

# Lista 8

① a)  $x^2 + 3xy + 4y^2 - 6x + 2y$

$$\frac{\partial}{\partial x} = 2x + 3y - 6$$

$$\frac{\partial}{\partial y} = 3x + 8y + 2$$

$$\begin{cases} 2x + 3y - 6 = 0 \\ 3x + 8y + 2 = 0 \end{cases} \rightarrow \begin{matrix} 6x + 9y - 18 \\ -6x - 16y - 4 \end{matrix} \rightarrow \begin{matrix} -7y - 22 \\ -7 \end{matrix} \rightarrow y = \frac{22}{7}$$

$$\begin{pmatrix} 54 & -22 \\ 7 & 7 \end{pmatrix} \rightarrow \begin{matrix} 16x + 24y - 48 \\ -9x - 24y - 6 \end{matrix} \rightarrow \begin{matrix} 7x - 54 \\ 7 \end{matrix} \rightarrow x = \frac{54}{7}$$

minimo local

②  $x^2 + y^2 + xy - 3x - 4y + 5$

$$\frac{\partial}{\partial x} = 2x + y - 3$$

$$\frac{\partial}{\partial y} = 3y^2 + x - 4$$

$$\frac{\partial^2}{\partial x^2} = 1$$

$$\frac{\partial^2}{\partial y^2} = 6y \quad (1,1)$$

③  $x^2 + 2xy + y^2 - 5x$

$$\frac{\partial}{\partial x} = 2x + 2y - 5 \rightarrow \frac{\partial}{\partial y} = 2x + 2y - 5 \quad (-1,1)$$

$$\frac{\partial^2}{\partial x^2} = 2 \rightarrow \frac{\partial^2}{\partial y^2} = 2$$

①  $-x^2 + y^2 + 2xy + 4x - 2y$

$\frac{\partial}{\partial x} = -2x + 2y + 4$

$\begin{cases} -2x + 2y + 4 = 2(-x + y + 2) = -x + y + 2 \\ 2x + 2y - 2 = 2(x + y - 1) = x + y - 1 \end{cases}$

$\frac{\partial}{\partial y} = 2y + 2x - 2$

$\begin{cases} -x + y + 2 = 2y + 1 + y = 1 \\ x + y - 1 = -2 \end{cases}$

$\begin{pmatrix} 2 & -1 \\ 2 & 2 \end{pmatrix}$

$\begin{cases} -x + y + 2 = -2x + 3 + x = 3 \\ -x + y + 1 = 2 \end{cases}$

②  $x^3 - 3x^2y + 27y$

$\frac{\partial}{\partial x} = 3x^2 - 6xy$

$\begin{pmatrix} 3 & 3 \\ & 2 \end{pmatrix}$

$\frac{\partial}{\partial y} = -3x^2 + 27$

$\begin{pmatrix} -3 & -3 \\ & 2 \end{pmatrix}$

③  $x^2 - 4xy + 4y^2 - x + 3y + 1$

$\frac{\partial}{\partial x} = 2x - 4y - 1$

Não admite ponto crítico

$\frac{\partial}{\partial y} = 4x + 8y + 3$

④  $\sqrt{x^2 + 2xy + 4y^2} - 6x - 12y$

(2,1) pontos de mínimos locais

⑤  $x^4 + y^4 - 2x^2 - 2y^2$

$\frac{\partial}{\partial x} = 4x^3 - 4x$

$$\frac{\partial}{\partial y} = 4y^3 - 4y \quad (0,0) \text{ ponto de máximo local}$$

$$\textcircled{1} x^4 + xy + y^2 - 6x - 5y$$

$$\frac{\partial}{\partial x} = 4x^3 + y - 6 \quad \begin{cases} 4x^3 + y - 6 \\ x + 2y - 5 \end{cases}$$

$$\frac{\partial}{\partial y} = x + 2y - 5 \quad (1,2) \text{ ponto de mínimo local}$$

$$\textcircled{2} x^4 + y^4 + 4x + 4y$$

$$\frac{\partial}{\partial x} = 4x^3 + 4 \quad \frac{\partial}{\partial y} = 4y^3 + 4 \quad (-1,-1) \text{ ponto de mínimo local}$$

$$\frac{\partial}{\partial y} = 4y^3 + 4$$

$$\textcircled{1} x^5 + y^5 - 5x - 5y \quad (1,1) \text{ e' o ponto de mínimo local}$$

$$\frac{\partial}{\partial x} = 5x^4 - 5$$

$$\frac{\partial}{\partial y} = 5y^4 - 5$$

$$\textcircled{m} \frac{1}{x^2} + \frac{1}{y} + xy \quad x > 0 \quad y > 0$$



③ a)  $x^2 + 2xy + 2y^2 - x + 2y$

$\frac{\partial}{\partial x} = 2x + 2y - 1$   $\begin{cases} 2x + 2y - 1 \\ 2x + 4y + 2 \end{cases}$

$\frac{\partial}{\partial y} = 2x + 4y + 2$   $\begin{cases} 2x + 2y - 1 \\ -2y - 3 \end{cases} \Rightarrow y = -3$

$\left( \begin{matrix} 2 \\ -3 \end{matrix} \right)$   $\begin{cases} -4x - 4y + 2 \\ 2x + 4y + 2 \end{cases} \Rightarrow x = 4$

④  $x^2 - y^2 - 3xy + x + 4y$

$\frac{\partial}{\partial x} = 2x - 3y + 1$   $\begin{cases} 2x - 3y + 1 \\ -3x + 2y + 4 \end{cases}$  Não admite

$\frac{\partial}{\partial y} = 2y - 3x + 4$

⑤  $x + 2y - 2xy - x^2 - 3y^2$

$\frac{\partial}{\partial x} = 1 - 2y - 2x$   $\begin{cases} 2x - 2y + 1 \\ -2x - 6y + 2 \end{cases} \left( \begin{matrix} 1 & 1 \\ 4 & 4 \end{matrix} \right)$

$\frac{\partial}{\partial y} = 2 - 2x - 6y$   $\begin{cases} 2x - 2y + 1 \\ -2x - 6y + 2 \end{cases} \Rightarrow y = 3$

⑥  $3x^2 + y^2 + xy - 2x - 2y$

$\frac{\partial}{\partial x} = 6x + y - 2$   $\begin{cases} 6x + y - 2 \\ x + 2y - 2 \end{cases} \begin{matrix} -12x - 2y + 4 \\ x + 2y - 2 \end{matrix} \Rightarrow x = 2$

$\frac{\partial}{\partial y} = 2y + x - 2$   $\begin{cases} 6x + y - 2 \\ -6x - 12y + 12 \end{cases} \Rightarrow y = -10$   $\left( \begin{matrix} 2 & -10 \\ 11 & 11 \end{matrix} \right)$

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$$\textcircled{e} \quad x^2 + 2y^2 + 3xy + 2x + 2y$$

Não admite

$$\textcircled{f} \quad x^2 + y^2 - 2x - 4y$$

$$\frac{\partial}{\partial x} = 2x - 2 \quad (1, 2) \text{ Ponto de mínimo Global}$$

$\frac{\partial}{\partial y}$

$$\frac{\partial}{\partial y} = 2y - 4$$

$\frac{\partial}{\partial y}$