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Remarks on specific sliding and Hertzian contact

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Why gears?

For power transmission:

$$\boxed{Power = \omega \times \mathrm{T}}$$

- Transmission ratio 8:1 ;
- Input velocity:1500 rpm
- Output velocity: 1500/8 rpm

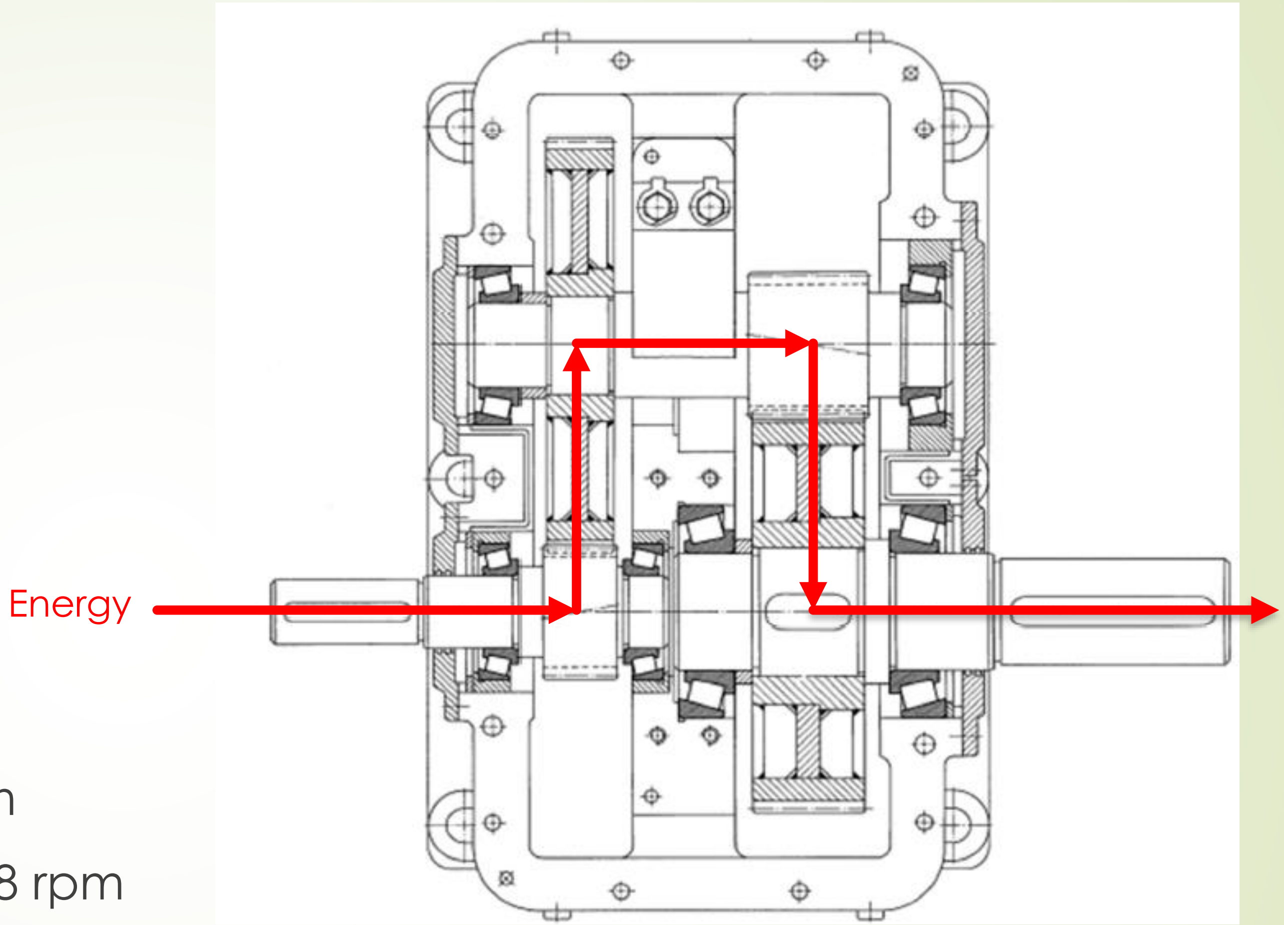
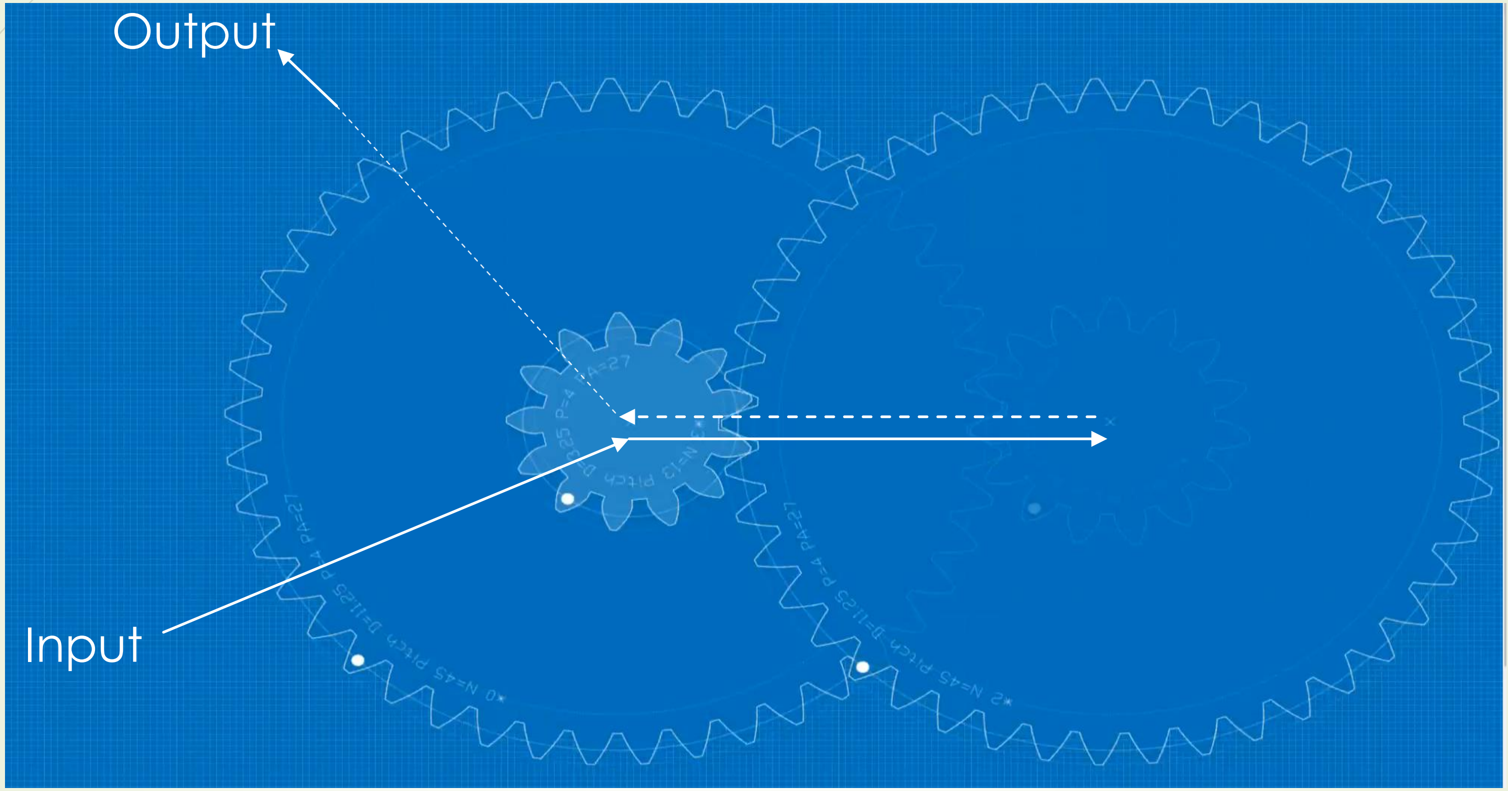


Figure 1 : Technical drawing of a speed reducer
Source: ‘Aplicaciones prácticas de rodamientos’, FAG Kugelfischer Georg Schäfer & Co Schweinfurt.

Why gears?



Animation 1: Animation obtained using https://geargenerator.com

Spur gears - Specific sliding - Introduction

- Specific sliding is directly linked to the lifetime of gears.

- It needs to be minimized.

And it's defined as :

$$gs_1B = \left| 1 - \frac{\sqrt{{ra_2}^2 - {rb_2}^2}}{a \times \sin\alpha - \sqrt{{ra_2}^2 - {rb_2}^2}} \times \frac{Z_1}{Z_2} \right| \qquad gs_2A = \left| \frac{\sqrt{{ra_1}^2 - {rb_1}^2}}{a \times \sin\alpha - \sqrt{{ra_1}^2 - {rb_1}^2}} \times \frac{Z_2}{Z_1} - 1 \right|$$

Spur gears - Specific sliding - Introduction

- To minimize the specific sliding we apply a profile shift (x_1 for the pinion and x_2 for the wheel):

$$gs_1B = \left|1 - \frac{\sqrt{\left(\frac{Z_2m}{2}+m+x_2\times m\right)^2-\left(\frac{Z_2m}{2}\cos\alpha\right)^2}}{a\times\sin\alpha-\sqrt{\left(\frac{Z_2m}{2}+m+x_2\times m\right)^2-\left(\frac{Z_2m}{2}\cos\alpha\right)^2}}\times\frac{Z_1}{Z_2}\right| \qquad gs_2A = \left|\frac{\sqrt{\left(\frac{Z_1m}{2}+m+x_1\times m\right)^2-\left(\frac{Z_1m}{2}\cos\alpha\right)^2}}{a\times\sin\alpha-\sqrt{\left(\frac{Z_1m}{2}+m+x_1\times m\right)^2-\left(\frac{Z_1m}{2}\cos\alpha\right)^2}}\times\frac{Z_2}{Z_1}-1\right|$$

- and equalize gs_1B and gs_2A:

$$\boxed{\left|1 - \frac{\sqrt{\left(\frac{Z_2m}{2}+m+x_2\times m\right)^2-\left(\frac{Z_2m}{2}\cos\alpha\right)^2}}{a\times\sin\alpha-\sqrt{\left(\frac{Z_2m}{2}+m+x_2\times m\right)^2-\left(\frac{Z_2m}{2}\cos\alpha\right)^2}}\times\frac{Z_1}{Z_2}\right| = \left|\frac{\sqrt{\left(\frac{Z_1m}{2}+m+x_1\times m\right)^2-\left(\frac{Z_1m}{2}\cos\alpha\right)^2}}{a\times\sin\alpha-\sqrt{\left(\frac{Z_1m}{2}+m+x_1\times m\right)^2-\left(\frac{Z_1m}{2}\cos\alpha\right)^2}}\times\frac{Z_2}{Z_1}-1\right|}$$

Spur gears - Specific sliding - Introduction

- $(z_1+z_2) \geq 60$:
  - Symmetrical profile shift ($x_1 = -x_2$)
  - Equation solved nummerically.

- $(z_1+z_2) < 60$:
  - Henriot's procedure
  - 4 equations and 4 unknowns
  - The center distance (a) will change

Spur gears - Specific sliding – Discussion

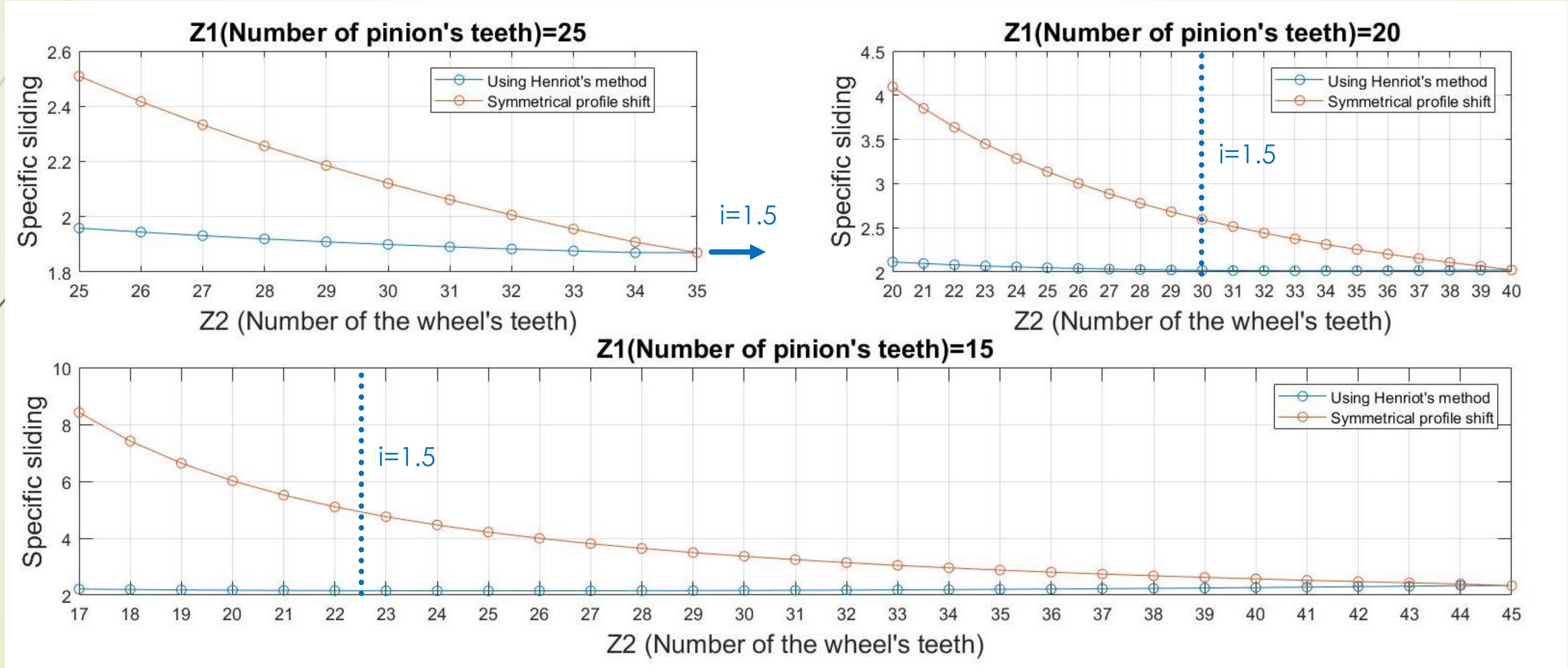


Figure 2 – Specific sliding representation for multiple gearings

Spur gears - Specific sliding – Discussion

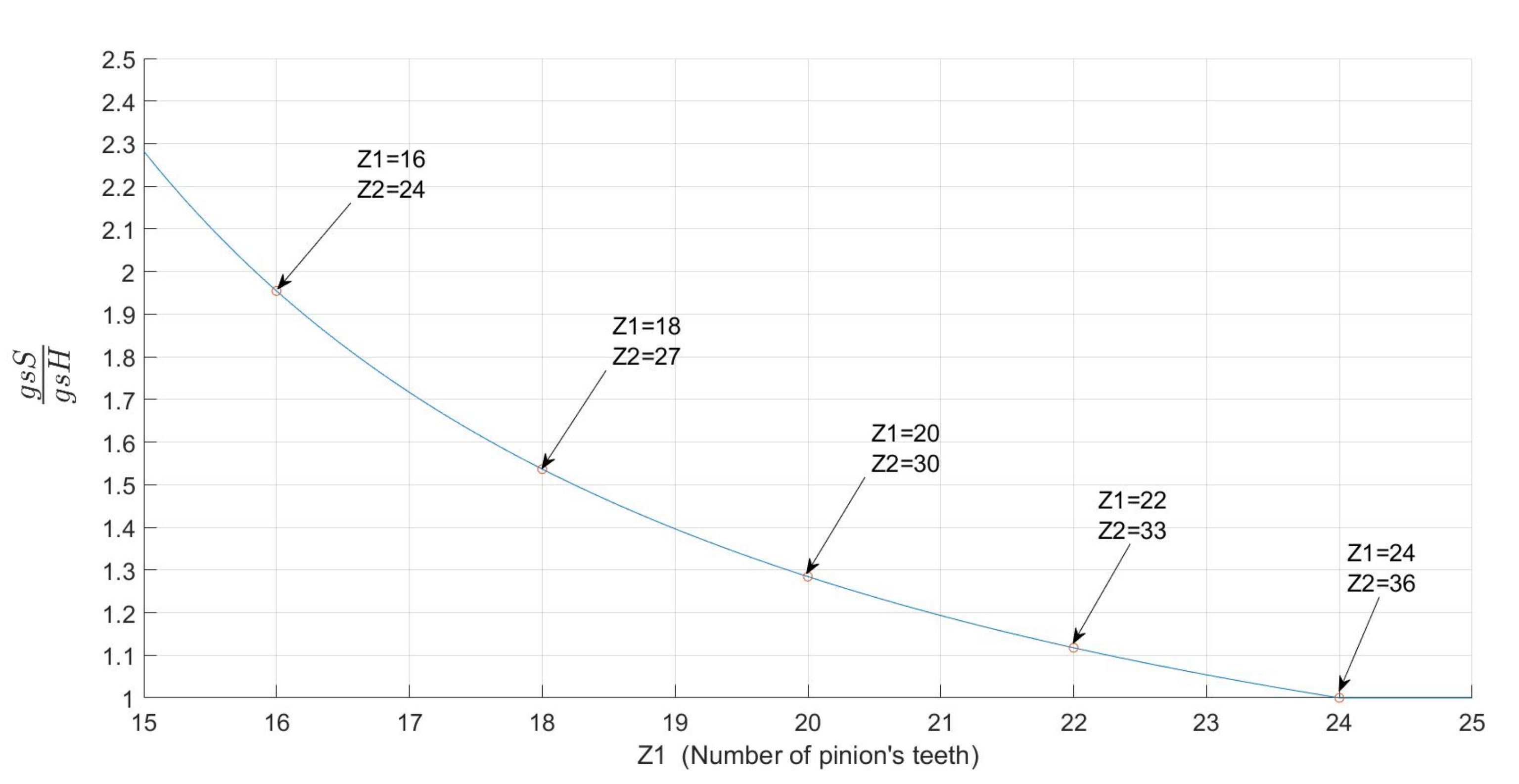


Figure 3 - Relation between specific sliding values obtained by each method (for the *i*=1.5 case)

- Maximum specific sliding obtained by the Henriot's procedure (***gsH***)
- Maximum specific sliding obtained using the symmetrial profile shift (***gsS***)

Hertzian contact pressure - Introduction

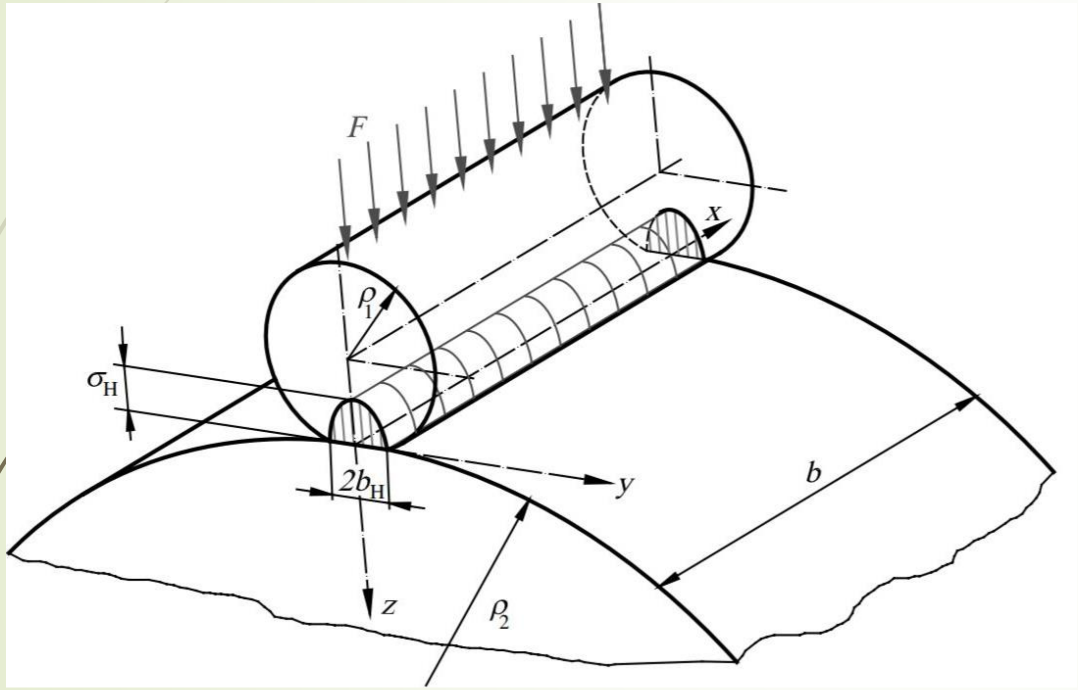


Figure 4 – Contact of two cylinders under load
Source: D. Jelaska, Gears and gear drives, John Wiley & Sons, 2012

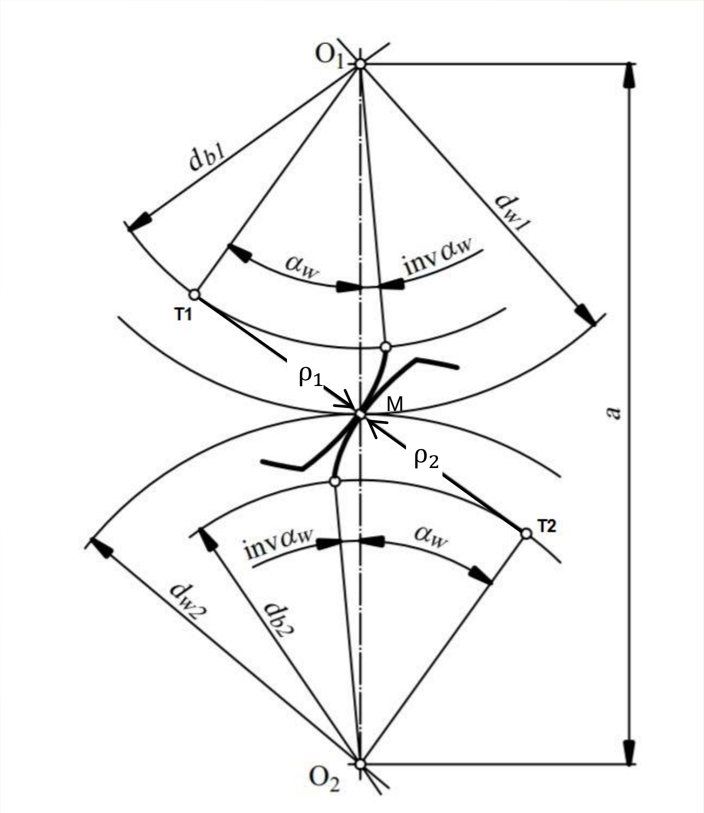


Figure 5 – Gearing squematic representation
Adapted from: D. Jelaska, Gears and gear drives, John Wiley & Sons, 2012

$$\rho_1 + \rho_2 = \overline{T_1T_2}$$

$$\overline{T_1T_2} = a \times \sin\alpha$$

Hertzian contact pressure - Introduction

- The general mathematical formulation:

$$\sigma_H = \sqrt{\frac{F_{nu}}{\pi} \times \frac{\frac{1}{\rho_1} + \frac{1}{\rho_2}}{\frac{(1-\nu_1)^2}{E_1} + \frac{(1-\nu_2)^2}{E_2}}}$$

- For the same material for both the pinion and the wheel (with *E*=210 GPa and ν=0.3):

$$\boxed{\sigma_H = 192\sqrt{F_{nu} \times \frac{1}{\frac{1}{\rho_1} + \frac{1}{\rho_2}}}}$$

Hertzian contact pressure – Discussion

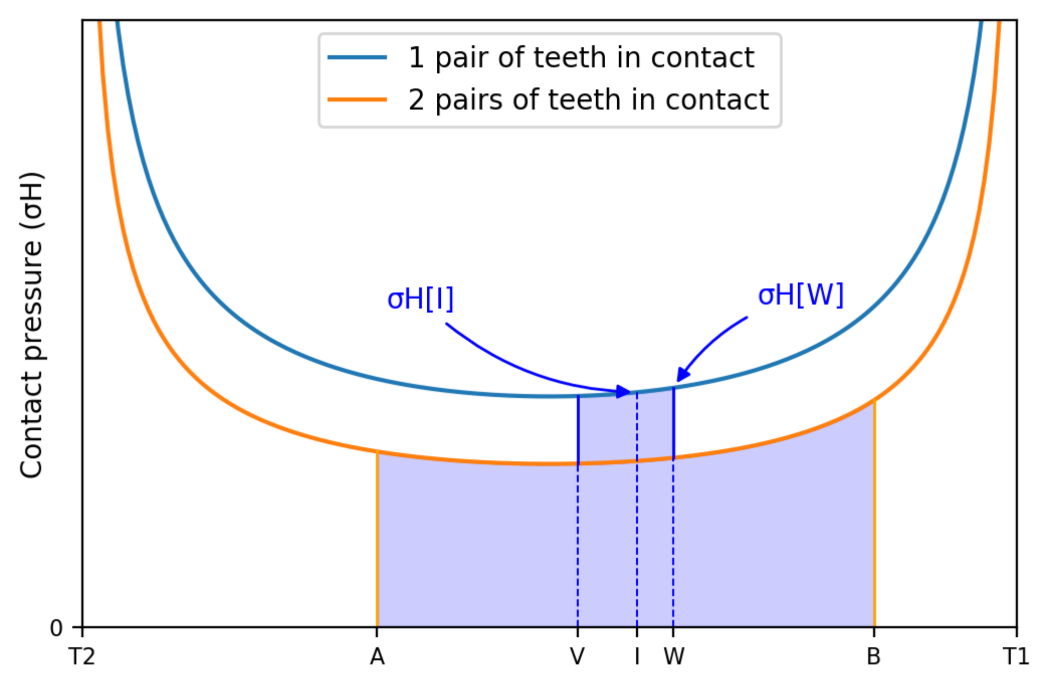
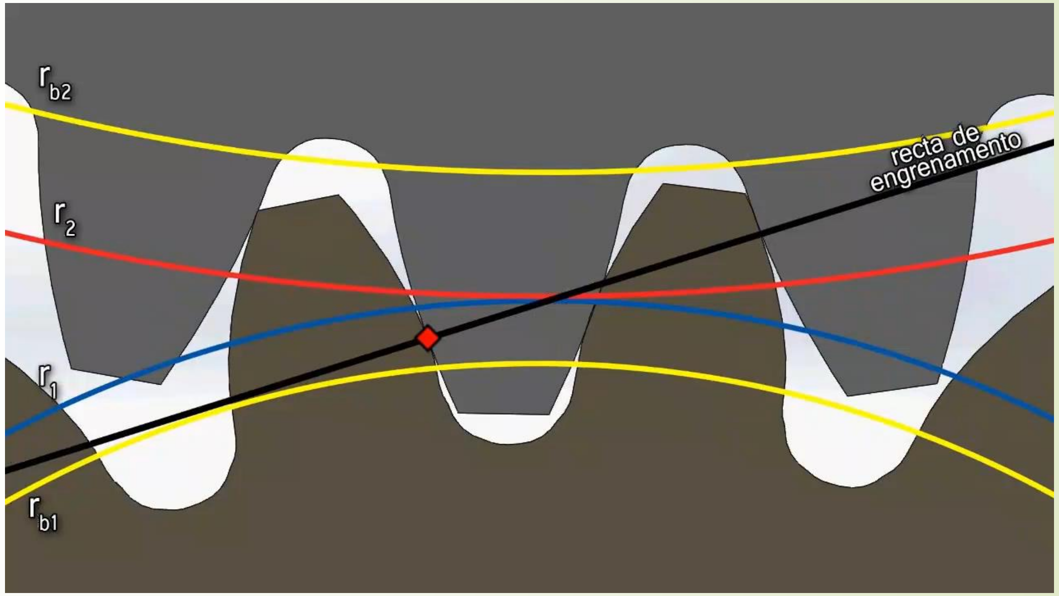


Figure 6 – General representation of the contact pressure along the gearing line



Animation 2: By Henrique Duarte, José Rafael Andrade and Rafael Tavares, in Orgãos de Máquinas, FEUP, 2012/2013

Hertzian contact pressure – Discussion

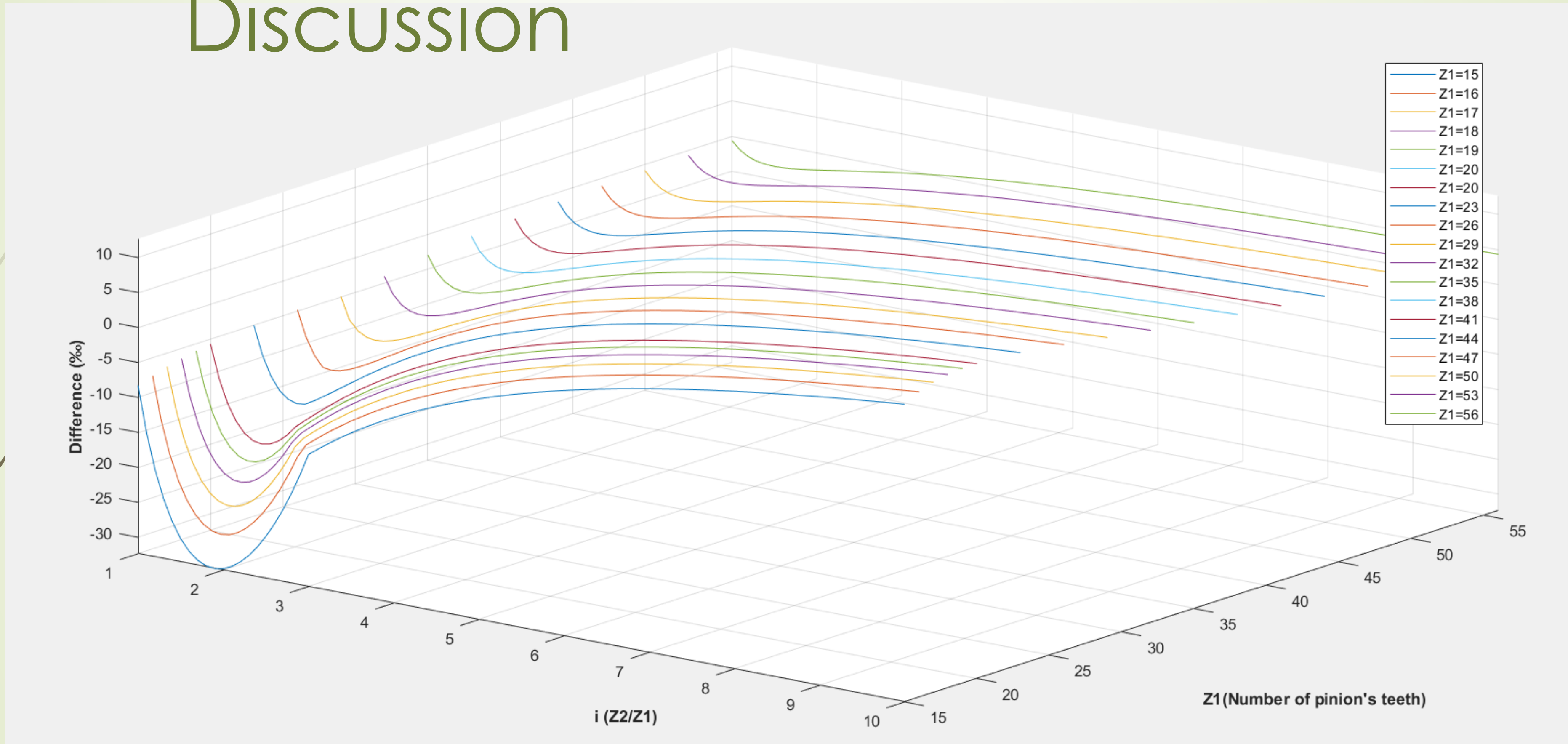


Figure 7 – Difference between $\sigma_H[\mathrm{I}]$ (standard ISO 6332-2) and $\sigma_H[\mathrm{W}]$

$$\text{Difference}=\frac{\sigma_H[\mathrm{I}]-\sigma_H[\mathrm{W}]}{\sigma_H[\mathrm{W}]}$$

Hertzian contact pressure – Discussion

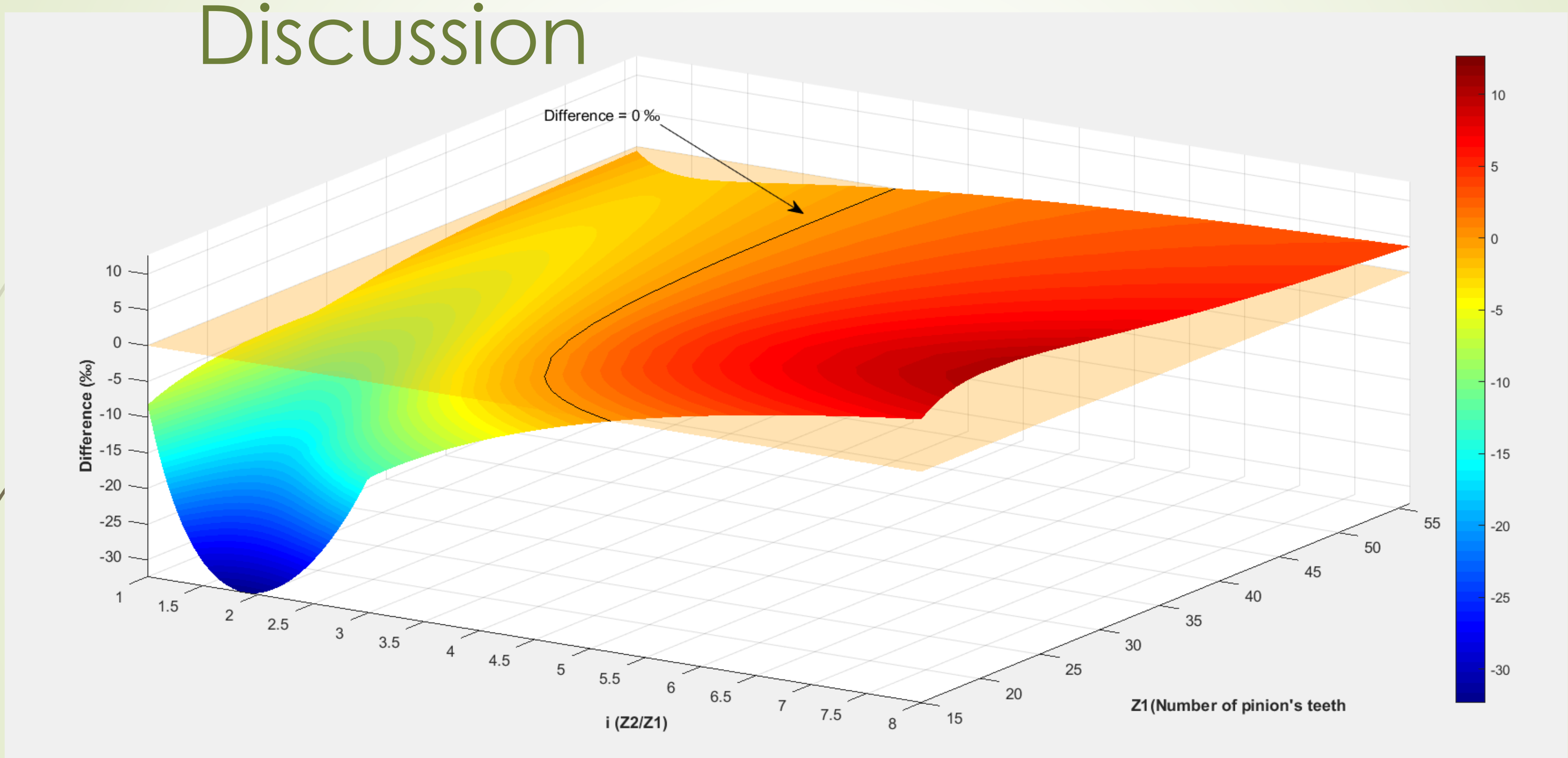


Figure 8 - Difference between $\sigma_H[\mathrm{I}]$ (standard ISO 6332-2) and $\sigma_H[\mathrm{W}]$

$$\text{Difference} = \frac{\sigma_H[\mathrm{I}] - \sigma_H[\mathrm{W}]}{\sigma_H[\mathrm{W}]}$$

To sum up

Specific sliding:
max. specific sliding values obtained using symmetrial profile shift (*gsS*), and the Henriot's procedure (*gsH*), were systematically compared over a wide range of situations. The resulting plot allows to quickly identify the penalty involved in the use x_1=-x_2 for low values of z_1+z_2.

Hertzian contact pressure:
Difference between σ_H[I] (as specified in ISO 6332-2 standard) and σ_H[W]

- was calculated over an wide range of z_1 and *i* values and
- is presented in graphical form enabling to quickly identify the approximations involved in the use of σ_H[I]

Thank you for your attention