Tackling the Challenges of Big Data
Big Data Analytics
Fast Algorithms I

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Tackling the Challenges of Big Data
Big Data Analytics
Fast Algorithms I:
Introduction

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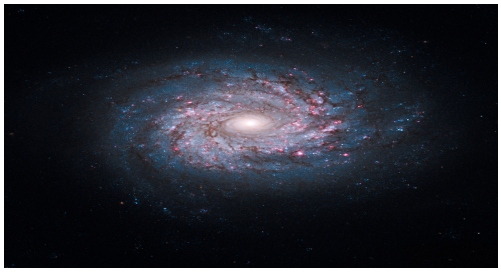
What to do About REALLY Big Data?



Spiral galaxy NGC 3982. Credit: NASA, ESA, and the Hubble Heritage Team (STScI/AURA)
Read more: http://www.universetoday.com/30168/galaxies/#ixzz2lgecrxox
SOURCE: Google

No Time

What can we hope to do without viewing most of the data?

Small World Phenomenon

- The social network is a graph:
  - "node" is a person
  - "edge" between people that know each other
- "6 degrees of separation"
  - are all pairs of people connected by path of distance at most 6?

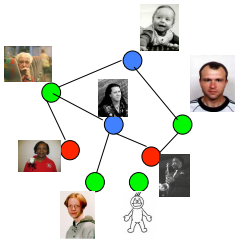


Photo & Image Source: Google

Really Big Data

- Impossible to access all of it?
- Accessible data is too enormous to be viewed by a single individual?
- Once accessed, data can change?

The Gold Standard

- linear time algorithms:
  - for inputs encoded by *n* bits/words, allow *cn* time steps (constant *c*)
- Inadequate…

What Can We Hope To Do Without Viewing Most of the Data?

- **Can't answer "for all" or "exactly" type statements:**
  - *exactly* how many individuals on earth are left-handed?
  - are *all* individuals connected by at most 6 degrees of separation?
- **Compromise?**
  - *approximately* how many individuals on earth are left-handed?
  - is there *a large group* of individuals connected by at most 6 degrees of separation?

What Types of Approximation?

- **Property Testing** – distinguish data that has a certain property from data that is *far* from having the property
- **Classical Approximation** – approximate correct output of computational problem

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Property Testing Algorithms

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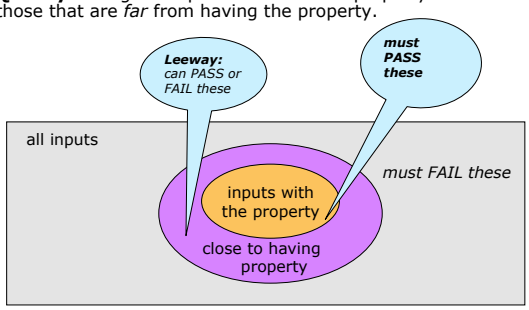
What Types of Approximation?

Property Testing

Distinguish data that *has* a certain property from data that is *far* from having the property.

Property Testing:

Quickly distinguish inputs that *have* the property from those that are *far* from having the property.



Why Property Testing?

- **Can often answer such questions much faster**
- **May be the natural question to ask**
  - *When some "noise" always present*
  - *When data constantly changing*
  - *Gives fast sanity check to rule out very "bad" inputs*
  - *Model selection problem in machine learning*

Which Properties?

- **Properties of any object, e.g.,**
  - *Functions, graphs, strings, matrices, codewords, …*
- **Examples:**
  - *clusterability, small diameter graph, increasing order, close to a codeword, linear or low degree polynomial function*
  - *Lots and lots more…*

Property Testing

- **Model must specify**
  - *representation of object and allowable queries*
  - *notion of close/far, e.g., number of bits/ words that need to be changed*

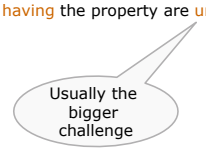
Examples

- Can test if the social network has 6 degrees of separation in CONSTANT TIME
- Can test if data is clusterable in CONSTANT TIME

[Parnas Ron][Alon Dar Parnas Ron]

Constructing a Property Tester:

- Find characterization of property that is
  - Efficiently testable
  - Robust
    - objects that have the property satisfy characterization,
    - and objects far from having the property are unlikely to PASS



Example: 6 Degrees of Separation

- **A "bad" testing characterization:**
  - For *every* node, *all* other nodes within distance 6.
- **Another bad one:**
  - For *most* nodes, *all* other nodes within distance 6.
- **Good characterization**: *[Parnas Ron]*
  - For *most* nodes, there are *many other* nodes within distance 6.

Many More Properties Studied!

- Graphs, functions, point sets, strings, …
- Amazing characterizations of problems testable in graph and function testing models in constant time!

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Sublinear Time Approximation Algorithms

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"Classical" Approximation

- **Output number close to value of the optimal solution (not enough time to construct a solution)**
- **Some examples:**
  - Minimum spanning tree,
  - vertex cover,
  - max cut,
  - positive linear program,
  - edit distance, …

What is Close?

- ***c*-multiplicative approximation**
  - output a number that is within a multiplicative factor of *c* of the best solution
    - e.g., *output < 2 x OPTIMAL*
- ***c*-additive approximation**
  - output a number that is additively within *c* of best solution
    - e.g., *output < OPTIMAL + c*

Example: Vertex Cover

- **Given graph G(V,E), a vertex cover (VC) C is a subset of V such that it "touches" every edge.**
- **What is minimum size of a vertex cover?**
  - NP-complete
  - Polynomial time multiplicative 2-approximation based on relationship of VC and maximal matching

"Classical" Approximation Examples

- **Can get CONSTANT TIME approximation for vertex cover on sparse graphs!**
  - Output y which is at most $2 \cdot OPT + \epsilon n$
- **How?**
- **Oracle reduction framework [Parnas Ron]**
  - Construct "oracle" that tells you if node u in 2-approximate vertex cover
  - Use oracle + standard sampling to estimate size of cover

But how do you implement the oracle?

Implementing The Oracle – Two Approaches:

- Sequentially simulate computations of a fast distributed algorithm [Parnas Ron]
- Figure out what a greedy maximal matching algorithm would do on u [Nguyen Onak]

Vertex Cover & Maximal Matching

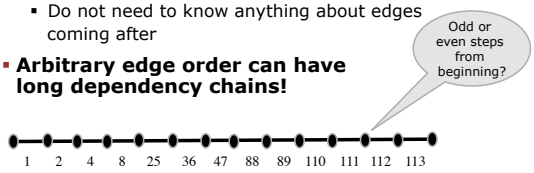
- **Maximal Matching:**
  - $M \subseteq E$ is a matching if no node is involved in more than one edge.
  - M is a maximal matching if adding any edge violates the matching property
- **Well known fact: Nodes in any maximal matching M give a pretty good Vertex Cover!**
  - That is, $|M| \leq VC \leq 2|M|$

Greedy Algorithm for Maximal Matching

- **Algorithm:**
  - Consider every edge (u,v) in arbitrary order:
    - If neither of u or v matched
      - Add (u,v) to M
  - Output M
- **Why is M maximal?**
  - If (u,v) not in M then either u or v already matched

Implementing the Oracle via Greedy

- **To decide if edge e in matching:**
  - Must know if adjacent edges that come before e in the ordering are in the matching
  - Do not need to know anything about edges coming after
- **Arbitrary edge order can have long dependency chains!**



Breaking Long Dependency Chains

- **Assign random ordering to edges**
  - Greedy works under any ordering
  - Important fact: random order has short dependency chains

[Nguyen Onak]

Better Complexity for VC

- Additional ideas yield query complexity nearly linear in average degree for general graphs [Yoshida Yamamoto Ito] [Onak Ron Rosen Rubinfeld]

Further Work

- More complicated arguments for sparse maximum matching, set cover, positive Linear Programming… [Parnas Ron + Kuhn Moscibroda Wattenhofer] [Nguyen Onak]
- Even better results for special classes of graphs [Hassidim Kelner Nguyen Onak] [Newman Sohler]
  - e.g., planar, hyperfinite

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Local Computation Algorithms

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Big inputs

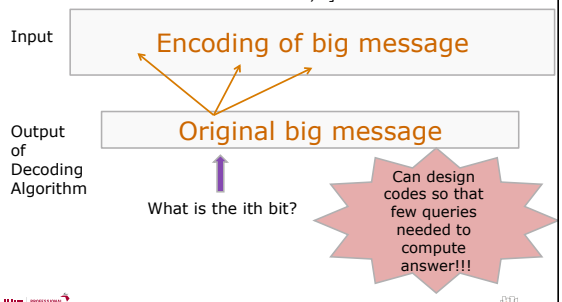
Big outputs

When We Don't Need to See All the Output…

do we need to see all the input?

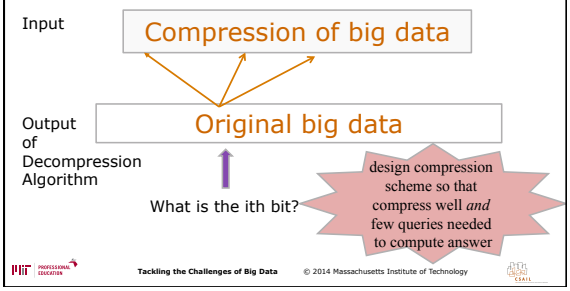
Locally (list-)Decodable Codes

[Sudan-Trevisan-Vadhan, Katz-Trevisan,…]

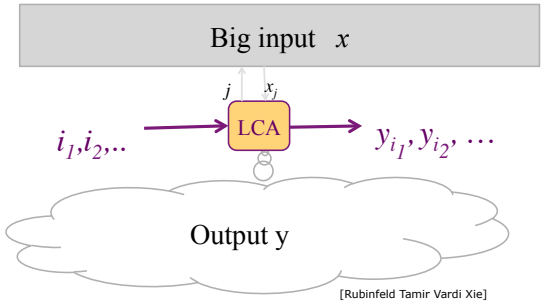


Local Decompression Algorithms

[Muthukrishnan Strauss Zheng][Chandar Shah Wornell][Sadakane Grossi] [Gonzalez Navarro] [Ferragina Venturini] [Kreft Navarro] [Billie Landau Raman Sadakane Satti Weimann] [Dutta Levi Ron Rubinfeld]



Local Computation Algorithms (LCAs)



[Rubinfeld Tamir Vardi Xie]

What More Can be Done in This Model?

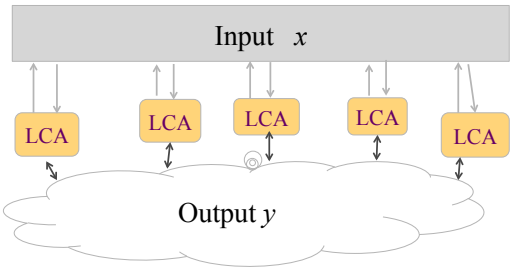
- **Optimization problems** [Rubinfeld Tamir Vardi Xie] [Alon Rubinfeld Vardi Xie]
  - Maximal independent set, certain constraint satisfaction problems,…
- **Graph sparsification** [Campagna Guo Rubinfeld] [Levi Ron Rubinfeld]

Even more:

- **Local algorithms for**
  - Ranking webpages
  - Graph partitioning

  [Andersen Borgs Chayes Hopcroft Mirrokni Teng] [Andersen Chung Lang] [Spielman Teng] [Borgs Brautbar Chayes Lucier]
- **Property preserving data reconstruction** [Ailon Chazelle Comandur Liu] [Comandur Saks] [Jha Raskhodnikova][Campagna Guo R.] [Awasthi Jha Molinaro Raskhodnikova] …

LCAs and the Cloud



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Big Distributions

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No Samples

What if data only accessible via random samples?

Distributions

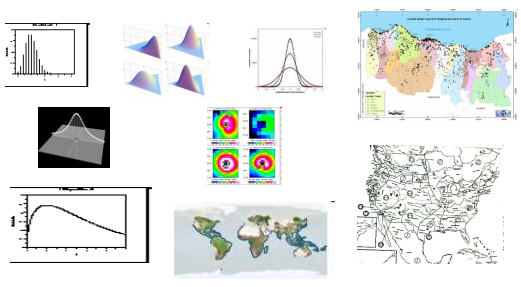


Photo & Image Source: Google

Play The Lottery?

Image Source: Google

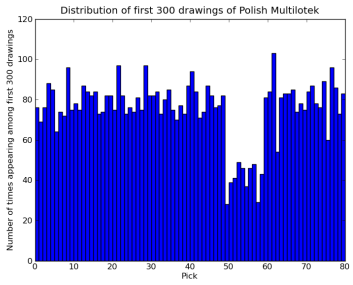
Is The Lottery Unfair?

- From Hitlotto.com: Lottery experts agree, past number histories can be the key to predicting future winners.

True Story!

- **Polish lottery Multilotek**
  - Choose "uniformly" at random distinct 20 numbers out of 1 to 80.
  - Initial machine biased
    - e.g., probability of 50-59 too small
- **Past results: http://serwis.lotto.pl:8080/archiwum/wyniki_wszystkie.php?id_gra=2**

Image: Lotto, Multi Multi, Mini Lotto, Twój Szczęśliwy Numerek ...www.wynikilotto.mortin.pl
Image Source: Google



Thanks to Krzysztof Onak (pointer) and Eric Price (graph)

New Jersey Pick 3, 4 Lottery

- **New Jersey Pick k (=3,4) Lottery.**
  - Pick k digits in order.
  - 10^k possible values.
  - Assume lottery draws iid
- **Data:**
  - Pick 3 - 8522 results from 5/22/75 to 10/15/00
    - χ^2-test gives 42% confidence
  - Pick 4 - 6544 results from 9/1/77 to 10/15/00.
    - fewer results than possible values
    - χ^2-test gives no confidence

Distributions on BIG domains

- **Given samples of a distribution, need to know, e.g.,**
  - entropy
  - number of distinct elements
  - "shape" (monotone, bimodal,…)
  - closeness to uniform, Gaussian, Zipfian…
- **No assumptions on shape of distribution**
  - i.e., smoothness, monotonicity, Normal distribution,…

Distributions on BIG domains, ctd.

- **Considered in statistics, information theory, machine learning, databases, algorithms, physics, biology,…**

Key Question

- **How many samples do you need in terms of domain size?**
  - Do you need to estimate the probabilities of each domain item?
  - Can sample complexity be sublinear in size of the domain?

Rules out standard statistical techniques, learning distribution

Our Aim:

Algorithms with ***sublinear*** sample complexity

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Similarity of Big Distributions?

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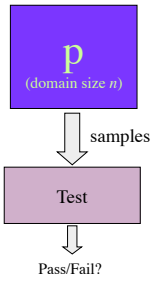
Our Aim:

Algorithms for understanding distributions with ***sublinear*** sample complexity

Similarities of Distributions

- **Are two distributions *p* and *q close* or *far*?**
  - *p* is given via samples
  - *q* is either
    - known to the tester (e.g. uniform)
    - given via samples

Is p uniform?



Theorem: ([Goldreich Ron] [Batu Fortnow Rubinfeld Smith White] [Paninski]) Sample complexity of distinguishing

$p=U$

from $||p-U||1>$ ☒ is ☒$(n\uparrow 1/2)$

Upper Bound for L_2 Distance

- L_2 distance:

$||p-U||_2^2 = \Sigma_{i\in[1..n]} (p_i - 1/n)^2$

$= \Sigma_i p_i^2 - 2\Sigma p_i/n + \Sigma 1/n^2$

$= \Sigma p_i^2 - 1/n$

- Estimate collision probability to estimate L_2 distance from uniform

[Goldreich Ron]

Testing Uniformity

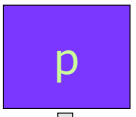
- **Upper bound: Estimate collision probability and use known relation between L1 and L2 norms**
  - Issues:
    - Collision probability of uniform is 1/n
    - Use O(sqrt(n)) samples via recycling
  - Comment: [Paninski] uses different estimator
- **Easy lower bound: $\Omega(n^{1/2})$ samples needed**
  - Can improve to $\Omega(n^{1/2}/\varepsilon^2)$ [P]

[Goldreich Ron, Batu Fortnow Rubinfeld Smith White]

Back to The Lottery…

plenty of samples!

Is p Uniform?



samples

Test

Pass/Fail?

Theorem: Sample complexity of distinguishing

$p=U$

from $||p-U||_1>\varepsilon$ is $\theta(n^{1/2})$

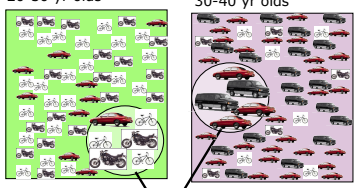
[Goldreich Ron][Batu Fortnow Rubinfeld Smith White] [Paninski]

Nearly same complexity to test if p is any known distribution
[Batu Fischer Fortnow Kumar Rubinfeld White]

Testing Closeness of Two Distributions

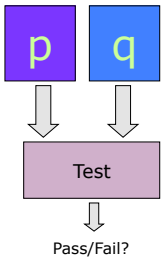
Transactions of 20-30 yr olds

Transactions of 30-40 yr olds



trend change?

Testing Closeness



Pass/Fail?

Theorem: Sample complexity of distinguishing

$p=q$

from $||p-q||_1 >\epsilon$

is $\theta(n^{2/3})$

[BFRSW] [P. Valiant][Chan Diakonikolas Valiant Valiant]

Why So Different?

- Collision statistics are all that matter
- Collisions on "heavy" elements can hide collision statistics of rest of the domain

Additively Estimate Distance?

New goal: Output $||p-q||_1 \pm \epsilon$

much harder, but still sublinear!

need $\theta(n/\log n)$ samples [G. Valiant P. Valiant]

Collisions Tell All

- **Algorithms:**
  - Algorithms use collisions to determine "wrong" behavior
    - E.g., too many collisions implies far from uniform [GR,BFSRW]
  - Use Linear Programming to determine if there is a distribution with the right collision probabilities and the right property [G. Valiant P. Valiant]

Collisions Tell All

- **Lower bounds:**
  - For symmetric properties, collision statistics are only relevant information [BFRSW] (see also [Orlitsky Santhanam Zhang] [Orlitsky Santhanam Viswanthan Zhang])
  - Need new analysis tools since not independent
    - Central limit theorem for generalized multinomial distributions [G. Valiant P. Valiant]

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Big Distributions – Final Words

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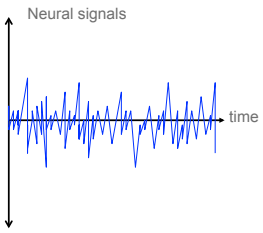
Information Theoretic Quantities

Entropy

Support size

Information in Neural Spike Trails

[Strong, Koberle, de Ruyter van Steveninck, Bialek '98]



- Each application of stimuli gives sample of signal (spike trail)
- Entropy of (discretized) signal indicates which neurons respond to stimuli

Compressibility of Data

GPS Tracking Reduces Illegal Garbage Dumping | GPS Tracking Blogtrackingtheworld.com

Image source: Google

Can We Get Multiplicative Approximations?

- **In general, no….**
- **What if entropy is at least some constant?**
  - Can γ-multiplicatively approximate the entropy with $\tilde{O}(n^{1/\gamma 2})$ samples (when entropy $>2g/\epsilon$) [Batu Dasgupta Rubinfeld Kumar]
  - requires $\Omega(n^{1/\gamma 2})$ [Valiant]
  - better bounds when support size is small [Brautbar Samorodnitsky]
  - Similar bounds for estimating support size [Raskhodikova Ron Rubinfeld Smith] [Raskhodnikova Ron Shpilka Smith]

Additive Approximations for Entropy and Support Size

- need $\theta(n/\log n)$ samples [Raskhodnikova, Ron, Shpilka, Smith] [Valiant Valiant]

More Properties:

- Independence and Limited Independence [Batu Fischer Fortnow Kumar Rubinfeld White] [Alon Andoni Kaufman Matulef R. Xie] [Haviv Langberg]
- K-histogram distributions [Levi Indyk Rubinfeld]
- K-modal distributions [Daskalakis Diakonikolas Servedio]
- Poisson Binomial Distributions [Daskalakis Diakonikolas Servedio]
- Monotonicity over general posets [Batu Kumar Rubinfeld] [Bhattacharyya Fischer Rubinfeld P. Valiant]
- Properties of multiple distributions [Levi Ron Rubinfeld]

Conclusion

- **For many problems, we need a lot less time and samples than one might think!**
- **Many cool ideas and techniques have been developed**
- **Lots more to do!**

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Big Distributions – Final Words

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