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## 1 string

### 1.1 kmp

```

1 642 vector<int> getBorder(string str) {
1 555     int n = str.size();
1 7AD     vector<int> border(n, -1);
2 199     for(int i = 1, j = -1; i < n; i++) {
2 904         while(j >= 0 && str[i] != str[j + 1]) {
330             j = border[j];
2 A09         }
2 AE4         if(str[i] == str[j + 1]) {
3 04B             j++;
3 B31         }
805         border[i] = j;
3 4D9     }
887     return border;
3 EA7 }
4 971 int matchPattern(const string &txt, const string &pat, const vector<int> &border) {
4 13D     int freq = 0;
4 035     for(int i = 0, j = -1; i < txt.size(); i++) {
4 8DA         while(j >= 0 && txt[i] != pat[j + 1]) {
5 330             j = border[j];
5 00A         }
5 53D         if(pat[j + 1] == txt[i]) {
04B             j++;
6 8CC         }
6 99B         if(j + 1 == (int) pat.size()) {
D41             //found occurence
6 CB5             freq++;
6 330             j = border[j];
7 F5F         }
D55     }
7 E33     return freq;
7 065 }

```

### 1.2 aho

```

9 123 struct AhoType {
847     static const int ALPHA = 26;
1E3     static int f(char c) { return c - 'A'; } // ver se ta maiusculo ou minusculo aqui
10 D13 };
10 29B template<typename AhoType>
11 51F struct AhoCorasick {
11 BF2     struct Node {
11 64C         int nxt[AhoType::ALPHA] {};
11 0D9         int p = 0, ch = 0, len = 0;
E7A         int link = 0;
79F         int occ_link = 0;
F4F         Node(int p = 0, int ch = 0, int len = 0) : p(p), ch(ch), len(len) {}
12 8EE     };
12 8ED     vector<Node> tr;
13 69B     AhoCorasick() : tr(1) {}
13 1F7     template<typename Iterator>
13 2CA     void add_word(Iterator first, Iterator last) {
13 AC3         int cur = 0, len = 1;
68C         for(; first != last; ++first) {
ED9             auto ch = AhoType::f(*first);
14 4F3             if(tr[cur].nxt[ch] == 0) {
9BF                 tr[cur].nxt[ch] = int(tr.size());
6CC                 tr.emplace_back(cur, ch, len);
15 B7C             }
16 BEE             cur = tr[cur].nxt[ch];
250             ++len;
16 159         }
16 D91         tr[cur].occ_link = cur;

```

```

FA3     }
OA8     void build() {
A36         vector<int> bfs(int(tr.size()));
2AA         int s = 0, t = 1;
D33         while(s < t) {
B21             int v = bfs[s++], u = tr[v].link;
F9E             if(tr[v].occ_link == 0) {
99B                 tr[v].occ_link = tr[u].occ_link;
E75             }
609             for(int ch = 0; ch < AhoType::ALPHA; ++ch) {
31D                 auto& nxt = tr[v].nxt[ch];
9FA                 if(nxt == 0) {
2CA                     nxt = tr[u].nxt[ch];
95C                 } else {
FE1                     tr[nxt].link = v > 0 ? tr[u].nxt[ch] : 0;
47D                     bfs[t++] = nxt;
353                 }
D85             }
FBE         }
7FF     }
1F7     template<typename Iterator>

7F0     vector<pair<int,int>> get_all_matches(Iterator first, Iterator last) const {
FD9     vector<pair<int,int>> occs;
E09         for(int cur = 0, i = 0; first != last; ++i, ++first) {
ED9             auto ch = AhoType::f(*first);
BEE             cur = tr[cur].nxt[ch];
F2C             for(int v = tr[cur].occ_link; v > 0; v = tr[tr[v].link].occ_link) {
D41                 // i = pos text, v = state
D1A                 occs.push_back({1+i-tr[v].len, i});
34C             }
08F         }
23F     return occs;
DD5     }
67A     template<typename T>
578     int get_next(int cur, T ch) const { return tr[cur].nxt[AhoType::f(ch)]; }
3F9 };

```

## 1.3 trie

```

CFC int trie[ms][sigma], terminal[ms], z = 1;

```

```

33B void insert(string &p) {
B3D     int cur = 0;
E2E     for(int i = 0; i < p.size(); i++) {
1BF         int id = p[i] - 'a';
919         if(!trie[cur][id]) {
869             trie[cur][id] = z++;
45C         }
3AD         cur = trie[cur][id];
D9E     }
B07     terminal[cur]++;
5EC }

```

```

684 int count(string &p) {
B3D     int cur = 0;
E2E     for(int i = 0; i < p.size(); i++) {
1BF         int id = p[i] - 'a';
919         if(!trie[cur][id]) {
D1F             return false;
F06         }
3AD         cur = trie[cur][id];
532     }
89E     return terminal[cur];
B27 }

```

## 1.4 zfunc

```

403 vector<int> Zfunction(string &s){
163     int n = s.size();
2B1     vector<int> z (n, 0);
A5C     for(int i=1, l=0, r=0; i<n; i++) {
76D         if(i <= r) z[i] = min(z[i-1], r-i+1);
F61         while(z[i] + i < n && s[z[i]] == s[i+z[i]]) z[i]++;
EAF         if(r < i+z[i]-1) l = i, r = i+z[i]-1;
0CD     }
070     return z;
D58 }

```

## 2 dp

### 2.1 cht2

```

72C struct Line{
12D     ll a, b;
028     double x_inter;
01D     Line(ll a, ll b, double x_inter = inf) : a(a), b(b), x_inter(x_inter){}

D82     bool operator < (double x){
80C         return x_inter < x;
1CB     }
30B     ll eval(ll x){
F27         return a*x + b;
480     }
7DA };

88F double intersect(Line x, Line y){
BBA     assert(x.a != y.a);
CBE     return (1.0d * x.b - y.b) / (1.0d * y.a - x.a);
65D }

```

```

4B5 struct CHT{
398     deque<Line> lines;
DC7     void insert_right(ll a, ll b){
D85         while(lines.size() >= 2){
595             Line x = lines[lines.size() - 2], y = lines[lines.size() - 1];
CBB             if(intersect(y, {a,b,0}) > intersect(x,y)) break;
501             lines.pop_back();
04A         }
4CB         if(!lines.empty()) lines[lines.size() - 1].x_inter = intersect(lines.back(), {a,b
,0});
748         lines.push_back(Line(a, b));
E42     }
7F4     void insert_left(ll a, ll b){
D85         while(lines.size() >= 2){
24D             Line x = lines[1], y = lines[0];
461             if(intersect(y, {a,b,0}) < intersect(x,y)) break;
688             lines.pop_front();
07A         }
29A         lines.push_front(Line(a, b));
359         if(!lines.empty()) lines.back().x_inter = inf;
BBF         if(lines.size() > 1) lines[0].x_inter = intersect(lines[0], lines[1]);
1E9     }
BF9     ll qry(ll x){ // todo: fazer two pointers pra ficar linear
349         auto lb = lower_bound(begin(lines), end(lines), x);
7F9         return (*lb).eval(x);
327     }
AAD     void dbg(){
DDE         for(Line l : lines)
F09             cout << l.a << "x + " << l.b << " " << l.x_inter << br;
C97     }

```

```
F51 };
```

## 2.2 cht

```
72C struct Line {
A3B     ll m, c;
D2C     Line(ll m, ll c) : m(m), c(c) {}
30B     ll eval(ll x) {
255         return m * x + c;
3CD     }
E9C };
4B5 struct CHT {
B57     vector<Line> lines;
1D1     bool bad(Line a, Line b, Line c) {
D41         // trocar pra < se for max
62B         return 1.d * (c.c - a.c)*(a.m - b.m) > 1.d * (b.c - a.c)*(a.m - c.m);
0B3     }
7CE     void insert(Line line) { // sortar antes de inserir
544         int sz = (int)lines.size();
7D8         for(; sz > 1; --sz) {
DBD             if(bad(lines[sz - 2], lines[sz - 1], line)) {
501                 lines.pop_back();
5E2                 continue;
578             }
C2B             break;
FF1         }
770         lines.emplace_back(line);
4FD     }
4AD     ll query(ll x) {
82D         int l = 0, r = (int)lines.size() - 1;
40C         while(l < r) {
EE4             int m = (l+r)/2;
D41             // trocar pra < se for max
A32             if(lines[m].eval(x) > lines[m+1].eval(x)) {
16D                 l = m + 1;
568             } else {
3E2                 r = m;
476             }
E56         }
348         return lines[l].eval(x);
571     }
7AC };
```

## 2.3 lis

```
D41 // Longest Increasing Sequence
514 int lis(vector<ll>& nums){
F64     int n = nums.size();
CF7     vector<ll> s;
603     for(int i = 0; i < n; i++){
EEB         auto it = lower_bound(s.begin(), s.end(), nums[i]);
BA0         if(it == s.end()){
719             s.pb(nums[i]);
C60         }
4E6         else{
570             *it = nums[i];
AD4         }
358     }
8B9     return (int)s.size();
0B2 }
```

## 3 math

### 3.1 mint

```
67A template<typename T>
56C T bin_exp(T a, long long e) {
DAC     T r(1);
D0E     for(; e > 0; e >= 1) {
EEE         if(e & 1) {
1C8             r *= a;
D4B         }
70C         a *= a;
EF5     }
4C1     return r;
D51 }
016 template<const uint32_t MOD>
BB6 struct Mod {
622     uint32_t x;
77D     Mod() : x(0) {};
67A     template<typename T>
EA0     Mod(T x) : x(uint32_t((int64_t(x) % MOD) + MOD) % MOD) {}
ECC     Mod& operator+=(Mod rhs) {
393         x += rhs.x;
290         if(x >= MOD) x -= MOD;
357         return *this;
7F3     }
1BD     Mod& operator-=(Mod rhs) {
C2B         x += MOD - rhs.x;
290         if(x >= MOD) x -= MOD;
357         return *this;
51D     }
EAD     Mod& operator*=(Mod rhs) {
4E6         auto y = lull * x * rhs.x;
2AA         if(y >= MOD) y %= MOD;
A6E         x = uint32_t(y);
357         return *this;
89A     }
4B8     Mod& operator/=(Mod rhs) { return *this *= bin_exp(rhs, MOD - 2); }
CE9     friend Mod operator+(Mod lhs, Mod rhs) { return lhs += rhs; }
16B     friend Mod operator-(Mod lhs, Mod rhs) { return lhs -= rhs; }
D5C     friend Mod operator*(Mod lhs, Mod rhs) { return lhs *= rhs; }
5B7     friend Mod operator/(Mod lhs, Mod rhs) { return lhs /= rhs; }
2B2     bool operator==(Mod rhs) const { return x == rhs.x; }
D50     bool operator!=(Mod rhs) const { return x != rhs.x; }
17E     friend ostream& operator<<(ostream& os, const Mod& o) { return os << o.x; }
52F     friend istream& operator>>(istream& is, Mod& o) {
C23         int64_t x;
AF7         is >> x;
84C         o = Mod(x);
FED         return is;
F1B     }
A9E };
```

### 3.2 extendedEuclidean

```
89C int gcd(int a, int b, int& x, int& y) {
A30     if (b == 0) {
483         x = 1;
01D         y = 0;
3F5         return a;
433     }
608     int x1, y1;
E8B     int d = gcd(b, a % b, x1, y1);
711     x = y1;
```

```

A2A     y = x1 - y1 * (a / b);
BE2     return d;
AF0 }

```

```

D41 // inverso modular de a
43C int inv, y;
7A8 int g = gcd(a,mod,inv,y);
37A inv = (inv % m + m) % m;

```

### 3.3 crt

```

C22 ll euclid(ll a, ll b, ll&x, ll&y){
1EE     if(!b) return x = 1, y = 0, a;
E3D     ll d = euclid(b, a % b, y, x);
0A4     return y -= a/b * x, d;
33B }
A4B ll crt(vector<ll>& rem, vector<ll>& mod){
1BB     int n = rem.size();
233     if(n == 0) return 0;
2F3     ll ans = rem[0], m = mod[0];
6F5     for(int i = 1; i < n; i++){
0BE         ll x,y;
168         ll g = euclid(mod[i],m,x,y);
D41         // if((ans - rem[i]) % g != 0) return -5;
865         assert((ans - rem[i]) % g == 0);
263         ans = ans + 1LL*(rem[i]-ans)*(m/g)*y;
B68         m = (mod[i]/g)*(m/g)*g;
6AD     }
BA7     return ans;
A58 }

```

### 3.4 pollardrho

```

F4C typedef unsigned long long ull;
F85 ull modmul(ull a, ull b, ull M) {
2DD     ll ret = a * b - M * ull(1.L / M * a * b);
964     return ret + M * (ret < 0) - M * (ret >= (1ll)M);
E93 }
4F6 ull modpow(ull b, ull e, ull mod) {
C1A     ull ans = 1;
A18     for (; e; b = modmul(b, b, mod), e /= 2)
9E8         if (e & 1) ans = modmul(ans, b, mod);
BA7     return ans;
100 }
DA4 bool isPrime(ull n) {
C16     if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
43A     ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
C17     s = __builtin_ctzll(n-1), d = n >> s;
E80     for (ull a : A) {
6B4         ull p = modpow(a%n, d, n), i = s;
274         while (p != 1 && p != n - 1 && a % n && i--)
C77             p = modmul(p, p, n);
E28         if (p != n-1 && i != s) return 0;
EDF     }
6A5     return 1;
60D }
7EB ull pollard(ull n) {
222     ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
5F5     auto f = [&](ull x) { return modmul(x, x, n) + i; };
F51     while (t++ % 40 || gcd(prd, n) == 1) {
BE9         if (x == y) x = ++i, y = f(x);
70F         if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
B78         x = f(x), y = f(f(y));
BF8     }
756     return __gcd(prd, n);
791 }

```

```

591 vector<ull> factor(ull n) {
1B9     if (n == 1) return {};
6B5     if (isPrime(n)) return {n};
BC6     ull x = pollard(n);
52A     auto l = factor(x), r = factor(n / x);
98A     l.insert(end(l), begin(r), end(r));
792     return l;
2E4 }

```

### 3.5 frac

```

D41 // de tfg

```

```

4FC template<class T>
28A T gcd(T a, T b) { return b == 0 ? a : gcd(b, a % b); }

4FC template<class T>
67A struct Frac {
4A6     T p, q;
124     Frac() {
FF3         p = 0, q = 1;
DDF     }
735     Frac(T x) {
E94         p = x;
B4A         q = 1;
11B     }
2B4     Frac(T a, T b) {
A30         if(b == 0) {
954             a = 0;
102             b = 1;
597         }
C53         p = a;
008         q = b;
39B         fix();
E90     }
50D     Frac<T> operator + (Frac<T> o) const { return Frac(p * o.q + o.p * q, q * o.q); }
DF1     Frac<T> operator - (Frac<T> o) const { return Frac(p * o.q - o.p * q, q * o.q); }
DE5     Frac<T> operator * (Frac<T> o) const { return Frac(p * o.p, q * o.q); }
1EC     Frac<T> operator / (Frac<T> o) const { return Frac(p * o.q, q * o.p); }

01A     void fix() {
4C3         if(q < 0) {
A71             q = -q;
1A2             p = -p;
729         }
BF8         auto g = gcd(max(p, -p), q);
5C4         p /= g;
EC3         q /= g;
698     }

6EA     bool operator < (Frac<T> o) const { return ((*this) - o).p < 0; }
CB2     bool operator > (Frac<T> o) const { return ((*this) - o).p > 0; }

EA8     friend ostream& operator << (ostream &os, const Frac<T> &f) {
603         return os << f.p << '/' << f.q;
E4F     }
5A1     friend istream& operator >> (istream &is, Frac<T> &f) {
B5D         char trash;
F9E         return is >> f.p >> trash >> f.q;
654     }
3F1 };

```

### 3.6 fastexp

```

D41 // Fast Exp
031 const ll mod = 1e9+7;

```

```

8D8 ll fexpll(ll a, ll n){
D54 ll ans = 1;
02A while(n){
A19     if(n & 1) ans = (ans * a) % mod;
4E2     a = (a * a) % mod;
9D3     n >>= 1;
CAB }
BA7 return ans;
D19 }
D41 // matriz quadrada
BE9 class Matrix{
673 public:
21E vector<vector<ll>> mat;
2E6 int m;
1D7 Matrix(int m): m(m){
593     mat.resize(m);
2BC     for(int i = 0; i < m; i++) mat[i].resize(m,0);
809 }
215 Matrix operator * (const Matrix& rhs){
8EB     Matrix ans = Matrix(m);
94F     for(int i = 0; i < m; i++){
A75         for(int j = 0; j < m; j++){
800             for(int k = 0; k < m; k++){
1F7                 ans.mat[i][j] = (ans.mat[i][j] + (mat[i][k] * rhs.mat[k][j]) % mod) % mod;
BA7             return ans;
2E6         }
A70 };

```

```

E2E Matrix fexp(Matrix a, ll n){
71E int m = a.m;
8EB Matrix ans = Matrix(m);
642 for(int i = 0; i < m; i++) ans.mat[i][i] = 1;
02A while(n){
A50     if(n & 1) ans = ans * a;
476     a = a * a;
9D3     n >>= 1;
CDF }
BA7 return ans;
966 }

```

## 3.7 divtrick

```

79C for(int l = 1, r; l <= n; l = r + 1) {
746     r = n / (n / l);
D41     // n / i has the same value for l <= i <= r
D41     // O(sqrt(n)) different floor(n/i) values
5BF }

```

## 3.8 phi

```

A8C const int LIM = 1e6+5;
C75 int phi[LIM];
8E0 void sieve(){
9A6     iota(phi, phi + LIM, 0);
C35     for(int i = 2; i < LIM; i++){
729         if(phi == i){
EBC             for(int j = i; j < LIM; j += i){
A9B                 phi[j] -= phi[j] / i;
4BC             }
37B         }
02A     }
953 }

```

```

67A template<typename T>

```

```

E6F T phi(T n) {
FC4     T ans = n;
D24     for(T p = 2; p * p <= n; p++) {
80A         if(n % p == 0) {
B7F             ans -= ans / p;
03E             while(n % p == 0) {
F4A                 n /= p;
91F             }
D76         }
4BB     }
B26     if(n > 1) {
675         ans -= ans / n;
C1B     }
BA7     return ans;
427 }

```

## 3.9 gaussianElim

```

67A template<typename T>
029 struct GaussElim{
757     vector<vector<T>> rows;
AF2     vector<int> where;
BDF     vector<bool> hasRow;
B5C     int m, n;
75D     GaussElim(int vars) : m(vars){
E53         where.assign(m,-1);
84E         n = 0;
9CB     }
FC0     void add_eq(vector<T> row){ // colocar o b aqui tb
6D8         rows.push_back(row);
15A         hasRow.push_back(false);
015         n++;
CA0     }
71A     int status(){
D41         // 0: no solution, 1: unique, 2: infinite
830         for(int i = 0; i < n; i++){
673             if(!hasRow[i] && rows[i].back() != T(0))
BB3                 return 0;
A75         for(int j = 0; j < m; j++){
EA5             if(where[j] == -1)
18B                 return 2;

6A5         return 1;
2A2     }
C44     int go(){
CA3         int n = rows.size();
891         for(int j = 0; j < m; j++){
603             for(int i = 0; i < n; i++){
A03                 if(rows[i][j] != T(0) && !hasRow[i]){
33C                     where[j] = i;
258                     hasRow[i] = 1;
C2B                     break;
4D1                 }
E80             }
DC1             if(where[j] == -1) continue;
D41             // fix linha where[j]
F71             T div = rows[where[j]][j];
38C             for(int k = 0; k <= m; k++){
24C                 rows[where[j]][k] /= div;
603             for(int i = 0; i < n; i++){
853                 if(i == where[j]) continue;
F58                 T mul = -1*rows[i][j];
38C                 for(int k = 0; k <= m; k++){
B6E                     rows[i][k] += mul*rows[where[j]][k];
D94                 }
8AD             }
D22             return status();
12F         }

```

```

05B     vector<T> get() {
D41         // assert status = 1
DF8         vector<T> ret(m);
94F         for(int i = 0; i < m; i++)
5F9             ret[i] = rows[where[i]].back();
EDF         return ret;
5BF     }
D7C };

```

## 4 geometry

### 4.1 minkowski

```

F05 using P = PT<double>;
B3C vector<P> minkowskiSum(vector<P> p, vector<P> q) {
D27     if(p.empty() || q.empty()) return {};
B23     auto fix = [](vector<P>& x) {
48D         rotate(x.begin(), min_element(x.begin(), x.end()), x.end());
72A         x.push_back(x[0]), x.push_back(x[1]);
D06     };
00A     fix(p); fix(q);
2E3     vector<P> ret;
692     int i = 0, j = 0;
2EE     while (i < p.size()-2 or j < q.size()-2) {
898         ret.push_back(p[i] + q[j]);
132         auto c = ((p[i+1] - p[i]).cross(q[j+1] - q[j]));
EBC         if (c >= 0) i = min<int>(i+1, p.size()-2);
81E         if (c <= 0) j = min<int>(j+1, q.size()-2);
40F     }
EDF     return ret;
D08 }

```

```

312 double segDist(P s, P e, P p) {
BD2     if (s==e) return (p-s).len();
564     auto d = (e-s).dist2(), t = min(d,max(.01, (p-s).dot(e-s)));
9E6     return ((p-s)*d-(e-s)*t).len()/d;
824 }

```

```

638 double dist_convex(vector<P> p, vector<P> q) {
116     for (P& i : p) i = i * -1;
029     auto s = minkowskiSum(p, q);
B2F     if (isInside(s, P(0,0))) return 0;
49D     double ans = 1e18; // INF
1DC     int ssz = s.size();
F26     for(int i = 0; i < ssz; i++){
06B         int j = (i+1)%ssz;
6AA         ans = min(ans, segDist(s[i], s[j], P(0,0)));
809     }
BA7     return ans;
EE2 }

```

### 4.2 mincircle

```

16E typedef PT<double> P;
406 double ccRadius(P& A, P& B, P& C) {
D82     return (B-A).len()* (C-B).len()* (A-C).len()/
83F         abs((B-A).cross(C-A))/2.0;
6BC }

07B P ccCenter(P& A, P& B, P& C) {
28A     P b = C-A, c = B-A;
680     return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
24F }
D41 // mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());

```

```

006 pair<P, double> mec(vector<P>& pts){
03B     shuffle(begin(pts),end(pts),rng);
185     P o = pts[0];
FCB     const double EPSS = 1+1e-8;
996     double r = 0;
743     for(int i = 0; i < pts.size(); i++) if((o-pts[i]).len() > r * EPSS){
759         o = pts[i], r = 0;
6E9         for(int j = 0; j < i; j++) if((o-pts[j]).len() > r * EPSS){
FD8             o = (pts[i]+pts[j])/2.0;
DAE             r = (o - pts[i]).len();
FE0             for(int k = 0; k < j; k++) if((o-pts[k]).len() > r * EPSS){
ECD                 o = ccCenter(pts[i],pts[j],pts[k]);
DAE                 r = (o - pts[i]).len();
5AB             }
102         }
509     }
645     return {o, r};
8E0 }

```

### 4.3 closestpair

```

531 pii ClosestPair(vector<PT<ll>>& pts) {
62D     ll dist = (pts[0]-pts[1]).dist2();
1A2     pii ans(0, 1);
CA0     int n = pts.size();
47B     vector<int> p(n);
469     iota(begin(p),end(p),0);
F85     sort(p.begin(), p.end(), [&](int a, int b) { return pts[a].x < pts[b].x; });
3E7     set<pii> points;
526     auto sqr = [](long long x) -> long long { return x * x; };
637     for(int l = 0, r = 0; r < n; r++) {
39E         while(sqr(pts[p[r]].x - pts[p[l]].x) > dist) {
9F7             points.erase(pii(pts[p[l]].y, p[l]));
63B             l++;
FFB         }
7CF         ll delta = sqrt(dist) + 1;
92B         auto itl = points.lower_bound(pii(pts[p[r]].y - delta, -1));
6C3         auto itr = points.upper_bound(pii(pts[p[r]].y + delta, n + 1));
901         for(auto it = itl; it != itr; it++) {
C66             ll curDist = (pts[p[r]] - pts[it->second]).dist2();
0CF             if (curDist < dist) {
1E3                 dist = curDist;
8E7                 ans = pii(p[r], it->second);
AE0             }
C07         }
EEA         points.insert(pii(pts[p[r]].y, p[r]));
9B6     }
EBB     if(ans.first > ans.second)
6FE         swap(ans.first, ans.second);
BA7     return ans;
D1D }

```

### 4.4 point

```

D41 // hypot, atan2, gcd
1D5 const double PI = acos(-1);
48B template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
67A template<typename T>
C19 struct PT{
645     T x, y;
7AB     PT(T x=0, T y=0) : x(x),y(y){}
425     bool operator < (PT o) const { return tie(x,y) < tie(o.x,o.y); }
987     bool operator == (PT o) const { return tie(x,y) == tie(o.x,o.y); }
EB1     PT operator + (PT o) const { return PT(x+o.x,y+o.y); }
D02     PT operator - (PT o) const { return PT(x-o.x,y-o.y); }
EAB     PT operator * (T k) const { return PT(x*k,y*k); }

```

```

593 PT operator / (T k) const { return PT(x/k,y/k); }
6F1 T cross(PT o) const { return x*o.y - y*o.x; }
F53 T cross(PT a, PT b) const { return (a-*this).cross(b-*this); }
A65 T dot(PT o) const { return x*o.x + y*o.y; }
F68 T dist2() const { return x*x + y*y; }
0A7 double len() const { return hypot(x,y); }
559 PT perp() const { return PT(-y,x); }
2B2 PT rotate(double a) const { return PT(x*cos(a)-y*sin(a), x*sin(a)+y*cos(a)); }
950 int quad() { return (x<0)^3*(y<0); }
CF5 bool ccw(PT<T> q, PT<T> r){ return (q-*this).cross(r-q) > 0; }
E63 };
67A template<typename T>
306 bool cmp_ang(PT<T> p, PT<T> q){
6E2     if (p.quad() != q.quad()) return p.quad() < q.quad();
B84     return q.ccw(PT<T>(0,0),p);
FD9 }

BA2 ostream &operator<<(ostream &os, const PT<int> &p) {
D80     return os << "(" << p.x << ", " << p.y << ")";
3EB }

```

## 4.5 convexhull

```

D41 // retorna poligono no sentido anti horario, trocar pra < se quiser horario
67A template<typename T>
F1C vector<PT<T>> convexHull(vector<PT<T>>& pts, bool sorted = false){
EC1     if(!sorted) sort(begin(pts),end(pts));
661     vector<PT<T>> h;
89A     h.reserve(pts.size() + 1);
7A4     for(int it = 0; it < 2; it++){
5C9         int start = h.size();
BB9         for(PT<T>& c : pts){
C31             while((int)h.size() >= start + 2){
184                 PT<T> a = h[h.size()-2], b = h.back();
D41                 // '>=' pra nao descartar pontos colineares
630                 if((b-a).cross(c-a) > 0) break;
BFB                 h.pop_back();
748             }
476             h.push_back(c);
A0A         }
A8B         reverse(begin(pts),end(pts));
BFB         h.pop_back();
AAD     }
E30     if(h.size() == 2 && h[0] == h[1]) h.pop_back();
81C     return h;
AD6 }

```

```

D41 // nao funciona se tem pontos colineares!!!!
D41 // considera ponto na aresta como dentro
67A template<typename T>
319 bool isInside(vector<PT<T>>& hull, PT<T> p) {
ACE     int n = hull.size();
8DD     PT<T> v0 = p - hull[0], v1 = hull[1] - hull[0], v2 = hull[n-1] - hull[0];
6A6     if(v0.cross(v1) > 0 || v0.cross(v2) < 0){
D1F         return false;
769     }
9D9     int l = 1, r = n - 1;
E55     while(l != r){
264         int mid = (l + r + 1) / 2;
549         PT<T> v0 = p - hull[0], v1 = hull[mid] - hull[0];
2C9         if(v0.cross(v1) < 0)
229             l = mid;
295         else
982             r = mid - 1;
406     }
87D     v0 = hull[(l+1)%n] - hull[l], v1 = p - hull[l];
57C     return v0.cross(v1) >= 0;
76A }

```

```

D41 // poligonos
36E ll polygon_area_db(const vector<Point>& poly){
604     ll area = 0;
F6A     for(int i = 0, n = (int)poly.size(); i < n; ++i) {
A91         int j = i + 1 == n ? 0 : i + 1;
140         area += cross(poly[i], poly[j]);
A87     }
199     return abs(area);
315 }
D41 // Teorema de Pick para lattice points
D41 // Area = insidePts + boundPts/2 - 1
D41 // 2A - b + 2 = 2i
D41 // usar gcd dos lados pra contar bound pts
E82 ll cntInsidePts(ll area_db, ll bound){
97C     return (area_db + 2LL - bound)/2;
CDC }

```

## 5 data-structures

### 5.1 maxqueue

```

2D5 template <class T, class C = less<T>>
CF4 struct MaxQueue {
8E5     MaxQueue() { clear(); }
75A     void clear() {
B08         id = 0;
4B6         q.clear();
5DD     }
3FC     void push(T x) {
9FB         pair<int, T> nxt(1, x);
285         while(q.size() > id && cmp(q.back().second, x)) {
7A6             nxt.first += q.back().first;
AB1             q.pop_back();
F02         }
986         q.push_back(nxt);
FAF     }
577     T qry() { return q[id].second;}
42D     void pop() {
62D         q[id].first--;
097         if(q[id].first == 0) { id++; }
3CB     }
BF2 private:
4A7     vector<pair<int, T>> q;
53E     int id;
1B8     C cmp;
961 };

```

### 5.2 segtree-lazy

```

D41 // Lazy SegTree ta meio desatualizado mas sei modificar
35A const int mx = 2e5+5;
131 vector<ll> seg(4*mx);
3F0 vector<ll> lazy(4*mx,0);
C6B vector<ll> nums(mx);
3B9 int n,q;

353 void build(int l = 0, int r = n-1, int idx = 0){
893     if(l == r){
389         seg[idx] = nums[l];
824         lazy[idx] = 0;
505         return;
821     }
EE4     int m = (l+r)/2;

```

```

EED    int left = 2*idx+1;
680    int right = 2*idx+2;
84F    build(l,m,left);
C3B    build(m+1,r,right);
5B2    seg[idx] = seg[left] + seg[right];
67B }

175 void prop(int l = 0, int r = n-1, int idx = 0){
A39    seg[idx] += (ll)(r-l+1)*lazy[idx];
579    if(l != r){ // nao for folha
EED        int left = 2*idx+1;
680        int right = 2*idx+2;
774        lazy[left] += lazy[idx];
AAE        lazy[right] += lazy[idx];
4D4    }
824    lazy[idx] = 0;
5EF }

BFE void update(int L, int R, ll val, int l = 0, int r = n-1, int idx = 0){
483    if(R < l || L > r) return;
E1B    prop(l,r,idx);
761    if(L <= l && r <= R){
F6E        lazy[idx] = val;
E1B        prop(l,r,idx);
64B    }
4E6    else{
EE4        int m = (l+r)/2;
EED        int left = 2*idx+1;
680        int right = 2*idx+2;
AC4        update(L,R,val,l,m,left);
CDB        update(L,R,val,m+1,r,right);
5B2        seg[idx] = seg[left] + seg[right];
01D    }
375 }

6F7 ll query(int L, int R, int l = 0, int r = n-1, int idx = 0){
E1B    prop(l,r,idx);
1BA    if(R < l || L > r) return 0;
761    if(L <= l && r <= R){
A9D        return seg[idx];
2BD    }
EE4    int m = (l+r)/2;
EED    int left = 2*idx+1;
680    int right = 2*idx+2;
1FE    return query(L,R,l,m,left) + query(L,R,m+1,r,right);
F3A }

```

## 5.3 kd-tree

```

605 template<class T, const int D = 2>
21E class KD_Tree {
673 public:
E72    using PT = array<T, D>;

BF2    struct Node {
BB6        PT point;
795        Node *left, *right;
41D    };

D02    void init(vector<PT> pts) {
BA3        if(pts.size() == 0) {
505            return;
6AD        }
443        int n = 0;
9FC        tree.resize(2 * pts.size());
C8F        build<0>(pts.begin(), pts.end(), n);
D41        //assert(n <= (int) tree.size());
7BD    }

```

```

4C7    pair<T, PT> nearestNeighbor(PT point) {
D41        // assert(tree.size() > 0);
916        pair<T, PT> ans;
465        ans.first = 8.1e18; // BE CAREFUL HERE
010        removedEqual = false;
2B2        nearestNeighbor<0>(&tree[0], point, ans);
BA7        return ans;
8E7    }
BF2 private:
8A5    vector<Node> tree;

C5A    template<const int d>
256    Node* build(auto l, auto r, int &n) {
0B1        if(l >= r) {
EA9            return NULL;
248        }
AB9        int id = n++;
652        if(r - l == 1) {
270            tree[id].left = tree[id].right = NULL;
547            tree[id].point = *l;
C82        } else {
263            auto mid = l + ((r - l) / 2);
FF2            nth_element(l, mid - 1, r, [](const PT &u, const PT &v) { return u[d] < v[d];

});
6FF            tree[id].point = *(mid - 1);
D41            // BE CAREFUL!
D41            // DO EVERYTHING BEFORE BUILDING THE LOWER PART!
2B4            tree[id].left = build<(d+1)%D>(l, mid-1, n);
57F            tree[id].right = build<(d+1)%D>(mid, r, n);
4B5        }
FAB        return &tree[id];
BEC    }
491    bool removedEqual;
C5A    template<const int d>
F9C    void nearestNeighbor(Node* node, PT point, pair<T, PT> &ans) {
260        if(!node) {
505            return;
08D        }
7DC        const T dist = sqrtDist(point, node->point);
9A9        if((point != node->point || removedEqual) && dist < ans.first) {
D41            // THIS WAS FOR A PROBLEM
D41            // THAT YOU DON'T CONSIDER THE DISTANCE TO ITSELF!
5BE            ans = {dist, node->point};
BB0        } else if(point == node->point) {
60E            removedEqual = true;
7B6        }
B67        T delta = point[d] - node->point[d];
5DE        if(delta <= 0) {
871            nearestNeighbor<(d+1)%D>(node->left, point, ans);
09A            if(ans.first > delta * delta) {
008                nearestNeighbor<(d+1)%D>(node->right, point, ans);
74C            }
A87        } else {
008            nearestNeighbor<(d+1)%D>(node->right, point, ans);
09A            if(ans.first > delta * delta) {
871                nearestNeighbor<(d+1)%D>(node->left, point, ans);
9F1            }
212        }
B7A    }

363    T sqrtDist(const PT &u, const PT &v) {
966        T ans = 0;
772        for(int i = 0; i < D; i++) {
ED8            T delta = u[i] - v[i];
928            ans += delta * delta;
B96        }
BA7        return ans;
1B7    }
03D };

```



## 5.4 colorupdate

```
DEE template <class InfoType = int32_t, class RangeType = int32_t>
F1D struct ColorUpdate {
673 public:
3D4     struct Range {
032         Range(RangeType _l = 0) : l(_l) {}
9A5         Range(RangeType _l, RangeType _r, InfoType _v) : l(_l), r(_r), v(_v) {}
A41         RangeType l, r;
0C1         InfoType v;

DBA
F3C     };

2A4     vector<Range> erase(RangeType l, RangeType r) {
D02         vector<Range> ans;
AF6         if(l >= r) return ans;
EFB         auto it = ranges.lower_bound(l);
A43         if(it != ranges.begin()) {
049             it--;
86E             if(it->r > l) {
BF0                 auto cur = *it;
649                 ranges.erase(it);
206                 ranges.insert(Range(cur.l, l, cur.v));
FD9                 ranges.insert(Range(l, cur.r, cur.v));
C39             }
FC5         }
7D1         it = ranges.lower_bound(r);
A43         if(it != ranges.begin()) {
049             it--;
E91             if(it->r > r) {
BF0                 auto cur = *it;
649                 ranges.erase(it);
D16                 ranges.insert(Range(cur.l, r, cur.v));
3A3                 ranges.insert(Range(r, cur.r, cur.v));
2E9             }
7DB         }
444         for(it = ranges.lower_bound(l); it != ranges.end() && it->l < r; it++) {
3BD             ans.push_back(*it);
992         }
E1D         ranges.erase(ranges.lower_bound(l), ranges.lower_bound(r));
BA7         return ans;
39A     }

B86     vector<Range> upd(RangeType l, RangeType r, InfoType v) {
F3D         auto ans = erase(l, r);
ED3         ranges.insert(Range(l, r, v));
BA7         return ans;
F56     }

8DB     bool exists(RangeType x) {
3D4         auto it = ranges.upper_bound(x);
ED2         if(it == ranges.begin()) return false;
049         it--;
03C         return it->l <= x && x < it->r;
E44     }
BF2 private:
4A2     set<Range> ranges;
7C5 };
```

## 5.5 sparsetable

```
D41 // use const, principalmente no log
D41 // log = maior msb a[i]
B55 const int LOG = 20;
D41 // query [L,R]
```

```
67A template<typename T>
7E9 struct SparseTable{
1A8     int n;
F9A     vector<vector<T>> table;
8EE     SparseTable(){}
407     SparseTable(vector<T>& v){
D6C         n = v.size();
310         table = vector<vector<T>>(LOG + 1, vector<T>(n));
760         table[0] = v;
F89         for(int lg = 0; lg < LOG; lg++){
603             for(int i = 0; i < n; i++){
5FE                 if (i + (1 << lg) >= n) break;
F88                 table[lg + 1][i] = min(table[lg][i], table[lg][i + (1 << lg)]);
8A6             }
F11         }
A47     }
C90     T qry(int l, int r){
884         int k = 31 - __builtin_clz(r-l+1);
DC9         return min(table[k][l], table[k][r - (1 << k) + 1]);
6EF     }
EAB };
```

## 5.6 fenwick-tree

```
35A const int mx = 2e5+5;
C14 ll bit[mx];
3B9 int n, q;

2AF ll qry(int i){ // [1,i] 1 indexado
B73     ll ret = 0;
FBD     for(; i > 0; i -= i & -i)
7CA         ret += bit[i];
EDF     return ret;
1D1 }

9A6 void increment(ll i, ll v){ // 1 indexado (+= v)
A6D     for(; i <= n; i += i & -i)
963         bit[i] += v;
28C }
```

## 5.7 dynamic-median

```
4B4 const ll inf = 1e18 + 5;
283 struct DynamicMedian{
E01     multiset<ll> left, right;
A2C     ll leftsum = 0, rightsum = 0;
6DE     ll get(){
D41         // if(left.empty()) return -1; // cuidar aqui
668         return *left.rbegin();
833     }
F4C     ll qry(){ // somatorio de distancia absoluta pra mediana
239         ll m = get();
D41         // if(m == -1) return -1;
099         return left.size()*m - leftsum + rightsum - right.size()*m;
AF5     }
01A     void fix(){
D41         // (L,R) ou (L+1,R)
770         while(right.size() + 1 < left.size()){
D41             // tirar do l e colocar no r
BF5             auto lst = --left.end();
FC3             rightsum += *lst;
FCD             leftsum -= *lst;
64F             right.insert(*lst);
47B             left.erase(lst);
761         }
49B         while(right.size() > left.size()){
```

```

D41      // tirar do r e colocar no l
D50      leftsum += *right.begin();
9C1      rightsum -= *right.begin();
449      left.insert(*right.begin());
5E3      right.erase(right.begin());
C41    }
78A    }

6CD    void insert(ll x){
6A4      ll m = (left.empty() ? inf : get());
D3B      if(x <= m){
BE2        left.insert(x);
00B        leftsum += x;
23E      }else{
AD2        right.insert(x);
D20        rightsum += x;
941      }
39B      fix();
3E3    }
C95    void erase(ll x){
F83      auto l = left.find(x);
FE1      if(l != left.end()){
CDF        leftsum -= *l;
898        left.erase(l);
138      }
4E6      else{
6DA        auto r = right.find(x);
519        rightsum -= *r;
74F        right.erase(r);
DD8      }
39B      fix();
326    }
781  };

```

## 5.8 segtree-topdown

```

D41 // SegTree
35A const int mx = 2e5 + 5;
ADA ll seg[4*mx];
56A ll a[mx];
3B9 int n,q;
4B5 ll join(ll a, ll b){
534   return a+b;
2D6 }

353 void build(int l = 0, int r = n - 1, int idx = 0){
893   if(l == r){
B28     seg[idx] = a[l];
505     return;
3A6   }
AE0   int mid = (l + r)/2;
8E4   build(l, mid, 2*idx + 1);
2F1   build(mid + 1, r, 2*idx + 2);
AED   seg[idx] = join(seg[2*idx + 1], seg[2*idx + 2]);
830 }

6F7 ll query(int L, int R, int l = 0, int r = n - 1, int idx = 0){
1BA   if(R < l || L > r) return 0; // elemento neutro
FA9   if(L <= l && r <= R) return seg[idx];
AE0   int mid = (l + r)/2;
9D8   return join(query(L, R, l, mid, 2*idx + 1), query(L, R, mid + 1, r, 2*idx + 2));
579 }

61D void update(int i, ll val, int l = 0, int r = n - 1, int idx = 0){
893   if(l == r){
873     seg[idx] = val;
505     return;
741   }

```

```

AE0   int mid = (l + r)/2;
8B0   if(i <= mid) update(i, val, l, mid, 2*idx + 1);
AAB   else update(i, val, mid + 1, r, 2*idx + 2);
AED   seg[idx] = join(seg[2*idx + 1], seg[2*idx + 2]);
EA9 }

```

## 6 etc

### 6.1 mo

Mo em árvore: queries em caminhos. Olhar carinhas que aparecem quantidade ímpar de vezes  
Seja u menor tin  
se u for lca de v: range = [tin[u], tin[v]]  
c.c: range = [tout[u], tin[v]] U [tin[lca],tin[lca]] -- tratar lca separado quando tiver respondendo

```

D41 // Mo apelao
D41 // Ordering based on the Hilbert curve
905 inline int64_t hilbertOrder(int x, int y, int pow, int rotate){
51A   if(pow == 0) return 0;
A6E   int hpow = 1 << (pow - 1);
01F   int seg = (x < hpow) ? ( (y < hpow) ? 0 : 3) : ( (y < hpow) ? 1 : 2);
6D9   seg = (seg + rotate) & 3;
F96   const int rotateDelta[4] = {3, 0, 0, 1};
D0B   int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
385   int nrot = (rotate + rotateDelta[seg]) & 3;
7AC   int64_t subSquareSize = int64_t(1) << (2*pow - 2);
98B   int64_t ans = seg * subSquareSize;
B22   int64_t add = hilbertOrder(nx, ny, pow - 1, nrot);
7C5   ans += (seg == 1 || seg == 2) ? add : (subSquareSize - add - 1);
BA7   return ans;
E55 }
670 struct Query{
738   int l, r, idx;
CE8   int64_t ord;
36F   Query(int l, int r, int idx) : l(l), r(r), idx(idx) {
11F     ord = hilbertOrder(l, r, 21, 0);
B25   }
E07   bool operator < (Query &other){
328     return ord < other.ord;
007   }
912 };
D41 // Mo normal
A47 const int MXN = 2e5;
D24 const int B = sqrt(MXN) + 1;
670 struct Query {
738   int l, r, idx;
665   bool operator<(Query o) const{
736     return make_pair(l / B, ((l/B) & 1) ? -r : r) < make_pair(o.l / B, ((o.l/B) & 1) ? -o
.r : o.r);
A4B   }
868 };

CD3 ll a[MXN];
C3D ll resp = 0;
EAC void add(int x);
292 void remove(int x);

E8D int main(){
C5F   int n, q; cin >> n >> q;
830   for(int i = 0; i < n; i++){
788     cin >> a[i];
240   vector<Query> queries;
ABF   for(int i = 0; i < q; i++){
5DA     int l, r; cin >> l >> r;

```

```

29D     queries.push_back(Query(l-1,r-1,i));
08A     }
671     sort(begin(queries),end(queries));
153     vector<ll> answers(q);
70A     int L = 0, R = -1;
ECD     for(Query qr : queries){
981         while (L > qr.l) add(--L);
FF1         while (R < qr.r) add(++R);
167         while (L < qr.l) remove(L++);
A20         while (R > qr.r) remove(R--);
620         answers[qr.idx] = resp;
1BD     }
EDF     for(int i = 0; i < q; i++)
04D         cout << answers[i] << "\n";
459 }

```

## 6.2 bitset

```

D41 // Comando hash de codigo :w !sha256sum

```

```

D41 // Bitset operations
99C __builtin_popcount(int x);
65C __builtin_popcountll(ll x);
302 const int SZ = 1e6;
596 bitset<SZ> b;
155 b.reset(); // 00 ... 00
29C b.set(); // 11 ... 11
98B b.flip();
C72 b._Find_first(); // retorna SZ se nao tiver
235 b._Find_next(i);
B9B b.to_ulong();
ED7 b.to_string();
E9E b.count();

D41 // rng
C8A mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
8A3 shuffle(begin(x),end(x),rng);
598 uniform_int_distribution<int>(0,x)(rng);

```

## 6.3 ternary-search

```

D5D double f(double t){
D41 // alguma funcao unimodal -> maximo ou minimo
D41 //      /\
D41 //    /  \
D41 //  __/    \__
392 }

CD1 double tern_search(double l, double r){
D1A     for(int it = 0; it < 300; it++){
A50         double m1 = l + (r-l)/3;
7E7         double m2 = r - (r-l)/3;
87F         double f1 = f(m1), f2 = f(m2);
19D         if(f1 < f2) l = m1; //change to > to find maximum
064         else r = m2;
ABB     }
792     return l;
058 }

D41 // golden section search
036 double gss(double a, double b) {
2B1     const double r = (sqrt(5)-1)/2, eps = 1e-7;
B87     double x1 = b - r*(b-a), x2 = a + r*(b-a);
47D     double f1 = f(x1), f2 = f(x2);
F2C     for(int it = 0; it < 250 && b-a > eps; it++)

```

```

F4D     if (f1 < f2) { //change to > to find maximum
DA5         b = x2; x2 = x1; f2 = f1;
DFB         x1 = b - r*(b-a); f1 = f(x1);
451     } else {
D6E         a = x1; x1 = x2; f1 = f2;
815         x2 = a + r*(b-a); f2 = f(x2);
505     }
3F5     return a;
894 }

D41 // retorna mais a esquerda no empate
E15 int int_tern_search(int l, int r){
7AD     int lo = l - 1, hi = r;
FF8     while(hi - lo > 1){
591         int m = (lo+hi)/2;
42D         if(f(m) < f(m+1)){ //
04A             lo = m;
FA4         }else{
89E             hi = m;
DAB         }
30C     }
480     return lo + 1;
88D }

```

## 6.4 formulas

Soma de pg:  $= a1 \cdot (q^n - 1) / (q - 1)$   
Soma dos impares:  $= n^2$   
Soma de  $i^2$ :  $= n(n+1)(2n+1)/6$

Number theory:

$\gcd(a+k \cdot b, b) = \gcd(a, b)$   
 $\phi(n) = \# \text{coprimos com } n \leq n$   
 $\phi(n) \geq \log_2(n)$   
 $\phi(\phi(n)) \leq n/2$   
 $a^{\phi(n)} \equiv 1 \pmod n$   
 $a^{-1} \equiv a^{(m-2)} \pmod m$

Conjectura de Goldbach's: todo numero par  $n > 2$  pode ser representado com  $n = a + b$  onde  $a$  e  $b$  sao primos

Twin prime: existem infinitos pares  $p, p + 2$  onde ambos sao primos

Legendre's: sempre tem um primo entre  $n^2$  e  $(n+1)^2$

Lagrange's: todo numero inteiro pode ser inscrito como a soma de 4 quadrados

Wilson's:  $n$  eh primo quando  $(n-1)! \pmod n = n - 1$

McNugget: Para dois coprimos  $x, y$  a quantidade de inteiros que nao pode ser escrito como  $ax + by$  eh  $(x-1)(y-1)/2$ ,  
o maior inteiro que nao consegue eh  $x \cdot y - x - y$

Geometria:

$V + F = A + 2$

Formula de heron:  $\sqrt{s(s-a)(s-b)(s-c)}$ ,  $s$  = semiperimetro

Volume de esfera:  $4/3 \pi \cdot r^3$

Area da esfera:  $4 \pi \cdot r^2$

Volume tetraedro:  $l^3 \cdot \sqrt{2}/12$

Projecao u em v =  $(u \cdot v) / (v \cdot v) \cdot v$

## 6.5 composite

number	divisors	factorization
120	16	$2^3 \cdot 3 \cdot 5$
1.260	36	$2^2 \cdot 3^2 \cdot 5 \cdot 7$
10.080	72	$2^5 \cdot 3^2 \cdot 5 \cdot 7$
110.880	144	$2^5 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$
1.081.080	256	$2^3 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$
10.810.800	480	$2^4 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$
110.270.160	800	$2^4 \cdot 3^4 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17$
1.102.701.600	1440	$2^5 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17$

## 7 graph

### 7.1 kosaraju

```
D41 // Kosaraju
F9F const int ms = 1e5 + 5;
9CB vector<int> G[ms], Gt[ms];
9F5 vector<int> id, order, root;
B57 vector<bool> vis;
1A8 int n;
CB7 void dfs1(int u) { // ordem de saida
B9C     vis[u] = true;
E44     for(int v : G[u])
C2D         if(!vis[v])
3BA             dfs1(v);
C75     order.push_back(u);
7A7 }
6A1 void dfs2(int u, int idx) {
846     id[u] = idx;
51D     for(int v : Gt[u])
BBC         if(id[v] == -1)
039             dfs2(v, idx);
594 }
D41 // retorna quantidade de componentes
973 int kosaraju() {
543     vis.assign(n, false);
C2F     id.assign(n, -1);
830     for(int i = 0; i < n; i++)
F69         if(!vis[i])
6D1             dfs1(i);
3B9     reverse(begin(order), end(order));
503     int idx = 0;
87A     for(int u : order)
98B         if(id[u] == -1)
B4B             dfs2(u, idx++), root.push_back(u);
449     return idx;
359 }
```

### 7.2 dinic

```
D41 //O(V^2 E), O(E sqrt V) in unit networks
67A template<typename T>
E9B struct Edge {
791     int to;
D90     T cap, flow;
112     Edge(int to, T cap) : to(to), cap(cap), flow(0) {}
338     T res() const { return cap - flow; }
E92 };
67A template<typename T>
14D struct Dinic {
4D8     using E = Edge<T>;
05B     int m = 0, n;
976     vector<E> ed;
789     vector<vector<int>>> g;
3B3     vector<int> dist, ptr;
085     Dinic(int n) : n(n), g(n), dist(n), ptr(n) {}
555     void add_edge(int u, int v, T cap) {
DF5         if(u != v) {
65F             ed.emplace_back(v, cap);
F0D             ed.emplace_back(u, 0);
329             g[u].emplace_back(m++);
4C9             g[v].emplace_back(m++);
688         }
680     }
```

```
123 bool bfs(int s, int t) {
FD5     fill(begin(dist), end(dist), n + 1);
A93     dist[s] = 0;
0B4     queue<int> q({s});
14D     while(!q.empty()) {
E4A         int u = q.front();
833         q.pop();
4B5         if(u == t) break;
10F         for(int id : g[u]) {
22E             E& e = ed[id];
D9E             if(e.res() > 0 && dist[e.to] > dist[u] + 1) {
29B                 dist[e.to] = dist[u] + 1;
A78                 q.emplace(e.to);
08B             }
3D5         }
53A     }
8B6     return dist[t] != n + 1;
B70 }
D6A T dfs(int u, int t, T flow) {
3B2     if(u == t || flow == 0) {
99D         return flow;
B48     }
C53     for(int& i = ptr[u]; i < (int)g[u].size(); ++i) {
04D         E& e = ed[g[u][i]];
A8D         E& eRev = ed[g[u][i] ^ 1];
AF4         if(dist[e.to] == dist[eRev.to] + 1) {
4CF             T amt = min(flow, e.res());
F17             if(T ret = dfs(e.to, t, amt)) {
786                 e.flow += ret;
CEB                 eRev.flow -= ret;
EDF                 return ret;
95A             }
C73         }
B2E     }
BB3     return 0;
C50 }
9C4 T max_flow(int s, int t) {
C80     T total = 0;
8CE     while(bfs(s, t)) {
197         fill(begin(ptr), end(ptr), 0);
419         while(T flow = dfs(s, t, numeric_limits<T>::max())) {
810             total += flow;
136         }
70C     }
994     return total;
EB4 }
159 bool cut(int u) const { return dist[u] == n + 1; }
064 };
```

### 7.3 kruskal

```
6D0 int n = 1e5;
E81 DSU dsu = DSU(n+5);
C5C using tp = tuple<ll, int, int>
820 vector<tp> edges(e);
0F4 for(auto& [w, u, v] : edges) {
52D     cin >> u >> v >> w;
7B8 }
F80 sort(begin(edges), end(edges));
854 ll cost = 0;
AC9 int cnt = 0;
2A3 for(auto [w, u, v] : edges) {
AC3     if(dsu.unite(u, v)) {
45F         cost += w;
F65         cnt++;
6CE     }
0B3 }
D41 // if(cnt != n-1) cout << "IMPOSSIBLE" << br;
```

## 7.4 smallToLarge

```
D41 // nlogn small to large (offline)
```

```
1C7 int tin[mxn], tout[mxn];
9E5 vector<int> g[mxn];
05B int clr[mxn];
A0E int inv[mxn];
5BC int sz[mxn];
910 int cnt[mxn];
A10 void calcsz(int u, int p){
267     sz[u] = 1;
73E     tin[u] = ++timer;
5E0     inv[timer] = u;
D76     for(int v : g[u]) if(v != p){
086         calcsz(v,u);
CC3         sz[u] += sz[v];
91B     }
4F8     tout[u] = timer;
CAA }

D78 void dfs(int u, int p, bool mata = 0){
599     int big = -1;
A8C     int howBig = 0;
D76     for(int v : g[u]) if(v != p){
987         if(sz[v] > howBig){
737             big = v;
005             howBig = sz[big];
E24         }
F73     }
181     for(int v : g[u]) if(v != p && v != big){
427         dfs(v,u,1);
02D     }
A5F     if(big != -1)
003         dfs(big,u,0);
181     for(int v : g[u]) if(v != p && v != big){
9AD         for(int l = tin[v]; l <= tout[v]; l++){
3BC             int who = inv[l];
029             cnt[clr[who]]++;
7C4         }
D64     }
D34     cnt[clr[u]]++;
D41     // solve queries aqui
D41     //
35E     if(mata){
21E         for(int l = tin[u]; l <= tout[u]; l++){
3BC             int who = inv[l];
733             cnt[clr[who]]--;
C46         }
4D2     }
51F }
```

## 7.5 policyBased

```
774 #include <ext/pb_ds/assoc_container.hpp> // Common file
30F #include <ext/pb_ds/tree_policy.hpp> // Including tree_order_statistics_node_update
0D7 using namespace __gnu_pbds;
DBF typedef tree<int, null_type, less<int>, rb_tree_tag,
39F tree_order_statistics_node_update> ordered_set;
609 ordered_set X;
766 X.insert(1); X.find_by_order(0); // Acha a key na ordem Y
9C3 X.order_of_key(-5); // Acha a ordem da key Y
FEC end(X), begin(X);
```

## 7.6 centroid

```
A3D bool vis[mxn];
03A int par[mxn]; // parent na centroid tree, par do primeiro centroid eh -1
5BC int sz[mxn];
3A0 int find_centroid(int u, int p, int s){
F5E     for(int v : g[u]) if(v != p && !vis[v] && sz[v] > s / 2){
062         return find_centroid(v,u,s);
8E4     }
03F     return u;
1C8 }

39F void calc_size(int u, int p){
D3C     if(vis[u]){
94A         sz[u] = 0;
505         return;
039     }
267     sz[u] = 1;
D76     for(int v : g[u]) if(v != p){
51D         calc_size(v,u);
CC3         sz[u] += sz[v];
6A2     }
C4D }

889 void init_centroid_tree(int u, int p = -1){
716     calc_size(u,u);
B3F     int c = find_centroid(u,u,sz[u]);
929     vis[c] = 1;
14D     par[c] = p;
45B     for(int v : g[c]) if(!vis[v]){
F15         init_centroid_tree(v,c);
811     }
E67 }
```

## 7.7 articulation

```
14E int n, m;
21F const int mxn = 1e5 + 5;
9E5 vector<int> g[mxn];
B1B int tin[mxn], low[mxn];
A34 vector<int> art;
813 int timer = 1;

FB6 void dfs(int u, int p){
406     tin[u] = timer++;
E99     low[u] = tin[u];
612     int ch = 0;
E67     int fw = 0;
D76     for(int v : g[u]) if(v != p){
85A         if(tin[v]) // lowlink direta
4EC             low[u] = min(tin[v], low[u]);
4E6         else{
95E             dfs(v,u);
AD9             fw++;
E7F             low[u] = min(low[v], low[u]);
918             ch = max(low[v], ch);
534         }
B36     }
9EC     if(u == p && fw > 1) art.push_back(u);
7BD     else if(u != p && ch && tin[u] <= ch) art.push_back(u);
2E6 }
```

## 7.8 mcmf

```
39C template<typename Cap, typename Cost>
6F3 struct MCMF{
```

```

523 const Cost INF = numeric_limits<Cost>::max();
E9B struct Edge {
791     int to;
F23     Cap cap, flow;
CB9     Cost cost;
A6D     Edge(int to, Cap cap, Cost cost) : to(to), cap(cap), flow(0), cost(cost) {}
8FD     Cap res() const { return cap - flow; }
1C8 };
05B int m = 0, n;
321 vector<Edge> edges;
789 vector<vector<int>> g;
ADE vector<Cap> neck;
35F vector<Cost> dist, pot;
E3B vector<int> from;
8C3 MCMF(int n) : n(n), g(n), neck(n), pot(n) {}
780 void add_edge(int u, int v, Cap cap, Cost cost) {
DF5     if(u != v) {
C1A         edges.emplace_back(v, cap, cost);
FB6         edges.emplace_back(u, 0, -cost);
329         g[u].emplace_back(m++);
4C9         g[v].emplace_back(m++);
A0D     }
451 }
E3A void spfa(int s) {
E59     vector<bool> inq(n, false);
0B4     queue<int> q({s});
14D     while(!q.empty()) {
352         auto u = q.front();
833         q.pop();
E0A         inq[u] = false;
B35         for(auto e : g[u]) {
5F3             auto ed = edges[e];
CDD             if(ed.res() == 0) continue;
B5D             Cost w = ed.cost + pot[u] - pot[ed.to];
BAC             if(pot[ed.to] > pot[u] + w) {
CB4                 pot[ed.to] = pot[u] + w;
863                 if(!inq[ed.to]) {
113                     inq[ed.to] = true;
FC4                     q.push(ed.to);
586                 }
5CB             }
B95         }
3F4     }
58B }
9DB bool dijkstra(int s, int t) {
EF2     dist.assign(n, INF);
0B5     from.assign(n, -1);
2DE     neck[s] = numeric_limits<Cap>::max();
C6F     using ii = pair<Cost, int>;
D9A     priority_queue<ii, vector<ii>, greater<ii>> pq;
6BD     pq.push({dist[s] = 0, s});
502     while(!pq.empty()) {
E18         auto [d_u, u] = pq.top();
716         pq.pop();
624         if(dist[u] != d_u) continue;
829         for(auto i : g[u]) {
D26             auto ed = edges[i];
B5D             Cost w = ed.cost + pot[u] - pot[ed.to];
C6C             if(ed.res() > 0 && dist[ed.to] > dist[u] + w) {
A3F                 from[ed.to] = i;
1A3                 pq.push({dist[ed.to] = dist[u] + w, ed.to});
68F                 neck[ed.to] = min(neck[u], ed.res());
7D8             }
D59         }
291     }
85D     return dist[t] < INF;
CCB }
47B pair<Cap, Cost> mcmf(int s, int t, Cap k = numeric_limits<Cap>::max()) {
717     Cap flow = 0;
247     Cost cost = 0;

```

```

FBB     spfa(s);
C28     while(flow < k && dijkstra(s, t)) {
D00         Cap amt = min(neck[t], k - flow);
0D7         for(int v = t; v != s; v = edges[from[v] ^ 1].to) {
2AE             cost += edges[from[v]].cost * amt;
3B4             edges[from[v]].flow += amt;
60F             edges[from[v] ^ 1].flow -= amt;
48F         }
2E8         flow += amt;
F5D         fix_pot();
7C0     }
884     return {flow, cost};
AA4 }
2C0 void fix_pot() {
19F     for(int u = 0; u < n; ++u) {
35E         if(dist[u] < INF) {
AB7             pot[u] += dist[u];
BC9         }
AC5     }
011 }
18C };

```

## 7.9 dsu

```

D56 struct DSU{
1A8     int n;
2E3     vector<int> p,sz;
2EB     DSU(int n) : n(n){
1C4         p.resize(n);
529         sz.resize(n,1);
469         iota(begin(p), end(p), 0);
701     }
686     int size(int a){ return sz[root(a)]; }
E22     int root(int a){ return p[a] = (p[a] == a ? a : root(p[a])); }
765     bool unite(int a, int b){
6CE         int ra = root(a), rb = root(b);
5C1         if(ra == rb) return 0;
F13         if(sz[ra] < sz[rb]) swap(ra,rb);
484         p[rb] = ra;
9FA         sz[ra] += sz[rb];
6A5         return 1;
DC9     }
CA7 };

```

## 7.10 dijkstra

```

1BC const int mx = 1e5+5;
E55 using pii = pair<ll,int>;
93C vector<pii> g[mx];
57C const ll inf = 8e18;
FF9 ll dist[mx]; // setar tudo inf

38F void dijkstra(ll src){
E13     dist[src] = 0;
F37     priority_queue<pii,vector<pii>, greater<pii>> pq;
E8D     pq.push({0,src});
502     while(!pq.empty()){
2F9         auto [d, u] = pq.top();
716         pq.pop();
3E1         if(d > dist[u]) continue;
78E         for(auto [w, v] : g[u]){
C48             ll cur = dist[u] + w;
F65             if(cur < dist[v]){
D84                 dist[v] = cur;
E9A                 pq.push({cur,v});
AAA             }

```

```
E5F    }
E9F    }
67C }
```

## 7.11 twosat

```
usar ~ para negacao
regras logica
A->B = ~B->~A (contrapositiva)
A->B = ~A | B (lei da implicacao)
~(A|B) = ~A & ~B (de morgan)
A & (B|C) = (A&B) | (A&C) (distributiva)
```

```
8B2 #define PB push_back
```

```
D9D struct TwoSat{
1A8     int n;
99F     vector<vector<int>>> G, Gt;
556     vector<int> id, order, ans;
B57     vector<bool> vis;
E53     TwoSat(){}
4B1     TwoSat(int n) : n(n){
FC7         G.resize(2*n);
E60         Gt.resize(2*n);
8B0         id.assign(2*n,-1);
CD7         ans.resize(n);
A89     }
D41     // negativos na esquerda
58B     void add_edge(int u, int v){
8EB         u = (u < 0 ? -1-u : u + n);
E41         v = (v < 0 ? -1-v : v + n);
A23         G[u].PB(v);
03D         Gt[v].PB(u);
276     }
B58     void add_or(int a, int b){
FED         add_edge(~a,b);
65F         add_edge(~b,a);
878     }
D41     // Apenas algum ser 1
D9B     void add_xor(int a, int b){
23B         add_or(a,b);
6A8         add_or(~a,~b);
B18     }
D41     // set(a) = 1, set(~a) = 0
F75     void set(int a){ // (a/a)
679         add_or(a,a);
46C     }
D41     // Mesmo valor
AA6     void add_xnor(int a, int b) {
170         add_xor(~a,b);
D41     }
CB7     void dfs1(int u){
B9C         vis[u] = true;
E44         for(int v : G[u])
C2D             if(!vis[v])
3BA                 dfs1(v);

26F         order.PB(u);
92F     }
6A1     void dfs2(int u, int idx){
846         id[u] = idx;
51D         for(int v : Gt[u])
BBC             if(id[v] == -1)
039                 dfs2(v,idx);
594     }
DB8     void kosaraju() {
```

```
DF8         vis.assign(2*n,false);
3DF         for(int i = 0; i < 2*n; i++){
F69             if(!vis[i])
6D1                 dfs1(i);
3B9         reverse(begin(order),end(order));
503         int idx = 0;
882         for(int u : order){
98B             if(id[u] == -1)
96A                 dfs2(u, idx++);
D65         }
656     }
271     bool satisfiable(){
75D         kosaraju();
603         for(int i = 0; i < n; i++){
C8B             if(id[i] == id[i + n]) return false;
3C8             ans[i] = (id[i] < id[i + n]);
1B8         }
8A6         return true;
35D     }
3E3 };
```

## 7.12 lca

```
90A const int mxn = 2e5+5;
853 const int LOG = 22;
3B9 int n, q;
1C7 int tin[mxn], tout[mxn];
5FE vector<vector<int>>> up; // up[v][k] = 2^k-esimo ancestor de v
9E5 vector<int> g[mxn];
8E0 int lvl[mxn];
2DC int timer = 0;
FB6 void dfs(int u, int p){
73E     tin[u] = ++timer;
D0F     lvl[u] = lvl[p] + 1;
22B     up[u][0] = p;
C64     for(int i = 1; i <= LOG; i++){
88A         up[u][i] = up[ up[u][i-1] ][i-1];
378     }
4D5     for(int v : g[u]){
6F3         if(v != u && !tin[v])
95E             dfs(v,u);
D69     }
5EF     tout[u] = ++timer;
557 }
```

```
F31 bool is_ancestor(int u, int v){
B6F     return tin[u] <= tin[v] && tout[u] >= tout[v];
88C }
```

```
7BE int lca(int a, int b){
727     if(is_ancestor(a,b)) return a;
9ED     if(is_ancestor(b,a)) return b;
E70     for(int i = LOG; i >= 0; i--){
AF6         if(!is_ancestor(up[a][i], b)){
3F4             a = up[a][i];
EEF         }
B9D     }
E6F     return up[a][0];
8A8 }
```

## 7.13 floydWarshall

```
9F9 const int mxn = 505;
3BF const ll inf = 1e18;
```

```

3F3 ll g[mxn][mxn]; // setar tudo infinito menos (i,i) como 0
1A8 int n;
3F0 void addEdge(int u, int v, ll w){
E24     g[u][v] = min(g[u][v],w);
9C7     g[v][u] = min(g[v][u],w); // tirar se for 1 dir
CBD }

EDA void floyd(){
E22     for(int k = 0; k < n; k++) // << k
830         for(int i = 0; i < n; i++)
F90             for(int j = 0; j < n; j++)
6A8                 if(g[i][k] + g[k][j] < g[i][j]) // cuida overflow aqui (inf)
FE5                     g[i][j] = g[i][k] + g[k][j];
99E }

```

## 7.14 bridges

```

14E int n, m;
21F const int mxn = 1e5 + 5;
9E5 vector<int> g[mxn];
B1B int tin[mxn], low[mxn];
C83 vector<pii> bridges;
813 int timer = 1;

FB6 void dfs(int u, int p){
406     tin[u] = timer++;
E99     low[u] = tin[u];
612     int ch = 0;
D76     for(int v : g[u]) if(v != p){
85A         if(tin[v]) // lowlink direta
4EC             low[u] = min(tin[v], low[u]);
4E6         else{
95E             dfs(v,u);
E7F             low[u] = min(low[v], low[u]);
E80             if(tin[u] < low[v]) bridges.push_back({u,v});
6A0         }
CFF     }
2DD }

```

# 8 Extra

## 8.1 Hash Function

Call

```

g++ hash.cpp -o hash
./hash < code.cpp

```

to get the hash of the code.

The hash ignores comments and whitespaces.

The hash of a line whith } is the hash of all the code since the { that opens it. (is the hash of that context)

(Optional) To make letters upperCase: for(auto&c:s)if('a'<=c) c^=32;

```

DE3 string getHash(string s){
909     ofstream ip("temp.cpp"); ip << s; ip.close();
EE9     system("g++ -E -P -dD -fpreprocessed ./temp.cpp | tr -d '[:space:]' | md5sum > hsh.temp")
;
CEF     ifstream fo("hsh.temp"); fo >> s; fo.close();
A15     return s.substr(0, 3);
17A }

```

```

E8D int main(){

```

```

973 string l, t;
3DA vector<string> st(10);
C61 while(getline(cin, l)){
54F     t = l;
242     for(auto c : l)
F11         if(c == '{') st.push_back(""); else
2F0         if(c == '}') t = st.back() + l, st.pop_back();
C33     cout << getHash(t) + " " + l + "\n";
1ED     st.back() += t + "\n";
D1B }
B65 }

```