}

20. Compute the determinant of the following using cofactor 's method.

a) $\begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 5 & -2 & 4 \\ 0 & 3 & -5 \\ 2 & -4 & 7 \end{bmatrix}$ d) $\begin{bmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{bmatrix}$

21. Use the method below to find the determinant of the following

$$\left[\begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{array}\right] \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{array}$$

a) $\begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{bmatrix}$

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DEPARTMENT OF MATHEMATICS

[redacted] Academic year

MA110: MATHEMATICAL METHODS

Tutorial sheet 9

1. Assuming that the rate of inflation is 7% per year, the equation $P = P_0(1.07)^t$ yields the prediction price P of an item in t years if it presently costs P_0. Find the predicted price of each of the following items for the indicated years ahead.
   a) \$ 55 can of soup in 3 years
   b) \$ 500 TV set in 7 years.
2. Suppose that it is estimated that the value of a car depreciated 20% per year for the first 5 years. The equation $A = P_0(0.8)^t$ yields the value (A) of a car after t years if the original price is P_0 . Find the value(to the nearest dollar) of each of the following cars after the indicated time.
   a) \$9000 car after 4 years.
   b) \$5295 car after 2 years.
3. Use the formula $A = P\left[1 + \frac{r}{n}\right]^{nt}$ to find the total amount of money accumulated at the end of the indicated time period for each of the following investments. Estimate to the nearest cent.
   a) \$250 for 5 years at 9% compounded annually
   b) \$300 for 6 year at 8% compounded semiannually
   c) \$750 for 15year at 9% compounded quarterly
4. Use the formula $A = Pe^{rt}$ to find the total amount of money accumulated at the end of the indicated time period by compounding continuously. Use 2.718 as an approximation for e.
   a)\$500 for 5 years at 8%
   b) \$800 for 10 years at 10%
5. Sketch each of the exponential functions and determine its domain and range.

   a) $f(x) = 3^{2x+1} - 2$ b) $f(x) = (-4)^{x-1} + 1$ c) $f(x) = \left(\frac{1}{3}\right)^{x+1} + 2$

   d) $f(x) = \left(-\frac{1}{4}\right)^{x-1} - 1$ e) $f(x) = 2^{x^2} - 1$ f) $f(x) = 3^{x^3} + 1$

   g) $f(x) = e^{x+1} + 2$ h) $f(x) = 3e^{x} + 1$ i) $f(x) = 2^{-x}$ j) $f(x) = e^{2x-1} + 1$

6. a)Suppose that in a certain bacterial culture , the equation $Q(t) = 1000e^{0.4t}$ expresses the Number of bacteria present as a function of the time , where t is expressed in hours . How many bacteria are present at the end of 2 hours , 3 hours.

7. Write each of the following in logarithmic forms.

a) $3^2 = 9$ b) $2^5 = 32$ c) $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$ d) $2^{-4} = \frac{1}{16}$

8. Write each of the following in exponential form

a)$\log_2 64 = 6$ b)$\log_5 \left(\frac{1}{25}\right) = -2$ c) $\log_{10} 0.1 = -1$ d) $\log_2 \left(\frac{1}{16}\right) = -4$

9. Evaluate each of the following

a) $\log_{1/2} \left(\frac{\sqrt[4]{8}}{2}\right)$ b) $5^{\log_5 13}$ c) $\log_6(\log_2 64)$ d)$\log_5 \sqrt[3]{25}$

10. Solve each of the following equations.

a) $\log_5 x = 2$ b) $\log_4 m = \frac{3}{2}$ c) $\log_b 3 = \frac{1}{2}$ d) $\log_{10} x = 0$

11. Given that $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, Evaluate each of the following

a) $\log_{10} 14$ b) $\log_{10} \sqrt{2}$ c) $\log_{10} \left(\frac{7}{2}\right)$ d) $\log_{10}(7)^{4/3}$

12. Express each of the following as the sum or difference of simpler logarithmic quantities

a) $\log_b \left(\frac{x^2}{y}\right)$ b) $\log_b x^{2/3} y^{3/4}$ c) $\log_b \left[x\left(\sqrt{\frac{x}{y}}\right)\right]$ d) $\log_b \frac{x\sqrt{y}}{z}$

e)$\log_e \sqrt{\left(\frac{x+1}{x-1}\right)}$

e) Given that $\log_a x^2 y = p$ and that $\log_a \left(\frac{x}{y^2}\right) = q$,

(i) find $\log_a x$ and $\log_a y$ in terms of p and q.

(ii) Hence express $\log_a (xy)$ in terms of p and q.

f) i) If $xy = 64$ and $\log_x y + \log_y x = \frac{5}{2}$, find x *and* y.

ii) Given that $3 + 2\log_2 x = \log_2 y$, Show that $y = 8x^2$

13. . Express each of the following as simple logarithm

a) $2\log_b x - 4\log_b y$ b) $\log_b x + \log_b y - \log_b z$ c) $2\log_b x + \frac{1}{2}\log_b(x-1) - 4\log_b(2x+5)$

d) $ln x + ln y - 1$, $ln\left(\frac{e^x}{x}\right) + ln\left(\frac{x}{e}\right)$

12. Solve each of the following logarithmic equations

a) $\log_{10}(x-4) + \log_{10}(x-1) = 1$ b) $\ln(3t-4) - \ln(t+1) = \ln 2$ c) $\log\sqrt{x} = \sqrt{\log x}$

d) $\log_2 x = 8 + 9\log_x 2$ e) $\ln(x+2) = \ln(x-4) + \ln 3$

f) $\frac{3}{2}\log_{10} a^3 - \log_{10}\sqrt{a} - 2\log_{10} a = 4$ g) $\log_{10} y - 4\log_y 10 = 0$ h) $\log_{10} a + \log_a 100 = 3$

i) $\log_{10} x + \log_{10} y = 1,\ x + y = 11$ j) $\log_{10}(19x^2+4) - 2\log_{10} x - 2 = 0$

k) $\log_2 x + \log_4 x = 2$ l) $\log_3 x - \dfrac{4}{\log_3 x} + 3 = 0$

m) $9\log_x 5 = \log_5 x$ n) $\log_8 \dfrac{x}{2} = \dfrac{\log_8 x}{\log_8 2}$

o) i) If a and b are both positive and unequal, and $\log_a b + \log_b a^2 = 3$, find b in terms of a.

ii) Express $\log_9 xy$ in terms of $\log_3 x$ and $\log_3 y$. Hence solve for x and y the simultaneous equations

$$\log_9 xy = \frac{5}{2}$$
$$\log_3 x \log_3 y = -6$$

expressing your answers as simply as possible.

13. Sketch each of the logarithmic functions and determine its domain and range.

a) Graph $y = \log_3 x$ by changing the equation to exponential form

b) Graph $f(x) = \log_{1/2} x$ by reflecting the graph of $g(x) = \left(\frac{1}{2}\right)^x$ across the line $y = x$.

c) $f(x) = 1 + \log_{10} x$

d) $f(x) = \log_{10}(x + 2)$

e) $f(x) = 2 - \log_3(x - 1)$

f) $y = 4\log_2(x + 1)$

g) $f(x) = \log_2(x - 1) + 2$

h) $f(x) = 2 + \ln x$

14. Solve each of the following exponential equations

a) $5^{3t+1} = 9$ b) $3^{2x+1} = 2^{3x+2}$ c) $3e^x - 1 = 17$ d) $2^{2x} + 3(2^x) - 4 = 0$

e) $4(3^{2x+1}) + 17(3^x) - 7 = 0$ f) $6^{x+2} = 2(3^{2x})$ g) $2^{2x+1} - 15(2^x) - 8 = 0$

15 a) Prove that $\log_a x = \frac{\log_b x}{\log_b a}$ b) Prove that $\log_b rs = \log_b r + \log_b s$

c) Prove that $\log_b\left(\frac{r}{s}\right) = \log_b r - \log_b s$ d) Prove that $\log_a r = \frac{1}{\log_r a}$

e) Prove that $\log_2 10 = \frac{1}{\log_{10} 2}$

f) Verify that $f(x) = b^x$ and $y = \log_b x$ where b and $x > 0$ are inverses of each other

16. Solve each of the following problems

a) How long will it take \$1000 to double itself if it is invested at 9% interest compounded semiannually?

b) How long will it take \$750 to be worth \$1000 if it is invested at 12% interest compounded quarterly?

c) How long will it take \$500 to triple it itself if it is invested at 9% interest compounded continuously?

d) How long will it take \$2000 to double it itself if it is invested at 13% interest compounded continuously?

e) At what rate of interest compounded continuously will an investment of \$500 grow to \$1000 in 10 years?

f) A piece of machinery valued at \$30 000 depreciates at a rate of 10% yearly. How long will it take until the machinery has a value of \$15 000?

g) The number of grams of a certain radioactive substance present after t hours is given by the equation $Q = Q_0 e^{-0.45t}$, where Q_0 represents the initial number of grams. How long will it take 2500 grams to be reduced to 1250

i) . The value of a car varies according to the formula

$$V = 20\,000e^{-\frac{t}{12}}$$ Where V is the value in \$ S and t is its age in years from new.

i) State its value when new .

ii) Find its value (to the nearest \$) after 4 years.

iii) Sketch the graph of V againts t

(j) The number of units N, of electricity used by a household after t months is given by

$$N = 30\left(1 + e^{kt}\right)$$ Where k is a constant.

i) Find the value of k if 270 units were used by the household at the end of one month.

ii) Find in simplified form ,the exact value of the number of units used by the household at the end of three months.

k) The sales S of a new product after it has been on the market for t years is $S = 30\,000\left(1 - e^{kt}\right)$.

(i) Find S as a function of t if 5 000 units have been sold after one year.

(ii) Find expressing your answer as a fraction in its lowest terms how many units will have been sold after 3 years.

L) In a group project in learning theory, a mathematical model for the proportion P of correct responses after n trials was found to be

$$P = \frac{0.83}{1 + e^{-0.2n}}.$$

Find the rate at which P is changing after $n = 3$ trials and $n = 10$ trials.

m) If the annual rate of inflation averages 5% over the next 10 years, the approximate cost C of goods or services during any year in that decade is

$$C(t) = P(1.05)^t$$

where t is the time in years and P is the present cost.

(i) If the price of an oil change for your car is presently \$24.95, estimate the price 10 years from now.

(ii) Find the rate of change of C with respect to t when $t = 1$ and $t = 8$.

(iii) Verify that the rate of change of C is proportional to C and find the constant of proportionality.

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SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

[redacted] Academic year

MA110: MATHEMATICAL METHODS

Tutorial sheet 10

1. The points A and B have coordinates $(-4,6)$ and $(2,8)$ respectively. A line p is drawn through B perpendicular to AB to meet the $y-axis$ at the point C
   a) Find the equation of the line p
   b) Determine the coordinates of C
2. The line l has equation $2x-y-1=0$. The line m passes through the point $A(0,4)$ and is perpendicular to the line l.
   a) Find an equation of m and show that the lines l and m intersect at the point $P(2,3)$.
   The line n passes through the point $B(3,0)$ and is parallel to the line m
   b) Find the equation of n and hence find the coordinates of the point Q where the lines l and n intersect.
3. The straight line passing through the point $P(2,1)$ and the point $Q(k,11)$ has the gradient $-\frac{5}{12}$.
   a) Find the equation of the line in terms of x and y only.
   b) Determine the coordinates of the point C
4. The points $(-1,-2)$, $B(7,2)$ and $C(k,4)$, where k is a constant, are the vertices of ΔABC. Angle ABC is a right angle.
   a) Find the gradient of AB.
   b) Calculate the value of k
   c) Find an equation of the straight line passing through B and Give your answer in the form $ax+by+c=0$, where a, b and c are integers.
5. The straight line l passes through $A(1,3\sqrt{3})$ and $B(2+\sqrt{3}, 3+4\sqrt{3})$
   a) Calculate the gradient of l giving your answer as a surd in it's simplest form.
   b) Give the equation of l in the form $y=mx+c$, where constants m and c are surds given in their simplest form.
   c) Show that l meets the x-axis at the point $C(-2,0)$
6. The straight line l_1 has the equation $4y+x=0$. The straight line l_2 has the equation $y=2x-3$.
   a) On the same axes, Sketch the graphs of l_1 and l_2. Show clearly the coordinates of all points at which the graphs meet the coordinate axes.
   The lines l_1 and l_2 intersect at the point A.
   b) Calculate, as exact fractions, the coordinates of A

7. a) The line $y = 2x - 8$ meets the coordinate axes at A and B.The line AB is a diameter of the circle . Find the equation of the circle.

b)The circle centre (8,10) meets the $x - axis$ at (4,0) and $(a, 0)$

i) find the radius of the circle. ii) find the value of a.

c) The circle $(x+3)^2 + (y+8)^2 = 100$ meets the positive coordinate axes at $A(a, o)$ and $B(o, b)$.

i) Find the value of a and b ii) Find the equation of the line AB

8. a) The circle ,centre (p, q) radius 25, meets the x-axis at $(-7,0)$ and $(7,0)$, where $q > 0$.

i) Find the value of p and q

ii) Find the coordinates of the points where the circle meets the $y-$ axis

b)The points $A(-1,0), B\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $C\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ are the vertices of a triangle

i)Show that the circle $x^2 + y^2 = 1$ passes through the vertices of the triangle

ii) Show that ΔABC is equilateral

c) The points $A(-3,-2), B(-6,0)$ and $C(p, q)$ lie on the centre $\left(-\frac{5}{2}, 2\right)$. The line BC is a diameter of the circle.

i) Find the value of P and q ii) Find the gradiet of i) AB ii) AC

ii) Show that AB is perpendicular to AC

9. a)Write the equation of the following circles. Express the final equation in the form $x^2 + y^2 + Dx + Ey + F = 0$

i) Centre at (2,3) and $r = 5$ ii) Centre at $(-1,-5)$ and $r = 3$

iii) Centre at $(-3,4)$ and $r = 2$ iv) Centre at $(0,-4)$ and $r = 6$

b) Find the center and the length of a radius of each of the following circles and graph them

i) $x^2 + y^2 - 6x - 10y + 30 = 0$

ii) $x^2 + y^2 - 10x = 0$

iii) $4x^2 + 4y^2 - 4x - 8y - 11 = 0$

iv) $x^2 + y^2 - x - 3y - 2 = 0$

c)i Find the equation of the circle for which the line segment determined by $(-4,9)$ and $(10,-3)$ is the diameter .

ii) Find the center and the radius of the circle given by the equation

$$4x^2 + 4y^2 - 6x + 10y - 1 = 0.$$

10. a) A is the point $(-1 . 2)$, B is the point $(2 , 3)$ and C is the point $(3 , 5)$. P is a point which divides BC in the ratio 3 : 4 and Q lies on AB such that $AQ = \frac{2}{5}AB$.

(i) Find the coordinates of P

(ii) Find the coordinates of Q.

b) I)) find the equation of the perpendicular from the point $A(5 , 3)$ to the line $2x - y + 4 = 0$.

(II) A point $P(a , b)$ is equidistant from the y – axis and from the point $(4 , 0)$. Find a relationship between a and b.

(III) M is the point $(-1 , -4)$ and N is the point $(-5 , 6)$. Find the coordinates of a point P which divides the line segment MN internally in the ratio 4 : 1

(IV) A line is drawn through the point $A(1 , 2)$ to cut the line $2y = 3x - 5$ at P and the line $x + y = 12$ at Q. If AQ = 2AP, find the coordinates of P and Q

C) Find the radius of a circle with center at $C(-2 , 5)$ if the line $x + 3y = 9$ is a tangent line.

11. Find the area of the triangle formed by the points $P(1, -1)$, $Q(2 , 2)$ and $R(3 , 7)$

12. Find the perpendicular distance from the given point to the given line:

a) $(2,0)$: $3x - 2y + 1 = 0$ b) $(1, -2)$: $5x + 12y - 7 = 0$ c) $(-2,5)$: $y = 7x + 11$

d) The line joining $(1 , 7)$ to $(4 , 2)$; $(-2 , 1)$ e) $y = 7$; $(2 , -4)$

13. a) Find the equation of the circle that passes through the three points,

i) $A(2,3)$ $B(3,2)$ and $C(-4,3)$

ii) $Q(6 , -1)$, $R(-2 , 5)$, $S(-6 , -7)$.

b) Find the equation of a circle which is tangent to the line $x + 3y = 9$ if its center is at $C(-2 , 5)$.

c) The line $L : 4x - 5y + 20 = 0$ cuts the x – axis at A and cuts the y – axis at B.

(i) Find the coordinates of the points A and B

(ii) Find the equation of a line perpendicular to L and passing through the origin.

d) A point $P(a , b)$ is equidistant from the y – axis and from the point $(4 , 0)$. Find a relationship between a and b.

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

2022/23 Academic year

MA110: MATHEMATICAL METHODS

Tutorial sheet 11

1. If the measurement is given in degrees-minute-second form, Change it to decimal form to the nearest one –hundredth of a degree. If the measurement is given in decimal form, change it to degree-minute-second form.

   a) $14^\circ 30'$ b) $62^\circ 15'$ c) $8^\circ 45' 18''$ d) $35^\circ 50' 30''$ e) 114.6° f) 73.47°

2. Each angle is expressed in radians. Change each angle to degrees without using a calculator.

   a) $\frac{2\pi}{5} rad$ b) $\frac{5\pi}{2}$ c) $\frac{7\pi}{12}$ d) $-\frac{7\pi}{6}$

3. Change each angle to radians. Do not use a calculator.

   a) 120° b)300° c)-330° d)-570°

4. a) Find, to the nearest tenth of an inch, the length of the arc intercepted by a central angle of $2\pi/3$ radians given that a radius of the circle is 22 inches long.

   b) Find, to the nearest tenth of meter, the length of the arc intercepted by a central angle of 130°radians given that a radius of the circle is 8 meters long.

   c) What is the length of an arc which subtends an angle of 0.8 rad at the centre of the circle of radius $10cm$

   d) An arc of the circle subtends an angle of 0.5 rad at the centre . Find the radius of the circle, if the length of the arc is 3 cm

   e) The chord AB of the circle subtends an angle of 60° at the centre . What is the ratio of the chord to the arc AB ?

5. a) Show that the length of an arc is $s = r\theta$

b) Show that the area of the sector of a circle with radius r is $A = \frac{1}{2}r^2\theta$

c) Show that the area of the segment in a circle of radius r is $A = \frac{1}{2}r^2(\theta - sin\theta)$

d) Show that the area of the triangle is $A = \frac{1}{2}ab\ sinC$

6. (i) A sector of a circle of radius r cm contains an angle of 1.2radians. Given that the sector has the same perimeter as a square of area $36cm^2$, find the value of r.

(ii) The arc AB of a circle, centre O and radius r cm, is such that $< AOB = 0.5\ radians$. Given that the perimeter of the minor sector AOB is 30 cm

a) Calculate the value of r.

b) Show that the area of the minor sector AOB is 36 cm^2

c) Calculate the area of the segment enclosed by the chord AB and the minor arc AB

7. (i) Suppose that a 20-foot ladder is leaning a against a building and makes an angle of 60° with the ground. How far upon the building does the top of the ladder reach?

(ii) Point P is on the terminal side of (x, y), and θ is a positive angle in standard position and less than 360°. Draw θ, and determine the values of the six trigonometric functions of θ.

a) $P(3, -4)$ b) $P(-5,12)$ c) P $(12,5)$ d) $P(-1.-1)$

(iii) Point P is on the terminal side of θ, and θ is in standard position and $0^\circ > \theta > -360^\circ$

Draw and determine the values of the six trigonometric functions of θ.

a) $P(2,4)$ b) $P(1, -3)$ c) $P(0, -1)$ d) $P(-1,0)$

8. Determine $\sin\theta$, $\cos\theta$ and $\tan\theta$ for the following

a) $\theta = -210^\circ$ b) $\theta = 660^\circ$ c) $-\frac{\pi}{4} = \theta$ d) $\theta = \frac{11\pi}{4}$

9. Complete the following table

θ	θ IN RADIANS	SIN θ	COS θ	tan θ	csc θ	sec θ	cot θ
0°							
30°							
45°							
60°							
90°							
180°							
270°							

9. a) Find $\sin\theta$ if the terminal side of θ lies on the line $y = x$ in the third quadrant .

b) Find $\cos\theta$ if the terminal side of θ lies on the line $y = -x$ in the second quadrant.

c) Find $\tan\theta$ if the terminal side of θ lies on the line $y = -2x$ in the fourth quadrant.

10. a) If $\sin\theta = -\frac{4}{5}$ and the terminal side of θ is in the fourth quadrant , find $\cos\theta$ and $\tan\theta$.

b) If $\tan\theta = -\frac{5}{12}$ and the terminal side of θ is in the second quadrant, find $\sin\theta$ and $\cos\theta$.

c) If $\cos\theta = -\frac{4}{5}$ and the terminal side of θ is in the third quadrant , find $\sin\theta$ and $\cot\theta$.

11. a) The area of a triangle is $10cm^2$. The angle between two of the sides, of length 6cm and 8cm respectively, is obtuse. Work out :

i) The size of this angle

ii) The length of the third side.

b) In ΔABC, $AB = 10cm$, $BC = a\sqrt{3}\ cm, AC = 5\sqrt{13}$ cm and $< ABC = 150^\circ$ .Calculate :

i) The value of a

ii) The exact area of ΔABC

c). In a triangle , the largest side has length 2cm and one of the other side's has length $\sqrt{2}$ cm . Given that the area of the triangle is $1cm^2$, show that the triangle is right –angled and isosceles.

b) The longest side of a triangle has length $(2x-1)cm$. The other sides have lengths $(x-1)cm\ and\ (x+1)cm$ .Given that the largest angle is 120°, Work out:

i) the value of x and

ii) the area of the triangle.

12. a) Prove the $sine$ rule for a general triangle ABC

b) Prove the $cosine$ rule for a general triangle ABC

c) In the cosine rule, substitute $cosA = 2cos^2\frac{A}{2} - 1$, and hence prove that $cos\frac{A}{2} = \sqrt{\left\{\frac{s(s-a)}{bc}\right\}}$

where $s = \frac{1}{2}(a+b+c)$

d) In the cosine rule, substitute $cosA = 1 - 2cos^2\frac{A}{2}$, and hence prove that $cos\frac{A}{2} = \sqrt{\left\{\frac{(s-b)(s-c)}{bc}\right\}}$

13. a) A function f is an even function if $f(-\theta) = f(\theta)$.

A function f is odd function if $f(-\theta) = -f(\theta)$.

Given that θ is an acute angle measured in degrees, express in terms of $sin\theta$,$cos\theta$ and $tan\theta$

and state whether $sin\theta$,$cos\theta$ and $tan\theta$ are odd or even functions.

i) $\sin(-\theta)$ ii) $\cos(-\theta)$ iii) $\tan(-\theta)$

b) Simplify each of the following expressions :

i) $1 - cos^2\frac{1}{2}\theta$ ii) $5sin^2 3\theta + 5cos^2 3\theta$ iii) $sin^2 A - 1$

iv) $\frac{sin\theta}{tan\theta}$ iii) $\frac{\sqrt{1-cos^2 x}}{cosx}$ iv) $\frac{\sqrt{1-cos^2 3A}}{\sqrt{1-sin^2 3A}}$

v) $sin^4\theta + 2sin^2\theta cos^2\theta + cos^4\theta$

c) Given that $2sin\theta = 3cos\theta$, find the value of $tan\theta$

d) Given that $sinxcosy = 3cosxsiny$, express $tanx$ in terms of $tany$

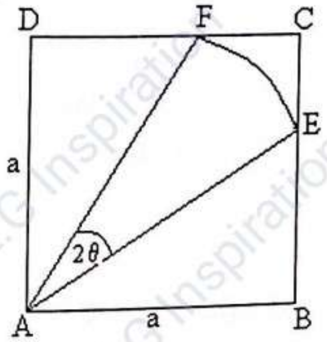
e) . Using the identities $sin^2 A + cos^2 A \equiv 1$ and / or $\tan A \equiv \frac{sinA}{cosA}$ $(cosA \neq 0)$, prove that :

i) $(sin\theta + cos\theta)^2 \equiv 1 + 2sin\theta cos\theta$ ii) $\frac{1}{cos\theta} - cos\theta \equiv sin\theta tan\theta$

iii) $tanx + \frac{1}{tanx} \equiv \frac{1}{sinxcosx}$ iv) $cos^2 A - sin^2 A \equiv 2cos^2 A - 1 \equiv 1 - 2sin^2 A$

v) $sin^2 xcos^2 y - cos^2 xsin^2 y = sin^2 x - sin^2 y$

14.



In the diagram, ABCD is a square of side a and AFE is a sector of a circle center A and angle 2θ. Show that AE = $\dfrac{a}{\cos\left(\frac{\pi}{4}-\theta\right)}$ and $AE^2 = \dfrac{2a^2}{1+\sin 2\theta}$.

Given that the area of the sector is half the area of the square, deduce that $1+\sin 2\theta = 4\theta$. Writing this equation in the form $1+\sin x = 2x$, draw the graphs of $y = 1+\sin x$ and $y = 2x$ for $0 \le x \le \dfrac{\pi}{2}$ to find an approximate value of x and hence of θ.

Team 3

THE COPPERBELT UNIVERSITY

SCHOOL OF MATHEMATICS AND NATURAL SCIENCES

DEPARTMENT OF MATHEMATICS

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MA110: MATHEMATICAL METHODS

Tutorial sheet 12

1. Sketch the unit circle and indicate the given arc s along with its initial and terminal points. Then determine the exact values of the six circular functions of s without using a calculator.
   a) $s = \frac{3\pi}{4}$ b) $s = -\frac{\pi}{4}$ c) $s = \frac{7\pi}{6}$ d) $s = -\frac{11\pi}{4}$
2. Sketch each of the functions in the indicated interval
   a) $f(x) = 2 - sinx, \quad -2\pi \leq x \leq 2\pi$
   b) $f(x) = -cos\left(x + \frac{\pi}{2}\right), \quad -\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$
   c) $f(x) = -1 + \cos(x + \pi), \quad -\pi \leq x \leq \pi$
   d) $f(x) = 1 - \sin(-x), \quad 0 \leq x \leq 2\pi$
3. Sketch the given function in the indicated interval.
   a) $f(x) = -2sin\frac{1}{2}x, \quad 0 \leq x \leq 4\pi$
   b) $f(x) = 3cos\frac{1}{2}x, 0 \leq x \leq 4\pi$
   c) $f(x) = -3\cos(-x), \quad 0 \leq x \leq 2\pi$
   d) $f(x) = sin3x, -\frac{2\pi}{3} \leq x \leq \frac{2\pi}{3}$
4. Find the period, amplitude, and phase shift of the given function and sketch.
   a) $f(x) = 2 - 3cos\left(x + \frac{\pi}{2}\right)$
   b) $f(x) = -2 + 2sin\left(2x - \frac{\pi}{2}\right)$
5. Sketch each of the functions in the indicated interval
   a) $f(x) = \tan(-x), -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
   b) $f(x) = 3csc\pi x, \quad 0 \leq x \leq 2$
   c) $f(x) = -2 + cotx, 0 \leq x \leq 2\pi$
   d) $f(x) = 1 + secx, -\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$
6. Find the period ,and phase shift of the given function and sketch graph through two periods

a) $f(x) = 2tan\left(x - \frac{\pi}{4}\right)$

b) $f(x) = -\frac{1}{2}\cot\left(x + \frac{\pi}{2}\right)$

c) $f(x) = \tan(2x + \pi)$

7. Find the period, and phase shift of the given function and sketch graph through one periods.

a) $f(x) = 2sec\left(x - \frac{\pi}{2}\right)$

b) $f(x) = \csc(2x + \pi)$

8. Solve for y and express y in radian measure. Do not use a calculator

a) $y = sin^{-1}\frac{\sqrt{2}}{2}$ b) $y = cos^{-1}\frac{\sqrt{3}}{2}$ c) $y = \arctan(-1)$

9. Solve for y and express y in degree measure. Do not use a calculator

a) $y = tan^{-1}\frac{\sqrt{3}}{3}$ b)$y = sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ c) $y = cos^{-1}\left(\frac{\sqrt{3}}{3}\right)$

10. Evaluate each expression without using a calculator.

a) $sin\left(cos^{-1}\left(-\frac{1}{2}\right)\right)$ b) $tan\left(sin^{-1}\frac{\sqrt{2}}{2}\right)$ c) $sin(tan^{-1}\sqrt{3})$

11. Sketch $y = sin^{-1}x$ by reflecting $y = sinx, where\ {}^{-\pi}/_{2} \leq x \leq {}^{\pi}/_{2}\ a\ cross\ the\ line\ y = x$

12. Use the definition to prove that

a) $sec\theta = \frac{1}{cos\theta}$ b) $cot\theta = \frac{1}{tan\theta}$ c) $\frac{cos\theta}{sin\theta} = cot\theta$

13. Simplify the given trigonometric expression to a single trigonometric function or a constant.

a) $\frac{sec\theta - cos\theta}{tan\theta}$ b) $\frac{tanxsinx}{sec^2x - 1}$ c) $cos\theta + tan\theta sin\theta$

d)$(1 - sin^2x)sec^2$

14. Verify each of the following identities

a) $\frac{cosx + tanx}{sinxcosx} = cscx + sec^2x$ b) $\frac{sinx}{1+cosx} = \frac{1-cosx}{sinx}$

c) $\frac{1}{1-cosx} + \frac{1}{1+cosx} = 2csc^2x$ d)$\frac{cscx-1}{cotx} = \frac{cotx}{cscx+1}$

15. Verify each of the following identities

a) $\tan^2 x + 1 = \sec^2 x$ b) $\sec x - \cos x = \sin x \tan x$

c) $(\sin\theta+\cos\theta)^2 = 1+\sin 2\theta$ d) $\frac{1}{1-\sin x}-\frac{1}{1+\sin x} = 2\tan x\sec x$

16. State the period, the amplitude and the phase shift of each function below and sketch the curve.

(i) $y=-2\cos\frac{1}{2}x$

(ii) $y=1+\frac{1}{3}\sin 2\left(x+\frac{\pi}{2}\right)$

(iii) $f(x)=3\sin\pi\left(x-\frac{1}{4}\right)-1$

17. Solve each of the following equations for θ, if $\theta^\circ \le x \le 360^\circ$. Do not use a calculator.

a) $2sin\theta+\sqrt{2}=0$ b) $3cos\theta+1=cos\theta+3$ c) $3tan\theta+3\sqrt{3}=0$ d) $tan^2\theta=3$

18. Solve each of the following equations for θ, if $0\le x\le 2\pi$. Do not use a calculator.

a) $tanxsinx=0$ b) $(2sinx+1)(tanx-1)=0$ c) $-2cosx=\sqrt{2}$ d) $sin^2x-1=0$

19. Solve each of the following equations. If the variable is θ, Find all solutions such that $0^0\le\theta\le 360^\circ$. If the variable is x, find all solutions such that $0\le x\le 2\pi$. Do not use a calculator.

a) $2sin^2x=sinx$ b) $sinxtan^2x=sinx$ c) $2sin^2x-cosx-1=0$

d) $sin\theta cos\theta-cos\theta+sin\theta-1=0$ e) $tanx+1=secx$ f) $2tan\theta sec\theta-tan\theta=0$

g) $2cos^2x+3cosx+1=0$

20. Find the general solutions for the following equations

a) $2cos\theta=\sqrt{3}$ b) $cot^2x-cotx=0$ c) $csc^2x-cscx-2=0$ d) $tanx+1=0$

e) $2sinx+\sqrt{3}=0$ f) $sec^2x=secx$

21. Sketch the following; State the domain and the range if the following inverses exist at certain interval

a) $y=sin^{-1}x$ b) $y=cos^{-1}x$ c) $y=tan^{-1}x$ d) $y=cot^{-1}x$

e) $y=csc^{-1}x$ f) $y=sec^{-1}x$

21. Complete the following table and show how you got the results.

θ	θ IN RADIANS	SIN θ	COS θ	tan θ	csc θ	sec θ	cot θ
0°							
30°							
45°							
60°							
90°							
180°							
270°							
360°							

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MA110: MATHEMATICAL METHODS

Tutorial sheet 13

1. Find the exact values without using a calculator.

   a) $sin105^{\circ}$ b) $cos\frac{5\pi}{12}$ c) $tan75^{\circ}$ d) $sin\frac{11\pi}{12}$

2. a) Given that $cos\alpha = \frac{3}{5}$, with α in the first quadrant , and $sin\beta = \frac{15}{17}$ with β in the second quadrant , find $\sin(\alpha - \beta)$ and $\tan(\alpha + \beta)$

   b)If α and β are acute angles such that $\cos\alpha = \frac{4}{5}$ and $tan\beta = \frac{8}{15}$, find $\cos(\alpha + \beta)$ and $\sin(\alpha + \beta)$

   c)Given that $\tan\alpha = -\frac{2}{3}$ with α in the second quadrant, and $tan\beta = \frac{3}{5}$ with β in the third quadrant , find $\tan(\alpha + \beta)$ and $\tan(\alpha - \beta)$

   d) Given that $tan\alpha = \frac{8}{15}$ with α in the first quadrant , and $cos\beta = \frac{7}{25}$ with β in the fourth quadrant , find $\sin(\alpha - \beta)$ and $\cos(\alpha - \beta)$

   e) Given that $\tan x = -\frac{3}{4}$ and that x is obtuse, find the exact value of $\sec x$ and $\cos x$

   (ii) Given that $p\sin x = 4$ and $p\cos x = 4\sqrt{3}$ $p > 0$, find p and the smallest positive value of x.

   (iii) If $\sin A = \frac{\sqrt{3}}{2}$ and $\cos B = \frac{\sqrt{3}}{2}$ where A is obtuse and B is acute, find the exact value of $\sin(A+B)$, $\cos(A-B)$ and $\cot(A+B)$

3. Find exact values without using a calculator .

   a) $sin\left(tan^{-1}\frac{3}{4} + cos^{-1}\frac{24}{25}\right)$ b)tan $\left(arcsin\frac{15}{17} + arccos\frac{4}{5}\right)$ c)$sin\left(tan^{-1}\frac{1}{2} - cos^{-1}\frac{4}{5}\right)$

4. Verify each of the following identities

a) $\sin(\alpha + 90^\circ) = cos\alpha$ b) $\cos(\alpha + 90^\circ) = -sin\alpha$ c) $\sin(\alpha + \pi) = -sin\alpha$

d) $\cos(\alpha - \pi) = -cos\alpha$ e) $tan(\alpha + \pi) = tan\alpha$ f) $tan(\alpha - \pi) = tan\alpha$

g) $tan\left(\alpha + \frac{\pi}{4}\right) = \frac{1+tan\alpha}{1-tan\alpha}$

5. a) Derive the formula $\cot(\alpha + \beta) = \frac{cot\alpha cot\beta - 1}{cot\alpha + cot\beta}$

b) Derive the formula $\cot(\alpha - \beta) = \frac{cot\alpha cot\beta + 1}{cot\beta - cot\alpha}$

6. Use the identities

$$\cos(\alpha + \beta) = cos\alpha cos\beta - sin\alpha sin\beta$$

$$\cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta$$

To produce the following product identities.

a) $cos\alpha cos\beta = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)]$

b) $sin\alpha sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

7. Use the identities

$$\sin(\alpha + \beta) = sin\alpha cos\beta + cos\alpha sin\beta$$

$$\sin(\alpha - \beta) = sin\alpha cos\beta - cos\alpha sin\beta$$

To produce the following product identities.

a) $sin\alpha cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

b) $cos\alpha sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

8. Use the identities established in problems 6 and 7 to express each of the following products as sum or difference.

a) $cos3\theta cos2\theta$ b) $2sin9\theta cos3\theta$ c) $sin3\theta sin\theta$

9. Use the identities established in problems 6 and 7 ,verify the following sum and difference identities

a) $cosA + cosB = 2cos\frac{A-B}{2}cos\frac{A-B}{2}$ b) $cosB - cosA = 2sin\frac{A+B}{2}sin\frac{A-B}{2}$

c) $sinA + sinB = 2sin\frac{A+B}{2}cos\frac{A-B}{2}$ c) $sinA - sinB = 2cos\frac{A+B}{2}sin\frac{A-B}{2}$

10. Use the identities established in problems 9 , write each of the following as a product

a) $cos4\theta + cos2\theta$ b) $sin3\theta - sin\theta$ c) $sin6\theta + sin2\theta$ d) $cos\theta - cos5\theta$

11. Prove the following identities

a) $\cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta$

b) $\cos(\alpha + \beta) = cos\alpha cos\beta - sin\alpha sin\beta$

c) $\sin(\alpha + \beta) = sin\alpha cos\beta + cos\alpha sin\beta$

d) $\sin(\alpha - \beta) = sin\alpha cos\beta - cos\alpha sin\beta$

e) $\tan(\alpha + \beta) = \frac{tan\alpha + tan\beta}{1 - tan\alpha tan\beta}$

f) $\tan(\alpha - \beta) = \frac{tan\alpha - tan\beta}{1 + tan\alpha tan\beta}$

12. Find the exact values for $sin2\theta, cos2\theta$, and $tan2\theta$. Do not use a calculator.

a) $cos\theta = \frac{4}{5}$ and θ is in a first quadrant angle.

b) $cot\theta = \frac{12}{5}$ and θ is in a first quadrant angle.

c) $sin\theta = -\frac{4}{5}$ and θ is in a third quadrant angle

d) $tan\theta = -\frac{3}{2}$ and θ is in a second quadrant angle

e) $cos\theta = \frac{15}{17}$ and θ is a fourth quadrant angle

13. Use the half-angle formulas to find exact values . Do not use a calculator.

a) $sin67.5^\circ$ b) $tan157.5^\circ$ c) $cos\frac{7\pi}{12}$ d) $sin\frac{5\pi}{8}$

14. Find the exact values for $sin\left(\frac{\theta}{2}\right)$, $cos\left(\frac{\theta}{2}\right)$ and $tan\left(\frac{\theta}{2}\right)$ Do not use a calculator

a) $sin\theta = \frac{3}{5}$ and $0^\circ < \theta < 90^\circ$ b) $cos\theta = -\frac{3}{5}$ and $180^\circ < \theta < 270^\circ$

c) $tan\theta = -\frac{12}{5}$ and $90^\circ < \theta < 180^\circ$ d) $sec\theta = -\frac{4}{3}$ and $180^\circ < \theta < 270^\circ$

15. Solve each equation for θ, where $0^\circ \le \theta < 360^\circ$. Do not use a calculator

a) $sin\frac{\theta}{2} + cos\theta = 1$ b) $sin2\theta sin\theta + cos\theta = 0$ c) $cos2\theta + 3sin\theta - 2 = 0$

d) $\tan 2\theta = \tan\theta$ e) $2 - \cos^2\theta = 4\sin^2\frac{\theta}{2}$ f) $\cos 4\theta = \cos 2\theta$

16. Solve each equation for x, where $0 \le x < 2\pi$. Do not use a calculator.

a) $\tan 2x + \sec 2x = 1$ b) $\sin 2x + \sqrt{2}\cos x = 0$ c) $2 - \sin^2 x = 2\cos^2\frac{x}{2}$

17. Verify each identity.

a) $(\sin\theta + \cos\theta)^2 - \sin 2\theta = 1$ b) $\frac{1-\tan^2 x}{1+\tan^2 x} = \cos 2\theta$ c) $2\cos^2\frac{x}{2}\tan x = \tan x + \sin x$

d) $\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}$ e) $\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} = \sec\theta$

18. Verify each identity.

a) $\sin 2\alpha = 2\sin\alpha\cos\alpha$ b) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$

c) $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$ d) $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ e) $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

f) $\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$

19. Express the following as a single *sine, cosine or tangent*:

a) $\sin 15^\circ \cos 20^\circ + \cos 15^\circ \sin 20^\circ$ b) $\frac{\tan 76^\circ - \tan 45^\circ}{1+\tan 76^\circ \tan 45^\circ}$

c) $\cos\left(\frac{3x+2y}{2}\right)\cos\left(\frac{3x-2y}{2}\right) - \sin\left(\frac{3x+2y}{2}\right)\sin\left(\frac{3x-2y}{2}\right)$

d) Use sum and difference formulas to simplify $\sin\left(x+\frac{\pi}{4}\right)+\sin\left(x-\frac{\pi}{4}\right)$. Hence find all solutions of $\sin\left(x+\frac{\pi}{4}\right)+\sin\left(x-\frac{\pi}{4}\right) = -1$ in the interval $[\,0\,,\,2\pi)$.

20 .Show that :

a) $\cos\theta + \sin\theta = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$ b) $\sqrt{3}\sin 2\theta - \cos 2\theta \equiv 2\sin\left(2\theta - \frac{\pi}{6}\right)$

b) $\cos 2\theta - \sqrt{3}\sin 2\theta \equiv 2\cos\left(2\theta + \frac{\pi}{3}\right) \equiv -2\sin\left(2\theta - \frac{\pi}{6}\right)$

21. Find the value of R, where $R > 0$, and the value of $\propto$, where $0 < \alpha < 90^0$, in each of the following cases:

a) $sin\theta + 3cos\theta \equiv Rsin(\theta + \alpha)$ b) $3sin\theta - 4cos\theta \equiv Rsin(\theta - \alpha)$

c)) $2cos\theta + 7sin\theta \equiv Rcos(\theta - \alpha)$ d) $cos2\theta + 2sin\theta \equiv Rcos(2\theta + \alpha)$

22. a) Show that $cos\theta - \sqrt{3}sin\theta$ can be written in the form $Rcos(\theta + \alpha)$, with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

b) Hence sketch the graph of $cos\theta - \sqrt{3}sin\theta$, $0 < \alpha < 2\pi$, giving the coordinates of points of intersection with the axes

THE TRIGONOMETRIC FORM OF COMPLEX NUMBER

23. Express each complex number in trigonometric form, where $0 \leq \theta \leq 2\pi$

a) $-2 + 2i$ b) $-4 - 4i$ c) $-3i$ d) -4

24. Express each complex number in trigonometric form, where $0 \leq \theta < 360$

a) $-\sqrt{3} - i$ b) $-4 - 4\sqrt{3}i$ c) -2

25. Change the given complex number from trigonometric form to $a + bi$ form

a) $4(co30^0 + isin30^0)$ b) $5(cos120^0 + isin120^0)$ c) $3\left(cos\frac{5\pi}{4} + isin\frac{5\pi}{4}\right)$

d) $1\left(cos\frac{11\pi}{6} + isin\frac{11\pi}{6}\right)$

26. Find the product z_1z_2 by using the trigonometric form of the numbers. Express the final results in $a + bi$ form . Check by using directly method

a) $z_1 = \sqrt{3} + i$ $z_2 = -2\sqrt{3} - 2i$ b) $z_1 = 1 + \sqrt{3}i$ $z_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

c) $z_1 = 8 + 0i$ $z_2 = 0 - 3i$ d) $z_1 = 5\sqrt{3} + 5i$ $z_2 = 6\sqrt{3} + 6i$

27. Use De Moivre's theorem to find the indicated powers. Express results in $a + bi$

a) $(1 + i)^{20}$ b) $\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^{15}$

c) $[2(cos50^0 + isin50^0)]^6$ d) $\left(cos\frac{\pi}{8} + isin\frac{\pi}{8}\right)^{10}$

28. Find the indicated roots. Express the roots in $a + bi$ form if they are exact. Otherwise, leave them in trigonometric form

a) The three cube roots of 8 b) The four fourth roots of $-8 + 8\sqrt{3}i$

c) The five fifth roots of $1 - i$ d) The two square roots of $\frac{9}{2} + \frac{9\sqrt{3}i}{2}$

29. (a) Use DeMoivre's theorem to simplify the following expressions

(i) $(\cos 2\theta + i\sin 2\theta)(\cos 5\theta + i\sin 5\theta)$

(ii) $\dfrac{\cos 7\theta + i\sin 7\theta}{\cos 2\theta - i\sin 2\theta}$

(iii) $\dfrac{\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^5 \left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)^4}{\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^2}$

30. (a) Find the cube roots of $2 + 2\,i$

(b) (i) Express $\dfrac{2\sqrt{3} + 2\,i}{\sqrt{3} - i}$ in the form $r(\cos\theta + i\sin\theta)$

(ii) Hence find fourth roots of $\dfrac{2\sqrt{3} + 2\,i}{\sqrt{3} - i}$.

HYPERBOLIC FUNCTIONS

31. i) Prove the following identities

a) $\coth^2 x - 1 = \operatorname{cosech}^2 x$

b) $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

c) $\cosh 3x = 4\cosh^3 x - 3\cosh x$

d) $\tanh^2 x = \dfrac{\cosh 2x - 1}{\cosh 2x + 1}$.

ii) Solve for real values of *x*, each equation below:

a) $\sinh x + 4 = 4\cosh x$

b) $4\tanh^2 x - \operatorname{sech} x = 1$

c) $\cosh 2x - 7\cosh x + 7 = 0$

d) $\sinh^2 x - 5\cosh x + 5 = 0$

e) $\sinh^2 x - 3\cosh x = 3$

POLAR COORDINATES

32. Plot the indicated ordered pairs as points in a polar coordinate system. Then find all the polar

coordinates of each Point

a) $\left(3,\frac{\pi}{4}\right)$ b) $\left(4,\frac{\pi}{3}\right)$ c) $\left(-2,\frac{2\pi}{3}\right)$ d) $\left(-3,\frac{5\pi}{6}\right)$

33. Plot the following points (given in polar coordinates) . Then find all the polar coordinates of each Point

a) $(2,{}^{\pi}/_{2})$ b) $(2,0)$ c) $(-2,{}^{\pi}/_{2})$ d) $(-2,0)$

34. Sketch the graph of each of the polar equation

a) $r = 1 + 2sin\theta$ b) $r = 2 - 2cos\theta$ c) $r^2 = cos\theta$ d) $r = cos({}^{\theta}/_{2})$

e) $r^2 = 4cos2\theta$ f) $r^2 = -sin2\theta$

35. Change each equation to polar form

a) $5x + 4y = 10$ b) $x^2 + y^2 + x = \sqrt{x^2 + y^2}$ c) $x^2 + y^2 + 6y = 0$ d) $x^2 = 4y$

36 . Change each polar equation to rectangular form

a) $r = 2sin\theta$ b) $r = 2cos\theta + 3sin\theta$ c) $r = \frac{4}{2+cos\theta}$ d) $r = \frac{5}{2-3sin\theta}$