CS542 Machine Learning

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Problem set 5

8.3

$$P(a,b) = \sum_{c\in\{0,1\}} P(a,b,c)$$

$$P(a) = \sum_{b\in\{0,1\}} \sum_{c\in\{0,1\}} P(a,b,c) \text{ and } P(b) = \sum_{a\in\{0,1\}} \sum_{c\in\{0,1\}} P(a,b,c)$$

$$P(a,b|c) = \frac{P(a,b,c)}{\sum_{a\in\{0,1\}} \sum_{b\in\{0,1\}} P(a,b,c)}$$

Similarly for the conditionals $P(a|c)$ and $P(b|c)$ we have

$$P(a|c) = \frac{\sum_{b\in\{0,1\}} P(a,b,c)}{\sum_{a\in\{0,1\}} \sum_{b\in\{0,1\}} P(a,b,c)}$$

$$P(b|c) = \frac{\sum_{a\in\{0,1\}} P(a,b,c)}{\sum_{a\in\{0,1\}} \sum_{b\in\{0,1\}} P(a,b,c)}$$

Table 2 compares $P(a,b|c)$ with $P(a|c)\,P(b|c)$, showing that these are equal for the given joint distribution $P(a,b,c)$ for both $c=0$ and $c=1$.

8.4. $P(a)$ in (2.31) was computed and $P(b|c)$ in (2.32).

$$P(c|a) = \frac{\sum_{b\in\{0,1\}} p(a,b,c)}{\sum_{b\in\{0,1\}}\sum_{c\in\{0,1\}} p(a,b,c)}$$

The required distributions are given in Table 3.

Table 2. Comparison of the conditional distribution $P(a,b|c)$ with the product of marginals $P(a|c)\,P(b|c)$ showing that these are equal for the given distribution.

a	b	c	P(a,b | c)
0	0	0	0.400
0	1	0	0.100
1	0	0	0.400
1	1	0	0.100
0	0	1	0.277
0	1	1	0.415
1	0	1	0.123
1	1	1	0.185

a	b	c	P(a|c) P(b|c)
0	0	0	0.400
0	1	0	0.100
1	0	0	0.400
1	1	0	0.100
0	0	1	0.277
0	1	1	0.415
1	0	1	0.123
1	1	1	0.185

Table 3

a	P(a)
0	0.600
1	0.400

c	a	P(c|a)
0	0	0.400
1	0	0.600
0	1	0.600
1	1	0.400

b	c	P(b|c)
0	0	0.800
1	0	0.200
0	1	0.400
1	1	0.60

$$p(a,b,c) = p(a)\,p(c|a)\,p(b|c).$$

8.11.

$$P(F=0|D=0) = \frac{P(D=0|F=0)\,P(F=0)}{P(D=0)}$$

To evaluate $P(D=0|F=0)$, we marginalize over B and G

$$P(D=0|F=0) = \sum_{B,G} P(D=0|G)\,P(G|B,F=0)\,P(B) = 0.748 \quad (2.33)$$

and to evaluate $P(D=0) = \sum$

$$P(D=0) = \sum_{B,G,F} P(D=0|G)\,P(G|B,F)\,P(B)\,P(F) = 0.352$$

Combining the results with $P(F=0)$, we get

$$P(F=0|D=0) = 0.213$$

with (8.32)

B in (233) and (234)

$$P(F=0|D=0,\ B=0) = 0.110$$

8.14. (5)

The most probable configuration corresponds to the configuration with the lowest energy. Since η is a positive constant (and $h = \beta = 0$) and $x_i, y_i \in \{-1, +1\}$, this will be obtained

$x_i = y_i$ for all $i = 1, \dots, D$