

$$P(F_h \geq 140 | p_h = 0.5, N = 250) = \sum_{F_h=140}^{250} \binom{250}{F_h} 0.5^{250} = 0.0332 \quad (1)$$

$\mathcal{H}_0$  : Coin is fair

$\mathcal{H}_1$  : Coin is biased

$D$  : Data of 140 heads in 250 spins

$F_h$  : 140 (frequency of heads)

$F_t$  : 110 (frequency of tails)

$p_h$  : Probability of heads

Getting model comparison.

$$P(\mathcal{H}_0 | D) = \frac{P(D | \mathcal{H}_0) P(\mathcal{H}_0)}{P(D)} \quad (2)$$

$$P(\mathcal{H}_1 | D) = \frac{P(D | \mathcal{H}_1) P(\mathcal{H}_1)}{P(D)} \quad (3)$$

$$P(\mathcal{H}_0) = P(\mathcal{H}_1) = \frac{1}{2} \quad (4)$$

$$\frac{P(\mathcal{H}_1 | D)}{P(\mathcal{H}_0 | D)} = \frac{P(D | \mathcal{H}_1)}{P(D | \mathcal{H}_0)} \quad (5)$$

Done getting model comparison.

Getting  $P(D | \mathcal{H}_1)$  :

$$P(D | \mathcal{H}_1) = \int_0^1 dp_h P(D | p_h, \mathcal{H}_1) P(p_h | \mathcal{H}_1) \quad (6)$$

$$P(D | p_h, \mathcal{H}_1) = p_h^{F_h} (1 - p_h)^{F_t} \quad (7)$$

$$P(p_h | \mathcal{H}_1) = 1 \quad (8)$$

$$P(D | \mathcal{H}_1) = \int_0^1 dp_h P(D | p_h, \mathcal{H}_1) = \int_0^1 dp_h p_h^{F_h} (1 - p_h)^{F_t} \quad (9)$$

$$B(x, y) = \int_0^1 dt t^{x-1} (1-t)^{y-1} = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \quad (10)$$

$$P(D|\mathcal{H}_1) = B(F_h + 1, F_t + 1) = \frac{\Gamma(F_h + 1)\Gamma(F_t + 1)}{\Gamma(F_h + F_t + 2)} \quad (11)$$

$$\Gamma(x) = (n-1)! \quad (12)$$

$$P(D|\mathcal{H}_1) = \frac{(F_h + 1)!(F_t + 1)!}{(F_h + F_t + 1)!} = \frac{F_h!F_t!}{(F + 1)!} \quad (13)$$

End of getting  $P(D|\mathcal{H}_1)$ .

Getting  $P(D|\mathcal{H}_0)$  :

$$p_f = \frac{1}{2} \quad (14)$$

$$P(D|\mathcal{H}_0) = \left(\frac{1}{2}\right)^{F_h} \left(1 - \left(\frac{1}{2}\right)^{F_t}\right) = \left(\frac{1}{2}\right)^F \quad (15)$$

Done getting  $P(D|\mathcal{H}_0)$ .

Computing model comparison:

$$\frac{P(\mathcal{H}_1|D)}{P(\mathcal{H}_0|D)} = \frac{\frac{F_h!F_t!}{(F+1)!}}{\left(\frac{1}{2}\right)^F} = \frac{\frac{140!110!}{251!}}{\left(\frac{1}{2}\right)^{250}} \cong 0.48 \quad (16)$$

End computing model comparison.