Linear Regression

Machine Learning and Pattern Recognition

(Largely based on slides from Andrew Ng)

Prof. Sandra Avila

Institute of Computing (IC/Unicamp)

MC886/MO444, August 14, 2018

House Price Prediction

$ 70 000

https://www.youtube.com/watch?v=IpGxLWOIZy4

House Price Prediction

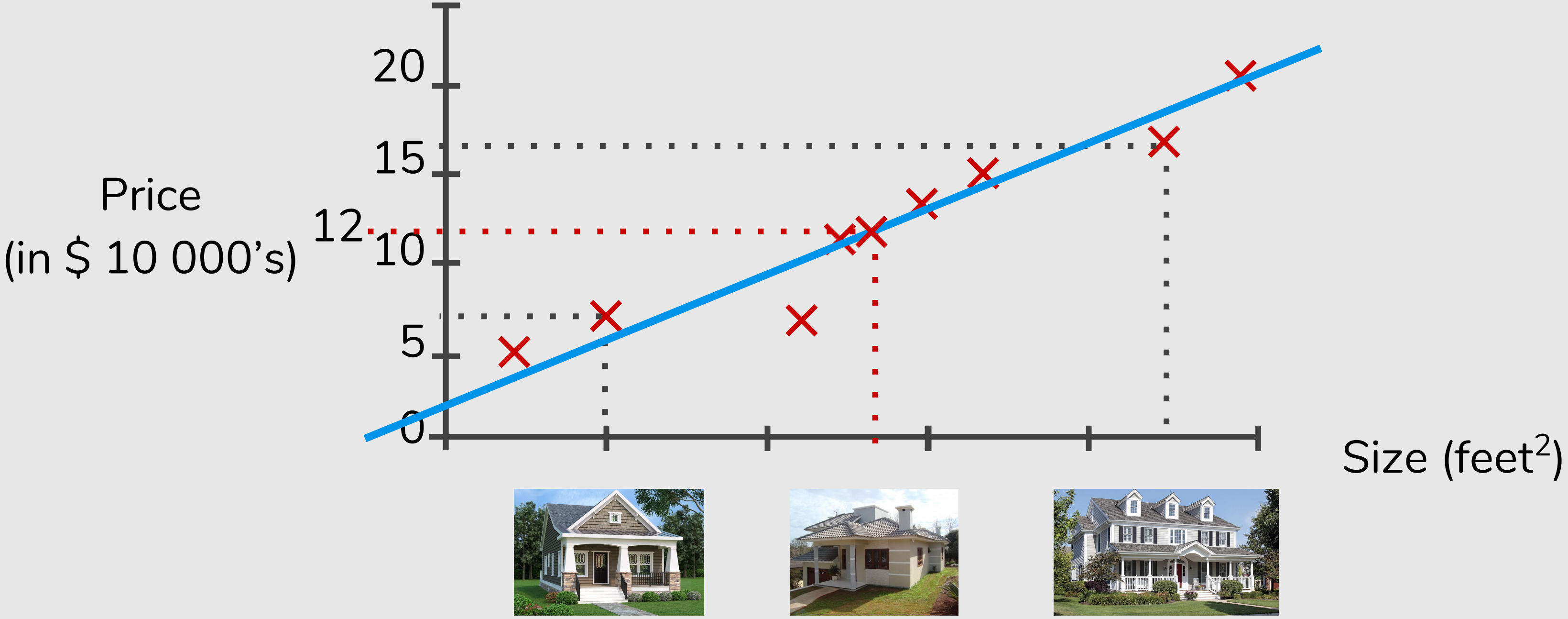
$ 160 000

House Price Prediction

???

House Price Prediction

Linear Regression



Today's Agenda

- **Linear Regression with One Variable**
    - **Model Representation**
    - **Cost Function**
    - **Gradient Descent**
- **Linear Regression with Multiple Variables**
    - **Gradient Descent for Multiple Variables**
    - Feature Scaling
    - Learning Rate
    - Features and Polynomial Regression
    - Normal Equation

Model Representation

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House Sales in King County, USA

Predict house price using regression

108

harlfoxem • last updated a year ago

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Tags: finance home small featured

Kernels

Feature Ranking w Random... run 2 days ago	55 votes
Step by Step House Price Pre... run 7 months ago	41 votes
House_Price_Prediction_Part_1 run a year ago	29 votes

Discussion

Variable explanation 6 days ago	15 replies
King County Geoclustering... 9 days ago	7 replies
RF, RFE, linear models|特征... 10 days ago	3 replies

Top Contributors

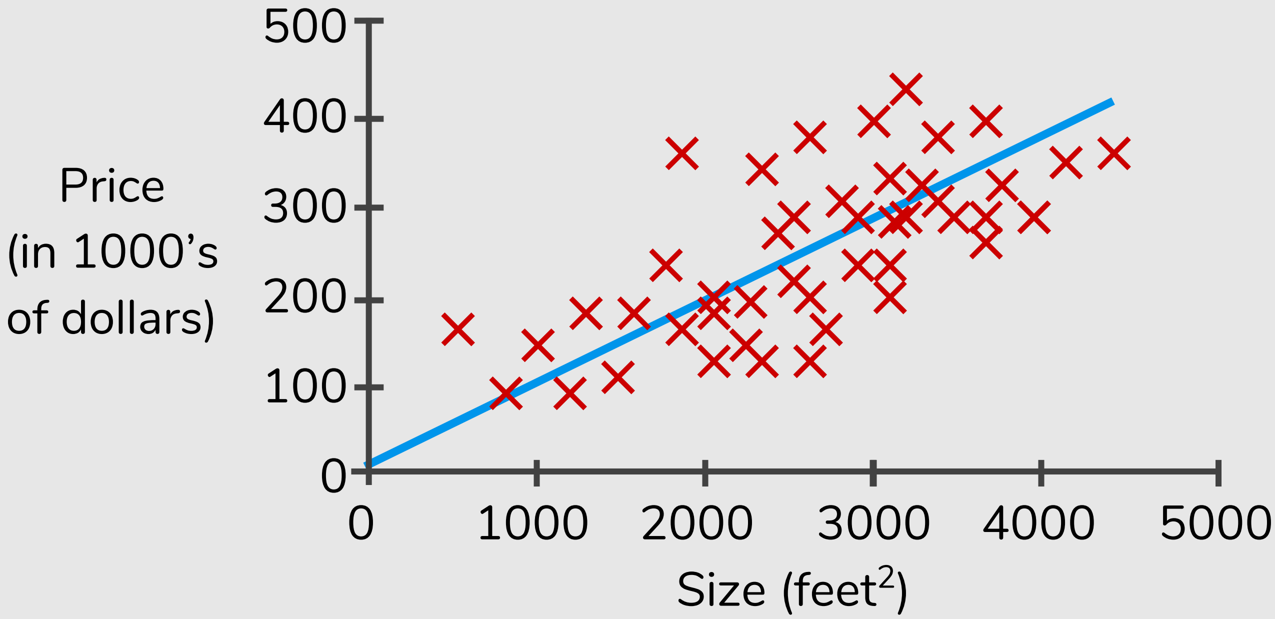
harlfoxem	1st
Anisotropic	2nd
ArmanUygur	3rd

Recent Activity

Jois Leonida Lobo	Ran version 10 of kernel **HousePricePrediction_SimpleLinearRegression**	13 hours ago
Anisotropic	Ran version 41 of kernel **Feature Ranking w RandomForest, RFE, linear models**	2 days ago
DavidTan	Commented on dataset discussion **Variable explanation**	6 days ago
Harsh Tyagi	Ran version 3 of kernel **King County's Housing Market, various techniques.**	8 days ago

https://www.kaggle.com/harlfoxem/housesalesprediction

Housing Prices



Supervised Learning

Given the "right answer" for each example in the data.

Regression Problem

Predict real-valued output

Training set of housing prices

Size in feet^2 (x)	Price ($) in 1000's ($y$)
2104	460
1416	232
1534	315
852	178
...	...

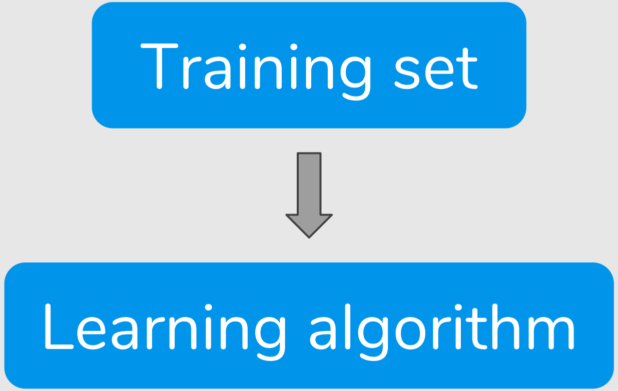
Notation:

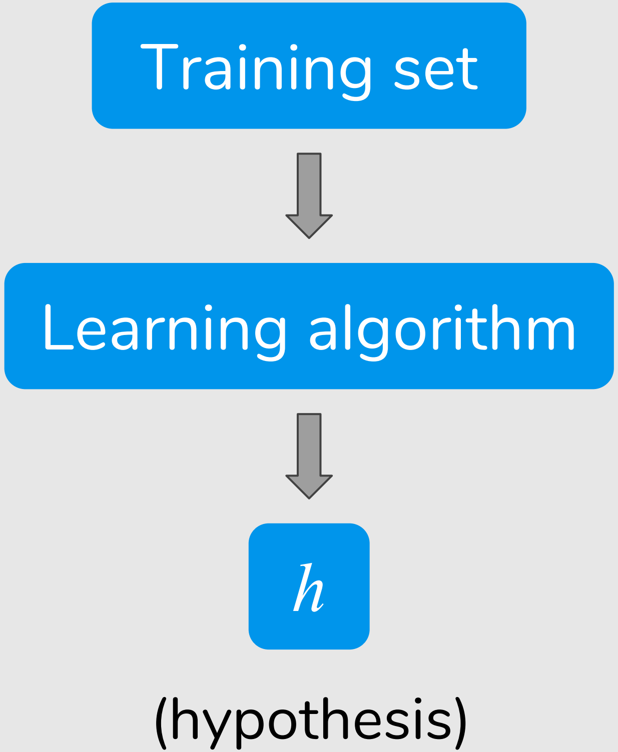
m = Number of training examples

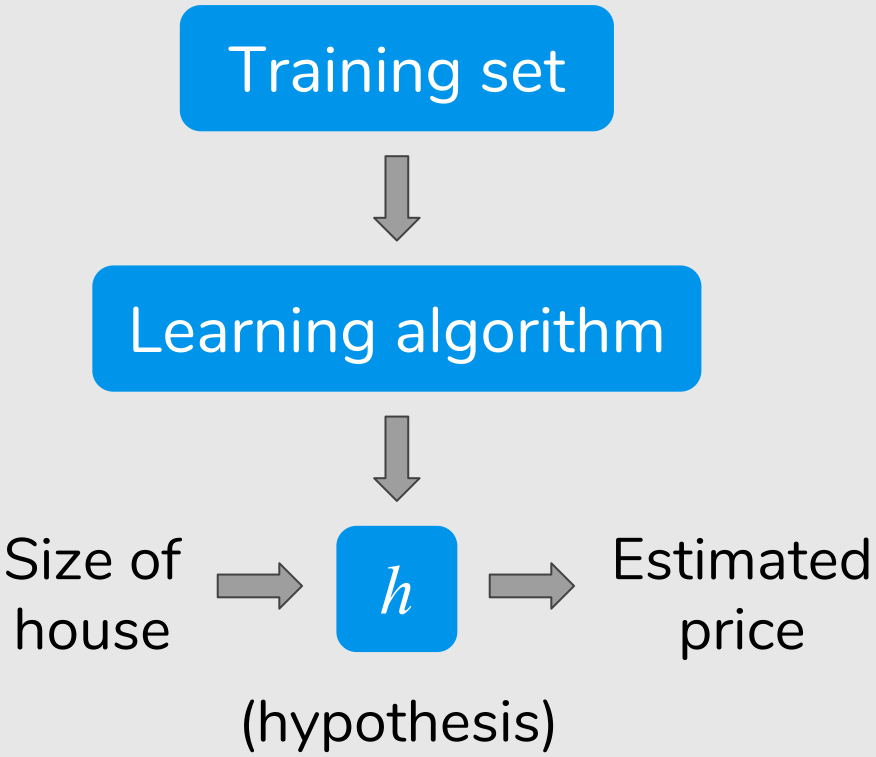
x's = "input" variable / features

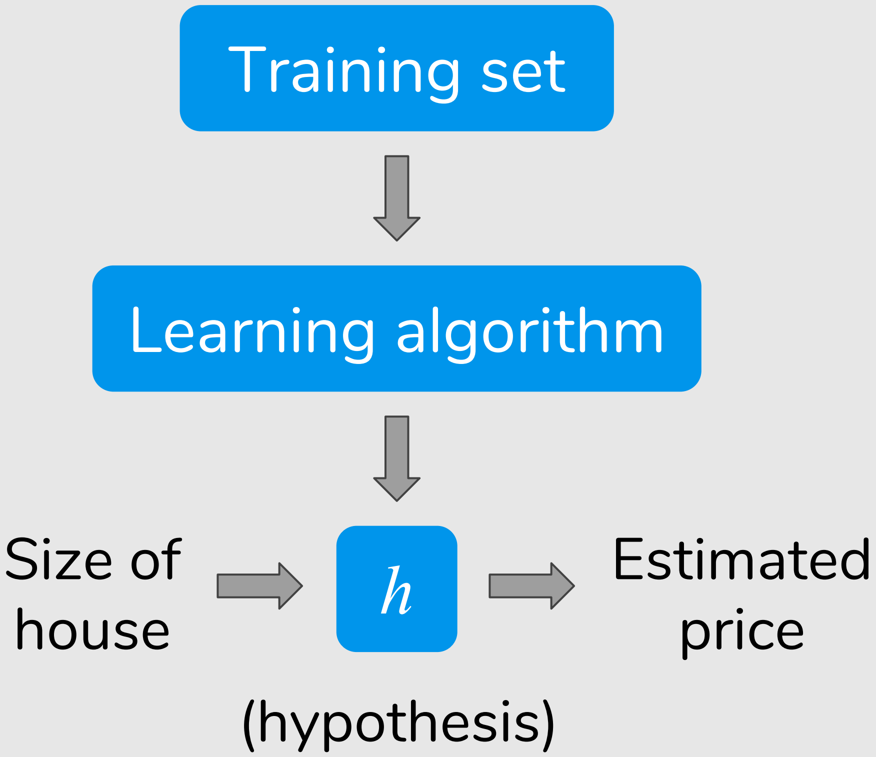
y's = "output" variable / "target" variable

Training set


Training set
Learning algorithm

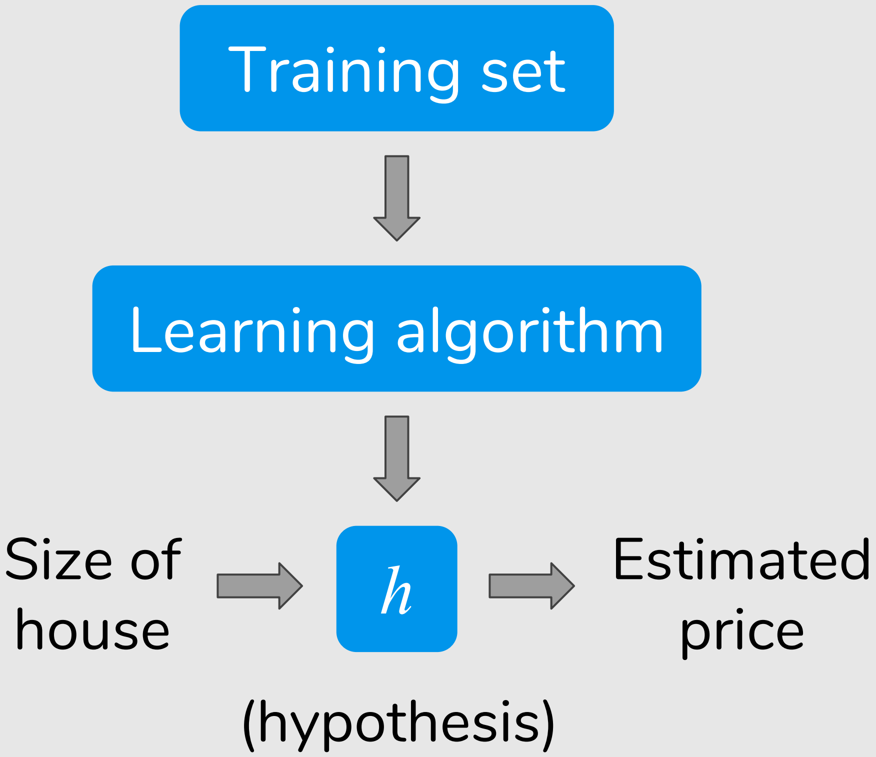

Training set
Learning algorithm
h
(hypothesis)


Training set
Learning algorithm
Size of house
h
(hypothesis)
Estimated price



h maps x's to y's

How do we represent h ?



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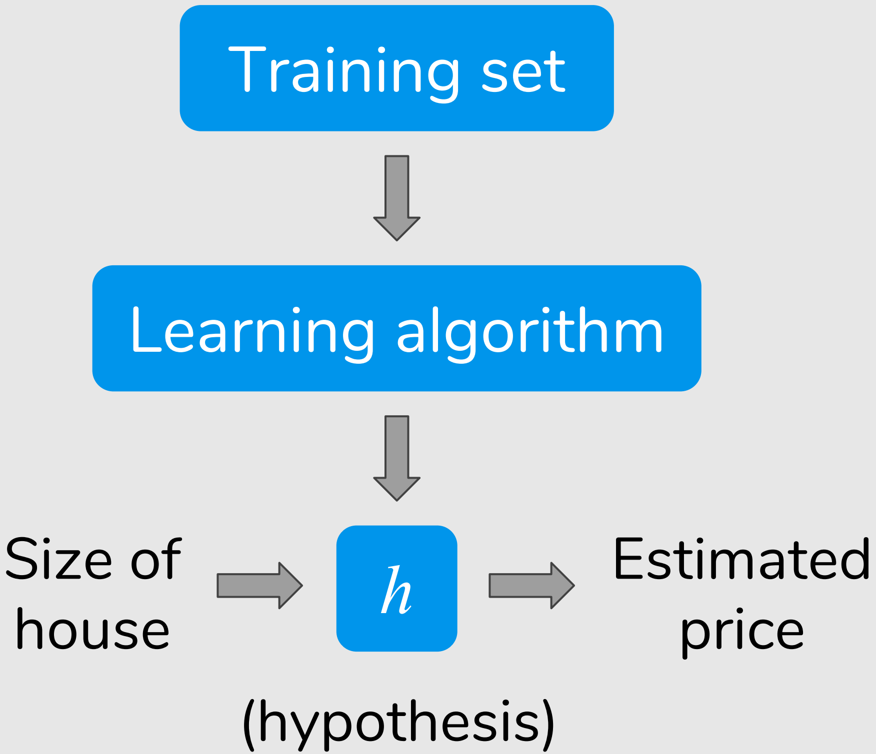
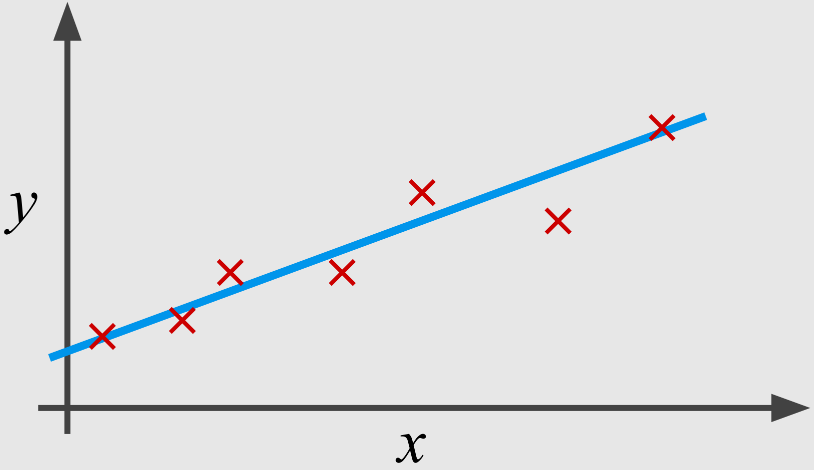


h maps x's to y's

How do we represent h ?

$$h_\theta(x) = \theta_0 + \theta_1 x$$



How do we represent h ?

Training set

↓

Learning algorithm

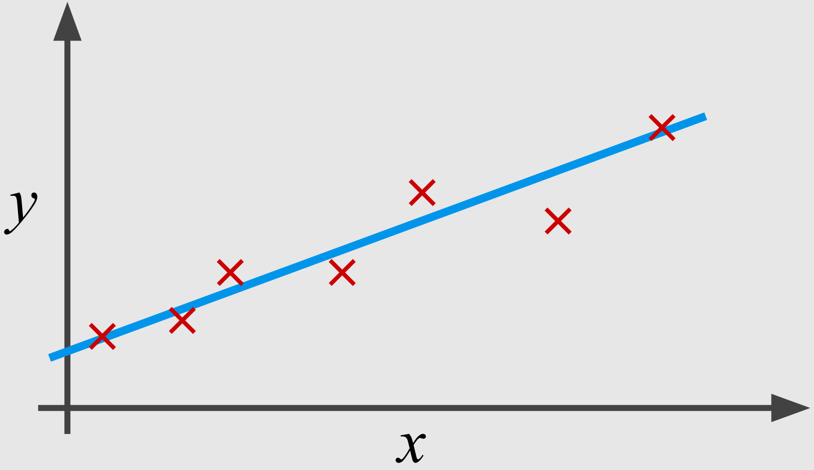
↓

Size of house → h → Estimated price

(hypothesis)

h maps x's to y's

$$h_\theta(x) = \theta_0 + \theta_1 x$$



Linear regression with one variable.

Univariate linear regression.

Cost Function

Training Set

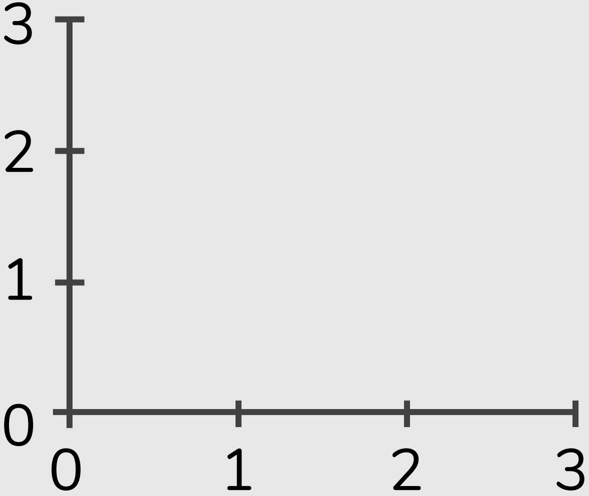
Size in feet^2 (x)	**Price $(\$)$ in 1000's (y)**
2104	460
1416	232
1534	315
852	178
...	...

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

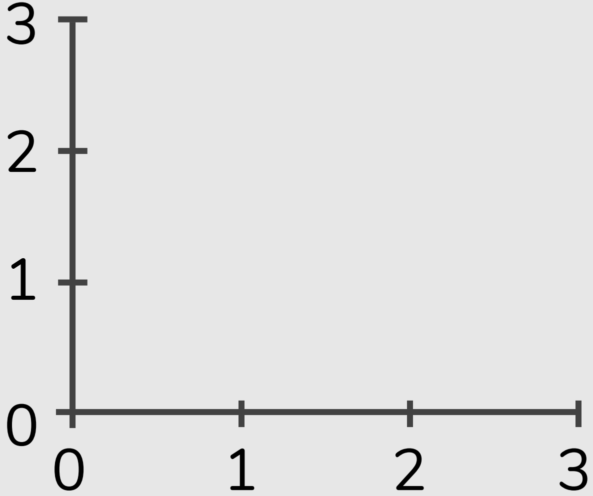
θi's: Parameters

How to choose θi's ?

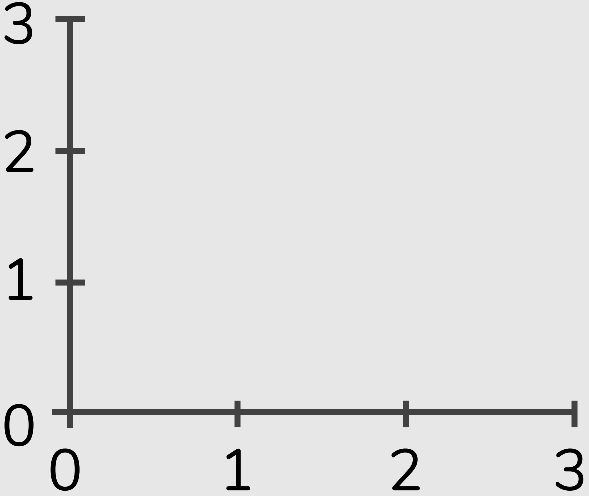
$$h_\theta(x) = \theta_0 + \theta_1 x$$



$\theta_0 = 1.5$
$\theta_1 = 0$

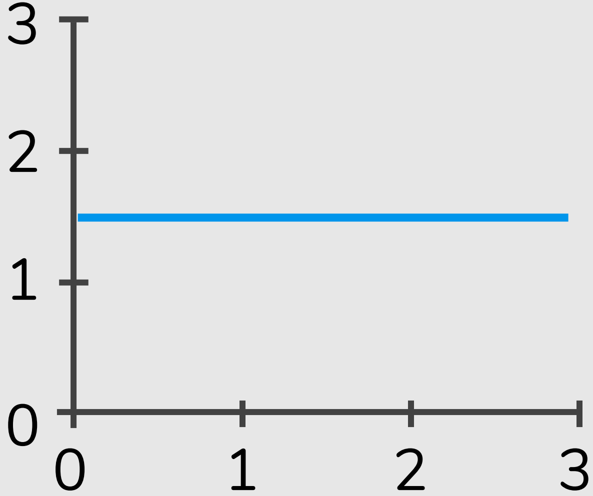


$\theta_0 = 0$
$\theta_1 = 0.5$

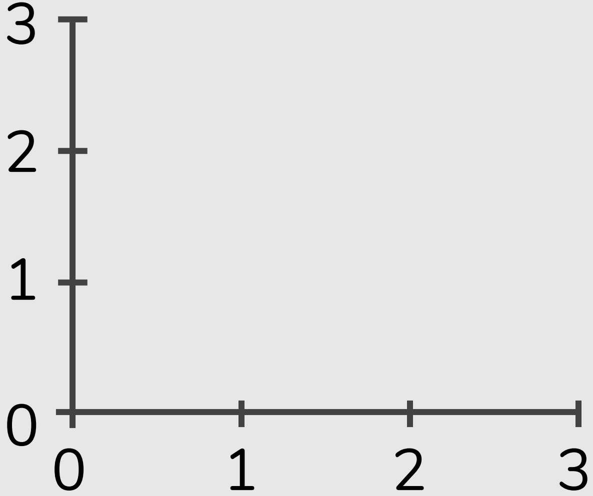


$\theta_0 = 1$
$\theta_1 = 0.5$

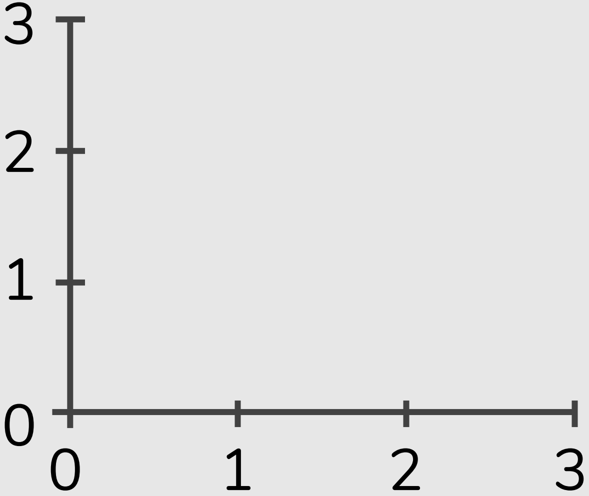
$$h_\theta(x) = \theta_0 + \theta_1 x$$



$$\theta_0 = 1.5$$
$$\theta_1 = 0$$

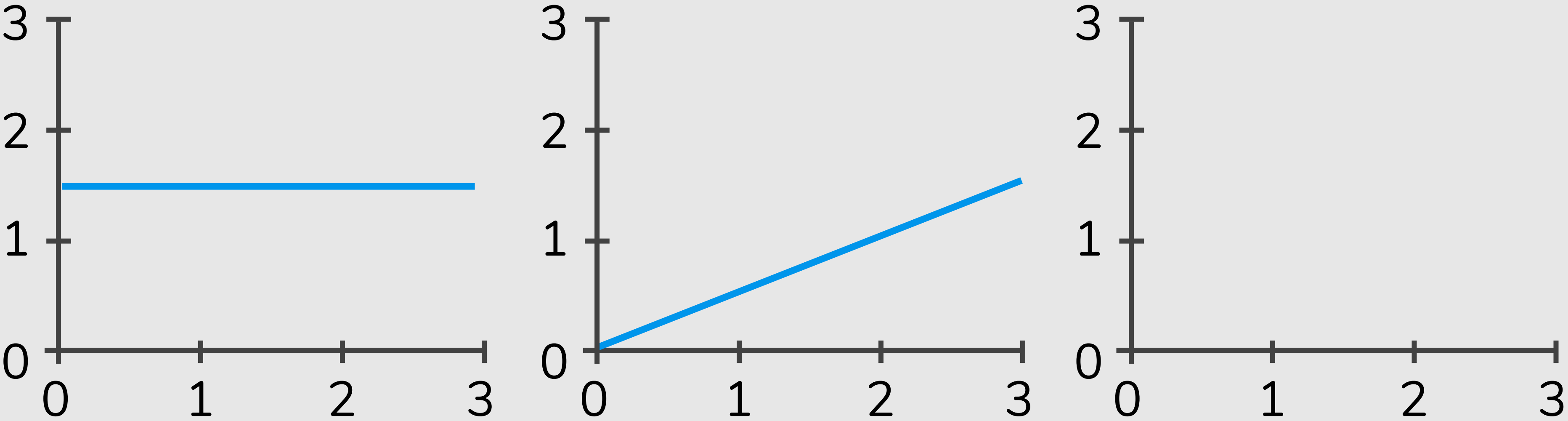


$$\theta_0 = 0$$
$$\theta_1 = 0.5$$



$$\theta_0 = 1$$
$$\theta_1 = 0.5$$

$$h_\theta(x) = \theta_0 + \theta_1 x$$



$\theta_0 = 1.5$
$\theta_1 = 0$

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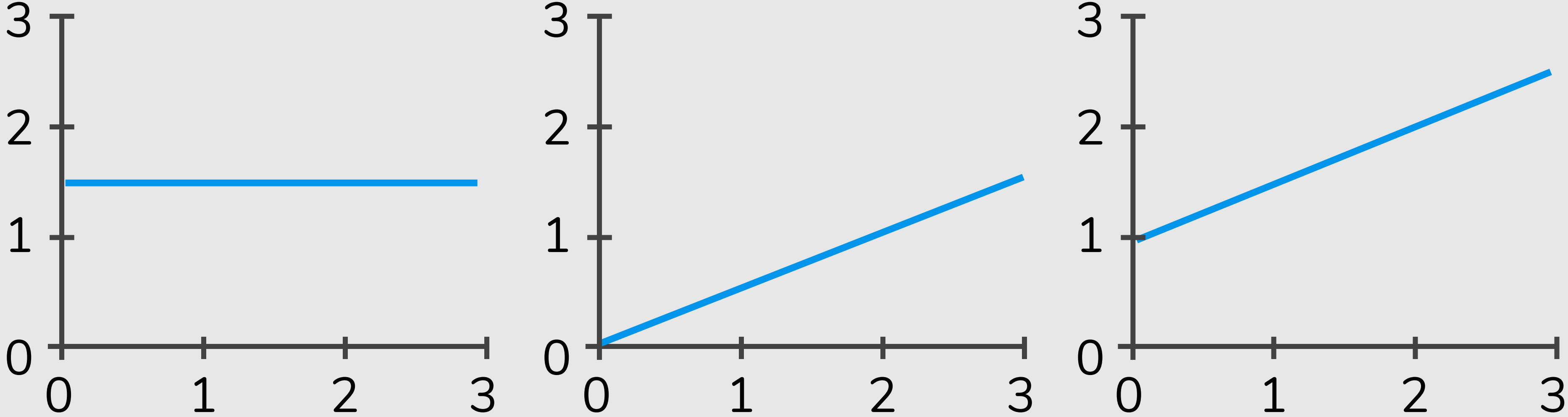
$\theta_1 = 0.5$

$\theta_0 = 1$
$\theta_1 = 0.5$

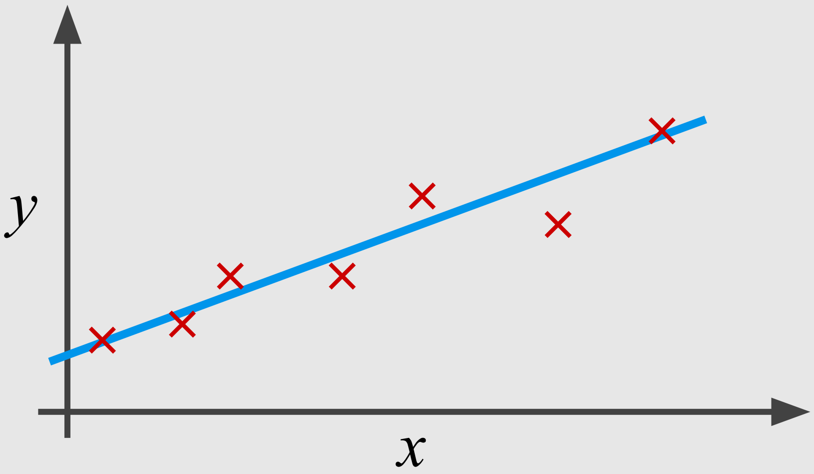
$$h_\theta(x) = \theta_0 + \theta_1 x$$



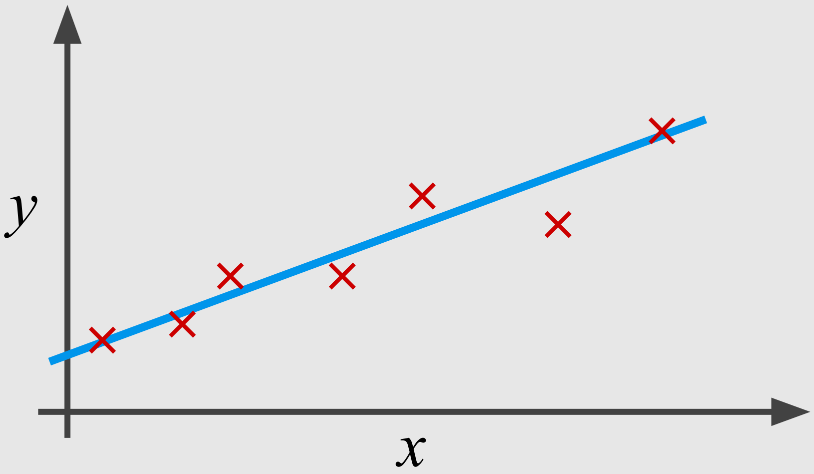
$\theta_0 = 1.5$
$\theta_1 = 0$

$\theta_0 = 0$
$\theta_1 = 0.5$

$\theta_0 = 1$
$\theta_1 = 0.5$

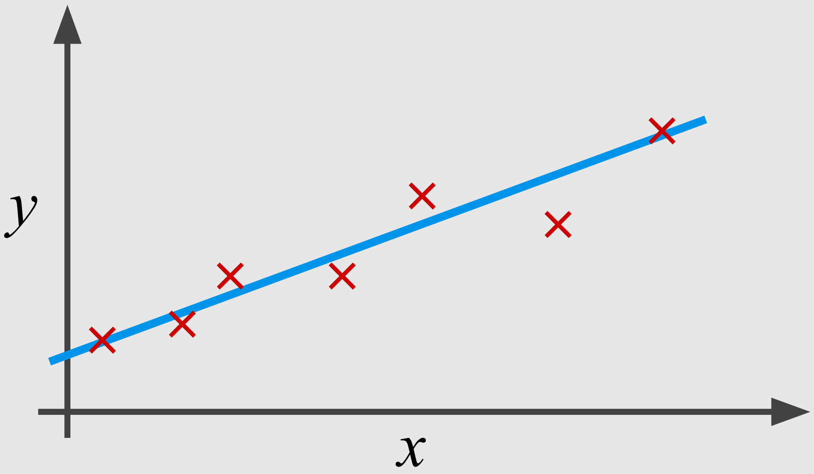


Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



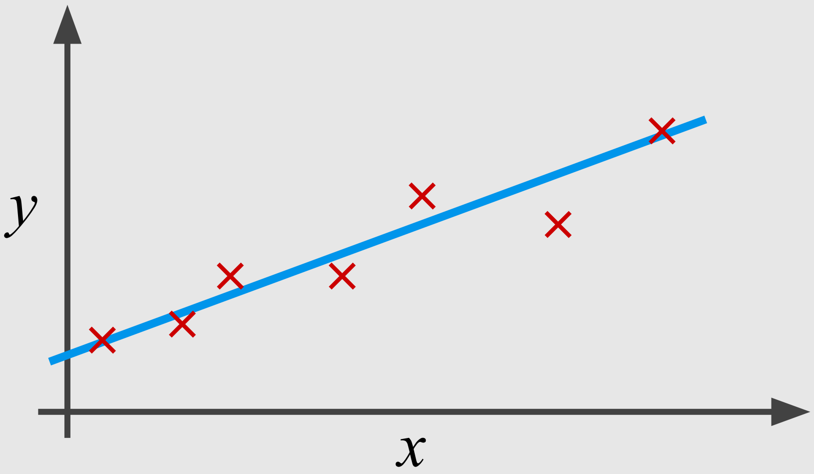
$$\underset{\theta_0,\theta_1}{\text{minimize}}$$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



$$\underset{\theta_0,\theta_1}{\text{minimize}} \qquad (h_\theta(x \quad) - y \quad)^2$$

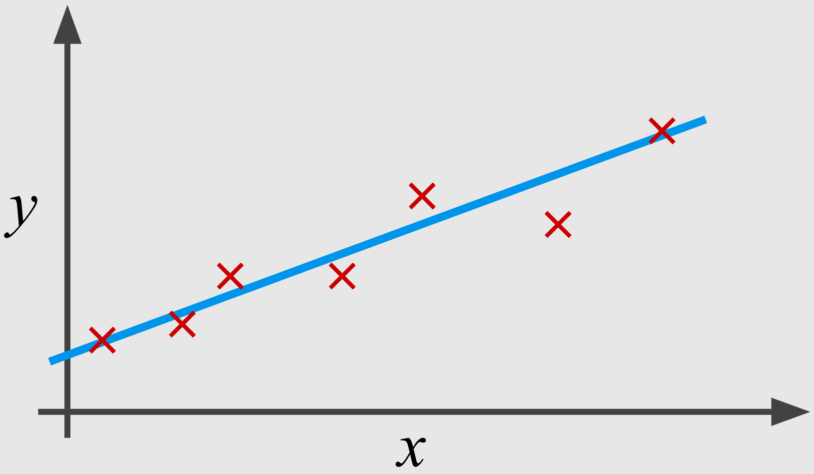
Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



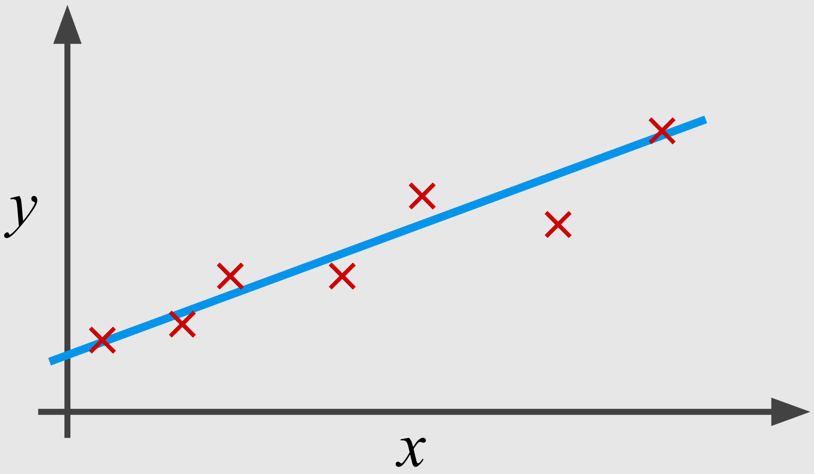
$$\underset{\theta_0,\theta_1}{\text{minimize}} \quad \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)

$$\underset{\theta_0,\theta_1}{\text{minimize}} \ \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$



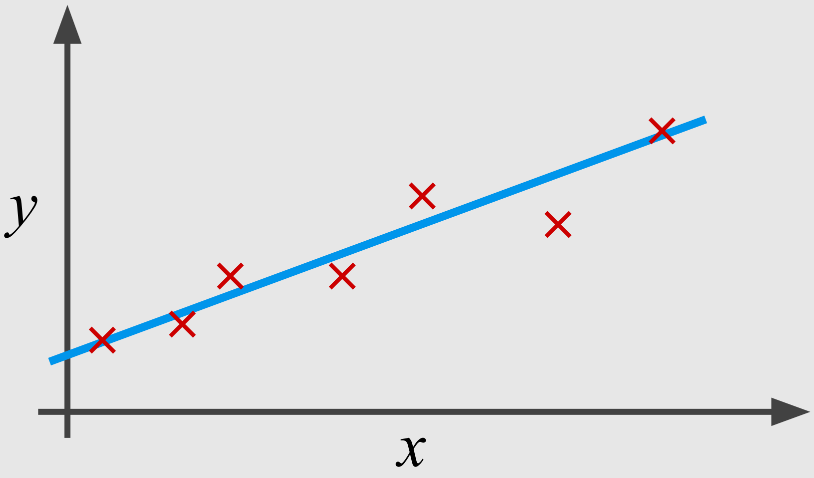
Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



$$\underset{\theta_0,\theta_1}{\text{minimize}} \ \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



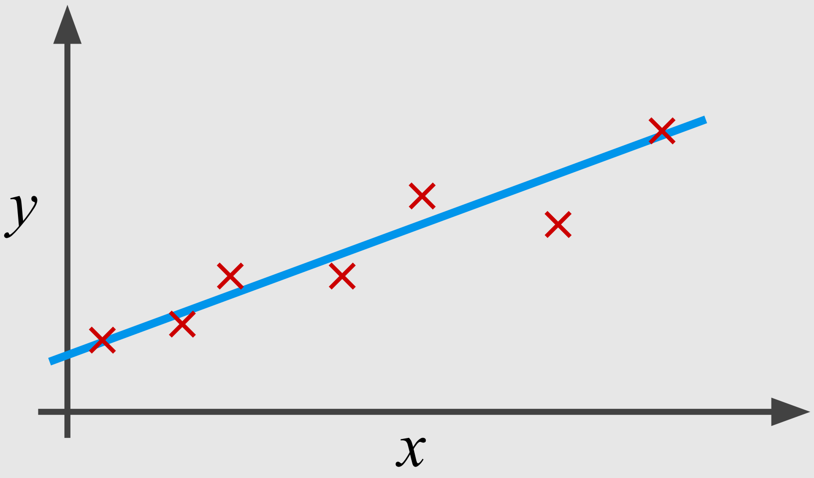
$$\underset{\theta_0,\theta_1}{\text{minimize}} \ \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\Downarrow$$

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0,\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)



Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ close to y for our training examples (x,y)

$$\underset{\theta_0,\theta_1}{\text{minimize}} \; \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\underset{\theta_0,\theta_1}{\text{minimize}} \; J(\theta_0,\theta_1)$$

Cost function
(Squared error function)

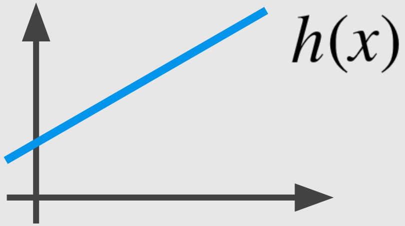
Cost Function Intuition I

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

$$\theta_0 , \theta_1$$



Cost Function:

$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

Goal:

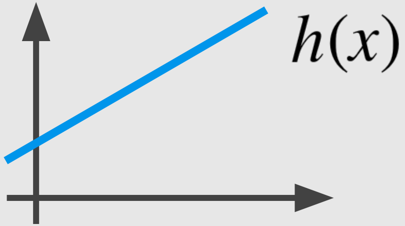
$$\underset{\theta_0,\theta_1}{\text{minimize}}\, J(\theta_0,\theta_1)$$

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

$$\theta_0 , \theta_1$$



Cost Function:

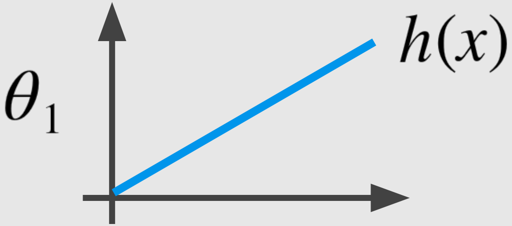
$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

Goal:

$$\underset{\theta_0,\theta_1}{\text{minimize}}\ J(\theta_0,\theta_1)$$

Simplified

$$h_\theta(x) = \theta_1 x$$



$$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

$$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$$

$h_\theta(x)$

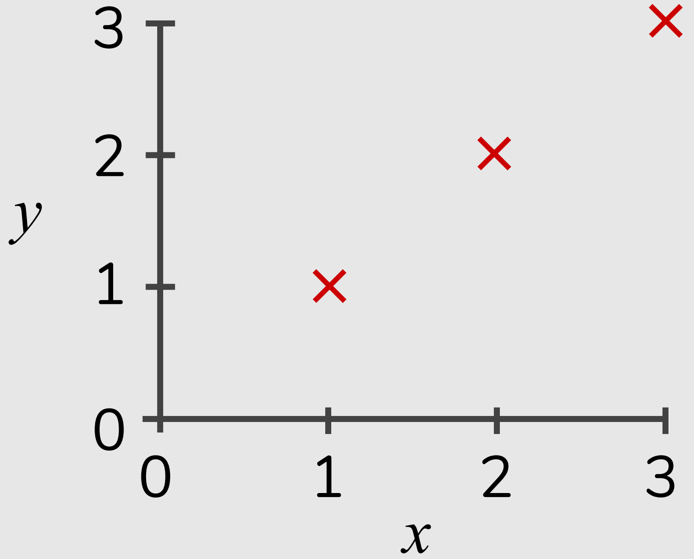
(for fixed θ_1, this is a function of x)

$J(\theta_1)$

(function of the parameters θ_1)

$h_\theta(x)$

(for fixed θ_1, this is a function of x)

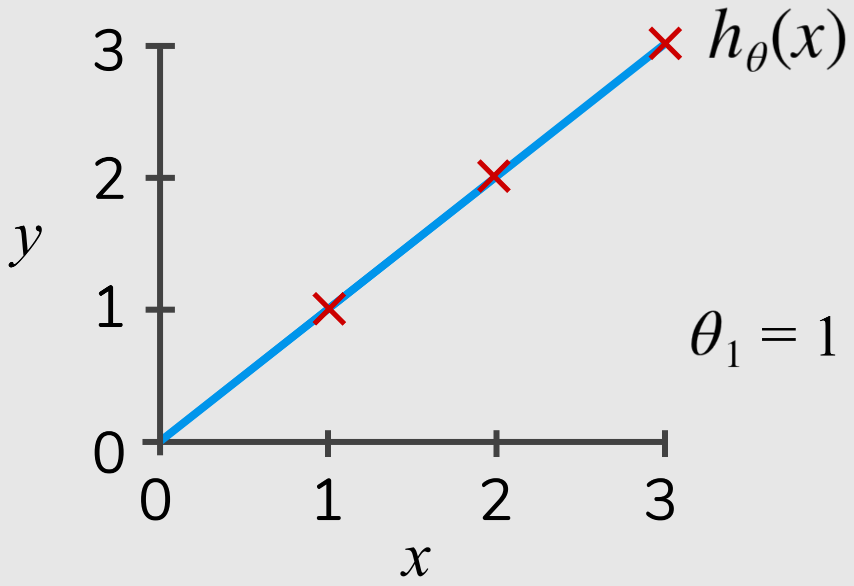


$J(\theta_1)$

(function of the parameters θ_1)

$h_\theta(x)$

(for fixed θ_1, this is a function of x)



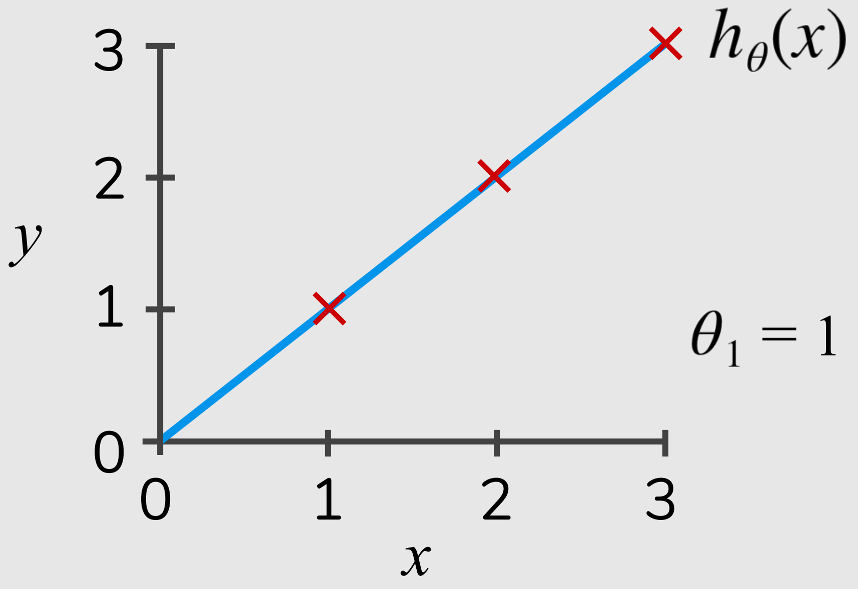
$J(\theta_1) = J(1) = ?$

$J(\theta_1)$

(function of the parameters θ_1)

$h_\theta(x)$

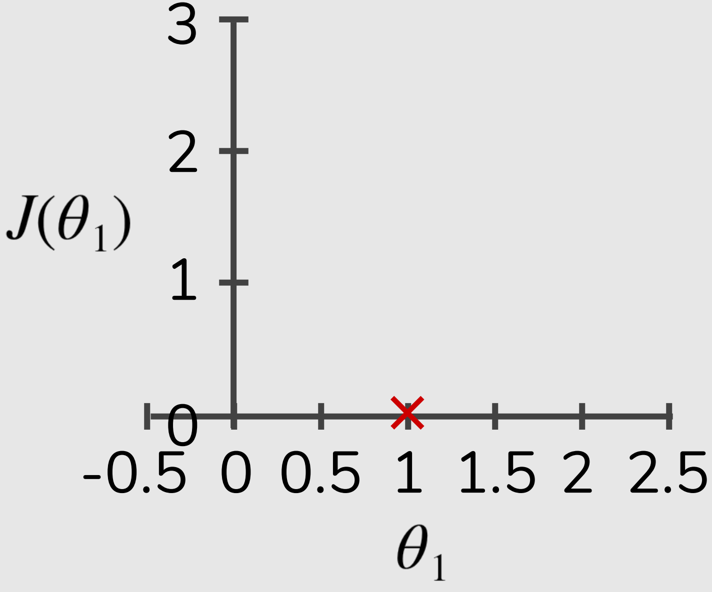
(for fixed θ_1, this is a function of x)



$J(\theta_1) = J(1) = 0$

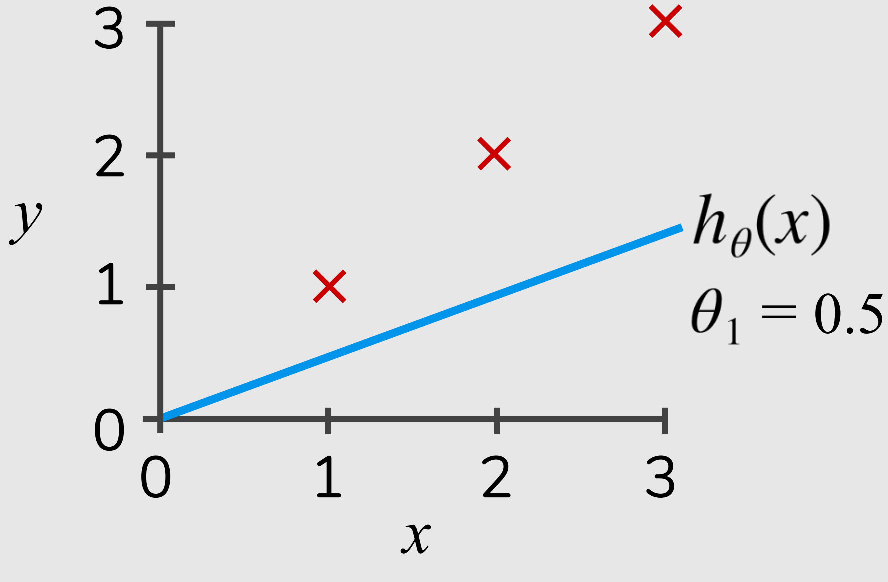
$J(\theta_1)$

(function of the parameters θ_1)



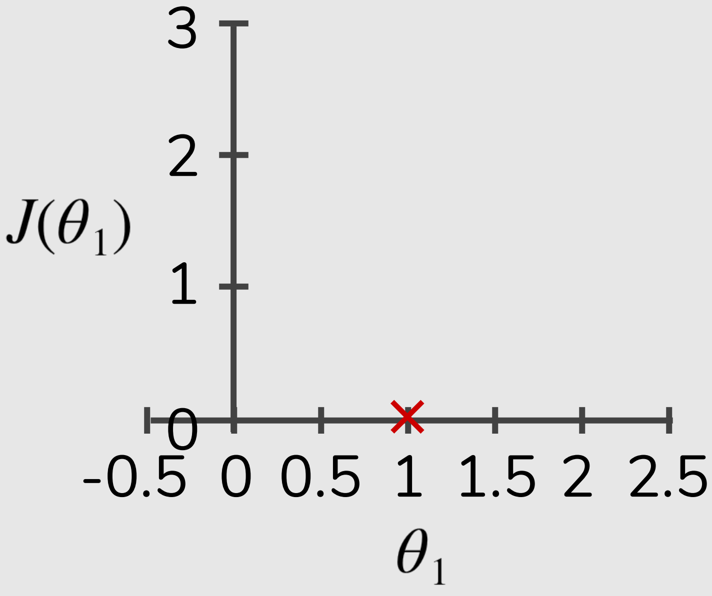
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



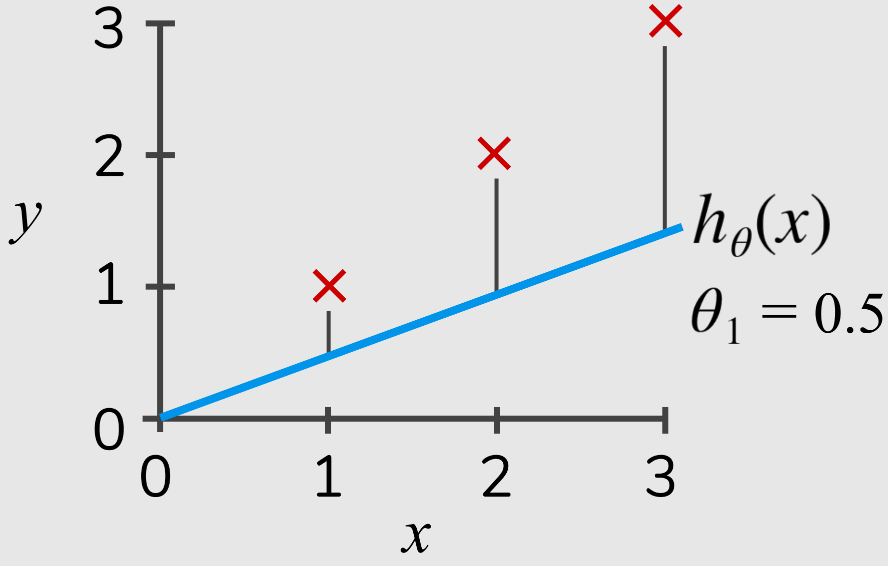
$J(\theta_1)$

(function of the parameters θ_1)



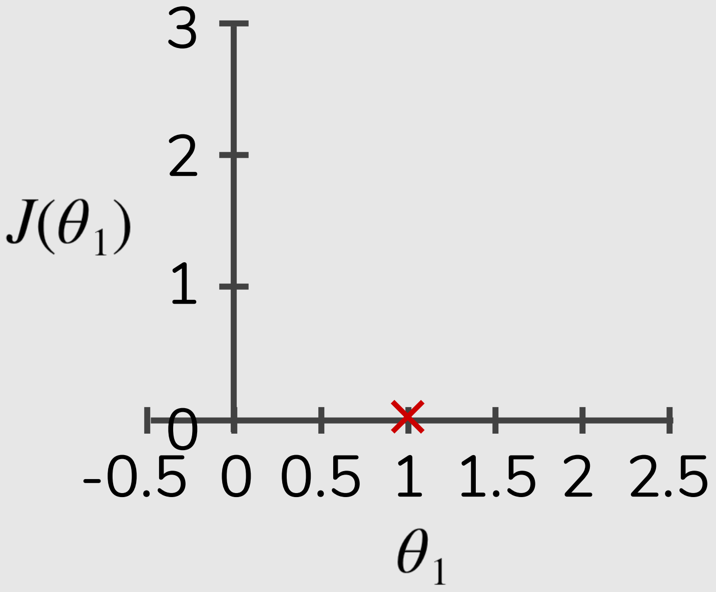
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



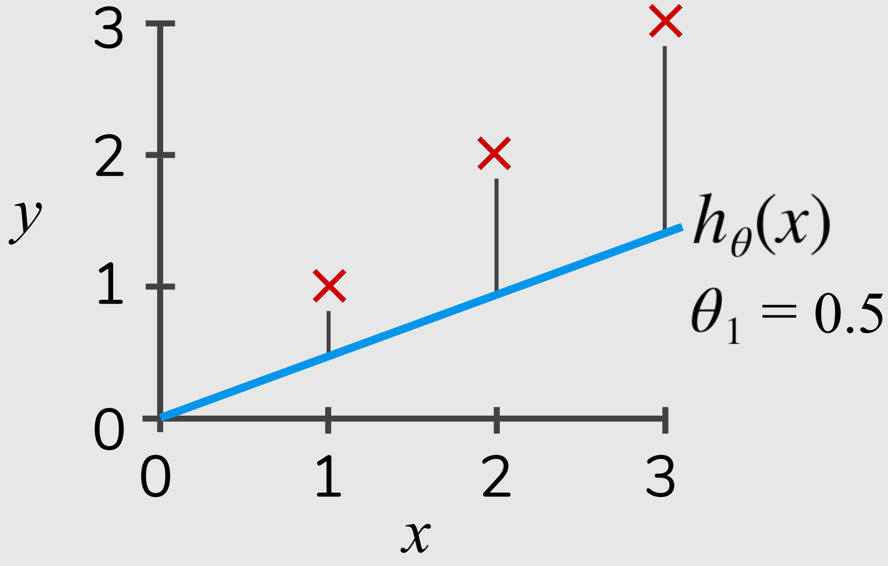
$J(\theta_1)$

(function of the parameters θ_1)



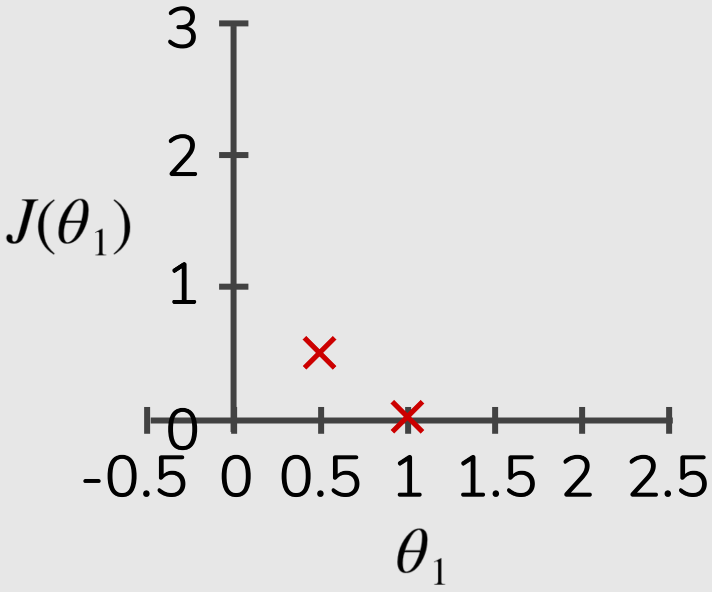
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



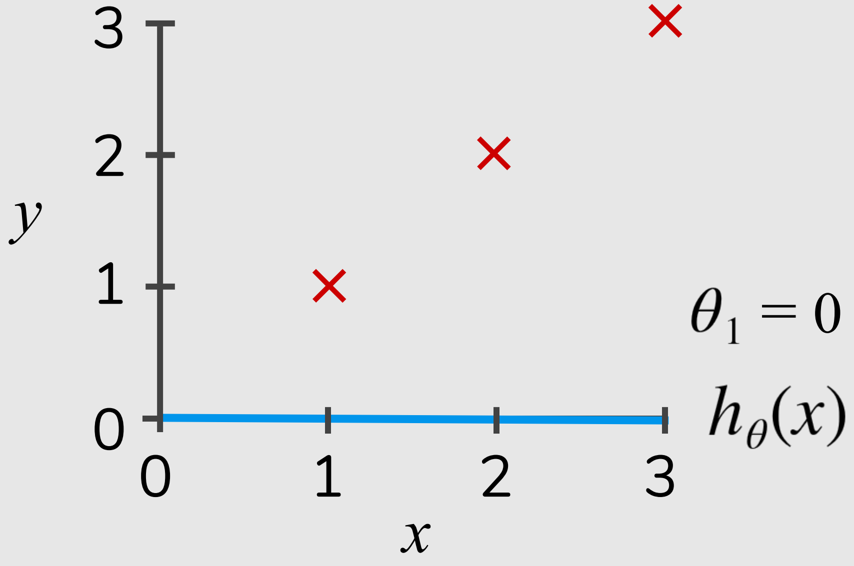
$J(\theta_1)$

(function of the parameters θ_1)



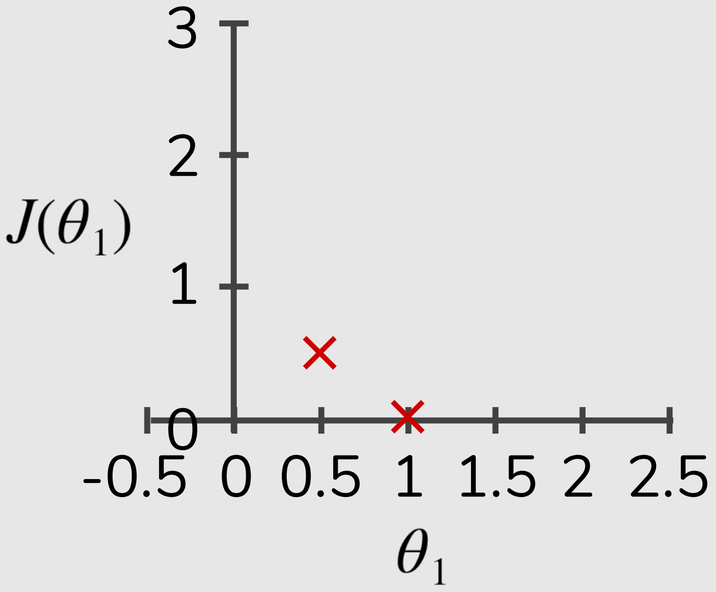
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



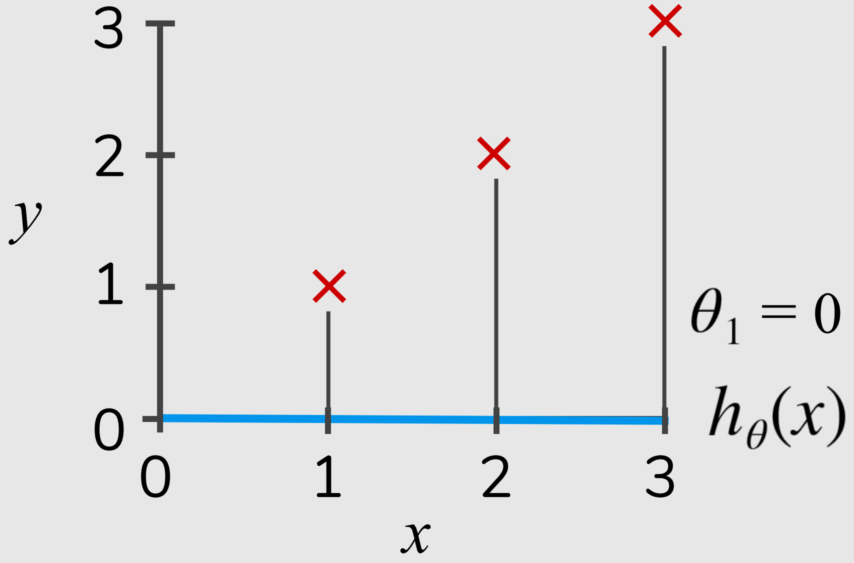
$J(\theta_1)$

(function of the parameters θ_1)



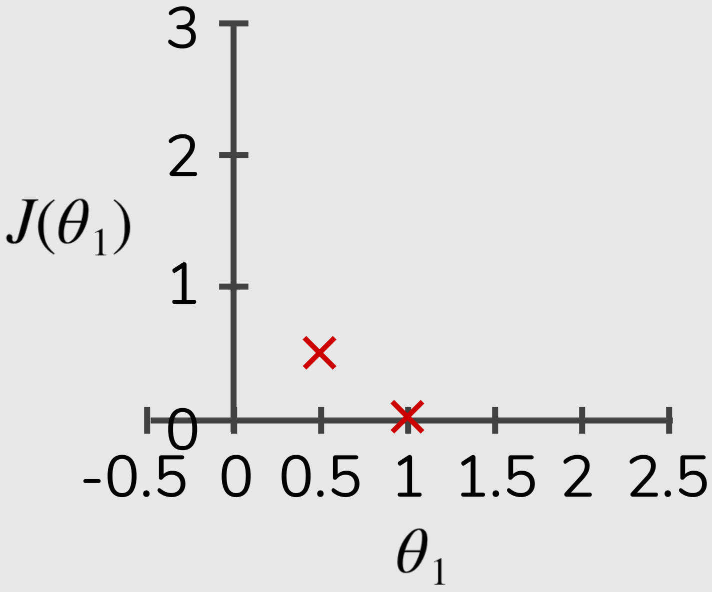
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



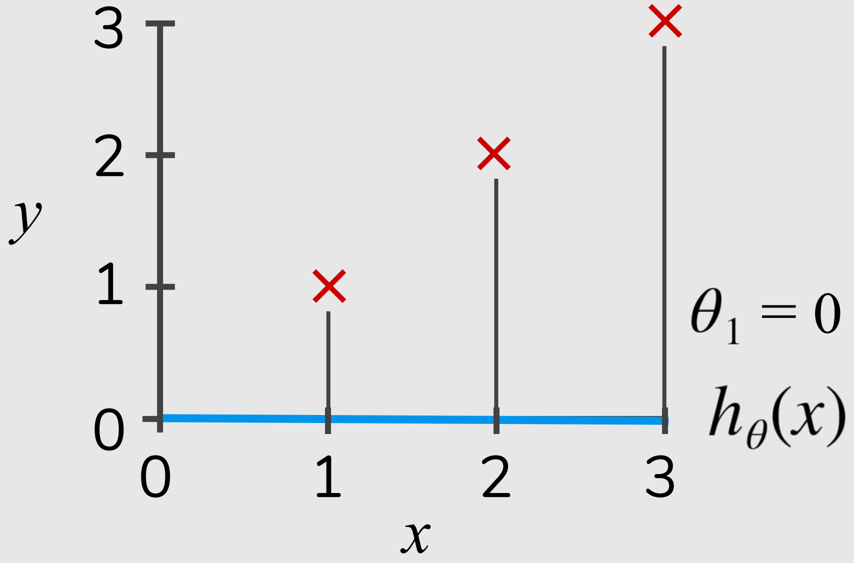
$J(\theta_1)$

(function of the parameters θ_1)



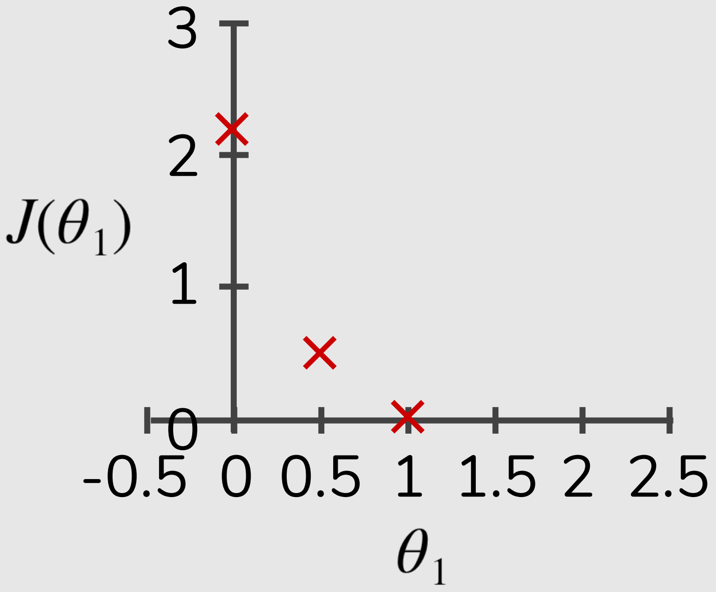
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



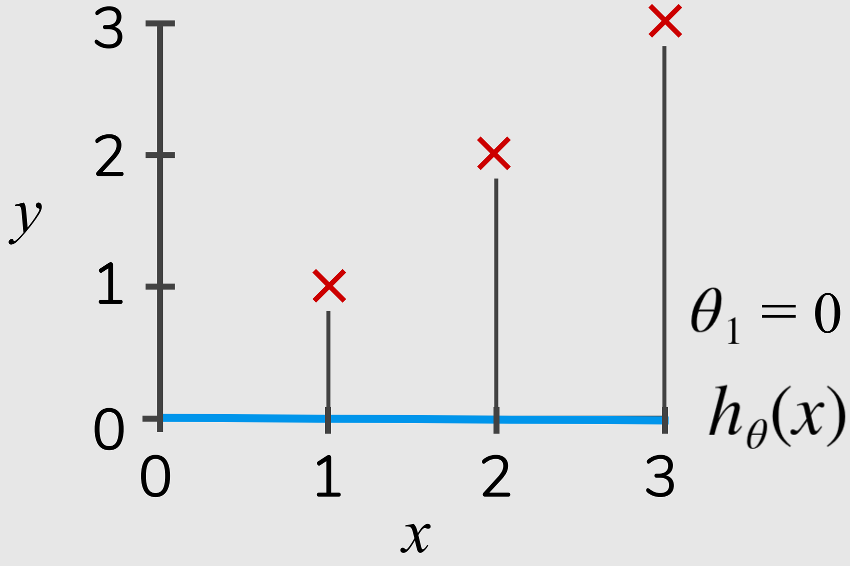
$J(\theta_1)$

(function of the parameters θ_1)



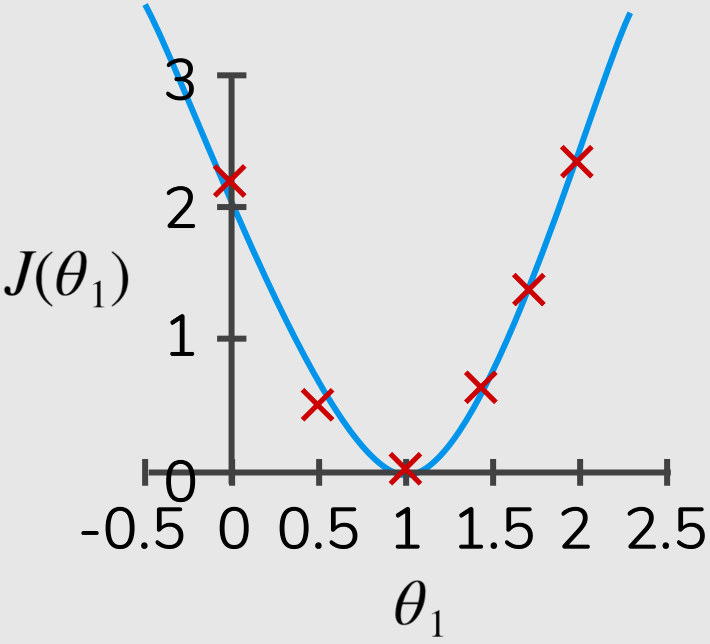
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameters θ_1)



Cost Function Intuition II

$h_\theta(x)$

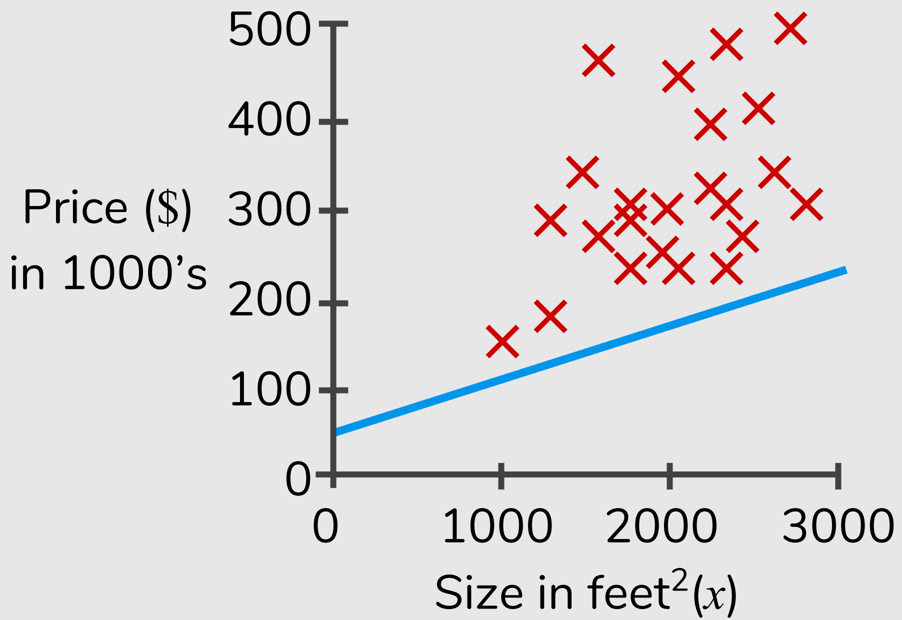
(for fixed θ_0, θ_1, this is a function of x)

$J(\theta_0,\theta_1)$

(function of the parameters θ_0, θ_1)

$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$h_\theta(x) = 50 + 0.06x$

$\theta_0 = 50$

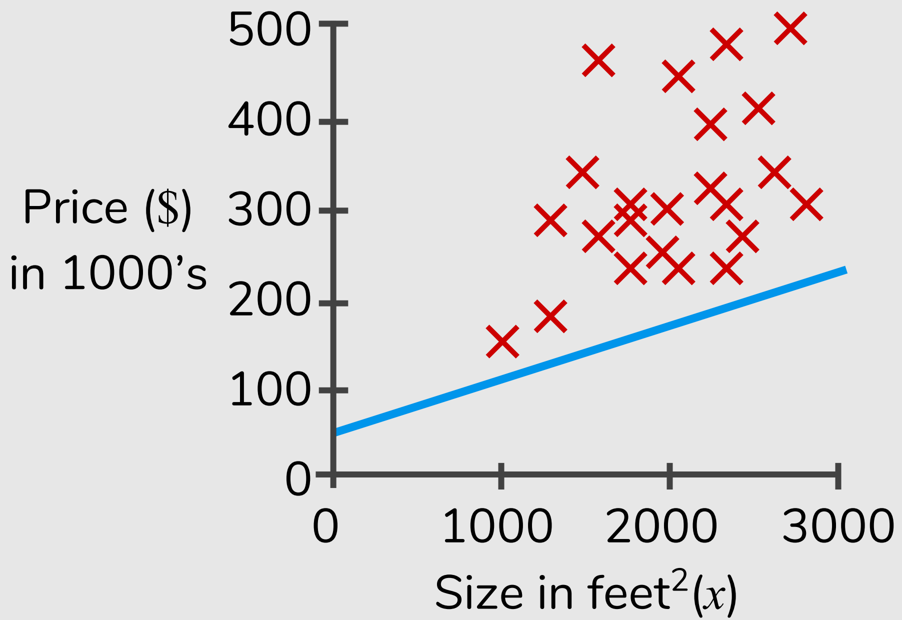
$\theta_1 = 0.06$

$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)

$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



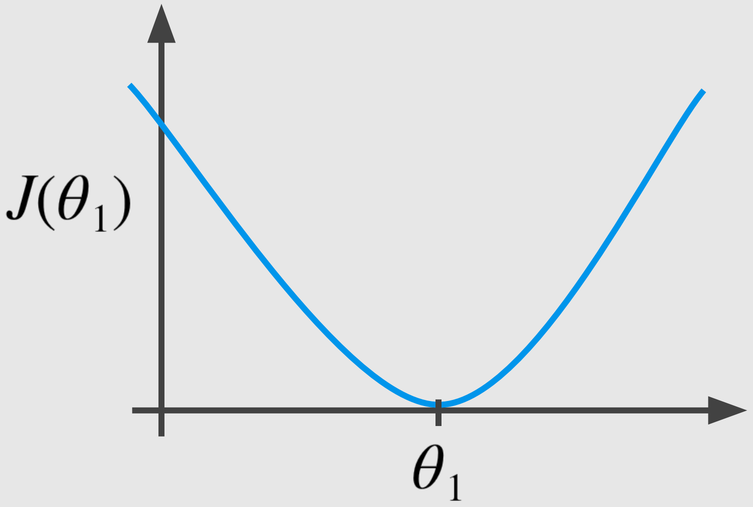
$$h_\theta(x) = 50 + 0.06x$$

$\theta_0 = 50$

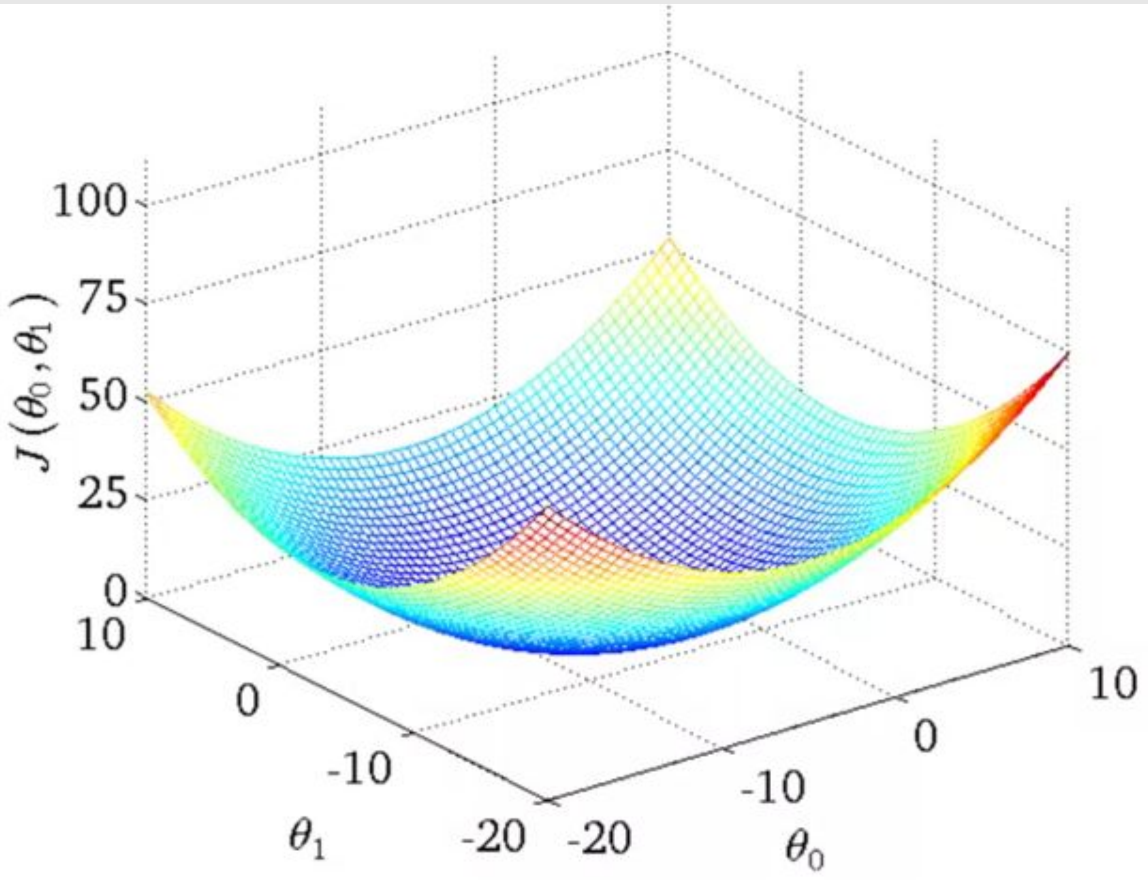
$\theta_1 = 0.06$

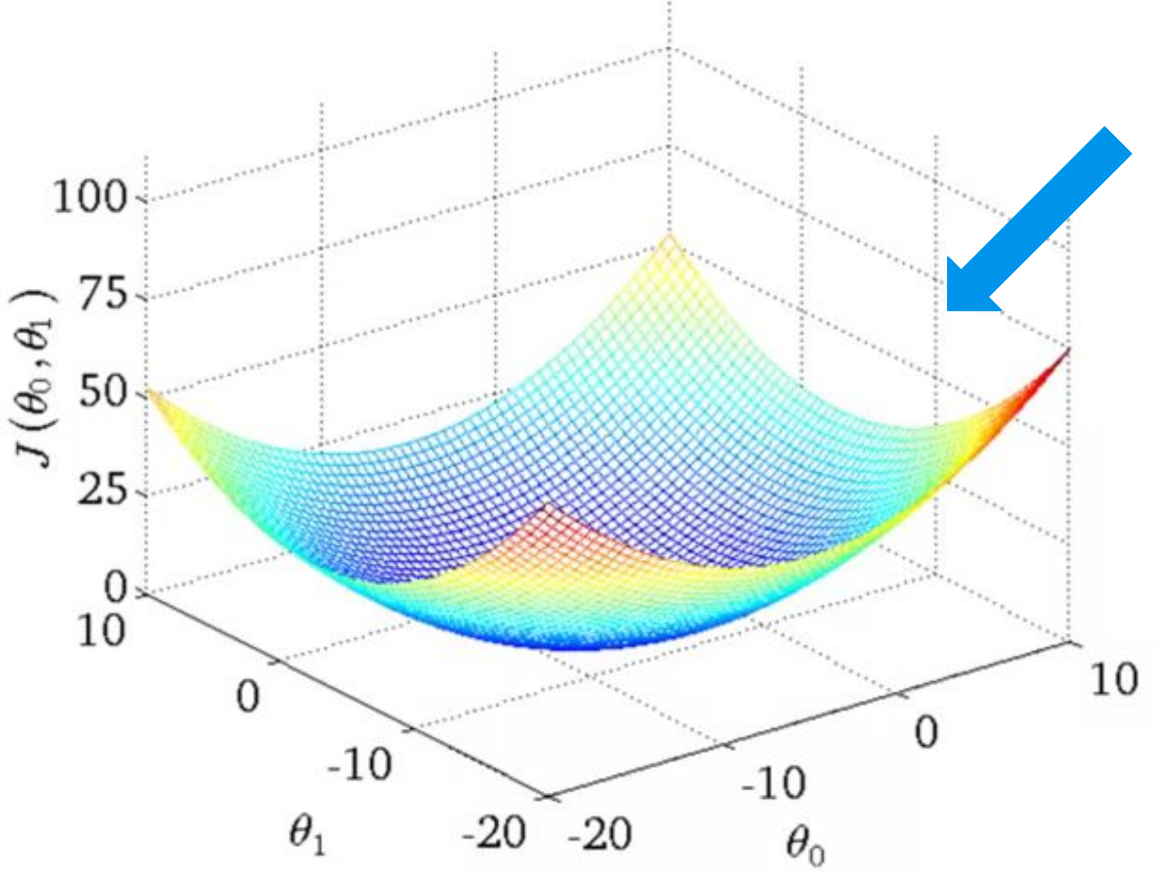
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



θ_0 and θ_1?

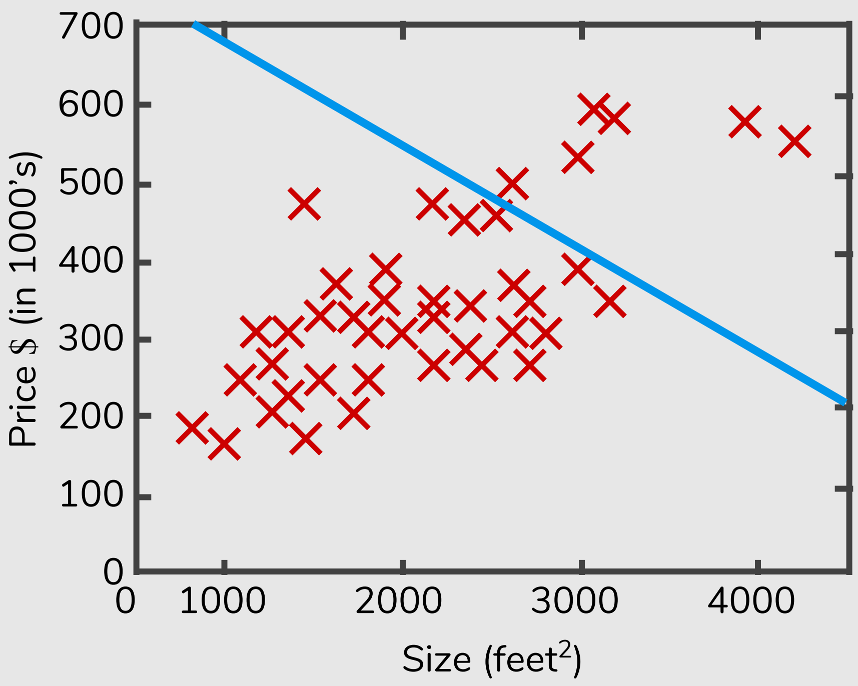

100
75
50
25
0
$J(\theta_0, \theta_1)$
10
0
-10
-20
θ_1
-20
-10
0
10
θ_0



Convex Function

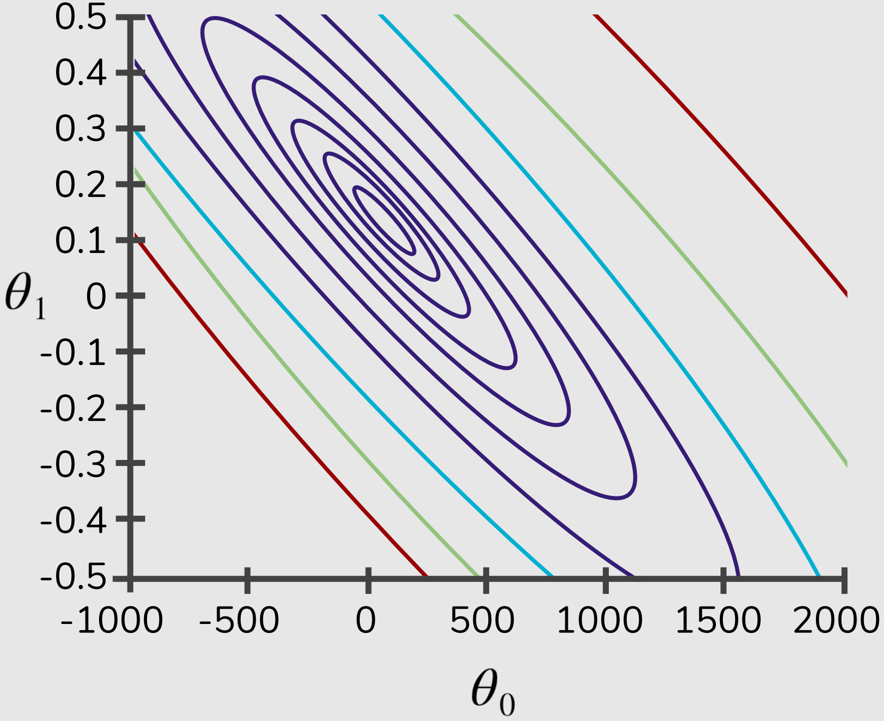
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



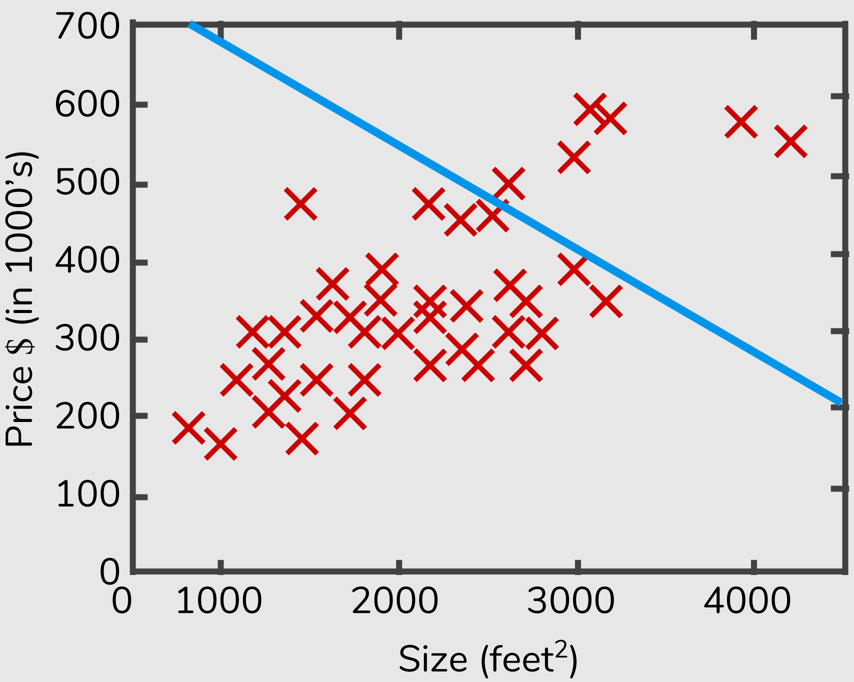
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



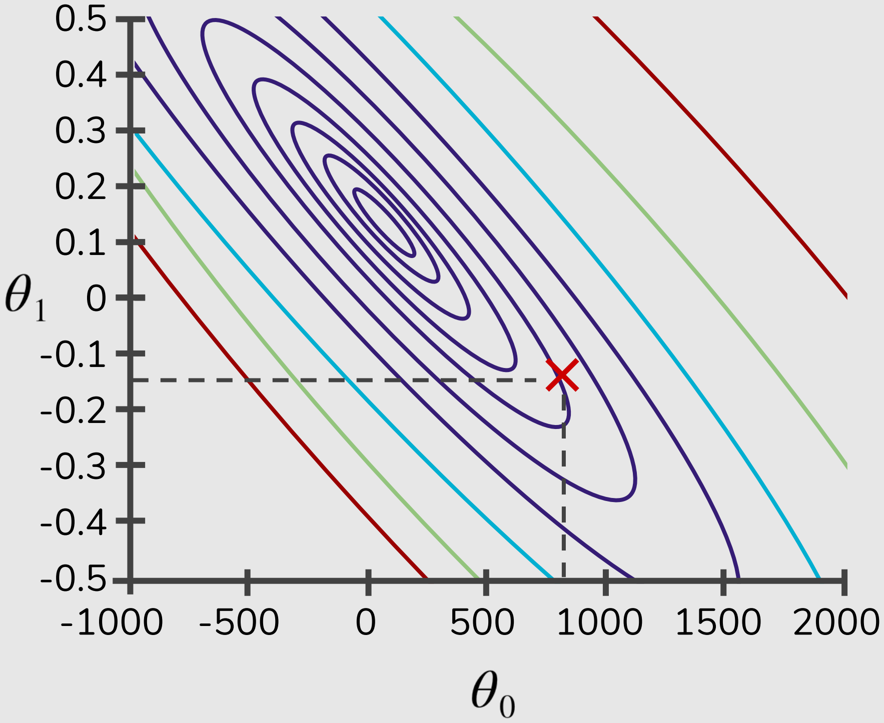
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



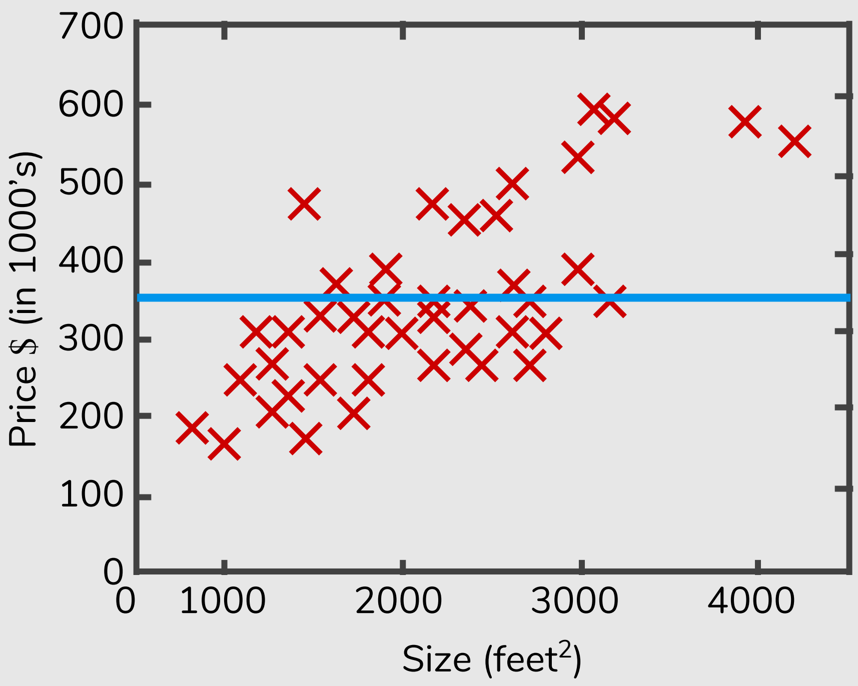
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



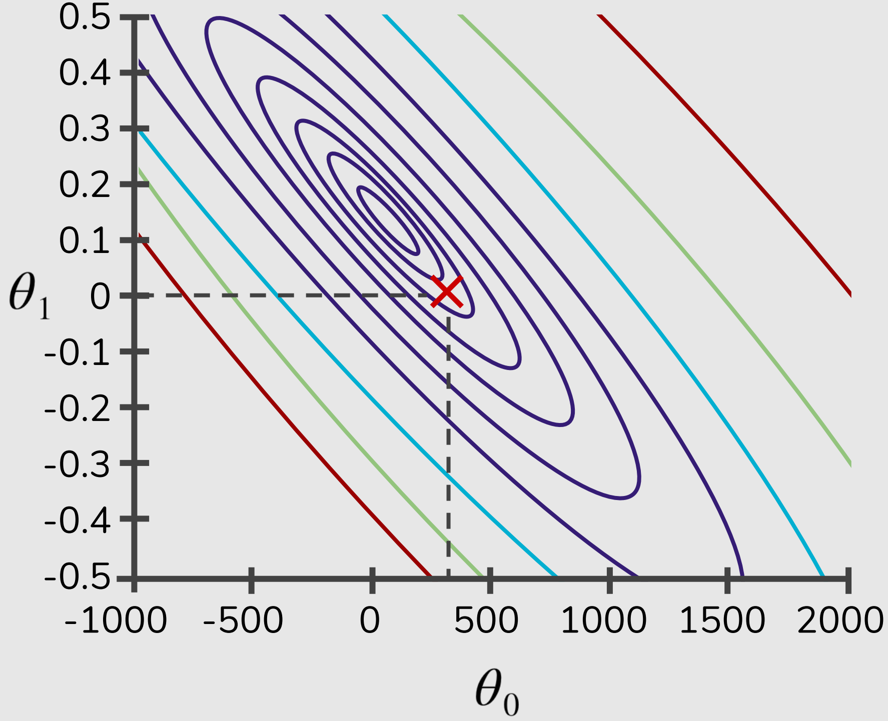
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



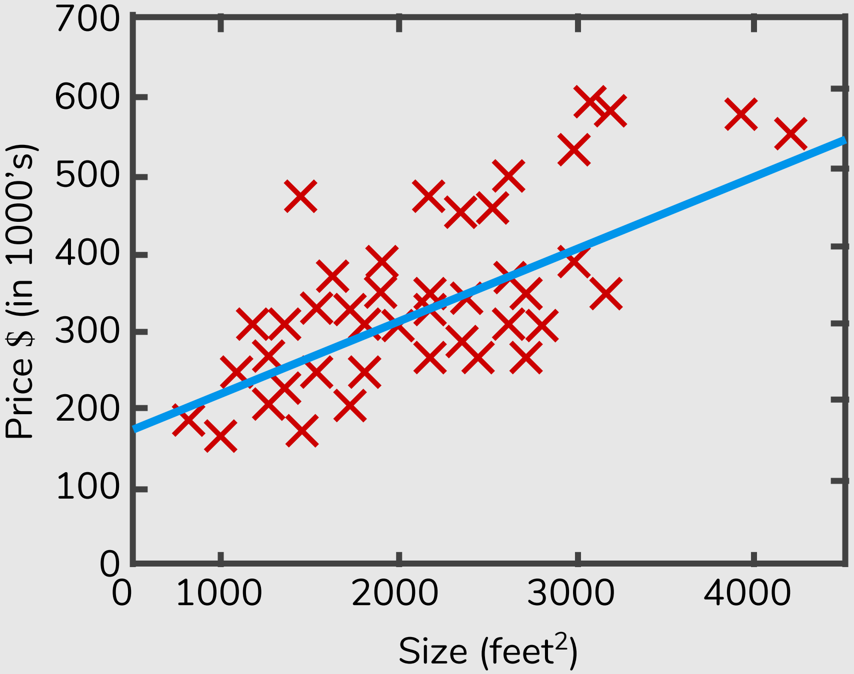
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



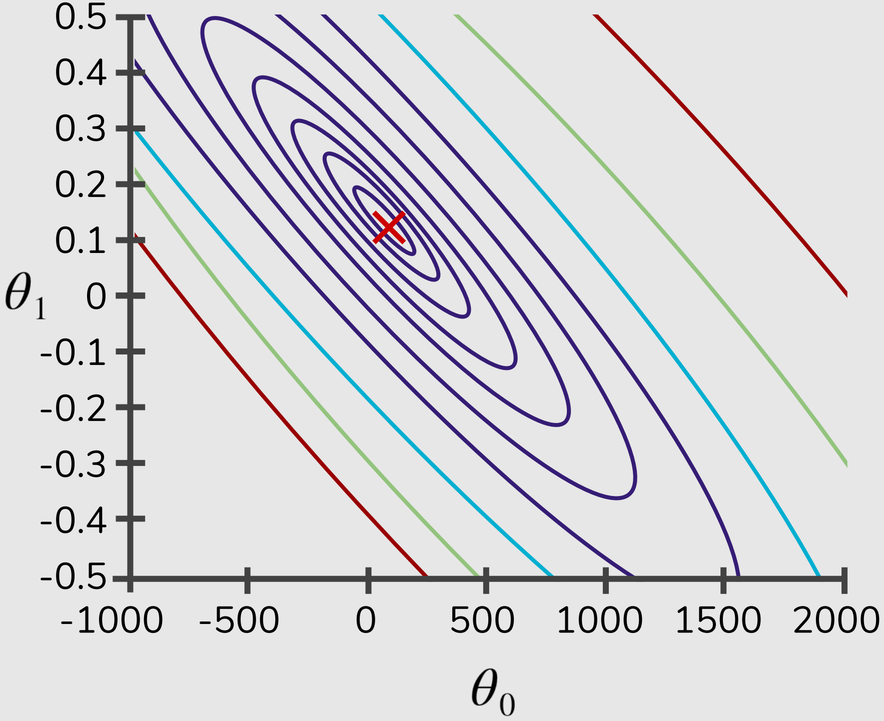
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



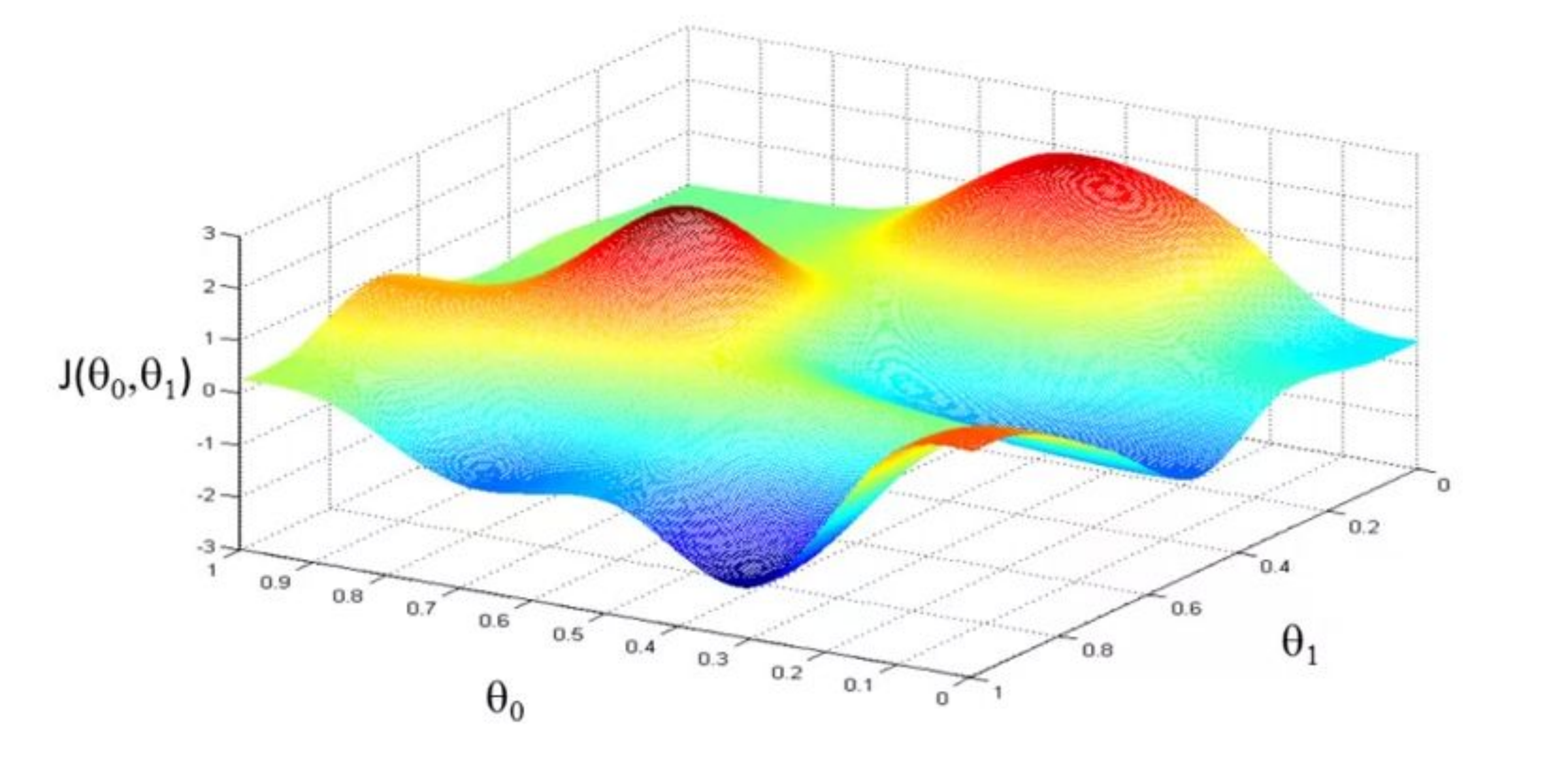
Gradient Descent

Have some function $J(\theta_0,\theta_1)$

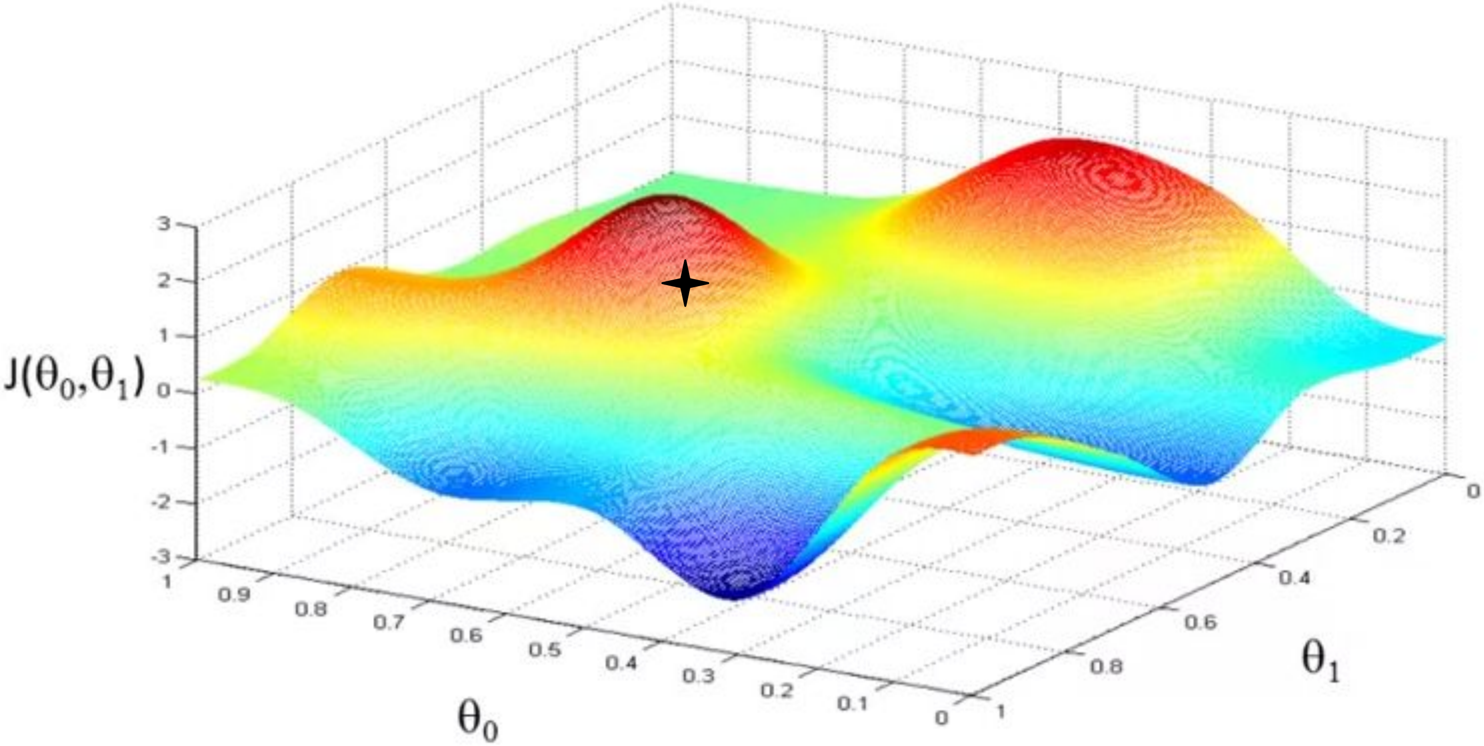
Want $\underset{\theta_0,\theta_1}{\text{minimize}}\, J(\theta_0,\theta_1)$

Outline:

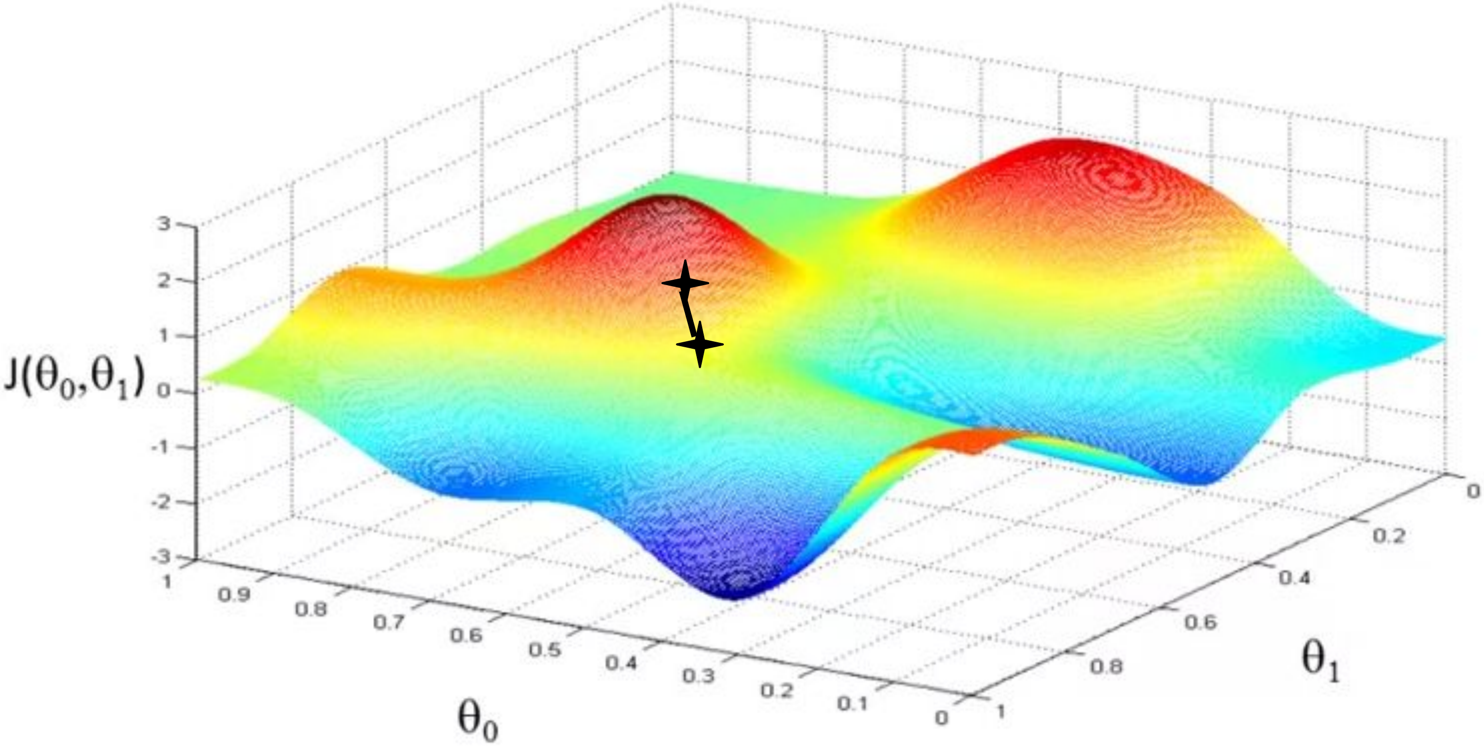
- Start with some θ_0, θ_1
- Keep changing θ_0, θ_1 to reduce $J(\theta_0,\theta_1)$ until we hopefully end up at a minimum



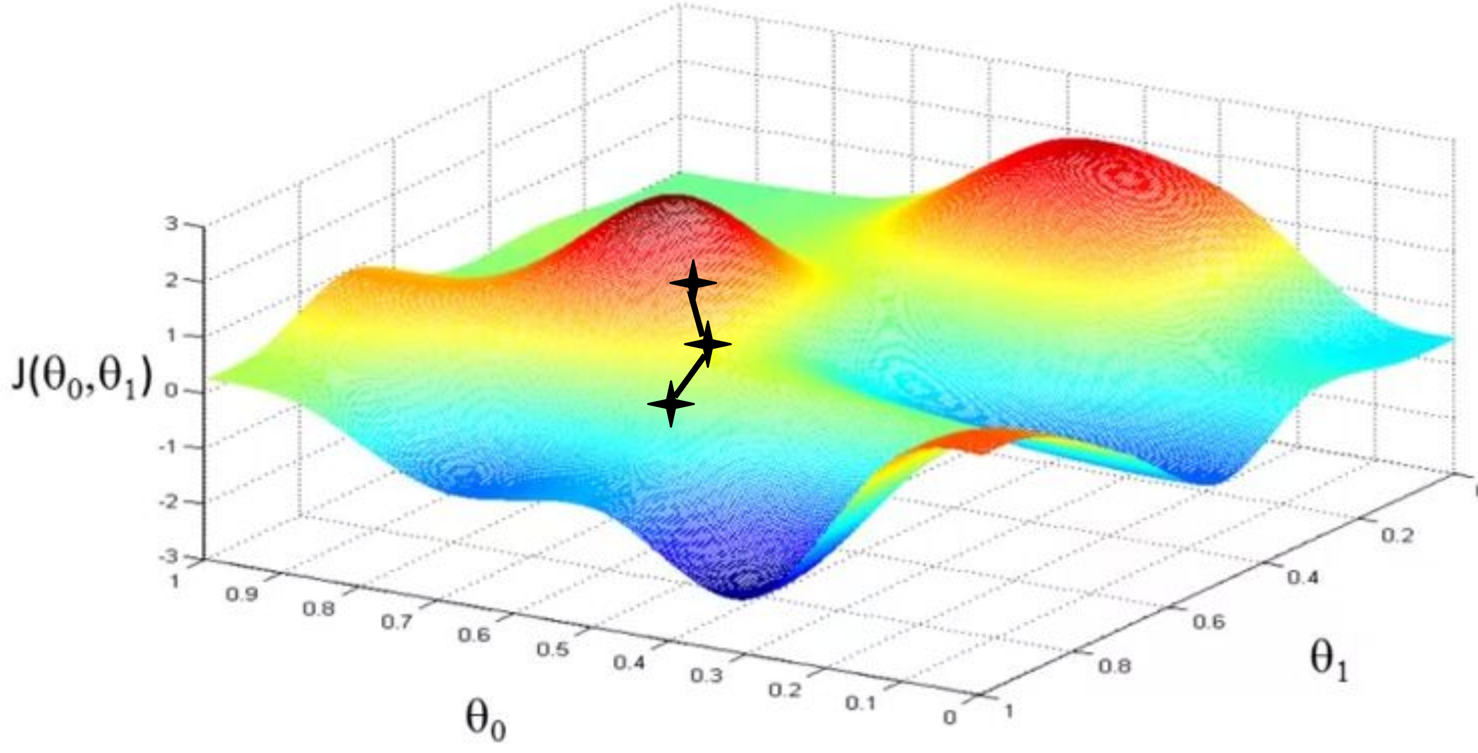
$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



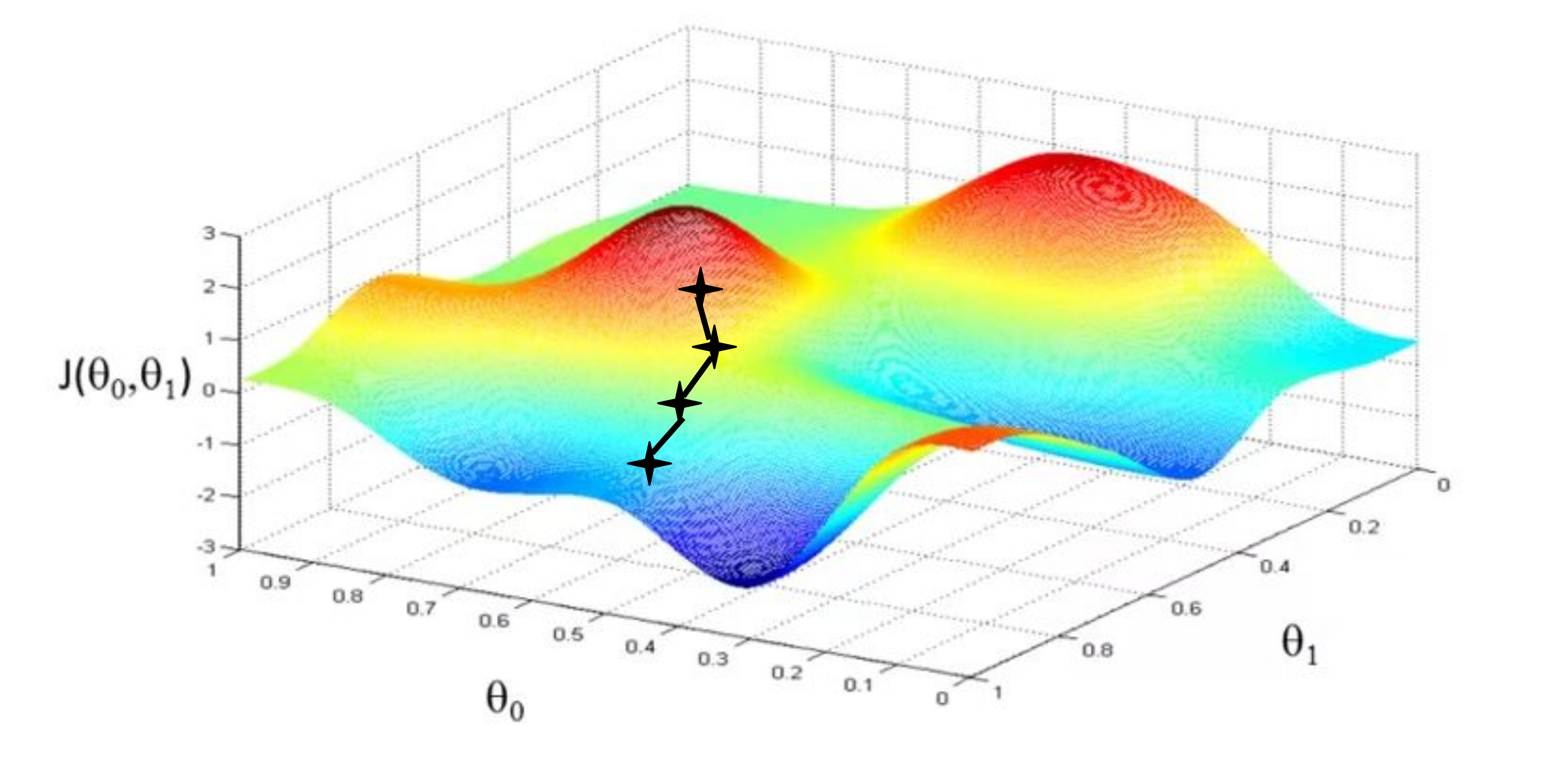
$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



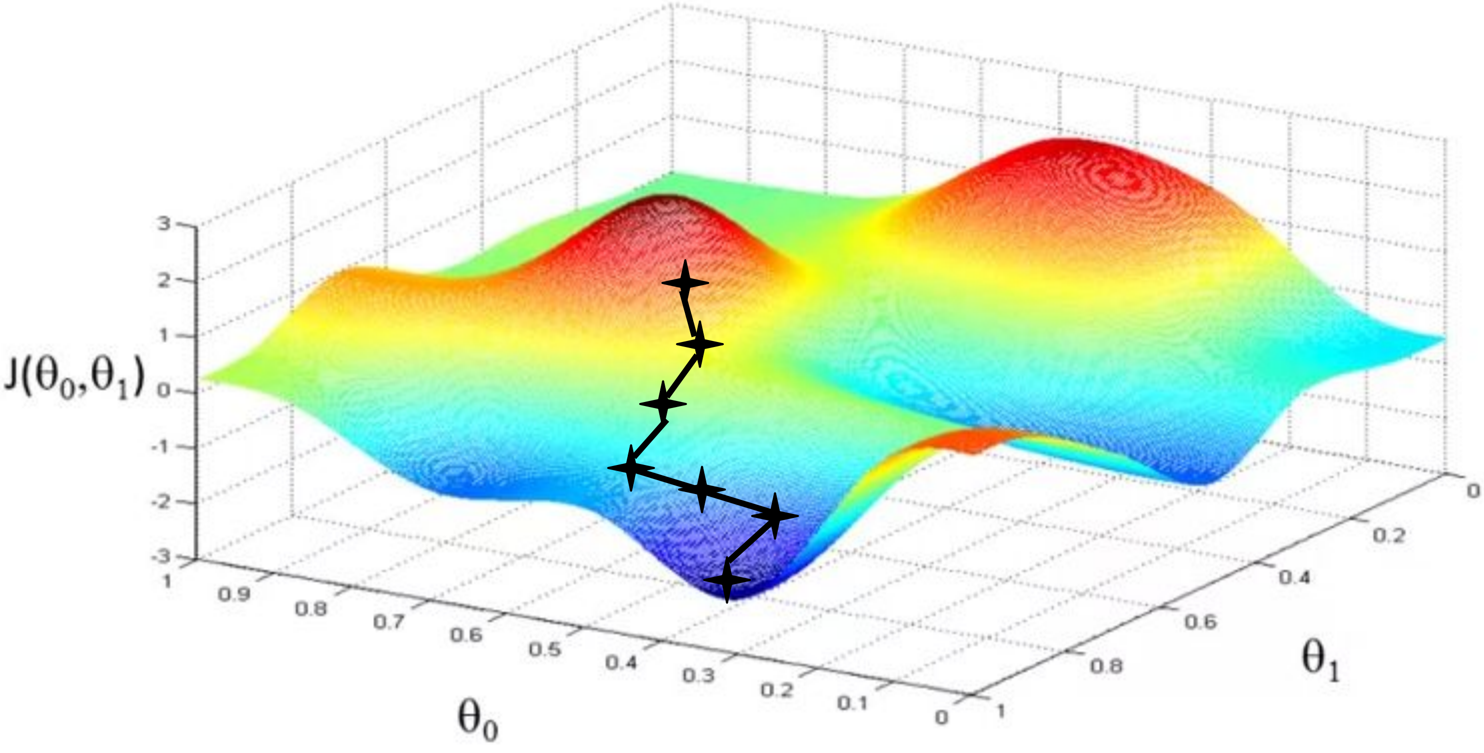
J(θ0,θ1)
3
2
1
0
-1
-2
-3
θ0
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ1
0
0.2
0.4
0.6
0.8
1



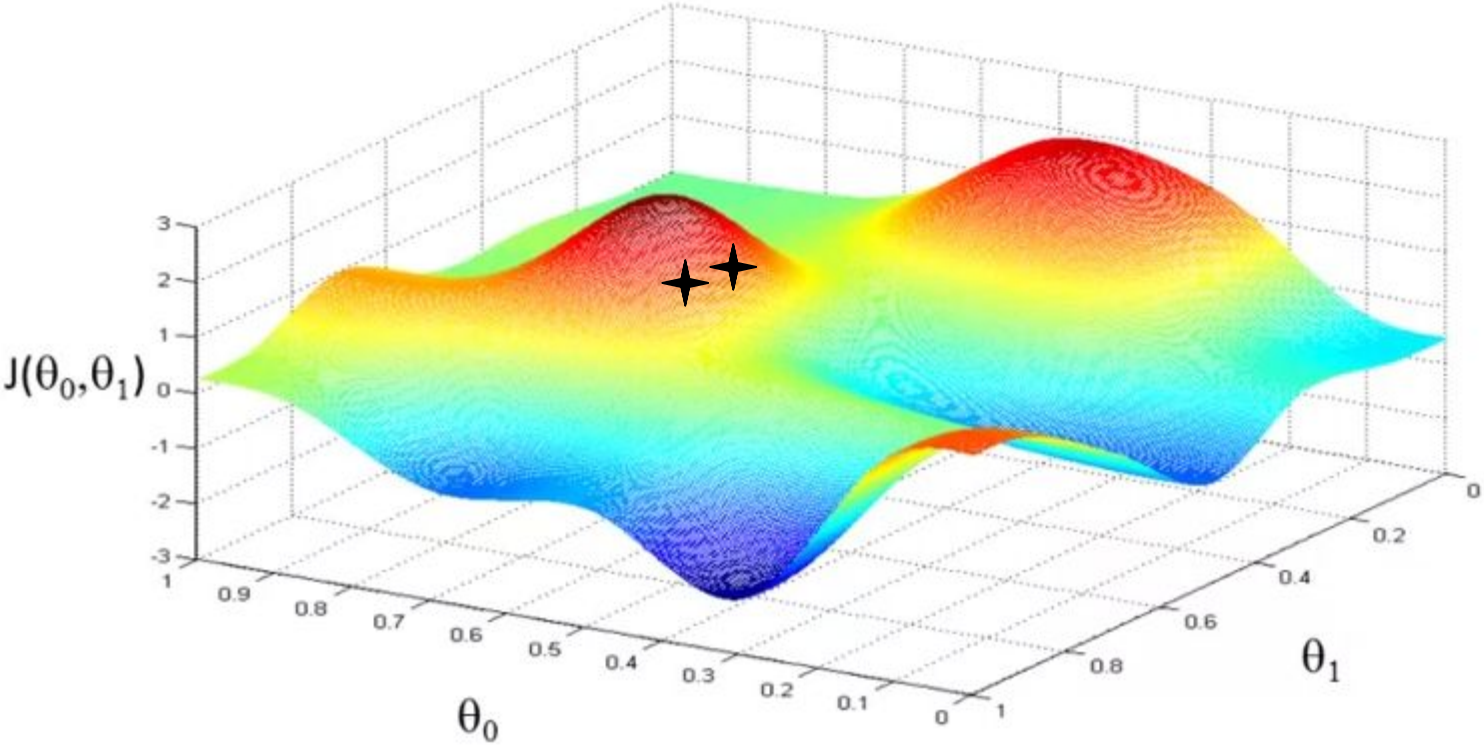
3
2
1
J(θ0,θ1)
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ0
0
0.2
0.4
0.6
0.8
1
θ1



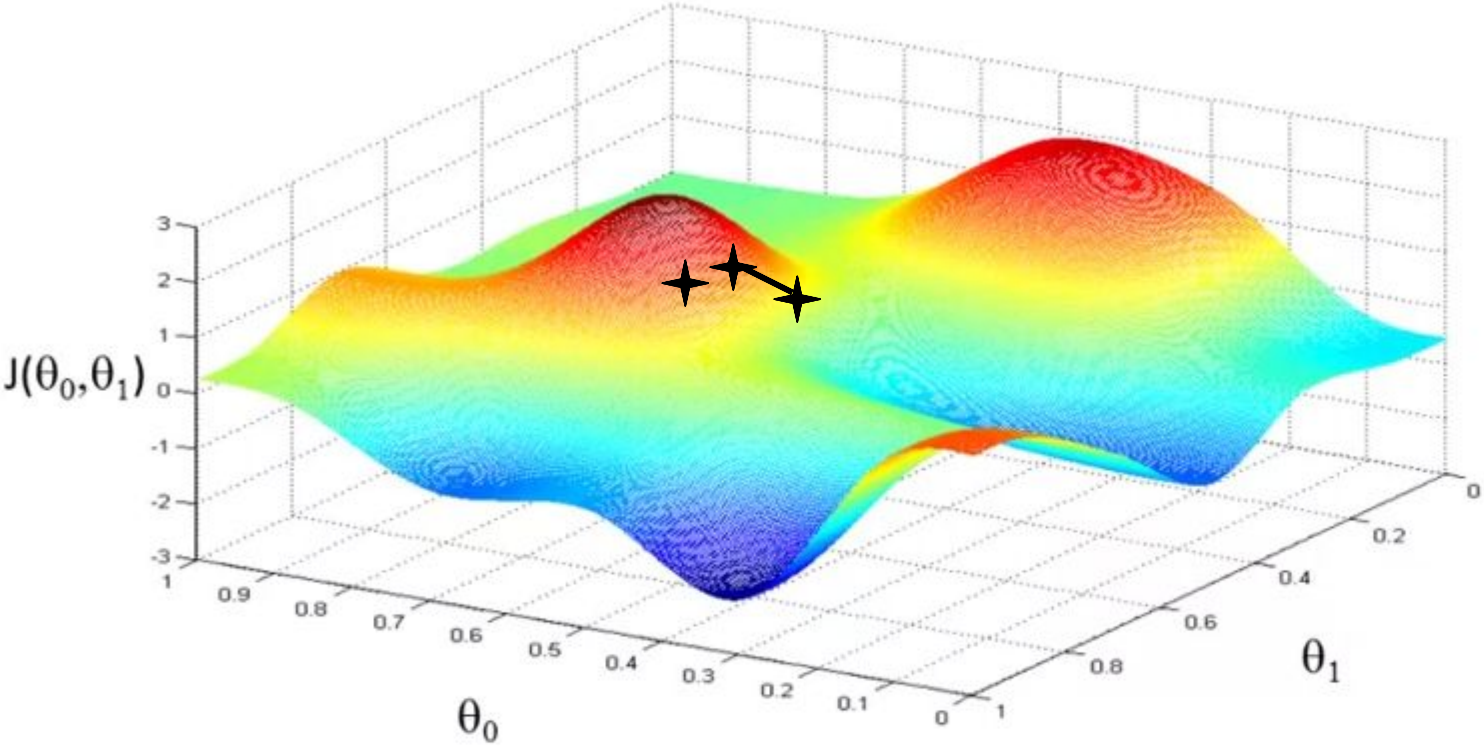
$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



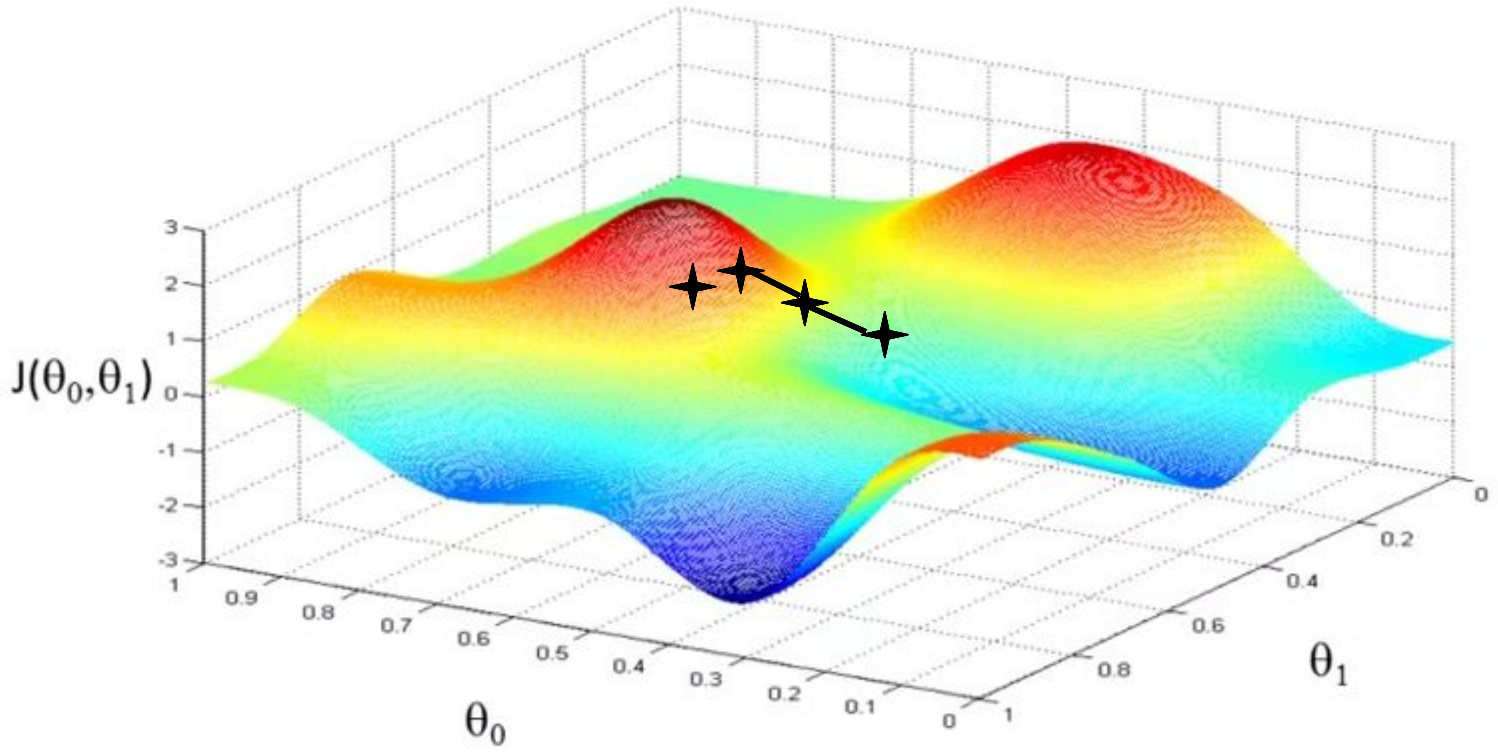
J(θ0,θ1)
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ0
0
0.2
0.4
0.6
0.8
1
θ1

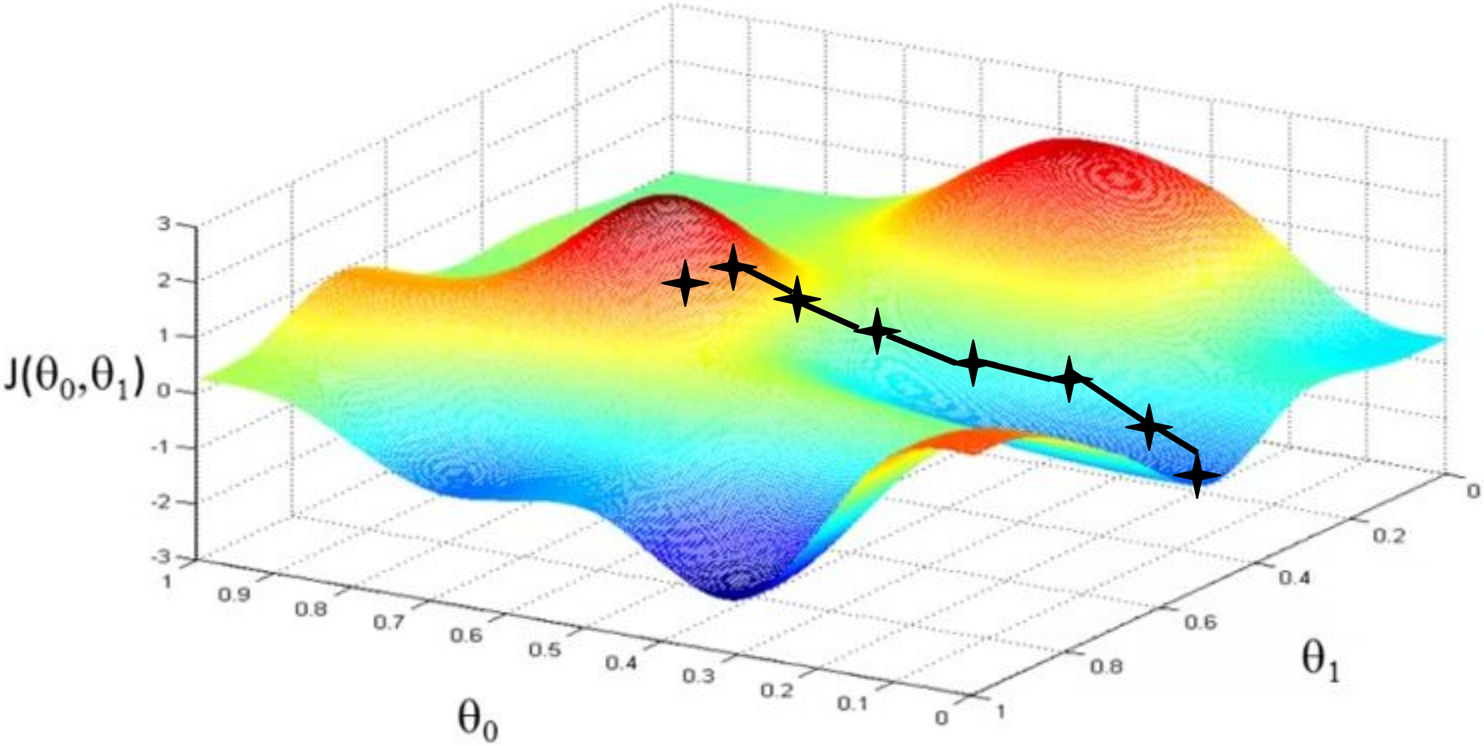


$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1


J(θ0,θ1)
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ0
0
0.2
0.4
0.6
0.8
1
θ1



J(θ0,θ1)
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ0
0
0.2
0.4
0.6
0.8
1
θ1

Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

}

(simultaneously update $j = 0$ and $j = 1$)

Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \boxed{\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)}$$

(simultaneously update

$j = 0$ and $j = 1$)

}

Learning rate

Derivative term

Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad \text{(for } j = 0 \text{ and } j = 1\text{)}$$

}

Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

$$\theta_0 := \text{temp0}$$

$$\theta_1 := \text{temp1}$$

Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad \text{(for } j = 0 \text{ and } j = 1\text{)}$$

}

Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

$$\theta_0 := \text{temp0}$$

$$\theta_1 := \text{temp1}$$

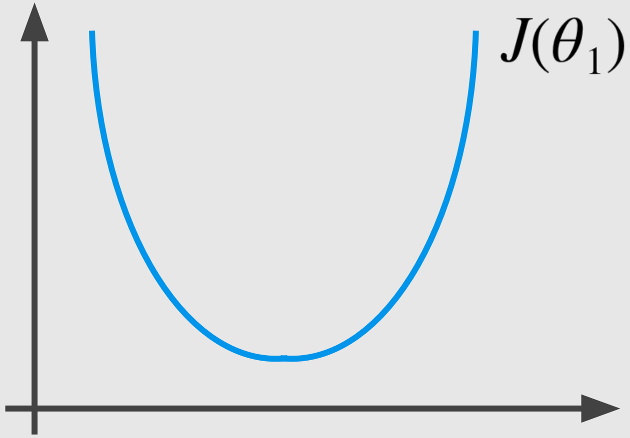
Incorrect

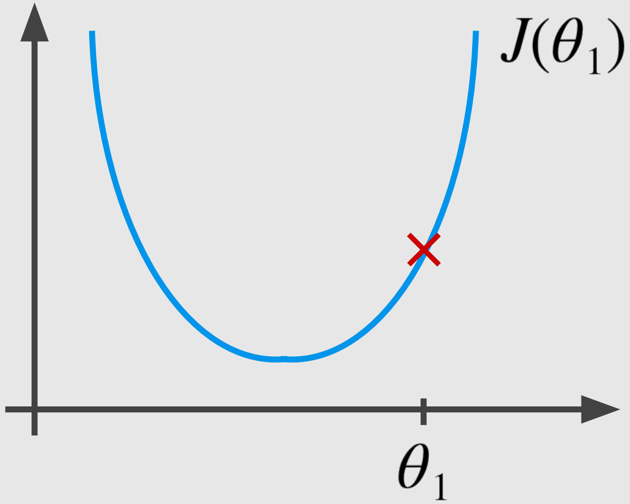
$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

$$\theta_0 := \text{temp0}$$

$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

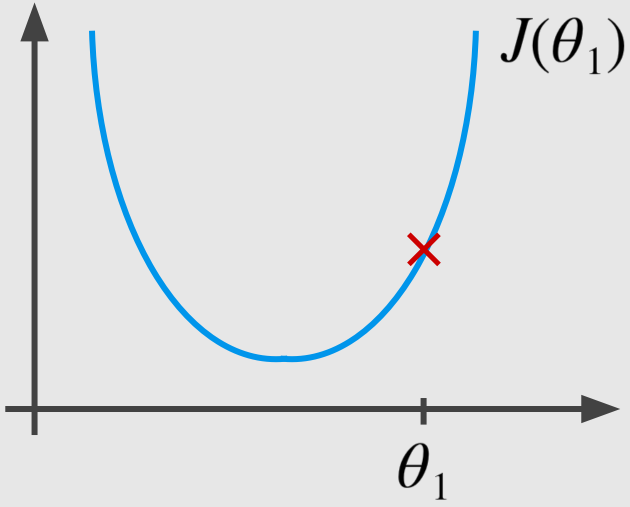
$$\theta_1 := \text{temp1}$$


$J(\theta_1)$



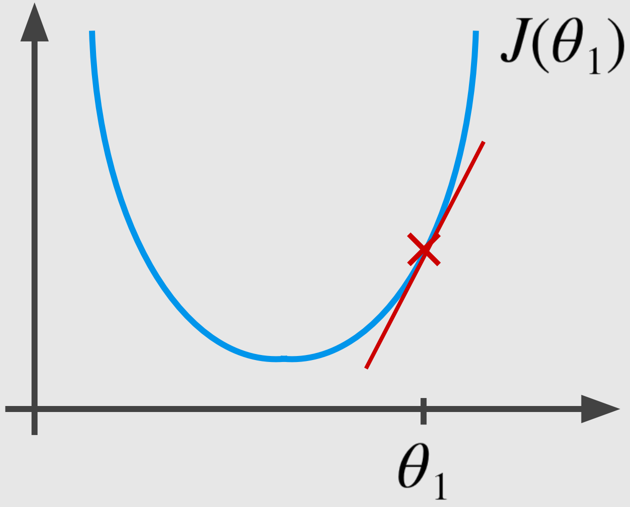
$\theta_1 \in \mathbb{R}$

$\theta_1 \in \mathbb{R}$



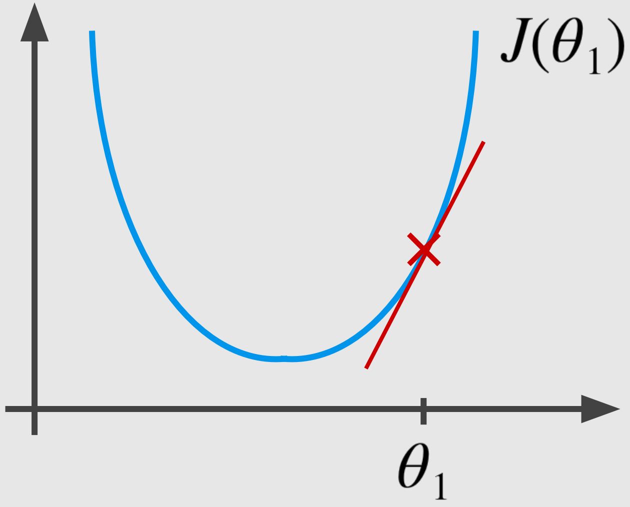
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

$$\theta_1 \in \mathbb{R}$$



$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

$\theta_1 \in \mathbb{R}$

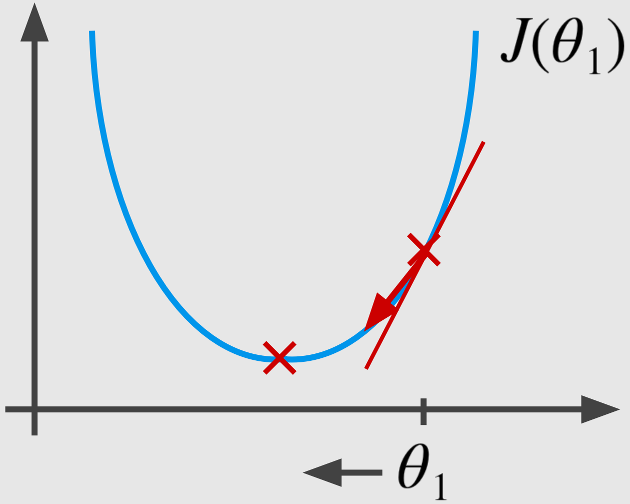


≥ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$$\theta_1 := \theta_1 - \alpha \cdot \text{(positive number)}$$

$\theta_1 \in \mathbb{R}$

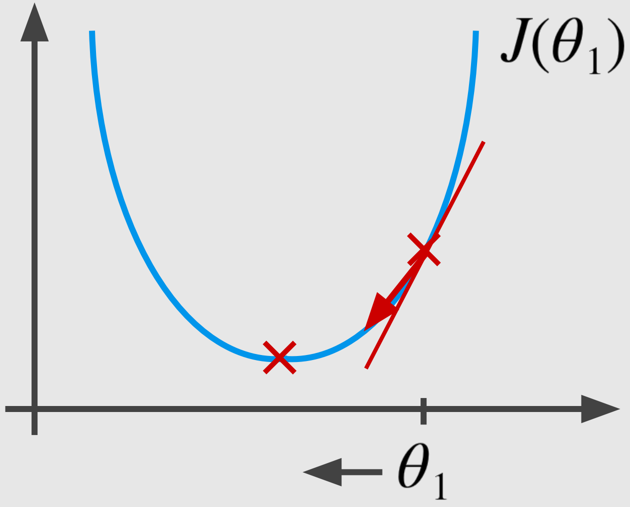


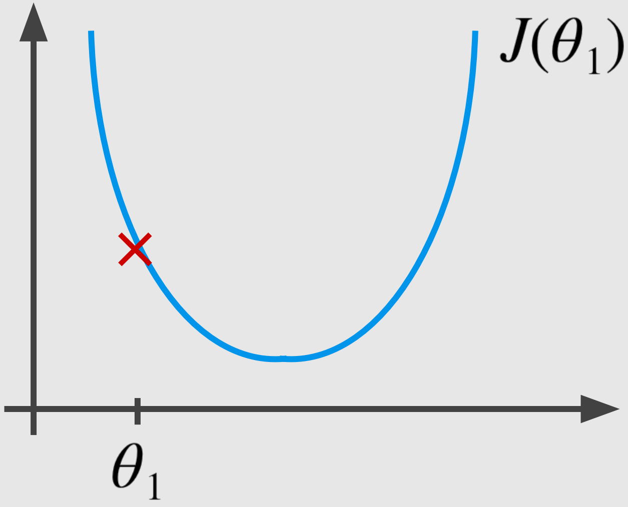
≥ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$\theta_1 := \theta_1 - \alpha \cdot$ (positive number)

$$\theta_1 \in \mathbb{R}$$



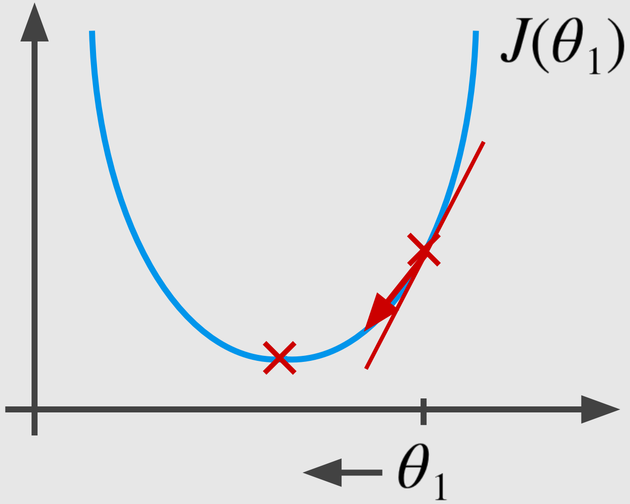


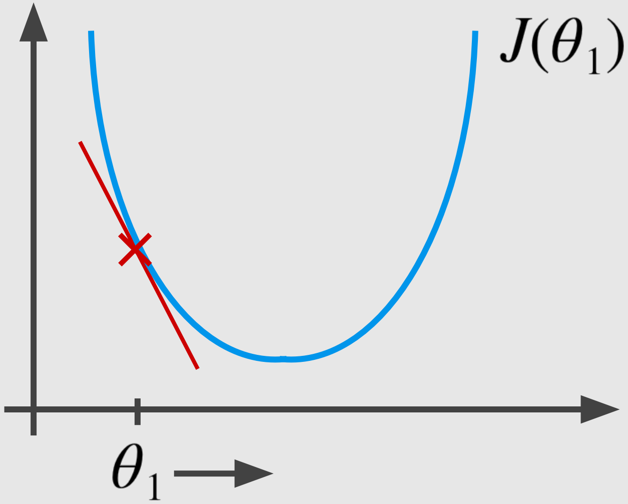
≥ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$$\theta_1 := \theta_1 - \alpha \cdot \text{(positive number)}$$

$$\theta_1 \in \mathbb{R}$$





≥ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

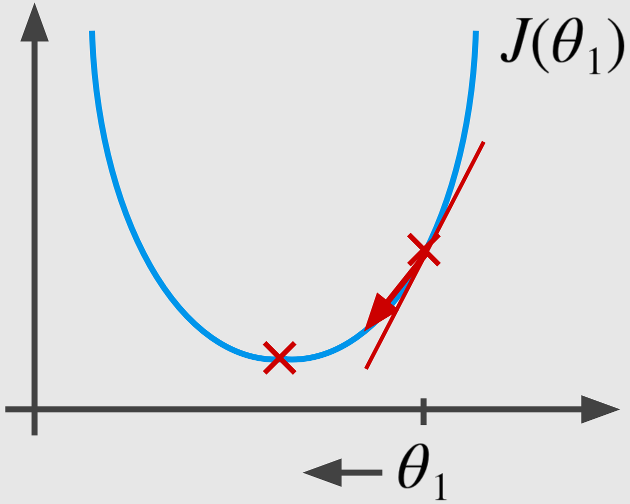
$$\theta_1 := \theta_1 - \alpha \cdot \text{(positive number)}$$

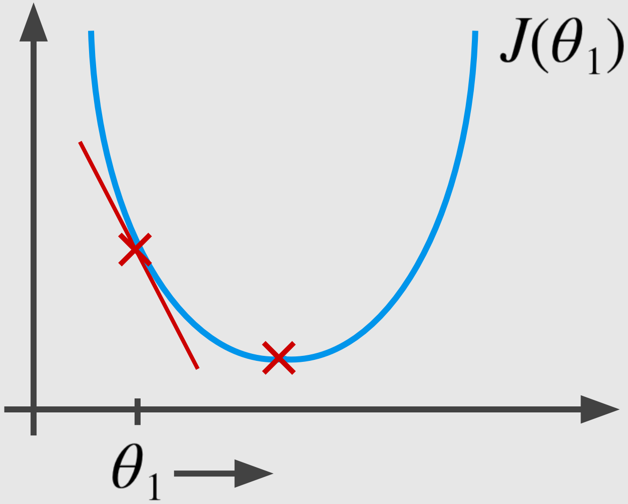
≤ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$$\theta_1 := \theta_1 - \alpha \cdot \text{(negative number)}$$

$$\theta_1 \in \mathbb{R}$$





≥ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$$\theta_1 := \theta_1 - \alpha \cdot \text{(positive number)}$$

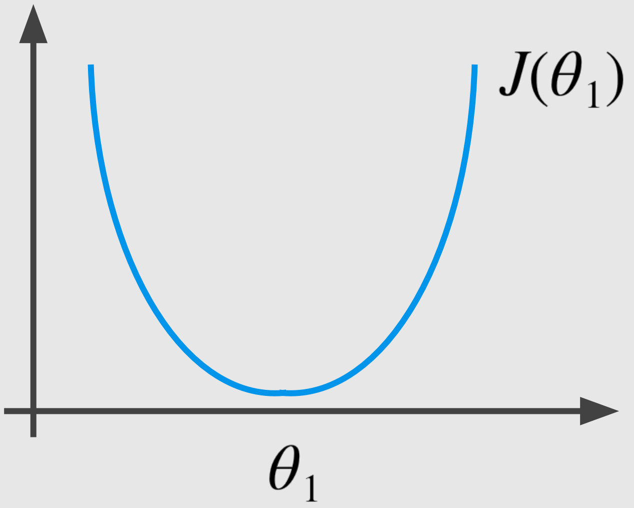
≤ 0

$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

$$\theta_1 := \theta_1 - \alpha \cdot \text{(negative number)}$$

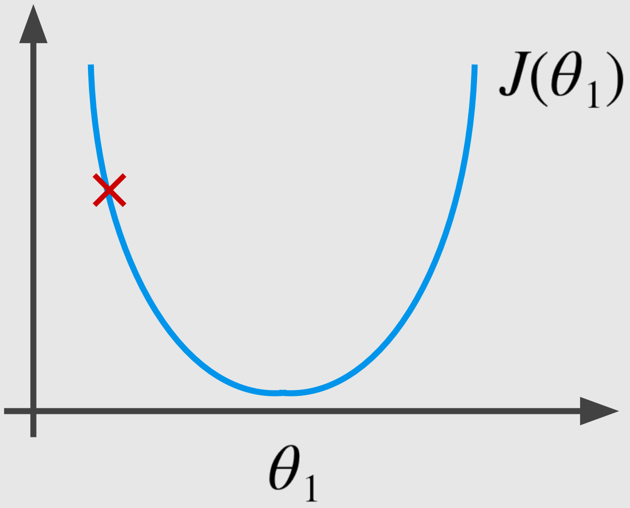
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be ...



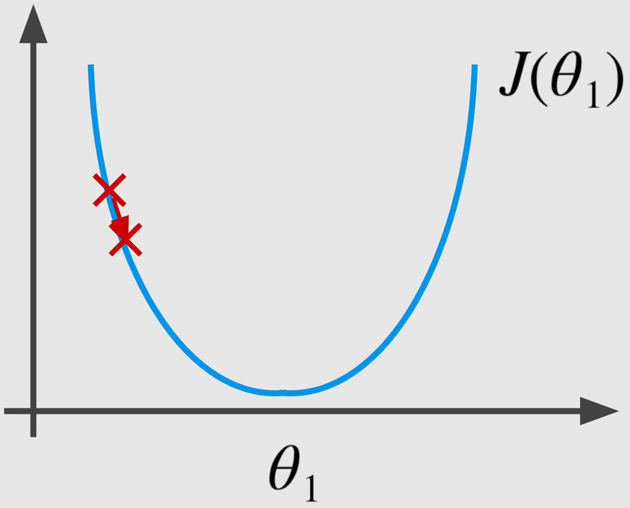
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be ...



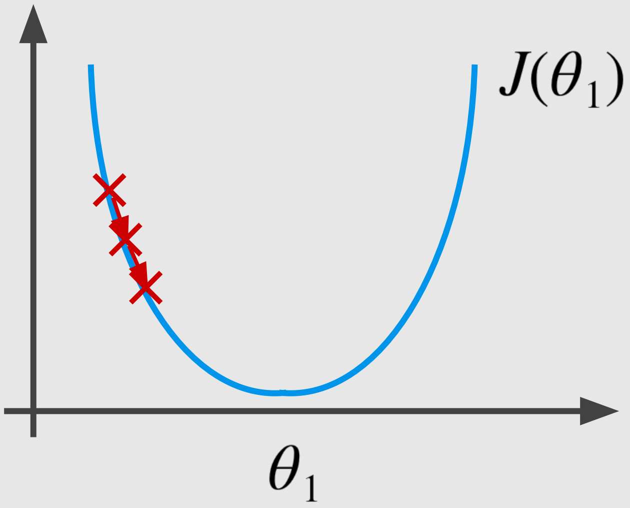
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.



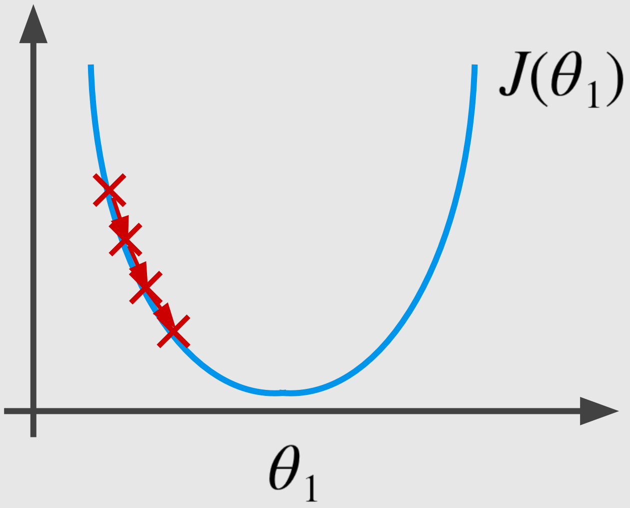
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.



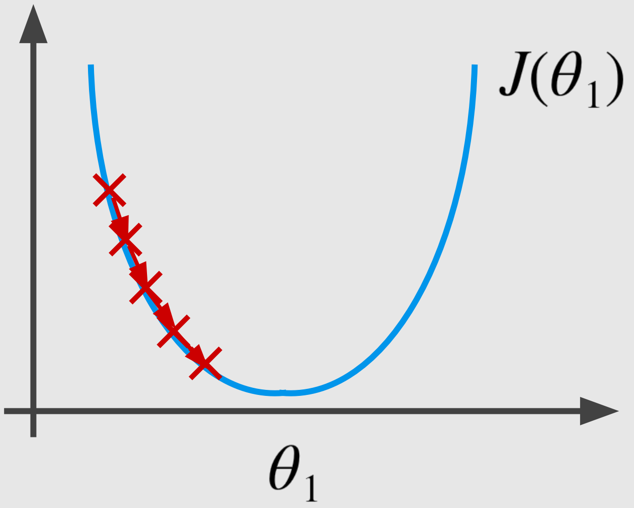
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.



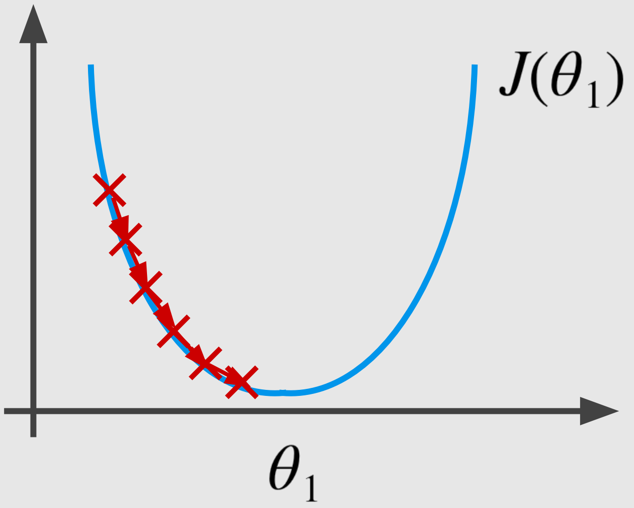
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.



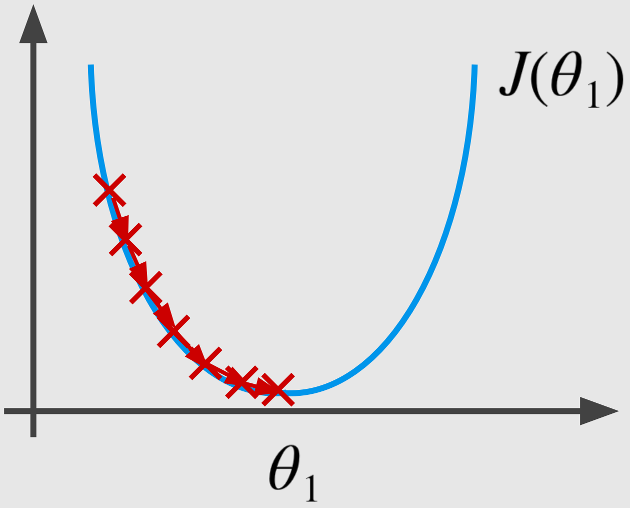
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.



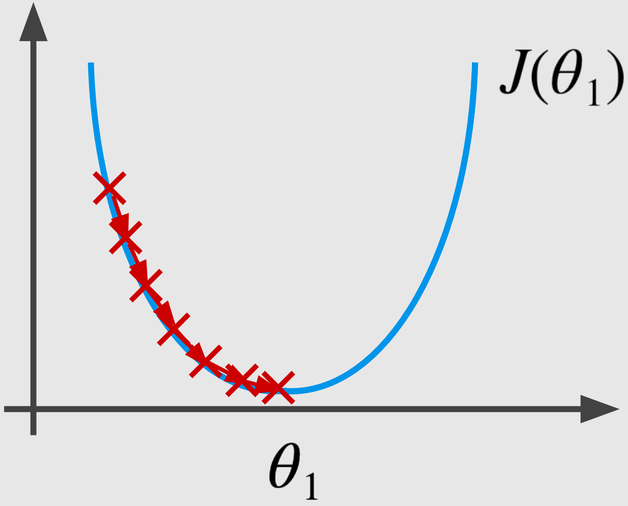
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.

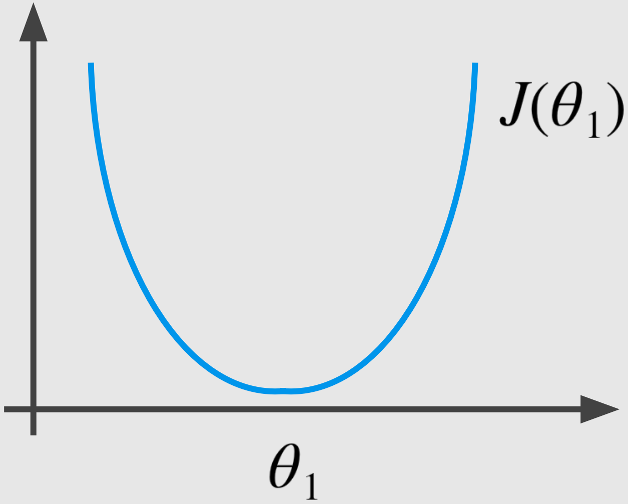


$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

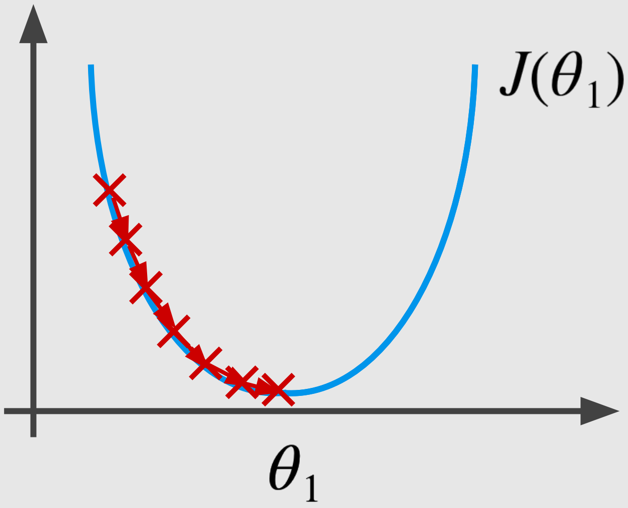
If α is too small, gradient descent can be slow.



If α is too large, gradient descent can be ...

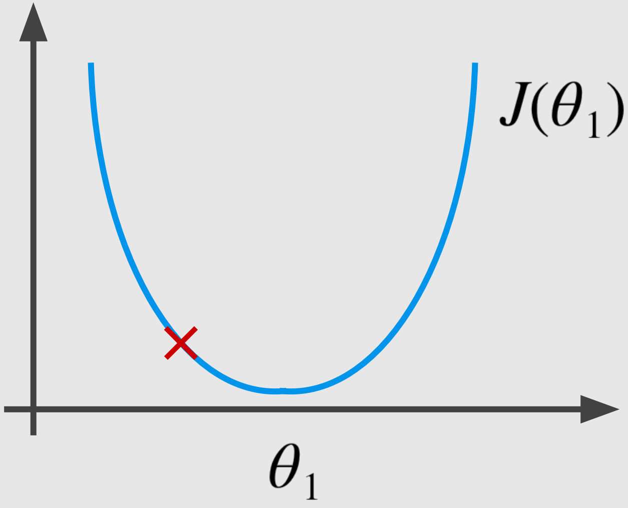


$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$



If α is too small, gradient descent can be slow.

If α is too large, gradient descent can be ...



$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

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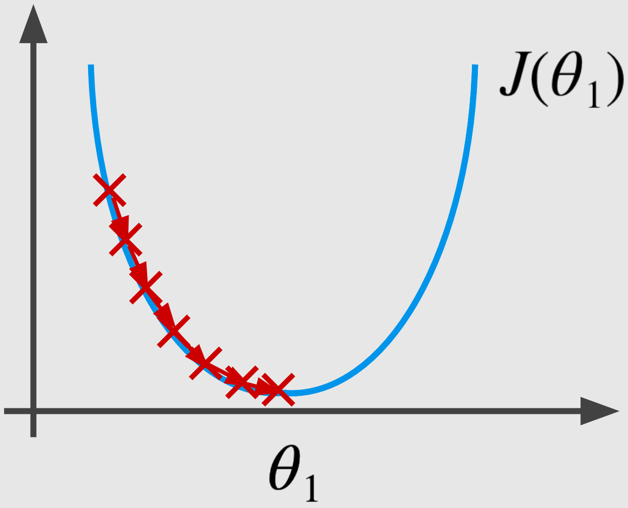
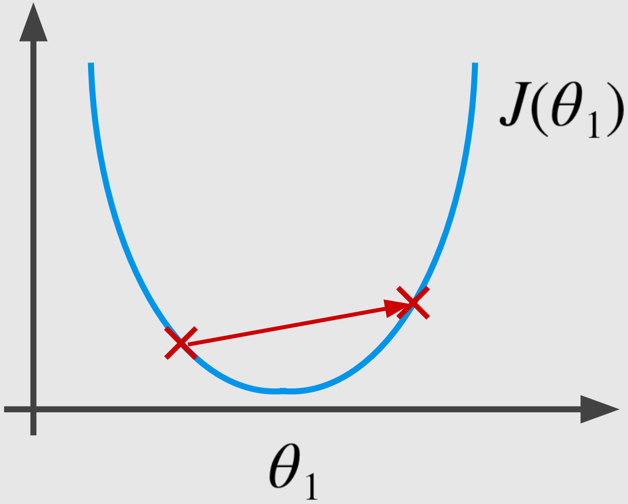


If α is too large, gradient descent can be overshoot the minimum. It may fail to converge, or even diverge.



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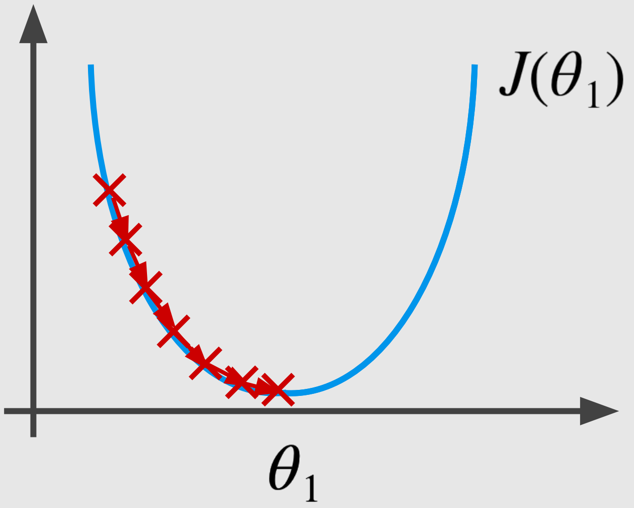

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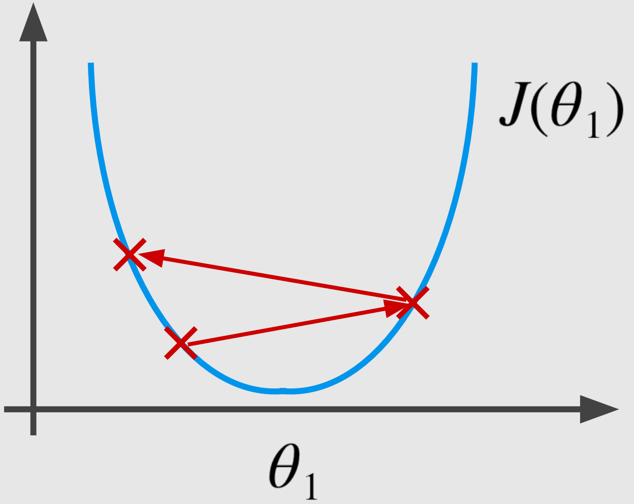

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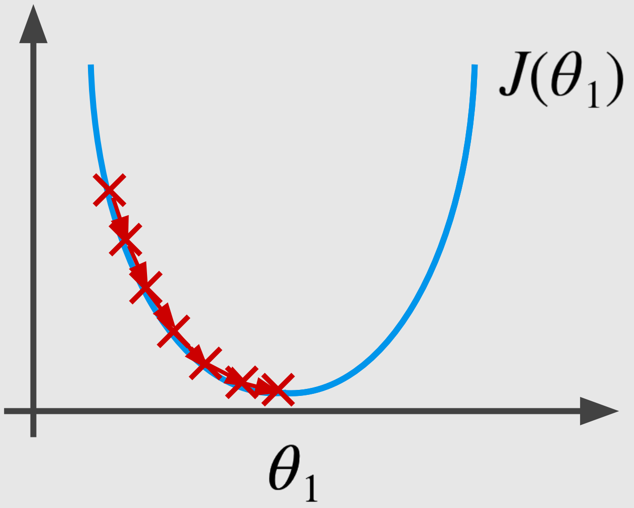

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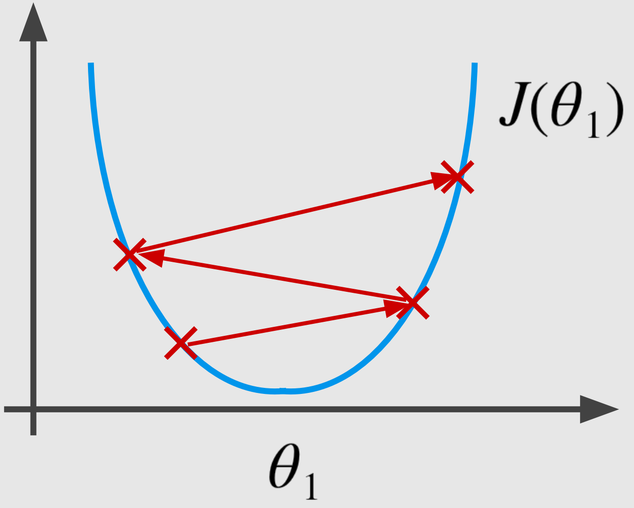

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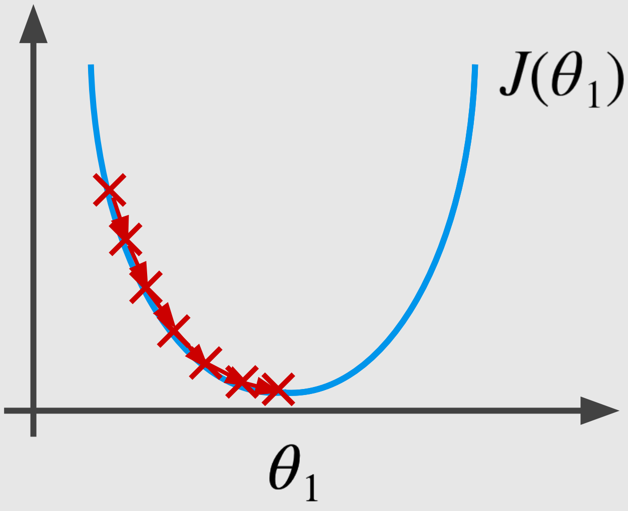

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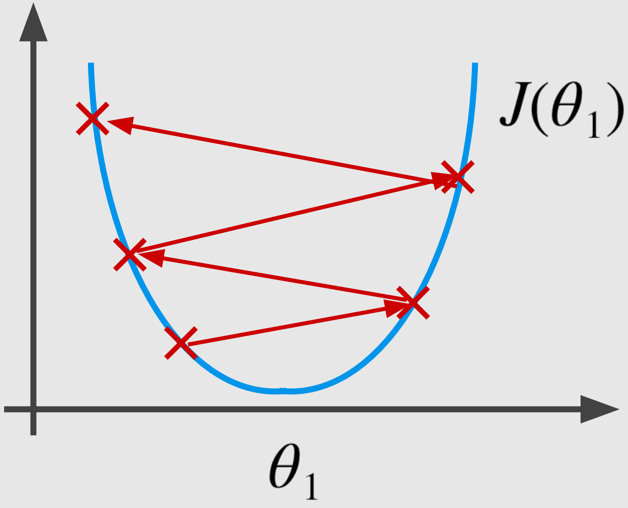
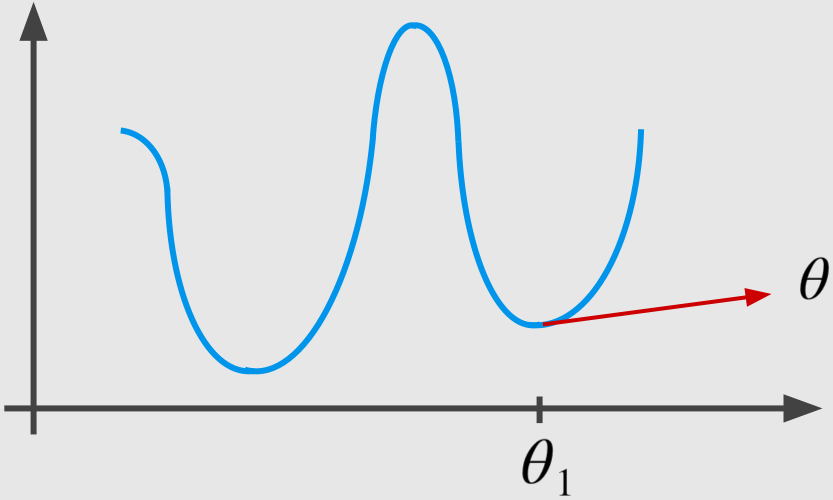


What will one step of gradient descent $\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$ do?



θ_1 at local optima

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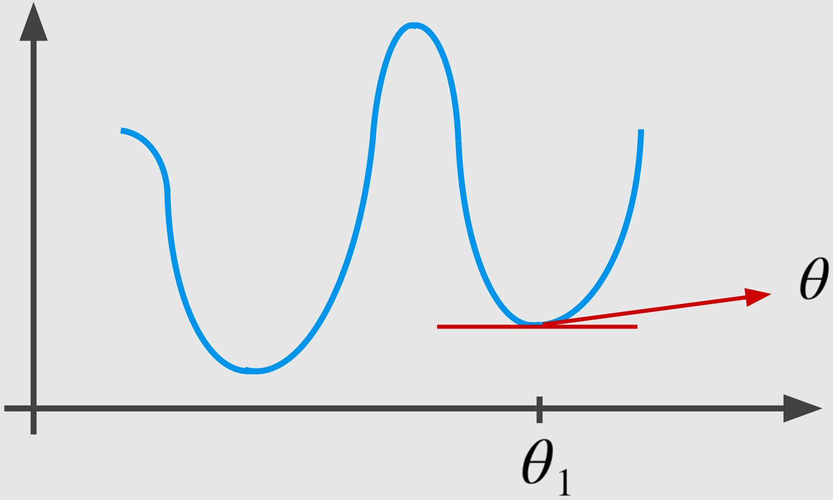


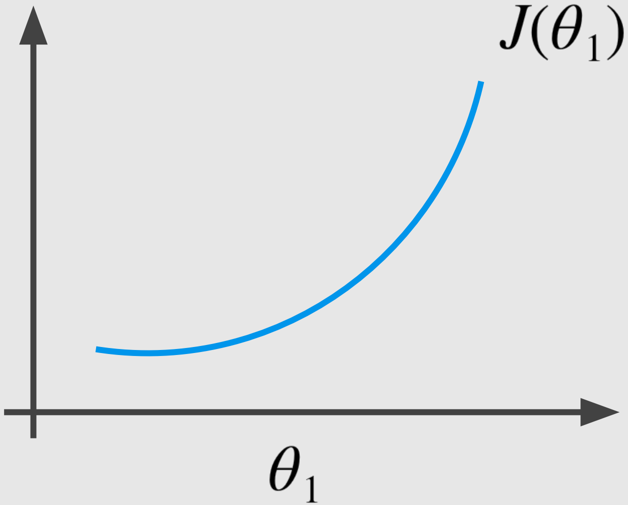
$$\theta_1 := \theta_1 - \alpha \boxed{\frac{d}{d\theta_1} J(\theta_1)}$$

θ_1 at local optima

$$= 0$$

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$



As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.

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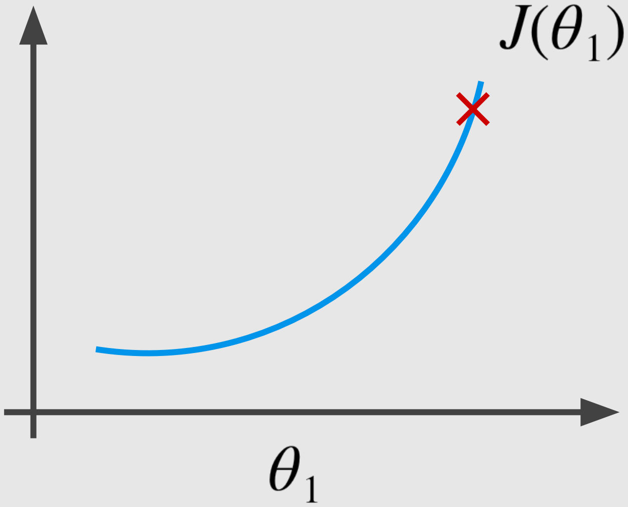

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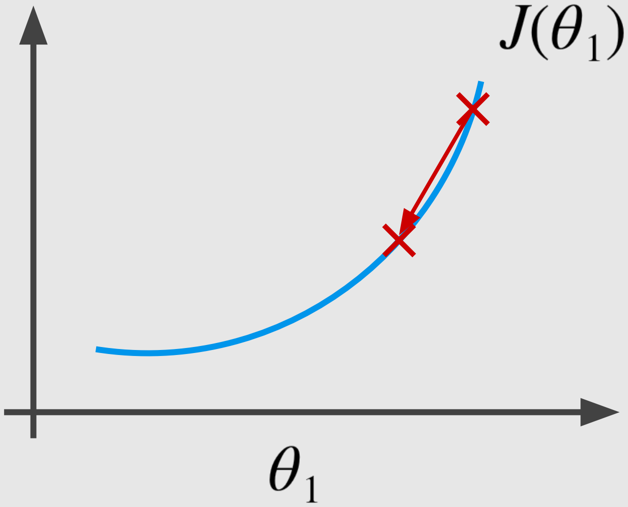

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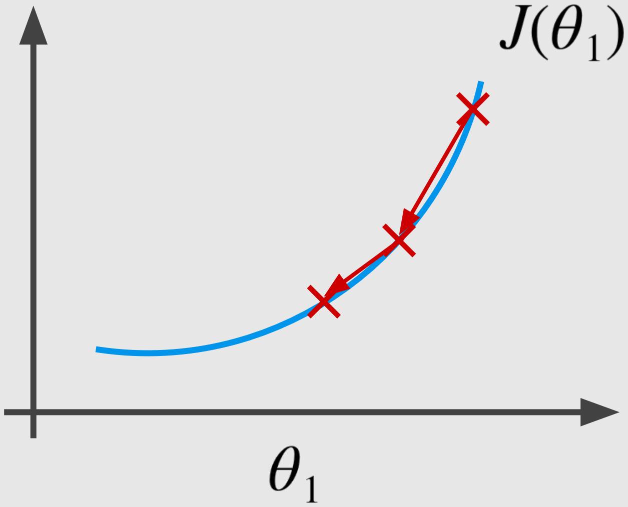

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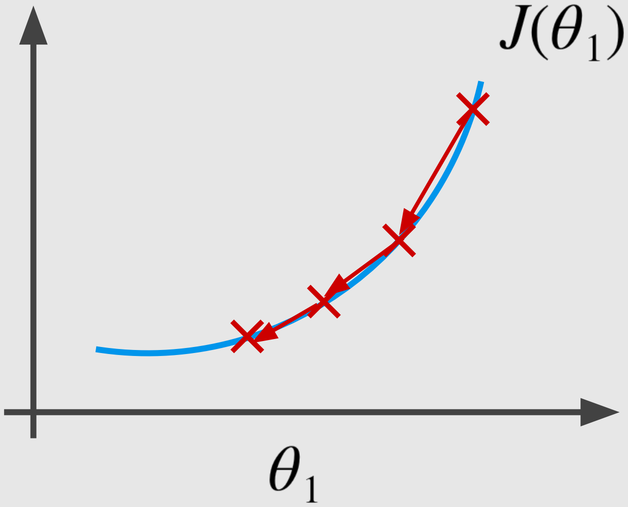

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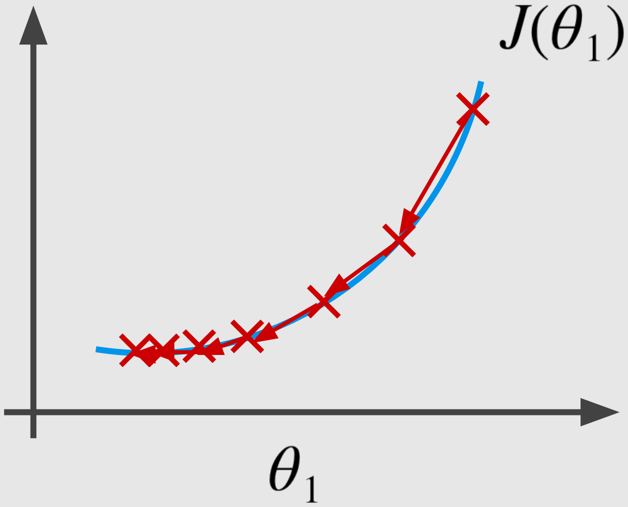

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.

Gradient Descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

(for $j = 0$ and $j = 1$)

}

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

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$$\boxed{\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)}$$

$$\frac{\partial}{\partial\theta_j} J(\theta_0, \theta_1)$$

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_j} \cdot \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

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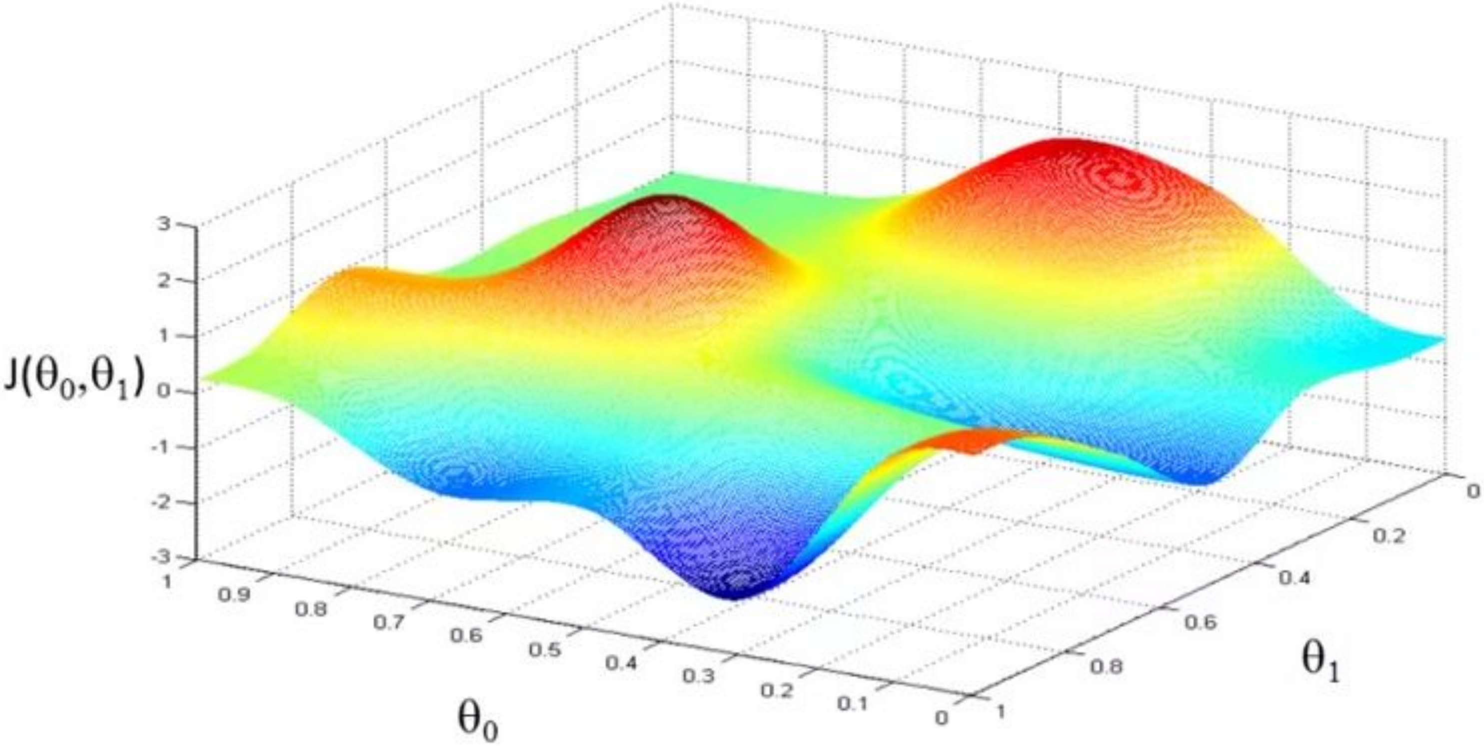
Gradient Descent algorithm

repeat until convergence {

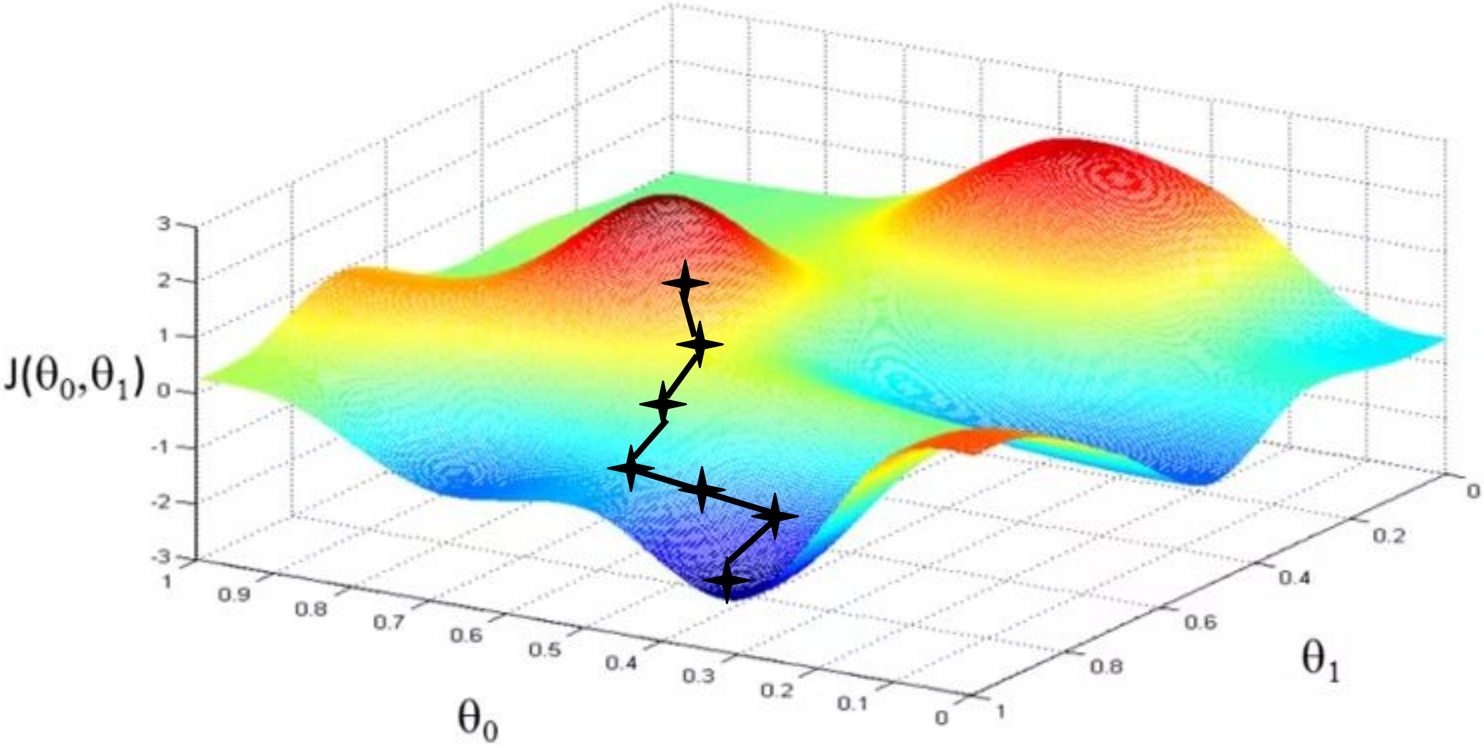
$$\left.\begin{aligned}\theta_0 &:= \theta_0 - \alpha\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)\\ \theta_1 &:= \theta_1 - \alpha\frac{1}{m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)x^{(i)}\end{aligned}\right\}$$

update θ_0 and θ_1 simultaneously

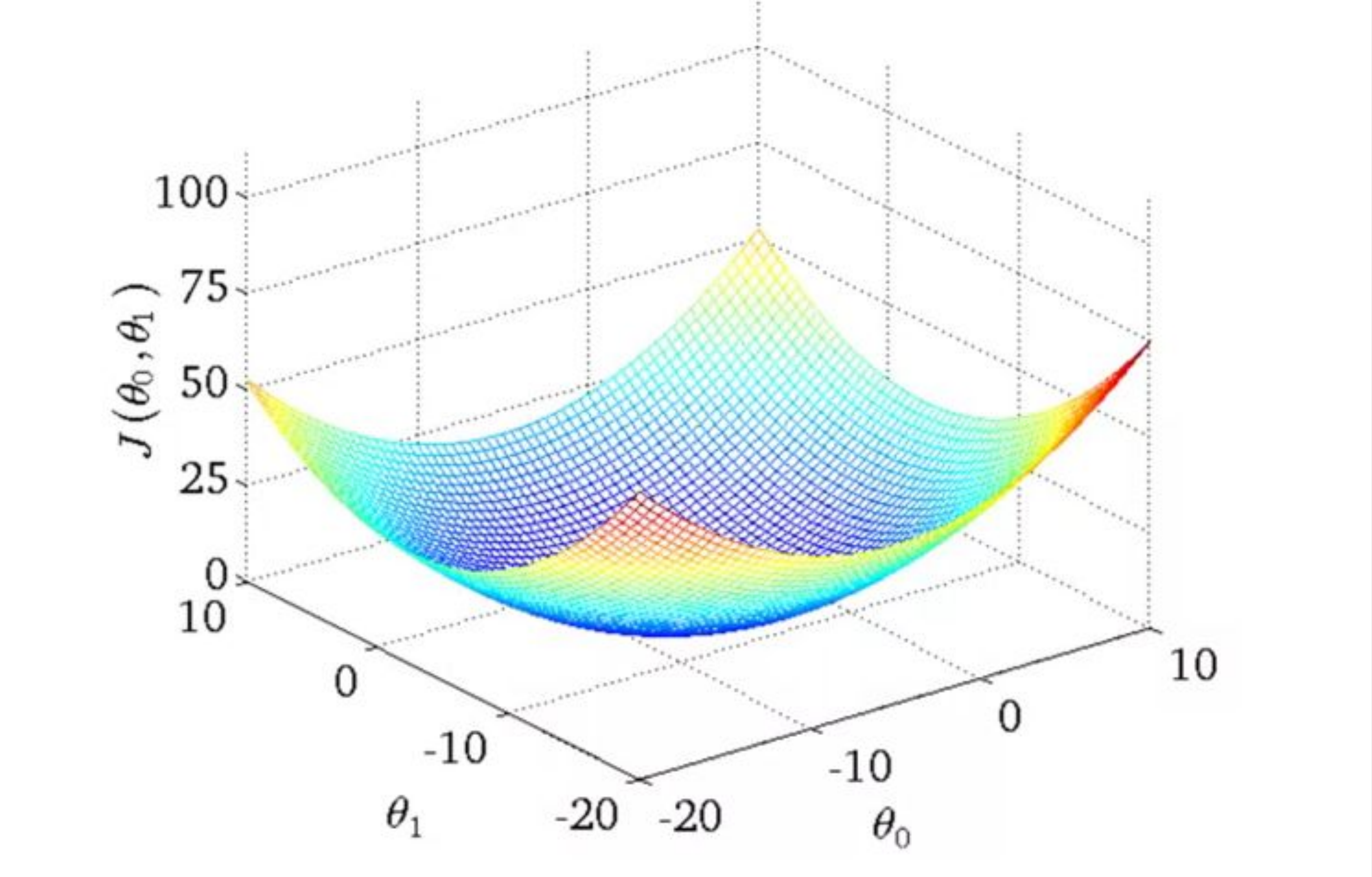
}



3
2
1
0
-1
-2
-3
$J(\theta_0,\theta_1)$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



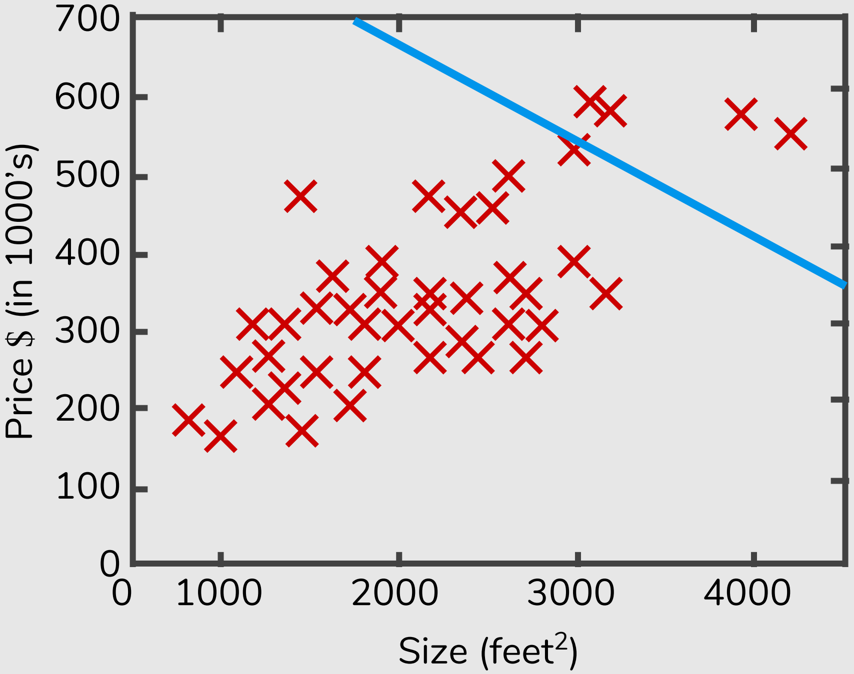
$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1



100
75
50
25
0
$J(\theta_0, \theta_1)$
10
0
-10
-20
θ_1
-20
-10
0
10
θ_0

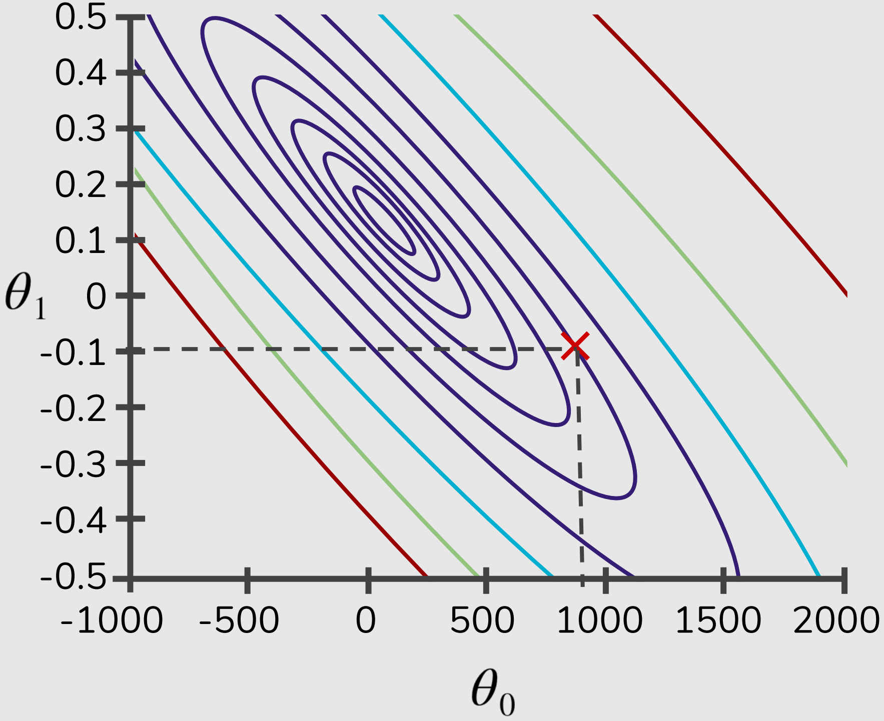
$$h_\theta(x)$$

(for fixed θ_0, θ_1, this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



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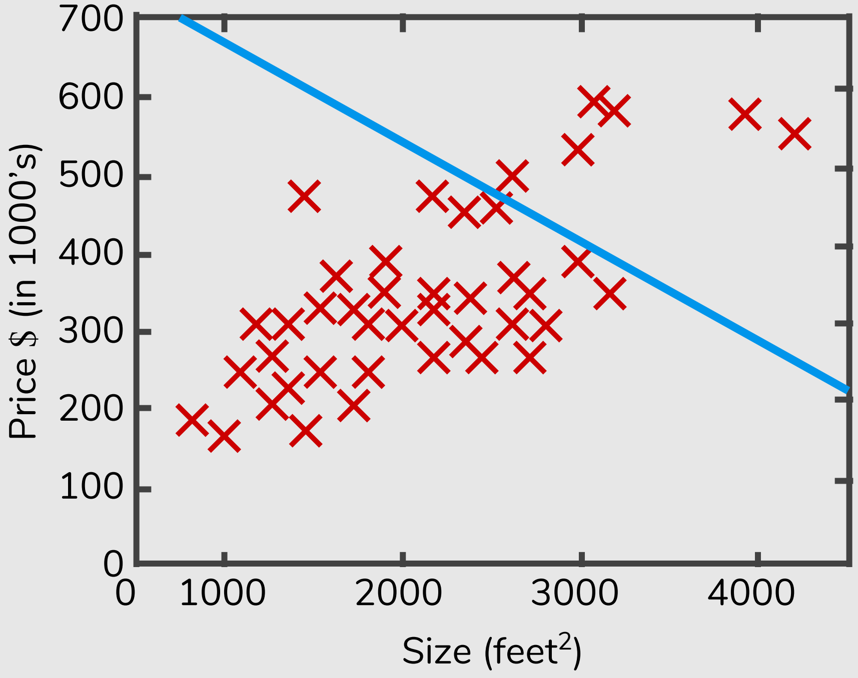

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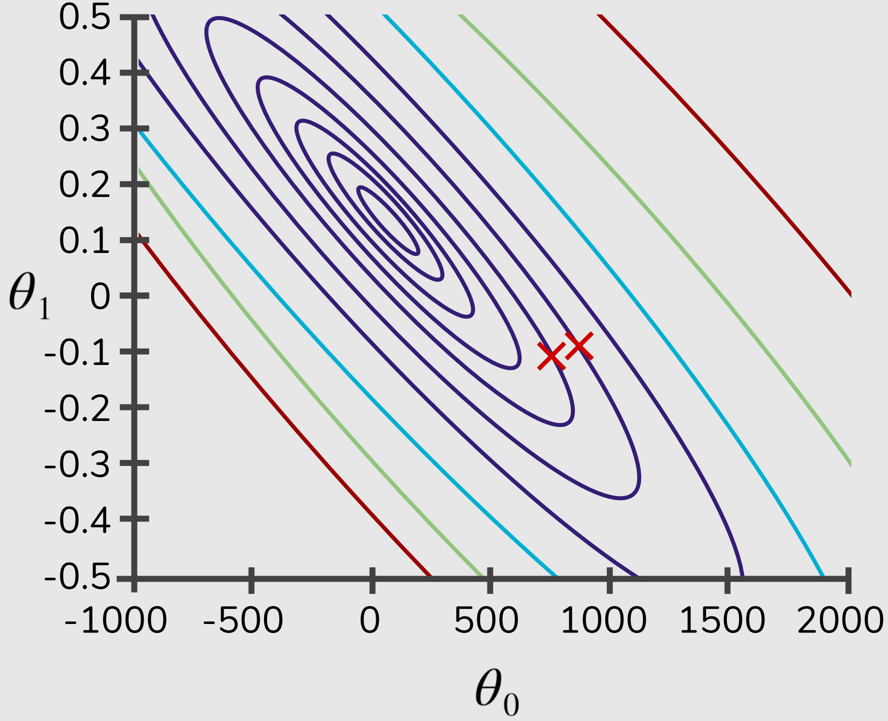

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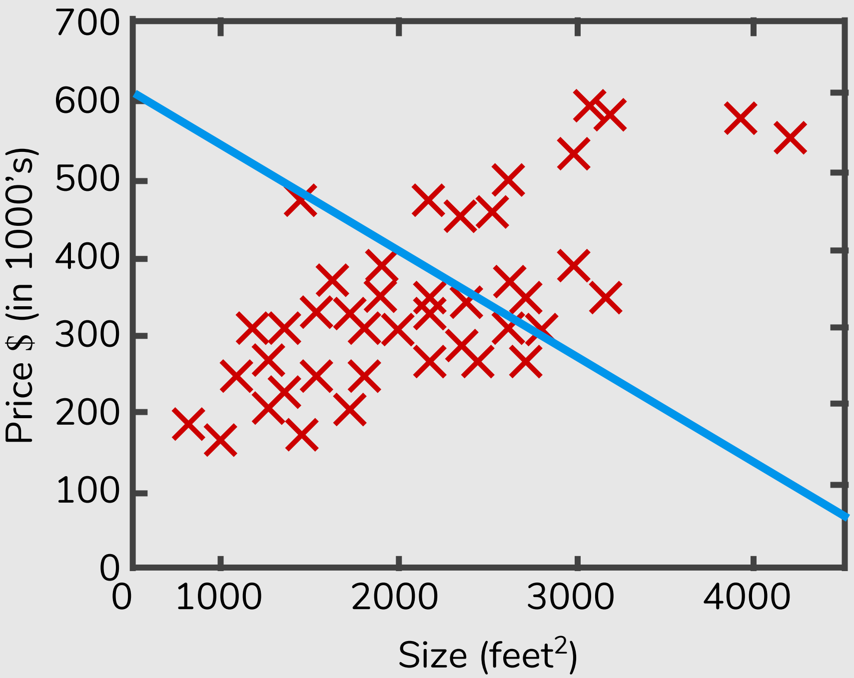

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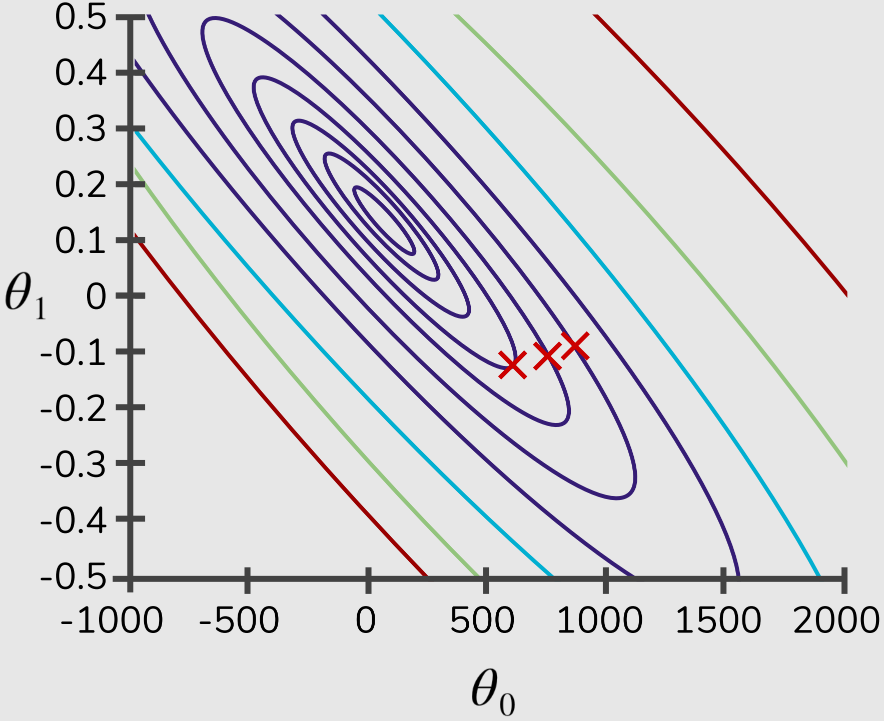

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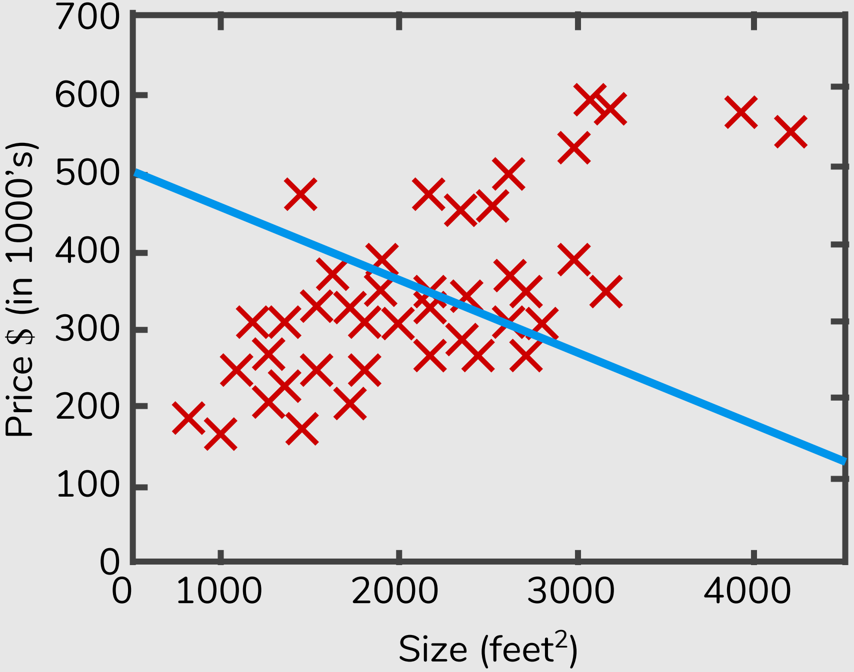

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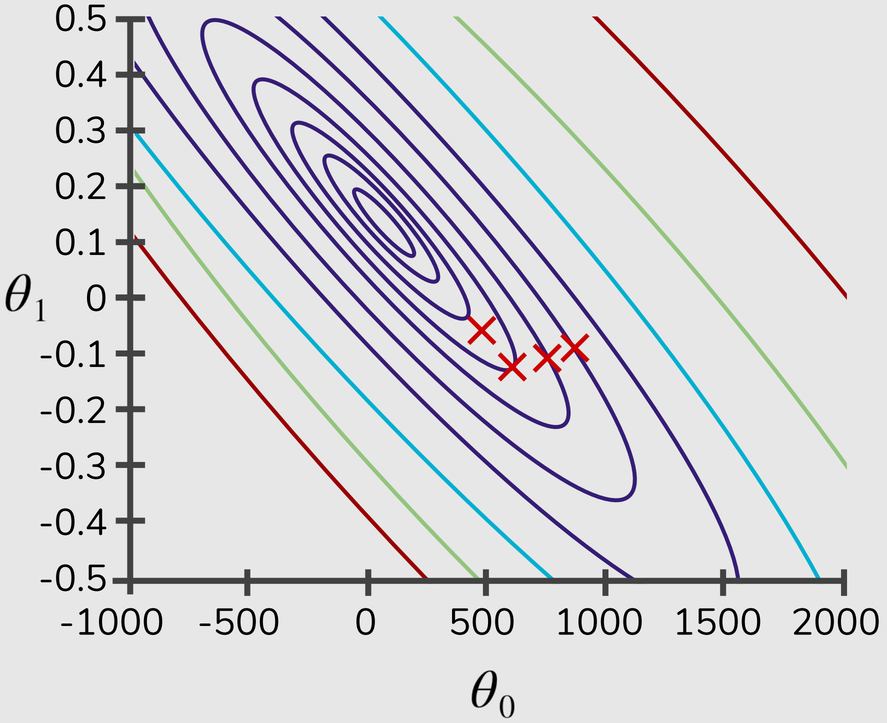

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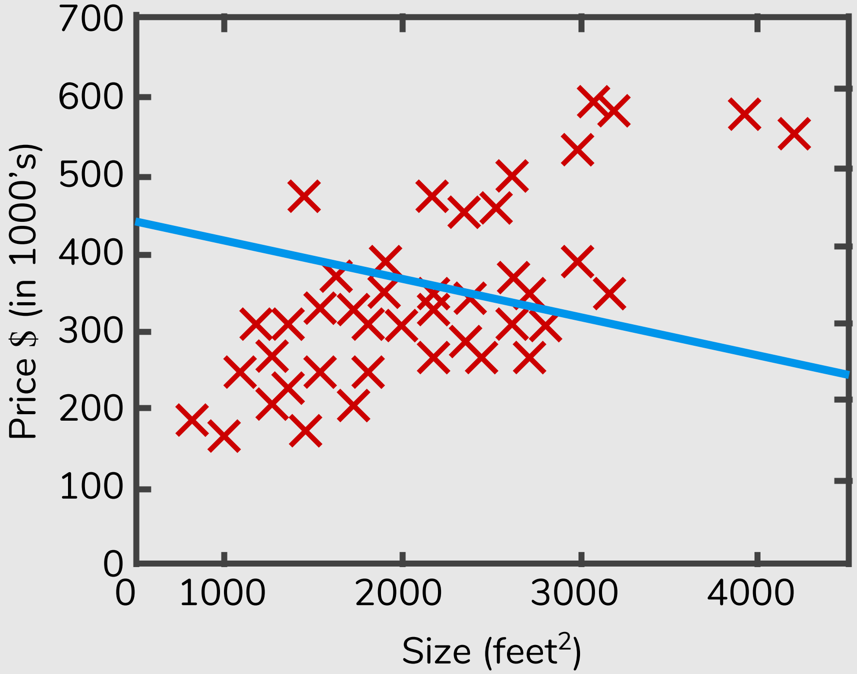
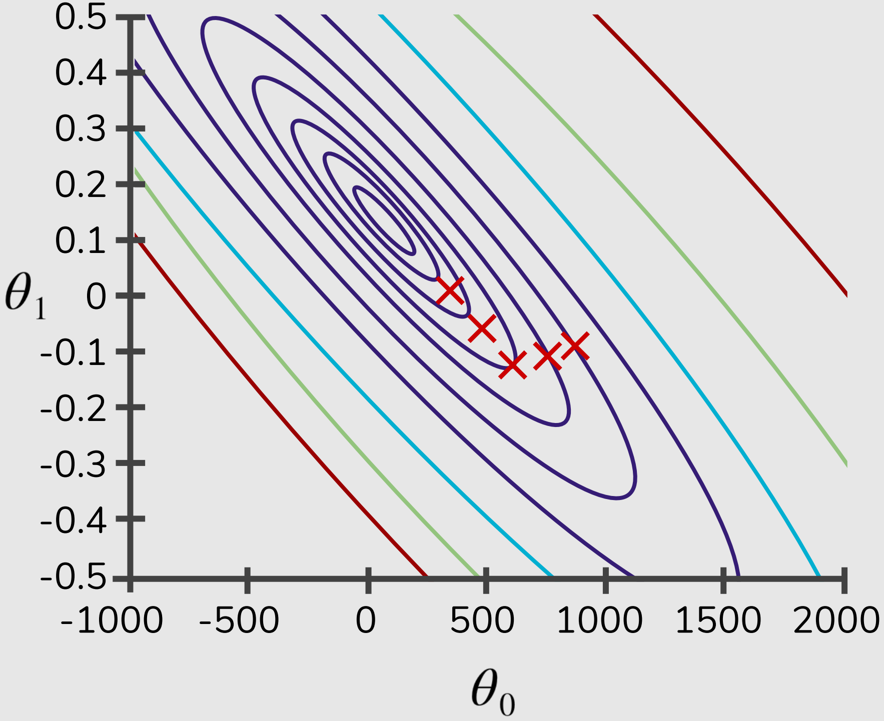


$J(\theta_0,\theta_1)$

(function of the parameters θ_0, θ_1)



$h_\theta(x)$

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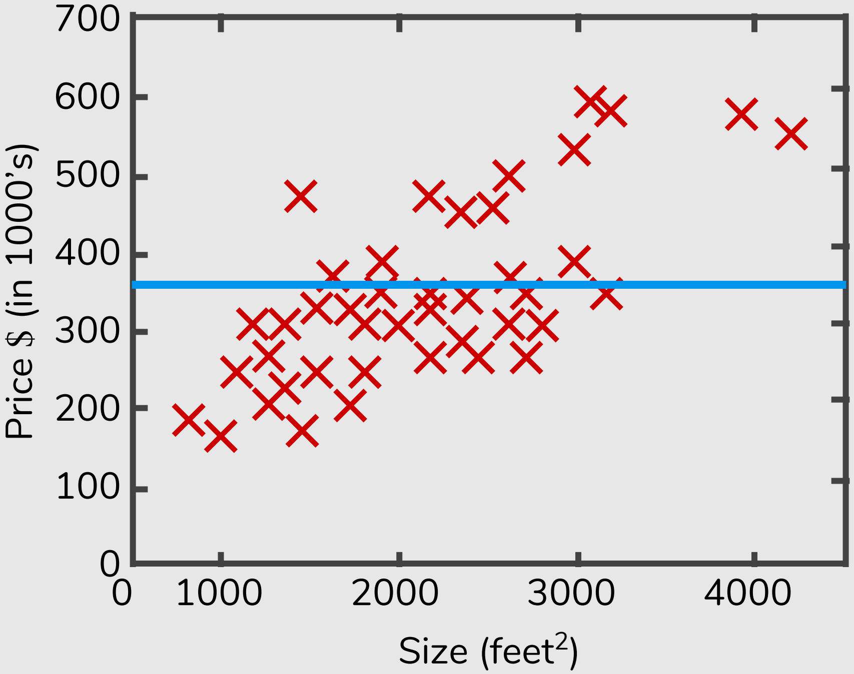

$J(\theta_0, \theta_1)$

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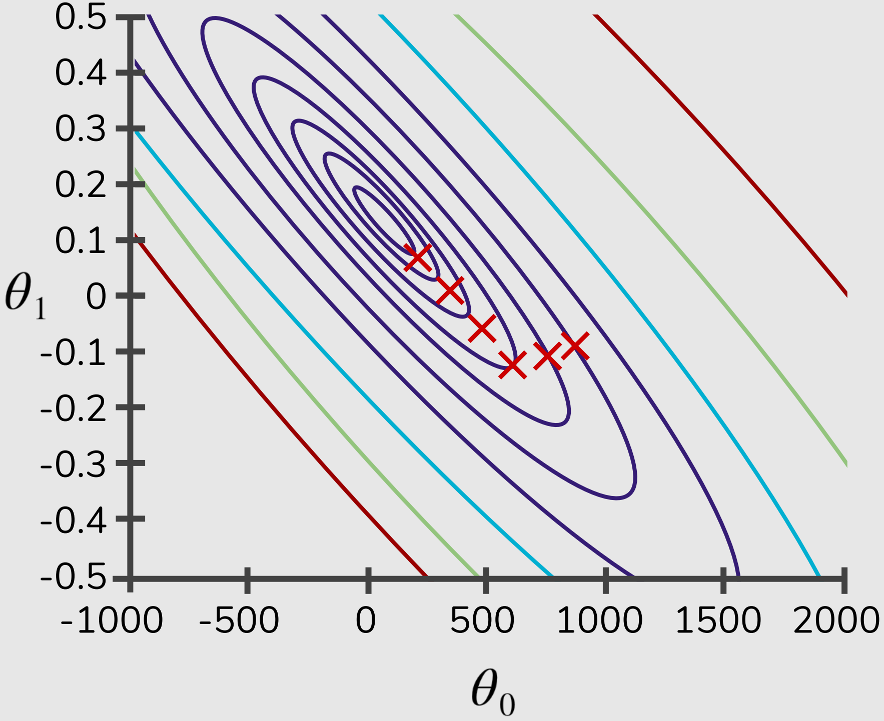

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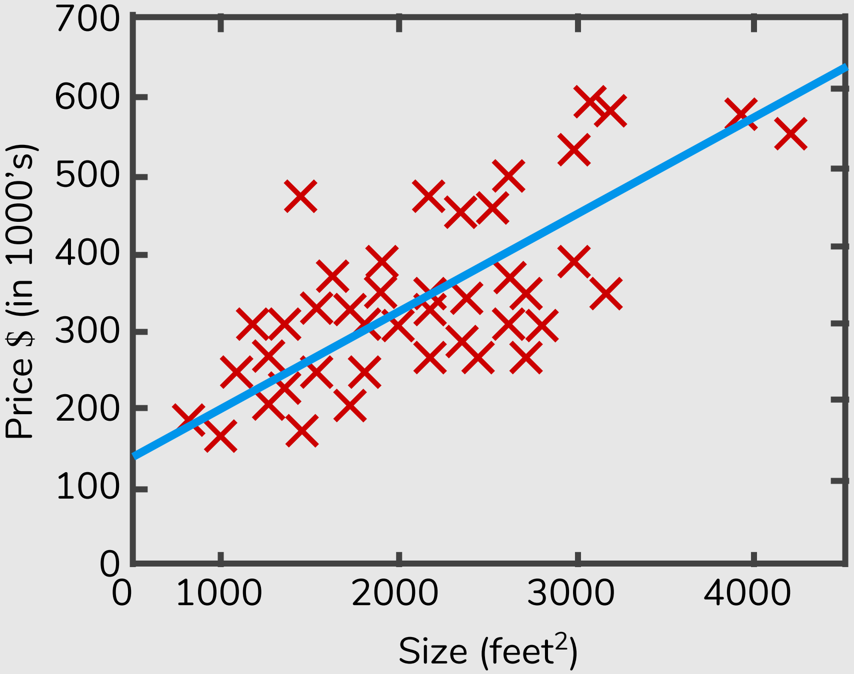

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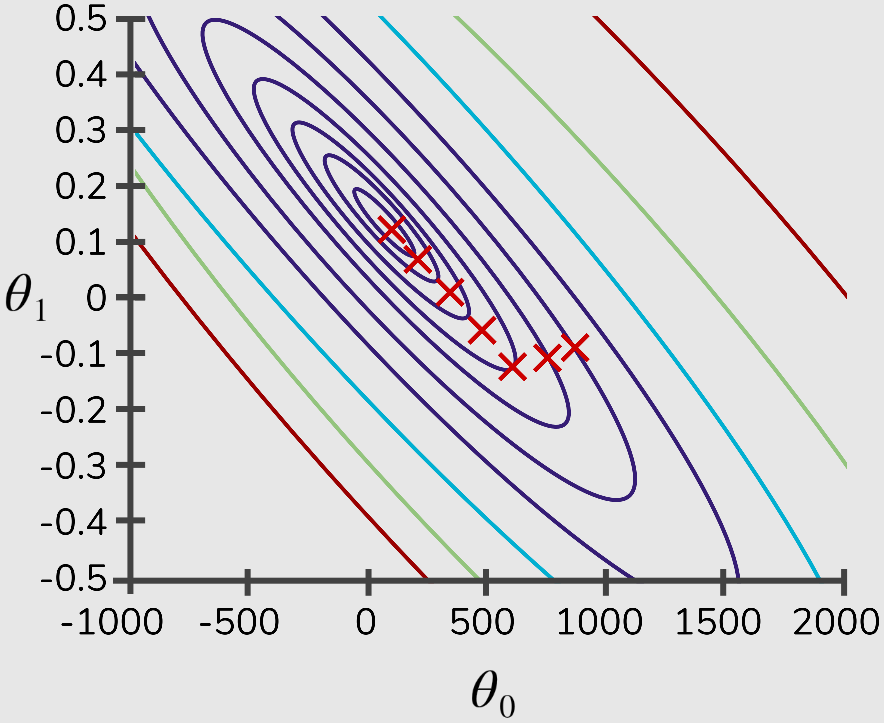
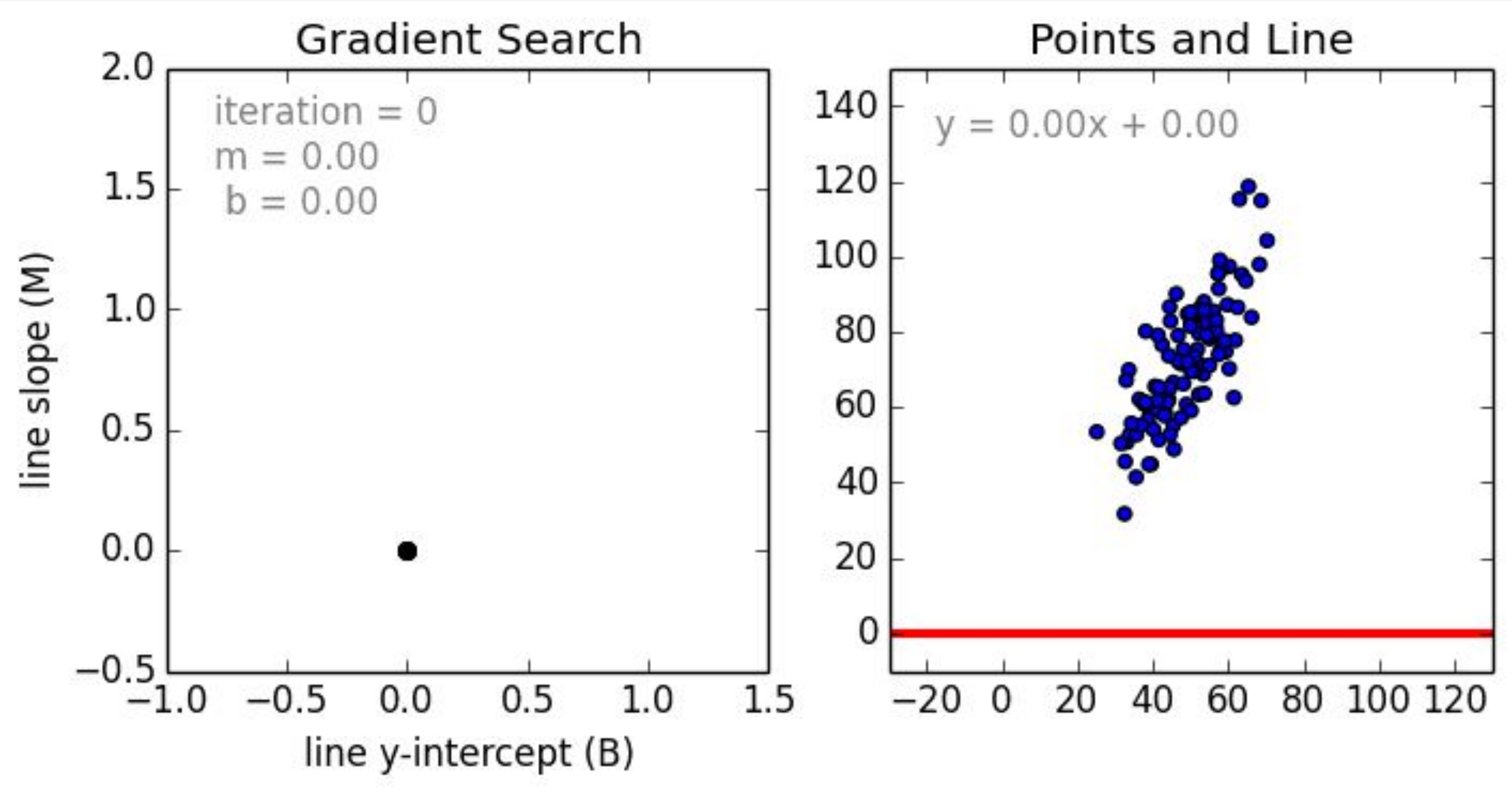


$$h_\theta(x) = \theta_0 + \theta_1 x \quad \Rightarrow \quad y = b + mx$$



Credit: https://github.com/mattnedrich/GradientDescentExample/raw/master/gradient_descent_example.gif

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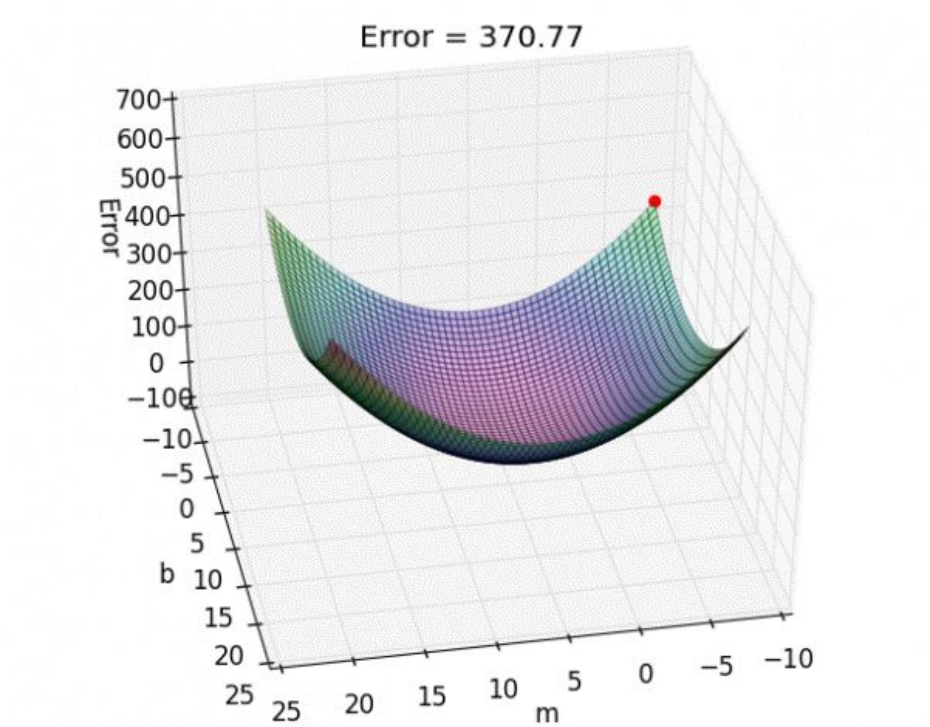

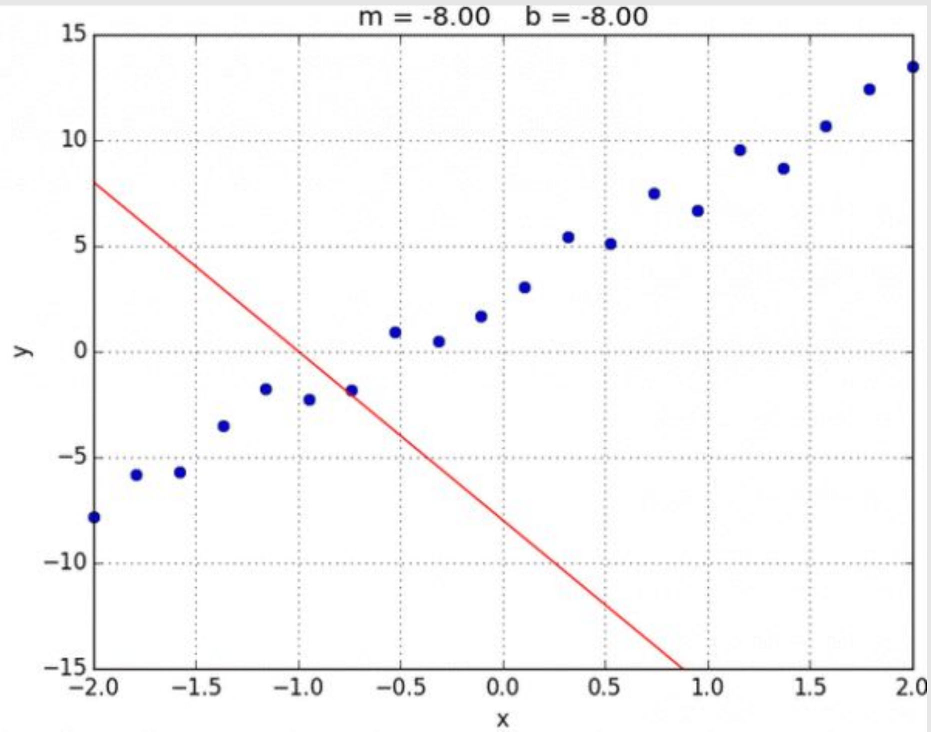

Credit: https://alykhantejani.github.io/a-brief-introduction-to-gradient-descent/

“Batch” Gradient Descent

“Batch”: Each step of gradient descent uses **all the training examples**.

“Batch” Gradient Descent

“Batch”: Each step of gradient descent uses **all the training examples**.

- Stochastic Gradient Descent
- Mini-batch Gradient Descent

“Batch” Gradient Descent

repeat until convergence {

$$\left.\begin{array}{l} \theta_0 := \theta_0 - \alpha \dfrac{1}{m} \displaystyle\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) \\[2em] \theta_1 := \theta_1 - \alpha \dfrac{1}{m} \displaystyle\sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})\, x^{(i)} \end{array}\right\} \begin{array}{c}\text{update } \theta_0 \text{ and } \theta_1 \\ \text{simultaneously}\end{array}$$

}

Stochastic Gradient Descent

Each step of gradient descent uses **one training example**.

repeat until convergence {

for $i := 1, \ldots, m$ {

$$\theta_0 := \theta_0 - \alpha(h_\theta(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha(h_\theta(x^{(i)}) - y^{(i)})x^{(i)}$$

}

}

Mini-batch Gradient Descent

Each step of gradient descent uses b **training examples**.

Say $b = 10, m = 1000.$
repeat until convergence {

for $i \coloneqq 1, 11, 21 \ldots, 991$ {

$$\theta_0 := \theta_0 - \alpha \frac{1}{10} \sum_{i=k}^{i+9} (h_\theta(x^{(k)}) - y^{(k)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{10} \sum_{i=k}^{i+9} (h_\theta(x^{(k)}) - y^{(k)})\, x^{(k)}$$

} }

Linear Regression with multiple variables

Multiple ~~Variables~~ Features

Size in feet^2 (x)	**Price ($) in 1000's ($y$)**
2104	460
1416	232
1534	315
852	178
…	…

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Multiple ~~Variables~~ Features

Size in feet^2 x_1	**Number of bedrooms** x_2	**Number of floors** x_3	**Age of home (years)** x_4	**Price (\$) in 1000's** y
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	2	36	178
...	...	...	...	...

Notation:

n = number of features

$x^{(i)}$ = input (features) of i^{th} training example

$x_j^{(i)}$ = value of features j in i^{th} training example

Hypothesis

Previously: $h_\theta(x) = \theta_0 + \theta_1 x$

Hypothesis

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$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$$

Hypothesis

Previously: $h_\theta(x) = \theta_0 + \theta_1 x$

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$$

$$h_\theta(x) = 80 + 0.1 x_1 + 10 x_2 + 3 x_3 - 2 x_4$$

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

For convenience of notation, define $x_0 = 1$.

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

For convenience of notation, define $x_0 = 1$.

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{n+1}$$

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

For convenience of notation, define $x_0 = 1$.

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^{n+1} \quad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{n+1}$$

$$\begin{bmatrix} \theta_0 & \theta_1 & \cdots & \theta_n \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$h_\theta(x) = \theta^T x$$

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$$\begin{bmatrix} \theta_0 & \theta_1 & \cdots & \theta_n \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$h_\theta(x) = \theta^T x$$

Multivariate linear regression.

Hypothesis: $h_\theta(x) = \theta^T x = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$

Parameters: $\theta_0, \theta_1, \ldots, \theta_n$

Cost Function: $J(\theta_0, \theta_1, \ldots, \theta_n) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$

Gradient Descent:

repeat {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1, \ldots, \theta_n)$$

} (simultaneously update for every $j = 0, 1, \ldots, n$)

Gradient Descent

Previously ($n = 1$):

repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})\, x^{(i)}$$

(simultaneously update θ_0, θ_1)

}

Gradient Descent

Previously ($n = 1$):

repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})\, x^{(i)}$$

(simultaneously update θ_0, θ_1)

}

New Algorithm ($n \geq 1$):

repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update θ_j for $j = 0,\ 1,\ \ldots,\ n$)

}

Gradient Descent

Previously ($n = 1$):

repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$$

(simultaneously update θ_0, θ_1)

}

New Algorithm ($n \geq 1$):

repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update θ_j for $j = 0, 1, \ldots, n$)

}

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

...

References

Machine Learning Books

- Hands-On Machine Learning with Scikit-Learn and TensorFlow, Chap. 2 & 4
- Pattern Recognition and Machine Learning, Chap. 3
- Machine Learning: a Probabilistic Perspective, Chap. 7

Machine Learning Courses

- https://www.coursera.org/learn/machine-learning, Week 1 & 2