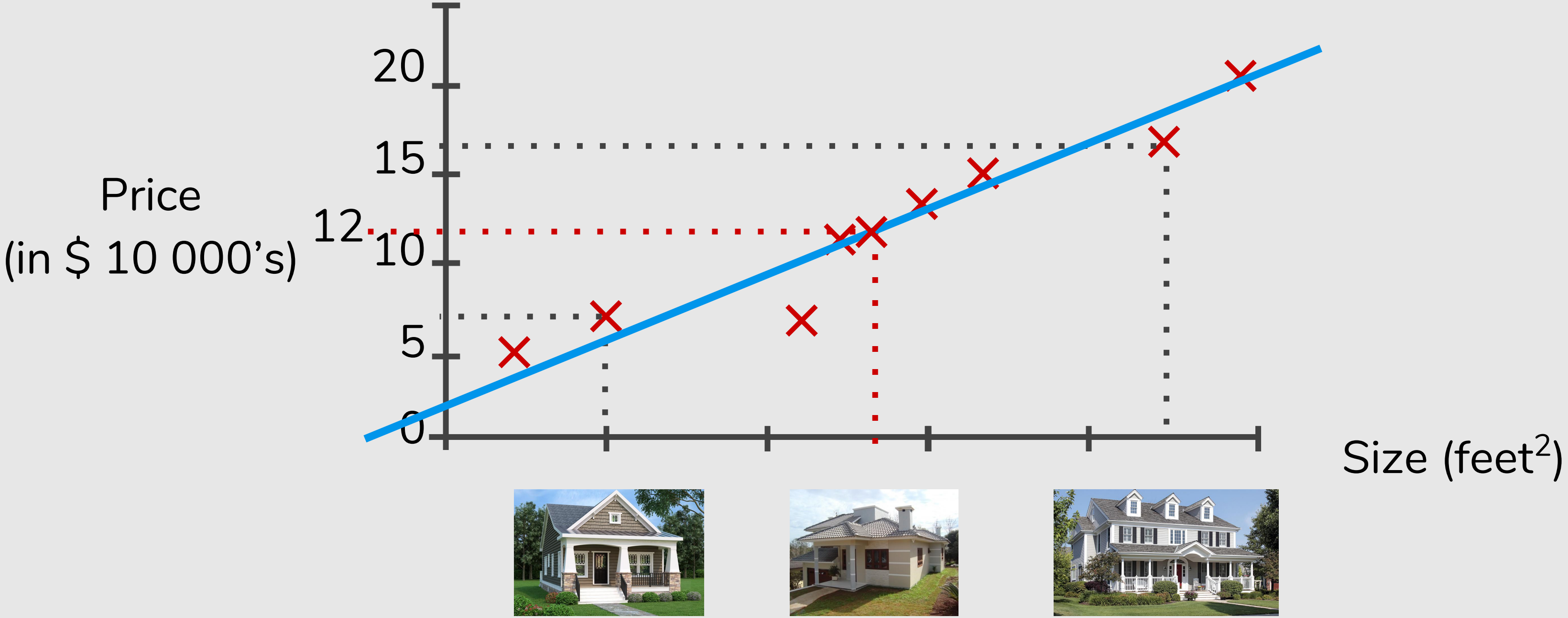
Recall from last time …

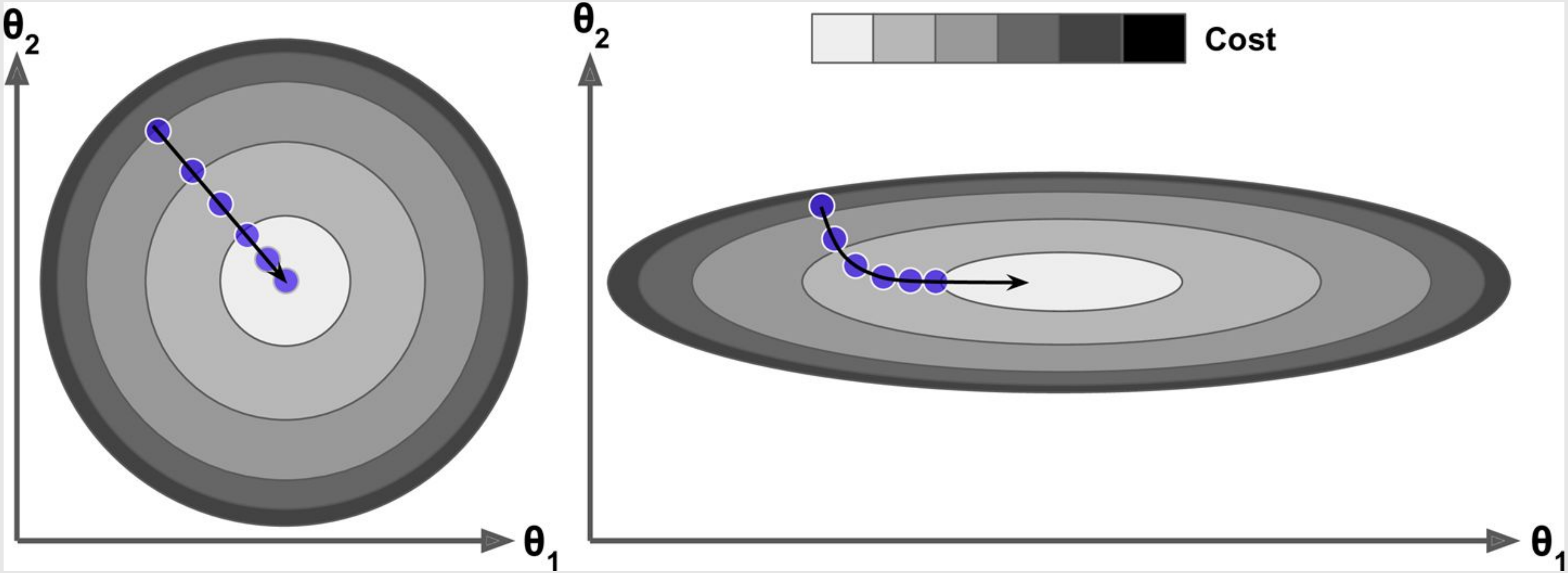
Linear Regression



Feature Scaling

Feature Scaling

Idea: Make sure features are on similar scale.



Mean Normalization

Replace x_i with $x_i - \mu_i$ to make features have approximately zero mean (do not apply to $x_0 = 1$).

E.g. $x_1 = \dfrac{\text{size} - 1000}{2000}$ ➡ $-0.5 \le x_1 \le 0.5$

$x_2 = \dfrac{\#\text{bedrooms} - 2.5}{5}$ ➡ $-0.5 \le x_2 \le 0.5$

$$x_1 = \frac{x_1 - \mu_1}{s_1} \qquad x_2 = \frac{x_2 - \mu_2}{s_2}$$

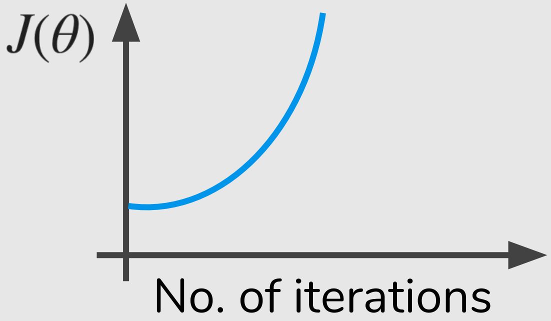
Learning Rate

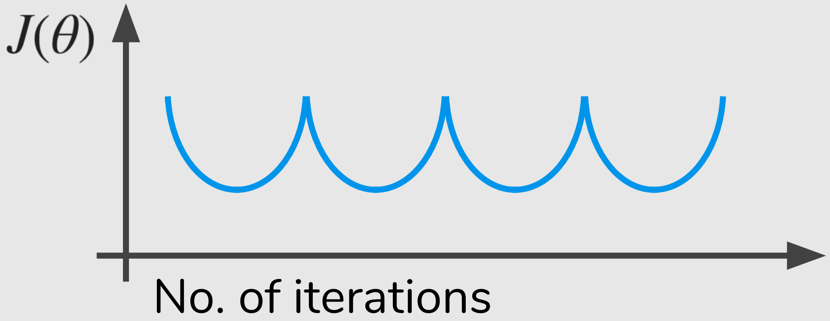
Gradient Descent

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

- "Debugging" : How to make sure gradient descent is working correctly.
- How to choose learning rate α.

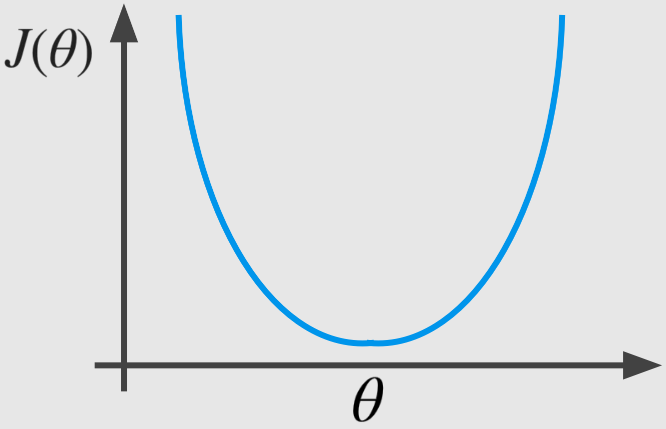
Making sure gradient descent is working correctly.





Gradient descent not working.

Use smaller α.



- For sufficiently small α, $J(\theta)$ should decrease on every iteration.
- But if α is too small, gradient descent can be slow to converge.

Features and Polynomial Regression

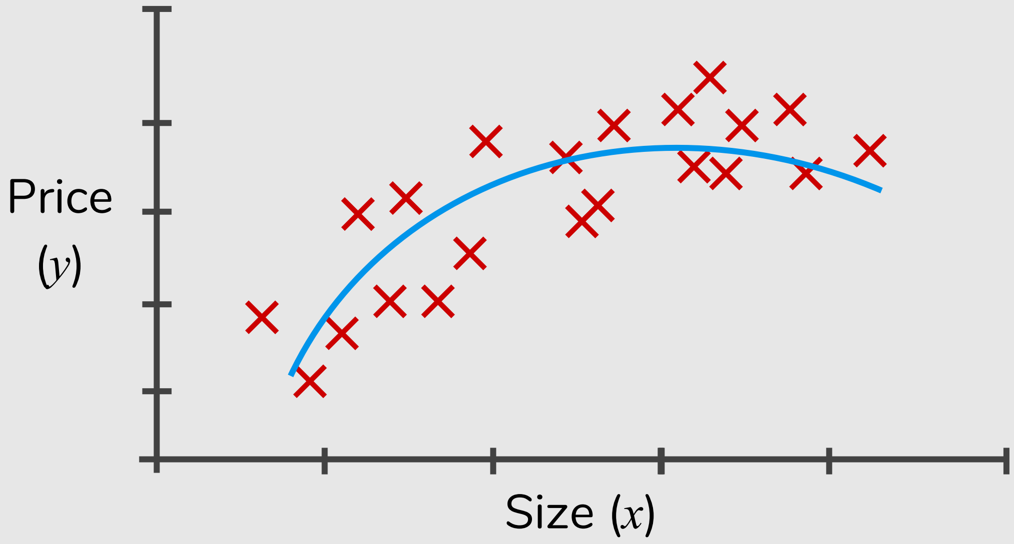
Housing prices prediction

$$h_\theta(x) = \theta_0 + \theta_1 \times \underbrace{\text{frontage}}_{x_1} + \theta_2 \times \underbrace{\text{depth}}_{x_2}$$

Area x = frontage $\times$ depth

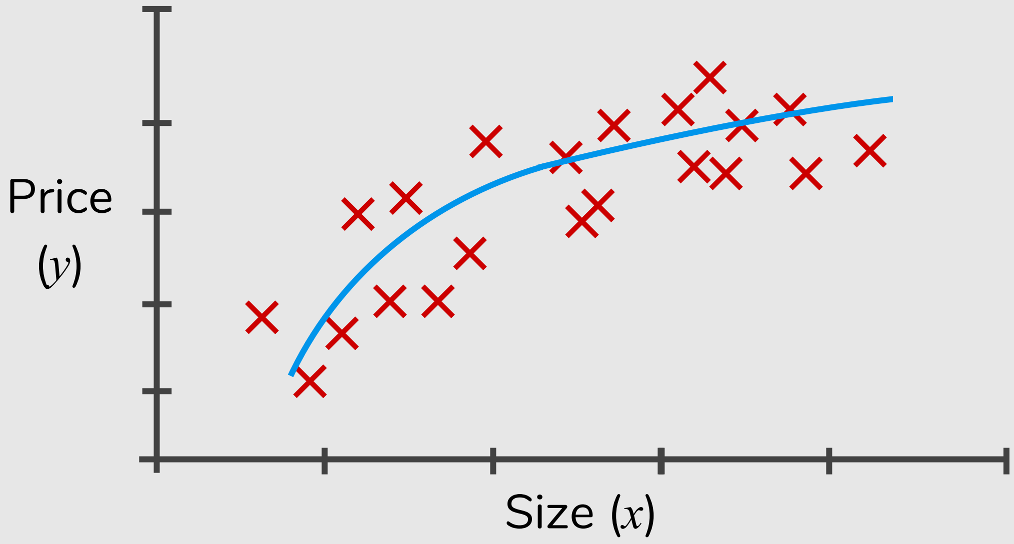
$$h_\theta(x) = \theta_0 + \theta_1 x$$

Choice of Features



$$h_\theta(x) = \theta_0 + \theta_1(\text{size}) + \theta_2(\text{size})^2$$

Choice of Features

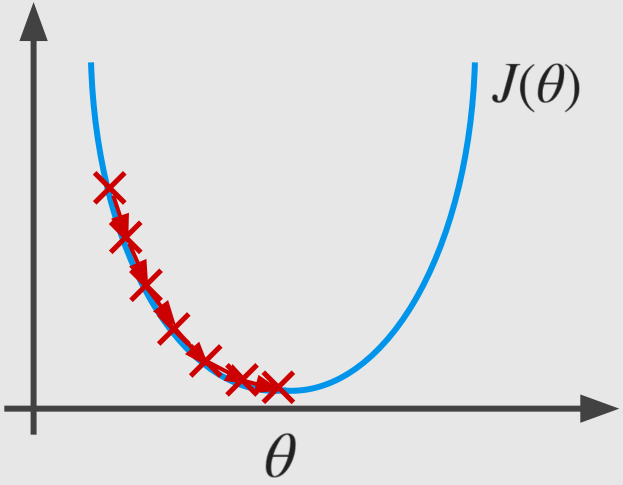


$$h_\theta(x) = \theta_0 + \theta_1(\text{ size }) + \theta_2(\text{ size })^2$$

$$h_\theta(x) = \theta_0 + \theta_1(\text{ size }) + \theta_2\sqrt{(\text{ size })}$$

Normal Equation

Gradient Descent



Normal equation: Method to solve θ **analytically**.

Examples: $m = 4$.

Size (feet^2) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price ($\$$) in 1000's y
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178

Examples: $m = 4$.

x_0	Size (feet^2) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price (\$) in 1000's y
1	2104	5	1	45	460
1	1416	3	2	40	232
1	1534	3	2	30	315
1	852	2	1	36	178

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

For convenience of notation, define $x_0 = 1$.

Examples: $m = 4$.

x_0	Size (feet^2) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price (\$) in 1000's y
1	2104	5	1	45	460
1	1416	3	2	40	232
1	1534	3	2	30	315

X = features/variables

y = target

θ = parameters

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix}_{m \times (n+1)} \qquad y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}_{m}$$

$$\boxed{\theta = (X^T X)^{-1} X^T y}$$

$$\theta = (X^T X)^{-1} X^T y$$

$(X^T X)^{-1}$ is inverse of matrix $X^T X$.

Deriving the Normal Equation using matrix calculus ...

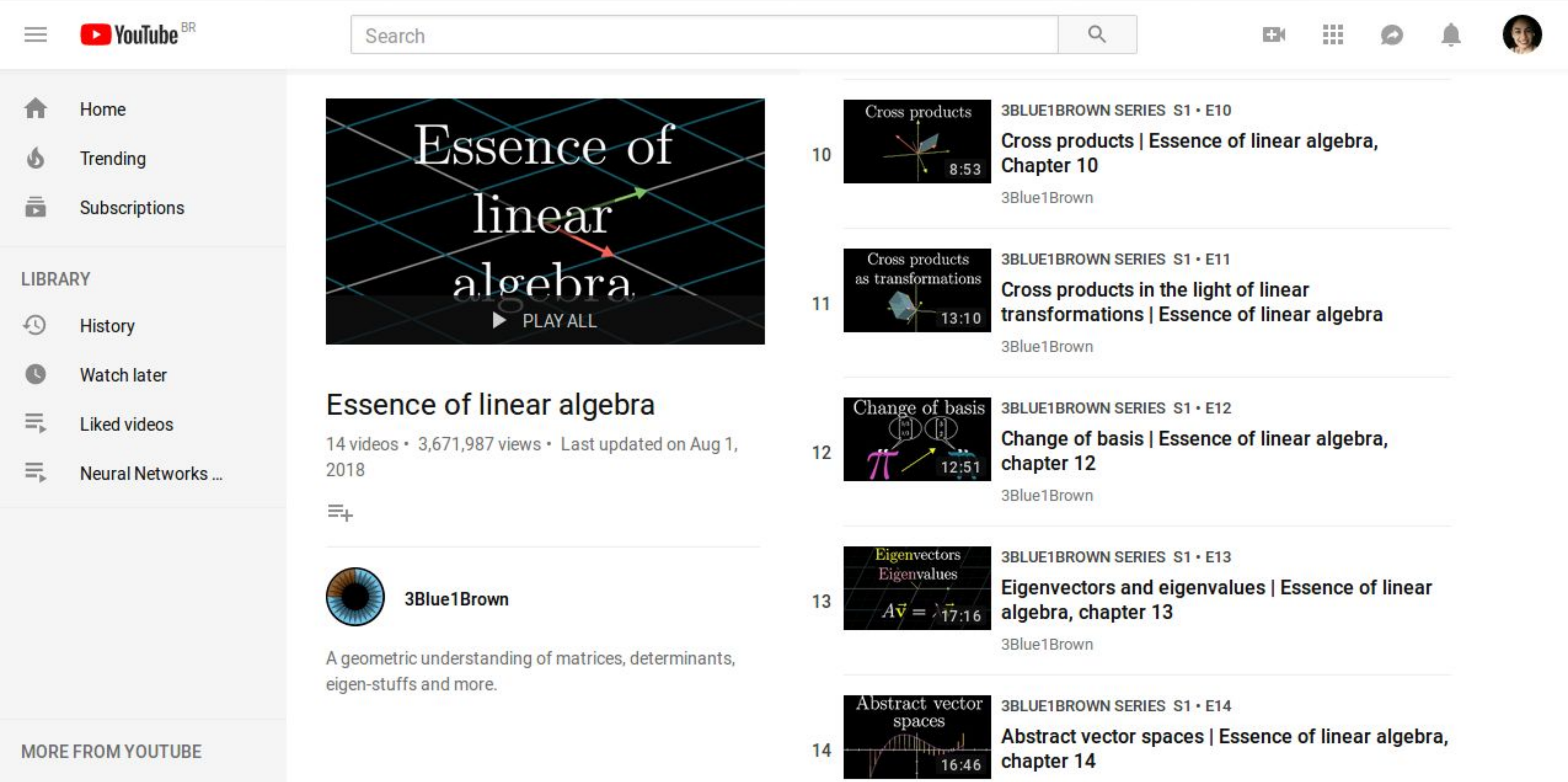
☞ https://ayearofai.com/rohan-3-deriving-the-normal-equation-using-matrix-calculus-1a1b16f65dda

What if $X^T X$ is noninvertible?

The common causes might be having :

- Redundant features, where two features are very closely related (i.e. they are linearly dependent).
- Too many features (e.g. $m \leq n$). In this case, delete some features or use "regularization".

https://www.youtube.com/playlist?list=PLZHQObOWTQDPD3MizzM2xVFitgF8hE_ab



Gradient Descent

- Need to choose α.
- Needs many iterations.
- Works well even when n is large.

Normal Equation

- No need to choose α.
- Don't need to iterate.
- **Don't need to scale.**
- Need to compute $(X^T X)^{-1} \rightarrow O(n^3)$.
- Slow if n is very large.

m examples and n features

Categorical/Nominal Variables

Size (feet^2) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Color x_5	Price (\$) in 1000's y
2104	5	1	45	blue	460
1416	3	2	40	white	232
1534	3	2	30	pink	315
852	2	1	36	green	178

Categorical/Nominal Variables

Dummy coding & One-hot encoding

http://www.statisticssolutions.com/dummy-coding-the-how-and-why/

https://en.wikiversity.org/wiki/Dummy_variable_(statistics)

Categorical/Nominal Variables

Dummy coding & One-hot encoding

- blue = 1, white = 2, pink = 3, and green = 4.

http://www.statisticssolutions.com/dummy-coding-the-how-and-why/

https://en.wikiversity.org/wiki/Dummy_variable_(statistics)

Categorical/Nominal Variables

Dummy coding & **One-hot encoding**

color	blue	white	pink	green
blue	1	0	0	0
white	0	1	0	0
pink	0	0	1	0
green	0	0	0	1

In this simplified data set, if we know that color is not Blue, not White, and not Pink, then it is Green.

So we only need to use three of these four.

Logistic Regression

Machine Learning and Pattern Recognition

(Largely based on slides from Andrew Ng)

Prof. Sandra Avila
Institute of Computing (IC/Unicamp)

MC886/MO444, August 21, 2018

Today's Agenda

- Logistic Regression
  - Classification
  - Hypothesis Representation
  - Decision Boundary
  - Cost Function
  - Simplified Cost Function and Gradient Descent
  - Multiclass Classification

Classification

Spam Filtering

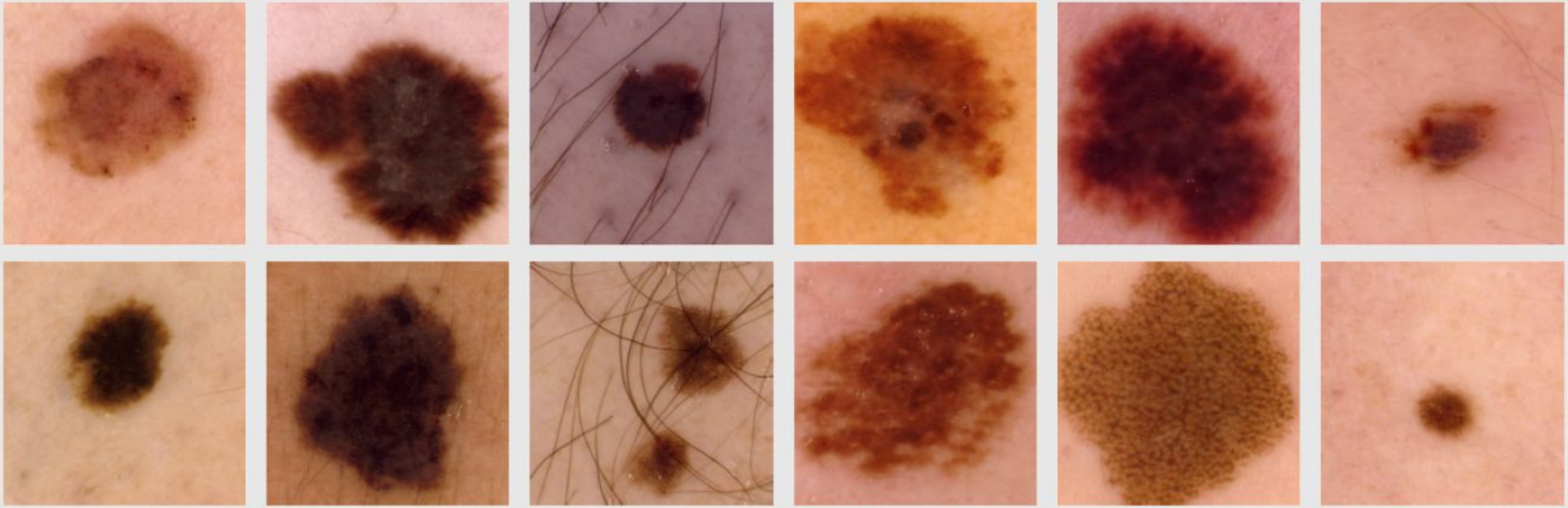
Bad Cures fast and effective! - Canadian *** Pharmacy #1 Internet Inline Drugstore Viagra Cheap Our price $1.99 …

Good Interested in your research on graphical models - Dear Prof., I have read some of your papers on probabilistic graphical models. Because I …

Sensitive Content Classification

Image Credit (left) : http://ehow-blog.com/how-to-avoid-internet-addiction/
Image Credit (right) : http://www.telegraph.co.uk/culture/tvandradio/9840832/Children-under-5-should-not-watch-TV-alone-Jackanory-creator-argues.html

Skin Cancer Classification



Melanomas (top row) and benign skin lesions (bottom row)

Image Credit: M. Fornaciali, S. Avila, M. Carvalho, E. Valle, Statistical learning approach for robust melanoma screening, SIBGRAPI, 2014

Classification

Email: **Spam** / **Not Spam**?

Content Video: **Sensitive** / **Non-sensitive**?

Skin Lesion: **Malignant** / **Benign**?

Classification

Email: **Spam** / **Not Spam**?

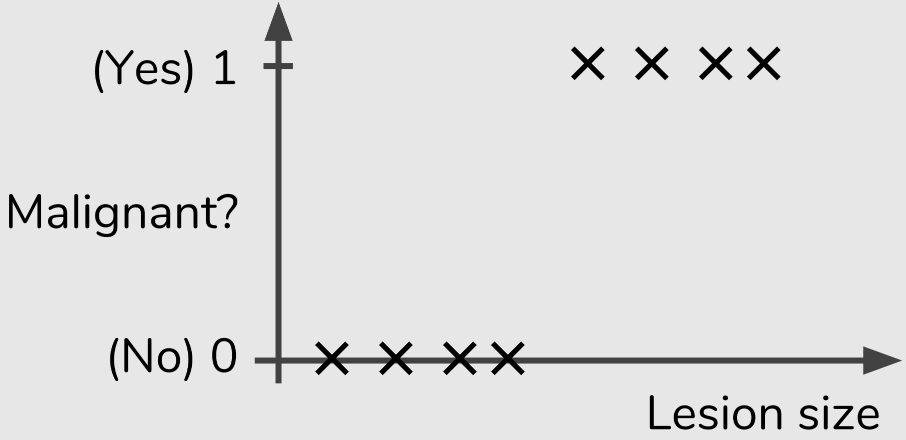
Content Video: **Sensitive** / **Non-sensitive**?

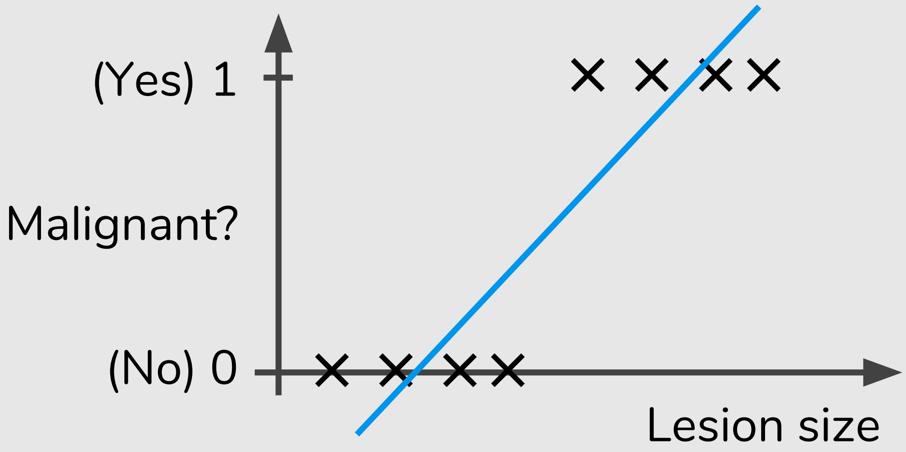
Skin Lesion: **Malignant** / **Benign**?

$y \in \{0,1\}$

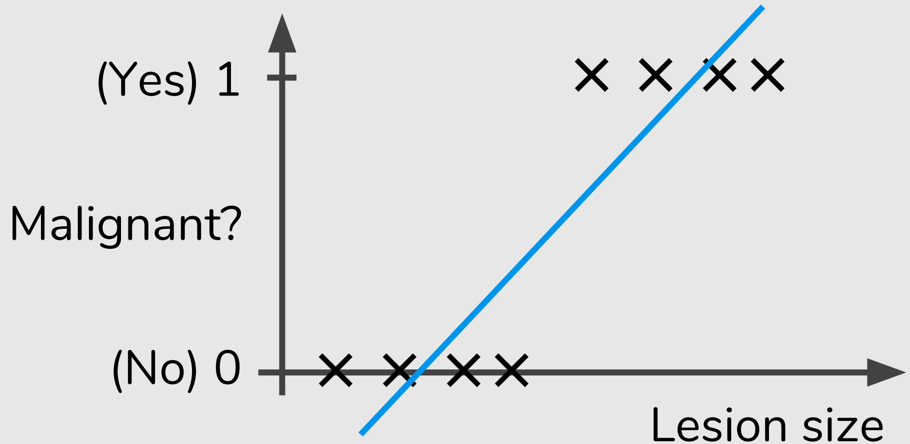
0: "Negative Class" (e.g., Benign skin lesion)

1: "Positive Class" (e.g., Malignant skin lesion)


(Yes) 1
Malignant?
(No) 0
Lesion size



$$h_\theta(x) = \theta^T x$$

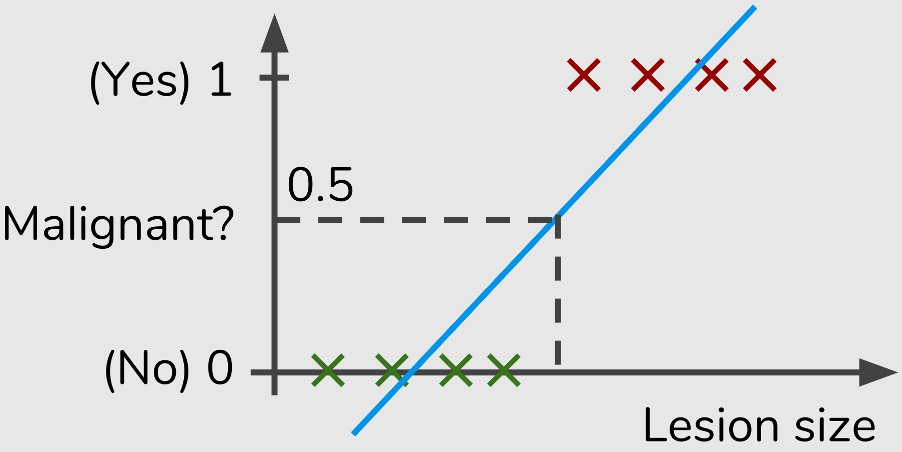


$$h_\theta(x) = \theta^T x$$

Threshold classifier output $h_\theta(x)$ at 0.5:

If $h_\theta(x) \geq 0.5$, predict "$y = 1$"

If $h_\theta(x) < 0.5$, predict "$y = 0$"



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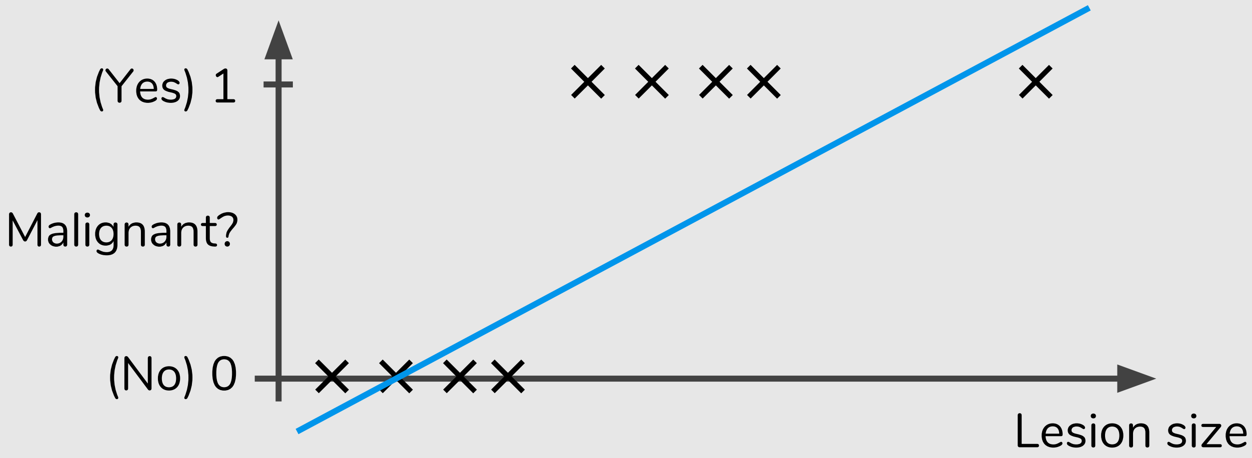

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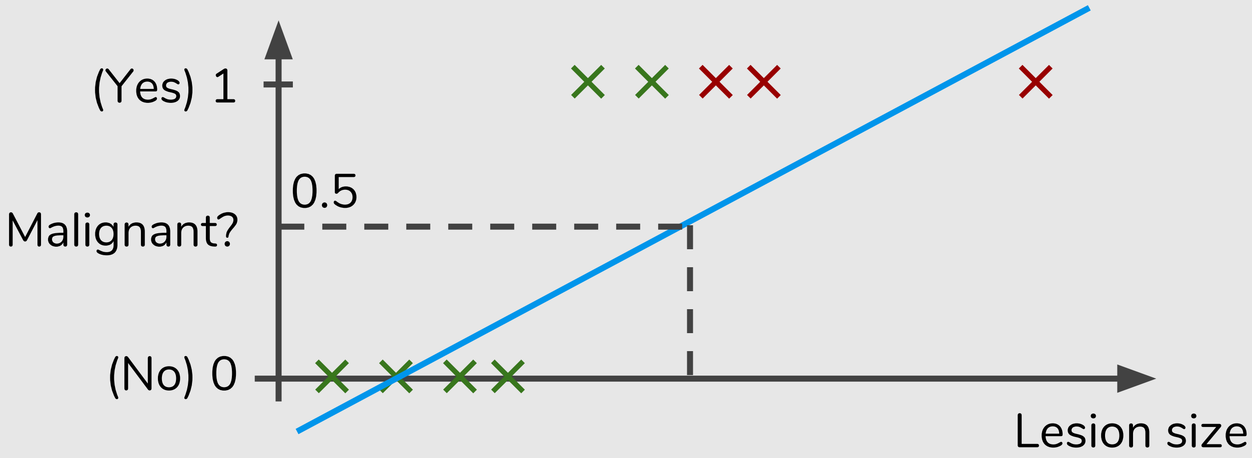

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Threshold classifier output $h_\theta(x)$ at 0.5:

If $h_\theta(x) \geq 0.5$, predict "$y = 1$"

If $h_\theta(x) < 0.5$, predict "$y = 0$"

Classification: $y = 0$ or $y = 1$

$h_\theta(x)$ can be > 1 or < 0

Logistic Regression: $0 \leq h_\theta(x) \leq 1$

Hypothesis Representation

Logistic Regression Model

Want $0 \leq h_\theta(x) \leq 1$

Logistic Regression Model

Want $0 \leq h_\theta(x) \leq 1$

$$h_\theta(x) = \quad \theta^{\mathrm{T}} x$$

Logistic Regression Model

Want $0 \leq h_\theta(x) \leq 1$

$$h_\theta(x) = g(\theta^{\mathrm{T}} x)$$

Logistic Regression Model

Want $0 \leq h_\theta(x) \leq 1$

$$h_\theta(x) = g(\theta^{\mathrm{T}} x)$$

$$g(z) = \frac{1}{1 + e^{-z}}$$

Logistic Regression Model

Want $0 \leq h_\theta(x) \leq 1$

$$h_\theta(x) = g(\theta^{T}x)$$

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Sigmoid Function

Logistic Function

Logistic Regression Model

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

Want $0 \leq h_\theta(x) \leq 1$

$$h_\theta(x) = g(\theta^T x)$$

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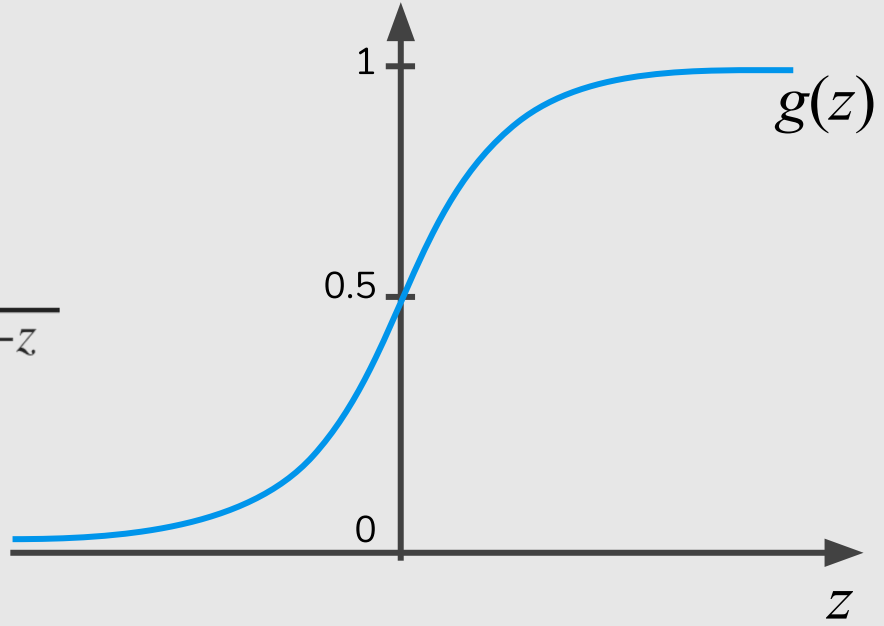

Interpretation of Hypothesis Output

$h_\theta(x)$ = estimated probability that $y = 1$ on input x

Interpretation of Hypothesis Output

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Example: If $x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{tumorSize} \end{bmatrix}$

$h_\theta(x) = 0.7$

Tell patient that 70% chance of tumor being malignant

Interpretation of Hypothesis Output

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Tell patient that 70% chance of tumor being malignant

$h_\theta(x) = P(y = 1 \mid x;\theta)$

Interpretation of Hypothesis Output

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Tell patient that 70% chance of tumor being malignant

"probability that $y = 1$, given x, parameterized by θ"

$h_\theta(x) = P(y = 1 \mid x; \theta)$

Interpretation of Hypothesis Output

$h_\theta(x)$ = estimated probability that $y = 1$ on input x

Example: If $x = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{tumorSize} \end{bmatrix}$

$h_\theta(x) = 0.7$

Tell patient that 70% chance of tumor being malignant

"probability that $y = 1$, given x, parameterized by θ"

$$P(y = 0 \mid x;\theta) + P(y = 1 \mid x;\theta) = 1$$

$$P(y = 1 \mid x;\theta) = 1 - P(y = 0 \mid x;\theta)$$

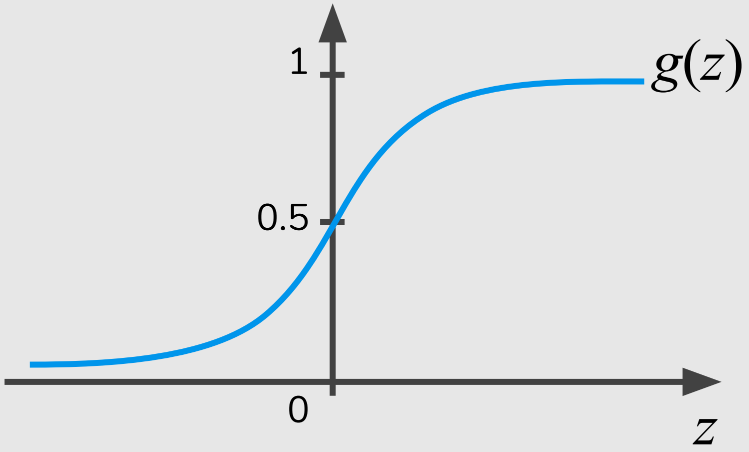
$$h_\theta(x) = P(y = 1 \mid x;\theta)$$

Decision Boundary

Logistic Regression

$$h_\theta(x) = g(\theta^{\mathrm{T}} x)$$

$$g(z) = \frac{1}{1 + e^{-z}}$$



Logistic Regression

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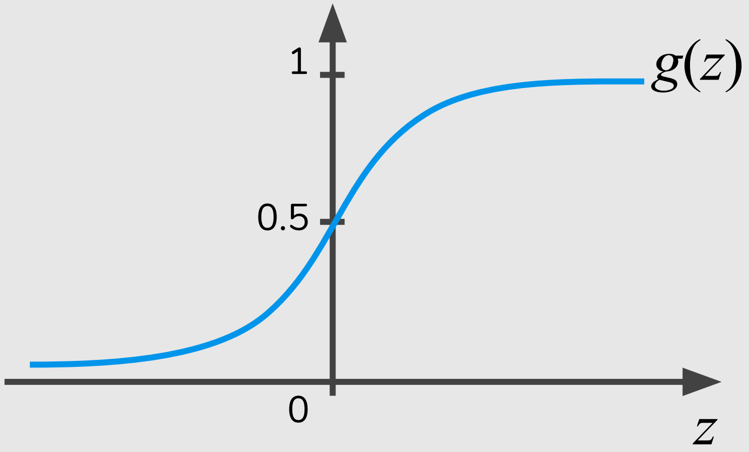

Suppose predict "$y = 1$" if $h_\theta(x) \geq 0.5$

predict "$y = 0$" if $h_\theta(x) < 0.5$

Logistic Regression

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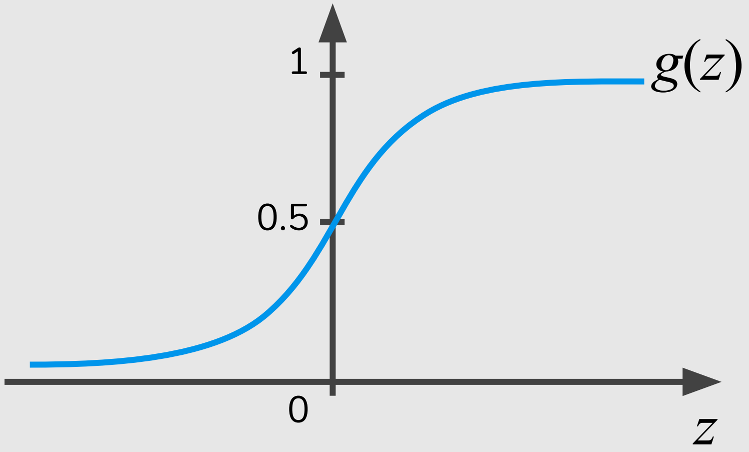

Suppose predict "$y = 1$" if $h_\theta(x) \geq 0.5$

predict "$y = 0$" if $h_\theta(x) < 0.5$

$g(z) \geq 0.5$ when $z \geq 0$

Logistic Regression

$$h_\theta(x) = g(\theta^{\mathrm{T}} x)$$

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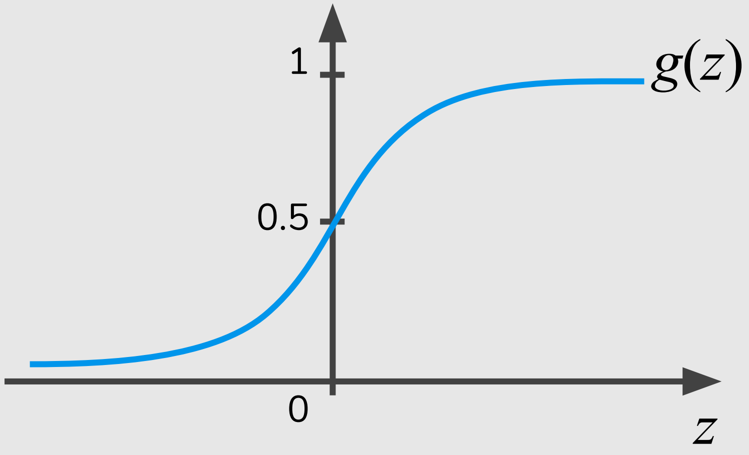


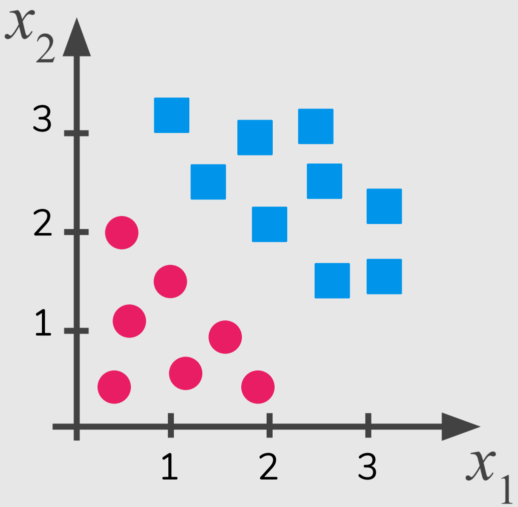
Suppose predict "$y = 1$" if $h_\theta(x) \geq 0.5$

predict "$y = 0$" if $h_\theta(x) < 0.5$

$g(z) \geq 0.5$ when $z \geq 0$

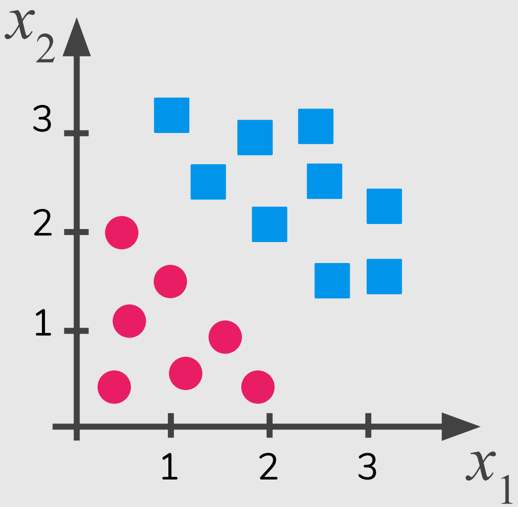
$g(z) < 0.5$ when $z < 0$

Decision Boundary



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

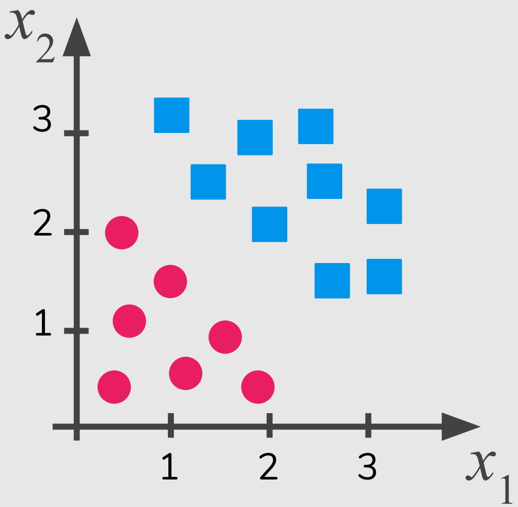
Decision Boundary



$$\overset{-3}{\uparrow}\quad\overset{1}{\uparrow}\quad\overset{1}{\uparrow}$$

$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

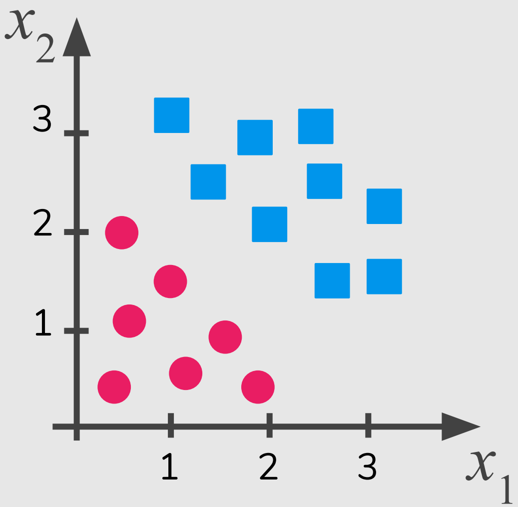
Decision Boundary



$$\begin{array}{c} \quad\quad\quad -3 \quad\quad 1 \quad\quad\quad 1 \\ h_\theta(x) = g(\underset{\uparrow}{\theta_0} + \underset{\uparrow}{\theta_1} x_1 + \underset{\uparrow}{\theta_2} x_2) \end{array}$$

Predict "$y = 1$" if $-3 + x_1 + x_2 \geq 0$

Decision Boundary

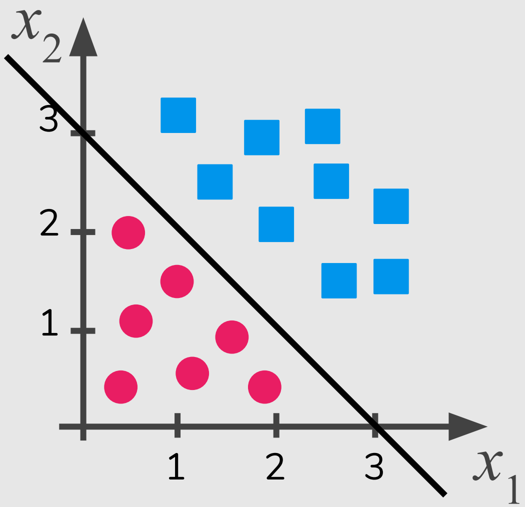


$$\begin{array}{c} \qquad\qquad\quad -3 \qquad\;\; 1 \qquad\quad\;\; 1 \\ \qquad\qquad\quad \Uparrow \qquad\;\; \Uparrow \qquad\quad\;\; \Uparrow \\ h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2) \end{array}$$

Predict "$y = 1$" if $-3 + x_1 + x_2 \geq 0$

$$x_1 + x_2 \geq 3$$

Decision Boundary

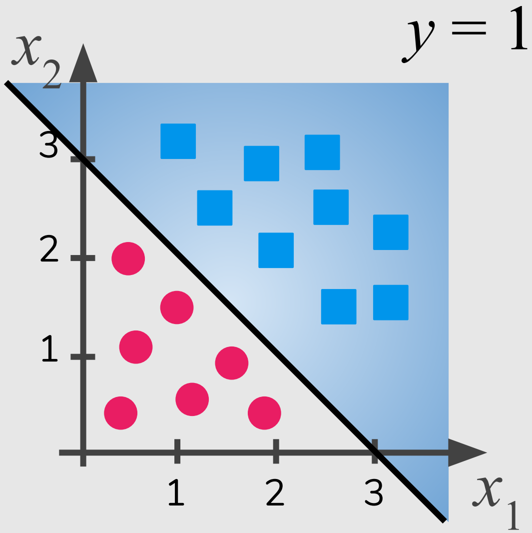


$$\begin{array}{c} \quad\quad\quad\quad\quad\quad -3 \quad\quad 1 \quad\quad\quad 1 \\ \quad\quad\quad\quad\quad\quad \Uparrow \quad\quad \Uparrow \quad\quad\quad \Uparrow \\ h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2) \end{array}$$

Predict "$y = 1$" if $-3 + x_1 + x_2 \geq 0$

$$x_1 + x_2 \geq 3$$

Decision Boundary



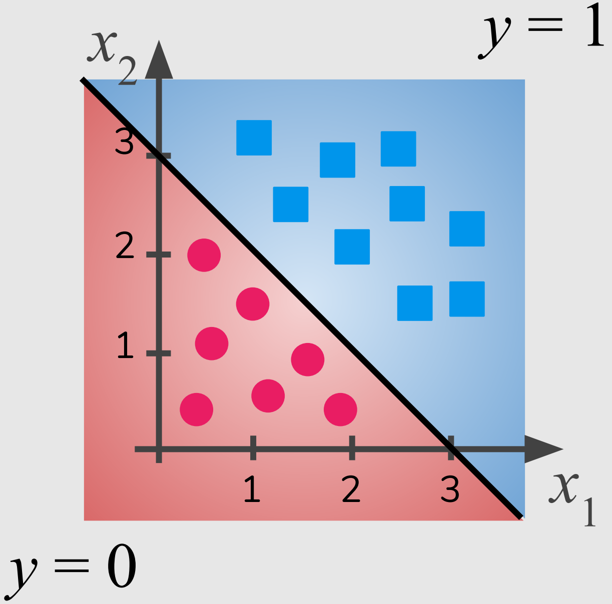
$$\begin{array}{c} \quad\quad\quad -3 \quad\quad 1 \quad\quad\quad 1 \\ \quad\quad\quad \Uparrow \quad\quad \Uparrow \quad\quad\quad \Uparrow \end{array}$$

$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

Predict "$y = 1$" if $-3 + x_1 + x_2 \geq 0$

$$x_1 + x_2 \geq 3$$

Decision Boundary



$$-3 \quad 1 \quad 1$$

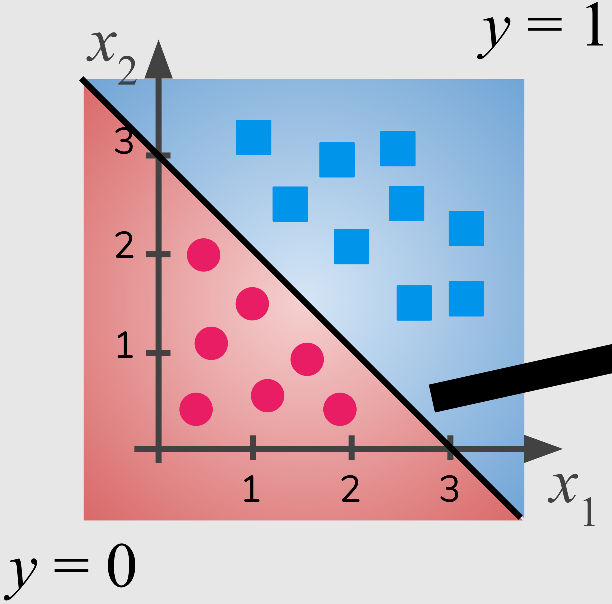
$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

Predict "$y = 1$" if $-3 + x_1 + x_2 \geq 0$

$$x_1 + x_2 \geq 3$$

$$y = 0,\ x_1 + x_2 < 3$$

Decision Boundary



$$h_\theta(x) = g(\overset{-3}{\theta_0} + \overset{1}{\theta_1} x_1 + \overset{1}{\theta_2} x_2)$$

Decision Boundary $x_1 + x_2 = 3$

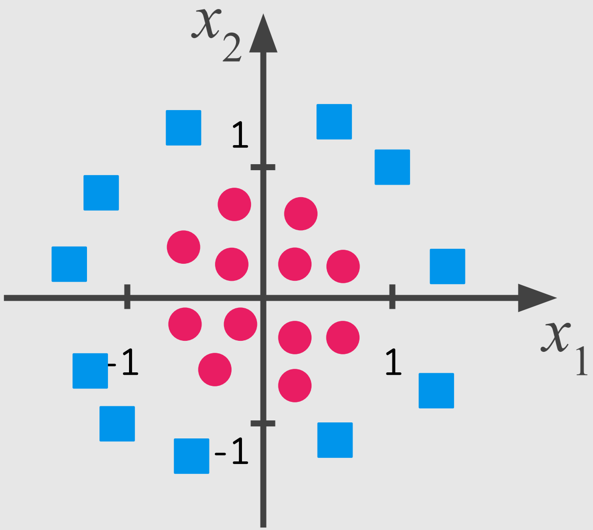
$$h_\theta(x) = 0.5$$

Predict "$y = 1$" if $-3 + x_1 + x_2 \geq 0$

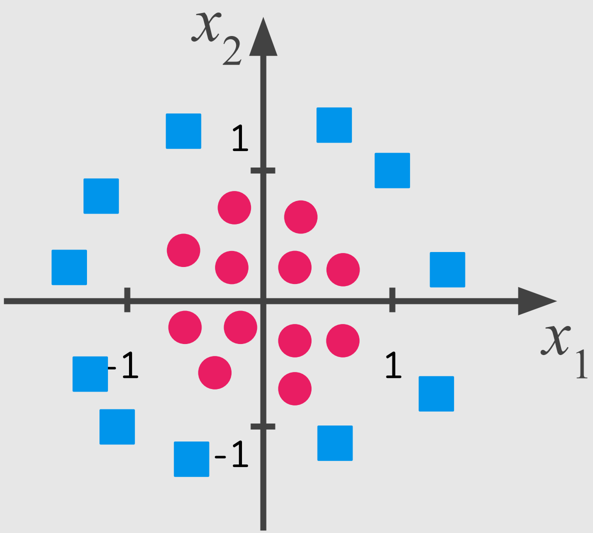
$$x_1 + x_2 \geq 3$$

$$y = 0,\ x_1 + x_2 < 3$$

Non-linear Decision Boundaries

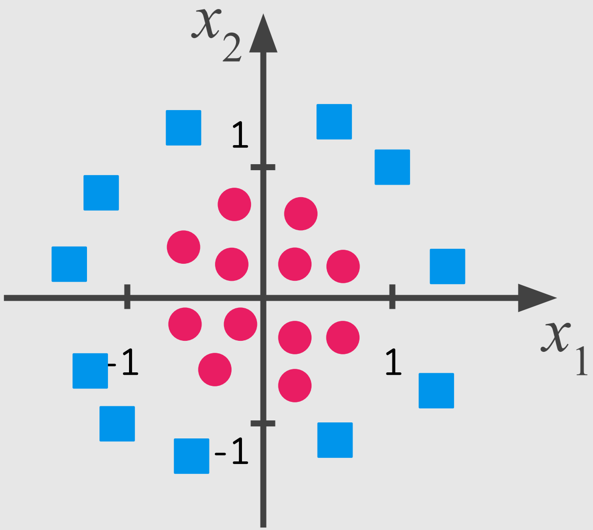


Non-linear Decision Boundaries



$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

Non-linear Decision Boundaries



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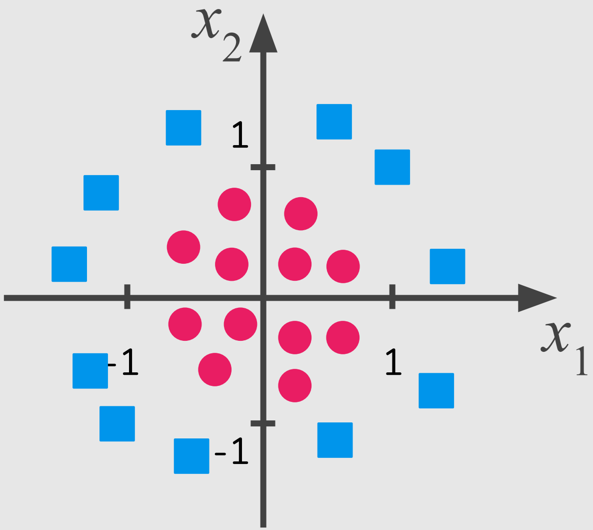
$$\begin{array}{ccccc} \theta_0 & \theta_1 & \theta_2 & \theta_3 & \theta_4 \\ \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow \\ -1 & 0 & 0 & 1 & 1 \end{array}$$

Non-linear Decision Boundaries



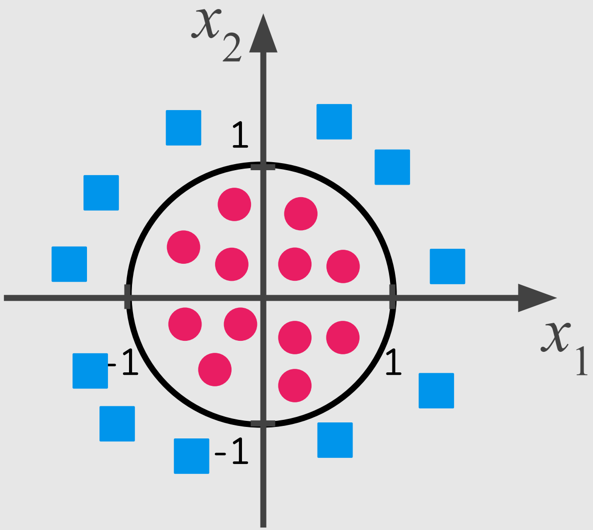
$$h_\theta(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

$$\theta_0 \downarrow -1 \quad \theta_1 \downarrow 0 \quad \theta_2 \downarrow 0 \quad \theta_3 \downarrow 1 \quad \theta_4 \downarrow 1$$

Predict "$y = 1$" if $-1 + x_1^2 + x_2^2 \geq 0$

$$x_1^2 + x_2^2 \geq 1$$

Non-linear Decision Boundaries



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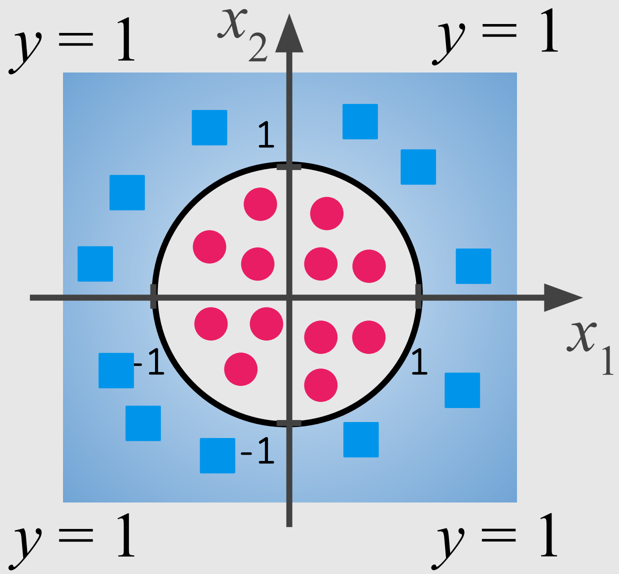

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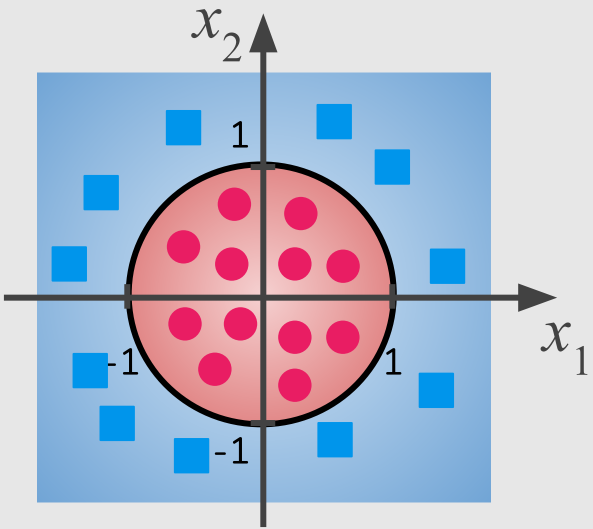

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Predict "$y = 1$" if $-1 + x_1^2 + x_2^2 \geq 0$

$$x_1^2 + x_2^2 \geq 1$$

Cost Function

Training set: $\{(x^{(1)},y^{(1)}),\ (x^{(2)},y^{(2)}),\ \ldots,\ (x^{(m)},y^{(m)})\}$

$$h_\theta(x) = \frac{1}{1+\mathrm{e}^{-\theta^{\mathrm{T}}x}}$$

$$x \in \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix} \quad x_0 = 1,\ y \in \{0,1\}$$

How to choose parameters θ ?

Cost Function

Linear regression: $J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{2} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$

Cost Function

$$\mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

Linear regression: $J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \boxed{\frac{1}{2}(h_\theta(x^{(i)}) - y^{(i)})^2}$

$$\mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)}) = \frac{1}{2}(h_\theta(x^{(i)}) - y^{(i)})^2$$

Cost Function

$$\mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

Linear regression: $J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \boxed{\frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2}$

$$\mathrm{Cost}(h_\theta(x), y) = \frac{1}{2}(h_\theta(x) - y)^2$$

Cost Function

$$\mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

~~Linear~~ regression: $J(\theta) = \frac{1}{m}\sum_{i=1}^{m}\boxed{\frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2}$

Logistic

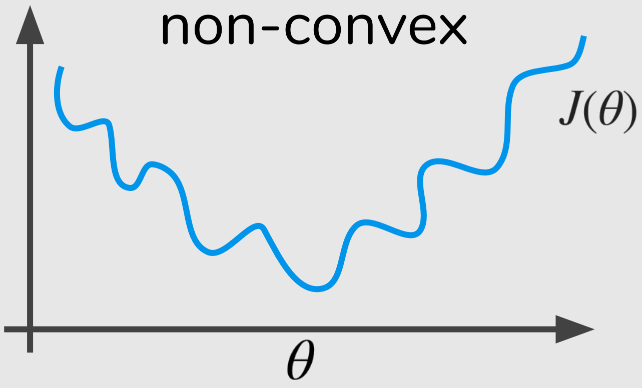
$$\mathrm{Cost}(h_\theta(x), y) = \frac{1}{2}(h_\theta(x) - y)^2 \qquad h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

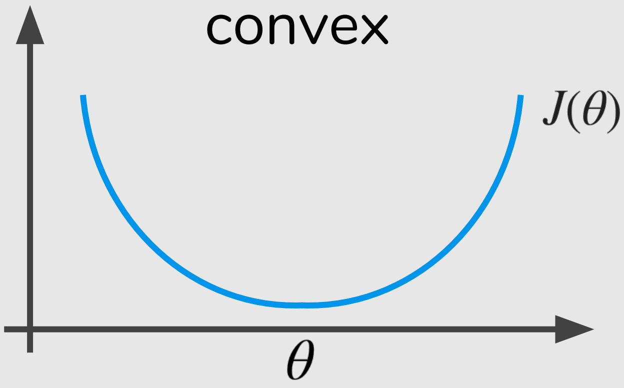
Cost Function

$$\mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

Logistic regression: $J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \boxed{\frac{1}{2}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2}$

$$\mathrm{Cost}(h_\theta(x), y) = \frac{1}{2}(h_\theta(x) - y)^2 \qquad h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$





Derivative of Logistic Function

$$\boxed{g(z)=\frac{1}{1+e^{-z}}}$$

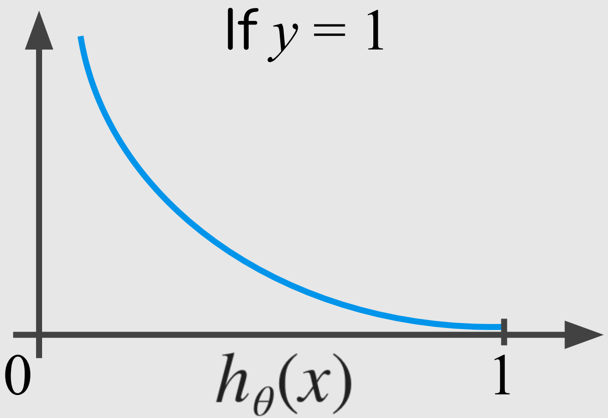
$$
\begin{aligned}
g'(z) &= \frac{d}{dz}\,\frac{1}{1-e^{-z}} \\
&= \frac{0\cdot(1-e^{-z})-1\cdot(-e^{-z})}{(1-e^{-z})^2} \qquad \text{(quotient rule)} \\
&= \frac{e^{-z}}{(1-e^{-z})^2} \\
&= \left(\frac{1}{1-e^{-z}}\right)\left(1-\frac{1}{1-e^{-z}}\right) \\
&= g(z)(1-g(z))
\end{aligned}
$$

Logistic Regression Cost Function

$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

Logistic Regression Cost Function

$$\mathrm{Cost}(h_\theta(x),\ y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



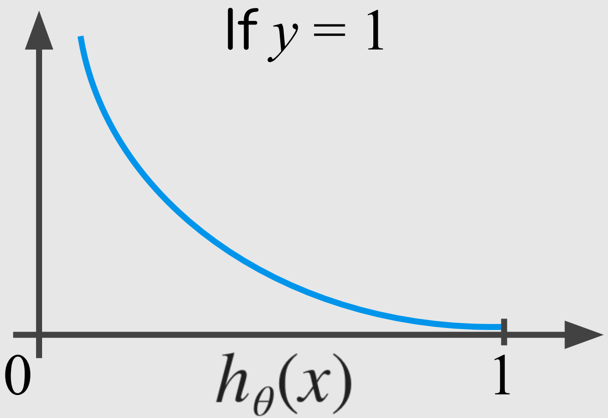
Cost $= 0$ if $y = 1,\ h_\theta(x) = 1$

But as $h_\theta(x) \rightarrow 0$

$\mathrm{Cost} \rightarrow \infty$

Logistic Regression Cost Function

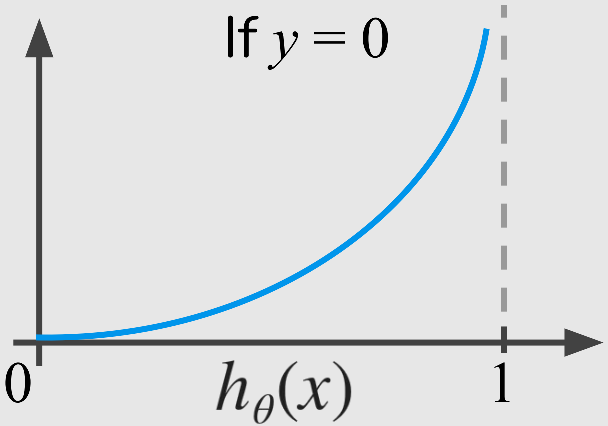
$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



Captures intuition that if $h_\theta(x) = 0$, (predict $P(y = 1 \mid x;\theta) = 0$), but $y = 1$, we'll penalize learning algorithm by a very large cost.

Logistic Regression Cost Function

$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$



Simplified Cost Function and Gradient Descent

Logistic Regression Cost Function

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \mathrm{Cost}(h_\theta(x^{(i)}),\ y^{(i)})$$

$$\mathrm{Cost}(h_\theta(x),\ y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

Logistic Regression Cost Function

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$$\mathrm{Cost}(h_\theta(x),\ y) = -y\log(h_\theta(x)) - (1-y)\log(1 - h_\theta(x))$$

Logistic Regression Cost Function

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \mathrm{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$\mathrm{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

$$\mathrm{Cost}(h_\theta(x), y) = -y\log(h_\theta(x)) - (1-y)\cancel{\log(1 - h_\theta(x))}$$

$$y = 1$$

Logistic Regression Cost Function

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \mathrm{Cost}(h_\theta(x^{(i)}),\ y^{(i)})$$

$$\mathrm{Cost}(h_\theta(x),\ y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

$$\mathrm{Cost}(h_\theta(x),\ y) = \underbrace{-y\cancel{\log(h_\theta(x))}}_{y=0} - (1-y)\log(1 - h_\theta(x))$$

Logistic Regression Cost Function

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \mathrm{Cost}(h_\theta(x^{(i)}),\ y^{(i)})$$

$$= -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log(h_\theta(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)})) \right]$$

Logistic Regression Cost Function

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To fit parameters θ : $\min_\theta J(\theta)$

Logistic Regression Cost Function

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To fit parameters θ : $\min_{\theta} J(\theta)$

To make a new prediction given new x: Output $h_\theta(x) = \dfrac{1}{1 + \mathrm{e}^{-\theta^{\mathrm{T}} x}}$

Gradient Descent

$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)}\log(h_\theta(x^{(i)})) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right]$$

Want $\min_{\theta} J(\theta)$:

repeat {

$$\theta_j := \theta_j - \alpha\frac{\partial}{\partial\theta_j}J(\theta)$$

} (simultaneously update θ_j for $j = 0, 1, \ldots, n$)

Gradient Descent

$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)}\log(h_\theta(x^{(i)})) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right]$$

Want $\min_\theta J(\theta)$:

repeat {

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial\theta_j}J(\theta)}$$

$$\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$$

} (simultaneously update θ_j for $j = 0, 1, \ldots, n$)

Gradient Descent

https://math.stackexchange.com/questions/477207
/derivative-of-cost-function-for-logistic-regrssion

Want $\min_{\theta} J(\theta)$:

repeat {

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Gradient Descent

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Algorithm looks identical to linear regression!

Gradient Descent

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$$h_\theta(x) = \theta^T x \Rightarrow h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

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Algorithm looks identical to linear regression!

Multiclass Classification: One-vs-all

Classification

Email tagging: Work, Friends, Family

Skin Lesion: Melanoma, Carcinoma, Nevus, Keratosis

Video: Pornography, Violence, Gore scenes, Child abuse

Classification

Email tagging: Work, Friends, Family

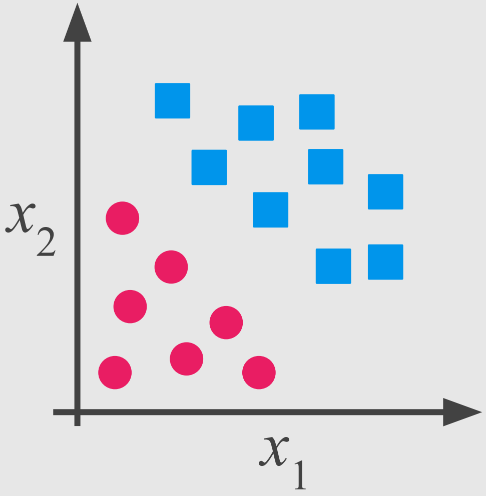
$y = 1 \quad y = 2 \quad y = 3$

Skin Lesion: Melanoma, Carcinoma, Nevus, Keratosis

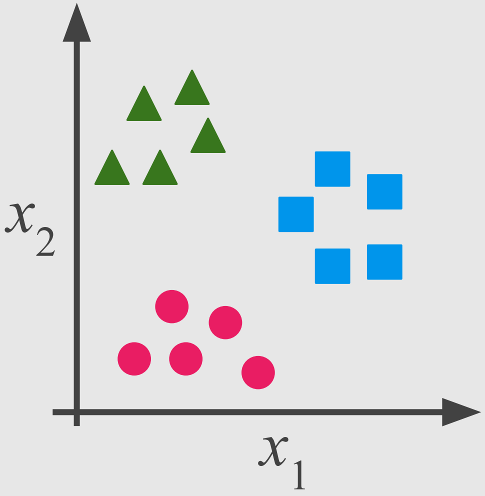
$y = 1 \quad y = 2 \quad y = 3 \quad y = 4$

Video: Pornography, Violence, Gore scenes, Child abuse

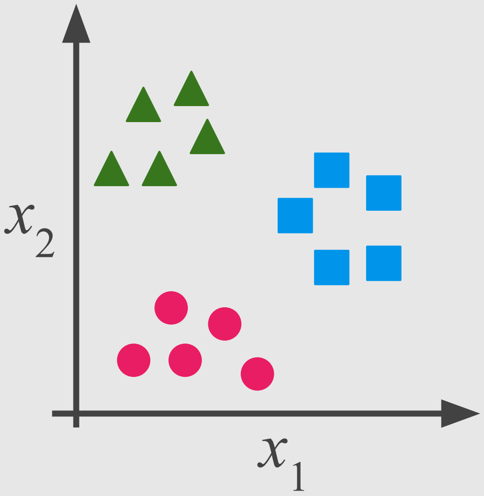
Binary Classification



Multi-class Classification

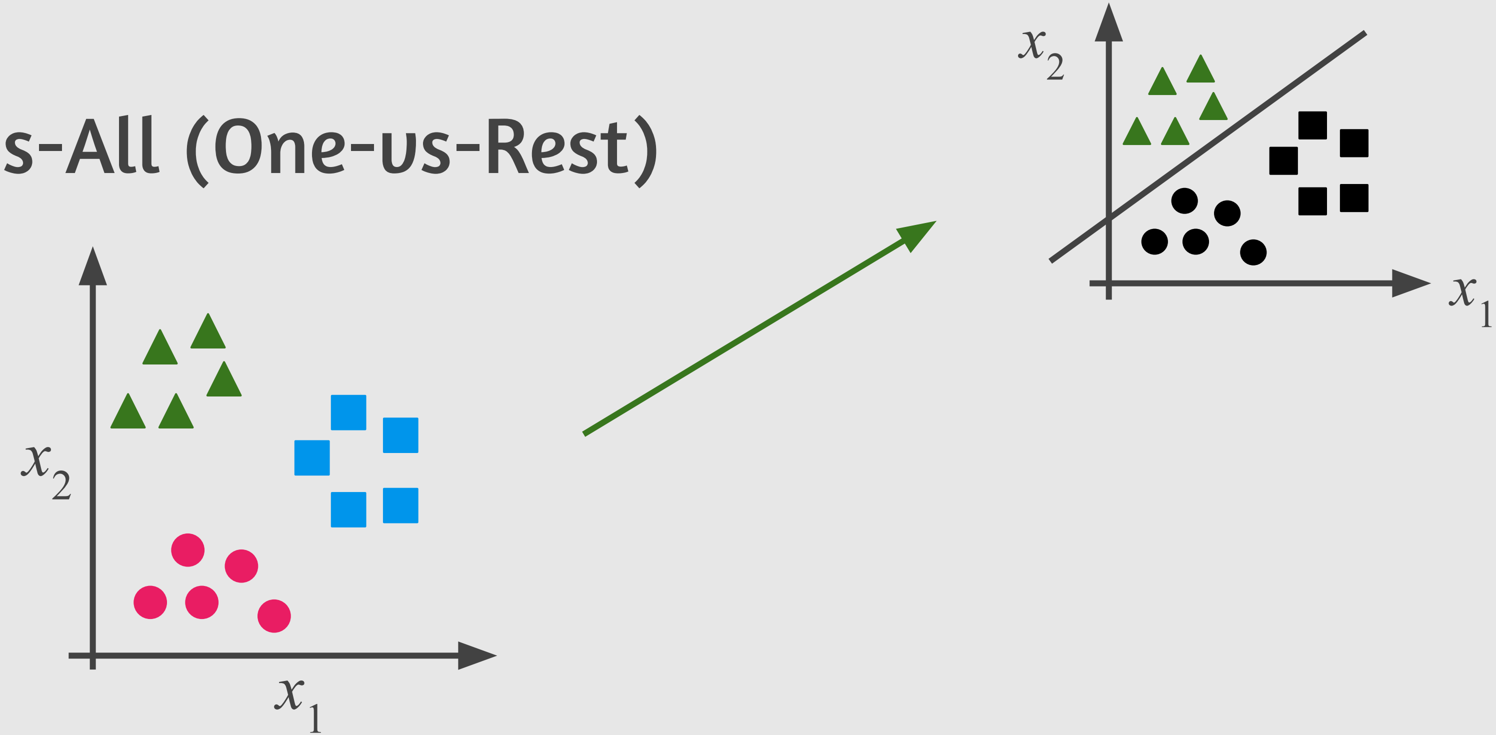


One-vs-All (One-vs-Rest)



Class 1: ▲
Class 2: ■
Class 3: ●

One-vs-All (One-vs-Rest)

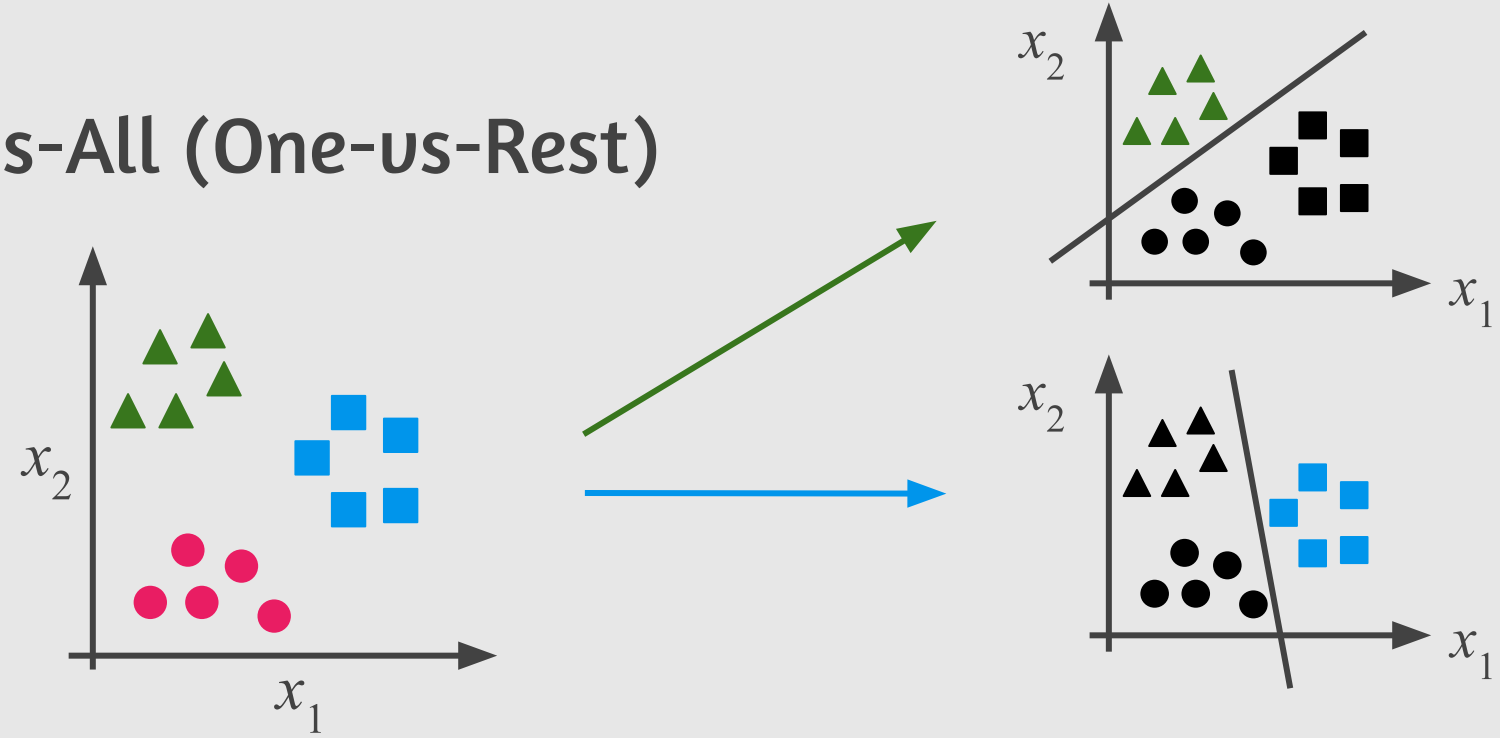


Class 1: ▲

Class 2: ■

Class 3: ●

One-vs-All (One-vs-Rest)

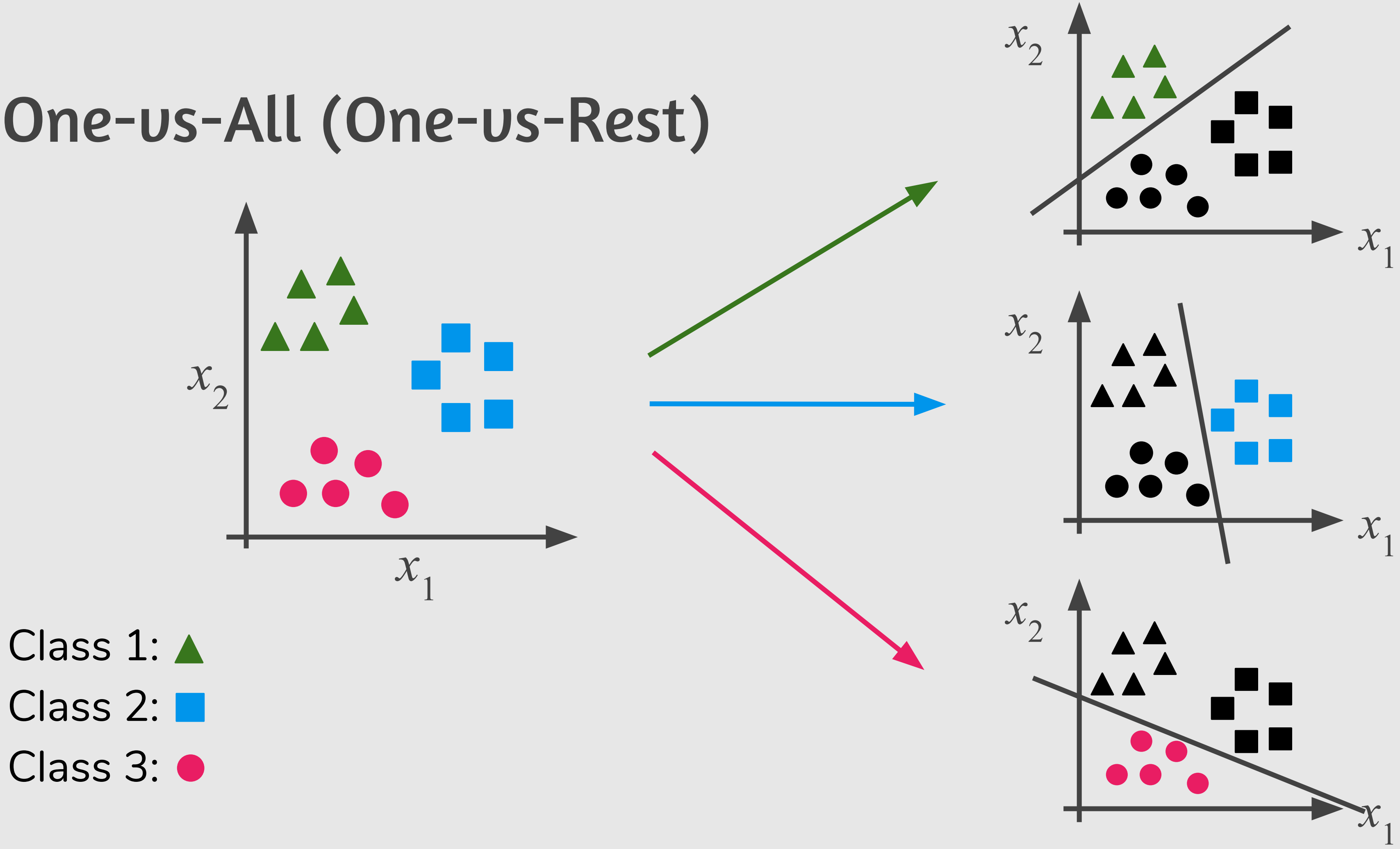


Class 1:

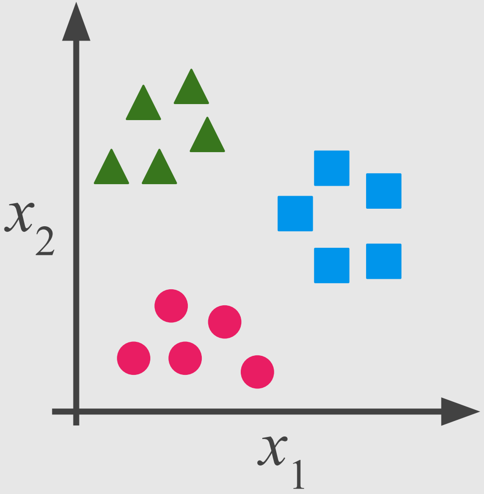
Class 2:

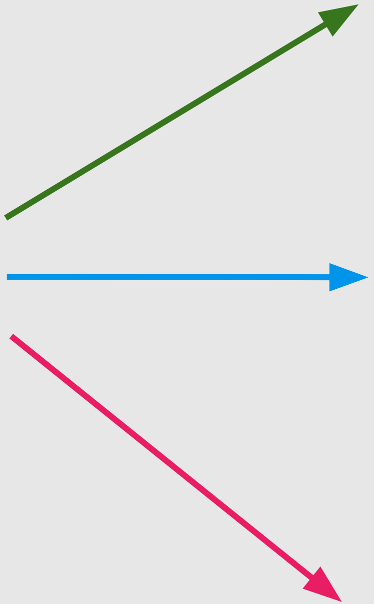
Class 3:

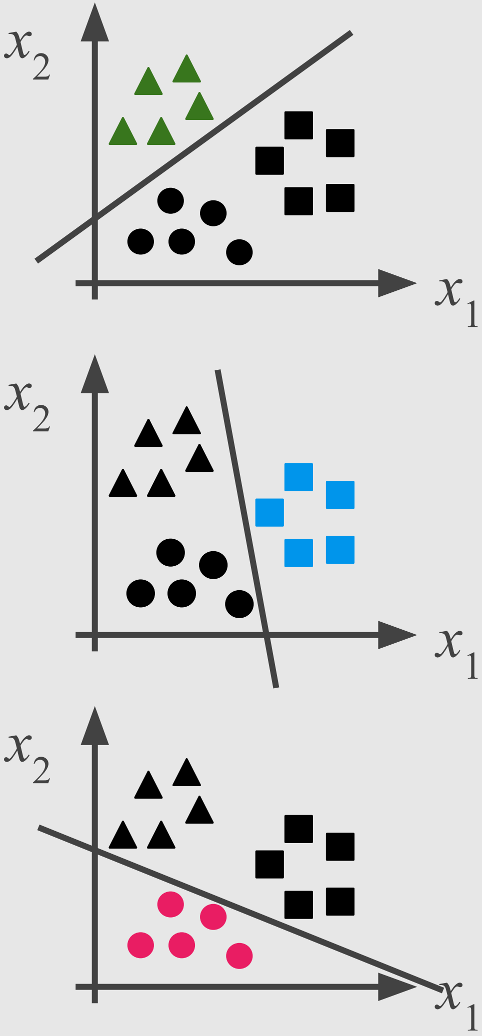
One-vs-All (One-vs-Rest)



One-vs-All (One-vs-Rest)







Class 1: ▲

Class 2: ■

Class 3: ●

$$h_\theta^{(i)}(x) = P(y=i \mid x;\theta) \quad (i=1,2,3)$$

One-vs-All (One-vs-Rest)

Train a logistic regression classifier $h_\theta^{(i)}(x)$ for each class i to predict the probability that $y = i$.

One a new input x, to make a prediction, pick the class i that maximizes

$$\max_i \; h_\theta^{(i)}(x)$$

References

Machine Learning Books

- Hands-On Machine Learning with Scikit-Learn and TensorFlow, Chap. 4
- Pattern Recognition and Machine Learning, Chap. 4

Machine Learning Courses

- https://www.coursera.org/learn/machine-learning, Week 3
- Logistic Regression — The Math of Intelligence (Week 2): https://youtu.be/D8alok2P468
- http://cs229.stanford.edu/notes/cs229-notes1.pdf

Logistic Regression — The Math of Intelligence (Week 2) by Siraj Raval https://youtu.be/D8alok2P468