Minimize Cost

There are various objective functions but which to choose is a controversy.

Landing time cost is non-linear

Linearise it by splitting before and after Target time

Integer programming formulation

Notation:

P -> Number of planes

E_i -> earliest landing time for plane i

L_i -> the latest landing time for plane i

T_i -> Target landing time for plane i.

S_{ij} -> the required separation time between i and j

g_i -> penality cost for landing before for i

g_i → " " " after for i

$[E_i, L_i]$ → Time window to land

$$E_i \leq T_i \leq L_i$$

Variables:

x_i → the landing time for plane i

α_i → how soon plane i lands before T_i

β_i → " " " " " after T_i

δ_{ij} $\begin{cases} 1 & \text{if } i \text{ lands before } j \\ 0 & \text{else} \end{cases}$

E_i, L_i, δ_{ij} are integers —

Objective function

$$\text{Min: } \sum_{i=1}^{P} (g_i \alpha_i + h_i \beta_i)$$

Constraints:

(1) $E_i \leq x_i \leq L_i \quad i = 1 \ldots P$

(2) $\delta_{ij} + \delta_{ji} = 1 \quad i = 1 \ldots P \quad j = 1 \ldots P$

U → set of uncertain planes i lands before j

V → set of definite planes i lands before j
separation not satisfied

W → same as V but separation automatically satisfied

(3) $W = [(i,j) \mid L_i < E_j \text{ and } L_i + S_{ij} \leq E_j]$

$i = 1 \ldots P \quad j = 1 \ldots P \quad i \neq j$

(4) $V = [(i,j) \mid L_i < E_j \text{ and } L_i + S_{ij} > E_j]$

$i = 1 \ldots P \quad j = 1 \ldots P \quad i \neq j$

(5) $U = [(i,j) \mid E_j \leq E_i \leq L_j \text{ or } E_j \leq L_i < L_j$
$\text{or } E_i \leq E_j \leq L_i \text{ or } E_i \leq L_j \leq L_i]$

$i = 1 \ldots P \quad j = 1 \ldots P \quad i \neq j$

(6) $\delta_{ij} = 1 \quad \forall (i,j) \in W \cup V$

(7) $x_j \geq x_i + s_{ij} \quad \forall (i,j) \in V$

(8) $x_j \geq E_j + (x_i - L_i)$

(9) $\alpha_i \geq T_i - x_i \quad i = 1 \ldots P$

(10) $0 \leq \alpha_i \leq T_i - E_i \quad i = 1 \ldots P$

(11) $\beta_i \geq x_i - T_i \quad i = 1 \ldots P$

(12) $0 \leq \beta_i \leq L_i - T_i \quad i = 1 \ldots P$

(13) $x_i = T_i - \alpha_i + \beta_i \quad i = 1 \ldots P$

(14) x_i, alpha i and beta i are integers