

# Circular Motion Problem Solving

Acceleration or a change in velocity is caused by a net force: Newton's 2<sup>nd</sup> Law  $\vec{a} = \sum \vec{F} / m$   
 An object accelerates when either the *magnitude* or the *direction* of the velocity changes

We saw in the last unit that an object speeds up or slows down when a net force acts in the same or opposite the direction of its motion (tangential to the motion)

$$\sum F_x = F_{netx} = ma_x \quad \text{and} \quad \sum F_y = F_{nety} = ma_y$$

In this unit, we examine the acceleration due to a change in *direction* of the velocity. An object changes direction or moves in a curve when a net force acts perpendicular to the direction of motion, towards the center of the curve. A net force perpendicular to the direction of motion is called a centripetal (center-seeking) force,  $F_c$ , and the acceleration it gives rise to is a centripetal acceleration,  $a_c$ . Centripetal acceleration is *a change in the velocity direction only* and it can be shown to be equal to  $v^2/r$  where  $v$  is the speed and  $r$  is the radius of the curve. The centripetal force is found by adding all the forces in the radial direction (perpendicular to the curve)

$$\sum F_r = F_c = ma_c = m(v^2 / r) = m\omega^2 r$$

Note that "Centripetal force" is just a fancy name for the radial component of the net force. It is not a new kind of force and is NOT drawn on force diagrams.

A net force could have both tangential and radial components; the component tangential to the direction of motion causes the object to speed up or slow down and the radial component causes the object to move in a curve.

In this unit, we focus on the acceleration associated with a change in direction only. In other words, an object that moves with constant speed but continuously changes direction – UNIFORM CIRCULAR MOTION. An object in uniform circular motion has constant speed,  $v$ , centripetal acceleration,  $a_c$ , and undergoes periodic motion with period  $T$  and frequency,  $f$ .

$$\left. \begin{array}{l} \text{tangential or linear speed: } v = d/\Delta t = 2\pi r/T = 2\pi r f \\ \text{angular speed: } \omega = \Delta\theta/\Delta t = 2\pi/T = 2\pi f \\ \text{centripetal acceleration: } a_c = v^2/r = \omega^2 r \end{array} \right\} v = \omega r$$

Problems:

1. The angular speed of a record player is 33 rotations per minute. It has a diameter of 12 inches (0.305m).

a) What is the period and frequency of rotation of the record?

$$f = 33 \text{ rot} / \text{min} = 0.55 / \text{sec} = 0.55 \text{ Hz}$$

$$T = 1 / f = 1.82 \text{ sec}$$

b) What is the angular speed in rad/s

$$\omega = 2\pi f = 3.45 \text{ rad} / \text{sec}$$

c) What is the tangential speed of the outer most part of the record?

$$v = \omega r = \frac{2\pi r}{T} = 3.45(0.155) = 0.53 \text{ m} / \text{s}$$

d) What is the tangential speed halfway out to the edge of the record? **tangential speed is directly proportional to  $r$  so the speed would be half what it is on the outer part:  $v = 0.265 \text{ m/s}$**



2. A 900 kg race car makes one lap around a track of radius 50. m in 11.0 s

a) What is the car's average speed?

$$v_{av} = \frac{2\pi r}{T} = \frac{2\pi(50)}{11} = 28.6 \text{ m/s}$$

b) What is the car's angular speed?

$$\omega = \frac{2\pi}{T} = \frac{v}{r} = \frac{28.6}{50} = 0.57 \text{ rad/s}$$

c) What is the car's centripetal acceleration?

$$a_c = \frac{v^2}{r} = \frac{28.6^2}{50} = 16.3 \text{ m/s}^2$$

d) What is the centripetal force acting on the car and which force provides it? **Static friction between the tires and the road provide the 14,670N centripetal force**

$$F_C = ma_c = (900)(16.3) = 14,670 \text{ N}$$

3. a) A car goes around a curve at 20.0 m/s. If the radius of the curve is 50.0 m, what is the centripetal acceleration of the car?

$$a_c = \frac{v^2}{r} = \frac{20^2}{50} = 8 \text{ m/s}^2$$

b) If the speed of the car were halved, how much different would  $a_c$  be to travel the same radius curve?

**Because the centripetal acceleration depends on the square of the speed, if the speed were halved and the radius stayed the same,  $a_c$  would decrease by a factor of 4**

4. A speed skater goes around a turn with a 31 m radius. The skater has a speed of 14m/s and experiences a centripetal force of 460N. What is the mass of the skater?

$$F_C = ma_c = m \frac{v^2}{r}$$

$$460 = \frac{m(14^2)}{31}$$

$$m = 72.8 \text{ kg}$$

5. Norman Neutron swings a rubber ball attached to a string over his head in a horizontal, circular path. The piece of string is 1.5 m long and the ball makes 120 complete turns each minute.

a) What is the average speed of the ball?

$$T(s) = \frac{1 \text{ min}}{120 \text{ cycles}} = \frac{60 \text{ s}}{120 \text{ cycles}} = 0.5 \text{ s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(1.5)}{0.5} = 18.8 \text{ m/s}$$

b) What is the ball's centripetal acceleration?

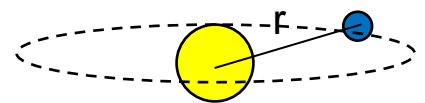
$$a_c = \frac{v^2}{r} = \frac{18.8^2}{1.5} = 236 \text{ m/s}^2$$

6. Calculate the centripetal acceleration of the Earth towards the Sun. ( $r = 1.5 \times 10^{11} \text{ m}$ )

**don't know the speed of the Earth, but we know the period (1 year)**

$$T(s) = 1 \text{ yr} \times \frac{365 \text{ days}}{1 \text{ yr}} \times \frac{24 \text{ hr}}{1 \text{ yr}} \times \frac{3600 \text{ s}}{1 \text{ hr}} = \frac{60 \text{ s}}{120 \text{ cycles}} = 3.154 \times 10^7 \text{ s}$$

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 (1.5 \times 10^{11})}{(3.154 \times 10^7)^2} = 0.0060 \text{ m/s}^2$$

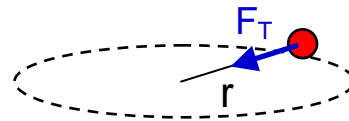


7. A 0.50 kg mass is attached to a string 1.0 m long and moves in a horizontal circle at a rate of 2.0 Hz.

a) Calculate the centripetal acceleration of the mass.

$$f = 2\text{Hz. } T = 1/f = 0.5\text{s}$$

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 (1)}{0.5^2} = 158\text{m/s}^2$$



b) Calculate the tension in the string. **The tension in the string is what provides the centripetal force.**

$$F_c = \sum F_r = ma_c$$

$$F_T = (0.5)(158) = 79\text{N}$$

8. A penny, placed on a spinning record, rotates at  $33\frac{1}{3}$  revolutions per minute when it is placed 10 cm from the center of the record (mass of penny = 2.5 grams).

a) What is the tangential and angular speed of the penny as it moves around the center of the record?

$$r = 10\text{cm} = 0.10\text{m}$$

$$f = 33.3\text{rev/min} = 0.555\text{rev/s} \quad v = \frac{2\pi r}{T} = \frac{2\pi(0.10)}{1.8} = 0.35\text{m/s}$$

$$T = 1/f = 1.80\text{sec}$$

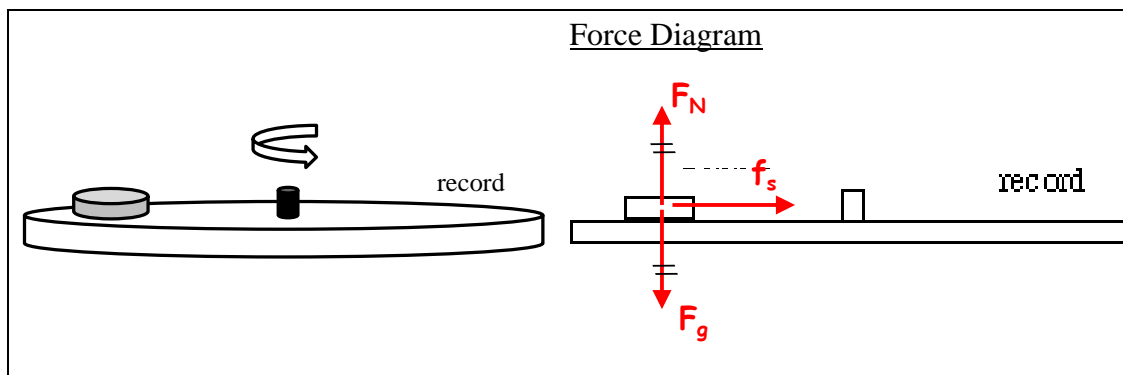
$$\omega = \frac{2\pi}{T} = \frac{v}{r} = 3.5\text{rad/s}$$

b) What is the magnitude of the centripetal force necessary to keep the penny moving in a circular path?

$$F_c = ma_c = m \frac{v^2}{r} = 0.0025 \frac{0.35^2}{0.1} = 0.00306\text{N}$$

c) How much frictional force is applied to the penny by the surface of the record? **It is static friction between the penny and turntable that provides all of the centripetal force. Therefore,  $f_s = F_c = 0.00306\text{N}$**

d) Draw a free body diagram showing all the forces acting on the rotating penny



e) What is the minimum coefficient of static friction required to keep the penny from sliding off the record?

Newton Second Law Application

$$F_c = \sum F_r = ma_c = mv^2 / r$$

$$f_s = \frac{mv^2}{r} = 0.00306$$

$$\mu_s F_N = 0.00306$$

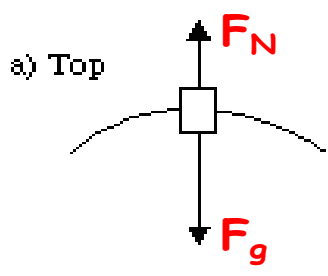
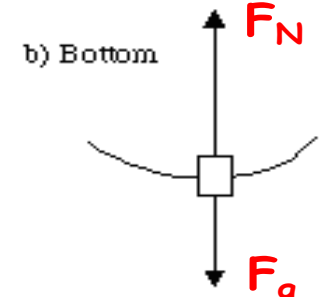
$$\mu_s mg = 0.00306$$

$$\mu_s = \frac{0.00306}{mg} = 0.125$$

9. A Ferris wheel with a 38 m radius and tangential speed of 5.0 m/s has a 76 kg passenger riding it.
- a) What is the magnitude of the centripetal force acting on the Ferris wheel passenger at the top and at the bottom? Same  $m$ ,  $v$  and  $r$  at top and bottom, so  $F_c$  is the same magnitude at the top and bottom.

$$F_c = ma_c = m \frac{v^2}{r} = 76 \frac{5^2}{38} = 50 \text{ N}$$

- b) Complete the freebody diagrams showing all the forces acting on the passenger at the top and at the bottom of the Ferris wheel. Label the forces. Indicate whether the apparent weight of the passenger (the normal force) is greater, equal to, or less than their true weight at the top and at the bottom.

<u>Force Diagram for top</u> Note that at the top, the centripetal force must be directly down, towards the center of the circle. Therefore $F_N < F_g$ to get a net force down (like the elevator accelerating downwards).	<u>Force Diagram for bottom</u> Note that at the bottom, the centripetal force must be directly up, towards the center of the circle. Therefore $F_N > F_g$ to get a net force up (like the elevator accelerating upwards).
<p>a) Top</p>  <p>Apparent wt <u>&lt;</u> True wt</p>	<p>b) Bottom</p>  <p>Apparent wt <u>&gt;</u> True wt</p>

- c) How big is the normal force exerted on the passenger the top and at the bottom?

**Note:** The positive radial direction is taken as the direction towards the center of the circle because forces pointed into the circle ADD to centripetal force. The **negative radial direction** is away from the center of the circle.

#### NSL Application

$\sum F_r = F_c = ma_c = (mv^2)/r$  Start with Newton's 2<sup>nd</sup> Law for the radial component of the net force. Use your FBD above to guide you in summing up the radial forces to find the centripetal force and the normal force.

#### At the top

$$F_c = \sum F_r = ma_c$$

$$F_c = F_g - F_N = \frac{mv^2}{r}$$

$$F_N = mg - \frac{mv^2}{r}$$

$$= 744.8 - 50 = 695 \text{ N}$$

Centripetal force is supplied by the combination of gravity pointing into the center of the circle and the normal force pointing out of the circle ( $F_g - F_N$ ). It is **50 N** ( $mv^2/r$ )

The normal force is what a scale would read of your weight. Here the normal force is your true weight **minus** a term due to the centripetal acceleration. Your apparent weight is lighter due to the acceleration, just as in the elevator

At the bottom

$$F_c = \sum F_r = ma_c$$

$$F_c = F_N - F_g = \frac{mv^2}{r}$$

$$F_N = mg + \frac{mv^2}{r}$$

$$= 744.8 + 50 = 795N$$

Centripetal force is supplied by the combination of gravity pointing out of the circle and the normal force pointing into the circle ( $F_N - F_g$ ). It is **50 N** ( $mv^2/r$ )

The normal force is what a scale would read of your weight. Here the normal force is your true weight **plus** a term due to the centripetal acceleration. Your **apparent weight** is *heavier* due to the acceleration, just as in the elevator

- d) How fast would the Ferris wheel have to go for the passenger to feel double their true weight at the bottom of the Ferris wheel?

What the passenger feels is the normal force pushing on them. If the normal force (apparent weight) was double the true weight (2G) at the bottom, the net force pointing into the center of the circle (the centripetal force), would be greater due to the Ferris wheel moving faster (since the radius and mass of passenger cannot change)

$$F_c = \sum F_r = ma_c$$

$$F_c = F_N - F_g = \frac{mv^2}{r}$$

$$F_c = 2F_g - F_g = \frac{mv^2}{r}$$

$$F_c = F_g = mg = \frac{mv^2}{r}$$

$$v = \sqrt{gr} = 14m/s$$

- e) At the speed found in part d) how heavy would the passenger feel at the top of the Ferris wheel (this is the normal force)?

At the speed found above, the centripetal force at the bottom (the net radial force) is  $F_g$

Since  $F_c = F_g$  is the same at every point on the Ferris wheel (what changes is the contribution of  $F_N$  and  $F_g$  to the centripetal force), at the top where  $F_g$  is pointing into the circle, the **normal force must be zero and the passenger feels weightless**. You can also find this by calculating

At the top

$$F_c = \sum F_r = ma_c = m \frac{v^2}{r} = \frac{(70)(14^2)}{20} = 686N$$

$$686 = F_g - F_N = 686 - F_N$$

$$F_N = 0$$

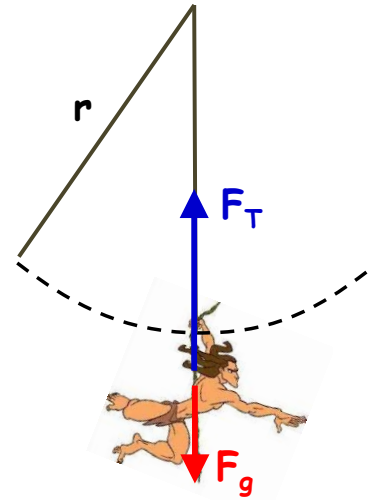
10. Tarzan ( $m = 85 \text{ kg}$ ) tries to cross a river by swinging from a 10-m-long vine. His speed at the bottom of the swing (as he just clears the water) is  $8.0 \text{ m/s}$ . Tarzan doesn't know that the vine has a breaking strength of  $1000 \text{ N}$ . Does he make it safely across the river? Justify your answer

$$F_c = \sum F_r = ma_c$$

$$F_c = F_T - F_g = \frac{mv^2}{r}$$

$$F_T = mg + \frac{mv^2}{r}$$

$$F_T = (85)(9.8) + \frac{(85)8^2}{10}$$
$$= 1377 \text{ N}$$



No, Tarzan doesn't make it since the tension required to provide the necessary centripetal force exceeds  $1000 \text{ N}$