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Exemplo (19)

$f(r,\theta) = f(x,y)$ onde $x = r\cos\theta$ e $y = r\,\text{sen}\,\theta$. Sendo $f(x,y)$ uma função diferenciável dada. Verifique que

$$\frac{\partial f}{\partial y}(x,y) = \frac{\cos\theta}{r}\frac{\partial f}{\partial\theta} + \text{sen}\,\theta\,\frac{\partial f}{\partial r}(r,\theta)$$

Solução

$$\frac{\partial f}{\partial r}(r,\theta) = \frac{\partial f}{\partial x}(x,y)\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}(x,y)\frac{\partial y}{\partial r}$$

ou

(1) $$\frac{\partial f}{\partial r}(r,\theta) = \cos\theta\,\frac{\partial f}{\partial x}(x,y) + \text{sen}\,\theta\,\frac{\partial f}{\partial y}(x,y)$$

$$\frac{\partial f}{\partial\theta}(r,\theta) = \frac{\partial f}{\partial x}(x,y)\frac{\partial x}{\partial\theta} + \frac{\partial f}{\partial y}(x,y)\frac{\partial y}{\partial\theta}$$

ou $$\frac{\partial f}{\partial\theta}(r,\theta) = -r\,\text{sen}\,\theta\,\frac{\partial f}{\partial x}(x,y) + r\cos\theta\,\frac{\partial f}{\partial y}(x,y)$$

ou

(2) $$\frac{1}{r}\frac{\partial f}{\partial\theta}(r,\theta) = -\text{sen}\,\theta\,\frac{\partial f}{\partial x}(x,y) + \cos\theta\,\frac{\partial f}{\partial y}(x,y)$$

Multiplicando (1) por $\text{sen}\,\theta$, (2) por $\cos\theta$ e somando membro a membro obtemos a relação que queríamos

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① ⓐ $z = \text{sen}\, xy$

$x = 3t$ $\quad \frac{Dz}{Dt} = -\text{sen}\, xy$

$y = t^2$

$= \text{sen}(3t\ t^2)$

$U = 3t^{3}$ $\quad = \text{sen}(3t^3)$

$= \cos(3t^3)\ \frac{D}{Dt}(3t^3)$

$\frac{D}{DU} = \text{sen}(U) \to \cos(U)$

$= 3\left(\frac{D}{Dt}(t^3)\right) \cos(3t^3)$

$= 3 \cdot 3t^2 \cdot \cos(3t^3)$

$= 9t^2 \cos(3t^3)$

ⓑ $z = x^2 + 3y^2$

$x = \text{sen}\ t$ $\quad \frac{Dz}{Dt} = x^2 + 3y^2 = \text{sen}(t)^2 + 3\cos(t)^2$

$y = \cos\ t$

Substituir

$\frac{DU^2}{DU}\ \frac{DU}{Dt}$

$U = \cos(t)$

$\frac{D}{Dt} = U^2 \to 2U$

$= \text{sen}(t)^2 + \underline{3\cos(t)^2}$

$= 3 \cdot 2\cos(t)\left(\frac{D}{Dt}(\cos(t))\right)$

$= 6\cos(t)\left(\frac{D}{Dt}(\cos(t))\right)$

$= -\left(\frac{D}{Dt}(t)\right)\text{sen}(t)\ 6\cos(t)$

Substituir

$\frac{D\cos(U)}{DU}\ \frac{DU}{Dt}$

$= -1 \cdot 6\cos(t)\,\text{sen}(t)$

$= -6\cos(t)\,\text{sen}(t)$

$U = t$

$\frac{D}{DU} = \cos(U) \to -\text{sen}(U)$

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$= -6\cos(t)\,sen(t) + \underbrace{sen(t)^2}$

$= 2\left(\frac{D}{Dt}(sen(t))\right) sen(t)$

$= \cos(t)\left(\frac{D}{Dt}(t)\right) 2\,sen(t)$

$= 1\; 2\cos(t)\; sen(t)$

$= 2\cos(t)\; sen(t)$

$= -6\cos(t)\,sen(t) + \cancel{2\cos(t)}\; sen(t)$

$= -4\cos(t)\; sen(t)$

Substituir

$\frac{Du^2}{Du}\; \frac{Du}{Dt}$

$U = sen(t)$

$\frac{D}{Du}(U^2) = 2U$

Substituir

$\frac{D\,sen(U)}{DU}\; \frac{DU}{Dt}$

$U = sen(t)$

$\frac{D}{DU}(sen(U)) = \cos(U)$

Ⓒ $z = \ln(1 + x^2 + y^2)$

$x = sen\; 3t$

$y = \cos\; 3t$

$\frac{Dz}{Dt} = \ln(1 + x^2 + y^2$

$= \ln(1 + sen(3t)^2 + \cos(3t)^2)$

$= 0$

③ ⓐ $x = t^2$
$y = 3t$ $\quad = 2t \dfrac{\partial f}{\partial x}(t^2, 3t) + 3 \dfrac{\partial f}{\partial y}(t^2, 3t)$

ⓑ $x = sen(3t)$
$y = \cos(2t)$ $\quad = 3\cos 3t \dfrac{\partial f}{\partial x}(x,y) - 2\, sen\, 2t \dfrac{\partial t}{\partial y}(x,y)$

④ $f(t^2, 2t) = t^3 - 3t \quad = 2t \dfrac{\partial f}{\partial x}(t^2, 2t) + 2 \dfrac{\partial f}{\partial y}(t^2, 2t) = 3t^2 - 3$

① ⓐ $G(t) = \dfrac{\partial f}{\partial x}(x,y)$, $x = t^2$, $y = sen\, t$

$= 2t \dfrac{\partial^2 f}{\partial x^2}(x,y) + \cos t \dfrac{\partial^2 f}{\partial y \partial x}(x,y)$

ⓑ $G(t) = t^3 \dfrac{\partial f}{\partial x}(3t, 2t)$

$= 3t^2 \dfrac{\partial f}{\partial x}(3t, 2t) + t^3 \left[3 \dfrac{\partial^2 f}{\partial x^2}(3t, 2t) + 2 \dfrac{\partial^2 f}{\partial y \partial x}(3t, 2t) \right]$

ⓒ $G(t) = \dfrac{\partial f}{\partial x}(t^2, 2t) + 5 \dfrac{\partial f}{\partial y}(sen\, 3t, t)$

$= 2t \dfrac{\partial^2 f}{\partial x^2}(t^2, 2t) + 2 \dfrac{\partial^2 f}{\partial y \partial x}(t^2, 2t) + 5 \left[3\cos 3t \dfrac{\partial^2 f}{\partial x \partial y}(sen\, 3t, t) + \right.$

$\left. + \dfrac{\partial^2 f}{\partial y}(sen\, 3t, t) \right]$

② $G'(t) \qquad G(t) = (5t, 4t)$

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$$G'(t) = 5\frac{\partial f}{\partial x}(x,y) + 4\frac{\partial f}{\partial y}(x,y)$$

$$G''(t) = 25\frac{\partial^2 f}{\partial x^2} + 40\frac{\partial^2 f}{\partial x \partial y}(x,y) + 16\frac{\partial^2 f}{\partial y^2}(x,y)$$

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