Lista 6 -

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142 b) Resolução:

$$f(x) = -\frac{7}{5}x^3 - \frac{\sqrt{3}}{7} \rightarrow f'(x) = \overbrace{\frac{d}{dx}\left(-\frac{7}{5}\cdot x^3\right)}^{\text{derivada do produto}} - \frac{d}{dx}\left(\frac{\sqrt{3}}{7}\right) \rightarrow$$

constante

$$\rightarrow \cancel{0\cdot x^3}^{\,0} + \left(-\frac{7}{5}\right)\cdot 3x^2 - 0 \rightarrow -\frac{21}{5}x^2$$

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144 b) Resolução:

$$b)\ f(x) = x^2\cdot(x+x^4)(1+x+x^3) \rightarrow f(x) = (x^3+x^6)(1+x+x^3)$$

$$\therefore f'(x) = (3x^2+6x^5)(1+x+x^3) + (x^3+x^6)(3x^2+1)$$

$$= 3x^2+3x^3+6x^5+6x^6+6x^8+3x^5+x^3+3x^8+x^6$$

$$= 9x^8+7x^6+12x^5+4x^3+3x^2$$

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144 g) $f(x) = x^4 \cdot a^{2x}$ ← produto

$$f'(x) = 4x^3\cdot a^{2x} + x^4\cdot a^{2x}\cdot \ln a^2$$

$$= 4x^3\cdot a^{2x} + x^4\cdot a^2\cdot 2\cdot \ln a$$

$$= 2x^3a^{2x}\left(2 + x\cdot \ln a\right)$$

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Resolução:

149 e) $f(x) = \dfrac{x+3}{x-1} + \dfrac{x+2}{x+1} \rightarrow f'(x) = \dfrac{1+0\cdot(x-1) - (x+3)\cdot 1+0}{(x-1)^2} + \dfrac{1+0\cdot(x+1) - (x-2)\cdot 1+0}{(x+1)^2}$

$$\rightarrow \frac{(\cancel{x}-1) - (\cancel{x}+3)}{(x-1)^2} + \frac{(\cancel{x}+1) - (\cancel{x}-2)}{(x+1)^2} \rightarrow \frac{-4}{(x-1)^2} + \left(-\frac{1}{(x+1)^2}\right) \rightarrow -\frac{4}{(x-1)^2} - \frac{1}{(x+1)^2} \rightarrow$$

$$\frac{-4(x+1)^2 - (x-1)^2}{(x-1)^2\cdot(x+1)^2} \rightarrow \frac{-4(x^2+2x+1) - (x^2-2x+1)}{(x^2-2x+1)\cdot(x^2+2x+1)} \rightarrow \frac{\overbrace{-4x^2-8x-4-x^2+2x-1}^{-5x^2-6x-5}}{\underbrace{x^4+\cancel{3x^3}+x^2-\cancel{3x^3}-4x^2-\cancel{2x}+x^2+\cancel{2x}+1}_{x^4-2x^2+1}} \rightarrow$$

$$\rightarrow -\frac{5x^2-6x-5}{x^4-2x^2+1}$$

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(150) Resolução:

g) $f(x) = \dfrac{tg\,x}{sen\,x + cos\,x} \rightarrow f'(x) = \dfrac{sec^2 x \cdot (sen\,x + cos\,x) - tg\,x \cdot (cos\,x - sen\,x)}{(sen\,x + cos\,x)^2}$

Lembrando:

derivada de $tg\,x = sec^2 x$

Derivada do quociente $\dfrac{U'(x) \cdot V(x) - U(x) \cdot V'(x)}{[V(x)]^2}$

(155) Resolução:

h) $F(x) = cotg(3x-1)$

Sabemos que $\dfrac{d[sen\,x]}{dx} = cos\,x$ e que $\dfrac{d[cos\,x]}{dx} = -sen\,x$

outra coisa é: $\dfrac{1}{cos\,x} = sec\,x$ e $\dfrac{1}{sen\,x} = cossec\,x$ e que $tg = \dfrac{sen\,x}{cos\,x}$

$\therefore \dfrac{d[cotg\,x]}{dx} = \dfrac{d\left[\frac{cos\,x}{sen\,x}\right]}{dx} = \dfrac{-sen\,x \cdot sen\,x - cos\,x \cdot cos\,x}{(sen\,x)^2} =$

$\dfrac{-sen^2 x - cos^2 x}{sen^2 x} = \dfrac{-(sen^2 x + cos^2 x)}{sen^2 x} = \dfrac{-1}{sen^2 x} = \underline{-cossec^2 x}$

Assim: $F'(x) = -cossec^2(3x-1) \cdot 3$

$\therefore F'(x) = -3 \cdot cossec^2(3x-1)$ ///

K) $F(x) = tg^3\,2x = tg\,2x \cdot tg\,2x \cdot tg\,2x$

Assim $F'(x) = \underline{sec^2(2x) * 2 * tg(2x) * tg(2x)} + \underline{sec^2(2x) \cdot 2 * tg(2x) * tg(2x)}$

$+ \underline{sec^2(2x) * 2 * tg(2x) \cdot tg(2x)}$.

$F'(x) = 3 * sec^2(2x) \cdot tg(2x) \cdot tg(2x) * 2$

$\boxed{F'(x) = 6 * sec^2(2x) \cdot tg^2(2x)}$

Resolução:

(161) b) $f(x) = x^n \cdot \ln x \quad (n \in \mathbb{N})$

Produto

$$f'(x) = n \cdot x^{n-1} \cdot \ln x + x^n \cdot \frac{1}{x} \rightarrow f'(x) = n \cdot x^{n-1} \cdot \ln x + \frac{x^n}{x} \rightarrow$$

$$\rightarrow f'(x) = n \cdot x^{n-1} \cdot \ln x + x^n \cdot x^{-1} \rightarrow f'(x) = n \cdot x^{n-1} \cdot \ln x + x^{n-1}$$

$$\rightarrow f'(x) = x^{n-1}(n \cdot \ln x + 1)$$ ///

Resolução:

f) $f(x) = \ln(ax^2 + bx + c)$

$f(x) = \ln(ax^2+bx+c)$ ↝ $h(x) = ax^2+bx+c \Rightarrow g(h(x))$

↳ $g(h) = \ln h$

$$f'(x) = g'(h) \cdot h'(x) \Rightarrow \frac{1}{ax^2+bx+c} \cdot 2ax + b + 0 \Rightarrow$$

$$\frac{2ax+b}{ax^2+bx+c}$$ ///

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(162) Resolução:

g) $f(x) = \sqrt{ax+b} \quad (a, b \in \mathbb{R})$

$f(x) = \sqrt{ax+b}$ ↳ $h(x) = ax+b \Rightarrow g(h(x)) \Rightarrow f'(x) = g'(h) \cdot h'(x)$

↳ $g(h) = \sqrt{h} = h^{1/2}$

$$\frac{1}{2} \cdot h^{1/2 - 1} = \frac{1}{2} \cdot h^{-1/2}$$

$$\Rightarrow \frac{1}{2} \cdot \frac{1}{h^{1/2}} \Rightarrow \frac{1}{2} \cdot \frac{1}{\sqrt{h}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{h}} \cdot a \Rightarrow$$

$$\Rightarrow \frac{1}{2} \cdot \frac{1}{\sqrt{ax+b}} \cdot a \Rightarrow$$

$$\frac{a}{2\sqrt{ax+b}}$$ ///

Resolución!

(163)

c) $f(x) = \arctan \frac{1}{x} = x^{-1}$

$$f'(x) = \frac{1}{1+\left(\frac{1}{x}\right)^2} = \frac{1}{1+\frac{1}{x^2}} \cdot (-1) \cdot x^{-2} \rightarrow -\frac{x^{-2}}{1+x^{-2}} \rightarrow$$

$$-\frac{x^{-2}}{1+\frac{1}{x^2}} \rightarrow -\frac{x^{-2}}{\frac{x^2+1}{x^2}} \rightarrow -\frac{x^{-2}}{1} \cdot \frac{x^2}{x^2+1} \rightarrow \frac{-1}{x^2+1}$$

$\frac{x^2+1}{x^2}$

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h) $f(x) = \underbrace{\ln \underbrace{(\arccos \underbrace{x}_{p(x)})}_{h(p)}}_{g(h)}$ $\quad g(h(p(x))) \Rightarrow$

$$f'(x) = g'(h) \cdot h'(p) \cdot p'(x)$$

$$= \frac{1}{\arccos x} \cdot \left(-\frac{1}{\sqrt{1-x^2}}\right) \cdot 1 \rightarrow -\frac{1}{\arccos x \cdot \sqrt{1-x^2}}$$
