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Review

A review on CT image noise and its denoising

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ABSTRACT

CT imaging is widely used in medical science over the last decades. The process of CT image reconstruction depends on many physical measurements such as radiation dose, software/hardware. Due to statistical uncertainty in all physical measurements in Computed Tomography, the inevitable noise is introduced in CT images. Therefore, edge-preserving denoising methods are required to enhance the quality of CT images. However, there is a tradeoff between noise reduction and the preservation of actual medical relevant contents. Reducing the noise without losing the important features of the image such as edges, corners and other sharp structures, is a challenging task. Nevertheless, various techniques have been presented to suppress the noise from the CT scanned images. Each technique has their own assumptions, merits and limitations. This paper contains a survey of some significant work in the area of CT image denoising. Often, researchers face difficulty to understand the noise in CT images and also to select an appropriate denoising method that is specific to their purpose. Hence, a brief introduction about CT imaging, the characteristics of noise in CT images and the popular methods of CT image denoising are presented here. The merits and drawbacks of CT image denoising methods are also discussed.

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1. Introduction

Computed Tomography (CT), invented by Hounsfield [60] in 1972, was the first method that allowed to generate non-overlapping axial slices of the internal structure of a humans body without opening it [13,102]. The Nobel prize awarded to Hounsfield [60] and Cormack [29] for their work on the initial CT device which signifies the importance of CT imaging [61].

1.1. CT imaging

Today, CT is associated with high competency in radiologic diagnostics and has become an important tool in medical examinations. Traditional X-ray technique has many limitations such as it cannot distinguish between muscles, ligaments or vessels and other tissues. Another major disadvantage is the superposition and compression of 3D information into a 2D image. Compare to traditional X-ray technique, the computed tomography is more accurate technique that adds a mathematical reconstruction theory of an object from its projections to multi-angular X-ray scanning. The computed tomography involves mainly the presence of a computer, which processes the information received through the passage of an X-ray beam through an anatomical area. The benefits of CT imaging are speed, accuracy, small scanning time and many more.

Computed tomography (CT), originally known as computed axial tomography (CAT or CT scan), is a medical imaging method employing tomography where digital geometry processing is used to generate a three-dimensional image of the internals of an object from a large series of two-dimensional X-ray images taken around a single axis of rotation [114]. Computed Tomography (CT) is a powerful nondestructive evaluation (NDE) technique for producing 2-D and 3-D cross-sectional images of an object from flat X-ray images. The characteristics of the internal structure of an object such as dimensions, shape, internal defects and density are readily available from CT images. The process through radon and inverse radon transform on the information received through the passage of an X-ray beam, an image can be reconstructed, is known as CT imaging.

As X-rays pass through the body, X-rays are absorbed or attenuated at different levels, creating a profile of beams of different intensities. A detector, that further transforms the profile in an image, analyzes this profile. The CT creates images representing cross sectional slices of the exposed area. The absorption of X-rays due to differences in elemental composition has an important effect on CT imaging. The absorption process is proportional to the atomic number which helps to differentiate gray from white matter in the CT images. Very little contrast is observable between blood and muscle as the density and the elemental compositions are very similar. To provide image contrast between them, usually a dense fluid with elements of high atomic number can be injected or swallowed during X-ray exposures.

2. CT image reconstruction

In Computed Tomography, a motorized X-ray source is used to transmit the X-rays over the patient with various angles. CT scanners use special digital X-ray detectors, which are located directly opposite the X-ray source. As the X-rays move through the patient, the detectors received the X-rays and change into electrical signals. These electrical signals are converted into the digital data by analog/digital converter. The data in the digital matrix can be changed into small boxes ranging from black to white gray through digital/analog converter. Finally, raw data (collected from various angles) are further mathematically computed using Radon transform to reconstruct the CT images. Backprojection is the most common concept for image reconstruction. The back projecting to the raw data with respective directions is generally known as backprojection [70].

Backprojection is conceptually simple but it does not correctly solve the inverse problem. To reconstruct the CT image, a simulation based example is shown in Fig. 1. In this example, a phantom image as shown in Fig. 1(a) is taken as an input object and Radon transform is applied to the phantom image by choosing $\theta = 180$. The raw data are collected from each unit angle. This raw data can be represented in the form of the image which is known as Sinogram as shown in Fig. 1(b). Finally, inverse Radon transform is performed over the Sinogram to obtain the reconstructed CT image as shown in Fig. 1(c). To overcome the blurry results (as in Fig. 1(c)), filter backprojection is widely used in CT image reconstructions. To correct this, the projections are first filtered in the Fourier domain by a filter such as ramp filter and then the filtered projections are projected using radon transform. An example of image reconstruction using filtered backprojection is shown in Fig. 2.

After, image reconstruction, the major issues and factors affecting the quality of CT images must be analyzed. A brief overview of these issues are given below:

2.1. Major issues on CT image reconstruction

Generally, CT images are obtained either from filtered back projection (FBP) technique or IR technique. There are some major issues which must understand or analyzed to get good quality of CT images over the low dose-data.

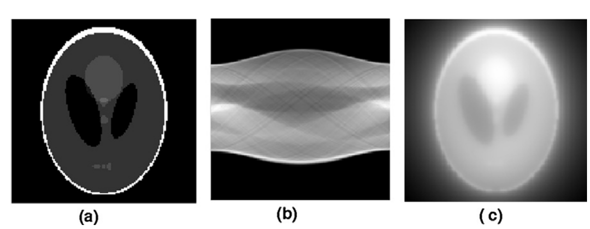

Fig. 1. Image reconstruction using Radon transform.

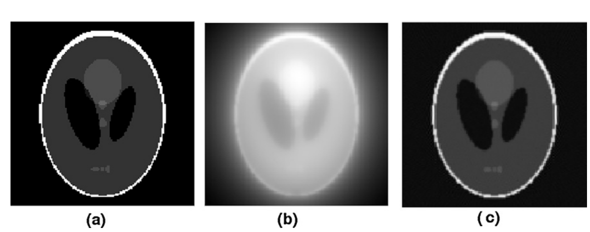

Fig. 2. Image reconstruction using filtered backprojection: (a) phantom image, (b) unfiltered image, and (c) filtered image.

- Noise power spectrum (NPS) is one of the important metrics to understand the noise in terms of amplitude and texture in images which is obtained from a uniform field of radiation. However, shape of NPS affected due to the detector coefficients, reconstruction algorithms and the different kernels used in the reconstruction. To get good quality CT image, the reconstruction has been done with a proper combination of reconstruction algorithms and the different kernels. Choi [28] performed reconstruction using eight different reconstruction kernels (B10, B20, B30, B40, B50, B60, B70 and B80) to get good quality images. Author separately analyzed the parameters of raw data and CT different kernels, and performed NPS and reconstruction kernel the quality control of CT systems.
- To get good quality CT images, the effect of dose reduction and the potential of noise reduction filters on image quality and the detection of lesions must be analyzed. Punwani et al. [106] analyzed low dose CT image reconstructions based on edge preserving noise reduction filters. They concluded that edge preserving noise reduction filters have potential advantages to detect the lesion over the low dose CT images.
- To optimize a novel adaptive noise reduction filter, reconstruction must be analyzed in terms of patient body weight, age and gender. Funama et al. [48] analyzed CT image reconstructions based on patient body weight and gender. They performed analysis over the 45 patients where CT image reconstruction is performed using Filtered Backprojection in hepatic multidetector computed tomography, and concluded that noise suppression filter effectively reduce noise which confirm the effectiveness of noise suppression filter in clinical hepatic images obtained at low doses.
- The impact of iterative CT reconstruction, Filtered Backprojection and diagnostic performance must be analyzed on low dose CT images. McLaughlin et al. [90] performed analysis over the 33 patients where CT image reconstruction is performed using iterative reconstruction and Filtered Backprojection. They analyzed the diagnostic performance based on filtered backprojection, Iterative backprojection and with the combination of filtered backprojection and Iterative backprojection. From comparative analysis, it was analyzed that filtered backprojection has greater noise in compare to Iterative backprojection technique, and the combination of filtered backprojection and Iterative backprojection performs well by giving less amount of noise in CT images.
- To get better quality of CT images, the effects of reconstruction parameters in Cone-beam Computed Tomography, MDCT, LDCT and Dual Source Computed tomography must be analyzed. Lee et al. [80] analyzed CT image reconstruction based on five different reconstruction filters: Ram-Lak, Shepp-Logan, Cosine, Hamming, and Hann filters. They concluded that the reconstruction filters affects on ideal Ramp filter in the frequency domain. By changing the reconstruction filter, the image noise and spatial resolution are also get affected. Eisentopf et al. [42] analyzed the evaluation of image quality and diagnostic accuracy using very low-dose based on dual-source computed tomography (DSCT). They concluded that Iterative reconstruction based CT image reconstruction in Dual Source Computed tomography leads to a significant improvement of image quality, mainly by reducing higher image noise.

2.2. Major factors affecting the quality of CT images

There are many factors which affect the quality of reconstructed CT images. Some of major factors affecting image quality are:

- Blurring: CT image may be blurred due to various reasons such as appropriate protocol factor values, patient movement, etc. Due to patient movement, blurring may occurs in CT reconstructed images. The patient movement can be happened due to various reasons such as uncooperative patient, breathing, heart beating, etc. The reconstruction algorithms are bit more complicated because the patient movement along *z*-direction needed to be taken into account. To reduce this complexity, image reconstruction algorithms reconstruct CT images by taking arbitrary positions at arbitrary spacing. However, the *z*-direction movement of patient should be controlled because the degree of blurring of image depends on the speed with which the patient is moved. Some major reasons of blurring are:
  - (i) How the equipment is operated.
  - (ii) Appropriate protocol factor values.
  - (iii) Blurring of image due to patient movement.
  - (iv) Fluctuation of CT number between pixels in the image for a scan of uniform material.
  - (v) Some of the filter algorithms or bad parameters of filter algorithms (to reduce noise) blur the image.
- Field of view (FOV): A selected region to reconstruct the CT image is known as field of view. If a reconstructed CT image is too small or too big, it is hard to find the abnormalities and the quality of CT image may be degraded.
- Artifacts: Distortion or an error in an image and unrelated from object is known as an artifact. Artifacts are the discrepancies between the CT numbers represented in the image and the expected CT numbers. Some common artifacts are: Beam hardening, partial volume effect, Metal artifact, Patient motion, etc.
  - (i) Beam hardening: Beam hardening occurs due to an average energy of an X-ray beam passing through the patient increases. This artifact is also called cupping. To avoid that some solutions can be applied such as increasing kvp, decreasing slice thickness, pre-filtering X-rays, avoiding high X-ray absorbing regions if possible, applying appropriate algorithms.
  - (ii) Metal artifact: Metal materials can cause the streaking artifacts due to block parts of projection data such as dental fillings, prosthetic devices, surgical clip, etc. To reduce this artifact, remove the metal material if possible.
  - (iii) Patient motion: Voluntary and involuntary motion can cause streaking artifacts in the reconstructed image. To avoid that some solutions can be applied such as motion reduction, shortening scan time, immobilization and positioning aid.
  - (iv) Software and hardware based artifacts: Due to improper inputs of software and poor equipment, the artifacts in CT images may also be generated. Some variety of artifacts may be generated due to mechanical failure, poor gantry rigidity, mechanical assignment, aliasing, poor sampling of the detector, staircase, tube arcing, etc. The quality of CT images is also get degraded due to bad parameterizations of CT image reconstruction. In computed tomography, there are some fundamental CT scan parameters that can be altered or optimized to improve the quality of CT images such as detector configuration, tube current, tube potential, reconstruction algorithm, patient positioning, scan range, reconstructed slice thickness, and pitch.
- Visual noise: Visual noise is an undesired information that degrades the visual effect of an image. Noise in CT images may appear due to various reasons such as acquisition, transmission and mathematical computations error and variation in attenuation coefficients between voxels. Visual noise affects the visual quality especially on low contrast objects.

In this paper, the problem of noise in CT images and the noise suppression techniques to enhance the quality of noisy CT images are being focused. There are various reasons to generate the noise in CT images. Radiation dose is one of the important factors which affect the quality of CT images in terms of noise. Radiation dose can be reduced under the condition that the quality of diagnostic image quality should not be affected. The main effect of reducing radiation dose on CT image is increasing noise level which can hamper the quality of CT image. To reconstruct a good quality CT image, the CT scanner has two important characteristics:

(1) Geometric efficiency: When X-rays are transmitted to the human body and some absorbed data are not received by the active detectors, it means geometric efficiency is reduced.
(2) Absorption efficiency: When X-rays are transmitted to the human body and some absorbed data are not captured by the active detectors, it means absorption efficiency is reduced.

The above mentioned efficiencies are affected to the CT images based on the performance of the CT scanner. Therefore, the relationship between noise and radiation dose in CT scanner must be analyzed.

- Detector: The major impact of radiation dose depends on the performance of detector in the CT system. It contains both characteristics, geometric and absorption efficiencies. By increasing the both characteristics, the quality of CT image can be improved without increasing the radiation dose.
- Collimators: It is a device used in two ways: pre-collimator and post-collimator. Pre-collimator is set between X-ray source and patient, and helps to avoid the unnecessary radiation dose to patients. To avoid the wastage of radiation dose or to improve the quality of CT image, geometric efficiency was introduced, which depended on the detector collimation. By increasing the width of detector collimation, the quality of CT image can be improved. Post-collimator is set between patient and the detector, and helps to reject the scattered radiation and also helps to improve the image quality.
- Scan range: It is one of the simplest method which effects on the quality of CT images. It is directly related to the total amount of radiation dose which is delivered to the patients. It is important to keep the dose as low as required without sacrificing the diagnosis. Radiation dose depends on tube current (amperage), slice scan time, and tube peak kilovoltage. To understand the effect of radiation dose in CT images, each factor of radiation dose must be known.
- Tube current: By changing the tube current (mA) values, the beam intensity will also changes. For example, if the mA value is doubled, beam intensity will also be doubled. Hence, the detection of X-ray measurement will also be doubled. Thus, increasing the tube current (mA), reduces image noise.
- Scan (rotation) time: By changing the scan time, the duration of each measurement of X-rays detection is also changed which affects the quality of CT image. Thus, increasing the scan time, reduces image noise.
- Slice thickness: By changing the value of slice thickness, it changes the number of X-rays entering on each detector. For example, if the thickness value is doubled, beam intensity will also be doubled. Hence, the detection of X-ray measurement will also double. Thus, increasing the slice thickness, reduces the image noise.
- Peak kilovoltage (KVP): By changing the value of peak kilovoltage, it changes the number of X-rays penetrating the patient and reaching the detectors. Thus, increasing the kilovoltage, reduces image noise but can (slightly) reduce subject contrast as well.

There are two main approaches to reduce the visual noise without sacrificing the diagnostic image quality.

(1) By understanding the radiation dose and improving the dose efficiency of CT systems, the low dose CT image can be improved.
(2) In second approach, CT image quality can be improved by developing algorithms to reduce the noise from CT images. These algorithms can be further used in order to reduce the radiation dose. Generally, the process of noise suppression is known as image denoising.

3. Noise in CT reconstructed images

CT has a high contrast sensitivity characteristic which is used to differentiate among the soft tissues within the human body. This characteristic is affected by noise which harms the visualization of low contrast structure. Before image denoising, the types of source noise and general properties of noise in CT images must be known.

3.1. Types of source noise

Before considering methods for noise reduction in CT images, it is an important to get an overview of source noise. The CT image noise, however, is affected by many sources. Some majors are:

Random noise: It may arise from the detection of a finite number of X-ray quanta in the projection. It looks like a fluctuation in the image density. As a result, the change into image density is unpredictable and in random manner, this is known as random noise.

Statistical noise: The energy of X-rays are transmitted in the form of individual chunks of energy called quanta. Therefore, these finite number of X-ray quanta are detected by the X-ray detector. The number of detected X-ray quanta may differ with another measurement because of statistical fluctuation. The statistical noise in CT images may appear because of fluctuations in detecting a finite number of X-ray quanta. Statistical noise may also be called quantum noise. As more quanta are detected in each measurement, the relative accuracy of each measurement is improved. The only way to reduce the effects of statistical noise is to increase the number of detected X-ray quanta. Normally, this is achieved by increasing the number of transmitted X-rays through an increase in X-ray dose.

Electronic noise: There are electric circuits to receive analog signals which are also known as analog circuits. The process of receiving analog signals by the electronic circuits may be affected with some noise, which is referred as electronic noise. The latest CT scanners are well designed to reduce the electronic noise.

Roundoff errors: The analog signals are converted into digital signals using signal processing steps and then sent to the digital computer for CT image reconstruction. In digital computers, there are digital circuits to handle the process of discrete signals. Due to limited number of bits for storage of discrete signals in computer system, mathematical computation is not possible without roundoff. This limitation is referred as roundoff error.

3.2. Distribution of noise in CT image

Generally, noise in reconstructed CT images are introduced mainly by two reasons. First, a continuously varying error due to electrical noise or roundoff errors, can be modeled as a simple additive noise, and second reason is the possible error due to random variations in detected X-ray intensity. To differentiate tissues (soft and hard), CT numbers are defined by using Hounsfield unit (HU) [60] for CT image reconstruction. Hounsfield unit (HU) scale is displayed in Fig. 3, where some CT numbers are defined. The CT number for a given tissue is determined by the X-ray linear attenuation coefficient (LAC). Linearity is the ability of the CT image to assign the correct Hounsfield unit (HU) to a given tissue. A good linearity is essential for quantitative analysis of CT images.

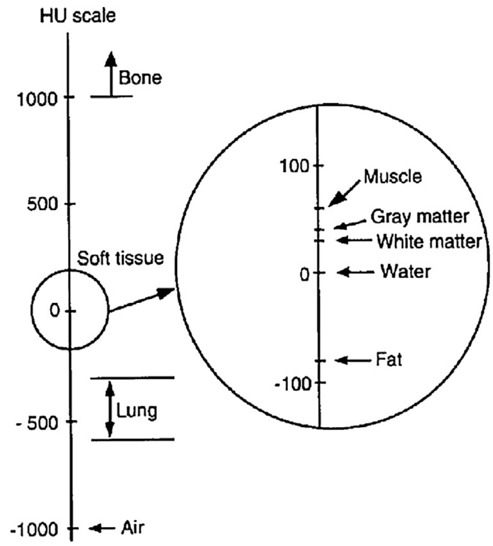

Fig. 3. Hounsfield unit (HU) scale [60].

Suppose there are two voxels of identical tissues and these voxels are transmitted into CT numbers for CT image reconstruction. Both CT numbers should be same but there is a small variation. This is a statistical variation which is considered as image noise. Because CT noise appears as fluctuations in CT numbers, a measurement of image noise is counted from these fluctuations, and such a measurement can be determined from a regions of interest (ROIs) on a CT scan images. The measurement can be performed using statistical ROI function which is calculated by the average and standard deviation (SD) of the CT numbers for the enclosed pixels. The resultant statistical distribution is most likely bell-shaped curve. The amount of noise can be determined by calculating the magnitude of random fluctuations in the CT numbers which is also know as standard deviation (SD). Larger value of SD means higher noise in CT images. This statistical distribution shows that the mean value of the noise is equal to the noise variance and it can be analyzed that the signal-to-noise ratio is proportional to the square root of the X-ray dose. Therefore, the relative amount of quantum noise at the detector elements can be decreased by increasing dose. With the known noise distribution at the detector, CT images are reconstructed using reconstruction method such as filtered back-projection method. The problem to identify noise in CT image is that all the intermediate steps, like interpolations or filtering with the convolution kernel, introduce correlations to the noisy data. Due to these dependencies, the noise distribution in the final CT image is usually unknown.

The distribution of noise in CT image can be determined by many approaches. By repeating scans of the same object can give an ideal noise distribution but impractical to perform on the patients. In another approach, noise is added to the raw data and multiple noise simulations and reconstructions are performed to analyze the distribution of noise in CT images. However, this approach is time consuming because of repeated simulations. A more elegant approach is to analyze the noise distribution analytically using a noise model at the time of the reconstruction process. In this approach, the distribution of noise in CT image can be derived by estimating the noise variance through reconstructions algorithms. However, the distribution of noise in CT image can be accurately characterized using the Poisson distribution. But for multi-detector CT (MDCT) scanner, the noise distribution is more accurately characterized by the Gaussian distribution. The literature [51,57,121,117] also confirms that the noise in CT images is generally an additive white Gaussian noise.

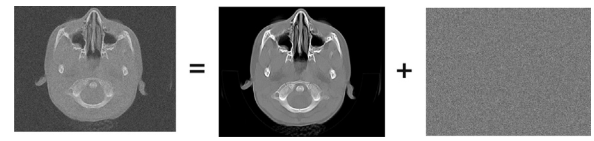

Fig. 4. A noisy image is the sum of the clean image and the noise component.

After noise analysis in CT images, CT image quality can be improved by developing algorithms to reduce the noise from CT images. These algorithms can be further used in order to reduce the radiation dose. Generally, the process of noise suppression is known as image denoising.

4. CT image denoising methods

Image denoising is the problem of finding a clean image from a noisy image [55]. In most of the cases, it is assumed that the noisy image is the summation of original image and a noise component, as shown in Fig. 4. Hence, denoising is a procedure which reduces the existing noise in an image and minimizes the loss of feature in the cleaned image. CT image denoising can be better performed with the prior knowledge about CT images and the noise. Without prior knowledge, the accuracy of CT image denoising is not effectively improved.

The goal of image denoising is to suppress the noise from the CT images with preserving clinical details so that CT image can be useful for diagnosis purpose. Digital image processing plays a key role to suppress the noise from the CT images [55,63]. Conventional smoothening and sharpening filters are the most popular filters for noise reduction in digital images. Smoothening filters are not sufficient for effective noise reduction, especially for higher noise, and it may also harm to detail parts such as edges due to smoothness factor. Sharpening filters have a tendency to reduce the noise and sharp the edges but, not impressive to preserve small details. This is the major problem of image denoising in medical image analysis because missing structures gives an inaccurate analysis of CT images which may harm to the life of a human. However, various methods have been investigated but still noise reduction is a challenging task. The main challenges for noise reduction in CT image are:

- Flat regions should be flat.
- Image boundaries should be preserved (no blurring).
- Texture details should not be lost.
- Global contrast should be preserved.
- New artifacts should not be generated.

With the invention of CT, the research on noise reduction in CT has come into existence. The very first articles [112,27] of CT image denoising were published shortly after the invention of CT, where the concept of low-pass filtering was used. They concluded that the noise is effectively reduced and also enhanced the detectability of big objects from the noisy CT image. However, these methods have major problems such as over-smoothing of edges and decreasing the detectability of small structures. Various techniques [7,99,135,136] were investigated for controlling the noise in CT scan imaging. Projection based techniques in CT works on raw data or sinogram which comes through Radon transform where noise filtering was applied on raw data using linear or non-linear filters followed by filtered back-projection (FBP) [46,75,81]. Many iterative reconstruction approaches for noise suppression in CT were also investigated [43,77] by optimizing statistical objective functions. Iterative reconstruction techniques have the advantage that the noise statistics in the projections can directly be taken into account during the reconstruction process. The disadvantage, however, is the high computational cost of iterative methods. The other popular techniques are based on post-processing approaches. The main goal of these techniques are the structure-preserving reduction of pixel noise in reconstructed CT-images and improved signal-to-noise ratio (SNR) without increasing the radiation dose. A very important requirement for any noise reduction in CT images is that all clinically relevant image contents must be preserved. Especially, edges and small structures should not be affected.

Various methods were investigated for image denoising [95,64]. Generally, images can be denoised into two domains: (i) spatial and (ii) transform domains.

4.1. Spatial domain filtering

A traditional method to remove or reduce noise from an image is spatial filtering. Spatial filters reduce noise by applying filtering directly on original noisy image. Generally, Spatial filters can be classified into linear and non-linear filters.

4.1.1. Linear and non-linear filters

Initially, in spatial domain, linear filters were used to denoise the images but it was not successful for preserving details over the images. Mean filtering was used to reduce the Gaussian noise but for high noise it produces blurry images. To overcome that, wiener filtering [93] was further used. With non-linear filters such as median filtering in spatial domain, the noise is removed without any attempt to explicitly identify it. Spatial filters employ a low pass filtering on group of pixels with the assumption that the noise occupies the higher region of frequency spectrum. Generally, linear filters are used to remove noise but blurring problem may occur [2]. Papageorgiou [103] analyzed the problem of robust regression for image denoising for both the linear and the nonlinear cases. Here, Greedy Algorithm for Robust Denoising (GARD), which is a sparse optimization technique, is derived and also analyzed the identification of the outliers and a bound on the estimation error. Author also consider nonlinear case by assuming the unknown nonlinear function belongs to a Reproducing Kernel Hilbert Space (RKHS). Finally, Kernel Greedy Algorithm for Robust Denoising (KGARD) has been proposed with the concept of GARD. The experimental results verify that KGARD method improves the process of noise reduction significantly, when outliers are present.

Naidich et al. [97] analyzed the effect of low dose on CT images and found that low dose can help to provide CT images for diagnosis purpose. However, the quality of CT images was not as good as conducted by high dose CT images. Mayo et al. [89] also tested on low dose CT images and observed that the photon detectors adapting less data will create the visual noise in the reconstructed CT images. They concluded that low dose CT images also meet with diagnostic requirements and further image processing algorithms would help to improve the quality of CT images.

4.1.2. Tikohonov, anisotropic diffusion and total variation methods

Recently, least square fidelity minimization [139–142] concept is also popular to remove the Gaussian noise from images. To minimize the least square fidelity term, many regularization functions were investigated such as Tikohonov, anisotropic diffusion and Total variation methods. Tikohonov method [68,123] is the simplest one in which data fidelity term is minimized with L_2 norm regularization, but it over smooths the detail parts of images [38,98]. To overcome that, anisotropic diffusion based methods were proposed to enhance the details of the images by stopping the diffusion at the edges, but it blurred the noisy edges [20,44]. In parallel with anisotropic diffusion [105,132], Total variation (TV) based regularization was proposed to overcome the smoothness in denoised images. From various studies, it was found that TV methods perform better for noise suppression in medical images, but it had a problem of unwanted stair-artifacts [22,110,128,129]. The advantages of TV and anisotropic diffusion were combined to improve the drawbacks of TV method and the combination is called as anisotropic total variation method [111].

Rudin-Osher-Fatemi (R.O.F.) model [111] was proposed to provide the smooth and denoised images by TV method using L_2 norm fidelity optimization. Rudin et al. [111] also discussed the computational difficulty because of non-linearity and non-differentiability. Therefore, R.O.F. model was further modified to avoid these difficulties and improved the performance of image denoising [21]. A non-differential TV method was introduced using Bregman iteration and got the success to minimize the problems. Later, Bregman iteration was improved by using linearization function [53]. Further, it was extended to minimize the L_1 regularization by introducing Split Bregman method [33,82,101,133]. Recently, various methods [15–17,19,24] have been proposed using Split Bregman method by modifying norms as well as regularization terms. Some authors also proposed the mixed models such as $L_2 - L_1$ [84,146], higher degree based image denoising [62] and gradient-based algorithms for constrained total variation [6]. However, the results of mixed model based image denoising are improved over the classical TV method and some of the state-of-the-art methods, but still blocking effects can be observed near the noisy edges. To solve the blocking effects in TV model, a method [122] was proposed based on exponentially TV method. Further, image denoising was performed by Split Bregman using exponential TV function. This method gives better noise reduction.

Bayram et al. [5] and Jiang et al [66] addressed the directional derivatives of an images and image denoising was performed on the basis of choosing directional TV function. The directional derivatives gave better edge and texture preservation [73]. Recently, total variation methods were improved such as coefficients driven based total variation [3], nonnegativity constrained based TV [76], to attenuate the noise from the noisy CT images.

4.1.3. Dictionary learning method

Mechlem et al. [91] proposed a CT image denoising scheme based on dictionary learning method. Here, denoising was performed on dual energy computed tomography images. They examined dictionary learning algorithm for dual energy CT aimed at exploiting the high spatial correlation between images obtained from different energy spectra. Both low and high energy images were divided into small patches, and for denoising, they combined the respective patches with the help of weighted mean using linear transformation. These combined patches were further processed using conventional dictionary denoising method. The experimental evaluation indicated that their proposed algorithm gave better outcomes compared to conventional dictionary denoising in terms of qualitative and quantitative image quality measures.

4.1.4. Bilateral and non-local means (NLM) filters

In recent years, Bilateral [124] and Non-local means (NLM) [14] filters are very popular for image denoising. Bilateral filter is a non-linear, edge-preserving and noise-reducing smoothing filter used for image denoising. Intensity value at each pixel in an image was replaced by a weighted average of intensity values from nearby pixels. Unlike local smoothing filters, non-local means does not update a pixel value with an average of the neighbour pixels, instead that, NLM filter updates a pixel using a weighted average of the pixels judged to be most similar where neighborhood comparisons is done within a specified search window. Both bilateral and Non-local means filters gave good results in terms of noise reduction and preserving clinical information. Giraldo et al. [52] presented a comparative study of two noise reduction methods for computed tomography images: Bilateral filter [37] and nonlocal means.

They compared both techniques against those from a commercially available weighted filtered back-projection (WFBP) method. They tested their results over real CT images as well as simulated phantom images. It was observed from result analysis that both methods are providing better denoising by weighted filtered back-projection (WFBP) method. Manduca et al. [88] presented a novel locally adaptive projection space denoising algorithm for low-dose CT image where locally adaptive bilateral sinogram filtering with a CT noise model was used. Due to bilateral filtering, the edge information in the sinogram is preserved and it helps to provide smooth denoised CT images. Li et al. [83] presented an adaptive nonlocal means filtering based on local noise level for CT denoising where a modified nonlocal means (NLM) algorithm was proposed to denoise the CT image. Due to modification in NLM filtering, the cost computational is additionally increased while the resultant image is giving better outcomes in compare to standard NLM methods. Wu et al. [134] proposed a parallelized nonlocal means denoising method where the pixels were filtered depending on the self-similarity feature of the pixel with different noise reduction levels through large-scale areas. However, this method provides sharp and smooth denoised CT images. If the image size is large, the process time of this method is also much higher. Demerit of this method is that it discards the small details from the noisy CT image and also takes high operation time. To overcome that, Hashemi et al. [59] presented a study on CT image denoising using total variation (TV) method where they analyzed that some of the small structures was removed while denoising using TV method. To overcome that, they proposed a modified nonlocal TV method, called probabilistic NLTV (PNLTV) for CT image denoising. Here, Non-locality allows the proposed method to preserve the image texture by enabling proposed stopping criterion. From the result analysis, it was analyzed that PNLTV has a capability to keep fine details unchanged and provide sharp and smooth denoised images.

Generally, the images are denoised based on the analysis of neighbour pixels. With this motivation, Okada [100] proposed a method for CT image denoising using weight neighboring pixels depending on the intensity differences. The implementation time of this method was little bit higher however it provided less smoothed CT images with higher spatial resolution. In an another approach, Lanzolla et al. [78] analyzed the effects of different noise reduction filters on computed tomography (CT) images. They introduced a denoising method based on the combination of Gaussian and Prewitt operators for CT image denoising. Simulation results indicated that their algorithm enhanced the image quality and also permitted to use low radiation dose protocol in CT examinations. Zheng et al. [144] introduced a new method for low dose CT image denoising using Pointwise Fractal Dimension where a modified pointwise fractal dimension (PWFD) function, named as pointwise box-counting dimension (PWBCD) was performed for calculating weight value of each pixel using NLM. The function was further used to calculate the variance between the two comparable windows to estimate the required weights. This method gave sharp and smooth denoised images but may lose small structures.

4.1.5. Deep learning algorithms

Recently, various image denoising concepts have been introduced using deep learning algorithms. With the revolution of deep learning algorithms in image denoising, it improves the learning of high level features of images through the hierarchical network concept. The techniques of Auto encoders, stacked sparse denoising autoencoder (SSDA) and its variant have been introduced for image denoising. They have a great ability to exploit strong spatial correlations to improve the performance of image denoising. Kang et al. [67] introduced a deeper version of convolutional neural network (CNN) using wavelet transform. They reduced the reconstruction error by limited angle tomography where filtered back projection has been used for noise reduction. Chen et al [26] proposed a method for CT image denoising using the framework of light-weight convolutional neural network. Further in extended work, Chen et al. [25] enhanced the denoising work on CT images using deconvolution network and shortcut connections into a CNN model which is referred as a residual encoder decoder convolutional neural network (RED-CNN). They performed patch based training which helps to improve the results. Gondara [54] presented medical image denoising where denoising autoencoders using convolutional layers. He concluded that higher noise is well suppressed as well as edges are well preserved where most other denoising methods would fail.

4.2. Transform domain filtering

The most investigated transform in denoising is the Wavelet Transform [87] where the input data is decomposed into its scale-space representation. Wavelet transform has been widely used in scientific and engineering fields, traditional wavelets perform well for representing point singularity. Wavelets are mathematical functions that decompose data into different frequency components. Wavelet transform is a powerful tool for signal and image processing tasks because of multi-resolution analysis, subbanding and localized in frequency and time domain.

4.2.1. Wavelet based denoising

In wavelet thresholding, two important tasks must be performed which are responsible for the results and computational complexity. First, a wavelet basis is chosen for determining its decomposing layers such as haar, db2, etc. Second, decomposition level is chosen for thresholding on all subbands for all levels. Wavelet based noise shrinkage can be expressed with the following major steps:

- Apply wavelet transform on image to obtain low and high frequency coefficients.
- Perform the denoising using following steps:
  - i. Estimate noise variance.
  - ii. Apply thresholding on detail parts.
- Apply inverse wavelet transform to obtain denoised image.

Wavelet based image denoising contains some major properties, such as:

(i) It is simple and can be computed efficiently with arbitrary kernel sizes in constant time using a local linear model.
(ii) It has the nice property of edge-preserving smoothing which can remove noise while preserving fine details and geometrical structures in the original image.
(iii) It is completely automatic that means there is no need to tune the parameters manually during denoising process.
(iv) It is flexible and can be applied in various image processing tasks, including noise reduction, detail smoothing/enhancement and compression.

Linear filters in transform domain such as Wiener filter [35,34] in the wavelet domain yield optimal results when the signal corruption can be modeled as a Gaussian process and the accuracy criterion is the mean square error (MSE) [119]. However, designing a filter based on this assumption frequently resulted in a filtered image that was more visually displeasing than the original noisy signal, even though the filtering operation successfully reduces the MSE.

The most investigated domain in denoising using Wavelet Transform is the non-linear coefficient thresholding based methods [87]. In wavelet based thresholding, a threshold value is estimated and small wavelet coefficients in high frequency subbands are removed by using the estimated threshold value and large wavelet coefficients are kept preserve. Thresholding methods were introduced to erase insignificant coefficients and to preserve larger coefficient values. Various thresholding techniques are used for noise reduction [115]. Histogram shape-based methods are used to analyze the peaks, valleys and curvatures of the smoothed histogram. Clustering-based methods are used as the gray level samples and clustered in two parts as background and foreground (object) as two Gaussian distributions. An entropy-based method is also used for thresholding. In transform-domain denoising techniques, the input data is decomposed into its scale-space representation and have a property of energy compaction. It is observed from multi resolution based denoising techniques that

(i) The noise and clean signal behave differently.
(ii) Noise is detected on geometrical components and sharp transitions of images, and
(iii) Most of the noise can be de detected from low resolution images.

The noise contains very less value which is embedded with clean pixel value to make a pixel noisy. Thresholding is one of the strategies to clean the pixels in an image. In wavelet based thresholding, small wavelet coefficients are removed in high frequency bands and large wavelet coefficients are preserved. The strategy of calculating value to differentiate between small and large wavelet coefficients are known as threshold estimation.

4.2.2. Threshold estimation

For CT image denoising, selection of a threshold value is a cumbersome task for edge preservation and noise suppression. By selecting small threshold value, the resultant image may left noisy while large threshold value may produce blurring on the edges of resultant image. To deal with this situation, an appropriate algorithm is to be selected to estimate a threshold value. Three threshold estimation criteria, called VisuShrink, SureShrink and BayesShrink, are described as follows.

VISUShrink proposed by Donoho and Johnstone [39] is a non-adaptive universal threshold, which depends only on the number of pixels in the image. The same threshold is applied to all levels of decomposition. Although the resulting estimate is very smooth and has a pleasant visual appearance, it is known that VisuShrink tends to oversmooth the signal. SureShrink is a thresholding scheme that applies a subband adaptive threshold [85]. A separate threshold is computed for each subband based on Steins unbiased risk estimator (SURE). BayesShrink [65] uses a Bayesian mathematical framework and assumes generalized Gaussian distribution for the wavelet coefficients in each detail subband to find the threshold that minimizes the Bayesian risk.

4.2.3. Shrinkage rules

After estimating a threshold value, the process of thresholding is applied by selecting an appropriate algorithm [74]. The shrinkage rule defines how the threshold must be applied. In its basic form, each coefficient in the wavelet transform domain is compared against a threshold. If the coefficient is smaller than the threshold, it is assigned zero; otherwise, it is kept or modified. The motivation is that large coefficients are due to important signal features, while small coefficients can be thresholded without affecting the significant features of the image. Hard thresholding and soft thresholding methods are very popular for thresholding. In hard threshold, each coefficient value is compared with threshold value and lesser values are replaced by zero. A disadvantage of the hard thresholding is its abrupt discontinuity, which may cause artifacts in the reconstructed image. On the other hand, the soft thresholding tends to oversmooth the reconstructed image. In Soft threshold, replaced by zero process is same as in hard threshold, additionally rest of coefficients are modified by subtracting threshold values. In comparison of both, Soft thresholding gives better performance for visual appearance of images. But soft thresholding has a limitation with large coefficient values. To overcome those limitations, an optimal linear interpolation (OLI) shrink algorithm [45] is used for thresholding. The hyperbola function [127] is another thresholding function which is also a version of soft thresholding. To avoid the drawbacks of both hard and soft thresholding rules, firm thresholding function [50] was also introduced, which provides less sensitivity to small perturbations in the data and reduces overall mean-squared error. Wavelet coefficient thresholding scheme of denoising is based on the idea that the energy of the signal to be defined concentrates on some of the wavelet coefficients, while the energy of noise spreads throughout all wavelet coefficients. Similarity between the basic wavelet and the signal to be defined plays a very important role, making it possible for the signals to concentrate on fewer coefficients. The effectiveness of thresholding depends on the estimation of an appropriate threshold.

4.2.4. Intra and inter scale dependencies based denoising

For the modification of wavelet coefficients, numerous strategies have been proposed to improve denoising performance. These strategies can be broadly categorized into two categories: (i) intra-scale dependency based denoising, and (ii) inter-scale dependency based denoising. In intra-scale dependency based denoising, the wavelet coefficients are modified with in same scale. The inter-scale dependence defines that if parent coefficients are large, then its child coefficients are also large. With this consideration, the wavelet coefficients are modified across the scale. The dependency between parent and child coefficients provides better denoising performance. Sendur and Selesnick [116] have developed a wavelet shrinkage technique based on Maximum a-Posteriori (MAP) estimator to estimate the bivariate prior of a coefficient and its parent as a circularly symmetric Laplacian model.

Moulin et al. [96] introduced a method for image denoising using Gaussian mixture model (GMM) and the Generalized Gaussian Distribution (GGD) over the wavelet coefficients. Although GGD is more accurate, GMM is simpler to use. For better enhancement of images, Chang et al. [23] introduced the use of adaptive wavelet thresholding for image denoising, by modeling the wavelet coefficients as a generalized Gaussian random variable, whose parameters are estimated locally (i.e., within a given neighborhood). Sanches et al. [113] conducted a Bayesian denoising algorithm to suppress the additive white Gaussian and multiplicative noise described by Poisson and Rayleigh distributions from CT images. The algorithm was based on the Maximum a Posteriori (MAP) criterion, and edge preserved priors which suppressed the distortion of relevant anatomical details. The main contribution of their algorithm is provided a Bayesian denoising algorithm followed by the Sylvester Lyapunov equation to suppress additive and multiplicative noise from CT images. The experimental evaluation indicates that proposed algorithm works well for noise suppression and edge preservation. Recently, Fathi et al. [45] proposed an efficient algorithm for image denoising which performs adaptive thresholding methods in the wavelet packet transform domain using optimal linear interpolation. Experiments were evaluated on medical as well as natural test images which were corrupted by various noise levels. The analysis of the results indicate that the method performs well in most of the cases and more visually pleasant than other wavelet based denoising methods. The computational cost of the proposed method is also modest. Yasuda et al. [143] proposed a new and smooth shrinkage function using noise distribution approach for CT image denoising. They analyzed that spatial frequency of noise exists in the same frequency band with the signal, it is impossible to completely remove the noise. However, wavelet shrinkage rules enables position scale processing, and it is possible to reduce noise components effectively without degrading high frequency signal components. Denoising performance of this scheme was compared with a conventional thresholding and filtering method. They also discussed that the preservability of diagnostic information and reduction of X-ray dose. They concluded that preservability of signals and denoising performance were obtained better with the proposed shrinkage function, if this method is applied to low-dose noisy images. Also conclude that reduction in the patient radiation dose can be achieved without degrading diagnostic information.

Wang et al. [130] introduced a method to improve the robustness of denoising algorithm where noise was estimated based on the concept of the pixel pattern classifier. Moreover, robust adaptive directional lifting (RADL) algorithm was performed at each pixel level to enhance the denoising performance. The denoising performance of RADL algorithm is improved in terms of edges and texture preservation. Firoiu et al. [47] presented a method based on the Bayesian approach of wavelet based thresholding for image denoising. To perform denoising, hyper analytic wavelet transform (HWT) is performed with some intermediate wavelets thresholding using bishrink filter. The results have been analyzed using different wavelet basis such as Haar, DB2, etc. However, the results are slightly different by using other wavelet basis but wavelets Daubechies-10 performed better denoising in most of the cases. Mohideen et al. [94] presented a method using multi wavelet with hard thresholding over the mammographic images. Initially, the contrast and faint details are improved with some preprocessing concepts. Image suppression has been performed where multi wavelet transforms are performed for better edge preservation. To find the noise variance, Laplacian random model is generalized at each subbands and thresholding has been performed using hard thresholding function. The performance of denoising method depends on estimated threshold value. However the results are improved but for higher noise the results are not up to the mark. Max Migonette [92] introduced a method using wavelet based thresholding for image denoising. However, the noise was suppressed effectively but blurring problem still occurs. To avoid blurring, a post processing de-convolution step has been performed.

Based on inter-scale and intra-scale dependencies in wavelet coefficients, Romberg et al. [109] analyzed that Hidden Markov Models (HMM) effectively work for inter-scale based image denoising. While Random Markov Field (RMF) models are more efficient to capture intra-scale correlations in image denoising. The complexity of local structures is not well described by Random Markov Gaussian densities whereas Hidden Markov Models can be used to capture higher order statistics. The correlations between coefficients at same scale are modeled by Hidden Markov Chain Model whereas the correlation between coefficients across the chain is modeled by Hidden Markov Trees (HMT). Once the correlation is captured by HMM, maximization expectation is used to estimate the required parameters and from those, denoised signal is estimated from noisy observation using well known MAP estimator. A disadvantage of HMT is the computational burden of the training stage. To avoid that, Malfait et al. [86] proposed a model where each neighborhood of wavelet coefficients was described as a Gaussian scale mixture (GSM) and an independent hidden random scalar multiplier. In another approach, Knas et al. [69] proposed a denoising algorithm using Gaussian filter and adaptive Prewitt Mask for computed tomography images. The process of denoising was performed on the basis of Markov Random Field (MRF) model. The results obtained from proposed scheme gives better results than standard approach of using only the MRF. It takes less time to execute the proposed algorithm.

Rabbani [107] proposed image denoising algorithm where each coefficients in each subband were suppressed using steerable pyramid with a Laplacian probability density function (PDF) with local variance. Within this framework, they performed a method for image denoising based on designing a maximum a posteriori (MAP) estimator, which relied on the zero-mean Laplacian random variables with high local correlation. Results were analyzed with several published methods such as Bayes least squared Gaussian scale mixture (BLS-GSM) technique that was a state-of-the-art denoising technique and it was observed that proposed algorithm works well for image denoising. Further, Rabbani et al. [108] extended his work and proposed a novel noise reduction algorithms that was used to improve the image quality in various medical imaging modalities such as magnetic resonance and multi-detector CT. Here, the noisy captured 3D data were first transformed by discrete complex wavelet transform. Using a nonlinear function, they have modeled the data as sum of the clean data plus additive Gaussian or Rayleigh noise. They used a mixture of bivariate Laplacian probability density functions for the clean data in the transformed domain. The MAP and minimum mean-squared error (MMSE) estimators allowed them to efficiently reduce the noise. Furthermore, they have estimated the parameters of the model using local information. Their experiments on CT images show that among their derived shrinkage functions usually BiLapGausMAP produced images with higher peak SNR.

On the basis of correlation between two similar noisy CT images, Borsdorf et al. [11] proposed wavelet based method for noise reduction over the noisy CT images. Here, the proposed algorithm works on the two input noisy CT images which are similar with CT data but differ with noise. Wavelet coefficients were obtained and by identifying correlation values, thresholding was performed for CT image denoising. They also tested correlation analysis on the basis of gradient approximation. the results indicates that noise reduction of approximately 50 percent is possible without loss of structure information. Further, Borsdorf et al. [9] extended a scheme based on a possibility to compute precise orientation dependent noise estimates for every pixel position. Here, projection noise is estimated using indirect fan-beam projection (FBP) reconstruction. Further, denoising process has been performed through bilateral filtering method using estimated noise. With this method, noise is effectively reduced in compare of bilateral filtering method. In another extended work, Borsdorf et al. [10] proposed an image denoising algorithm using wavelet transform. They work on two input CT images with the assumptions that both CT images have similar data with uncorrelated noise. Wavelet coefficients are identified both input CT images, and correlation is analyzed. They provided a framework where small correlation values were suppressed which are considered as noise while high correlation values are preserved and considered. The final noise-suppressed image was reconstructed from the averaged and weighted wavelet coefficients of the input images. The quantitative and qualitative evaluation based on phantom as well as real clinical data showed that high noise reduction rates of around 40 percent could be achieved without noticeable loss of image resolution.

Silva et al. [120] presented denoising techniques on high-determination processed tomography (HRCT) images where denoising is performed for tumor discrimination. Here, they performed Wiener filtering, geometric mean filtering and wavelet based thresholding. After analysis of this work, they concluded that wavelet denoising performs well for denoising, but it is difficult to say that wavelet denoising works well for all CT reconstructed images.

In another approach, Shih et al. [118] proposed a method to reduce the noise from the low-dose computed tomography (LDCT) images using multiresolution total variation minimization (MRTV) method. Here, discrete wavelet transform was performed over the

CT images to decompose low and high frequency wavelet coefficients. These high frequency wavelet coefficients were further processed using total variation minimization with suitable tuning parameters. The CT image reconstructed by inverse wavelet transform is the denoised CT image. The results were tested on the Shepp–Logan phantom added with Gaussian white noise and also real head CT images. They concluded that results were improved in terms of signal-to-noise ratio in compare to total variation minimization methods. In another hybrid approach, Shao et al. [117] proposed a new approach where two or more denoising methods were combined together. They analyzed two methods named: constrained least square filter algorithm and Lucy–Richardson algorithm. In analysis, they concluded that both methods were good and effective for image resolution but not efficient for denoising. Similarly, they perform denoising using non-local means filter algorithm and wavelet filter. Here, both approaches work effectively for noise suppression but have less impact to enhance resolution.

Skiadopoulos et al. [121] analyzed a comparative study between multi-scale platelet denoising methods. They applied a Butterworth filter at the stage of pre and postprocessing on image reconstruction. The comparison with and/or without noise was made on cardiac phantom containing two different size cold defects and a pilot-verified clinical dataset of 15 patients with ischemic defects reduction. They concluded that denoising by platelet and Butterworth post-processing methods for without noise attenuation conditions outperformed on Butterworth pre-processing for large size defects. In another approach, Vandeghinste et al. [126] proposed an alternative TV minimization based on split-Bregman based algorithm to perform iterative CT reconstruction using shearlet regularization. Shearlet model contains the structure in image using a non-piecewise constant image model which leads to different artifacts than in the case of TV. However, on acquired CT data, the textures are more similar to the reference texture than TV.

4.2.5. Image denoising based on extended versions of transform

Currently, transform domain is one of the popular way to denoise the images such as wavelet thresholding. The main strength of wavelet thresholding is to estimate the true signal using estimation technique and process on different frequency components of the image, separately. Discrete Wavelet Transform (DWT) has several limitations, some majors are: aliasing, shift sensitivity and poor directional selectivity. Due to large changes in wavelet coefficients and downsampling, aliasing may occurs in DWT. The inverse DWT removes this aliasing only if the wavelet and scaling coefficients are unchanged. Because of shift sensitivity, the small shifts in input signals can cause an unpredictable change in the distribution of energy between DWT coefficients at different scales. DWT cannot distinguish between +45 degree and −45 degree spectral features because of poor directional selectivity. From various research papers, it is found that directional wavelet transforms have successfully boosted the denoising performance where as two-dimensional separable wavelet transforms are not upto the mark to represent other discontinuities such as contours and edges in images. To avoid limitations of wavelet transform, there are some extended versions of transforms such as wavelet packet transform, tetrolet transform (Haar-type wavelet transform), dual-tree complex wavelet, curvelet, contourlet, shearlet and directionlet transforms.

The idea of tetrolet transform comes from a famous computer game Tetris, where five geometric patterns are used with rotation and reflection properties [71]. These geometric patterns are known as tetrominoes. The Contourlet transform also provides better results for noise reduction and edge preservation but it has less directional features than curvelets [36,72]. The Shearlet transform is similar to Contourlet except the no limitation on number of directions. However, Shearlet transform represents the edges more efficiently because long range directions. but still Shearlet fails to overcome the shift invariance problem. Therefore, Easley et al. [41] proposed nonsubsampled shearlet transform (NSST) to provide the flexible directional selectivity and shift invariance. Some techniques based on thresholding in transform domain are popular for image denoising such as a new sure approach to image denoising: a denoising algorithm via Wiener filtering in the shearlet domain [137], image denoising by non-subsampled shearlet domain multivariate model and its method noise thresholding [49], etc.

Boubchir et al. [12] analyzed that noise-free curvelet coefficients are highly correlated in local intra-band neighborhoods in compare to inter-scale and inter-orientation counterparts. The NSST coefficients have similar properties with curvelet coefficients. In order to use the statistical dependency between NSST coefficients to improve the performance of image denoising, an effective NSST-based image denoising method by the maximum a posteriori (MAP) estimator has been derived. Experimental results effectively improved with some state of arts.

4.2.6. Block-matching and 3D filtering (BM3D)

Recently Block-matching and 3D filtering (BM3D) [30] is the current state of the art algorithm for denoising images corrupted by Additive White Gaussian noise (AWGN). In [58], it improves the BM3D method with the combination of wavelet transform. To improve the edge preservation, Dai et al. [31] enhance the BM3D method by processing with tetrolet transform. Here the results are improved and enhanced the structure of small detail parts. However, Wavelet transform has a limitation of directions, which reflects as a less efficiency in sharp transitions such as line and curve singularities. To enhance this limitations, ridgelet transform has been proposed which provides the orientation of the linear edges but it fails to represent the two dimensional singularities. To overcome that, curvelet transform [18] has been came with smooth curves and better edge preservation. Various methods have been performed to enhance the quality of images using curvelet transform such as denoising of PET images [79], medical image denoising using adaptive fusion [8], image denoising using curvelet [32] and many more.

5. Comparison of CT denoising methods

For comparative study of denoising methods, the results of some existing popular methods are analyzed using two factors: (i) visual analysis, (ii) performance metrics.

The quality of denoised CT image can be analyzed by visual analysis and also by evaluating performance metrics. To measure the visual analysis, there is no mathematical or specific method available. For better visual inspection, generally, four criteria are followed: (i) visibility of the artifacts; (ii) preservation of edge details; (iii) visibility of low contrast objects, and (iv) preservation of the texture. To measure the accuracy of CT image denoising algorithms, some standard performance metrics are used, such as: root mean square error (*RMSE*), peak signal to noise ratio (*PSNR*), Structural Similarity [131] (*SSIM*), Image Quality Index (*IQI*), Entropy Difference (*ED*), Difference in Variance (*DIV*) and Gradient Magnitude Similarity Deviation (*GMSD*) [138].

A typical confusion in CT image denoising is to assume noise to be additive with a zero-mean, constant-variance Gaussian distribution or to be Poisson distributed. However, different CT reconstruction algorithms can change the distribution of noise models. Thus by Central Limit Theorem (CLT), the noise can be best modeled using the Gaussian distribution. The CLT can be applied in CT images because of addition of values from many different projections into the voxels of CT images [56,104,107,108,145]. Hence, all image denoising schemes in this paper are applied on CT images (size 512×512) corrupted by simulated Gaussian noise at four different noise level $\sigma \in [10, 20, 30, 40]$. All experiments in this work are evaluated in MATLAB 7.12 on Pentium Dual Core 2 GHz.

5.1. Measures of image denoising performance

To evaluate the performance metrics of CT image denoising, some standard methods are used, which are explained below:

SSIM between two images *X* and *R* is defined as:

$$SSIM(X, R) = \frac{(2\mu_X\mu_R + C_1)(2\sigma_{XR} + C_2)}{(\mu_X^2 + \mu_R^2 + C_1)(\sigma_X^2 + \sigma_R^2 + C_2)} \quad (1)$$

where μ_X, μ_R, σ_X^2, σ_R^2 are the averages and variances of *X* and *R* respectively, σ_{XR} is the covariance between *X* and *R*, C_1 and C_2 are predefined constants.

SSIM provides the accuracy on the basis of structural information, while *GMSD* gives the accuracy by estimating pixel-wise gradient similarity. The outcome of *SSIM* lie between −1 and 1, the values closer to 1 show better results. However, *GMSD* values lie between 0 to 1 and values closer to 0 indicate better results.

For input image (*X*) and denoised image (*R*), the Image Quality Index (*IQI*) can be defined as:

$$IQI = \frac{4\sigma_{XR}\bar{X}\bar{R}}{(\sigma_X^2 + \sigma_R^2)[(\bar{X})^2 + (\bar{R})^2)]} \quad (2)$$

where $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$, $\bar{R} = \frac{1}{N}\sum_{i=1}^{N} R_i$, $\sigma_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$, $\sigma_R^2 = \frac{1}{N-1}\sum_{i=1}^{N}(R_i - \bar{R})^2$ and $\sigma_{XR} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})(R_i - \bar{R})$.

The quality of image index range lies between 1 and −1. The best value 1 represents an identical values of input image pixels and denoised image pixels.

Peak Signal-to-noise Ratio (*PSNR*) is an important factor to evaluate denoising performance. The high *PSNR* value represents more similarity between the denoising and original image than lower PSNR values. For clean image (*X*) and denoised image (*R*), the *PSNR* is expressed as:

$$PSNR = 10 \times \log_{10}\left(\frac{255 \times 255}{MSE}\right) \quad (3)$$

where mean square error (*MSE*) is defined as- $MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[X(i, j) - R(i, j)]^2$

Root mean square error (*RMSE*) can be estimated as:

$$RMSE = \sqrt{MSE} \quad (4)$$

Entropy is a statistical measure of randomness that can be used to characterize the texture of the input images. Between clean image (*X*) and denoised image (*R*), patch wise Shannon entropy is estimated. The difference of mean value represents as Entropy Difference (*ED*). Entropy Difference (*ED*) is calculated as follows:

$$ED = SE(X) - SE(R) \quad (5)$$

where *SE* represents Shannon entropy. Shannon entropy (*SE*) [105] is calculated as follows:

$$SE = -\sum_i X_i^2 \log(X_i^2) \quad (6)$$

The performance of algorithm can also be measured by estimating *DIV*, defined as follows:

$$DIV = 1 - \frac{\sigma_R^2}{\sigma_X^2} \quad (7)$$

The values closer to 0 indicate better results of *RMSE*, *ED* and *DIV*.

5.2. A comparative study

Ai et al. [1] proposed a new scheme based on an adaptive tensor-based principal component analysis (AT-PCA) algorithm for low-dose CT image denoising. Using neighborhood pixels, patches were selected and similar patches were searched by providing training the patches for further processing. Further, tensor-based PCA was used to obtain transformation matrices where coefficients were sequentially shrunk by the linear minimum mean square error. The shrunked patches were obtained and aggregation was performed on all shrunked patches to get the denoised images. The experimental evaluation was performed on CT images and it was observed that this method can suppress the noise, enhance the edges, and improve the image quality more effectively than NLM filtering. Here, the impact of PCA affects the local edge preservation of image by defining each pixel with its nearby neighbors, which are modeled as a patch. By estimating an adaptive searching window, the similar patches are defined and very diverse structures are excluded, which is directly related to help the edge preservation. But the invariance could not be estimated through PCA unless the training data explicitly provides the detail information.

Al-Ameen et al. [3] introduced a novel scheme to attenuate the noise from CT images and provide a better visual quality using total variational denoising algorithm. They detected the noise from other image components using four new noise distinguishing coefficients and reduce it using a novel minimization function. This method have a capability to preserve the small image details, simple structure, reduces the noise in an efficient way. Over the high textured images, the blocky (staircase) effect in the image may generate.

For better edge preservation, Al-Ameen et al. [4] proposed a phase-preserving algorithm to denoise CT images. The phase preserving is an excellent noise reduction method, but it suppresses the specific details from the processed images supposing them as noise. Therefore, phase-preserving algorithm is modified using Wiener filter that uses 2D Gaussian point spread function. The advantages of this algorithm are better preservation of the minor medical details and an improvement in noise suppression. Due to the estimation of convergence is a difficult task, phase preservation denoising method can remove specific details from the higher textured noisy images supposing them as noise.

Similarly, Duan et al. [40] presented a scheme based on second order total generalized variation mode to remove the noise from medical images. Here, FFT-based split Bregman algorithm was incorporated to provide the high computational efficiency. A quantitative evaluation on the synthetic medical data was evaluated and observed that this scheme suppresses the noise better than the existing state-of-the-art noise reduction methods. This method has computational efficiency through the use of the explicit FFT-based split Bregman algorithm. it reduces noise effectively without generating staircase effects. But over the higher noisy contrast image, it losing small structural features in the image.

A cluster based dictionary learning approach has also been combined in wavelet domain such as, Ghadrdan et al. [51] proposed a new scheme based on wavelet transform where denoising was performed using clustering and dictionary learning. They extracted the most suitable features using wavelet and clustering in the CT images to obtain accurate dictionary atoms for the denoising algorithm. Here, a single image noise level estimation was developed to update the cluster by observing higher PSNRs. Due to clustering and dictionary learning, the edges are well preserved over the high textured images also. It has features to remove noise while preserving fine details and geometrical structures. There is no need to set the parameters manually during denoising process.

Trinh et al. [125] introduced an optimal weight denoising method where patch wise filtering was performed based on defined database of standard image blocks. This method depends on the standard image data-set which is not applicable for all images, especially when there is only the noisy CT images. The edges are well preserved but some noise near the edges may left during the denoising process over the higher textured images.

To gain better edge preservation, Zheng et al. [144] proposed a method for low dose CT image denoising where pointwise fractal dimension function has been modified named as pointwise box-counting dimension (PWBCD) and, was performed for calculating weight value of each pixel using NLM. The function was further used to calculate the variance between the two comparable windows to estimate the required weights. This method gave sharp and smooth denoised images but may lose small structures.

Kang et al. [67] proposed a method for image denoising using convolutional neural network (CNN) in wavelet domain. Here, the reconstruction error has been reduced by limited angle tomography where filtered back projection has been used for noise reduction. Due to limited angle tomography and filtered back projection, the edges are well preserved.

Chen et al. [25] proposed a method for CT image denoising using deconvolution network and shortcut connections into a CNN model which is referred as a residual encoder decoder convolutional neural network (RED-CNN). In this method, patch based training has been performed which helps to preserve the edges.

Gondara [54] presented medical image denoising where denoising autoencoders using convolutional layers. The results indicated that higher noise is well suppressed as well as edges are well preserved where most other denoising methods would fail.

The experiments were conducted on simulated CT images obtained from public access database (https://eddie.via.cornell.edu/cgibin/datac/logon.cgi). From result analysis, it is analyzed that spatial filter eliminates high frequency noise at the cost of blurring fine details and sharp edges in CT images. The accuracy and efficiency of the existing denoising methods for diagnosis are based on the values of the quantitative performance measures. The existing CT denoising methods are compared based on the quantitative performance metrics. The average results of some popular CT image denoising methods are shown in Tables 1 and 2.

The spatial filter eliminates high frequency noise at the expense of blurring fine details and sharp edges in the CT images i.e., these filters are inadequate in suppressing truncation artifacts. According to experimental evaluation, the results of Duan et al. [40] indicates that noise is reduced effectively without generating staircase effects in homogeneous regions. It also observed that object contours, boundaries between different tissues are not only preserved and even enhanced. It also has a tendency to erase small structures due to its edge enhancement effect. The results of Ai et al. [1] has also great potential for noise suppression and edge preservation. The noise is effectively reduced, edges are preserved, and no extra artifact is introduced. Due to sequentially reduces the noise in each pixel by a patch, the computation burden has been generated. The results of Al-Ameen et al. [3] indicates fast computation speed, a simple structure, a relatively low computational cost and preserves the small image details while reducing the noise efficiently. This algorithm not only produced a natural appearance with no visible artifacts and an accepted visual quality but it also preserved the small image details. The Al-Ameen et al. [4] helps to attenuate the noise from CT images as it helps in conserving more medical details. The results show major improvement not only in noise attenuation but also in preserving the small details. It improves the quality of noisy images and contrast is also well preserved. The main drawback is to remove specific details from the higher noisy images supposing them as noise. The results of Zheng et al. [144] indicate that noise is well suppressed. It gives sharp and smooth results. Small features such as structure are not well preserved for higher noise. Due to the impact of NLM filtering, the computation burden is increased. In [125], the weight function has been introduced which impacts to results. From experiments, it was analyzed that the noise suppression is improved but edges are not effectively preserved. In [51], the results confirm that this denoising algorithm works well for CT images, improving the visual quality of the image for diagnosis. Other than visual quality, the performance metrics also confirm

Table 1
RMSE, PSNR and SSIM of CT denoised images

σ	RMSE				PSNR				SSIM			
	10	20	30	40	10	20	30	40	10	20	30	40
[40]	8.1727	18.1708	28.0817	37.9911	29.8509	22.9115	19.1317	16.5074	0.8822	0.6992	0.6077	0.5592
[1]	8.5437	18.5204	28.4952	38.6414	29.4675	22.7472	19.0078	16.3576	0.8737	0.6963	0.6075	0.5576
[3]	9.2709	19.2453	29.1603	39.0873	28.7581	22.4145	18.7997	16.2564	0.8563	0.6871	0.6020	0.5550
[4]	9.9575	19.9490	29.9086	39.7625	28.1380	22.0987	18.5870	16.1113	0.8429	0.6776	0.5983	0.5519
[144]	7.4668	17.4150	27.4157	37.3013	30.6353	23.2813	19.3408	16.6642	0.8954	0.7154	0.6133	0.5634
[125]	7.8725	17.8067	27.7561	37.7731	30.1796	23.0866	19.2330	16.5542	0.8879	0.7050	0.6109	0.5608
[51]	7.9736	17.9260	27.8551	38.9188	30.2315	22.5284	18.5213	16.7543	0.8078	0.7053	0.6176	0.5607
[67]	7.9201	17.1245	27.1091	37.7812	30.2136	23.1238	19.1290	16.4562	0.8912	0.7217	0.6054	0.5125
[25]	7.1561	17.6313	27.2951	37.2791	30.4621	23.9812	19.2312	16.1235	0.8878	0.7109	0.6190	0.5091
[54]	7.2341	17.1903	27.1078	38.2351	30.3091	22.1094	18.3467	16.0912	0.8091	0.7209	0.6091	0.5128

Table 2
ED, DIV and GMSD of CT denoised images.

σ	ED				DIV				GMSD			
	10	20	30	40	10	20	30	40	10	20	30	40
[40]	0.6618	0.5657	0.5396	0.5233	1.4230	3.2890	5.0488	6.9309	0.0794	0.2065	0.2826	0.3269
[1]	0.6547	0.5681	0.5367	0.5228	1.4849	3.1835	4.8124	6.4410	0.0833	0.2093	0.2834	0.3289
[3]	0.6436	0.5619	0.5368	0.5222	1.5318	3.5606	5.0399	6.7116	0.0955	0.2167	0.2873	0.3313
[4]	0.6276	0.5572	0.5359	0.5227	1.6782	3.4186	5.4367	6.9713	0.1053	0.2221	0.2921	0.3328
[144]	0.6771	0.5707	0.5375	0.5236	1.4013	2.8137	4.5955	6.6144	0.0688	0.1974	0.2770	0.3240
[125]	0.6691	0.5674	0.5368	0.5237	1.4874	3.1870	4.9947	6.5835	0.0738	0.2019	0.2805	0.3285
[51]	0.6213	0.5639	0.5373	0.5239	1.4270	2.9833	4.7102	6.4088	0.0617	0.1945	0.2940	0.3286
[67]	0.6222	0.5923	0.5897	0.5236	1.4852	2.8123	4.5478	6.6252	0.0612	0.1910	0.2712	0.3238
[54]	0.6678	0.5344	0.5268	0.5131	1.4823	3.1256	4.9124	6.3532	0.0782	0.2091	0.2876	0.3289
[25]	0.6256	0.5644	0.5312	0.5276	1.4234	2.9312	4.7212	6.4834	0.0712	0.1512	0.2012	0.3261

Table 3
Summary of some CT denoising methods.

CT denoising methods	Advantages	Disadvantages
[40]	It has computational efficiency through the use of the explicit FFT-based split Bregman algorithm. It reduces noise effectively without generating staircase effects.	Losing small structural features in the image.
[1]	Local image structure is preserved by defining each pixel with its nearby neighbors, which are modeled as a patch. An adaptive searching window is estimated in which the similar patches are defined and very diverse structures are excluded.	Similar patches for training data are insufficient. The simplest invariance could not be estimated through PCA unless the training data explicitly provides the detail information. Due to iterative procedure, computation cost is increased.
[3]	Fast application time, preserves the small image details, simple structure, reduces the noise in an efficient way and low computational cost.	Erases small features (blocky (staircase) effect in the image).
[4]	Improve the quality of noisy images, contrast is also well preserved.	The main drawback is to remove specific details from the higher noisy images supposing them as noise. In phase preservation denoising, the estimation of convergence is a difficult task.
[144]	It provides sharp and smooth images.	Due to the NLM, the computation cost is higher. Small structures are considered as noise and are removed.
[125]	Provide smooth and edge preserved images	Some noise near the edges may left during the denoising process.
[51]	Wavelet transform has an excellent property of edge-preserving smoothing which can remove noise while preserving fine details and geometrical structures in the original image. There is no need to set the parameters manually during denoising process.	Some noise near the edges may left during the denoising process.
[67]	It combines a deep convolution neural network with a directional wavelet approach.	High computation burden.
[54]	Good denoising performance can be achieved using small training dataset.	It is difficult to find an optimal architecture for small sample denoising.
[25]	Great potential of deep learning for noise suppression, structural preservation, and lesion detection at a high computational speed.	High computation burden.

the improvement of CT image denoising algorithms. Every image denoising algorithm has their own advantages and disadvantages as shown in Table 3. From Table 3, it can be concluded that if we go for sharp and smooth image, it means the some small structure may be lost. Similarly, if we follow iterative concept for denoising, the blocky effects or high computations cost may increase. Hence, there is tradeoff between the edge preservation and noise reduction.

6. Conclusions

Computed Tomography is a powerful diagnostic tool in the medical science. The noise in CT images may be introduced due to various reasons. However, the main reason is acquisition and transmission. The noise can be reduced or removed using appropriate denoising filters, while reconstruction process of CT images are performed. Therefore, denoising should be performed to improve the image quality for more accurate diagnosis. There have been several published CT image denoising techniques and their advantages and limitations are discussed in this paper. To deal with noise, the denoising procedure requires a prior knowledge of the noise map and adaptability. The ultimate goal of noise reduction methods in CT is to obtain piecewise constant, or slowly varying signals in homogeneous tissue regions while preserving tissue boundaries. This paper summarizes the reconstruction of CT image, noise in CT image, CT denoising techniques and comparative analysis based on their performance which is measured using quantitative performance metrics such as PSNR, RMSE, SSIM, MSE, ED and DIV. However, a single CT image denoising method is not capable to cover all benefits in terms of noise reduction, edge preservation, robustness, user interaction, applicability to the different CT acquisition techniques, and computation cost. Day by day, the imaging techniques of CT and their types are upgraded. Hence, the denoising techniques also need to be improved. The goal of this survey is to provide an overall view of noise in CT image and available CT denoising techniques. The analysis of noise will be help to the researchers to develop new denoising techniques for CT images.

References

[1] D. Ai, J. Yang, J. Fan, W. Cong, Y. Wang, Adaptive tensor-based principal component analysis for low-dose CT image denoising, PLOS ONE 10 (5) (2015) e0126914.

[2] Z. Al-Ameen, S. Al-Ameen, G. Sulong, Latest methods of image enhancement and restoration for computed tomography: a concise review, Appl. Med. Inform. 36 (1) (2015) 1.

[3] Z. Al-Ameen, G. Sulong, Attenuating noise from computed tomography medical images using a coefficients-driven total variation denoising algorithm, Int. J. Imaging Syst. Technol. 24 (4) (2014) 350–358.

[4] Z. Al-Ameen, G. Sulong, A. Rehman, M. Al-Rodhaan, T. Saba, A. Al-Dhelaan, Phase-preserving approach in denoising computed tomography medical images, Computer Methods in Biomechanics and Biomedical Engineering: Imaging & Visualization (2014) 1–11.

[5] I. Bayram, M.E. Kamasak, Directional total variation, IEEE Signal Process. Lett. 19 (12) (2012) 781–784.

[6] A. Beck, M. Teboulle, Fast gradient-based algorithms for constrained total variation image denoising and deblurring problems, IEEE Trans. Image Process. 18 (11) (2009) 2419–2434.

[7] F.J. Beekman, C. Kamphuis, Ordered subset reconstruction for X-ray CT, Phys. Med. Biol. 46 (7) (2001) 1835.

[8] H. Bhadauria, M. Dewal, Medical image denoising using adaptive fusion of transform and total variation, Comput. Electr. Eng. 39 (5) (2013) 1451–1460.

[9] A. Borsdorf, S. Kappler, R. Raupach, J. Hornegger, Analytic noise propagation for anisotropic denoising of CT images, in: 2008 IEEE Nuclear Science Symposium Conference Record, IEEE, 2008, pp. 5335–5338.

[10] A. Borsdorf, R. Raupach, T. Flohr, J. Hornegger, Wavelet based noise reduction in CT-images using correlation analysis, IEEE Trans. Med. Imaging 27 (12) (2008) 1685–1703.

[11] A. Borsdorf, R. Raupach, J. Hornegger, Wavelet based noise reduction by identification of correlations, in: Joint Pattern Recognition Symposium, Springer, 2006, pp. 21–30.

[12] L. Boubchir, J. Fadili, Multivariate statistical modeling of images with the curvelet transform, ISSPA (2005) 747–750.

[13] R.A. Brooks, G. Di Chiro, Principles of computer assisted tomography (CAT) in radiographic and radioisotopic imaging, Phys. Med. Biol. 21 (5) (1976) 689.

[14] A. Buades, B. Coll, J.-M. Morel, A review of image denoising algorithms, with a new one, Multiscale Model. Simul. 4 (2) (2005) 490–530.

[15] J.-F. Cai, S. Osher, Z. Shen, Convergence of the linearized Bregman iteration for $\ell 1$-norm minimization, Math. Comput. 78 (268) (2009) 2127–2136.

[16] J.-F. Cai, S. Osher, Z. Shen, Linearized Bregman iterations for compressed sensing, Math. Comput. 78 (267) (2009) 1515–1536.

[17] J.-F. Cai, S. Osher, Z. Shen, Split Bregman methods and frame based image restoration, Multiscale Model. Simul. 8 (2) (2009) 337–369.

[18] E.J. Candès, D.L. Donoho, New tight frames of curvelets and optimal representations of objects with piecewise C2 singularities, Commun. Pure Appl. Math. 57 (2) (2004) 219–266.

[19] E.J. Candes, M.B. Wakin, S.P. Boyd, Enhancing sparsity by reweighted $\ell 1$ minimization, J. Fourier Anal. Appl. 14 (5-6) (2008) 877–905.

[20] F. Catté, P.-L. Lions, J.-M. Morel, T. Coll, Image selective smoothing and edge detection by nonlinear diffusion, SIAM J. Numer. Anal. 29 (1) (1992) 182–193.
[21] A. Chambolle, An algorithm for total variation minimization and applications, J. Math. Imaging Vis. 20 (1–2) (2004) 89–97.
[22] A. Chambolle, T. Pock, A first-order primal-dual algorithm for convex problems with applications to imaging, J. Math. Imaging Vis. 40 (1) (2011) 120–145.
[23] S.G. Chang, B. Yu, M. Vetterli, Adaptive wavelet thresholding for image denoising and compression, IEEE Trans. Image Process. 9 (9) (2000) 1532–1546.
[24] H. Chen, C. Wang, Y. Song, Z. Li, Split Bregmanized anisotropic total variation model for image deblurring, J. Vis. Commun. Image Represent. 31 (2015) 282–293.
[25] H. Chen, Y. Zhang, M.K. Kalra, F. Lin, P. Liao, J. Zhou, G. Wang, Low-Dose CT with a Residual Encoder–Decoder Convolutional Neural Network (RED-CNN), 2017 arXiv preprint arXiv:1702.00288.
[26] H. Chen, Y. Zhang, W. Zhang, P. Liao, K. Li, J. Zhou, G. Wang, Low-dose CT via convolutional neural network, Biomed. Opt. Express 8 (2) (2017) 679–694.
[27] E. Chew, G. Weiss, R. Brooks, G. Di Chiro, Effect of CT noise on detectability of test objects, Am. J. Roentgenol. 131 (4) (1978) 681–685.
[28] J. Choi, Measurement of noise power spectra for CT images reconstructed with different kernels, J. Converg. Inf. Technol. 8 (16) (2013) 70.
[29] A.M. Cormack, Early two-dimensional reconstruction and recent topics stemming from it, Science 209 (4464) (1980) 1482–1486.
[30] K. Dabov, A. Foi, V. Katkovnik, K. Egiazarian, et al., Image denoising with block-matching and 3D filtering, Proceedings of SPIE, vol. 6064 (2006), 606414.
[31] L. Dai, Y. Zhang, Y. Li, Image denoising using BM3D combining tetrolet prefiltering, Inf. Technol. J. 12 (10) (2013) 1995–2001.
[32] J. Deng, H. Li, H. Wu, A CT image denoise method using curvelet transform, in: Communication Systems and Information Technology, Springer, 2011, pp. 681–687.
[33] L. Denis, F. Tupin, J. Darbon, M. Sigelle, SAR image regularization with fast approximate discrete minimization, IEEE Trans. Image Process. 18 (7) (2009) 1588–1600.
[34] M. Diwakar, M. Kumar, CT image noise reduction based on adaptive Wiener filtering with wavelet packet thresholding, in: 2014 International Conference on Parallel, Distributed and Grid Computing (PDGC), IEEE, 2014, pp. 94–98.
[35] M. Diwakar, M. Kumar, Edge preservation based CT image denoising using Wiener filtering and thresholding in wavelet domain, in: 2016 Fourth International Conference on Parallel, Distributed and Grid Computing (PDGC), IEEE, 2016, pp. 332–336.
[36] M. Diwakar, M. Kumar, A hybrid method based CT image denoising using nonsubsampled contourlet and curvelet transforms, in: Proceedings of International Conference on Computer Vision and Image Processing, Springer, 2017, pp. 571–580.
[37] M. Diwakar, M. Kumar, et al., CT image denoising based on complex wavelet transform using local adaptive thresholding and bilateral filtering., in: Proceedings of the Third International Symposium on Women in Computing and Informatics, ACM, 2015, pp. 297–302.
[38] D.C. Dobson, F. Santosa, Recovery of blocky images from noisy and blurred data, SIAM J. Appl. Math. 56 (4) (1996) 1181–1198.
[39] D.L. Donoho, J.M. Johnstone, Ideal spatial adaptation by wavelet shrinkage, Biometrika 81 (3) (1994) 425–455.
[40] J. Duan, W. Lu, C. Tench, I. Gottlob, F. Proudlock, N.N. Samani, L. Bai, Denoising optical coherence tomography using second order total generalized variation decomposition, Biomed. Signal Process. Control 24 (2016) 120–127.
[41] G. Easley, D. Labate, W.-Q. Lim, Sparse directional image representations using the discrete shearlet transform, Appl. Comput. Harmon. Anal. 25 (1) (2008) 25–46.
[42] J. Eisentopf, S. Achenbach, S. Ulzheimer, C. Layritz, W. Wuest, M. May, M. Lell, D. Ropers, L. Klinghammer, W.G. Daniel, et al., Low-dose dual-source CT angiography with iterative reconstruction for coronary artery stent evaluation, JACC: Cardiovasc. Imaging 6 (4) (2013) 458–465.
[43] I.A. Elbakri, J.A. Fessler, Efficient and accurate likelihood for iterative image reconstruction in X-ray computed tomography, in: Medical Imaging 2003, International Society for Optics and Photonics, 2003, pp. 1839–1850.
[44] S. Esedoglu, S.J. Osher, Decomposition of images by the anisotropic Rudin–Osher–Fatemi model, Commun. Pure Appl. Math. 57 (12) (2004) 1609–1626.
[45] A. Fathi, A.R. Naghsh-Nilchi, Efficient image denoising method based on a new adaptive wavelet packet thresholding function, IEEE Trans. Image Process. 21 (9) (2012) 3981–3990.
[46] J.A. Fessler, E.P. Ficaro, N.H. Clinthorne, K. Lange, Grouped-coordinate ascent algorithms for penalized-likelihood transmission image reconstruction, IEEE Trans. Med. Imaging 16 (2) (1997) 166–175.
[47] I. Firoiu, A. Isar, D. Isar, A Bayesian approach of wavelet based image denoising in a hyperanalytic multi-wavelet context, WSEAS Trans. Signal Process. 6 (2010) 155–164.
[48] Y. Funama, K. Awai, O. Miyazaki, T. Goto, Y. Nakayama, M. Shimamura, K. Hiraishi, S. Hori, Y. Yamashita, Radiation dose reduction in hepatic multidetector computed tomography with a novel adaptive noise reduction filter, Radiat. Med. 26 (3) (2008) 171–177.
[49] G. Gao, Image denoising by non-subsampled shearlet domain multivariate model and its method noise thresholding, Optik 124 (22) (2013) 5756–5760.
[50] H.-Y. Gao, A.G. Bruce, Waveshrink with firm shrinkage, Stat. Sin. (1997) 855–874.
[51] S. Ghadrdan, J. Alirezaie, J.-L. Dillenseger, P. Babyn, Low-dose computed tomography image denoising based on joint wavelet and sparse representation., in: 2014 36th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, IEEE, 2014, pp. 3325–3328.
[52] J.C.R. Giraldo, Z.S. Kelm, L.S. Guimaraes, L. Yu, J.G. Fletcher, B.J. Erickson, C.H. McCollough, Comparative study of two image space noise reduction methods for computed tomography: bilateral filter and nonlocal means., in: 2009 Annual International Conference of the IEEE Engineering in Medicine and Biology Society, IEEE, 2009, pp. 3529–3532.
[53] T. Goldstein, S. Osher, The split Bregman method for L1-regularized problems, SIAM J. Imaging Sci. 2 (2) (2009) 323–343.
[54] L. Gondara, Medical image denoising using convolutional denoising autoencoders., in: 2016 IEEE 16th International Conference on Data Mining Workshops (ICDMW), IEEE, 2016, pp. 241–246.
[55] R.C. Gonzalez, R.E. Woods, et al., Digital Image Processing, 2002.
[56] P. Gravel, G. Beaudoin, J.A. De Guise, A method for modeling noise in medical images, IEEE Trans. Med. Imaging 23 (10) (2004) 1221–1232.
[57] H. Hanzouli, J. Lapuyade-Lahorgue, E. Monfrini, G. Delso, W. Pieczynski, D. Visvikis, M. Hatt, PET/CT image denoising and segmentation based on a multi observation and a multi scale Markov tree model., in: 2013 IEEE Nuclear Science Symposium and Medical Imaging Conference (2013 NSS/MIC), IEEE, 2013, pp. 1–4.
[58] M.M. Hasan, Adaptive Edge-Guided Block-Matching and 3D Filtering (BM3D) Image Denoising Algorithm, 2014.
[59] S. Hashemi, S. Beheshti, R.S. Cobbold, N.S. Paul, Non-local total variation based low-dose computed tomography denoising., in: 2014 36th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, IEEE, 2014, pp. 1083–1086.
[60] G.N. Hounsfield, Computerized transverse axial scanning (tomography): Part 1. Description of system, Br. J. Radiol. 46 (552) (1973) 1016–1022.
[61] J. Hsieh, Computed Tomography: Principles, Design, Artifacts, and Recent Advances, SPIE Bellingham, WA, 2009.
[62] Y. Hu, M. Jacob, Higher degree total variation (HDTV) regularization for image recovery, IEEE Trans. Image Process. 21 (5) (2012) 2559–2571.
[63] A.K. Jain, Fundamentals of Digital Image Processing, Prentice-Hall, Inc, 1989.
[64] P. Jain, V. Tyagi, A survey of edge-preserving image denoising methods, Inf. Syst. Front. 18 (1) (2016) 159–170.
[65] M. Jansen, A. Bultheel, Empirical Bayes approach to improve wavelet thresholding for image noise reduction, J. Am. Stat. Assoc. (2011).
[66] W. Jiang, H. Cui, F. Zhang, Y. Rong, Z. Chen, Oriented total variation $\ell 1/2$ regularization, J. Vis. Commun. Image Represent. 29 (2015) 125–137.
[67] E. Kang, J. Min, J.C. Ye, A Deep Convolutional Neural Network Using Directional Wavelets for Low-Dose X-Ray CT Reconstruction, 2016 arXiv preprint arXiv:1610.09736.
[68] A.K. Katsaggelos, Digital Image Restoration, Springer Publishing Company, Incorporated, 2012.
[69] M. Knas, R. Cierniak, Computed tomography images denoising with Markov random field model parameterized by Prewitt Mask, in: Image Processing & Communications Challenges 6, Springer, 2015, pp. 53–58.
[70] M. Knaup, M. Kachelrieb, W.A. Kalender, 3D and 4D imaging from multi-threaded cone-beam CT scans., in: IEEE Nuclear Science Symposium Conference Record, 2005, vol. 4, IEEE, 2005, p. 5.
[71] M. Kumar, M. Diwakar, CT image denoising using locally adaptive shrinkage rule in tetrolet domain, J. King Saud Univ. Comput. Inf. Sci. (2016).
[72] M. Kumar, M. Diwakar, Edge preservation based CT image denoising using wavelet and curvelet transforms, in: Proceedings of Fifth International Conference on Soft Computing for Problem Solving, Springer, 2016, pp. 771–782.
[73] M. Kumar, M. Diwakar, A new exponentially directional weighted function based CT image denoising using total variation, J. King Saud Univ. Comput. Inf. Sci. (2016).
[74] M. Kumar, M. Diwakar, A new locally adaptive patch variation based CT image denoising, Int. J. Image Graph. Signal Process. 8 (1) (2016) 43.
[75] P.J. La Rivière, J. Bian, P.A. Vargas, Penalized-likelihood sinogram restoration for computed tomography, IEEE Trans. Med. Imaging 25 (8) (2006) 1022–1036.
[76] G. Landi, E.L. Piccolomini, An efficient method for nonnegatively constrained total variation-based denoising of medical images corrupted by Poisson noise, Comput. Med. Imaging Graph. 36 (1) (2012) 38–46.
[77] K. Lange, J.A. Fessler, Globally convergent algorithms for maximum a posteriori transmission tomography, IEEE Trans. Image Process. 4 (10) (1995) 1430–1438.
[78] A. Lanzolla, G. Andria, F. Attivissimo, G. Cavone, M. Spadavecchia, T. Magli, Denoising filter to improve the quality of CT images, in: Instrumentation and Measurement Technology Conference, 2009, I2MTC'09, IEEE, 2009, pp. 947–950.
[79] A. Le Pogam, H. Hanzouli, M. Hatt, C.C. Le Rest, D. Visvikis, Denoising of PET images by combining wavelets and curvelets for improved preservation of resolution and quantitation, Med. Image Anal. 17 (8) (2013) 877–891.
[80] S.-W. Lee, C.-L. Lee, H.-M. Cho, H.-S. Park, D.-H. Kim, Y.-N. Choi, H.-J. Kim, Effects of reconstruction parameters on image noise and spatial resolution

in cone-beam computed tomography, J. Korean Phys. Soc. 59 (4) (2011) 2825–2832.
[81] T. Li, X. Li, J. Wang, J. Wen, H. Lu, J. Hsieh, Z. Liang, Nonlinear sinogram smoothing for low-dose X-ray CT, IEEE Trans. Nucl. Sci. 51 (5) (2004) 2505–2513.
[82] W. Li, Q. Li, W. Gong, S. Tang, Total variation blind deconvolution employing split Bregman iteration, J. Vis. Commun. Image Represent. 23 (3) (2012) 409–417.
[83] Z. Li, L. Yu, J.D. Trzasko, D.S. Lake, D.J. Blezek, J.G. Fletcher, C.H. McCollough, A. Manduca, Adaptive nonlocal means filtering based on local noise level for CT denoising, Med. Phys. 41 (1) (2014) 011908.
[84] Y. Lou, T. Zeng, S. Osher, J. Xin, A weighted difference of anisotropic and isotropic total variation model for image processing, SIAM J. Imaging Sci. 8 (3) (2015) 1798–1823.
[85] F. Luisier, T. Blu, M. Unser, A new SURE approach to image denoising: interscale orthonormal wavelet thresholding, IEEE Trans. Image Process. 16 (3) (2007) 593–606.
[86] M. Malfait, D. Roose, Wavelet-based image denoising using a Markov random field a priori model, IEEE Trans. Image Process. 6 (4) (1997) 549–565.
[87] S.G. Mallat, A theory for multiresolution signal decomposition: the wavelet representation, IEEE Trans. Pattern Anal. Mach. Intell. 11 (7) (1989) 674–693.
[88] A. Manduca, L. Yu, J.D. Trzasko, N. Khaylova, J.M. Kofler, C.M. McCollough, J.G. Fletcher, Projection space denoising with bilateral filtering and CT noise modeling for dose reduction in CT, Med. Phys. 36 (11) (2009) 4911–4919.
[89] J. Mayo, T.E. Hartman, K.S. Lee, S. Primack, S. Vedal, N. Müller, CT of the chest: minimal tube current required for good image quality with the least radiation dose, Am. J. Roentgenol. 164 (3) (1995) 603–607.
[90] P. McLaughlin, K. Murphy, S. Hayes, K. Carey, J. Sammon, L. Crush, F. O'Neill, B. Normoyle, A. McGarrigle, J. Barry, et al., Non-contrast CT at comparable dose to an abdominal radiograph in patients with acute renal colic; impact of iterative reconstruction on image quality and diagnostic performance, Insights Imaging 5 (2) (2014) 217–230.
[91] K. Mechlem, S. Allner, K. Mei, F. Pfeiffer, P.B. Noël, Dictionary-based image denoising for dual energy computed tomography, SPIE Medical Imaging, International Society for Optics and Photonics (2016), 97830E.
[92] M. Mignotte, A post-processing deconvolution step for wavelet-based image denoising methods, IEEE Signal Process. Lett. 14 (9) (2007) 621.
[93] J. Mohan, V. Krishnaveni, Y. Guo, A survey on the magnetic resonance image denoising methods, Biomed. Signal Process. Control 9 (2014) 56–69.
[94] K. Mohideen, A. Perumal, N. Krishnan, M. Sathik, Image denoising and enhancement using multiwavelet with hard threshold in digital mammographic images, Int. Arab. J. e-Technol. 2 (1) (2011) 49–55.
[95] M.C. Motwani, M.C. Gadiya, R.C. Motwani, F.C. Harris, Survey of image denoising techniques., Proceedings of GSPX (2004) 27–30.
[96] P. Moulin, J. Liu, Analysis of multiresolution image denoising schemes using generalized Gaussian and complexity priors, IEEE Trans. Inf. Theory 45 (3) (1999) 909–919.
[97] D.P. Naidich, C.H. Marshall, C. Gribbin, R.S. Arams, D.I. McCauley, Low-dose CT of the lungs: preliminary observations, Radiology 175 (3) (1990) 729–731.
[98] M. Nikolova, Local strong homogeneity of a regularized estimator, SIAM J. Appl. Math. 61 (2) (2000) 633–658.
[99] J. Nuyts, B. De Man, P. Dupont, M. Defrise, P. Suetens, L. Mortelmans, Iterative reconstruction for helical CT: a simulation study, Phys. Med. Biol. 43 (4) (1998) 729.
[100] M. Okada, Noise evaluation and filter design in CT images, IEEE Trans. Biomed. Eng. 9 (1985) 713–719.
[101] S. Osher, M. Burger, D. Goldfarb, J. Xu, W. Yin, An iterative regularization method for total variation-based image restoration, Multiscale Model. Simul. 4 (2) (2005) 460–489.
[102] T. Pan, Computed tomography: from photon statistics to modern cone-beam CT, J. Nucl. Med. 50 (7) (2009), 1194.
[103] G. Papageorgiou, Robust Algorithms for Linear and Nonlinear Regression via Spare Modeling Methods: Theory, Algorithms and Applications to Image Denoising (PhD thesis), 2016.
[104] A. Perelli, M. Lexa, A. Can, M.E. Davies, Denoising message passing for X-ray computed tomography reconstruction., IEEE Trans. Comput. Imaging (2016).
[105] P. Perona, J. Malik, Scale-space and edge detection using anisotropic diffusion, IEEE Trans. Pattern Anal. Mach. Intell. 12 (7) (1990) 629–639.
[106] S. Punwani, J. Zhang, W. Davies, R. Greenhalgh, P. Humphries, Paediatric CT: the effects of increasing image noise on pulmonary nodule detection, Pediatr. Radiol. 38 (2) (2008) 192–201.
[107] H. Rabbani, Image denoising in steerable pyramid domain based on a local Laplace prior, Pattern Recognit. 42 (9) (2009) 2181–2193.
[108] H. Rabbani, R. Nezafat, S. Gazor, Wavelet-domain medical image denoising using bivariate Laplacian mixture model, IEEE Trans. Biomed. Eng. 12 (56) (2009) 2826–2837.
[109] J.K. Romberg, H. Choi, R.G. Baraniuk, Bayesian wavelet-domain image modeling using hidden Markov trees., in: 1999 International Conference on Image Processing, 1999, ICIP 99, Proceedings, vol. 1, IEEE, 1999, pp. 158–162.
[110] L.I. Rudin, S. Osher, Total variation based image restoration with free local constraints, in: IEEE International Conference, Image Processing, 1994, Proceedings, ICIP-94, vol. 1, IEEE, 1994, pp. 31–35.
[111] L.I. Rudin, S. Osher, E. Fatemi, Nonlinear total variation based noise removal algorithms, Phys. D: Nonlinear Phenom. 60 (1) (1992) 259–268.
[112] R. Rutherford, B. Pullan, I. Isherwood, Measurement of effective atomic number and electron density using an EMI scanner, Neuroradiology 11 (1) (1976) 15–21.
[113] J.M. Sanches, J.C. Nascimento, J.S. Marques, Medical image noise reduction using the Sylvester–Lyapunov equation, IEEE Trans. Image Process. 17 (9) (2008) 1522–1539.
[114] M. Sandborg, Computed Tomography: Physical Principles and Biohazards, 1995.
[115] B. Sankur, M. Sezgin, Image thresholding techniques: a survey over categories, Pattern Recognit. 34 (2) (2001) 1573–1583.
[116] L. Sendur, I.W. Selesnick, Bivariate shrinkage functions for wavelet-based denoising exploiting interscale dependency, IEEE Trans. Signal Process. 50 (11) (2002) 2744–2756.
[117] W.-j. Shao, J. Ni, C. Zhu, A hybrid method of image restoration and denoise of CT images., in: 2012 Sixth International Conference on Internet Computing for Science and Engineering, IEEE, 2012, pp. 117–121.
[118] C.-T. Shih, S.-J. Chang, Y.-l. Liu, J. Wu, Noise reduction of low-dose computed tomography using the multi-resolution total variation minimization algorithm, in: SPIE Medical Imaging, International Society for Optics and Photonics, 2013, 86682H.
[119] P.-L. Shui, Z.-F. Zhou, J.-X. Li, Image denoising algorithm via best wavelet packet base using Wiener cost function, IET Image Process. 1 (3) (2007) 311–318.
[120] J.S. Silva, A. Silva, B.S. Santos, Image denoising methods for tumor discrimination in high-resolution computed tomography, J. Dig. Imaging 24 (3) (2011) 464–469.
[121] S. Skiadopoulos, A. Karatrantou, P. Korfiatis, L. Costaridou, P. Vassilakos, D. Apostolopoulos, G. Panayiotakis, Evaluating image denoising methods in myocardial perfusion single photon emission computed tomography (SPECT) imaging, Meas. Sci. Technol. 20 (10) (2009) 104023.
[122] C. Sun, C. Tang, X. Zhu, H. Ren, Exponential total variation model for noise removal, its numerical algorithms and applications, AEU Int. J. Electron. Commun. 69 (3) (2015) 644–654.
[123] A. Tikhonov, V.Y. Arsenin, Methods for Solving Ill-Posed Problems, John Wiley and Sons, Inc, 1977.
[124] C. Tomasi, R. Manduchi, Bilateral filtering for gray and color images., in: Sixth International Conference on Computer Vision, 1998, IEEE, 1998, pp. 839–846.
[125] D.H. Trinh, M. Luong, J.-M. Rocchisani, C.D. Pham, H.D. Pham, F. Dibos, An optimal weight method for CT image denoising, J. Electron. Sci. Technol. 10 (2) (2012) 124–129.
[126] B. Vandeghinste, B. Goossens, R. Van Holen, C. Vanhove, A. Pižurica, S. Vandenberghe, S. Staelens, Iterative CT reconstruction using shearlet-based regularization, IEEE Trans. Nucl. Sci. 60 (5) (2013) 3305–3317.
[127] B. Vidakovic, Nonlinear wavelet shrinkage with Bayes rules and Bayes factors, J. Am. Stat. Assoc. 93 (441) (1998) 173–179.
[128] C.R. Vogel, A multigrid method for total variation-based image denoising, in: Computation and Control IV, Springer, 1995, pp. 323–331.
[129] C.R. Vogel, M.E. Oman, Iterative methods for total variation denoising, SIAM J. Sci. Comput. 17 (1) (1996) 227–238.
[130] X. Wang, G. Shi, Y. Niu, L. Zhang, Robust adaptive directional lifting wavelet transform for image denoising, IET Image Process. 5 (3) (2011) 249–260.
[131] Z. Wang, A.C. Bovik, H.R. Sheikh, E.P. Simoncelli, Image quality assessment: from error visibility to structural similarity, IEEE Trans. Image Process. 13 (4) (2004) 600–612.
[132] J. Weickert, Anisotropic Diffusion in Image Processing, vol. 1, Teubner Stuttgart, 1998.
[133] Y. Wotao, O. Stanley, G. Donald, D. Jerome, Bregman iterative algorithms for $\ell 1$-minimization with applications to compressed sensing, SIAM J. Imaging Sci. 1 (1) (2008) 143–168.
[134] H. Wu, W. Zhang, D. Gao, X. Yin, Y. Chen, W. Wang, Fast CT image processing using parallelized non-local means, J. Med. Biol. Eng. 31 (6) (2011) 437–441.
[135] F. Xu, K. Mueller, Accelerating popular tomographic reconstruction algorithms on commodity PC graphics hardware, IEEE Trans. Nucl. Sci. 52 (3) (2005) 654–663.
[136] F. Xu, K. Mueller, Real-time 3D computed tomographic reconstruction using commodity graphics hardware, Phys. Med. Biol. 52 (12) (2007) 3405.
[137] P. Xu, Q. Miao, X. Tang, J. Zhang, A denoising algorithm via Wiener filtering in the shearlet domain, Multimedia Tools Appl. 71 (3) (2014) 1529–1558.
[138] W. Xue, L. Zhang, X. Mou, A.C. Bovik, Gradient magnitude similarity deviation: a highly efficient perceptual image quality index, IEEE Trans. Image Process. 23 (2) (2014) 684–695.
[139] J. Yan, W.-S. Lu, New algorithms for sparse representation of discrete signals based on ℓ_p–ℓ_2 optimization, in: 2011 IEEE Pacific Rim Conference on Communications, Computers and Signal Processing (PacRim), IEEE, 2011, pp. 73–78.
[140] J. Yan, W.-S. Lu, Power-iterative strategy for lp-l2 optimization for compressive sensing: towards global solution., in: 2011 Conference Record of the Forty Fifth Asilomar Conference on Signals, Systems and Computers (ASILOMAR), IEEE, 2011, pp. 1153–1157.
[141] J. Yan, W.-S. Lu, Smoothed ℓ_p–ℓ_2 solvers for signal denoising, in: 2012 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), IEEE, 2012, pp. 3801–3804.
[142] J. Yan, W.-S. Lu, Image denoising by generalized total variation regularization and least squares fidelity, Multidimens. Syst. Signal Process. 26 (1) (2015) 243–266.

[143] N. Yasuda, Y. Kodera, Design of a noise-dependent shrinkage function in wavelet shrinkage of X-ray CT image, Int. J. Comput. Assisted Radiol. Surg. 4 (4) (2009) 353–366.
[144] X. Zheng, Z. Liao, S. Hu, M. Li, J. Zhou, Improving spatial adaptivity of nonlocal means in low-dosed CT imaging using pointwise fractal dimension., Computational and Mathematical Methods in Medicine 2013 (2013).
[145] F. Zhu, T. Carpenter, D.R. Gonzalez, M. Atkinson, J. Wardlaw, Computed tomography perfusion imaging denoising using Gaussian process regression, Phys. Med. Biol. 57 (12) (2012) N183.
[146] M. Zibulevsky, M. Elad, L1–L2 optimization in signal and image processing, IEEE Signal Process. Mag. 27 (3) (2010) 76–88.