Gabarito Prova 1

Física 3, 03/09/2014

questão 1

IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.74. F_e is the repulsive Coulomb force between the spheres. For small θ, $\sin\theta \approx \tan\theta$.

EXECUTE: $\sum F_x = T\sin\theta - F_e = 0$ and $\sum F_y = T\cos\theta - mg = 0$. So $\dfrac{mg\sin\theta}{\cos\theta} = F_e = \dfrac{kq^2}{d^2}$. But $\tan\theta \approx \sin\theta = \dfrac{d}{2L}$;

so $d^3 = \dfrac{2kq^2L}{mg}$ and $d = \left(\dfrac{q^2L}{2\pi\epsilon_0 mg}\right)^{1/3}$.

EVALUATE: d increases when q increases.

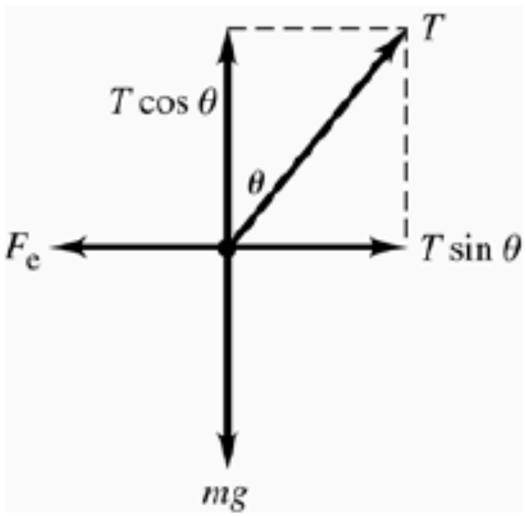

Figure 21.74

questão 2

IDENTIFY: There is a force on each electron due to the other electron and a force due to the sphere of charge. Use Coulomb's law for the force between the electrons. Example 22.9 gives E inside a uniform sphere and Eq.(21.3) gives the force.

SET UP: The positions of the electrons are sketched in Figure 22.53a.

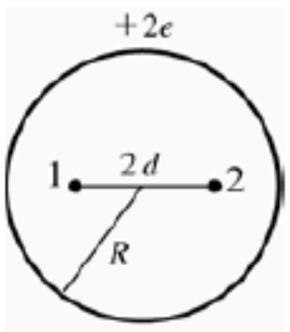

Figure 22.53a

If the electrons are in equilibrium the net force on each one is zero.

questão 2

EXECUTE: Consider the forces on electron 2. There is a repulsive force F_1 due to the other electron, electron 1.

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{(2d)^2}$$

The electric field inside the uniform distribution of positive charge is $E = \frac{Qr}{4\pi\epsilon_0 R^3}$ (Example 22.9), where $Q = +2e$.

At the position of electron 2, $r = d$. The force F_{cd} exerted by the positive charge distribution is $F_{cd} = eE = \frac{e(2e)d}{4\pi\epsilon_0 R^3}$

and is attractive.
The force diagram for electron 2 is given in Figure 22.53b.

Figure 22.53b

Net force equals zero implies $F_1 = F_{cd}$ and $\frac{1}{4\pi\epsilon_0}\frac{e^2}{4d^2} = \frac{2e^2 d}{4\pi\epsilon_0 R^3}$

Thus $(1/4d^2) = 2d/R^3$, so $d^3 = R^3/8$ and $d = R/2$.

EVALUATE: The electric field of the sphere is radially outward; it is zero at the center of the sphere and increases with distance from the center. The force this field exerts on one of the electrons is radially inward and increases as the electron is farther from the center. The force from the other electron is radially outward, is infinite when $d = 0$ and decreases as d increases. It is reasonable therefore for there to be a value of d for which these forces balance.

questão 3

IDENTIFY: The work needed to assemble the nucleus is the sum of the electrical potential energies of the protons in the nucleus, relative to infinity.

SET UP: The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is $U = (1/4\pi\epsilon_0)(qq_0/r)$. Each charge is e and the charges are equidistant from each other, so the total potential energy is $U = \frac{1}{4\pi\epsilon_0}\left(\frac{e^2}{r}+\frac{e^2}{r}+\frac{e^2}{r}\right) = \frac{3e^2}{4\pi\epsilon_0 r}$.

EXECUTE: Adding the potential energies gives

$$U = \frac{3e^2}{4\pi\epsilon_0 r} = \frac{3(1.60\times10^{-19}\ \text{C})^2(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)}{2.00\times10^{-15}\ \text{m}} = 3.46\times10^{-13}\ \text{J} = 2.16\ \text{MeV}$$

EVALUATE: This is a small amount of energy on a macroscopic scale, but on the scale of atoms, 2 MeV is quite a lot of energy.

questão 4

IDENTIFY: We must integrate to find the total energy because the energy to bring in more charge depends on the charge already present.

SET UP: If ρ is the uniform volume charge density, the charge of a spherical shell or radius r and thickness dr is $dq = \rho 4\pi r^2 dr$, and $\rho = Q/(4/3\ \pi R^3)$. The charge already present in a sphere of radius r is $q = \rho(4/3\ \pi r^3)$. The energy to bring the charge dq to the surface of the charge q is Vdq, where V is the potential due to q, which is $q/4\pi\epsilon_0 r$.

EXECUTE: The total energy to assemble the entire sphere of radius R and charge Q is sum (integral) of the tiny increments of energy.

$$U = \int V dq = \int \frac{q}{4\pi\epsilon_0 r} dq = \int_0^R \frac{\rho \frac{4}{3}\pi r^3}{4\pi\epsilon_0 r}\left(\rho 4\pi r^2 dr\right) = \frac{3}{5}\left(\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R}\right)$$

where we have substituted $\rho = Q/(4/3\ \pi R^3)$ and simplified the result.

EVALUATE: For a point-charge, $R \to 0$ so $U \to \infty$, which means that a point-charge should have infinite self-energy. This suggests that either point-charges are impossible, or that our present treatment of physics is not adequate at the extremely small scale, or both.