# palao - ICPC Library

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# 1 Data Structures

## 1.1 Fenwick

```
const int mx = 2e5+5;
11 bit[mx];
int n, q;
```

### 1.2 Segtree Lazy

```
// Lazy SegTree
const int mx = 2e5+5;
vector<ll> seq(4*mx);
vector<11> lazy(4*mx,0);
vector<11> nums(mx);
int n, q;
void build(int 1 = 0, int r = n-1, int idx = 0) {
        if(1 == r){
                seg[idx] = nums[1];
                lazy[idx] = 0;
                return;
        int m = (1+r)/2;
        int left = 2*idx+1;
        int right = 2*idx+2;
        build(1, m, left);
        build(m+1, r, right);
        seg[idx] = seg[left] + seg[right];
void prop(int 1 = 0, int r = n-1, int idx = 0) {
        seg[idx] += (ll) (r-l+1)*lazy[idx];
        if(1 != r) { // nao for folha
                int left = 2*idx+1;
                int right = 2*idx+2;
                lazy[left] += lazy[idx];
                lazy[right] += lazy[idx];
        lazy[idx] = 0;
void update(int L, int R, ll val, int l = 0, int r = n-1, int idx = 0) {
        if(R < 1 || L > r) return;
        prop(l,r,idx);
        if(L <= 1 && r <= R) {
                lazy[idx] = val;
                prop(l,r,idx);
        else{
                int m = (1+r)/2;
                int left = 2*idx+1;
                int right = 2*idx+2;
                update(L,R,val,l,m,left);
                update(L,R,val,m+1,r,right);
                seg[idx] = seg[left] + seg[right];
11 query (int L, int R, int 1 = 0, int r = n-1, int idx = 0) {
        prop(l,r,idx);
        if(R < 1 || L > r) return 0;
        if(L <= 1 && r <= R) {</pre>
                return seg[idx];
```

```
int m = (1+r)/2;
int left = 2*idx+1;
int right = 2*idx+2;
return query(L,R,1,m,left) + query(L,R,m+1,r,right);
}
```

### 1.3 Segtree

```
// SegTree
const int mx = 2e5 + 5;
11 \text{ seq}[4*mx];
11 nums[mx];
int n,q;
11 merge(ll a, ll b) {
        return a+b;
void build(int l = 0, int r = n-1, int idx = 0) {
        if(1 == r){
                seg[idx] = nums[l];
                return;
        int mid = 1 + (r-1)/2;
        int left = 2*idx + 1;
        int right = 2*idx + 2;
        build(1,mid,left);
        build(mid+1, r, right);
        seg[idx] = merge(seg[left], seg[right]);
11 query (int L, int R, int 1 = 0, int r = n-1, int idx = 0) {
        if(R < 1 || L > r) return 0; // elemento neutro
        if(L <= 1 && r <= R) return seg[idx];</pre>
        int mid = 1 + (r-1)/2;
        int left = 2*idx + 1;
        int right = 2*idx + 2;
        11 ql = query(L,R,l,mid,left);
        11 gr = query(L,R,mid+1,r,right);
        return merge(ql,qr);
void update(int pos, int num, int l = 0, int r = n-1, int idx = 0) {
        if(1 == r){
                 seq[idx] = num;
                return;
        int mid = 1 + (r-1)/2;
        int left = 2*idx + 1;
        int right = 2*idx + 2;
        if(pos <= mid) {</pre>
                update(pos, num, 1, mid, left);
        else update(pos,num,mid+1,r,right);
        seg[idx] = merge(seg[left],seg[right]);
```

## 1.4 SparseTable

```
vector<vector<1l>> table;
vector<1l> lg2;
void build(int n, vector<1l> v) {
   lg2.resize(n + 1);
   lg2[1] = 0;
   for (int i = 2; i <= n; i++) {
      lg2[i] = lg2[i >> 1] + 1;
```

```
} table.resize(lg2[n] + 1);
for (int i = 0; i < lg2[n] + 1; i++) {
    table[i].resize(n + 1);
}
for (int i = 0; i < n; i++) {
    table[0][i] = v[i];
}
for (int i = 0; i < lg2[n]; i++) {
    if (j + (1 << i) >= n) break;
    table[i + 1][j] = min(table[i][j], table[i][j + (1 << i)]);
}
}
}
ll get(int l, int r) { // (l,r) inclusivo
    int k = lg2[r - l + 1];
    return min(table[k][l], table[k][r - (1 << k) + 1]);
}
</pre>
```

## 1.5 Dynamic Median

```
struct DynamicMedian{
  priority_queue<1l> left;
  priority_queue<11, vector<11>, greater<11>> right;
  ll get(){
    assert(left.size());
    return left.top();
  void insert(ll x){
    if(left.emptv()){
      left.push(x);
      return;
    11 m = qet();
    if(x <= m) left.push(x);</pre>
    else right.push(x);
    if(left.size() > right.size() + 1){
      11 y = left.top();
      left.pop();
      right.push(y);
    if(right.size() > left.size()){
      ll y = right.top();
      right.pop();
      left.push(y);
  void removeMedian(){
    left.pop();
    if(right.size() > left.size()) {
      left.push(right.top());
      right.pop();
};
```

## 2 DP

### 2.1 CHT

```
struct Line {
    ll m, c;
    Line(ll m, ll c) : m(m), c(c) {}
    ll eval(ll x) {
        return m * x + c;
    }
}
```

```
struct CHT {
 vector<Line> lines;
 bool bad(Line a, Line b, Line c) {
        // trocar pra < se for max
    return 1.d * (c.c - a.c) * (a.m - b.m) > 1.d * (b.c - a.c) * (a.m - c.m);
 void insert(Line line) { // sortar antes de inserir
    int sz = (int)lines.size();
    for(; sz > 1; --sz) {
      if(bad(lines[sz - 2], lines[sz - 1], line)) {
        lines.pop_back();
        continue;
     break;
    lines.emplace_back(line);
  11 query(ll x) {
    int l = 0, r = (int) lines.size() - 1;
    while (1 < r) {
      int m = (1+r)/2;
      // trocar pra < se for max
      if(lines[m].eval(x) > lines[m+1].eval(x)) {
        \dot{1} = m + 1;
      } else {
       r = m;
    return lines[l].eval(x);
};
```

### 2.2 Knapsack

```
// Knapsack
const int MXW = 1e5+5;
const int MXN = 105;
int n, max_w;
vector<int> weight (MXN), value (MXN);
vector<vector<ll>>> dp(MXN, vector<ll>(MXW, -1));
11 solveDp(int i, int k){ // k -> peso atual
        if(i == n) return 0;
        if(dp[i][k] != -1) return dp[i][k];
        11 ignore = solveDp(i+1,k);
11 add = -1;
        if(weight[i] + k \le max_w){
                 add = value[i] + solveDp(i+1, weight[i] + k);
        return dp[i][k] = max(ignore, add);
// iterativo
11 knapsack(){
  vector<ll> dp(dpmx, 0);
  for (int i = 0; i < n; i++) {
    11 w = weight[i];
    11 v = value[i];
    for (int sz = max_w; sz >= w; sz--) {
      dp[sz] = max(dp[sz], dp[sz-w]+v);
  return *max_element(begin(dp),end(dp));
```

### 2.3 LIS

# 3 Geometry

### 3.1 Point

```
// hypot, atan2, gcd
const double PI = acos(-1);
template \langle class T \rangle int sgn(T x) \{ return (x > 0) - (x < 0); \}
template<typename T>
struct PT{
  T x, y;
  PT(T = 0, T = 0) : x(x), y(y) {}
  bool operator < (PT o) const { return tie(x,y) < tie(o.x,o.y); }</pre>
  bool operator == (PT o) const { return tie(x,y) == tie(o.x,o.y); }
  PT operator + (PT o) const { return PT(x+o.x,y+o.y);
  PT operator - (PT o) const { return PT(x-o.x,y-o.y); }
  PT operator * (T k) const { return PT (x*k,y*k); }
  PT operator / (T k) const { return PT(x/k, y/k);
  T cross(PT o) const { return x*o.y - y*o.x; }
  T cross(PT a, PT b) const { return (a-*this).cross(b-*this); }
  T dot(PT o) const { return x*o.x + v*o.v; }
  T dist2() const { return x*x + y*y; }
  double len() const { return hypot(x,y); }
  PT perp() const { return PT(-v,x);
  PT rotate(double a) const { return PT(x*cos(a)-y*sin(a), x*sin(a)+y*cos(a
      )); }
ostream & operator << (ostream & os, const PT < 11 > & p) {
  return os << "(" << p.x << "," << p.y << ")";
```

### 3.2 Convex Hull

```
// retorna poligono no sentido anti horario, trocar pra < se quiser horario
template<typename T>
vector<PT<T>>> convexHull(vector<PT<T>>>& pts, bool sorted = false) {
    if(!sorted) sort(begin(pts),end(pts));
    vector<PT<T>> h;
    h.reserve(pts.size() + 1);
    for(int it = 0; it < 2; it++) {
        int start = h.size();
        for(PTCT>& c : pts) {
        while((int)h.size() >= start + 2) {
            PT<T> a = h[h.size()-2], b = h.back();
            // '>=' pra nao descartar pontos colineares
            if((b-a).cross(c-a) > 0) break;
            h.pop_back();
```

```
h.push_back(c);
    reverse (begin (pts), end (pts));
    h.pop_back();
  if(h.size() == 2 && h[0] == h[1]) h.pop_back();
 return h;
// nao funciona se tem pontos colineares!!!!
// considera ponto na aresta como dentro
template<typename T>
bool isInside(vector<PT<T>>& hull, PT<T> p)
  int n = hull.size();
  PT < T > v0 = p - hull[0], v1 = hull[1] - hull[0], v2 = hull[n-1] - hull[0];
  if(v0.cross(v1) > 0 || v0.cross(v2) < 0){</pre>
    return false;
  int 1 = 1, r = n - 1;
  while (1 != r) {
    int mid = (1 + r + 1) / 2;
    PT < T > v0 = p - hull[0], v1 = hull[mid] - hull[0];
    if(v0.cross(v1) < 0)
      1 = mid;
    else
      r = mid - 1;
  v0 = hull[(1+1)%n] - hull[1], v1 = p - hull[1];
  return v0.cross(v1) >= 0;
// poligonos
11 polygon_area_db(const vector<Point>& poly) {
  11 area = 0;
  for (int i = 0, n = (int) poly.size(); <math>i < n; ++i) {
    int j = i + 1 == n ? 0 : i + 1;
    area += cross(poly[i], poly[j]);
  return abs (area);
// Teorema de Pick para lattice points
// Area = insidePts + boundPts/2 - 1
// 2A - b + 2 = 2i
// usar gcd dos lados pra contar bound pts
11 cntInsidePts(11 area db, 11 bound) {
  return (area_db + 2LL - bound) /2;
```

## 3.3 Min Enclosing Circle

```
typedef PT<double> P;
double ccRadius (P& A, P& B, P& C) {
  return (B-A).len() * (C-B).len() * (A-C).len()/
      abs ((B-A).cross(C-A))/2.0;
P ccCenter(P& A, P& B, P& C) {
 P b = C-A, c = B-A;
 return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
// mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
pair<P, double> mec(vector<P>& pts) {
        shuffle (begin (pts), end (pts), rng);
        P \circ = pts[0];
        const double EPSS = 1+1e-8;
        double r = 0;
        for(int i = 0; i < pts.size(); i++) if((o-pts[i]).len() > r * EPSS)
                o = pts[i], r = 0;
                for(int j = 0; j < i; j++) if((o-pts[j]).len() > r * EPSS){
```

### 3.4 Clostest Pair

```
pii ClosestPair(vector<PT<11>>& pts) 
  11 \text{ dist} = (pts[0]-pts[1]).dist2();
  pii ans(0, 1);
  int n = pts.size();
  vector<int> p(n);
  iota(begin(p), end(p), 0);
  sort(p.begin(), p.end(), [&](int a, int b) { return pts[a].x < pts[b].x;</pre>
      });
  set<pii> points;
  auto sqr = [](long long x) -> long long { return x * x; };
  for (int 1 = 0, r = 0; r < n; r++) {
    while (sqr(pts[p[r]].x - pts[p[l]].x) > dist) {
      points.erase(pii(pts[p[1]].y, p[1]));
      1++;
    11 delta = sqrt(dist) + 1;
    auto itl = points.lower_bound(pii(pts[p[r]].y - delta, -1));
    auto itr = points.upper_bound(pii(pts[p[r]].y + delta, n + 1));
    for(auto it = itl; it != itr; it++) {
      11 curDist = (pts[p[r]] - pts[it->second]).dist2();
      if(curDist < dist) {</pre>
        dist = curDist;
        ans = pii(p[r], it->second);
    points.insert(pii(pts[p[r]].y, p[r]));
  if(ans.first > ans.second)
    swap(ans.first, ans.second);
  return ans;
```

# 4 ETC

## 4.1 Bitset

```
// Bitset operations
_builtin_popcount(int x);
_builtin_popcountll(ll x);
const int SZ = 1e6;
bitset<SZ> b;
b.reset(); // 00 ... 00
b.set(); // 11 ... 11
b.flip();
b._Find_first(); // retorna SZ se nao tiver
b._Find_next(i);
b.to_ulong();
b.to_string();
b.count();
// rng
```

```
\label{limits} $$ mt19937 \ rng(chrono::steady\_clock::now().time\_since\_epoch().count()); $$ shuffle(begin(x),end(x),rng); $$ uniform\_int\_distribution<int>(0,x)(rng); $$
```

### 4.2 Ternary Search

```
double f(double t) {
        // alguma funcao unimodal -> maximo ou minimo
double tern_search(double 1, double r) {
  for(int it = 0; it < 300; it++) {</pre>
    double m1 = 1 + (r-1)/3;
    double m2 = r - (r-1)/3;
    double f1 = f(m1), f2 = f(m2);
    if(f1 < f2) 1 = m1; //change to > to find maximum
    else r = m2;
  return 1;
// golden section search
double gss (double a, double b) {
  const double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
double f1 = f(x1), f2 = f(x2);
  for(int it = 0; it < 250 && b-a > eps; it++)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
  return a;
// retorna mais a esquerda no empate
int int_tern_search(int 1, int r){
        int 10 = 1 - 1, hi = r;
        while (hi - lo > 1) {
                 int m = (lo+hi)/2;
                 if(f(m) < f(m+1)){ //
                         lo = m;
                 }else{
                         hi = m;
        return lo + 1;
}
```

# 4.3 Mo Algorithm

```
// Mo apelao
// Ordering based on the Hilbert curve
// inline int64_t hilbertOrder(int x, int y, int pow, int rotate) {
    if (pow == 0) return 0;
    int hpow = 1 << (pow - 1);
    int seg = (x < hpow) ? ( (y < hpow) ? 0 : 3) : ( (y < hpow) ? 1 : 2)
    ;

// seg = (seg + rotate) & 3;
    const int rotateDelta[4] = {3, 0, 0, 1};
    int nx = x & (x ^ hpow), ny = y & (y ^ hpow);
    int nrot = (rotate + rotateDelta[seg]) & 3;
    int64_t subSquareSize = int64_t(1) << (2*pow - 2);
    int64_t ans = seg * subSquareSize;
    int64_t add = hilbertOrder(nx, ny, pow - 1, nrot);</pre>
```

```
ans += (seg == 1 | | seg == 2) ? add : (subSquareSize - add - 1);
       return ans:
// struct Query{
       int l, r, idx;
       int64_t ord;
       Query(int 1, int r, int idx) : l(1), r(r), idx(idx) {
            ord = hilbertOrder(1, r, 21, 0);
       bool operator < (Query &other) {
            return ord < other.ord;
// };
const int MXN = 2e5;
const int B = sqrt(MXN) + 1;
struct Query {
    int 1, r, idx;
    bool operator<(Query other) const{</pre>
      return make_pair(1 / B, ((1/B) & 1) ? -r : r) < make_pair(other.1 / B
    , ((other.1/B) & 1) ? -other.r : other.r);</pre>
};
11 a[MXN];
11 resp = 0;
void add(int x);
void remove(int x);
int main(){
    ios_base::sync_with_stdio(false);
    cin.tie(NULL);
    int n, q; cin >> n >> q;
    for (int i = 0; i < n; i++)
        cin >> a[i];
    vector<Query> queries;
    for (int i = 0; i < q; i++) {
        int 1, r; cin >> 1 >> r;
         queries.push_back(Query(1-1,r-1,i));
    sort (begin (queries), end (queries));
    vector<11> answers(q);
    int L = 0, R = -1;
    for(Query qr : queries) {
      while (L > qr.1) add (--L);
      while (R < qr.r) add(++R);</pre>
      while (L < qr.1) remove(L++);</pre>
      while (R > qr.r) remove(R--);
      answers[qr.idx] = resp;
    for (int i = 0; i < q; i++)
         cout << answers[i] << "\n";</pre>
```

# 5 Graph

#### 5.1 Articulation Pts

```
int n, m;
const int mxn = 1e5 + 5;
vector<int> g[mxn];
int tin[mxn], low[mxn];
vector<int> art;
int timer = 1;

void dfs(int u, int p) {
  tin[u] = timer++;
  low[u] = tin[u];
  int ch = 0;
  int fw = 0;
```

```
for(int v : g[u]) if(v != p) {
   if(tin[v]) // lowlink direta
      low[u] = min(tin[v],low[u]);
   else{
      dfs(v,u);
      fw++;
      low[u] = min(low[v],low[u]);
      ch = max(low[v],ch);
   }
}
if(u == p && fw > 1) art.push_back(u);
else if(u != p && ch && tin[u] <= ch) art.push_back(u);</pre>
```

## 5.2 Bridges

```
int n, m;
const int mxn = 1e5 + 5;
vector<int> g[mxn];
int tin[mxn], low[mxn];
vector<pii> bridges;
int timer = 1;
void dfs(int u, int p) {
  tin[u] = timer++;
  low[u] = tin[u];
  int ch = 0;
  for(int v : g[u]) if(v != p){
    if(tin[v]) // lowlink direta
     low[u] = min(tin[v], low[u]);
    else{
      dfs(v,u);
      low[u] = min(low[v], low[u]);
      if(tin[u] < low[v]) bridges.push_back({u,v});</pre>
```

# 5.3 Dijkstra

```
const int mx = 1e5+5;
using pii = pair<11,int>;
vector<pii> g[mx];
const 11 inf = 8e18;
11 dist[mx]; // setar tudo inf
void dijkstra(ll src){
  dist[src] = 0;
  priority_queue<pii, vector<pii>, greater<pii>>> pq;
  pq.push({0,src});
  while(!pq.empty()){
    auto [d, u] = pq.top();
    pq.pop();
    if(d > dist[u]) continue;
    for(auto [w, v] : g[u]) {
      ll cur = dist[u] + w;
      if(cur < dist[v]){</pre>
        dist[v] = cur;
        pq.push({cur,v});
```

#### 5.4 **DSU**

```
struct DSU{
        vector<int> p;
        vector<int> sz;
        int n;
        DSU(int nodes) {
                n = nodes;
                p.resize(nodes);
                sz.resize(nodes,1);
                iota(begin(p), end(p), 0);
        int size(int a) { return sz[root(a)]; }
        int root(int a) { return p[a] = (p[a] == a ? a : root(p[a])); }
        bool unite(int a, int b) {
                int ra = root(a), rb = root(b);
                if(ra != rb) {
                         if(sz[ra] < sz[rb]) swap(ra,rb);</pre>
                         p[rb] = ra;
                         sz[ra] += sz[rb];
                         return 1;
                return 0;
};
```

### 5.5 Floyd Warshall

```
const int mxn = 505;
const l1 inf = 1e18;
l1 g[mxn] [mxn]; // setar tudo infinito menos (i,i) como 0
int n;
void addEdge(int u, int v, l1 w) {
    g[u][v] = min(g[u][v],w);
    g[v][u] = min(g[v][u],w); // tirar se for 1 dir
}

void floyd() {
    for(int k = 0; k < n; k++) // << k
        for(int i = 0; i < n; i++)
            for(int j = 0; j < n; j++)
            if(g[i][k] + g[k][j] < g[i][j]) // cuida overflow aqui (inf)
            g[i][j] = g[i][k] + g[k][j];
}</pre>
```

## 5.6 Kosaraju

```
// Kosaraju
const int ms = 1e5 + 5;
vector<int> G[ms], Gt[ms];
vector<int> id, order, root;
vector<bool> vis;
int n;
void dfs1(int u) { // ordem de saida
  vis[u] = true;
  for(int v : G[u])
    if(!vis[v])
      dfs1(v);
 order.push_back(u);
void dfs2(int u, int idx){
  id[u] = idx;
  for(int v : Gt[u])
    if(id[v] == -1)
      dfs2(v,idx);
// retorna quantidade de componentes
int kosaraju(){
```

```
vis.assign(n, false);
id.assign(n,-1);
for(int i = 0; i < n; i++)
    if(!vis[i])
    dfs1(i);
reverse(begin(order),end(order));
int idx = 0;
for(int u : order)
    if(id[u] == -1)
    dfs2(u, idx++), root.push_back(u);
return idx;</pre>
```

#### 5.7 Kruskal

## 5.8 Lowest Common Ancestor

```
const int mxn = 2e5+5;
const int LOG = 22;
int n, q;
int tin[mxn], tout[mxn];
vector<vector<int>> up; // up[v][k] = 2^k-esimo ancestor de v
vector<int> q[mxn];
int lvl[mxn];
int timer = 0;
void dfs(int u, int p) {
    tin[u] = ++timer;
    lvl[u] = lvl[p] + 1;
    up[u][0] = p;
    for (int i = 1; i <= LOG; i++) {</pre>
        up[u][i] = up[up[u][i-1]][i-1];
    for(int v : g[u]) {
        if(v != u && !tin[v])
            dfs(v,u);
    tout[u] = ++timer;
bool is_ancestor(int u, int v) {
    return tin[u] <= tin[v] && tout[u] >= tout[v];
int lca(int a, int b){
    if(is ancestor(a,b)) return a;
    if(is ancestor(b,a)) return b;
    for (int i = LOG; i >= 0; i--) {
        if(!is_ancestor(up[a][i], b)){
            a = up[a][i];
```

```
}
    return up[a][0];
}
```

### 5.9 Max Flow

```
template <class T = int>
class MCMF {
public:
  struct Edge {
    Edge(int a, T b, T c) : to(a), cap(b), cost(c) {}
    int to;
    T cap, cost;
  MCMF(int size) {
    n = size;
    edges.resize(n);
    pot.assign(n, 0);
    dist.resize(n);
    visit.assign(n, false);
  pair<T, T> mcmf(int src, int sink) {
    pair<T, T > ans(0, 0);
    if(!SPFA(src, sink)) return ans;
    fixPot();
    // can use dijkstra to speed up depending on the graph
    while(SPFA(src, sink)) {
      auto flow = augment(src, sink);
      ans.first += flow.first;
      ans.second += flow.first * flow.second;
      fixPot():
    return ans;
  void addEdge(int from, int to, T cap, T cost) {
    edges[from].push_back(list.size());
    list.push_back(Edge(to, cap, cost));
    edges[to].push_back(list.size());
    list.push_back(Edge(from, 0, -cost));
private:
  int n;
  vector<vector<int>> edges;
  vector<Edge> list;
  vector<int> from;
  vector<T> dist, pot;
  vector<bool> visit;
  /*bool dij(int src, int sink) {
   T INF = std::numeric_limits<T>::max();
    dist.assign(n, INF);
    from.assign(n, -1);
    visit.assign(n, false);
    dist[src] = 0;
    for(int i = 0; i < n; i++) {
      int best = -1;
      for (int j = 0; j < n; j++) {
        if(visit[j]) continue;
        if(best == -1 || dist[best] > dist[j]) best = j;
      if(dist[best] >= INF) break;
      visit[best] = true;
      for (auto e : edges[best]) {
        auto ed = list[e];
        if (ed.cap == 0) continue;
        T toDist = dist[best] + ed.cost + pot[best] - pot[ed.to];
        assert(toDist >= dist[best]);
```

```
if(toDist < dist[ed.to]) {</pre>
          dist[ed.to] = toDist;
          from[ed.to] = e;
    return dist[sink] < INF;
  pair<T, T> augment(int src, int sink) {
    pair<T, T> flow = {list[from[sink]].cap, 0};
    for(int v = sink; v != src; v = list[from[v]^1].to) {
      flow.first = min(flow.first, list[from[v]].cap);
      flow.second += list[from[v]].cost;
    for(int v = sink; v != src; v = list[from[v]^1].to) {
      list[from[v]].cap -= flow.first;
      list[from[v]^1].cap += flow.first;
    return flow;
  queue<int> q;
  bool SPFA(int src, int sink) {
    T INF = numeric_limits<T>::max();
    dist.assign(n, INF);
    from.assign(n, -1);
    q.push(src);
    dist[src] = 0;
    while(!q.empty()) {
      int on = q.front();
      q.pop();
      visit[on] = false;
      for(auto e : edges[on]) {
        auto ed = list[e];
        if(ed.cap == 0) continue;
        T toDist = dist[on] + ed.cost + pot[on] - pot[ed.to];
        if(toDist < dist[ed.to]) {</pre>
          dist[ed.to] = toDist;
          from[ed.to] = e;
          if(!visit[ed.to]) {
            visit[ed.to] = true;
            q.push(ed.to);
    return dist[sink] < INF;</pre>
  void fixPot() {
    T INF = numeric_limits<T>::max();
    for (int i = 0; i < n; i++) {
      if(dist[i] < INF) pot[i] += dist[i];</pre>
};
```

## 5.10 Policy Based

#### 5.11 2-sat

```
#define PB push_back
// usar ~ para negacao
regras logica
A \rightarrow B = ^B \rightarrow ^A (contrapositiva)
A \rightarrow B = A \mid B \text{ (lei da implicação)}
^{\sim}(A|B) = ^{\sim}A \& ^{\sim}B  (de morgan)
A \& (B|C) = (A\&B) \mid (A\&C) (distributiva)
struct TwoSat{
  int n;
  vector<vector<int>> G, Gt;
  vector<int> id, order, ans;
  vector<bool> vis;
  TwoSat(){}
  TwoSat(int n) : n(n) {
    G.resize(2*n);
    Gt.resize(2*n);
    id.assign(2*n,-1);
    ans.resize(n);
  // negativos na esquerda
  void add_edge(int u, int v) {
    u = (u < 0 ? -1-u : u + n);
    v = (v < 0 ? -1 - v : v + n);
    G[u].PB(v);
    Gt [v].PB(u);
  void add_or(int a, int b) {
    add_edge(~a,b);
    add_edge(~b,a);
  // Apenas algum ser 1
  void add_xor(int a, int b) {
    add_or(a,b);
    add_or(~a,~b);
  // set(a) = 1, set(~a) = 0
  void set (int a) { // (a/a)
    add_or(a,a);
  // Mesmo valor
  void add xnor(int a, int b) {
    add_xor(~a,b);
  void dfs1(int u) {
    vis[u] = true;
    for(int v : G[u])
      if(!vis[v])
        dfs1(v);
    order.PB(u);
  void dfs2(int u, int idx){
    id[u] = idx;
    for(int v : Gt[u])
      if(id[v] == -1)
         dfs2(v,idx);
  void kosaraju() {
    vis.assign(2*n, false);
    for (int i = 0; i < 2*n; i++)
      if(!vis[i])
        dfs1(i);
    reverse (begin (order), end (order));
    int idx = 0;
    for(int u : order) {
```

# 6 Math

#### 6.1 Extended Euclidean

```
int gcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}

// inverso modular de a
int inv, y;
int g = gcd(a, mod, inv, y);
inv = (inv % m + m) % m;
```

#### 6.2 Factorization

### 6.3 Fastexp

```
while(n) {
                if(n \& 1) ans = (ans * a) % mod;
                a = (a * a) % mod;
                n >>= 1;
        return ans;
// matriz quadrada
class Matrix{
        public:
        vector<vector<ll>> mat;
        int m:
        Matrix(int m): m(m) {
                mat.resize(m);
                for (int i = 0; i < m; i++) mat[i].resize(m,0);</pre>
        Matrix operator * (const Matrix& rhs) {
                Matrix ans = Matrix(m);
                for (int i = 0; i < m; i++)
                         for (int j = 0; j < m; j++)
                                 for (int k = 0; k < m; k++)
                                          ans.mat[i][j] = (ans.mat[i][j] + (
                                              mat[i][k] * rhs.mat[k][j]) %
                                              mod) % mod;
                return ans;
};
Matrix fexp(Matrix a, ll n) {
        int m = a.m;
        Matrix ans = Matrix(m);
        for(int i = 0; i < m; i++) ans.mat[i][i] = 1;</pre>
        while(n){
                if(n \& 1) ans = ans * a;
                a = a * a;
                n >>= 1;
        return ans;
```

## 6.4 Phi

```
const int LIM = 1e6+5;
int phi[LIM];
void sieve() {
         iota(phi, phi + LIM, 0);
         for(int i = 2; i < LIM; i++) {</pre>
                 if(phi == i){
                          for (int j = i; j < LIM; j += i) {
                                   phi[j] -= phi[j] / i;
template<typename T>
T phi(T n) {
  T ans = n;
  for (T p = 2; p * p <= n; p++) {
    if(n % p == 0) {
  ans -= ans / p;
      while (n % p == 0) {
        n /= p;
  if(n > 1) {
    ans -= ans / n;
  return ans;
```

#### 6.5 Pollard Rho

```
// from: https://github.com/kth-competitive-programming/kactl
typedef unsigned long long ull;
return ret + M * (ret < 0) - M * (ret >= (11)M);
ull modpow(ull b, ull e, ull mod) {
       ull ans = 1;
        for (; e; b = modmul(b, b, mod), e /= 2)
                if (e & 1) ans = modmul(ans, b, mod);
        return ans;
bool isPrime(ull n) {
        if (n < 2 \mid | n % 6 % 4 != 1) return (n \mid 1) == 3;
        ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
           s = \underline{\quad} builtin_ctzll(n-1), d = n >> s;
        for (ull a : A) { // ^ count trailing zeroes
               ull p = modpow(a%n, d, n), i = s;
                while (p != 1 && p != n - 1 && a % n && i--)
                       p = modmul(p, p, n);
                if (p != n-1 && i != s) return 0;
        return 1:
ull pollard(ull n) {
        ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
        auto f = [\&] (ull x) \{ return modmul(x, x, n) + i; \};
        while (t++ % 40 | | _{gcd}(prd, n) == 1) {
               if (x == y) x = ++i, y = f(x);
               if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
               x = f(x), y = f(f(y));
        return __gcd(prd, n);
vector<ull> factor(ull n) {
        if (n == 1) return {};
        if (isPrime(n)) return {n};
       ull x = pollard(n);
        auto l = factor(x), r = factor(n / x);
        l.insert(l.end(), begin(r),end(r));
```

## 6.6 Polynomial

```
template<typename T>
struct Poly {
 int n;
  vector<T> v;
  Poly(int sz) : n(sz+1) { v.resize(sz+1,0);}
  friend Poly operator*(const Poly& lhs, const Poly& rhs) {
    int grauL = (int)lhs.n - 1;
    int grauR = (int) rhs.n - 1;
    Poly ans (grauR+grauL);
    for (int i = 0; i <= grauL; ++i) {</pre>
      for(int j = 0; j <= grauR; ++j) {</pre>
        ans.v[i + j] += lhs.v[i] * rhs.v[j];
    return ans;
 void set_identity() { // 1
    v[0] = T(1);
    for (int i = 1; i < n; ++i) {
```

```
v[i] = T(0);
}

};
template<typename T>
Poly<T> poly_exp(Poly<T> a, long long e) {
    Poly<T> r(0);
    r.set_identity();
    for(; e > 0; e >>= 1) {
        if(e & 1) {
            r = r * a;
        }
        a = a * a;
    }
    return r;
}
```

# 7 String

#### 7.1 KMP

```
vector<int> getBorder(string str) {
  int n = str.size();
  vector<int> border(n, -1);
  for (int i = 1, j = -1; i < n; i++) {
    while (j \ge 0 \&\& str[i] != str[j + 1]) {
      i = border[i];
    if(str[i] == str[j + 1]) {
    border[i] = j;
  return border;
int matchPattern(const string &txt, const string &pat, const vector<int> &
    border) {
  int freq = 0;
  for(int i = 0, j = -1; i < txt.size(); i++) {
    while(j \ge 0 \&\& txt[i] != pat[j + 1]) {
      j = border[j];
    if(pat[j + 1] == txt[i]) {
     j++;
    if(j + 1 == (int) pat.size()) {
      //found occurence
      freq++;
      j = border[j];
  return freq;
```

# 7.2 RabinKarp

```
// Rabin Karp
const 11 base = 997;
const 11 mod[] ={1000000007, 1000000009};
const int MXSZ = 1e6+2;
/*
Some Big Prime Numbers:
37'139'213
const 11 MOD1 = 131'807'699; -> Big Prime Number for hash 1
const 11 MOD1 = 127'065'427; -> Big Prime Number for hash 2
const 11 base = 127; -> Random number larger than the Alphabet
```

```
11 pot[2][MXSZ];
void buildPots(){ // lembrar de chamar essa funcao
  pot[0][0] = 1;
  pot[1][0] = 1;
  for (int j = 0; j < 2; j++)
    for(int i = 1; i < MXSZ; i++)</pre>
      pot[j][i] = (pot[j][i-1]*base) % mod[j];
class RabinKarp{
public:
  string s;
  int sz;
  vector<11> has[2];
  RabinKarp() {}
  RabinKarp(const string& str): s(str){
    sz = str.size();
    has[0].resize(sz+1);
    has[1].resize(sz+1);
    build();
  void build() {
    has[0][0] = s[0], has[1][0] = s[0];
    for (int j = 0; j < 2; j++)
      for(int i = 1; i < sz; i++)</pre>
        has[j][i] = ((has[j][i-1]*base)+s[i])%mod[j];
  11 getKey(int 1, int r){ // inclusivo
    11 x = has[0][r], y = has[1][r];
    if(1 > 0){
      x = (((x - pot[0][r-l+1]*has[0][l-1])*mod[0] + mod[0])*mod[0]);
      y = (((y - pot[1][r-1+1]*has[1][1-1])*mod[1] + mod[1])*mod[1]);
    return (x<<32LL) | y;
};
```

### 7.3 Trie

```
int trie[ms][sigma], terminal[ms], z = 1;
void insert(string &p) {
  int cur = 0;
  for(int i = 0; i < p.size(); i++) {</pre>
    int id = p[i]-'a';
    if(!trie[cur][id])
      trie[cur][id] = z++;
    cur = trie[cur][id];
  terminal[cur]++;
int count(string &p) {
  int cur = 0:
  for(int i = 0; i < p.size(); i++) {</pre>
    int id = p[i]-'a';
    if(!trie[cur][id]) {
      return false;
    cur = trie[cur][id];
  return terminal[cur];
```

#### 7.4 Z Function

```
vector<int> Zfunction(string &s) {
    int n = s.size();
    vector(int> z (n, 0);
    for(int i=1, l=0, r=0; i<n; i++) {
        if(i <= r) z[i] = min(z[i-1], r-i+1);
        while(z[i] + i < n && s[z[i]] == s[i+z[i]]) z[i]++;
        if(r < i+z[i]-1) l = i, r = i+z[i]-1;
    }
    return z;
}</pre>
```

#### 7.5 Aho-Corasick

```
struct AhoType {
  static const int ALPHA = 26;
  static int f (char c) { return c = 'A'; } // ver se ta maiusculo ou
      minusculo aqui
template<typename AhoType>
struct AhoCorasick {
 struct Node {
   int nxt[AhoType::ALPHA] {};
   int p = 0, ch = 0, len = 0;
   int link = 0:
    int occ link = 0;
   Node (int p = 0, int ch = 0, int len = 0) : p(p), ch(ch), len(len) {}
  vector<Node> tr;
  AhoCorasick() : tr(1) {}
  template<typename Iterator>
  void add_word(Iterator first, Iterator last) {
    int cur = 0, len = 1;
    for(; first != last; ++first) {
      auto ch = AhoType::f(*first);
      if(tr[cur].nxt[ch] == 0) {
        tr[cur].nxt[ch] = int(tr.size());
        tr.emplace_back(cur, ch, len);
     cur = tr[cur].nxt[ch];
      ++len;
   tr[cur].occ_link = cur;
  void build() {
   vector<int> bfs(int(tr.size()));
    int s = 0, t = 1;
    while(s < t) {</pre>
      int v = bfs[s++], u = tr[v].link;
      if(tr[v].occ_link == 0) {
        tr[v].occ_link = tr[u].occ_link;
      for(int ch = 0; ch < AhoType::ALPHA; ++ch) {</pre>
        auto& nxt = tr[v].nxt[ch];
        if(nxt == 0) {
         nxt = tr[u].nxt[ch];
        } else {
          tr[nxt].link = v > 0 ? tr[u].nxt[ch] : 0;
          bfs[t++] = nxt;
  template<typename Iterator>
 vector<pair<int,int>> get_all_matches(Iterator first, Iterator last)
      const {
        vector<pair<int,int>> occs;
    for(int cur = 0, i = 0; first != last; ++i, ++first) {
      auto ch = AhoType::f(*first);
      cur = tr[cur].nxt[ch];
```

```
}
template<typename T>
int get_next(int cur, T ch) const { return tr[cur].nxt[AhoType::f(ch)]; }
};
```