Tackling the Challenges of Big Data
Big Data Analytics

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Tackling the Challenges of Big Data
Big Data Analytics
Fast Algorithms II & Streaming and Sampling
Introduction

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Streaming & Sampling

Two approaches to dealing with massive data using limited resources

Sampling: pick a random sample of the data, and perform the computation on the sample

8 2 1 9 1 9 2 4 6 3 9 4 2 9 4 9 3 9 5 9 5 6 …

Streaming: make a single pass over the whole data; maintain a `sketch' of the data set from which the desired properties can be inferred

8 2 1 9 1 9 2 4 6 3 9 4 2 9 4 9 3 9 5 9 5 6 …

Streaming vs. Sampling

Sampling pros:

- Computation is performed only on the sample - reduced storage and computation time
- Only the sampled data elements are needed; the remaining elements do not even need to be materialized

Streaming pros:

- Every data element is seen at least once, so `no element is left behind'.
- E.g., does the data set contain any 1 ?
- Reduced storage, computation time near-linear in the data size

Rest of This Lecture

Streaming:

- Estimate the number of distinct elements in the stream in limited space, up to $1 \pm \varepsilon$ error
- Need only 128 bytes for all works of Shakespeare with error $\varepsilon \approx 10\%$
- Other problems solvable using streaming algorithms

Sampling:

- Recent algorithms for sparse Fourier Transform that estimate large coefficients in the spectrum of a signal using few signal samples

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Streaming Algorithms

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Streaming Algorithms

Single pass over the data: $i_1, i_2, \ldots, i_n$

- Typically, we assume the number of data items n is known, at least approximately

Bounded storage (e.g. $n^{1/2}$ or $\log^2(n)$)

- Units of storage: bits, bytes or data elements

Fast processing time per element

Streaming Algorithms, ctd.

- Most algorithms are randomized (they use pseudo-random numbers) and approximate
- That is, they report an estimate such that
  $\Pr[\text{Estimate}=\text{Truth } (1\pm\varepsilon)]>1-P$
- This is often (but not always) necessary

Counting Distinct Elements

- Stream elements: integer numbers from 1…m
  8 2 1 9 1 9 2 4 6 3 9 4 2 9 4 9 3 9 5 9 5 6 …
- Goal: estimate the number of distinct elements DE in the stream
  - Up to $1\pm\varepsilon$
  - With probability 1-P
- Simpler goal: for a given T>0, provide an algorithm which, with probability 1-P:
  - Answers `DE>T' if DE> $(1+\varepsilon)$T
  - Answers `DE<T' if DE< $(1-\varepsilon)$T

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Streaming Algorithms

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Distinct Elements

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Counting Distinct Elements

- Stream elements: integer numbers from 1…m
- Goal: estimate the number of distinct elements DE in the stream
  - Up to 1±ε
  - With probability 1-P
- Simpler goal: for a given T>0, provide an algorithm which, with probability 1-P:
  - Answers `DE>T' if DE> (1+ε)T
  - Answers `DE<T' if DE< (1-ε)T

Count Vector Interpretation

- Stream: 8 2 1 9 1 9 2 4 4 9 4 2 5 4 2 5 8 5 2 5

Vector c:

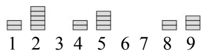
- Initially, c=0
- Arrival of i is interpreted as

$$c_i = c_i + 1$$

- Want to estimate the number of non-zero entries in c, denoted by NZ(c)

Estimating NZ(c)

Vector c:



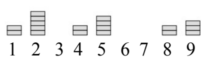
- Preprocessing:
  - Select k random sets $S_0 \ldots S_{k-1}$ of coordinates such that, for each i, we have

    $$\Pr[i \in S_j] = 1/T$$

  - Important: do not store S explicitly. Instead, to test if $i \in S_j$:
    - Set the pseudo-random seed to j+k*i
    - Generate a pseudo-random number R(i,j)
    - Check if R(i,j) mod T = 0

Estimating NZ(c), continued

Vector c:



- Computation: For each j, compute $sum_j = \Sigma_{i \in S_j} c_i$
  - For each j, let $sum_j=0$
  - For each stream element i
    - For each j, if $i \in S_j$ then $sum_j=sum_j+1$

Estimating NZ(c), continued

Vector c:

- After we computed $sum_j = \Sigma_{i \in S_j} c_i$ we estimate:
  - Let Z = number of sum_j that are equal to 0
  - If Z>k/e then report `NZ < T' else report `NZ>T'

- Intution: if NZ is small compared to T, then the non-zero coordinates are not likely to belong to sets S_j. Therefore Z will be large.

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Performance

- The described algorithm is a variant of [Flajolet-Martin, FOCS'83]
- Best theoretical result: $O(1/\varepsilon^2 + \log n)$ bits [Kane-Nelson-Woodruff, PODS'10]
- Practice: need only 128 bytes for all works of Shakespeare, with error $\varepsilon \approx 10\%$
- LogLog, HyperLogLog [Durand-Flajolet, ESA'03]

Other Tasks

- **Heavy hitters** (a.k.a. elephants) [Misra-Gries'82, Charikar-Chen-FarachColton'02, Estan-Varghese'03, Cormode-Muthukrishnan'04,'05, Cormode-Hadjieleftheriou'07,…]
  - Finds coordinates i such that $|c_i|$ is "large"
  - Estimates $c^*_i = c_i \pm \varepsilon n$
- **Entropy** [DoBa-Chakrabarti-Muthukrishnan'05, Guha-McGregor-Venkatasubramanian'05, Chakrabarti-Cormode-McGregor'06, Bhuvanagiri-Ganguly'06, Harvey-Nelson-Onak'08]
- **Independence testing** [Indyk-McGregor'08]
- **Median, quantiles, histograms** [Munro-Paterson'80, Manku-Rajagopalan-Lindsay'98,'99, Greenwald-Khanna'02, Gilbert-Guha-Indyk-Kotidis-Muthukrishnan-Strauss'02,…]

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Sampling Algorithms for the Sparse Fourier Transform

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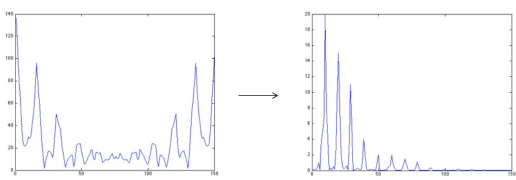
Discrete Fourier Transform

- Given a signal, compute its spectrum



Applications: Audio, Video, GPS, Radar, Sequencing,…

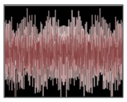
Computing Fourier Transform

- Naïve Algorithm $O(n^2)$
- In 1965, Cooley and Tukey introduced the FFT which computes the frequencies in $O(n \log n)$
- But … FFT is too slow for BIG Data problems

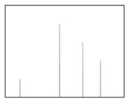
Can we design a sublinear Fourier algorithm?

Idea: Leverage Sparsity

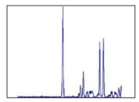
Often the Fourier Transform is dominated by a few peaks



Time Signal



Sparse Freqs.



Approximately Sparse Freqs.

Sparsity appears in video, audio, seismic data, telescope/satellite data, medical tests, genomics

Sparse FFT runs in sub-linear time on sparse data

Benefits of Sparse Fourier Transform

- Faster computation
  ◊Scalable to larger datasets
- Use only samples of the data
  ◊ Lower acquisition time
  ◊ Less communication bandwidth
- Lower power consumption

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Sparse Fourier Transform - Algorithm and Performance

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Basic Ideas

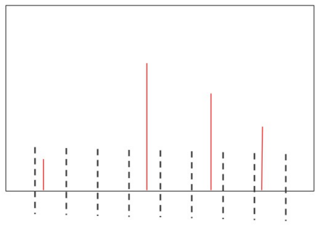
1- Bucketize

Divide spectrum into a few buckets

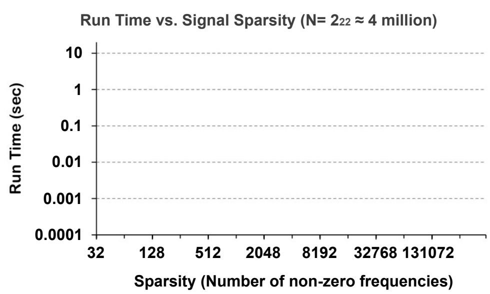
◊ Can ignore empty bucket

2- Estimate

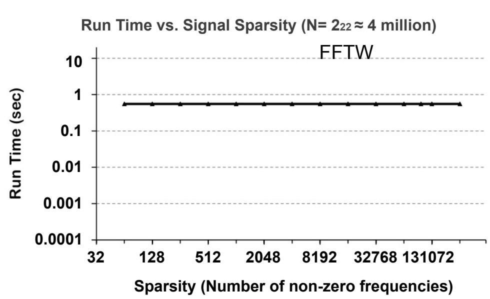
Estimate the large coefficient in each non-empty bucket



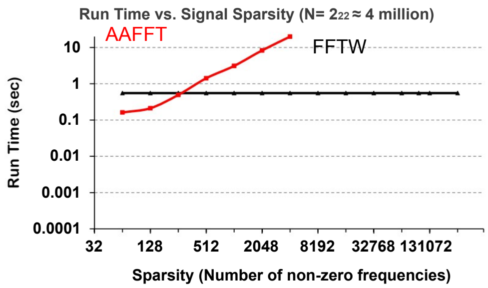
Results



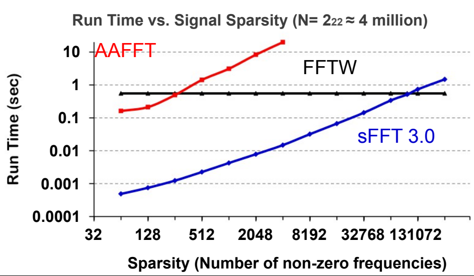
Results



Results



Results



- **Linear in sparsity, not signal length**
- **Orders of magnitude faster than FFT on sparse d**

Current Directions

- Hardware for a million-point Fourier Transform
- Applications:
  - GPS
  - Smaller and cheaper 3D cameras
  - Medical Imaging - NMR, and MRI

References

- SFFT 1.0, 2.0 [Hassanieh-Indyk-Katabi-Price, SODA'12]
- SFFT 3.0 [Hassanieh-Indyk-Katabi-Price, STOC'12]
- Sparse Fourier Transform website: http://groups.csail.mit.edu/netmit/sFFT/

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