Cryptography and Network Security

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(1) Public-Key Cryptography

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PK Cryptography

Using math and computational difficulty

for: confidentiality

& authentication

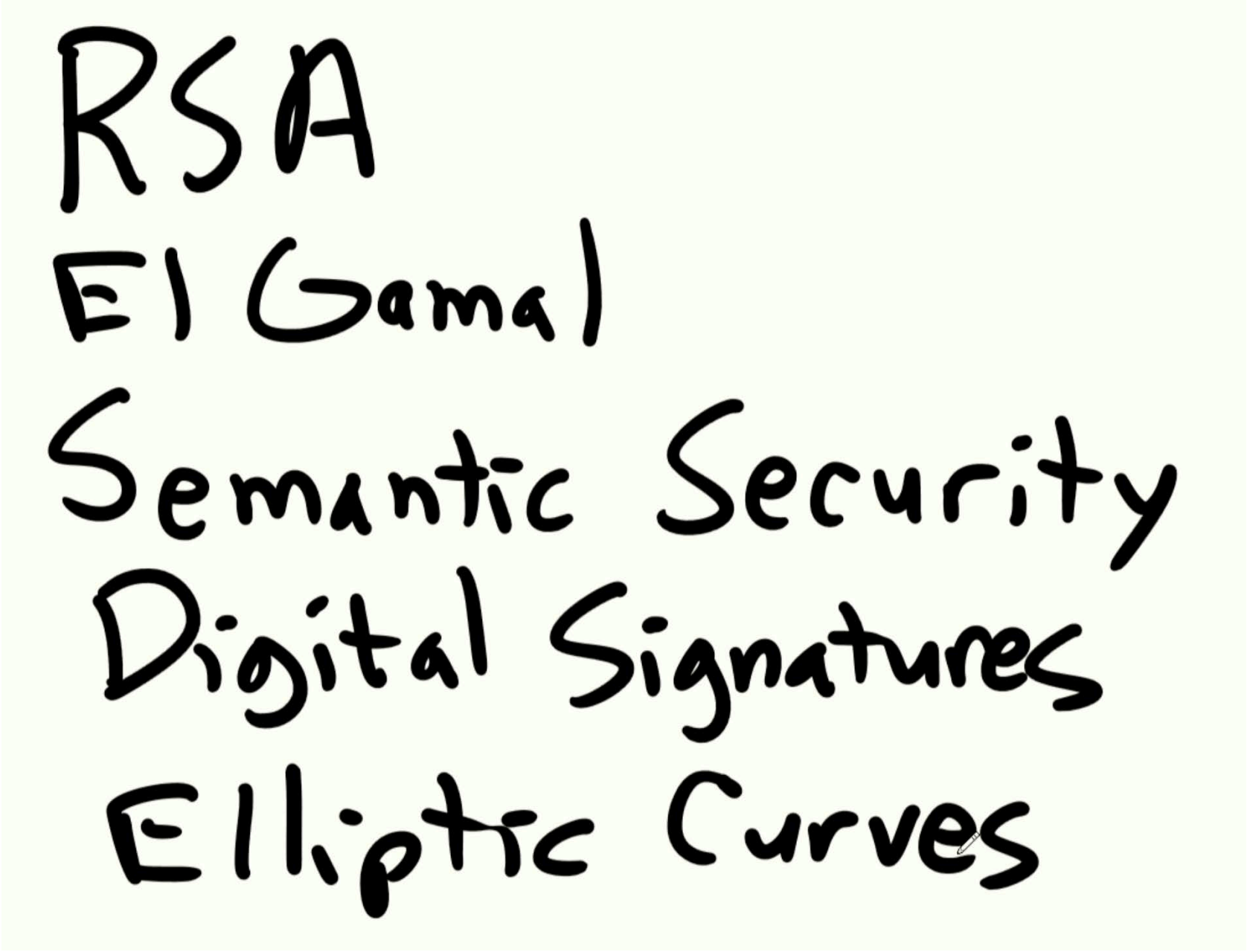
Context

Invention

Number Theory

Computational Complexity

Key Establishment



Homomorphic Properties Attacks & Security Looking Ahead

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(2) PK Cryptography: Context

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"Keyless" Crypto

hash functions: file → "digest"

should be:

collision resistant

no h(file1) = h(file2)

CR

one-way

can't find file from h(file)

OW

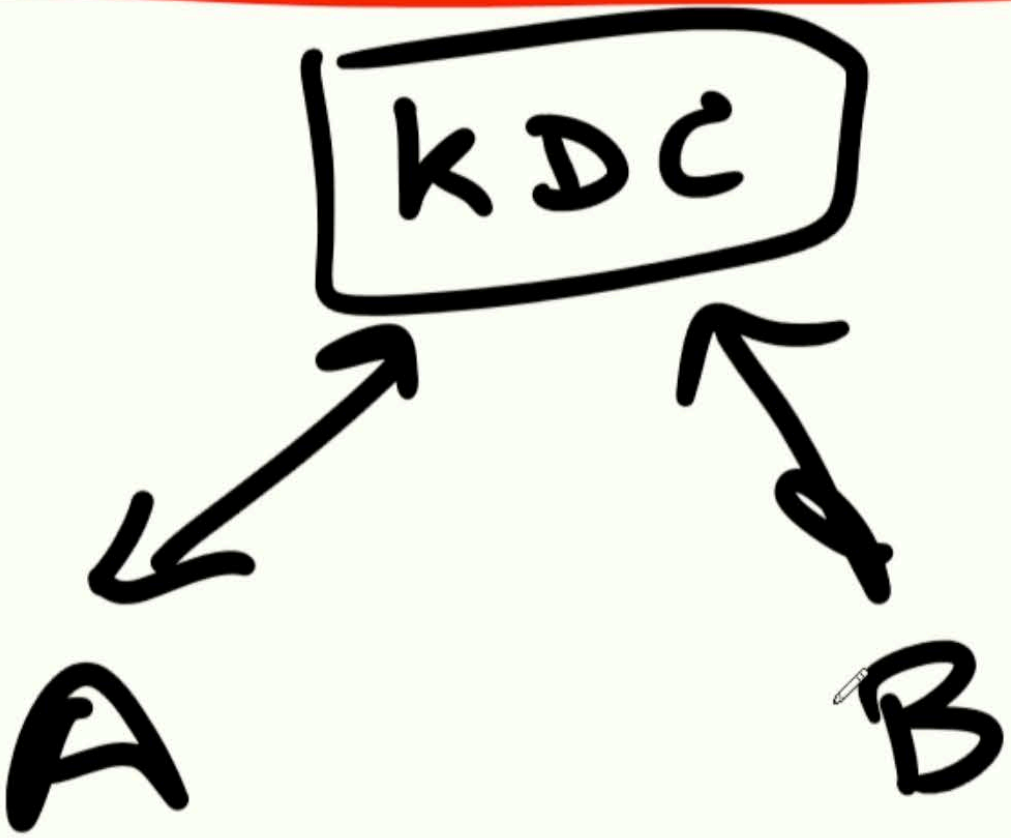
Symmetric crypto

Same key used by Alice & Bob

block ciphers (DES, AES)

message authentication codes (HMAC)

Key Distribution Centers



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(3) PK Cryptography: Invention

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Invention of PK Crypto

Diffie & Hellman "New Directions in Cryptography"

Merkle "Secure Communications over an Insecure Channel"

Different keys used by Alice & Bob

$$PK, Sk \leftarrow \text{keygen}$$

$$C \leftarrow E(PK, M)$$

C: ciphertext; M: plaintext

$$M \leftarrow D(sk, C)$$

Also digital signatures

$sig \leftarrow Sign(SK, M)$

$Verify(PK, M, sig)$

$\rightarrow$ True/False

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(4) PK Cryptography: Number Theory

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Modular Math

$$3^2 + 7 = 5 \pmod{11}$$

Powering Up (Repeated Squaring)

$$3^{101} \pmod{197} = 3 \cdot 3^{100} \pmod{197}$$

$$3^{100} \pmod{197} = (3^{50})^2 \pmod{197}$$

Primes

2, 3, 5, 7, 11, ...

To find a large prime:

Pick a large random integer p

accept p as prime if

$$3^{p-1} = 1 \pmod{p}$$

else restart

Generators

For every prime p there is a "generator" g such that

$$g^0, g^1, \ldots, g^{p-2}$$

give all nonzero mod p

Easy to find p & associated generator g

Example: mod p p=5 g=3

$3^0 = 1$

$3^1 = 3$

$3^2 = 4$

$3^3 = 2$

$3^4 = 1$

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(5) PK Cryptography: Computational Complexity

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Computational Complexity

Most crypto is based on assumptions

Examples:

$P \neq NP$

hash fn is CR or OW

Factoring

Given product $n = p \cdot q$
of two large primes p, q
it is (assumed) hard to find p, q

Stanley Jevons Challenge (1874)

$8616460799 = ?$
$89681 \cdot 96079$

Discrete Logarithm

Given p, g & $y = g^x \bmod p$

it is infeasible to find x

Example:

$14 = 3^x \pmod{31}$ what is x?

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(6) PK Cryptography: Key establishment

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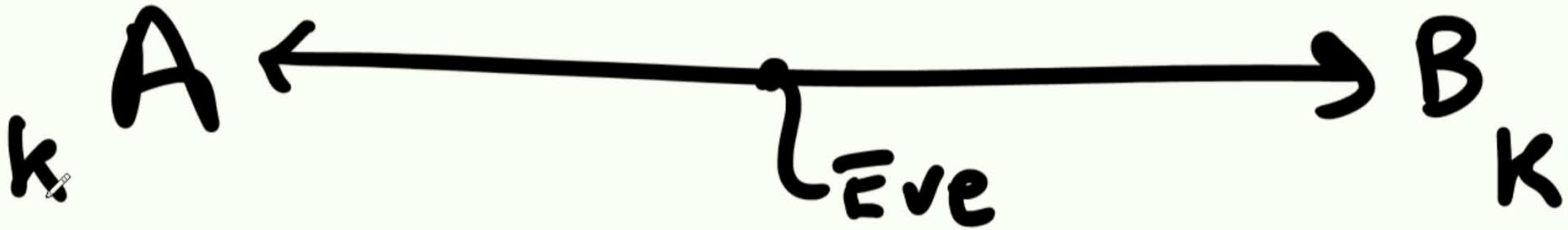
Diffie-Hellman Key Establishment

How can Alice & Bob establish a secure shared secret key over an insecure channel?



Method: prime p, generator g

$A \rightarrow B$: $g^a \bmod p$ (a secret to A)

$B \rightarrow A$: $g^b \bmod p$ (b secret to B)

$K = (g^a)^b = (g^b)^a = g^{ab} \bmod p$

Eve can't compute g^{ab} from g^a & g

Hybrid Encryption (TLS)

setup session key K using DH

use AES to encrypt msgs thereafter

Forward secrecy property:

once A & B erase K, a, b, M

Eve gains nothing by hacking into their laptops

a, b, K are "ephemeral" keys

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(7) PK Cryptography: RSA

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RSA (Rivest-Shamir-Adleman, 1977)

Based on difficulty of factoring

Setup:

$e = 3$ or $2^{16}+1$

p, q large random primes
($p-1, q-1$ not multiples of e)

$PK = (n, e)$ where $n = p \cdot q$

$SK = (n, d)$

where $d = e^{-1} \pmod{(p-1)\cdot(q-1)}$

To encrypt M with $PK(n,e)$:

$$C = M^e \pmod{n}$$

To decrypt C with $SK(n,d)$:

$$M = C^d \pmod{n}$$

Example: $p=5, q=11 \quad n=55$

$e=3, d=27$

Encrypt $m=7$

$$C = 7^3 \pmod{55} = 13$$

Decrypt $c=13$

$$m = 13^{27} \pmod{55} = 7$$

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(8) PK Cryptography: El Gamal

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El Gamal PK Cryptosystem (1985)

Based on discrete log problem

large prime p, generator g

SK x random in $0, 1, \ldots, p-2$

PK $y = g^x \pmod{p}$

To encrypt m:

pick r random in $0, 1, \ldots, p-2$

$$c = (a, b) = (g^r, m \cdot y^r)$$

To decrypt $c = (a, b)$

$$m = b / a^x \pmod{p}$$

Example:

$p = 5, g = 3, x = 2, y = 3^2 = 4$

$m = 2, r = 3$

$c = (a, b) = (3^3, 2 \cdot 4^3)$

$= (2, 3)$

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(9) PK Cryptography: Semantic Security

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Semantic Security

= indistinguishability under chosen plaintext attack IND-CPA

Goldwasser-Micali: 1982

Adv can't distinguish

E(a) from E(b)

even if he knows (or picks) a & b.

E("dog") or E("cat")

?

requires randomized encryption

El Gamal semantically secure

assuming DDH (Decision Diffie Hellman)

given g^x, g^r, and g^s

Adv can't tell if $g^s = g^{xr} \pmod p$

or not

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(10) PK Cryptography: Digital Signatures and Certs

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Digital signatures:

SK used to sign M (or hash of M):

$Sig = Sign(SK, M)$

M, sig → recipient

PK used to verify

$Verify(PK, M, sig) = True/False$

Security

Adv can't forge signatures on new messages, even after seeing many valid signatures on messages of his choice!

Certificates

A certificate is a signed statement that a PK belongs to a named party.

Example: Verisign says (and signs) stmt that "Google's Pk is $n = \ldots, e = \ldots$"

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(11) PK Cryptography: Elliptic Curves

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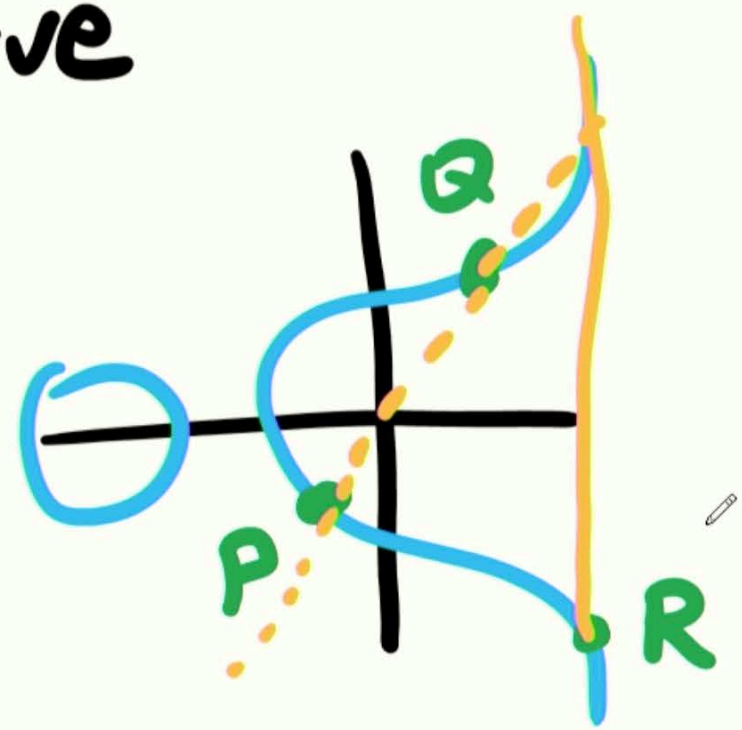
Elliptic Curves

Replaces "mod p" or "mod n" with operations over elliptic curve

$$(x,y): y^2 = x^3 + ax + b$$

$$P + Q = R$$



Advantages:

- more compact
- often faster
- admits new functionalities via "pairing functions"

$$e(g^a, g^b) = e(g,g)^{ab}$$

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(12) PK Cryptography: Homomorphic Properties

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Homomorphic Properties

operation on ciphertext ≡
operation' on plaintext

Example: $E(a+b) = E(a) \cdot E(b)$

Fully Homomorphic Encryption (FHE):
Can do for a "complete" set of operations

Voting Application

Each voter submits $E(v)$

$v=1$ for "for" & $v=0$ for "against"

$$E(v_1) \cdot E(v_2) \cdot \dots \cdot E(v_n)$$

$$= E(v_1 + v_2 + \dots + v_n)$$

decrypt

$$v_1 + v_2 + \dots + v_n$$

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(13) PK Cryptography: Attacks and Security

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What can Adversary do?

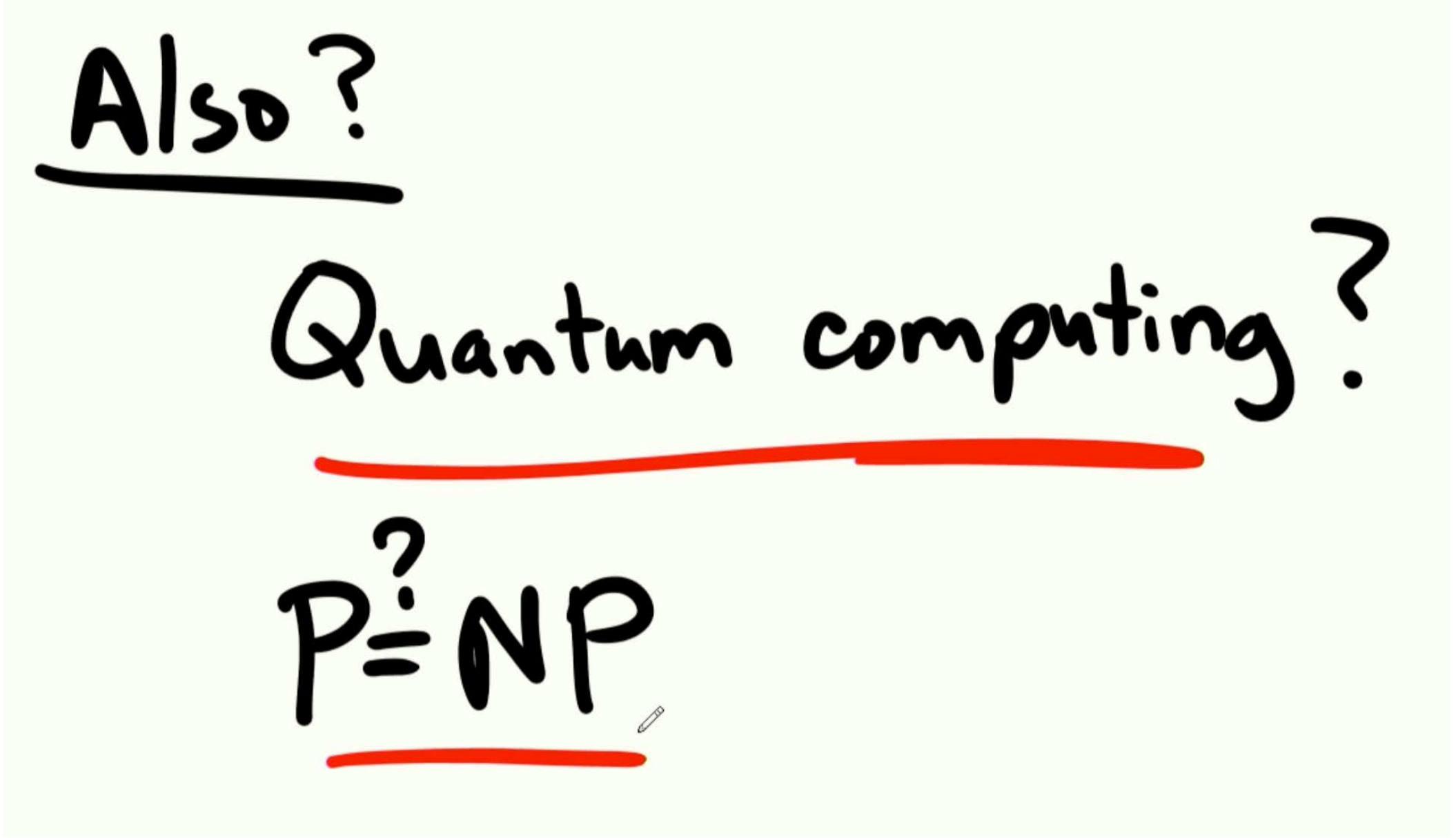
- Steal keys (malware)
- using PK
- get decryptions of chosen ciphertexts (IND-CCA)
- exploit implementation
- bugger implementation
- exploit protocol

Factoring (DL Similar)

768-bit # factored

1024-bit # - maybe in a few years

2048-bit # - many decades, if ever



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(14) PK Cryptography: Looking Ahead

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Shafi Goldwasser:
MPC, Secret Sharing, Distributed Trust

Vinod Vaikuntanathan: Homomorphic & Functional Encryption

Dave Clark: Network Security & Protocol Design

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