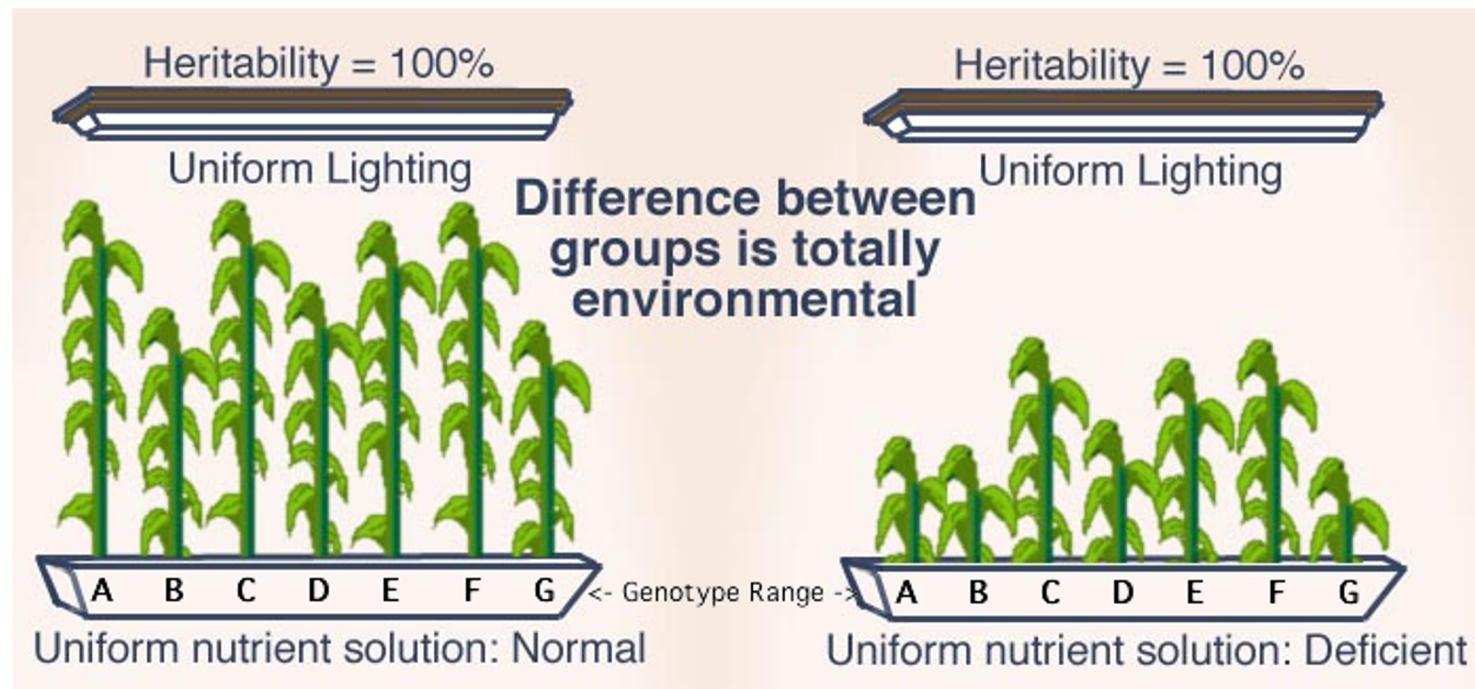


# **The Inheritance of Complex Traits**

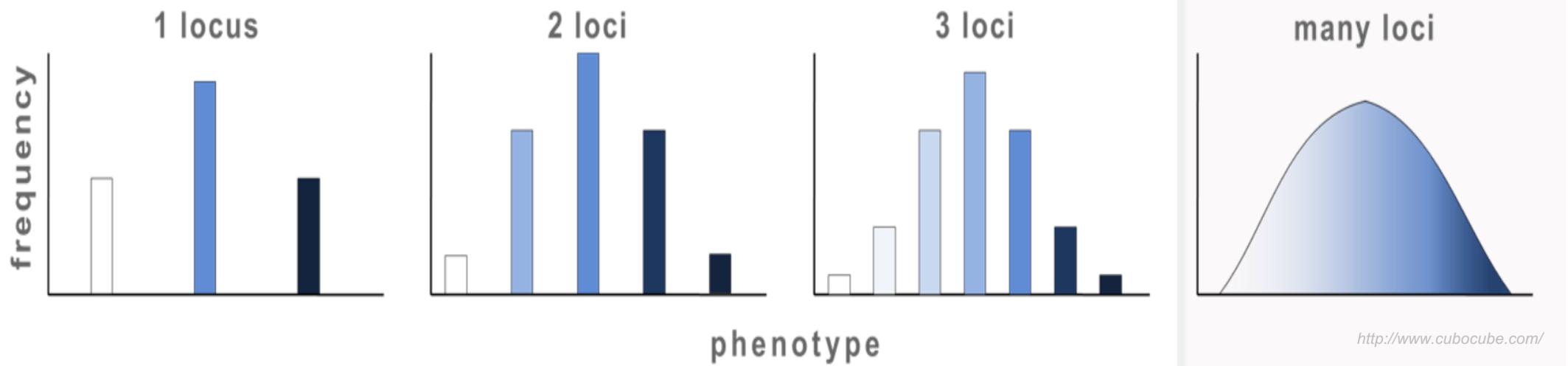
# The inheritance of complex traits

- Quantitative variation: Mean, variance, standard deviation
- Genetic model
- Genetic and environmental variances
- Heritability: Broad and narrow sense



# Quantitative variation

Traits that show a continuous range of variation and don't behave in simple Mendelian fashion are known as quantitative or complex traits



Quantitative genetics!

# Basic statistical concepts

## Mean

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

$$\bar{X} = \frac{X_1 + X_2 + X_3 \dots X_n}{n}$$

Where

$\bar{X}$  = mean

$X_1$  = first value

$X_2$  = second value

$X_3$  = third value

$X_n$  = last value

n = number of samples

# Basic statistical concepts

## Mean

$$X = \sum_{i=1}^k f_i X_i$$

Where

$\bar{X}$  = mean

k = classes

f = frequency

Height (cm)	Count	Frequency x Height
156	1	1.56
157	2	3.14
158	1	1.58
...	...	...
184	2	3.68
sum	100	170

Griffiths et al 2012

$$\bar{X} = (X_1 * f_1) + (X_2 * f_2) + (X_3 * f_3)$$

$$\bar{X} = (0.01 \times 156) + (0.02 \times 157) + \dots + (0.02 \times 184) = 170$$

# Basic statistical concepts

## Mean

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

The expected value is the average of all the values we would observe if we measured  $X$  many times:

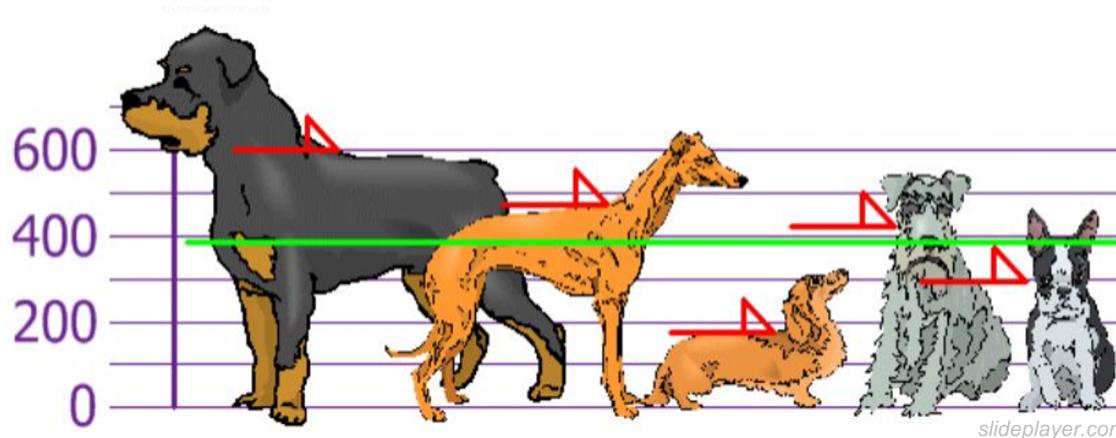
$$E(X) = \bar{X}$$

# Basic statistical concepts

## Variance:

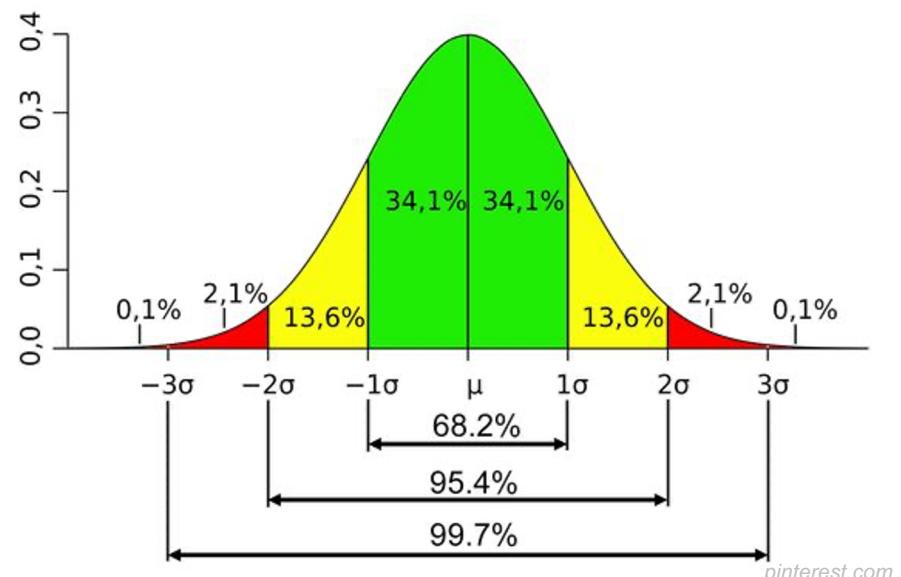
Measure of dispersion around the mean.

$$s^2 = \frac{1}{n-1} \sum_i (X_i - \bar{X})^2$$



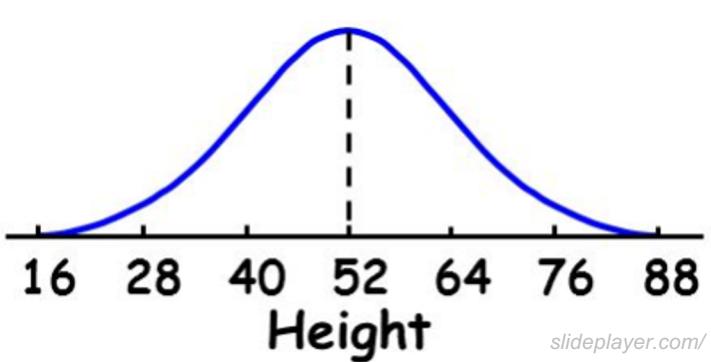
## Standard deviation:

$$\begin{aligned}s &= \sqrt{\frac{1}{n} \sum_i (X_i - \bar{X})^2} \\&= \sqrt{s^2}\end{aligned}$$

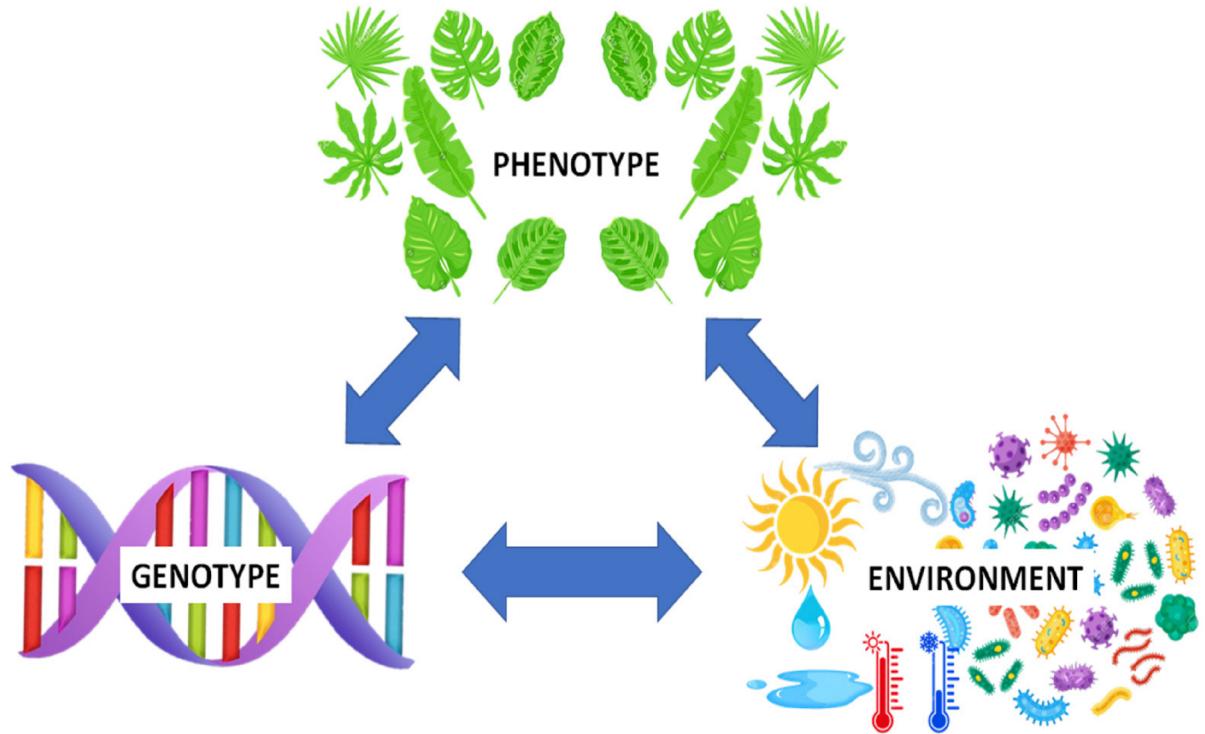


# Simple genetic model

- $X = \bar{X} + g + e$



[slideplayer.com/](http://slideplayer.com/)



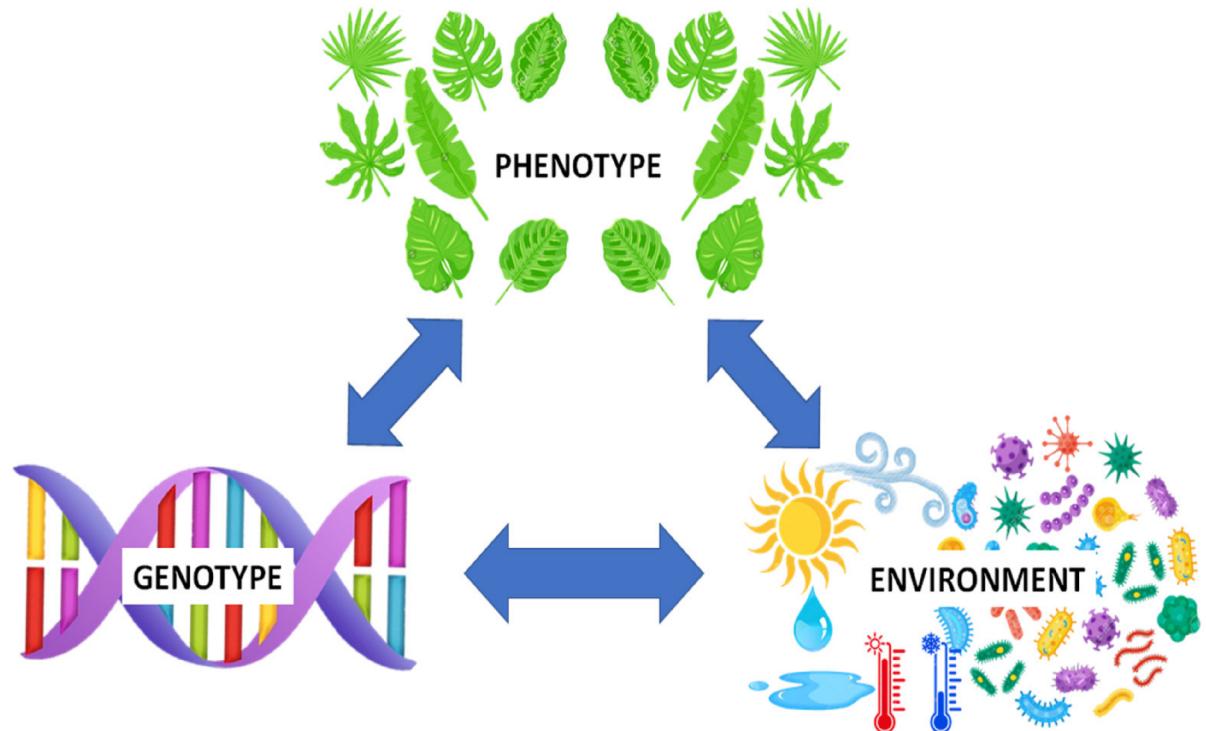
Djande et al 2020

# Simple genetic model

$$X = \bar{X} + g + e$$

- $x = g + e$

$x$  is the individual's phenotypic deviation.



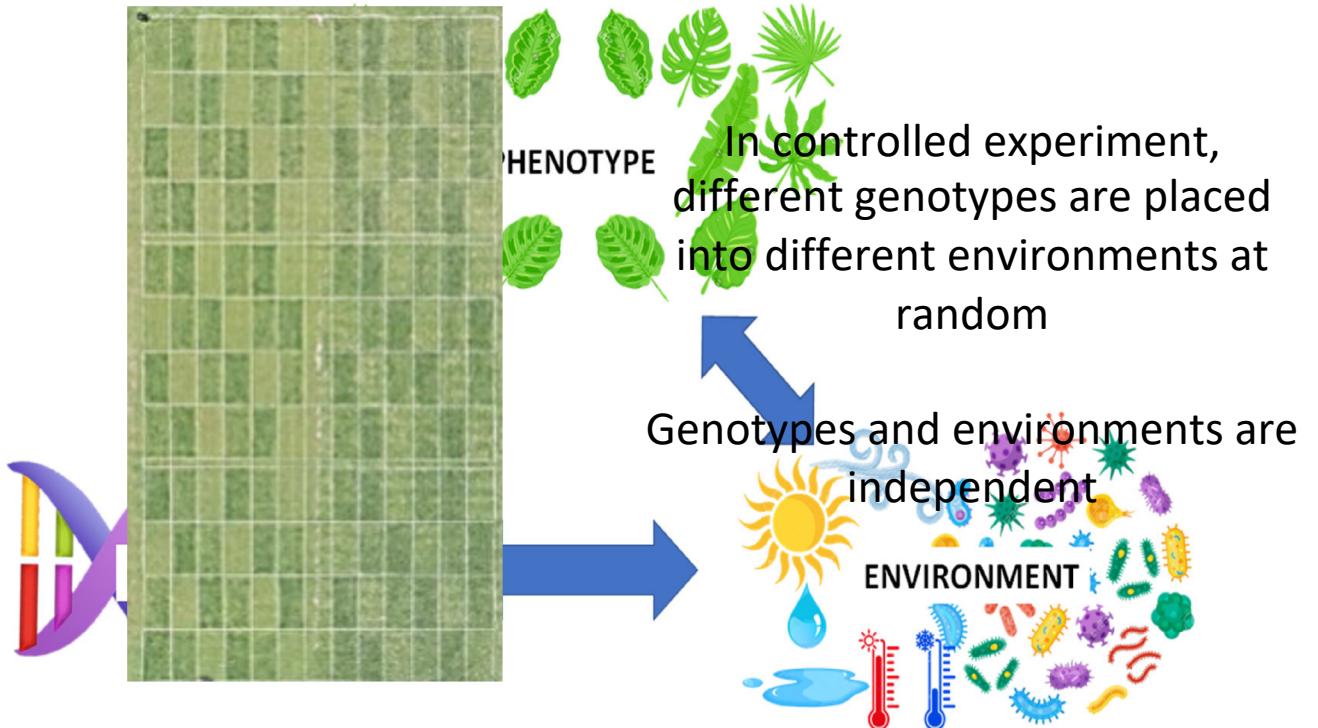
Djande et al 2020

# Genetic and environmental variances

$$x = g + e$$

$$V_x = Vg + Ve + 2\text{cov}_{ge}$$

$$V_x = Vg + Ve$$



Djande et al 2020

# Heritability

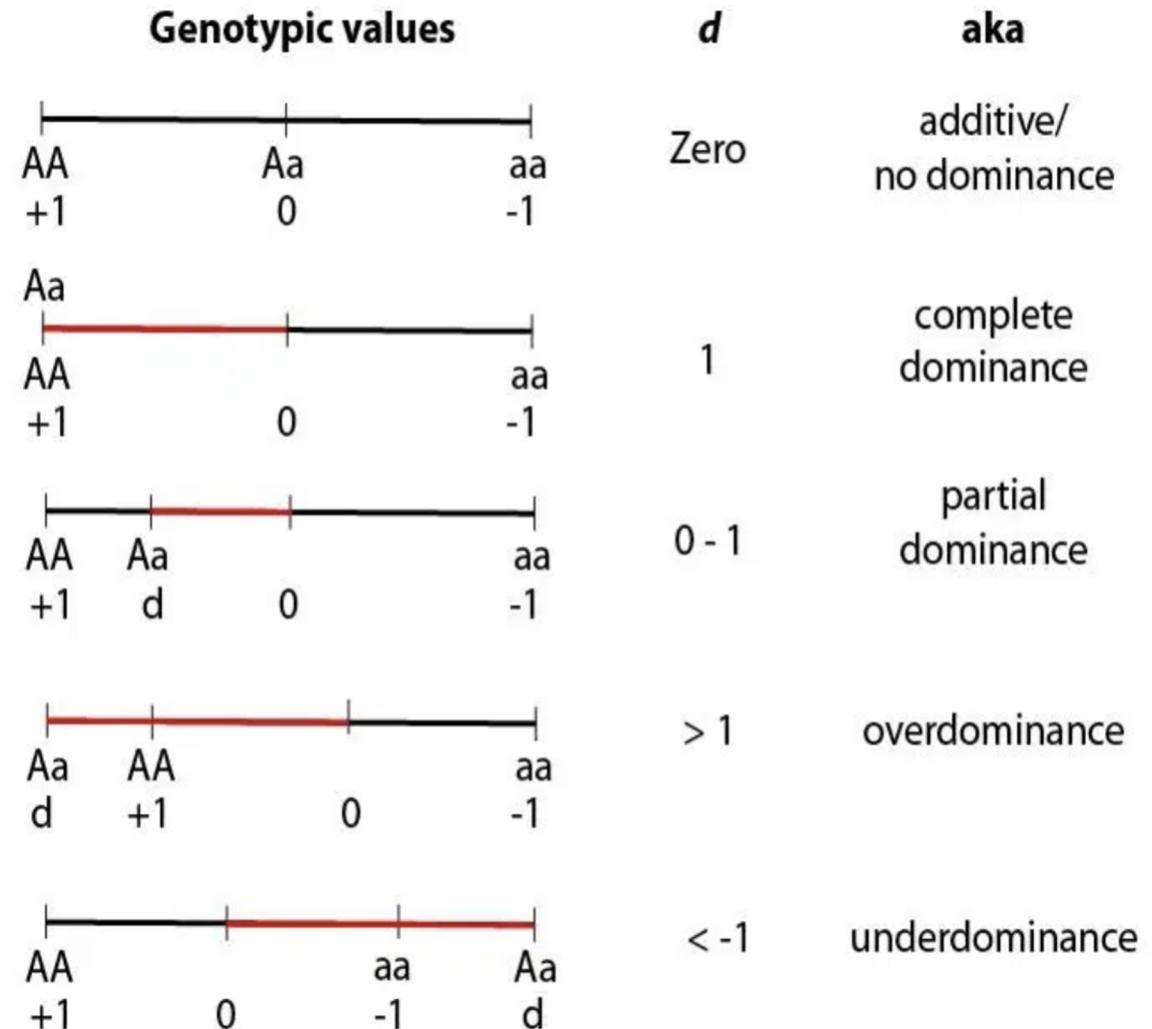
The degree of variation in a phenotypic trait in a population that is due to genetic variation between individuals in that population.

$$V_x = Vg + Ve$$

Broad-sense heritability ( $H^2$ )

$$H^2 = \frac{Vg}{Vx}$$

$$V_g = V_A + V_D + V_I$$



# Heritability

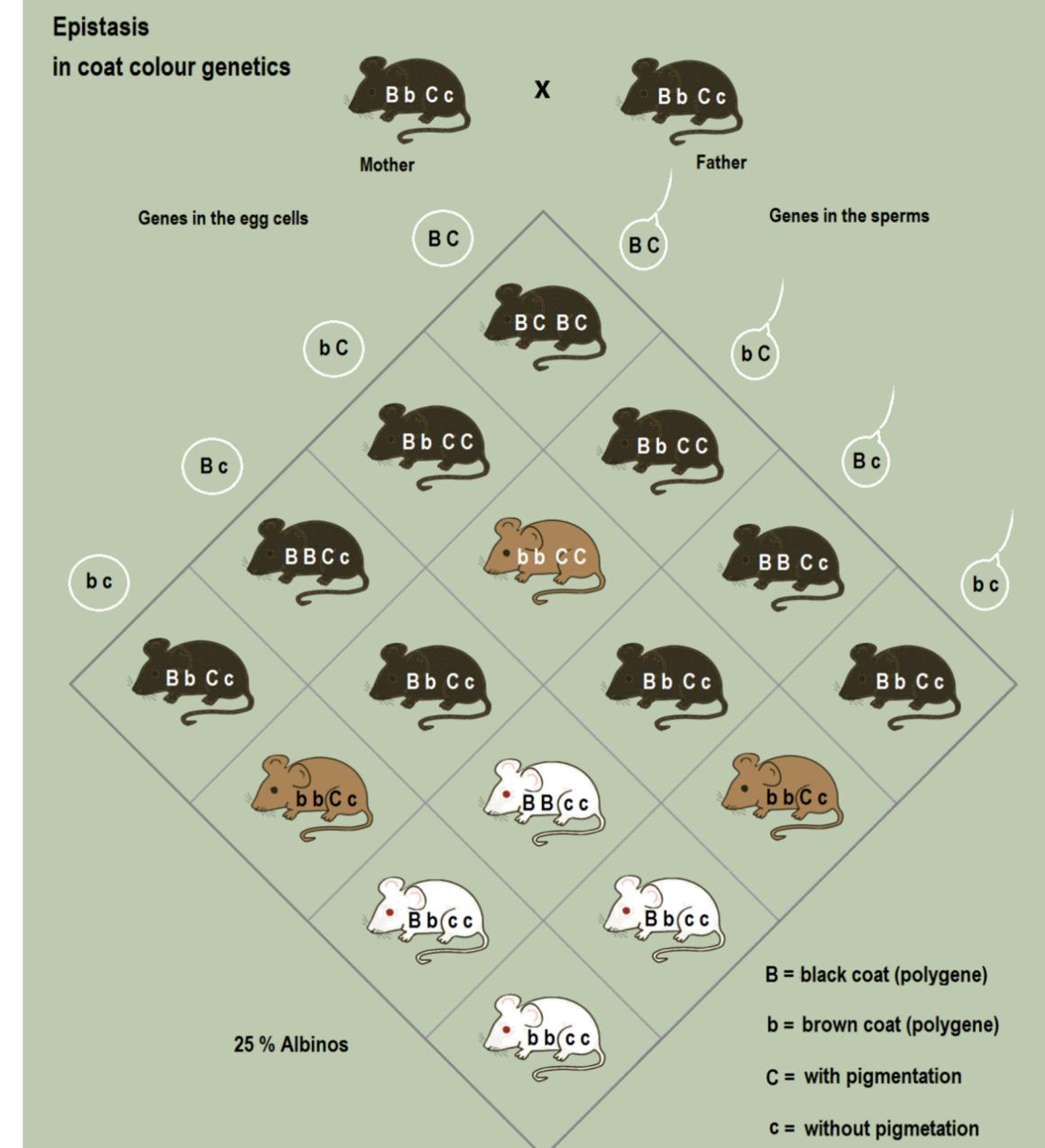
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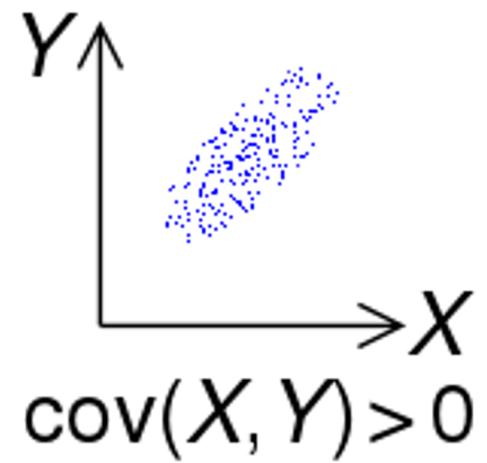
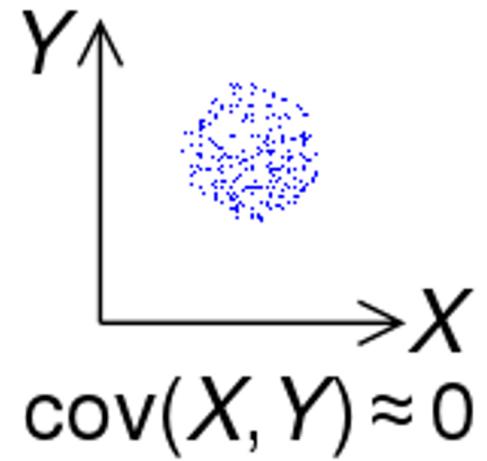
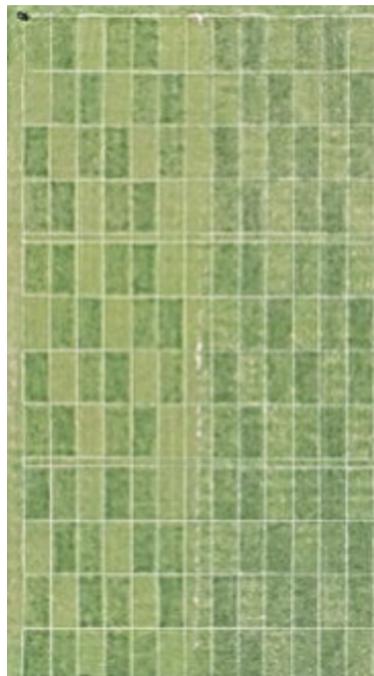
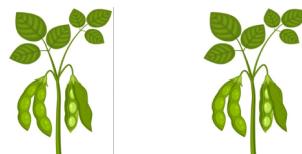


# Heritability

Broad-sense heritability ( $H^2$ )

$$H^2 = \frac{Vg}{Vx} = \frac{\text{Cov } x', x''}{Vx}$$

$$\text{Cov}_{xy} = \frac{\Sigma(x - \bar{x})(y - \bar{y})}{(n-1)}$$



[en.wikipedia.org](https://en.wikipedia.org)

# Heritability

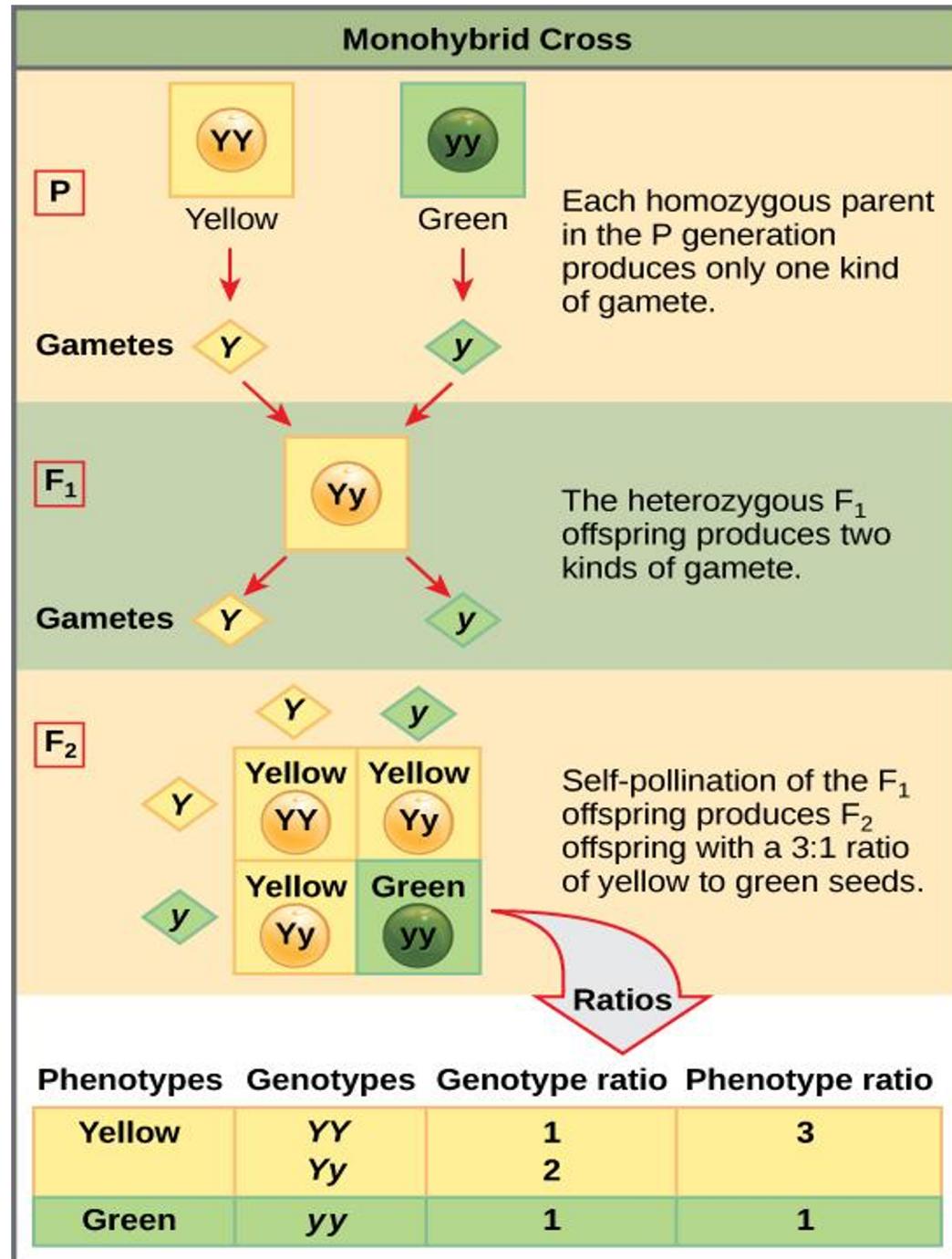
Broad-sense heritability ( $H^2$ )

$$H^2 = \frac{V_g}{V_x}$$

$$V_g = V_a + V_d + V_i$$

$H^2$  is not transmissible to the next generation in a predictive way!

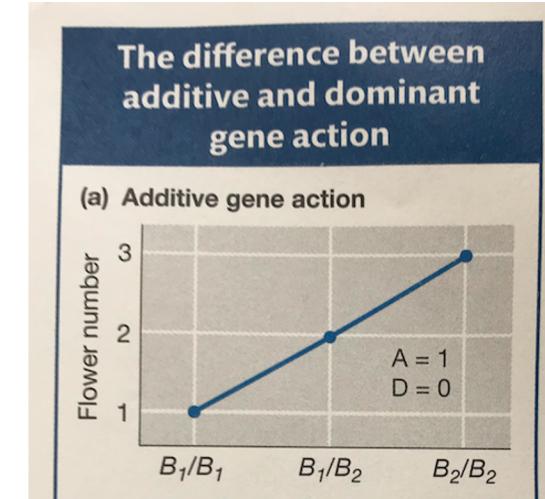
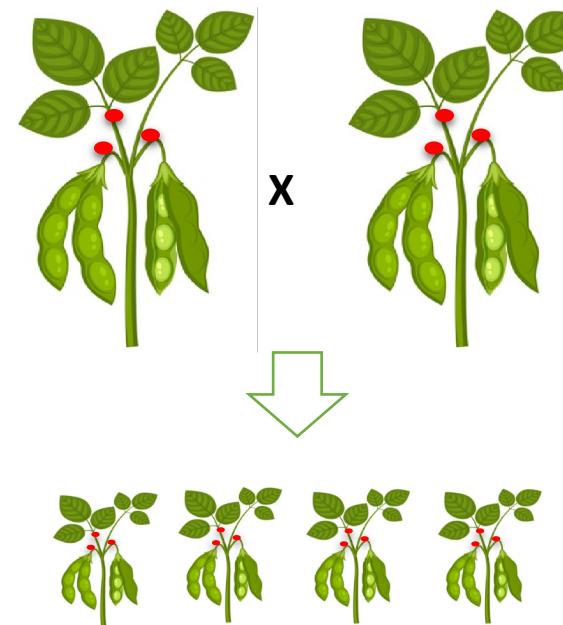
Parents transmit their genes **but not** their genotypes!



# Heritability

Narrow-sense heritability ( $h^2$ )

$$h^2 = \frac{Va}{Vx} = \frac{Va}{Va+Vd+Vi+Ve}$$



Griffiths et al 2012

$h^2$  is transmissible to the next generation in a predictive way!

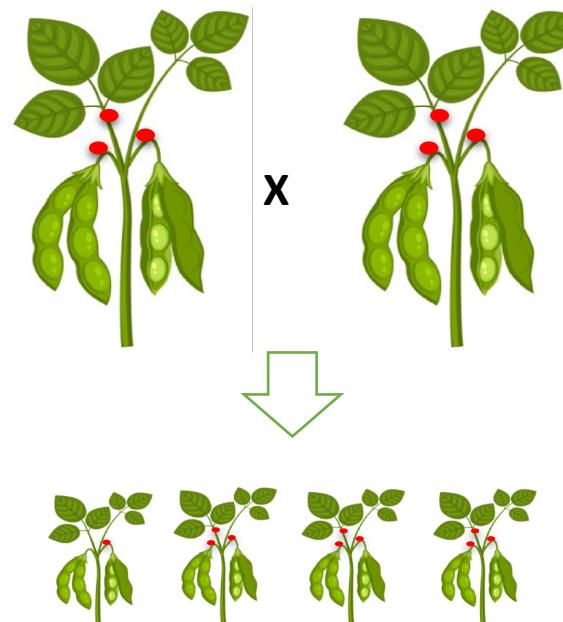
Genotype	Frequency	Trait value (no. of flowers)	Contribution to the mean (frequency × value)
B <sub>1</sub> /B <sub>1</sub>	0.25	1	0.25
B <sub>1</sub> /B <sub>2</sub>	0.50	2	1.0
B <sub>2</sub> /B <sub>2</sub>	0.25	3	0.75
Mean = 2.0			

# Heritability

Narrow-sense heritability ( $h^2$ )

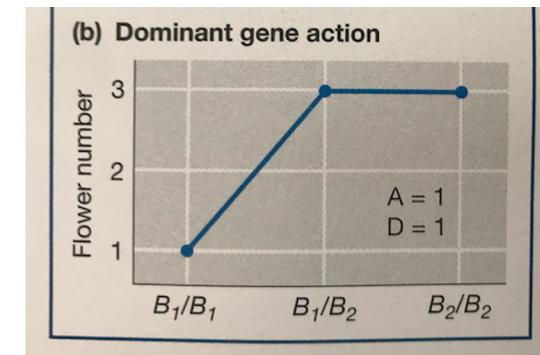
$$h^2 = \frac{Va}{Vx} = \frac{Va}{Va+Vd+Vi+Ve}$$

parents = 3



Offspring < 3 (2.78)  
The phenotype is not fully heritable!

The difference between additive and dominant gene action



Griffiths et al 2012

Genotype	Frequency	Phenotype	Contribution to the mean (frequency × value)
B <sub>1</sub> /B <sub>1</sub>	0.25	1	0.25
B <sub>1</sub> /B <sub>2</sub>	0.50	3	1.5
B <sub>2</sub> /B <sub>2</sub>	0.25	3	0.75
Mean = 2.5			

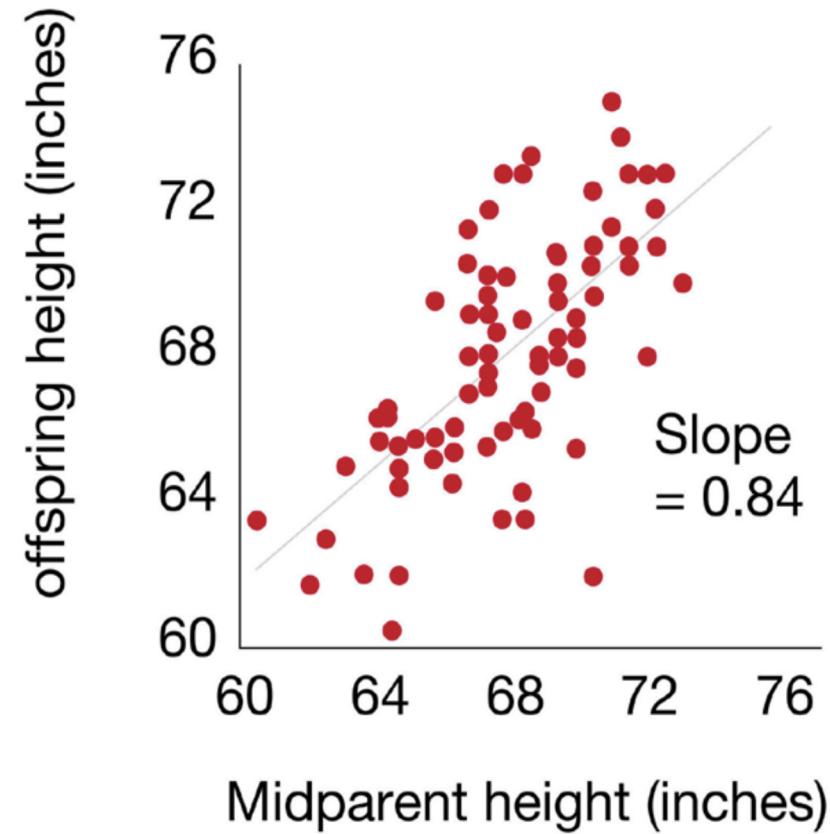
# Heritability

Narrow-sense heritability ( $h^2$ )

$$h^2 = \frac{Va}{Vx}$$

$h^2$  is transmissible to the next generation in a predictive way!

$$\beta_{O \sim Mp} = \frac{Cov(u_{ih}, u_i)}{Var(y_i)} = \frac{\sigma_a^2}{\sigma_p^2} = h^2$$

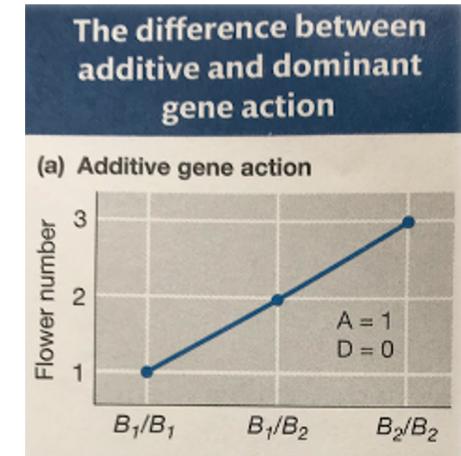
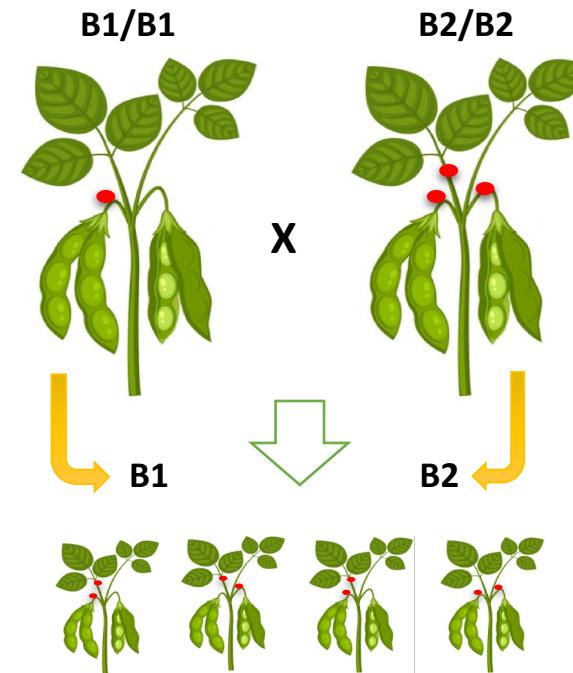
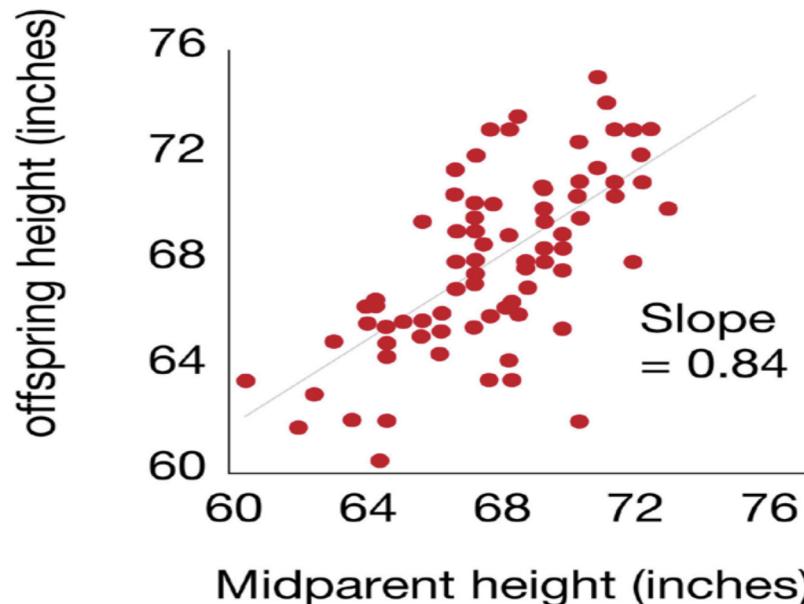


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# Heritability

Narrow-sense heritability ( $h^2$ )

$$\beta_{O \sim Mp} = \frac{Cov(u_{ih}, u_i)}{Var(y_i)} = \frac{\sigma_a^2}{\sigma_p^2} = h^2$$



Genotype	Frequency	Trait value (no. of flowers)	Contribution to the mean (frequency × value)
$B_1/B_1$	0.25	1	0.25
$B_1/B_2$	0.50	2	1.0
$B_2/B_2$	0.25	3	0.75
Mean = 2.0			

# Review

- Quantitative variation: Mean, variance, standard deviation

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \quad s^2 = \frac{1}{n-1} \sum_i (X_i - \bar{X})^2$$

- Genetic model

$$x = g + e$$

- Heritability: Broad and narrow sense

$$H^2 = \frac{Vg}{Vx}$$

$$h^2 = \frac{Va}{Vx}$$

