Statistical Inference Project Part 1: Simulation Exercise

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This document serves as the first part of the Statistical Inference course project.

```
set.seed(2)
##Define variables
n <- 40
lambda <- 0.2
nsim <- 1000

###Run Simulation
simmeans <- rowMeans(matrix(rexp(n*nsim, lambda), nsim, n))
```

1. Show where the distribution is centered at and compare it to the theoretical center of the distribution. The simulated distribution is centred at:

```
median(simmeans)
```

```
## [1] 4.986371
```

The theoretical center of the distribution is at 1/lambda:

```
1/lambda
```

```
## [1] 5
```

In the plot below, the theoretical center is illustrated with a vertical red line.

2. Show how variable it is and compare it to the theoretical variance of the distribution. The actual variance of the simulated distribution is:

```
var(simmeans)
```

```
## [1] 0.5776132
```

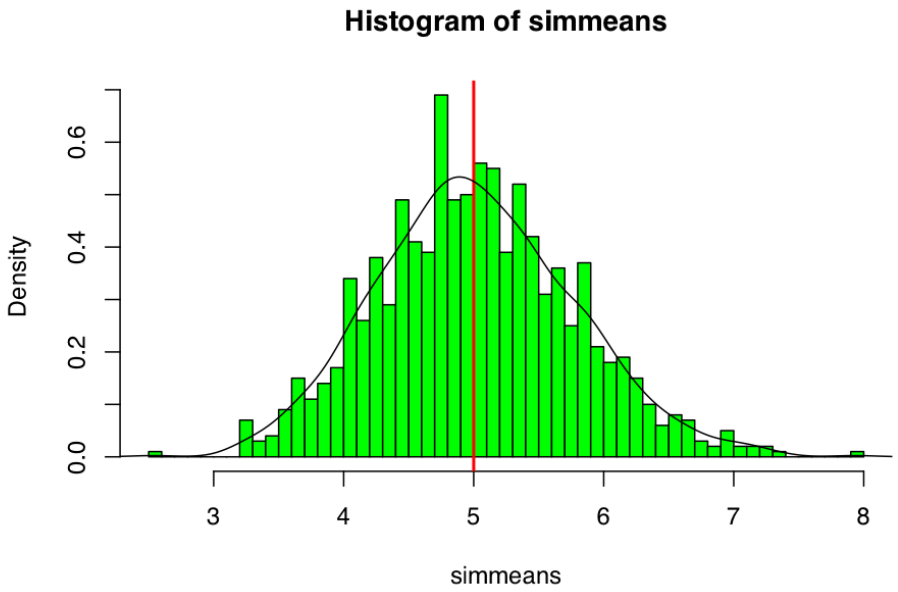
The theoretical variance of the simulated distribution is:

```
1/(lambda^2 * n)
```

```
## [1] 0.625
```

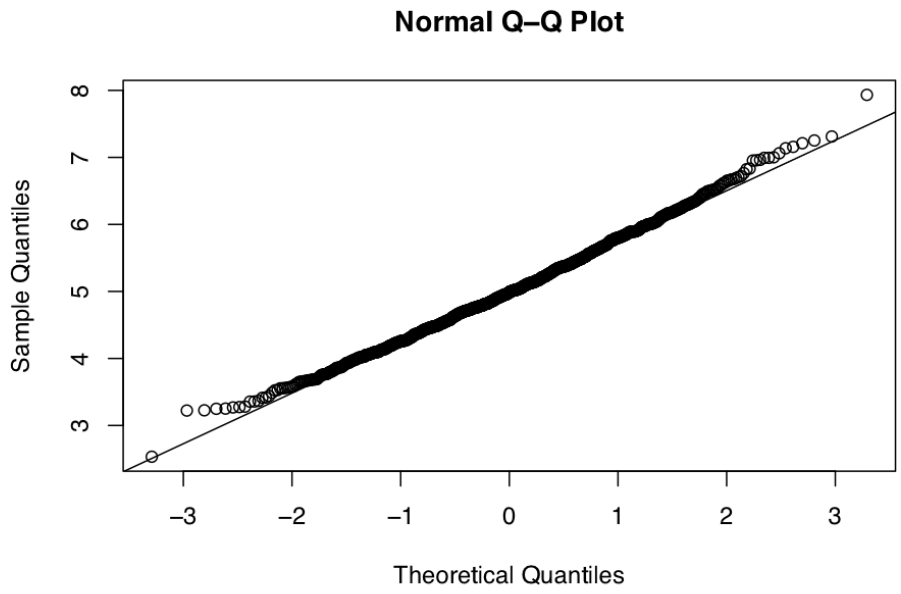
3. Show that the distribution is approximately normal. The simulated distribution is plotted below. A density curve is overlayed to demonstrate that the distribution is approximately normal.

```r
hist(simmeans, breaks = 50, col = "green", prob = TRUE)
abline(v = 1/lambda, lwd = 2, col = "red")
lines(density(simmeans))
```



Q-Q plot to further demonstrate approximate normality:

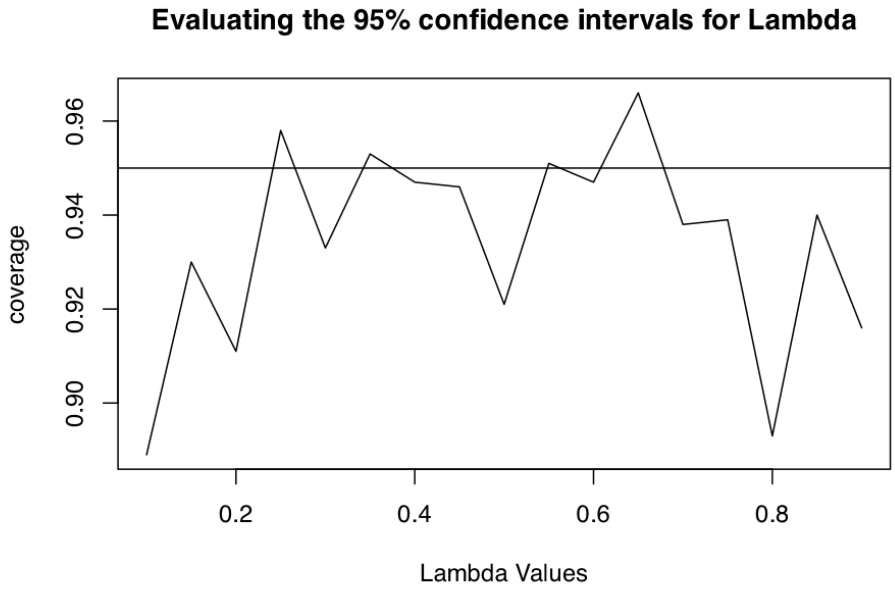
```r
qqnorm(simmeans)
qqline(simmeans, probs=c(0.25,0.75))
```



4. Evaluate the coverage of the confidence interval for 1/lambda: $\bar{X} \pm 1.96Sn$. Confidence interval plot for lambda:

```
lvals <- seq(0.1, .9, by = 0.05)
coverage <- sapply(lvals, function(p) {
  phats <- rbinom(nsim, prob = p, size = n)/n
  ll <- phats - qnorm(0.975) * sqrt(phats * (1 - phats)/n)
  ul <- phats + qnorm(0.975) * sqrt(phats * (1 - phats)/n)
  mean(ll < p & ul > p)
})

plot(lvals, coverage, type = "l", xlab = "Lambda Values",
     main = "Evaluating the 95% confidence intervals for Lambda")
abline(h = 0.95)
```



Confidence interval for lambda = 0.2

```
mean(simmeans) + c(-1,1)*1.96*sd(simmeans)/sqrt(length(simmeans))
```

```
## [1] 4.969250 5.063462
```